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Abstract

This paper proposes a probabilistic model based on comovements and nonlinearities useful to assess the type of shock affecting each phase of the business cycle. By providing simultaneous inferences on the phases of real activity and inflation cycles, contractionary episodes are dated and categorized into demand, supply and mix recessions. The impact of shocks originated in the housing market over the business cycle is also assessed, finding that recessions are usually accompanied by housing deflationary pressures, while expansions are mainly influenced by housing demand shocks, with the only exception occurred during the period surrounding the "Great Recession," affected by expansionary housing supply shocks.

Keywords: Business Cycles, Inflation Cycles, Housing Price Cycles, Dynamics Factors, Markov-Switching.

JEL Classification: E32, C22, E27

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1 Introduction

The National Bureau of Economic Research defines the notion of business cycles as periodic but irregular up-and-down movements in economic activity, typically observed in macroeconomic indicators such as real GDP and Industrial Production. However, its Business Cycle Dating Committee does not enter into the causes of these recessions, which are traditionally assumed to come from two different sources. On the one hand, recessions that start on the supply side of the economy are known to be caused by supply shocks which typically affect production costs. On the other hand, recessions that start on the demand side of the economy are known as caused by demand shocks which affect economy-wide expenditure levels. To discriminate between these two sources of business cycle downturns, it is worth to emphasize that although both types of shocks cause decreases in actual economic activity, their effects on the price level is different.

In a seminal work, Blanchard and Quah (1989) investigate if the joint behavior of U.S. real and nominal variables is consistent with the traditional interpretation of macroeconomic fluctuations, i.e. that aggregate demand (supply) shocks move output and prices in the same (opposite) direction, finding a qualified yes as an answer. In other words, while recessionary demand shocks tend to produce price declines, negative supply shocks tend to increase the price level.

Recently, Aruoba and Diebold (2010) examine the dynamic interactions between real activity and prices over the business cycle to extract information about the sources of the contractionary shocks. For this purpose, they propose two separate state-space linear dynamic-factor models and use the Kalman filter to produce optimal extractions of real and nominal activities. According to these authors the coherence of their respective movements and the business cycle chronology determined by the NBER are the key to determine whether the recessionary shocks are demand- or supply-driven.

Relying on the widely accepted view that recessions are caused by adverse shocks of different nature, with the corresponding mix varying substantially across recessions, Galí

(1992), Ireland (2010) and Forni and Gambetti (2010), this paper proposes a multistate Markov-Switching Dynamic bi-Factor (MSDbF) approach that improves the methodology used in Aruoba and Diebold (2010) in two directions, which allow us to make inference on the type of aggregate shocks hitting the business cycle in order to uncover the sources of recessionary episodes.

First, although the authors examine the interactions between real activity and prices, they use separate dynamic factor models to compute the real and nominal indexes, without taking into account the potential interrelation between these two concepts. The model in this paper extends the previous approach by considering a unified framework where two separate factors are extracted from the same set of real and nominal indicators. Hence, both real and nominal indexes are endogenously determined and the interactions between the indicators and the factors are estimated without strong restrictions.

Second, although one of the defining characteristics of the business cycle is its asymmetric nature, Burns and Mitchel (1946), the authors extract the factors from linear models. The proposed model in this paper accounts for nonlinearities by allowing the factors to be governed by two, potentially dependent, Markov-switching processes. Therefore, this proposal is a natural extension of the single-index Markov-switching dynamic factor model proposed in the late nineties by Kim and Yoo (1995), Chauvet (1998) and Kim and Nelson (1998), since it relaxes the restriction that all the indicators depend on a unique common nonlinear dynamics.[1] Accordingly, the algorithm used to estimate the model in this paper via maximum likelihood is extended to consider two factors that depend on two separate latent state variables, dealing with issues related to their dependence relation and the identification of the factors.

The multistate MSDbF model is applied to study the interrelation, first, between real activity and inflation cycles in order to assess the type of shock's contribution to the business cycle, and second, the interrelation between real activity and housing price cycles in order to assess the impact of the shocks originated in the housing market on the business cycle. On the one hand, inferences on the mix varying shock contribution across con-

[1] Kholodilin and Yao (2005) proposes a similar approach applied to the case of leading and coincident indicators, by setting one state variable to be the lag of the other.

tractionary episodes suggest that recessions occurred in 1960.II-1961.I and 2001.I-2001.IV can be categorized as "demand recessions", while the ones occurred in 1969.IV-1970.IV, 1973.IV-1975.I and 1980.I-1980.III as "supply recessions". Furthermore, recessions during the periods 1981.III-1982.IV and 1990.III-1991.I were given by similar amounts of both types of contractionary shocks, being named as "mix recessions". The "great recession", 2007.IV-2009.II, which is of special interest, enters in the mix category, agreeing with Ireland (2010) who shows that this recession had its origins in a combination of aggregate demand and supply disturbances.

On the other hand, the model that incorporates housing information reveals that most of recessionary periods have been accompanied by deflationary pressures in the housing market, the only exception occurred in the 2001's recession, where housing prices experimented continuously increasing growth rates. The results also reveal that expansion periods have been mainly influenced by expansionary housing demand shocks, with the only exception occurred in the periods before and after the "great recession", where expansionary housing supply shocks were mostly influencing such business cycle phases, making this recession different than the rest of contractionary episodes in this respect.

The paper is structured as follows. Section 2, develops the algorithm to estimate the proposed multistate MSDbF model, which can be straightforwardly generalized to the case of K factors. Section 3 examines the empirical results by analyzing the dynamic interaction between real economic activity, inflation, and housing price cycles. Section 4 concludes.

2 The model

2.1 Model's dynamics

In this section, I combine the dynamic-factor and Markov-switching frameworks to create a statistical model capturing both regime shifts and comovements. Specifically, the log-level of each of the N economic indicators, y_{it}, is modelled as composed of three stochastic autoregressive processes. The first component corresponds to the common factor among the real activity indicators, f_t^b, and refers to the business cycle conditions. The second component corresponds to the common factor among the nominal indicators, f_t^p, and refers to the underlying price evolution. Finally, the third component corresponds to the idiosyncratic dynamics, e_{it}, and refers to the particular evolution of the time series. According to previous studies (see Aruoba and Diebold, 2010, and references therein), a stochastic trend is not included in the dynamic factor based on assumption that each of the series studied are integrated but not cointegrated. Therefore, the empirical analysis is undertaken using the log of the first difference of the observable indicators

$$y_{it} = \gamma_i^b f_t^b + \gamma_i^p f_t^p + e_{it}, \tag{1}$$

where γ_i^b and γ_i^p refer to the factor loadings.

To complete the specification of the data generating process, the factors, f_t^b and f_t^p, are assumed to be governed by two unobserved regime-switching variables, S_t^b and S_t^p, respectively. Hence, the dynamics of these factors can be specified as

$$f_t^r = \mu_{S_t^r}^r + \sum_{h=1}^{k} b_r \left(f_{t-h}^r - \mu_{S_{t-h}^j}^r \right) + \omega_t^r, \tag{2}$$

where the errors, ω_t^r, are distributed as $N(0, \sigma_r^2)$, and $r = b, p$.[2] Within this framework, one can label $S_t^b = 1$ as expansions and $S_t^b = 2$ as recessions at time t if $\mu_1^b > \mu_2^b$. In addition, one can also label $S_t^p = 1$ as highly inflationary regimes and $S_t^p = 2$ as regimes of low inflation at time t if $\mu_1^p > \mu_2^p$. In these cases, the first coincident indicator is expected to exhibit high (usually positive) growth rates in expansions and low (usually negative) growth rates in recessions, while the second coincident indicator is expected to exhibit higher growth rates in inflationary regimes and low growth rates periods of price stability. In addition, each state variables is assumed to evolve according to a irreducible 2-state Markov chains whose transition probabilities are defined by

$$p(S_t^r = j | S_{t-1}^r = i, S_{t-2}^r = h, ..., \psi_{t-1}) = p(S_t^r = j | S_{t-1}^r = i) = p_{ij}^r, \tag{3}$$

[2] According to Albert and Chib (1993), an AR(0) Markov-switching model, provides a useful model of the U.S. quarterly output series, hence following Camacho and Perez-Quiros (2007), I use $k = 0$ in the empirical application. In this case, the economic indicators are modelled as a recurrent sequences of shifts between two fixed equilibria of high and low growth means.

where $r = b, p$, ψ_t is the information set up to period t, and $i, j, h = 0, 1$.[3]

The dynamics of the idiosyncratic components are stated as

$$e_{it} = \sum_{h=1}^{m} \phi_{ih} e_{it-h} + \varepsilon_{it}, \tag{4}$$

where ε_{it} are distributed as $N(0, \sigma_i^2)$, with $i = 1, ..., N$. Finally, all the shocks, ε_{it} and ω_t^r, are assumed mutually uncorrelated in cross-section and time-series dimensions.

2.2 Estimation procedure

For estimation purpose, it is convenient to cast model into state space form. More compactly, the *measurement equation* is defined as

$$y_t = H\beta_t + e_t, \tag{5}$$

where y_t is an N-vector that collects the observed indicators, and $e_t \sim i.i.d.N\,(0, R)$. In addition, the expression for the *transition equation* is defined as

$$\beta_t = \widetilde{\mu}_{S_t^b, S_t^p} + F\beta_{t-1} + \upsilon_t, \tag{6}$$

with $\upsilon_t \sim i.i.d.N\,(0, Q)$. An extensive description of what these equations look like for the empirical model and a the detailed form of H, F, $\widetilde{\mu}$, e_t, υ_t, and the state vector β_t is presented in the Appendix.

If the regimes that determine the evolution of the two factors were observable, then the system would be a linear Gaussian dynamic factor model and the standard Kalman filter combined with procedures based on the likelihood functions could be applied to obtain parameter estimates and the paths of the unobservable components. However, since the regimes are not directly observed, rather it must be inferred from the data, the usual Kalman filter cannot be employed. Instead, each iteration of the Kalman filter produces a fourfold increase in the number of cases to consider and approximations to the Kalman filter are unavoidable.

[3] The variances σ_b^2 and σ_p^2 are taken to be unity for identification of the model.

Based on the approximate maximum-likelihood estimation method of Kim (1994), I propose an algorithm to estimate the nonlinear dynamic bi-factor model. Basically, the method contains three unified stages which are run in each iteration of the Kalman and Hamilton filters. In the first stage, the algorithm computes one-step-ahead predictions of the state vector and its associated mean squared error matrices by using as inputs the joint probabilities of the Markov-switching processes and the state vector. Adding a new set of observations, the Kalman filter updates the state vector and its mean squared errors and evaluates the likelihood function conditional on the bivariate Markov processes. In the second stage, the algorithm applies the Hamilton's (1989) filter which involves an evaluation of the likelihood function and updates the filtered probabilities. Accordingly, the likelihood function can be maximized with respect to the model parameters. In the third stage, the algorithm collapses the posteriors using the probability terms according to the Kim's (1994) approximations.[4] Let us describe these three stages carefully.

Stage 1: The goal is to form a forecast of the state vector, β_t, and its associated mean squared error matrices, P_t, conditional on the information set ψ_{t-1}, and on present and past states of each unobservable variables S_t^b and S_t^p. Assuming that the state variables take on the values j^b and j^p at t, and take on the values i^b and i^p at $t-1$, the forecasts are computed from the prediction equations

$$\beta_{t|t-1}^{(i^b,i^p,j^b,j^p)} = \widetilde{\mu}_{j^b,j^p} + F\beta_{t-1|t-1}^{(i^b,i^p)}, \tag{7}$$
$$P_{t|t-1}^{(i^b,i^p,j^b,j^p)} = FP_{t-1|t-1}^{(i^b,i^p)}F' + Q, \tag{8}$$

where $i^b, i^p, j^b, j^p = 1, 2$.

Once a new set of observations is included, the algorithm computes the forecast error and its variance matrix that can be obtained as

$$\eta_{t|t-1}^{(i^b,i^p,j^b,j^p)} = y_t - H\beta_{t|t-1}^{(i^b,i^p,j^b,j^p)}, \tag{9}$$
$$f_{t|t-1}^{(i^b,i^p,j^b,j^p)} = HP_{t|t-1}^{(i^b,i^p,j^b,j^p)}H' + R. \tag{10}$$

[4]The algorithm can straightforwardly be generalized to a Markov-switching dynamic K-factor model where each factor is governed by M-state variables. For the empirical purposes of this paper, we focus just on the case of a bi-factor model which largely facilitates notation.

In addition, the conditional likelihood of the observable variables can be evaluated as a by-product of the algorithm at each t, which allows estimation of the unknown model parameters. The likelihood function at each t is:

$$f(y_t|\psi_{t-1}) = \sum_{i^b i^p j^b j^p} f(y_t|S^b_{t-1} = i^b, S^p_{t-1} = i^p, S^b_t = j^b, S^p_t = j^p, \psi_{t-1})$$
$$\times p(S^b_{t-1} = i^b, S^p_{t-1} = i^p, S^b_t = j^b, S^p_t = j^p|\psi_{t-1}), \quad (11)$$

where the first terms of these products are the conditional Gaussian

$$(2\pi)^{-\frac{N}{2}}|f^{(i^b,i^p,j^b,j^p)}_{t|t-1}|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}\eta^{(i^b,i^p,j^b,j^p)\prime}_{t|t-1}\left[f^{(i^b,i^p,j^b,j^p)}_{t|t-1}\right]^{-1}\eta^{(i^b,i^p,j^b,j^p)}_{t|t-1}\right] \quad (12)$$

and the second probability terms are computed in the next stage.

Stage 2: The goal is to compute inferences about the different states by using Hamilton's nonlinear filter. Since the dependence relationship between the two Markov-switching variables is unknown, in order to model the joint probability events associated to the possible realizations of each unobserved state variable, I rely on the two polar cases of dependence. First, the completely independent case, in which the joint probability event is just the product of the individual probabilities.

$$p(S^b_{t-1} = i^b, S^p_{t-1} = i^p, S^b_t = j^b, S^p_t = j^p|\psi_{t-1}) = p(S^b_{t-1} = i^b, S^b_t = j^b|\psi_{t-1})\times$$
$$p(S^p_{t-1} = i^p, S^p_t = j^p|\psi_{t-1}). \quad (13)$$

Second, the completely dependent or perfect synchronization case, in which both Markov-switching variables follow exactly the same pattern, implying that there is just one state variable governing the whole model's dynamics, i.e. $S^b_t = S^p_t = S_t$.

$$p(S^b_{t-1} = i^b, S^p_{t-1} = i^p, S^b_t = j^b, S^p_t = j^p|\psi_{t-1}) = p(S_{t-1} = i, S_t = j|\psi_{t-1}). \quad (14)$$

Then, I follow the line of Bengoechea et al. (2005), who suggest that in empirical applications such degree of dependence should be located somewhere in between these two extreme possibilities. Hence, it can be seen as a linear combination between them, given by a parameter δ which provides insights about the degree of synchronization between the state variables and that satisfies $0 \leq \delta \leq 1$.

$$p(S_{t-1}^b = i^b, S_{t-1}^p = i^p, S_t^b = j^b, S_t^p = j^p|\psi_{t-1}) = \delta \times p(S_{t-1} = i, S_t = j|\psi_{t-1}) + (1-\delta) \times$$
$$p(S_{t-1}^b = i^b, S_t^b = j^b|\psi_{t-1}) \times p(S_{t-1}^p = i^p, S_t^p = j^p|\psi_{t-1}). \tag{15}$$

The terms on the right hand side of equations (13) and (14) can easily be obtained by using the transition probabilities

$$p(S_{t-1}^r = i^r, S_t^r = j^r|\psi_{t-1}) = p(S_t^r = j^r|S_{t-1}^r = i^r)p(S_{t-1}^r = i^r|\psi_{t-1}) \tag{16}$$
$$p(S_{t-1} = i, S_t = j|\psi_{t-1}) = p(S_t = j|S_{t-1} = i)p(S_{t-1} = i|\psi_{t-1}), \tag{17}$$

where $r = d, b$.

Stage 3: Using the new set of observations at the end of time t, y_t, the probability terms can be updated using Bayes rule

$$p(S_{t-1}^r = i^r, S_t^r = j^r|\psi_t) = \frac{f(y_t, S_{t-1}^r = i^r, S_t^r = j^r|\psi_{t-1})}{f(y_t|\psi_{t-1})} \tag{18}$$
$$p(S_{t-1} = i, S_t = j|\psi_t) = \frac{f(y_t, S_{t-1} = i, S_t = j|\psi_{t-1})}{f(y_t|\psi_{t-1})}, \tag{19}$$

where

$$f(y_t, S_{t-1}^r = i^r, S_t^r = j^r|\psi_{t-1}) = \sum\nolimits_{i^{r'} j^{r'}} f(y_t|S_{t-1}^r = i^r, S_{t-1}^{r'} = i^{r'}, S_t^r = j^r, S_t^{r'} = j^{r'}, \psi_{t-1})$$
$$\times p(S_{t-1}^r = i^r, S_{t-1}^{r'} = i^{r'}, S_t^r = j^r, S_t^{r'} = j^{r'}|\psi_{t-1}),$$
$$f(y_t, S_{t-1} = i, S_t = j|\psi_{t-1}) = \sum\nolimits_{i^{r'} = j^{r'}} f(y_t|S_{t-1}^r = i^r, S_{t-1}^{r'} = i^{r'}, S_t^r = j^r, S_t^{r'} = j^{r'}, \psi_{t-1})$$
$$\times p(S_{t-1}^r = i^r, S_{t-1}^{r'} = i^{r'}, S_t^r = j^r, S_t^{r'} = j^{r'}|\psi_{t-1}),$$

with $r, r' = b, p$. By the law of total probability, the state probabilities become

$$p(S_t^r = j^r|\psi_t) = \sum_{i^r=1}^{2} p(S_{t-1}^r = i^r, S_t^r = j^r|\psi_t) \tag{20}$$

$$p(S_t = j|\psi_t) = \sum_{i^r=1}^{2} p(S_{t-1} = i, S_t = j|\psi_t) \tag{21}$$

with $r = b, p$.

The last step of the Kalman filter updates the inferences of the state vector and its variance matrix by using the updating equations

$$\beta_{t|t}^{(i^b,i^p,j^b,j^p)} = \beta_{t|t-1}^{(i^b,i^p,j^b,j^p)} + P_{t|t-1}^{(i^b,i^p,j^b,j^p)} H' \left[f_{t|t-1}^{(i^b,i^p,j^b,j^p)} \right]^{-1} \eta_{t|t-1}^{(i^b,i^p,j^b,j^p)}, \tag{22}$$

$$P_{t|t}^{(i^b,i^p,j^b,j^p)} = \left(I - P_{t|t-1}^{(i^b,i^p,j^b,j^p)} H' \left[f_{t|t-1}^{(i^b,i^p,j^b,j^p)} \right]^{-1} H \right) P_{t|t-1}^{(i^b,i^p,j^b,j^p)}. \tag{23}$$

It is worth noting that the algorithm calculates a battery of (2ˆ2)ˆ2 different inferences of the state vector and its mean square error matrix, corresponding to every possible value of the vector $\left(i^b, i^p, j^b, j^p\right)'$. This implies that after a few iterations the number of cases increases dramatically and the system become intractable.

To overcome this drawback, I extend the approximation to the filter suggested by Kim (1994) that reduces the number of different terms at each time t by collapsing the (2ˆ2)ˆ2 posteriors $\beta_{t|t}^{(i^b,i^p,j^b,j^p)}$ and $P_{t|t}^{(i^b,i^p,j^b,j^p)}$, into (2ˆ2) posteriors $\beta_{t|t}^{(j^b,j^p)}$ and $P_{t|t}^{(j^b,j^p)}$. In particular, I use

$$\beta_{t|t}^{(j^b,j^p)} = \frac{\sum_{i^b=1}^{2} \sum_{i^p=1}^{2} p(S_{t-1}^b = i^b, S_{t-1}^p = i^p, S_t^b = j^b, S_t^p = j^p|\psi_t) \beta_{t|t}^{(i^b,i^p,j^b,j^p)}}{p(S_t^b = j^b, S_t^p = j^p|\psi_t)}, \tag{24}$$

and

$$\begin{aligned} P_{t|t}^{(j^b,j^p)} &= \frac{1}{p(S_t^b = j^b, S_t^p = j^p|\psi_t)} \sum_{i^p=1}^{2} p(S_{t-1}^b = i^b, S_{t-1}^p = i^p, S_t^b = j^b, S_t^p = j^p|\psi_t) \\ &\times \left[P_{t|t}^{(i^b,i^p,j^b,j^p)} + \left(\beta_{t|t}^{(j^b,j^p)} - \beta_{t|t}^{(i^b,i^p,j^b,j^p)} \right) \left(\beta_{t|t}^{(j^b,j^p)} - \beta_{t|t}^{(i^b,i^p,j^b,j^p)} \right)' \right]. \end{aligned} \tag{25}$$

2.3 Weights

The weights implicitly used by the Kalman Filter to perform factor estimates from the coincident variables can be calculated by measuring the effects of units changes in the lags of individual observations on the inference of the state vector $\beta_{t|t}$. They are useful for identification purposes since they give insights regarding to which are the key variables governing the evolution of each factor. The weights can be obtained directly from the Kalman filter matrices β_t, P_t and f_t. However, contrary to the standard linear frameworks, these matrices are in this case state dependent. Since the Kalman filter is linear when the unobservable states are known, the expected value of the Kalman matrices conditional on the state variables is computed following the line used in Markov-switching impulse responses, that is

$$\Theta_{t|t-1} = \sum_{j^b=1}^{2} \sum_{i^b=1}^{2} \sum_{j^p=1}^{2} \sum_{i^p=1}^{2} \Theta_{t|t-1}^{(i^b,i^p,j^b,j^p)} p(S_t^b = j^b, S_{t-1}^b = i^b, S_t^p = j^p, S_{t-1}^p = i^p), \tag{26}$$

for $\Theta = \beta, P$ and f. According to Stock and Watson (1991) and Banbura and Rustler (2007), the weights are now easy to compute. Plugging the expression of the forecast errors into the forecasting equation leads:

$$\beta_{t|t} = \beta_{t|t-1} + P_{t|t-1} H' \left[f_{t|t-1} \right]^{-1} \left[y_t - H\beta_{t|t-1} \right]. \tag{27}$$

Then, replacing $\beta_{t|t-1}$ in the right hand side of the above equation by the prediction equation and denoting the Kalman gain by $G_{t|t-1} = P_{t|t-1} H' \left[f_{t|t-1} \right]^{-1}$, it can be obtained

$$\beta_{t|t} = \left[I - G_{t|t-1} H \right] F \beta_{t-1|t-1} + G_{t|t-1} y_t + \left[I - G_{t|t-1} H \right] \widetilde{\mu}, \tag{28}$$

where

$$\widetilde{\mu} = \sum_{j^b=1}^{2} \sum_{j^p=1}^{2} \widetilde{\mu}_{j^b,j^p} p[S_t^b = j^b, S_t^p = j^p | \psi_t]. \tag{29}$$

Since the matrix F in the transition equation of the state-space representation is time invariant, $G_{t|t-1}$ converges to the steady-state Kalman gain, G. Under these conditions and with some algebra Equation (28) can also be expressed as:

$$\beta_{t|t} = M(L) y_t + J \widetilde{\mu}, \tag{30}$$

where L denotes the lag-operator, $J = (I - (I - GH) FL)^{-1} (I - GH)$, and the elements of the matrix of lag polynomial $M(L) = (I - (I - GH) FL)^{-1} G$ measure the effect of changes in y_t on the inference of $\beta_{t|t}$, which can be decomposed into the weighted sum of observations by letting M_j be each of these matrices

$$\beta_{t|t} = \sum_{j=0}^{\infty} M_j y_{t-j} + J\widetilde{\mu}. \tag{31}$$

Accordingly, $M(1) = (I - (I - GH) F)^{-1} G$, is the matrix that contains the cumulative impacts of the individual observations in the inference of the state vector, Camacho and Perez -Quiros (2010).

2.4 Ragged Edges

The framework can also be extended to allow for missing observations in the data by following the approach in Mariano and Murasawa (2003). It consists in replacing missing observations with random draws ϵ_t from a $N(0, \sigma_\epsilon^2)$ that are independent from the model parameters that do not impact on the model estimation. As a consequence, some of the system matrices would be time-varying, remaining the elements in the measurement equation being replaced by the following expressions:

$$y_{it}^* = \begin{cases} y_{it} & \text{if } y_{it} \text{ observable} \\ \epsilon_t & \text{otherwise} \end{cases} \tag{32}$$

$$H_{it}^* = \begin{cases} H_{it} & \text{if } y_{it} \text{ observable} \\ \mathbf{0}_\kappa & \text{otherwise} \end{cases} \tag{33}$$

$$e_{it}^* = \begin{cases} 0 & \text{if } y_{it} \text{ observable} \\ \epsilon_t & \text{otherwise} \end{cases} \tag{34}$$

$$R_{iit}^* = \begin{cases} 0 & \text{if } y_{it} \text{ observable} \\ \sigma_\epsilon^2 & \text{otherwise} \end{cases} \tag{35}$$

where y_{it} is the i-th element in the vector y_t, R_{iit} its variance, H_{it} is the i-th row of the matrix H_t that contains κ columns, and $\mathbf{0}_\kappa$ a row vector of κ zeroes. Accordingly, Equation (5) would be replaced by

$$y_t^* = H^* \beta_t + e_t^*, \tag{36}$$

where $e_t^* \sim i.i.d.N\left(0, R^*\right)$.

3 Empirical results

3.1 Data

This section presents the estimates of two multistate MSDbF models. The first model study the interrelation between real activity cycles and inflation cycles by relying on information of price dynamics of the overall economy. The second model focuses particularly on disentangle shocks originated in the housing market affecting business cycle phases, this is done by analyzing the interrelation between real activity cycles and housing price cycles.

The indicators of real economic activity are real GDP (1959.I–2011.III), industrial production (1959.I–2011.III), real personal income less transfers (1959.I–2011.III), real manufacturing trading sales (1967.I–2011.III), and total non-farm labor (1959.I–2011.III). The indicators of inflation dynamics are deflator of GDP(1959.I–2011.III), consumer price index (1959.I–2011.III), producer price index (1959.I–2011.III), Standard and Poor's GSCI non-energy commodities price index (1970.I–2011.III), spot oil price (1959.I–2011.III), and hourly compensation in the non-farm business sector (1959.I–2011.III). These variables were taken from the Federal Reserve Bank of St. Louis and the Bureau of Economic Analysis databases, and their selection was based following the line of Aruoba and Diebold (2010). The indicators of housing prices are Price deflator index of new single-family houses under construction (1975.I-2011.III), Conventional mortgage home price index (1975.I-2011.III), Federal Housing Finance Administration (FHFA) All-transactions index (1975.I-2011.III), FHFA Purchase-only index (1991.I-2011.III), and S&P Cash-Shiller-10-cities home price index (1987.I-2011.III). These indicators were taken from the Federal Reserve Bank of St. Louis, Freddie Mac and Census Bureau databases, and the selection was based by following Ng and Moench (2011), who analyze the US housing market dynamics with hierarchical regional and national factors.

For all the indicators in logs, the standard tests for a unit root were unable to reject at standard significance levels. Accordingly, the empirical analysis uses the growth rates of the observable indicators.[5] Finally, the variables are standardized to have a zero mean and a variance equal to one before estimating the model.

3.2 Real Activity versus Inflation Cycles

In the first model to be analyzed, the vector of observed variables, y_t, contains all five indicators of real economic activity and all six indicators of inflation dynamics, which coincides with the database used in Aruoba and Diebold (2010). Several features of the model's parameters estimates, reported in Table 1, deserve attention. First, the loading factors of f_t^b that are associated to real activity and inflation indicators are positive and statistically significant, with the exception of GDP deflator and hourly compensation. However, the loading factors of f_t^b that are related to real activity indicators are much higher than those related to price indicators. This result indicates that the first factor can be interpreted as a coincident index of the US real economic activity. Second, the loading factors of f_t^p that are related to price indicators in the measurement equation are positive and statistically significant.[6] But, the loading factors of f_t^p that are related to real activity indicators are negative. These estimates suggest that the second factor can be interpreted as an inflation index. Note that although the indicators has not been a priory classified as real and nominal, the model assigns endogenously the indicators loads on each factor. Third, the degree of dependence between the phases of the two indexes given by δ, is equal to 0,32 and statistically significant at all levels. Since its interpretation refers to perfect synchronization when it is equal to one and total independence when it is equal to zero, it is suggesting that U.S. business cycles and inflation cycles coincide approximately 30% of the time.

Moreover, as it was pointed out in Section 2, the weights that variables have on each

[5] Recall that we assume that the series are not cointegrated.

[6] Again with the only exception of hourly compensation that has a negative loading factor related to f_t^p.

Table 1: Maximum likelihood estimates: Real activity versus Inflation

Param.	Estim.	St. D.	Param.	Estim.	St. D.	Param	Estim.	St. D.
γ_1^b	0.4414	0.0415	γ_8^p	0.5078	0.0580	σ_5	0.2775	0.0286
γ_2^b	0.5354	0.0407	γ_9^p	0.1897	0.0498	ϕ_{61}	0.6673	0.0670
γ_3^b	0.3410	0.0431	γ_{10}^p	0.3584	0.0465	ϕ_{62}	0.2422	0.0669
γ_4^b	0.2745	0.0496	γ_{11}^p	-0.3186	0.0493	σ_6	0.4303	0.0210
γ_5^b	0.3309	0.0276	μ_1^p	3.0261	0.3699	ϕ_{71}	1.7643	0.0607
γ_6^b	0.0438	0.0225	μ_2^p	-0.3546	0.1052	ϕ_{72}	-0.7782	0.0535
γ_7^b	0.2433	0.0394	p_{11}^p	0.7162	0.1484	σ_7	0.0364	0.0134
γ_8^b	0.2487	0.0468	p_{22}^p	0.9641	0.0217	ϕ_{81}	0.0242	0.1120
γ_9^b	0.2525	0.0493	ϕ_{11}	-0.1094	0.0802	ϕ_{82}	0.1962	0.1057
γ_{10}^b	0.2110	0.0447	ϕ_{12}	0.0393	0.0747	σ_8	0.5879	0.0346
γ_{11}^b	-0.0955	0.0470	σ_1	0.5856	0.0326	ϕ_{91}	0.0181	0.1028
μ_1^b	0.4662	0.1036	ϕ_{21}	-0.1122	0.1487	ϕ_{92}	-0.0001	0.0009
μ_2^b	-3.0542	0.3805	ϕ_{22}	-0.0031	0.0083	σ_9	0.8747	0.0485
p_{11}^b	0.9627	0.0221	σ_2	0.3335	0.0388	$\phi_{10,1}$	-0.0957	0.0716
p_{22}^b	0.9581	0.0792	ϕ_{31}	0.0026	0.0747	$\phi_{10,2}$	-0.0023	0.0034
γ_1^p	-0.2286	0.0480	ϕ_{32}	0.1715	0.0722	σ_{10}	0.8123	0.0405
γ_2^p	-0.2273	0.0452	σ_3	0.6745	0.0357	$\phi_{11,1}$	0.0153	0.0707
γ_3^p	-0.2735	0.0468	ϕ_{41}	0.4695	0.0611	$\phi_{11,2}$	0.0136	0.0698
γ_4^p	-0.1035	0.0456	ϕ_{42}	0.0001	0.0713	σ_{11}	0.8857	0.0438
γ_5^p	-0.1150	0.0302	σ_4	0.7288	0.0396	p_{11}^*	0.9666	0.0337
γ_6^p	0.0838	0.0228	ϕ_{51}	1.1629	0.0751	p_{22}^*	0.6622	0.1977
γ_7^p	0.5008	0.0399	ϕ_{52}	-0.3381	0.0437	δ	0.3187	0.0914

Note. Superindexes p and b refer to the first (or business cycle) and the second (or price index) factors. Subindexes in the loadings, γ, from1 to 11 refer to real GDP (1), Industrial Production (2), Personal Income less Net Transfers (3), Real Manufacturing Trading Sales (4), Total Nonfarm Labor (5), GDP Deflator (6), Consumer Price Index (7), Producer Price Index (8), Spot Oil Price (9), Standard and Poor's GSCI Non-energy Commodities Price Index (10), Hourly Compensation in the Non-farm Business Sector (11).

factor helps us to analyze further the extent to which indicators loads on each factor. These weights were computed, indicating that real activity variables have the 68% of the weight on the first factor dynamics, while price indicators have the 67% of the weight on the second factor. This result reinforces the interpretation of the first factor as an index of economic activity and the second factor as an index of inflation dynamics.

According to the results of the previous section, each of the eleven economic indicators is decomposed on two unobserved common dynamic factors plus an idiosyncratic component. The first factor is mainly driven by all five real activity indicators while the second factor is governed by the evolution of all six price indicators.

Figure 1. Real Activity versus Inflation: First Factor

Chart 1. First factor and the CFNAI index

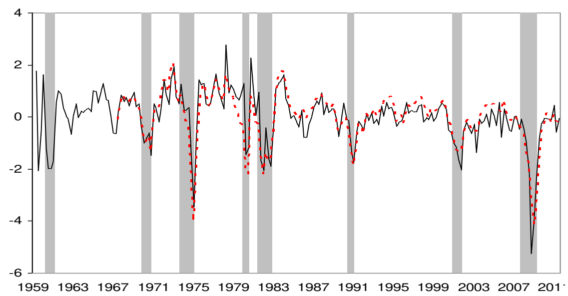
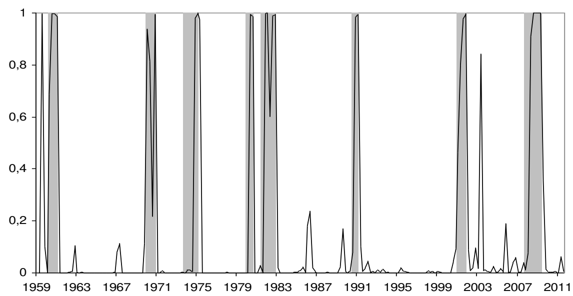


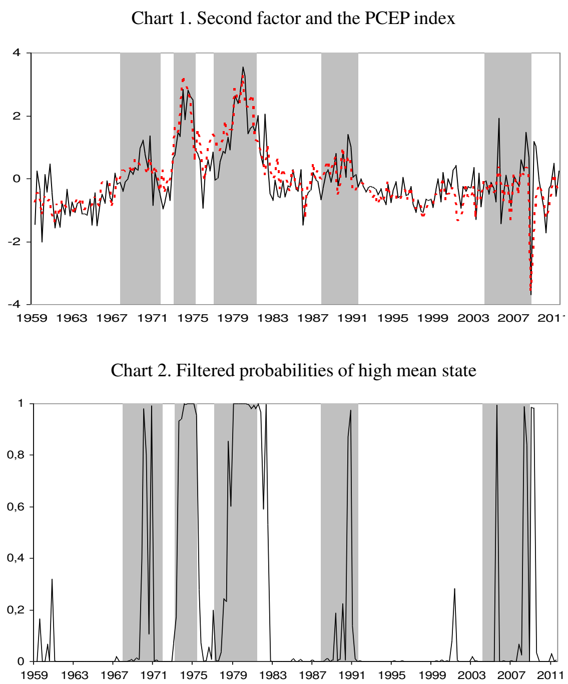
Chart 2. Filtered probabilities of low mean state



Notes. In Chart 1, the solid line refers to the first factor (1959.II-2011.III) and the dashed line refers to the CFNAI (1967.II-2011.III). To facilitate graphing, both series are standardized. The probabilities plotted in Chart 2 come from the state variable that governs the dynamics of the first factor. Shaded areas correspond to recessions as documented by the NBER.

The top chart of Figure 1 depicts the business cycle dynamics the first factor. While it fluctuates around its unconditional mean, the broad changes of direction in the factor seem to mark quite well the NBER-referenced business cycles. During expansions, the value of the factor rises up to about its estimated first-state mean of 0.46. During recessions, the factor drastically falls to its second-state mean of about -3.05. In addition, the figure also reveals the strong coherence of the second factor and the Chicago Fed National Activity Index (CFNAI) which is a leading index designed to gauge overall US economic activity.[7]

Figure 2. Real Activity versus Inflation: Second Factor



Notes. In Chart 1, solid line refers to the second factor (1959.II-2011.III) and the dashed line refers to the PCEPI (1959.II-2011.III). To facilitate graphing, both series are standardized. The probabilities plotted in Chart 2 come from the state variable that governs the dynamics of the second factor. Shaded areas correspond to high inflation periods documented by the Chicago Fed.

Finally, the bottom chart of Figure 1 displays the filtered probabilities of being in state 2 that come from the state variable that governs the evolution of the first factor of the

[7] To convert the monthly CFNAI into quarterly observations, the index is expressed as weighted averages

$$w_t = \frac{1}{3}z_t + \frac{2}{3}z_{t-1} + z_{t-2} + \frac{2}{3}z_{t-3} + \frac{1}{3}z_{t-4},$$

where w_t refers to quarterly and z_t to monthly.

multistate MSDbF model, $p(S_t^b = 2|\psi_t)$, along with the NBER recessions. According to this figure, it is easy to interpret state 2 as recessions and the series plotted in this chart as probabilities of being in recession. Therefore, one can interpret this factor an index of the broad economic activity which is much less noisy than the individual economic indicators.

The top chart of Figure 2 plots the second factor and reveals that the evolution of the factor does not follow as closely as the first factor the business cycle dynamics. This index takes negative values in the sixties, it sharply increases to during the seventies and mid-eighties and come back to negative values since then. According to the estimates of the conditional means of the state variable that governs the evolution of the second factor reported in Table 1, the first and last part of the sample is governed by state 1 (estimated mean of state 1 is 3.02) while the middle part of the sample is governed by state 2 (estimated mean of state 1 is -0.35). The chart also points out that the evolution of the second factor and PCEPI (Personal Consumption Expenditure Price Index) growth strongly cohere. Finally, the bottom chart of Figure 2 displays the filtered probabilities of being in state 1 that come from the state variable that governs the evolution of the second factor, $p(S_t^p = 2|\psi_t)$, along with the high inflation periods referenced by the Chicago Fed. According to this figure, one can interpret state 1 as periods of high inflationary pressures and the second factor a price index.

3.2.1 Inferences on Shocks

It is now widely accepted that fluctuations in economic activity are caused by a mix of several types of shocks, e.g. demand, supply, monetary or fiscal, as shown in Forni and Gambetti (2010) or technology and nontechnology shocks, Galí (1999), which can have simultaneous or lagged, soft or strong, short or long, positive or negative impact on it. Some seminal attempts to study the effects of some of these shocks using structural VARs are presented in Blanchard and Quah (1989) in which disturbances that have a temporary effect and the ones that have a permanent effect on output fluctuations are interpreted as demand and supply disturbances respectively. This work was extended by Galí (1992) to allow the inclusion of monetary components, finding that the four main sources of fluctuations are money supply, money demand, investment, and aggregate supply shocks.

However, there is some criticism about the identification strategy of shocks when structural VARs are used, as can be seen in Lippi and Reichlin (1993). They show that a very simple modification of the underlying model can lead to significant changes in results. The main point of their criticism is based on the fact that economic theory does not in general provide sufficient structure to choose the most appropriate moving-average representation to estimate the structural VAR model, carrying to the dilemma of fundamentalness of the representation to be issued, Blanchard and Quah (1993). Hence another way to identify shocks without imposing strong restrictions on the structure of the model and without loss of economic intuition seems needed.

Two of the most relevant types of shocks are aggregate demand and aggregate supply shocks since their features are of great importance to the study of business cycles. As suggested by Aruoba and Diebold (2010), prices and quantities are related over the business cycle, and the nature of this relationship contains information about the sources of shocks. While adverse demand shocks lead to periods of business cycle downturns and low inflation, adverse supply shocks lead to reductions in economic activity along with inflationary pressures. In an analogous way, expansionary demand shocks lead to increases of economic activity along with prices, but expansionary supply shocks lead to periods of business cycle upturns and low inflation.

The proposed multistate MSDbF model allows to perform inference on the four types of shocks since the probabilities of recession, $p(S_t^b = 2|\psi_t)$, can be additively decomposed into the probability of a recession consistent with an adverse demand shock, $p(S_t^b = 2, S_t^p = 2|\psi_t)$, and the probability of a recession consistent with a contractionary supply shock, $p(S_t^b = 2, S_t^p = 1|\psi_t)$. The same criterion applies for periods of expansions, that is

$$p(S_t^b = 1|\psi_t) = p(S_t^b = 1, S_t^p = 2|\psi_t) + p(S_t^b = 1, S_t^p = 1|\psi_t) \quad (37)$$

$$p(S_t^b = 2|\psi_t) = p(S_t^b = 2, S_t^p = 2|\psi_t) + p(S_t^b = 2, S_t^p = 1|\psi_t). \quad (38)$$

The top panel of Figure 3 plots the probabilities of recessions that are caused by demand shocks, i.e., probabilities of the joint event that characterizes periods of low activity and low prices. The figure shows that this probability tends to raise during the whole periods of the recessions 1960.II-1961.I and 2001.I-2001.IV, consistent with the view that these recessions are caused by adverse demand shocks. The bottom panel of Figure 3 shows the joint filtered probabilities of stagflation, i.e., low activity and high prices. This figure reveals that recessions in 1969.IV-1970.IV, 1973.IV-1975.I and 1980.I-1980.III show high probabilities of decreased real activity and increased inflation, consistent with adverse supply shocks as the source of these recessions. Moreover, the recessions occurred during the periods 1981.III-1982.IV and 1990.III-1991.I, start showing high probability of contractionary supply shocks, but they end showing high probability of contractionary demand shocks. This is consistent with the view that they were caused by a mix of aggregate a supply and demand shocks.

Figure 3. Real Activity versus Inflation: Contractionary Shocks

Chart 1. Probability of contractionary demand shock

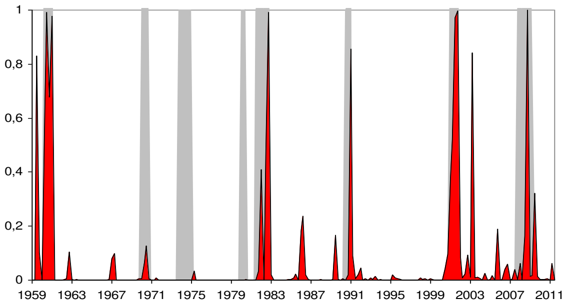
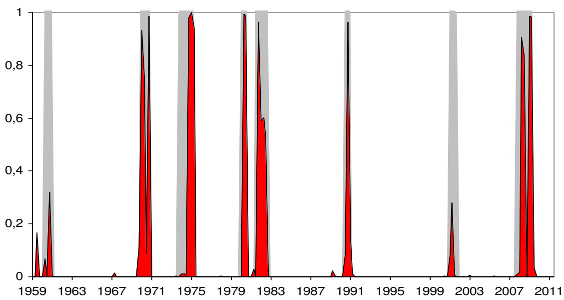


Chart 2. Probability of contractionary supply shock



Notes. Chart 1 plots the joint filtered probabilities of low economic activity and low prices. Chart 2 plots the joint filtered probabilities of low economic activity and high prices. Shaded areas correspond to recessions as documented by the NBER.

The analysis of the "great recession" is of special interest. According to Aruoba and Diebold (2010), in this recession inflation falls later than real activity, plunging only in summer 2008, whereas real activity begins its descent in 2007. This agrees with the high values of $p(S_t^b = 2, S_t^p = 1|\psi_t)$ observed at the beginning of this recession in the bottom panel of Figure 3. However, inflation follows the falls occurred in real activity within approximately six months, leading to the sharp increases on the joint probability of low real activity and prices, $p(S_t^b = 2, S_t^p = 2|\psi_t)$, during the third quarter of 2008 plotted in the top panel of Figure 3. This positive comovement of real activity and inflation during the recent recession is consistent with the adverse demand shock documented by these authors. Finally, since the end of 2008 prices start to increase while real activity was still falling, as can be seen with the high values of $p(S_t^b = 2, S_t^p = 1|\psi_t)$ during the last part of the "great recession," suggesting that it is consistent with a mix of contractionary supply and demand shocks.

Regarding the expansionary phases, the top panel of Figure 4 plots the probability of high real activity and high prices, $p(S_t^b = 1, S_t^p = 1|\psi_t)$, showing that some expansionary periods occurred in the 1970s and early 80s were caused by expansionary demand shocks. However, during the rest of expansionary periods in the sample, the probability of high real activity and low prices, $p(S_t^b = 1, S_t^p = 2|\psi_t)$, plotted in the bottom panel of Figure 4, remains high, indicating that the main source of these expansions are positive supply shocks. This result agrees with Galí (1992), who attributes a large estimate of the contribution of supply factors to short-run GNP fluctuations.

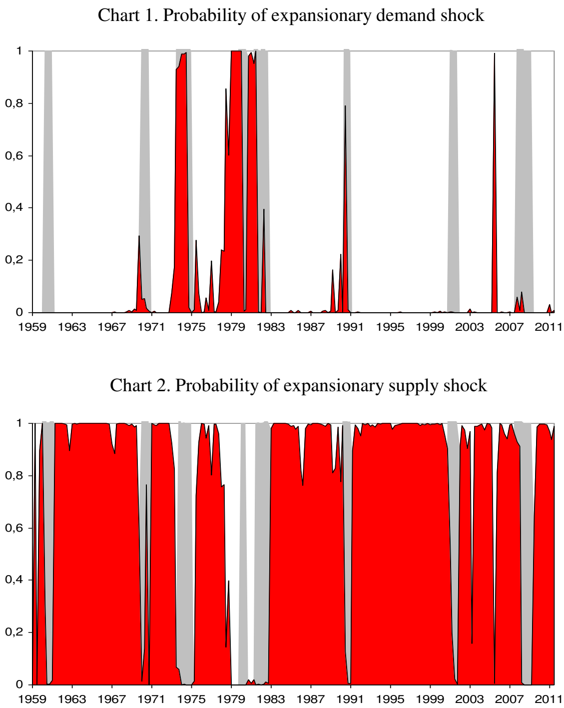
Finally, the proposed model allows quantifying the contribution of each type of shock on the phases of the U.S. business cycle by averaging the filtered probabilities of the joint events through each NBER-referenced recession periods. In order to obtain a better picture of the results described so far, Table 2 reports such contributions, calculated as

$$\alpha_\tau^{\text{supply}} = \sum_\tau \frac{\Pr(S_t^p = 1, S_t^b = 2)}{\Pr(S_t^b = 2)} \tag{39}$$

$$\alpha_\tau^{\text{demand}} = \sum_\tau \frac{\Pr(S_t^p = 2, S_t^b = 2)}{\Pr(S_t^b = 2)}, \tag{40}$$

where $\alpha_\tau^{\text{supply}}$ and $\alpha_\tau^{\text{demand}}$ denote the contribution of aggregate supply and demand shocks, respectively, to the recession occurred during the period τ.

Figure 4. Real Activity versus Inflation: Expansionary Shocks



Notes. Chart 1 plots the joint filtered probabilities of high economic activity and high prices. Chart2 plots the joint filtered probabilities of high economic activity and low prices. Shaded areas correspond to recessions as documented by the NBER.

The results reported in Table 2 support the heterogeneity of recessions showing a substantial mix of shocks varying across recessions and agreeing with Galí (1992), Forni and Gambetti (2010) and Ireland (2010). Specifically, contraction periods 1960.II-1961.I and 2001.I-2001.IV can be categorized as *Demand recessions*, while 1969.IV-1970.IV, 1973.IV-1975.I and 1980.I-1980.III as *Supply recessions*. Moreover contractions during 1981.III-1982.IV and 1990.III-1991.I can be referred as *Mix recessions*, since the contribution of one of the average shocks type, supply or demand, is less than or equal to 70%.[8] Regarding

[8]The 70% has been chosen based on the criterion of the author just for the purpose of defining an intermediate category.

Table 2: Contractionary shocks contributions

Recession Periods	Cont. Demand	Cont. Supply	Rec. type
1960.II - 1961.I	0.89	0.11	Demand
1969.IV - 1970.IV	0.15	0.85	Supply
1973.IV - 1975.I	0.08	0.92	Supply
1980.I - 1980.III	0.17	0.83	Supply
1981.III - 1982.IV	0.32	0.68	Mix
1990.III - 1991.I	0.30	0.70	Mix
2001.I - 2001.IV	0.87	0.13	Demand
2007.IV - 2009.II	0.33	0.67	Mix

Note: Average contribution of Contractionary Demand and Contractionary Supply shocks through periods of recession. If the contribution of one of the average shocks type, supply or demand, is less than or equal to 70% they are categorized as mix recessions.

the 2007.IV-2009.II recession, two thirds of the whole period were influenced by negative supply shocks, the beginning and the end, while one third was caused by negative demand shocks. As a result the "great recession" enters in the *mix* category.

Previous studies provide evidence that when an economy faces supply-driven recessions, such contractionary episodes are usually companied by increases in oil prices. Also, when the opposite occurs and the economy faces a demand-driven recession, credit conditions usually become tighter. In order to assess the validity of such statements, the top chart of Figure 5 plots the inferences on contractionary demand shocks, computed with the multistate MSDbF model, along with the National Financial Conditions Credit (NFCC) Index, computed by the Chicago Fed, and the bottom chart of the same figure plots the inferences on contractionary supply shocks along with the Spot oil prices.

Focusing first on the "great recession" (2007.IV-2009.II) at the beginning of the recession the inferences computed from the model suggest that contractionary supply shocks are the ones prevailing, that is consistent with the increase in oil price, reaching growth rates higher than 20%, while at the same time, credit conditions were "average" according to the NFCC index.[9] In the middle of such recession, there is a significant drop in

[9]Positive values of the NFCC index indicate financial conditions that are tighter than on average, while negative values indicate financial conditions that are looser than on average. Hence, average conditions occur when the index is equal to zero.

oil prices, reaching growth rates around -50%, while credit conditions reach the tightest position occurred in the last two decades, this is consistent with the computed inferences of contractionary demand shock, which rise up to one during that period. Finally, the last part of the contractionary episode is characterized by a drop in credit conditions and a rise in oil prices, this coincides with a high probability of contractionary supply shocks computed from the multistate MSDbF model.

Figure 5. Contractionary Demand versus Contractionary Supply Shocks

Chart 1. Demand-driven Recessions

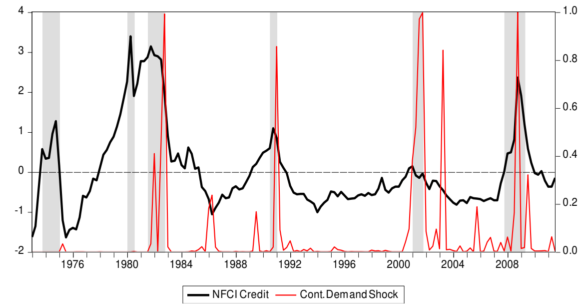
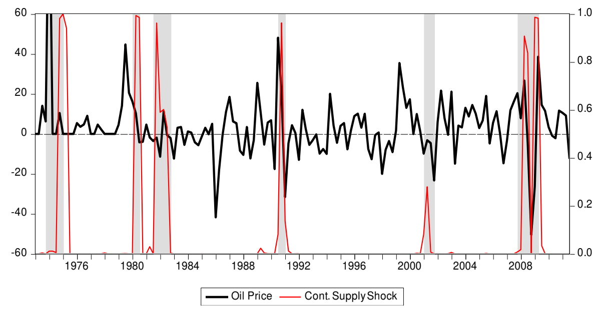


Chart 2. Supply-driven Recessions



Notes. Chart 1 plots the National Financial Conditions Credit Index, thick black line, and inferences on contractionary demand shocks, thin red line. Chart 2 plots Spot oil price, thick black line, and inferences on contractionary supply shocks, thin red line. Shaded areas correspond to NBER recessions.

The whole 2001's recession period is accompanied by a drop in oil prices and, although slightly, tighter credit conditions, this coincides with the high probability of contractionary demand shocks. During the 1990's recession, there is a temporary rise in oil price at the beginning, followed by an increase in credit conditions, consistent with a *Mix recession*, which starts with the supply side and ends with the demand side. This consistency with the contractionary demand or supply shocks and oil prices or credit conditions can be seen also in the previous recessions providing validity of the model's assignments on the sources of contractionary episodes.

3.3 Real Activity versus Housing Price Cycles

Particular interest has been placed in the evolution of housing market prices for the U.S. economy. Most of related studies focus on analyzing housing price movements across U.S. states, extracting regional and national components to assess the monetary policy effects on each of them, as in Del Negro and Otrok (2007) and Ng and Moench (2011), among others. However, those studies do not directly assess the relationship between the evolution of housing prices and the phases of the business cycle.

The second model to be implemented focuses on studying the interaction between real economic activity and housing price cycles in order to assess the impact of shocks originated in the housing market on the business cycle. For this purpose, in this model the vector of observed variables, y_t, contain all five real economic activity indicators and all five housing price indicators. Model's parameters estimates reported in Table 3 indicate, on the one hand, that loadings of f_t^bassociated to real activity indicators are all positive and statistically significant, while those associated to housing prices have lower magnitude, one of them is negative and some of them are not statistically significant, giving a first interpretation of f_t^b as a real activity factor. On the other hand, loadings of f_t^p associated to real activity indicators are of low magnitude, most of them are not statistically significant, with the exception of real GDP, but loadings associated to housing price indicators are all positive and most of them statistically significant, in particular the two indicators obtained from the Federal Housing Finance Agency are the ones showing higher influence an the factor, giving to f_t^p an interpretation of housing price factor. It is worth noting that the degree of dependence between the latent variable governing real activity and housing prices cycles equals to 0,38 showing a higher interdependence than in the case of real activity and inflation cycles.

The first factor extracted with the multistate MSDbF model is plotted in the top chart

Table 3: Maximum likelihood estimates: Real actvity versus Housing prices

Param.	Estim.	St. D.	Param.	Estim.	St. D.	Param	Estim.	St. D.
γ_1^b	0.4377	0.0447	γ_7^p	0.0765	0.0174	ϕ_{51}	1.1342	0.1305
γ_2^b	0.5614	0.0453	γ_8^p	0.1985	0.0306	ϕ_{52}	-0.2873	0.1256
γ_3^b	0.3816	0.0499	γ_9^p	0.2486	0.0228	σ_5	0.2745	0.0328
γ_4^b	0.2880	0.0582	γ_{10}^p	0.0355	0.0237	ϕ_{61}	0.4908	0.0808
γ_5^b	0.3236	0.0316	μ_1^p	0.7382	0.4768	ϕ_{62}	0.3125	0.0798
γ_6^b	0.0266	0.0427	μ_2^p	-5.5240	0.8558	σ_6	0.6363	0.0373
γ_7^b	0.0578	0.0250	p_{11}^p	0.9684	0.0226	ϕ_{71}	1.0123	0.0873
γ_8^b	-0.1428	0.0397	p_{22}^p	0.7573	0.1120	ϕ_{72}	-0.0930	0.0881
γ_9^b	0.0000	0.0316	ϕ_{11}	-0.1526	0.1006	σ_7	0.3313	0.0201
γ_{10}^b	0.1743	0.0416	ϕ_{12}	0.0149	0.0958	ϕ_{81}	0.6533	0.0849
μ_1^b	0.4839	0.1252	σ_1	0.5560	0.0377	ϕ_{82}	0.2071	0.0830
μ_2^b	-3.3693	0.4072	ϕ_{21}	0.0309	0.1628	σ_8	0.4701	0.0311
p_{11}^b	0.9998	0.0020	ϕ_{22}	0.0455	0.1421	ϕ_{91}	1.8072	0.0510
p_{22}^b	0.9856	0.0146	σ_2	0.3165	0.0429	ϕ_{92}	-0.8165	0.0461
γ_1^p	0.1152	0.0321	ϕ_{31}	-0.0301	0.0861	σ_9	0.0377	0.0070
γ_2^p	0.0491	0.0306	ϕ_{32}	0.1968	0.0838	$\phi_{10,1}$	1.1166	0.1215
γ_3^p	0.0394	0.0351	σ_3	0.7255	0.0445	$\phi_{10,2}$	-0.1956	0.1231
γ_4^p	-0.0636	0.0389	ϕ_{41}	0.5529	0.0898	σ_{10}	0.3054	0.0242
γ_5^p	0.0456	0.0229	ϕ_{42}	-0.0311	0.0879	p_{11}^*	0.7316	0.2441
γ_6^p	0.0341	0.0321	σ_4	0.6798	0.0414	p_{22}^*	0.9573	0.0258
						δ	0.3791	0.0858

Note. Superindexes p and b refer to the first (or business cycle) and the second (or housing prices) factors. Subindexes in the loadings, γ, from1 to 10 refer to real GDP (1), Industrial Production (2), Personal Income less Net Transfers (3), Real Manufacturing Trading Sales (4), Total Nonfarm Labor (5), Price deflator index of new single-family houses under construction (6), Conventional Mortgage Home Price Index (7), FHFA All-transactions Prices (8), FHFA Purchase-only Index (9), S&P Case-Shiller-10-cities Home Price Index (10).

of Figure 6, showing high similarity with the dynamics of the CFNAI and hence indicating that it can be interpreted as a real activity factor. Its corresponding probabilities of low mean, plotted in the bottom chart of Figure 6, closely track NBER recession, taking the interpretation of recession probabilities.

Figure 6. Real Activity versus Housing Price: First Factor

Chart 1. First factor and the CFNAI index

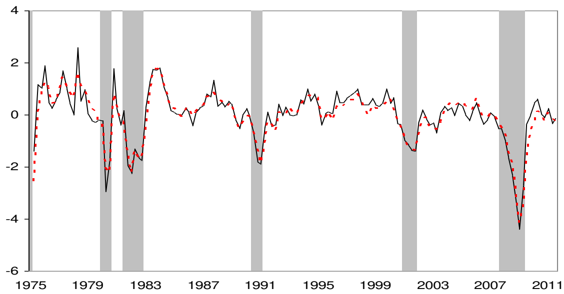
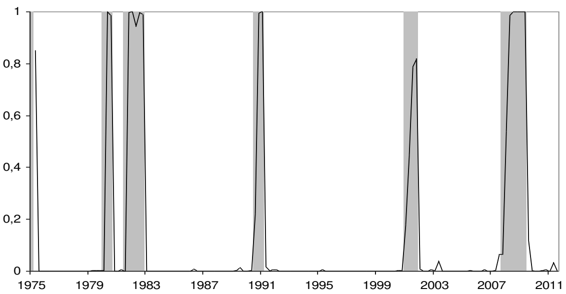


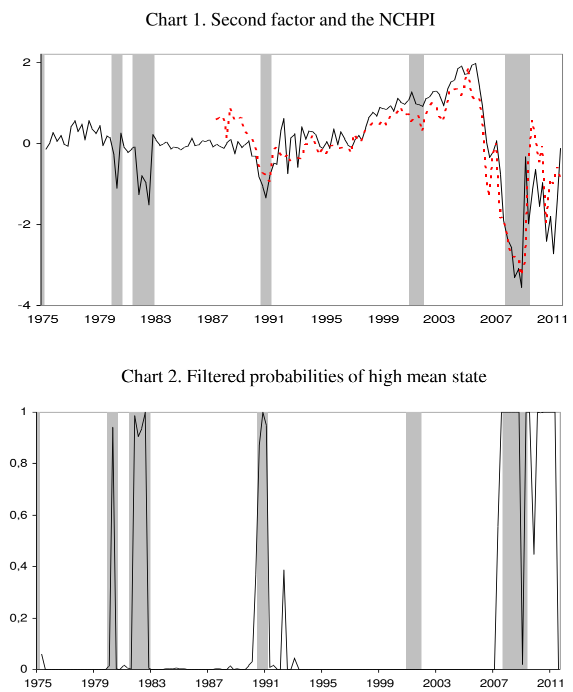
Chart 2. Filtered probabilities of low mean state



Notes. In Chart 1, the solid line refers to the first factor (1975.II-2011.III) and the dashed line refers to the CFNAI (1975.II-2011.III). To facilitate graphing, both series are standardized. The probabilities plotted in Chart 2 come from the state variable that governs the dynamics of the first factor. Shaded areas correspond to recessions as documented by the NBER.

The second factor is plotted in the top chart of Figure 7, along with the National Composite Home Price Index (NCHPI), which is based on the Cash-Shiller methodology, covering not only information about some cities, but about the overall national U.S. economy. Although data of NCHPI starts in 1987, the figure shows strong comovement between both series, which lead to the interpretation of the second factor as a housing price factor. It experiments increasing growth rates during the 90's and the early 2000's, followed by a deep and prolonged drop until the end of the sample. An interesting feature of the housing price factor is that during the 70's and 80's its dynamics are closely related to the business cycle. This is confirmed with its associated probabilities of low mean, plotted in the bottom panel of Figure 7, which follow a close relationship with the NBER recessions. Therefore, these probabilities can be interpreted as an indicator of deflationary pressures in the housing market, which are especially present during and after the "great recession".

Figure 7. Real Activity versus Housing Price: Second Factor

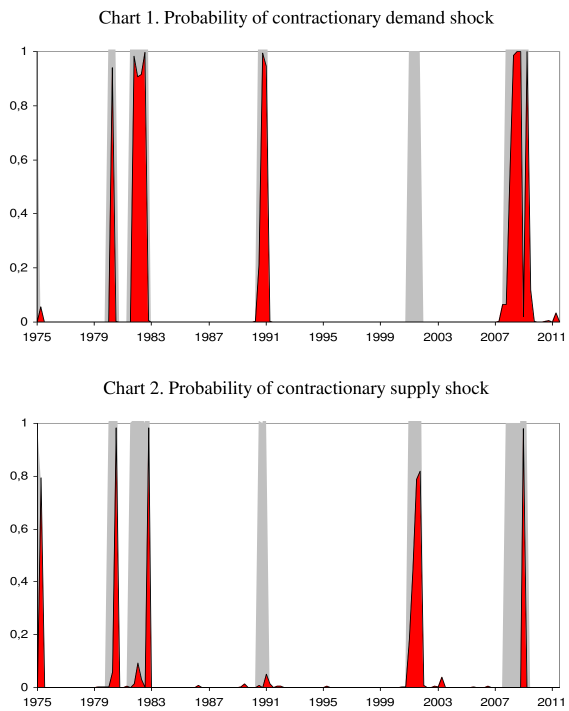


Notes. In Chart 1, the solid line refers to the second factor (1975.II-2011.III) and the dashed line refers to the National Composite Home Price Index (1987.II-2011.III). To facilitate graphing, both series are standardized. The probabilities plotted in Chart 2 come from the state variable that governs the dynamics of the second factor. Shaded areas correspond to NBER recessions.

3.3.1 Inferences on Housing Shocks

The probabilities of recession attached to the real activity factor can be additively decomposed in order to disentangle contractionary episodes which are accompanied by deflationary pressures in the housing market, low prices, from recessionary periods accompanied by high prices. By relying on the results obtained in the *real activity versus inflation* model in Section 3.2 and taking into account that housing prices correspond to a particular set of information contained in the economy's inflation dynamics, the decomposed probabilities obtained from the *real activity versus housing price* model can give insights on the impact of contractionary or expansionary shocks originated in the housing market over the business cycle. The top chart of Figure 8 plots the joint probability of low real economic activity and low housing prices, i.e. inferences on contractionary housing demand shocks, and the bottom chart of the same figure plots the probability of low real economic activity and high housing prices, i.e. contractionary housing supply shocks.

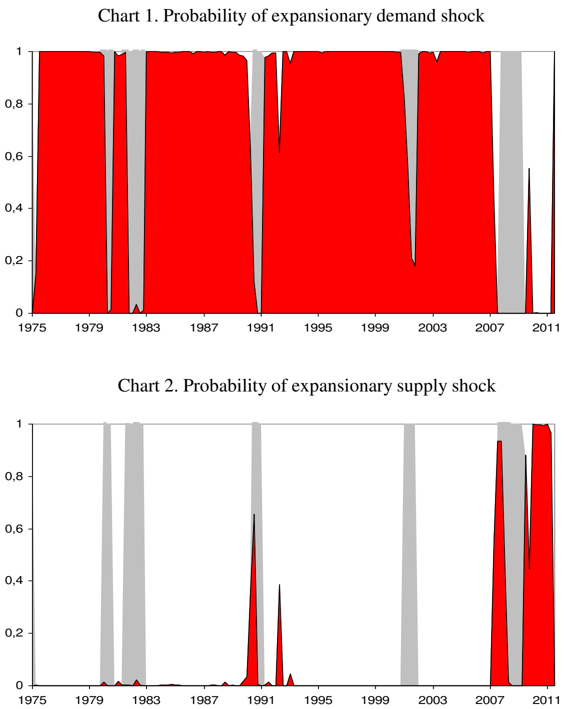
Figure 8. Real Activity versus Housing Price: Contractionary Shocks



Notes. Chart 1 plots the joint filtered probabilities of low economic activity and low housing prices. Chart2 plots the joint filtered probabilities of low economic activity and high housing prices. Shaded areas correspond to recessions as documented by the NBER.

The results show that recessionary periods are usually accompanied by deflationary pressures in the housing market, while expansionary periods are mainly influenced by expansionary housing demand shocks. Specifically, during the first half of the 1980's recession, contractionary demand shocks originated in the housing market were mainly influencing such period, while during the second half, contractionary housing supply shocks were the ones prevailing.

Figure 9. Real Activity versus Housing Price: Expansionary Shocks



Notes. Chart 1 plots the joint filtered probabilities of high economic activity and high housing prices. Chart2 plots the joint filtered probabilities of high economic activity and low housing prices. Shaded areas correspond to recessions as documented by the NBER.

The scenario during the 1981's recession is different, since almost the entire recession period was influenced by contractionary housing demand shocks. A somewhat similar situation occurs during the 1990's recession, which is entirely influenced by the demand side of the housing market. In the next recession, 2001's, the scenario is actually the opposite to the previous one, since this is the only recession, in the sample, which is not accompanied

Table 4: Housing shocks contributions

Expansion Periods	Exp. Housing Demand	Exp. Housing Supply
1975.II - 1979.IV	1.00	0.00
1980.IV - 1981.II	0.99	0.01
1983.I - 1990.II	0.99	0.01
1991.II - 2000.IV	0.99	0.01
2002.I - 2007.III	0.94	0.06
2009.III - 2011.III	0.19	0.81
Recession Periods	Cont. Housing Demand	Cont. Housing Supply
1980.I - 1980.III	0.49	0.51
1981.III - 1982.IV	0.78	0.22
1990.III - 1991.I	0.98	0.02
2001.I - 2001.IV	0.00	1.00
2007.IV - 2009.II	0.85	0.15

Note: Average contribution of Contractionary and Expansionary Housing Demand and Supply shocks through periods of recession based on housing prices.

by deflationary pressures in the housing market, this is consistent with the increasing growth rates that the housing factor experimented during the early 2000's. Finally, the contribution of the housing market shocks to the "great recession" is well-defined, since such period is mainly characterized by a deep and prolonged drop in the housing price factor, which is consistent with the high probability of contractionary housing demand shocks. There is just a sudden and short increase in housing prices, which is represented by the spike in the probabilities of supply shocks occurred almost at the end of the recession.

Regarding periods of expansions, in the top and bottom charts of Figure 9 are plotted the probabilities of expansionary housing demand shocks, i.e. high real activity and high housing prices, and expansionary housing supply shocks, i.e. high real activity and low housing prices, respectively. The figure clearly shows that almost all expansion periods have been free of deflationary pressures in the housing market, as can be seen in the top chart. The only notorious exception occurs during the period surrounding the "great recession", as can be seen in the bottom chart. These results suggest that the importance of shocks originated in the housing market have played a fundamental role in the fall and the recovery from such recession, making it different than the rest of contractionary episodes, in the sample, in this respect.

The average contribution of the type of housing shock affecting the business cycle are quantified and reported in Table 4, showing that all expansionary periods come from the supply side, with the exception of the one after the "great recession", and that contractionary periods have been more influenced by the housing demand side, with the exception of the 2001's recession, and the 1980's recession, which was almost equally influenced by the demand and supply side of the housing market.

4 Conclusions

By using the proposed multistate Markov-Switching Dynamic bi-Factor model, the interrelation between real activity and inflation cycles is assessed, providing a tool useful to infer economic recessions and periods of high inflation simultaneously. Relying on such inferences, the framework is able to categorize NBER contractionary episodes into *demand*, *supply* and *mix* recessions by quantifying the type of shock contribution to each period of time, finding the "great recession" in the *mix* category, since it is initially affected by supply, followed by demand, and finally again by supply contractionary shocks.

Moreover, by incorporating data of housing prices in the proposed model, it is assessed the impact of shocks originated in the housing market over the business cycle. The results show that recessionary periods are usually accompanied by deflationary pressures in the housing market, while expansions are mainly influenced by expansionary housing demand shocks, with only a notorious exception occurred during the period surrounding the "great recession" showing that housing market shocks have played a fundamental role in the fall and the recovery from such recessionary episode.

Appendix

It is assumed that $m = 2$, and $k = 0$. Focusing first on the *real activity versus inflation* model, according to the eleven-variable model used in the empirical application, the measurement equation, $y_t = H\beta_t + e_t$, with $e_t \sim N(0, R)$, is

$$\begin{bmatrix} \Delta GDP_t \\ \Delta IND_t \\ \Delta PIN_t \\ \Delta SAL_t \\ \Delta PAY_t \\ \Delta DEF_t \\ \Delta CPI_t \\ \Delta PPI_t \\ \Delta GSC_t \\ \Delta OIL_t \\ \Delta HCN_t \end{bmatrix} = \begin{bmatrix} \gamma_1^b & \gamma_1^p & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 0 & 0 \\ \gamma_2^b & \gamma_2^p & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 0 & 0 \\ \gamma_3^b & \gamma_3^p & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 0 & 0 \\ \gamma_4^b & \gamma_4^p & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \cdots & \cdots & 0 & 0 & 0 & 0 \\ \gamma_5^b & \gamma_5^p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 0 & 0 \\ \gamma_6^b & \gamma_6^p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 0 & 0 \\ \gamma_7^b & \gamma_7^p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 0 & 0 \\ \gamma_8^b & \gamma_8^p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 0 & 0 \\ \gamma_9^b & \gamma_9^p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 0 & 0 \\ \gamma_{10}^b & \gamma_{10}^p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \cdots & 1 & 0 & 0 & 0 \\ \gamma_{11}^b & \gamma_{11}^p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} f_t^b \\ f_t^p \\ e_{1t} \\ e_{1,t-1} \\ e_{2t} \\ e_{2,t-1} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ e_{11,t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{A.1}$$

where R is a matrix of zeroes. The notation of the variables is defined as: GDP is real GDP, IND is industrial production, PIN is real personal income less transfers, SAL is real manufacturing trading sales, PAY is total non-farm labor, DEF is deflator of GDP, CPI is consumer price index, PPI is producer price index, GSC is Standard and Poor's GSCI non-energy commodities price index, OIL is spot oil price, and HCN hourly compensation in the non-farm business sector.

The transition equation, $\beta_t = \widetilde{\mu}_{S_t^b, S_t^p} + F\beta_{t-1} + \upsilon_t$, with $\upsilon_t \sim N(0, Q)$, is

$$\begin{bmatrix} f_t^b \\ f_t^p \\ e_{1t} \\ e_{1,t-1} \\ e_{2t} \\ e_{2,t-1} \\ \vdots \\ \vdots \\ \vdots \\ e_{11,t-1} \end{bmatrix} = \begin{bmatrix} \mu^b_{S_t^b} \\ \mu^p_{S_t^p} \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 \\ 0 & 0 & \phi_{11} & \phi_{12} & 0 & 0 & \cdots & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & \phi_{21} & \phi_{22} & \cdots & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & \cdots & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \ddots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & & \phi_{11,1} & \phi_{11,2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \cdots & 1 & 0 \end{bmatrix} \begin{bmatrix} f_{t-1}^b \\ f_{t-1}^p \\ e_{1,t-1} \\ e_{1,t-2} \\ e_{2,t-1} \\ e_{2,t-2} \\ \vdots \\ \vdots \\ \vdots \\ e_{11,t-2} \end{bmatrix} + \begin{bmatrix} \omega_t^b \\ \omega_t^p \\ \varepsilon_{1t} \\ 0 \\ \varepsilon_{2t} \\ 0 \\ \vdots \\ \vdots \\ \vdots \\ 0 \end{bmatrix}, \tag{A.2}$$

where Q is a diagonal matrix in which the entries inside the main diagonal are collected in the vector

$$\left(\sigma_b^2, \sigma_p^2, \sigma_1^2, 0, \sigma_2^2, 0, \sigma_3^2, 0, \sigma_4^2, 0, \sigma_5^2, 0, \sigma_6^2, 0, \sigma_7^2, 0, \sigma_8^2, 0, \sigma_9^2, 0, \sigma_{10}^2, 0, \sigma_{11}^2, 0\right)' \tag{A.3}$$

For the *real activity versus housing prices* model, for the measurement and transition equations follow the same reasoning as in Equations (A.1) and (A.2), respectively. The main change, apart from adjusting the appropriate dimension of the matrices for 10 observed indicators, is that vector y_t, as defined in Equation (A.1), is replaced by

$$(\Delta GDP_t, \Delta IND_t, \Delta PIN_t, \Delta SAL_t, \Delta PAY_t, \Delta PDI_t, \Delta CMH_t, \Delta ATP_t, \Delta POI_t, \Delta SPC_t)'$$

The notation of the housing price indicators is defined as: PDI is Price deflator index of new single-family houses under construction, CMH is Conventional Mortgage Home Price Index, ATP is FHFA All-transactions Price Index, POI is FHFA Purchase-only Index, SPC is S&P Case-Shiller-10-cities Home Price Index.

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