Michael Sonis and Geoffrey Hewings theories and theirs multiplier effect on the Brazilian economy

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Abstract

It is hard to think about the contributions of Michael Sonis to Input-Output Analysis without taking into consideration his joint work with Geoffrey Hewings. Both are linked together into a type n, $n \to \infty$, multiplier of theory and knowledge. This paper makes a brief presentation of the various theories developed by Sonis and Hewings that gave a new meaning to the word input-output analysis, giving a revival on the use of input-output analysis to study the Brazilian economy. From the Fields of Influence approach to the Pure Linkages analyses, passing through the various decompositions, the Landscapes, and many other techniques, there is a great deal of researchers and academics working with these theories, making it possible to have a better understanding of the productive structure of Brazil and its regions.

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1. Introduction

Sometimes you need to go back in time and remembered how things happened so you could better understand how things are today. So far I think this is the better start to this paper.

When I was still a Ph.D. student in the mid 1980’s and Geoffrey Hewings was my advisor at the University of Illinois, I remember to be called to attend a seminar from a professor from Israel, which name was Michael Sonis.

By that time, I was more worried with my dissertation than anything else and for me it was an interesting seminar, about fields of influence, but I did not give the importance that it really needed to be giving.

By that time, this was the first time that Michael Sonis was visiting the University Illinois and it was really the beginning of an association with Geoffrey Hewings that would be one of the more brilliant associations in terms of creating new theories, which could be used to better understand the economies in which we live.

Michael Sonis certainly has a brilliant mathematical mind, very sharp and fast, and he for sure can come out with a solution much more fast than any one of us could thought. Geoffrey Hewings on the other as an unsurpassed mind in understand the economy, economic theory and the links that can be made with economic models and the way that the world works.

It was done the perfect combination, since that time, in the mid 1980´s, Geoffrey Hewings and Michael Sonis have started an association that continues until today and that will go for a long time, where new theories and ideas are popping out faster than anyone of us, mere mortals, could catch.

The construction of this new theories gave a new meaning to the word input-output analysis. Despite all of its power of analysis, the input-output approach was lacking structural theories that could extract, from the information contained it, a better understanding of how the economic system works. That was in the past, because the theories of Sonis and Hewings would come to fill in this lack.

And in filling this lack, these theories, on one hand attracted more people to work in the field, and on the o the other hand create the need for more and better input-output data.

That was what happen in Brazil, seeing the new theories being developed by Sonis and Hewings, the scholars started to apply these theories and suddenly realized their power of analysis and the need for more data. So, the construction of new input-output matrices, national and regional, started because with these new theories we could better understand how the economic system in which we live works. So things started to go in a spiral way, the construction of input-output matrix would be because of new theories are being developed and new theories are being developed because now there is more and better input-output data. And, fortunately, things will tend to go like that for a long time.

In the next section it will be made a brief presentation of some of the Sonis and Hewings theories that are already in use in the economic analysis of the Brazilian economy and of its regions. In the third section some examples of applications are presented, while in the last section the final comments are made.

2. Some of Sonis & Hewings Theories and Developments

In this section it is presented some of the theories developed by Sonis and Hewings and that were used in studies about the Brazilian economy, at the national and regional level.

This section first starts with a presentation of the Rasmussen and Hirschman approach that is the basis for the development of some of the Sonis and Hewings theories, like the the pure linkage approach, the field of influence, and the matrix product multiplier. Then, it follows the theory to study the synergetic interactions among regions and the one to study decomposition, source, and evolution of the output change.

2.1. The Rasmussen/Hirschman Approach

The work of Rasmussen (1956) and Hirschman (1958) led to the development of indices of linkage that have now become part of the generally accepted procedures for identifying key sectors in the economy. Define b_{ij} as a typical element of the Leontief inverse matrix, B; B^* as the average value of all elements of B, and if $B_{\bullet j}$ and $B_{i\bullet}$ are the associated typical column and row sums, then the indices may be developed as follows:

Backward linkage index (power of dispersion):

$$U_j = \left[B_{\bullet j} / n \right] / B^* \tag{1}$$

Forward linkage index (sensitivity of dispersion):

$$U_i = \left[B_{i\bullet} / n \right] / B^* \tag{2}$$

One of the criticisms of the above indices is that they do not take into consideration the different *levels* of production in each sector of the economy, what it is done by the pure linkage approach presented in the next section.

2.2. The Pure Linkage Approach

As presented by Guilhoto, Sonis and Hewings (1996) the pure linkage approach can be used to measure the importance of the sectors in terms of production generation in the economy.

Consider a two-region input-output system represented by the following block matrix, *A*, of direct inputs:

$$A = \left[\begin{array}{c|c} A_{jj} & A_{jr} \\ \hline A_{rj} & A_{rr} \end{array} \right] \tag{3}$$

where A_{jj} and A_{rr} are the quadrate matrices of direct inputs within the first and second region and A_{jr} and A_{rj} are the rectangular matrices showing the direct inputs purchased by the second region and vice versa.

From (3), one can generate the following expression:

$$B = (I - A)^{-1} = \begin{bmatrix} B_{jj} & B_{jr} \\ B_{rj} & B_{rr} \end{bmatrix} = \begin{bmatrix} \Delta_{jj} & 0 \\ 0 & \Delta_{rr} \end{bmatrix} \begin{bmatrix} \Delta_j & 0 \\ 0 & \Delta_r \end{bmatrix} \begin{bmatrix} I & A_{jr}\Delta_r \\ A_{rj}\Delta_j & I \end{bmatrix} \tag{4}$$

where:

$$\Delta_j = \left(I - A_{jj} \right)^{-1} \tag{5}$$

$$\Delta_r = \left(I - A_{rr} \right)^{-1} \tag{6}$$

$$\Delta_{jj} = \left(I - \Delta_j A_{jr} \Delta_r A_{rj}\right)^{-1} \tag{7}$$

$$\Delta_{rr} = \left(I - \Delta_r A_{rj} \Delta_j A_{jr}\right)^{-1} \tag{8}$$

By utilizing this decomposition (equation 4), it is possible to reveal the process of production in an economy as well as derive a set of multipliers/linkages.

From the Leontief formulation:

$$X = \left(I - A\right)^{-1} Y \tag{9}$$

and using the information contained in equations (4) through (8), one can derive a set of indexes that can be used: a) to rank the regions in terms of its importance in the economy; b) to see how the production process occurs in the economy.

From equations (4) and (9) one obtains:

$$\begin{pmatrix} X_j \\ X_r \end{pmatrix} = \begin{pmatrix} \Delta_{jj} & 0 \\ 0 & \Delta_{rr} \end{pmatrix} \begin{pmatrix} \Delta_j & 0 \\ 0 & \Delta_r \end{pmatrix} \begin{pmatrix} I & A_{jr}\Delta_r \\ A_{rj}\Delta_j & I \end{pmatrix} \begin{pmatrix} Y_j \\ Y_r \end{pmatrix} \tag{13}$$

which leads to the definitions for the Pure Backward Linkage (PBL) and for the Pure Forward Linkage (PFL), i.e.,

$$\begin{aligned} PBL &= \Delta_r A_{rj} \Delta_j Y_j \\ PFL &= \Delta_j A_{jr} \Delta_r Y_r \end{aligned} \tag{14}$$

where the PBL will give the pure impact on the rest of the economy of the value of the total production in region j, $\left(\Delta_j Y_j\right)$: i.e., the impact that is free from a) the demand inputs that region j makes from region j , and b) the feedbacks from the rest of the economy to region j and vice-versa. The PFL will give the pure impact on region j of the total production in the rest of the economy $\left(\Delta_r Y_r\right)$.

2.3. The Fields of Influence

The concept of field of influence was introduced and elaborated by Sonis and Hewings (1989, 1994). It is mainly concerned with the problem of coefficient change, namely the influence of a change in one or more direct coefficients on the associated Leontief inverse matrix.[2] Since, given an economic system, some coefficients are more "influential" than others, the sectors responsible for the greater changes in the economy can be determined. Together with the Rasmussen/Hirschman linkage indices and the pure linkage indices, it completes our analytical framework for the determination of key sectors in an economic system.

Considering a small enough variation, ε, in the input coefficient, a_{ij}, the presentation of the *basic* solution of the coefficient change problem proposed by Sonis and Hewings may be presented as follows. let $A = (a_{ij})$ be an *nxn* matrix of direct input coefficients; let $E(e_{ij})$ be a matrix of incremental changes in the direct input coefficients; let $B = (I - A)^{-1} = \|b_{ij}\|$, $B(E) = (I - A - E)^{-1} = \|b(e)_{ij}\|$ be the Leontief inverses before and after changes.

Using the notion of inverse-important input coefficients that is based on the conception of the field of influence associated with the change *in only one* input coefficient, assume that this change occurs in location (i_1, j_1), that is,

$$e_{ij} = \begin{cases} \varepsilon & i = i_1,\ j = j_1 \\ 0 & i \neq i_1 \text{ or } j \neq j_1 \end{cases} \tag{15}$$

then, the field of influence can be constructed as the matrix $F(e_{ij})$ generated by multiplication of the j^{th} column of the Leontief matrix, B, with the i^{th} row:

$$F(e_{ij}) = \begin{pmatrix} b_{1j} \\ b_{2j} \\ \vdots \\ b_{nj} \end{pmatrix} \begin{pmatrix} b_{i1} & b_{i2} & \ldots & b_{in} \end{pmatrix}, \tag{16}$$

where $F(e_{ij})$ is a *nxn* matrix, interpreted as the field of influence of the change on the input coefficient, a_{ij}. For every coefficient, a_{ij}, there will be an associated *nxn* field of influence matrix.

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[2]We considered here only the simplest case, i.e., the case in which the change occurs in only one input parameter. However, the analysis can be extended to the cases of changes in whole rows or columns.

In order to determine which coefficients have the greater field of influence, reference is made to the rank-size ordering of the elements, S_{ij} , from the largest to the smallest ones. Therefore, for every matrix $F\left(e_{ij}\right)$, there will be an associated value given by:

$$S_{ij} = \sum_{k}^{n} \sum_{l}^{n} f_{kl}\left(e_{ij}\right) . \tag{17}$$

It is possible to see that $S_{ij} = b_{\bullet j} b_{i\bullet}$ and thus provides a direct relationship with the intensity matrix defined in (16). Thus, from the values of S_{ij} , a hierarchy can be developed of the direct coefficients, based on their fields of influence, i.e., ranking sectoral relations in terms of their sensitivity to changes, in a sense that they will be responsible for more significant impacts on the economy.

2.4 The Structure of Production: Economic Landscapes[3]

This section introduces the notion of *artificial economic landscapes* and the corresponding multiplier product matrices representing the essence of key sector analysis. The definition of the multiplier product matrix is as follows: let $A = \left\|a_{ij}\right\|$ be a matrix of direct inputs in the usual input-output system, and $B = \left(I - A\right)^{-1} = \left\|b_{ij}\right\|$ the associated Leontief inverse matrix and let $B_{\bullet j}$ and $B_{i\bullet}$ be the column and row multipliers of this Leontief inverse. These are defined as:

$$B_{\bullet j} = \sum_{i=1}^{n} b_{ij}, \quad B_{i\bullet} = \sum_{j=1}^{n} b_{ij} \quad j = 1, 2, ..., n \tag{18}$$

The row and column vectors of column and row multipliers take the following form:

$$M_c(B) = \left[B_{\bullet 1} B_{\bullet 2} ... B_{\bullet p} \right], \qquad M_r(B) = \begin{bmatrix} B_{1\bullet} \\ B_{2\bullet} \\ \vdots \\ B_{n\bullet} \end{bmatrix} \tag{19}$$

Let V be the global intensity of the Leontief inverse matrix:

[3] The first part of this section draws on Sonis e*t al*., (1997a)

$$V = \sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij} \tag{20}$$

Then, the input-output multiplier product matrix (MPM) is defined as:

$$\begin{aligned} M &= \frac{1}{V}\left\|B_{i\bullet}B_{\bullet j}\right\| = \frac{1}{V}\begin{pmatrix} B_{1\bullet} \\ B_{2\bullet} \\ \vdots \\ B_{n\bullet} \end{pmatrix}\begin{pmatrix} B_{\bullet 1} & B_{\bullet 2} & \cdots & B_{\bullet n} \end{pmatrix} = \left\|m_{ij}\right\| \\ V &= \sum_{i=1}^{n} B_{i\bullet} = \sum_{j=1}^{n} B_{\bullet j} = \begin{pmatrix} B_{\bullet 1} & B_{\bullet 2} & \cdots & B_{\bullet n} \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} = (11\ldots1)\begin{pmatrix} B_{1\bullet} \\ B_{2\bullet} \\ \vdots \\ B_{n\bullet} \end{pmatrix} \end{aligned} \tag{22}$$

or, in vector notation:

$$M = \frac{1}{V} M_r(B) M_c(B); \;\; V = M_c(B) \times i' = i \times M_r(B) \tag{23}$$

The properties of the MPM that will now be considered will focus on (1) the hierarchy of backward and forward linkages and their economic landscape associated with the cross-structure of the MPM, and (2) the interpretation of MPM as a matrix of first order intensities of the fields of influence of individual changes in direct inputs.

The concept of key sectors is based on the notion of backward and forward linkages and has been associated with the work of both Rasmussen (1956) and Hirschman (1958). The major thrust of the analytical techniques, and subsequent modifications and extensions, has been towards the identification of sectors whose linkage structures are such that they create an above-average impact on the rest of the economy when they expand or in response to changes elsewhere in the system.

The definitions of backward and forward linkages provided by (1) and (2) imply that the rank-size hierarchies (rank-size ordering) of these indices coincide with the rank-size hierarchies of the column and row multipliers. It is important to underline, in this connection, that the column and row multipliers for MPM are the same as those for the Leontief inverse matrix. Thus, the structure of the MPM is essentially connected with the properties of sectoral backward and forward linkages.

The structure of the matrix, *M*, can be ascertained in the following fashion: consider the largest column multiplier, $B_{\bullet j}$, and the largest row multiplier, $B_{i\bullet}$, of the Leontief inverse, with the element, $m_{i_0 j_0} = \frac{1}{V} B_{i_0 \bullet} B_{\bullet j_0}$, located in the place (i_0, j_0) of the matrix, *M*. Moreover, all rows of the matrix, *M*, are proportional to the i_0^{th} row, and the elements of this row are larger than the corresponding elements of all other rows. The same property applies to the j_0^{th} column of the same matrix. Hence, the element located in (i_0, j_0) defines the center of the largest *cross* within the matrix, *M*. If this cross is excluded from *M*, then the second largest cross can be identified and so on. Thus, the matrix, *M*, contains the rank-size sequence of crosses. One can reorganize the locations of rows and columns of *M* in such a way that the centers of the corresponding crosses appear on the main diagonal. In this fashion, the matrix will be reorganized so that a descending *economic landscape* will be apparent.

This rearrangement also reveals the descending rank-size hierarchies of the Rasmussen-Hirschman indices for forward and backward linkages. Inspection of that part of the landscape with indices > 1 (the usual criterion for specification of key sectors) will enable the identification of the key sectors. However, it is important to stress that the construction of the economic landscape for different regions or for the same region at different points in time would create the possibility for the establishment of a taxonomy of these economies.

2.5. Hierarchical Inclusion of Economic Landscapes[4]

In this section, attention will be directed to a description of multiple shifts in intraregional backward and forward linkages and the associated changes in the positions of key sectors under the influence of interaction between the region and the rest of economy. The approach creates the possibility to evaluate immediately when economic sectors became more important for the regional economy under the influence of synergetic interactions with the rest of economy.

[4] This section draws on Sonis and Hewings (1999).

The main analytical tool of the hierarchical inclusion of the economic landscapes will now be revealed. Consider the product, $B = B^{/}B^{//}$, of two matrices, $B^{/}$ and $B^{//}$, of the respective sizes $n \times m,\;\; m \times p$. Let

$$
\begin{aligned}
B_{\bullet j} &= \sum_{i=1}^{n} b_{ij}; \quad B_{i\bullet} = \sum_{j=1}^{n} b_{ij} \\
B^{/}_{\bullet j} &= \sum_{i=1}^{n} b^{/}_{ij}; \quad B^{/}_{i\bullet} = \sum_{j=1}^{n} b^{/}_{ij} \\
B^{//}_{\bullet j} &= \sum_{i=1}^{n} b^{//}_{ij}; \quad B^{//}_{i\bullet} = \sum_{j=1}^{n} b^{//}_{ij}
\end{aligned}
\tag{24}
$$

be the column and row multipliers of these matrices. Using the definition of *V*, the global intensity of the matrix *B* from (22), the following multiplicative connections between the vectors of column and row multipliers of these matrices exist:

$$
\begin{aligned}
&\left[B_{\bullet 1}B_{\bullet 2}\ldots B_{\bullet p}\right] = \left[B^{'}_{\bullet 1}B^{'}_{\bullet 2}\ldots B^{'}_{\bullet m}\right] \times B^{''}; \quad \begin{bmatrix} B_{1\bullet} \\ B_{2\bullet} \\ : \\ B_{n\bullet} \end{bmatrix} = B^{'} \times \begin{bmatrix} B^{''}_{1\bullet} \\ B^{''}_{2\bullet} \\ : \\ B^{''}_{m\bullet} \end{bmatrix}; \\
&V = \left[B^{'}_{\bullet 1}B^{'}_{\bullet 2}\ldots B^{'}_{\bullet m}\right] \times \begin{bmatrix} B^{''}_{1\bullet} \\ B^{''}_{2\bullet} \\ : \\ B^{''}_{m\bullet} \end{bmatrix}
\end{aligned}
\tag{25}
$$

These expressions can be checked by direct calculations of the components of the corresponding vectors and matrices.

Further, specify the following vectors:

$$
\begin{aligned}
M_c(B) &= \left[B_{\bullet 1} B_{\bullet 2} \ldots B_{\bullet p} \right] \\
M_c(B') &= \left[B'_{\bullet 1} B'_{\bullet 2} \ldots B'_{\bullet m} \right] \\
M_c(B'') &= \left[B''_{\bullet 1} B''_{\bullet 2} \ldots B''_{\bullet m} \right]
\end{aligned} \tag{26}
$$

$$
M_r(B) = \begin{bmatrix} B_{1\bullet} \\ B_{2\bullet} \\ \vdots \\ B_{n\bullet} \end{bmatrix}, M_r(B') = \begin{bmatrix} B'_{1\bullet} \\ B'_{2\bullet} \\ \vdots \\ B'_{m\bullet} \end{bmatrix}, M_r(B'') = \begin{bmatrix} B''_{1\bullet} \\ B''_{2\bullet} \\ \vdots \\ B''_{m\bullet} \end{bmatrix}
$$

as the row vectors and column vectors with components that are the column and row multipliers of the matrices, B, B', B''. Using this notation, equation (26) may be presented in the following form:

$$
\begin{aligned}
&M_c(B) = M_c(B')B''; \\
&M_r(B) = B'M_r(B''); \\
&V = M_c(B')M_r(B'')
\end{aligned} \tag{27}
$$

Consider the economic system that is comprised of a region *r* and the rest of economy, *R*. The corresponding input-output system can be represented by the block matrix

$$
A = \begin{pmatrix} A_{rr} & A_{rR} \\ A_{Rr} & A_{RR} \end{pmatrix} \tag{28}
$$

Assume that the intra-regional matrix, A_{rr}, of the region *r* has the following incremental change E_{rr}, and A_{rR}, A_{Rr} are the inter-regional matrices representing direct input connections between region and the rest of the economy, while the matrix A_{RR} represents the intra-regional inputs within the rest of the economy.

The Leontief inverse $B = (I - A)^{-1}$ can be formally presented in the following block:

$$
B = \begin{bmatrix} B_{rr} & B_{rR} \\ B_{Rr} & B_{RR} \end{bmatrix} \tag{29}
$$

and this can be further elaborated with the help of the Schur-Banachiewicz formula (Schur, 1917; Banachiewicz, 1937; Miyazawa, 1966; Sonis and Hewings, 1993):

$$
B = \begin{bmatrix} B_{rr} & B_{rr}A_{rR}B_{R} \\ B_{RR}A_{Rr}B_{r} & B_{RR} \end{bmatrix} = \begin{bmatrix} B_{rr} & B_{r}A_{rR}B_{RR} \\ B_{RR}A_{Rr}B_{rr} & B_{RR} \end{bmatrix} \tag{30}
$$

where the matrices $B_r = (I - A_{rr})^{-1}$ and $B_R = (I - A_{RR})^{-1}$ represent the Miyazawa internal matrix multipliers for the region r and the rest of economy (revealing the interindustry propagation effects within the isolated region and isolated rest of economy) while the matrices $A_{Rr}B_r, B_r A_{rR}, A_{rR}B_R,$ and $B_R A_{Rr}$ show the induced effects on output or input between the two parts of input-output system (Miyazawa, 1966).

Further:

$$\begin{aligned} B_{rr} &= (I - A_{rr} - A_{rR}B_R A_{Rr})^{-1} \\ B_{RR} &= (I - A_{RR} - A_{Rr}B_r A_{rR})^{-1} \end{aligned} \tag{31}$$

are the extended Leontief multipliers for the region r and the rest of economy. The connections between these extended Leontief multipliers are:

$$\begin{aligned} B_{rr} &= B_r + B_r A_{rR} B_{RR} A_{Rr} B_r \\ B_{RR} &= B_R + B_R A_{Rr} B_{rr} A_{rR} B_R \end{aligned} \tag{32}$$

By using the Miyazawa decomposition, the extended Leontief inverses can be decomposed into the products of internal and external multipliers describing direct and induced self-influences (Miyazawa, 1966, 1976):

$$\begin{aligned} B_{rr} &= B_r B_{rr}^R = B_{rr}^L B_r \\ B_{RR} &= B_R B_{RR}^R = B_{RR}^L B_R \end{aligned} \tag{33}$$

where

$$\begin{aligned} B_{rr}^L &= (I - B_r A_{rR} B_R A_{Rr})^{-1}; \quad B_{rr}^R = (I - A_{rR} B_R A_{Rr} B_r)^{-1} \\ B_{RR}^L &= (I - B_R A_{Rr} B_r A_{rR})^{-1}; \quad B_{RR}^R = (I - A_{Rr} B_r A_{rR} B_R)^{-1} \end{aligned} \tag{34}$$

are the left and right Miyazawa external multipliers for the region r and the rest of economy.

It is easy to see that for the block Leontief inverse, the row vector $M_c(B)$ of the column multipliers has the following block form:

$$M_c(B) = \left[M_c(B_{rr}) + M_c(B_{Rr}) \quad M_c(B_{rR}) + M_c(B_{RR}) \right] \tag{35}$$

Using (30), one obtains:

$$
\begin{aligned}
M_c(B) &= \begin{bmatrix} M_c(B_{rr}) + M_c(B_{RR})A_{Rr}B_r & M_c(B_{rr})A_{rR}B_R + M_c(B_{RR}) \end{bmatrix} = \\
&= M_c(B_{rr})\begin{bmatrix} I & A_{rR}B_R \end{bmatrix} + M_c(B_{RR})\begin{bmatrix} A_{Rr}B_r & I \end{bmatrix}
\end{aligned}
\tag{36}
$$

Analogously, the column block vector of the row multipliers of the Leontief inverse B can be presented in the form:

$$
\begin{aligned}
M_r(B) &= \begin{bmatrix} M_r(B_{rr}) + M_r(B_{rR}) \\ M_r(B_{Rr}) + M_r(B_{RR}) \end{bmatrix} = \\
&= \begin{bmatrix} M_r(B_{rr}) + B_r A_{rR} M_r(B_{RR}) \\ B_R A_{Rr} M_r(B_{rr}) + M_r(B_{RR}) \end{bmatrix} = \\
&= \begin{bmatrix} I \\ B_R A_{Rr} \end{bmatrix} M_r(B_{rr}) + \begin{bmatrix} B_r A_{rR} \\ I \end{bmatrix} M_r(B_{RR})
\end{aligned}
\tag{37}
$$

Therefore, the expressions (22) and (23) yield the following form of the multiplier product matrix for the block matrix A of the multiregional input-output system and its Leontief inverse:

$$
\begin{aligned}
M(B) &= \frac{1}{V(B)} M_r(B) M_c(B) = \\
&= \frac{1}{V(B)} \left\{ \begin{bmatrix} I \\ B_R A_{Rr} \end{bmatrix} M_r(B_{rr}) + \begin{bmatrix} B_r A_{rR} \\ I \end{bmatrix} M_r(B_{RR}) \right\} \left\{ M_c(B_{rr}) \begin{bmatrix} I & A_{rR}B_R \end{bmatrix} + M_c(B_{RR}) \begin{bmatrix} A_{Rr}B_r & I \end{bmatrix} \right\} = \\
&= \frac{1}{V(B)} \begin{bmatrix} I \\ B_R A_{Rr} \end{bmatrix} M_r(B_{rr}) M_c(B_{rr}) \begin{bmatrix} I & A_{rR}B_R \end{bmatrix} + \frac{1}{V(B)} \begin{bmatrix} I \\ B_R A_{Rr} \end{bmatrix} M_r(B_{rr}) M_c(B_{rr}) \begin{bmatrix} A_{Rr}B_r & I \end{bmatrix} + \\
&+ \frac{1}{V(B)} \begin{bmatrix} B_r A_{rR} \\ I \end{bmatrix} M_r(B_{RR}) M_c(B_{rr}) \begin{bmatrix} I & A_{rR}B_R \end{bmatrix} + \frac{1}{V(B)} \begin{bmatrix} B_r A_{rR} \\ I \end{bmatrix} M_r(B_{RR}) M_c(B_{rr}) \begin{bmatrix} A_{Rr}B_r & I \end{bmatrix}
\end{aligned}
\tag{38}
$$

It is important to underline that the application of equations (22) and (23) to the extended Leontief inverses, B_{rr}, B_{RR}, will provide the following extended intraregional multiplier product matrices for the region *r* and the rest of economy:

$$
\begin{aligned}
M_{rr} &= M(B_{rr}) = \frac{1}{V(B_{rr})} M_r(B_{rr}) M_c(B_{rr}) \\
M_{RR} &= M(B_{RR}) = \frac{1}{V(B_{RR})} M_r(B_{RR}) M_c(B_{RR})
\end{aligned}
\tag{39}
$$

By analogy it is possible to define the interregional extended multiplier product matrices:

$$M_{rR} = \frac{1}{V(B_{rr})} M_r(B_{rr}) M_c(B_{RR})$$
$$M_{Rr} = \frac{1}{V(B_{RR})} M_r(B_{RR}) M_c(B_{rr}) \tag{40}$$

By analogy it is possible to define the interregional extended multiplier product matrices:

$$M_{rR} = \frac{1}{V(B_{rr})} M_r(B_{rr}) M_c(B_{RR})$$
$$M_{Rr} = \frac{1}{V(B_{RR})} M_r(B_{RR}) M_c(B_{rr}) \tag{41}$$

Therefore, the multiplier product matrix $M(B)$ for the block matrix A of the multiregional input-output system reveals the following structure:

$$\begin{aligned} M(B) = {} & \frac{V(B_{rr})}{V(B)} \begin{bmatrix} I \\ B_R A_{Rr} \end{bmatrix} M_{rr} \begin{bmatrix} I & A_{rR} B_R \end{bmatrix} + \frac{V(B_{rr})}{V(B)} \begin{bmatrix} I \\ B_R A_{Rr} \end{bmatrix} M_{rR} \begin{bmatrix} A_{Rr} B_r & I \end{bmatrix} + \\ & + \frac{V(B_{RR})}{V(B)}) \begin{bmatrix} B_r A_{rR} \\ I \end{bmatrix} M_{Rr} \begin{bmatrix} I & A_{rR} B_R \end{bmatrix} + \frac{V(B_{RR})}{V(B)} \begin{bmatrix} B_r A_{rR} \\ I \end{bmatrix} M_{RR} \begin{bmatrix} A_{Rr} B_r & I \end{bmatrix} \end{aligned} \tag{42}$$

Denote the four components of the decomposition (42) as: $M(B)[rr]; M(B)[rR]; M(B)[Rr]; M(B)[RR]$. Then:

$$M(B) = M(B)[rr] + M(B)[rR] + M(B)[Rr] + M(B)[RR] \tag{43}$$

where

$$M(B)[rr] = \frac{V(B_{rr})}{V(B)} \begin{bmatrix} I \\ B_R A_{Rr} \end{bmatrix} M_{rr} \begin{bmatrix} I & A_{rR} B_R \end{bmatrix} = \frac{V(B_{rr})}{V(B)} \begin{bmatrix} M_{rr} & M_{rr} A_{rR} B_R \\ B_R A_{Rr} M_{rr} & B_R A_{Rr} M_{rr} A_{rR} B_R \end{bmatrix} \tag{44}$$

$$M(B)[rR] = \frac{V(B_{rr})}{V(B)} \begin{bmatrix} I \\ B_R A_{Rr} \end{bmatrix} M_{rR} \begin{bmatrix} A_{Rr} B_r & I \end{bmatrix} = \frac{V(B_{rr})}{V(B)} \begin{bmatrix} M_{rR} A_{Rr} B_r & M_{rR} \\ B_R A_{Rr} M_{rR} A_{Rr} B_r & B_R A_{Rr} M_{rR} \end{bmatrix} \tag{45}$$

$$M(B)[Rr] = \frac{V(B_{RR})}{V(B)} \begin{bmatrix} B_r A_{rR} \\ I \end{bmatrix} M_{Rr} \begin{bmatrix} I & A_{rR} B_R \end{bmatrix} = \frac{V(B_{RR})}{V(B)} \begin{bmatrix} B_r A_{rR} M_{Rr} & B_r A_{rR} M_{Rr} A_{rR} B_R \\ M_{Rr} & M_{Rr} A_{rR} B_R \end{bmatrix} \tag{46}$$

$$M(B)[RR] = \frac{V(B_{RR})}{V(B)} \begin{bmatrix} B_r A_{rR} \\ I \end{bmatrix} M_{RR} \begin{bmatrix} A_{Rr} B_r & I \end{bmatrix} = \frac{V(B_{RR})}{V(B)} \begin{bmatrix} B_r A_{rR} M_{RR} A_{Rr} B_r & B_r A_{rR} M_{RR} \\ M_{RR} A_{Rr} B_r & M_{RR} \end{bmatrix} \tag{47}$$

Using the block structure of the components $M(B)[rr];M(B)[rR];M(B)[Rr];M(B)[RR]$, one can construct the block structure of the multiplier product matrix as:

$$M(B)=\begin{bmatrix}[M(B)]_{rr} & [M(B)]_{rR}\\ [M(B)]_{Rr} & [M(B)]_{RR}\end{bmatrix} \tag{48}$$

by summing the corresponding blocks from (44) - (47);

$$\begin{aligned}
[M(B)]_{rr} &= \frac{V(B_{rr})}{V(B)}M_{rr}+\frac{V(B_{rr})}{V(B)}M_{rR}A_{Rr}B_r+\frac{V(B_{RR})}{V(B)}B_rA_{rR}M_{Rr}+\frac{V(B_{RR})}{V(B)}B_rA_{rR}M_{RR}A_{Rr}B_r;\\
[M(B)]_{rR} &= \frac{V(B_{rr})}{V(B)}M_{rR}+\frac{V(B_{rr})}{V(B)}M_{rr}A_{rR}B_R+\frac{V(B_{RR})}{V(B)}B_rA_{rR}M_{RR}+\frac{V(B_{RR})}{V(B)}B_rA_{rR}M_{Rr}A_{rR}B_R;\\
[M(B)]_{Rr} &= \frac{V(B_{RR})}{V(B)}M_{Rr}+\frac{V(B_{RR})}{V(B)}M_{RR}A_{Rr}B_r+\frac{V(B_{rr})}{V(B)}B_RA_{Rr}M_{rr}+\frac{V(B_{rr})}{V(B)}B_RA_{Rr}M_{rR}A_{Rr}B_r;\\
[M(B)]_{RR} &= \frac{V(B_{RR})}{V(B)}M_{RR}+\frac{V(B_{RR})}{V(B)}M_{Rr}A_{rR}B_R+\frac{V(B_{rr})}{V(B)}B_RA_{Rr}M_{rR}+\frac{V(B_{rr})}{V(B)}B_RA_{Rr}M_{rr}A_{rR}B_R
\end{aligned} \tag{49}$$

Here, a modification of an earlier approach to the region versus the rest of the economy is provided that extends the interpretation to a broader context (see Sonis *et al*., 1996). If attention was directed only to the regional part, *M(B)[rr]*, of the economic landscape, *M(B)*, then (49) may be shown as:

$$[M(B)]_{rr}=\frac{V(B_{rr})}{V(B)}M_{rr}+\frac{V(B_{rr})}{V(B)}M_{rR}A_{Rr}B_r+\frac{V(B_{RR})}{V(B)}B_rA_{rR}M_{Rr}+\frac{V(B_{RR})}{V(B)}B_rA_{rR}M_{RR}A_{Rr}B_r$$

This part of (49) describes the spread of changes within the region *r* caused by (1) the changes in direct inputs within the region, $\frac{V(B_{rr})}{V(B)}M_{rr}$; (2) changes in regional forward linkages, $\frac{V(B_{rr})}{V(B)}M_{rR}A_{Rr}B_r$; (3) changes of the regional backward linkages, $\frac{V(B_{RR})}{V(B)}B_rA_{rR}M_{Rr}$ and, finally, (4) changes in the direct inputs within the isolated rest of economy, $\frac{V(B_{RR})}{V(B)}B_rA_{rR}M_{RR}A_{Rr}B_r$. This decomposition provides a summary of the changes differentiated into internal, forward, backward and external linkages.

2.6. Economic Landscapes – Version 2.0

Define a standard Leontief system:

$$X = AX + Y \tag{50}$$

where X is a vector (n x 1) of total production by sector; Y (n x 1) is the final demand; and A is a (n x n) matrix of technical coefficients.

The usual solution is:

$$X = BY \tag{51}$$

and

$$B = (I - A)^{-1} \tag{52}$$

where B (n x n) is the Leontief inverse matrix.

To construct an economic landscape, in production values, to show how the economic system will work directly and indirectly to produce one unit of the j^{th} sector for the final demand, the A matrix of direct coefficients is post multiplied by the diagonal of the j^th column of matrix B $\left[\hat{B}_{\bullet j}\right]$, i.e.:

$$L_j = A\left[\hat{B}_{\bullet j}\right] \tag{53}$$

where L_j is the economic landscape for sector j.

To estimate the economic landscape for a given sector in the economy, in terms of value added, employment, imports, etc. this can be done by first estimating the coefficient of the value added, for example, as:

$$w_k = \frac{VA_k}{X_k} \tag{54}$$

where w_k is the coefficient of the value added for sector k, VA_k is the value added of sector k, and X_k is the production level of sector k.

After which, it is obtained the value added generated, directly and indirectly, in each sector by the sale of one unit of sector j to the final demand (v_{kj}), i.e.,

$$v_{kj} = w_k b_{kj} \tag{55}$$

where b_{kj} is an element of the matrix B defined above.

To obtain the value added matrix of sector *j* that will be used to construct the value added economic landscape for this sector (L_j^w), one first get the share of each direct coefficient in a given row (e_{ij}), then the resulting matrix is post multiplied by the diagonal matrix of vector *v*. In that way:

$$e_{ij} = \frac{a_{ij}}{\sum_{i=1}^{n} a_{ij}} \tag{56}$$

$$L_j^w = E\hat{V}_j \tag{57}$$

2.7. Synergetic Interactions Among Regions

This methodological section will be divided into two parts: a) in the first one it is made reference to the theory originally developed for the two regions case; and b) in the second it is showed how this theory can be extended to the *n* regions case.

2.7.1. The Two Regions Case

A complete description for the 2 regions case is presented in Sonis, Hewings, and Miyazawa (1997b), which is the basis for this section.

Consider an input-output system represented by the following block matrix, A, of direct inputs:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{58}$$

where A_{11} and A_{22} are the quadrat matrices of direct inputs within the first and second regions, respectively, and A_{12} and A_{21} are the rectangular matrices showing the direct inputs purchased by the second region and vice versa.

The building blocks of the pair-wise hierarchies of sub-systems of intra/interregional linkages of the block-matrix Input-Output system are the four matrices A_{11}, A_{12}, A_{21} and A_{22}, corresponding to four basic block-matrices:

$$A_{11} = \begin{bmatrix} A_{11} & 0 \\ 0 & 0 \end{bmatrix}; \quad A_{12} = \begin{bmatrix} 0 & A_{12} \\ 0 & 0 \end{bmatrix}; \quad A_{21} = \begin{bmatrix} 0 & 0 \\ A_{21} & 0 \end{bmatrix}; \quad A_{22} = \begin{bmatrix} 0 & 0 \\ 0 & A_{22} \end{bmatrix} \qquad (59)$$

This section will usually consider the decomposition of the block-matrix (58) into the sum of two block-matrices, such that each of them is the sum of the block-matrices (59) A_{11}, A_{12}, A_{21} and A_{22}. From (1) 14 types of pair-wise hierarchies of economic sub-systems can be identified by the decompositions of the matrix of the block-matrix A (see Figure 1 and Table 1).

Consider the hierarchy of Input-Output sub-systems represented by the decomposition $A = A_1 + A_2$. Introducing the Leontief block-inverse $L(A) = L = (I - A)^{-1}$ and the Leontief block-inverse $L(A_1) = L_1 = (I - A_1)^{-1}$ corresponding to the first sub-system, the outer left and right block-matrix multipliers M_L and M_R are defined by equalities:

$$L = L_1 M_R = M_L L_1 \qquad (60)$$

The definition (3) implies that:

$$M_L = L(I - A_1) = (I - L_1 A_2)^{-1} \qquad (61)$$

$$M_R = L(I - A_1)L = (I - A_2 L_1)^{-1} \qquad (62)$$

The calculation of the outer block-multiplier M_L and M_R is based on the particular form of the Leontief block-inverse $L(A) = L$. This work will presented the application of formulas (60), (61) and (62) to the derivation of a taxonomy of synergetic interactions between regions. The possibilities for the A_1 matrix are presented in Table 1. Also, Figure 1 shows the schematic representation of the possible forms of the A_1 matrices.

Based on hierarchy of input-output sub-systems represented by the decomposition $A = A_1 + A_2$, their Leontief block-inverse $L(A) = L = (I - A)^{-1}$ and the Leontief block-inverse $L(A_1) = L_1 = (I - A_1)^{-1}$ corresponding to the first sub-system, the multiplicative decomposition of the Leontief inverse $L = L_1 M_R = M_L L_1$ can be converted to the sum:

$$L = L_1 + (M_L - I)L_1 = L_1 + L_1(M_R - I) \qquad (63)$$

If f is the vector of final demand and x is the vector of gross output, then from the decomposition (63) is possible to divide the gross output into two parts: $x_1 = L_1 f$ and the increment $Dx = x - x_1$. Such decomposition is important for the empirical analysis of the structure of actual gross output and for the contribution that the relations among the regions have to the total gross output.

While 14 types of pair-wise hierarchies of economic linkages have been developed (Figure 1 and Table 1), it is possible to suggest a typology of categories into which these types may be placed. The following characterization is suggested:

1. backward linkage type (VI, IX): power of dispersion
2. forward linkage type (V, X): sensitivity of dispersion
3. intra- and inter- linkages type (VII, VIII): internal and external dispersion
4. isolated region versus the rest of the economy interactions style (I, XIV, IV, XI)
5. triangular sub-system versus the interregional interactions style (II, XIII, III, XII).

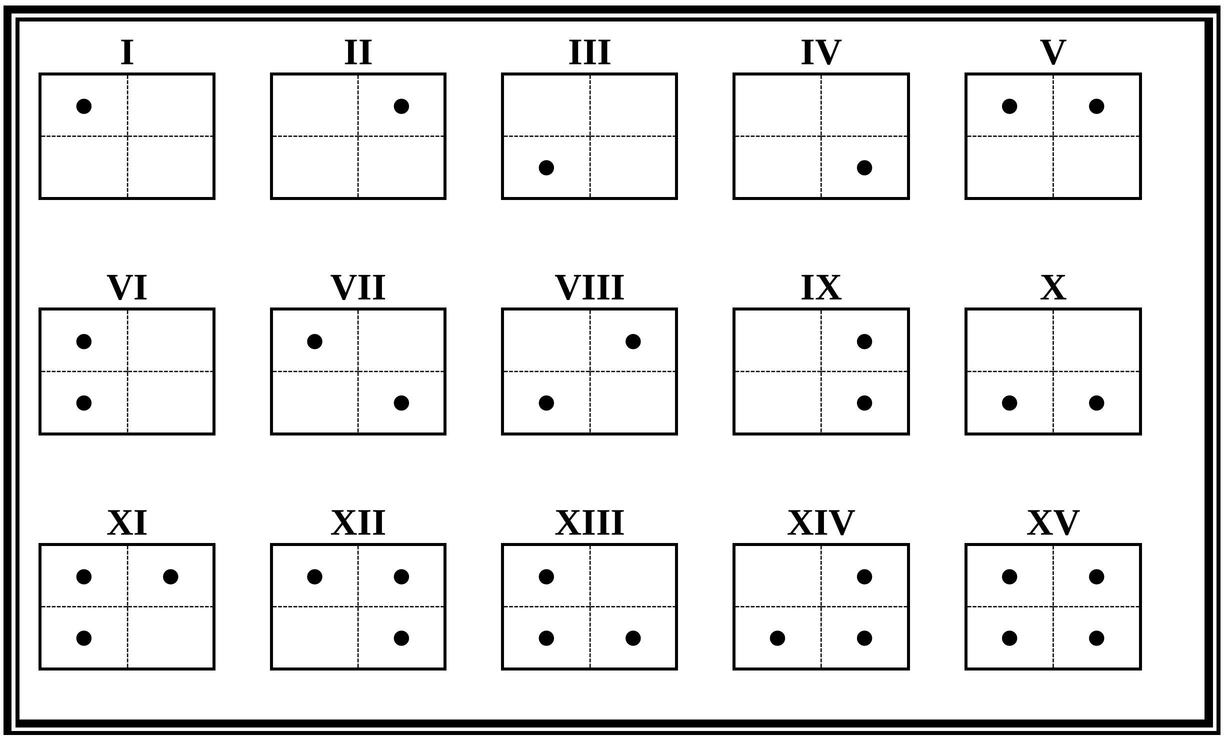

Figure 1. Schematic Representation of the Possible Forms of the A_1 Matrix – 2 Regions Case.

Table 1. Taxonomy of Synergetic Interactions between Economic Sub-Systems.

[Each entry presents a description of the structure and the corresponding form of the A_1 matrix]

I. *Hierarchy of isolated region versus the rest of economy*	$A_1 = \begin{bmatrix} A_{11} & 0 \\ 0 & 0 \end{bmatrix}$
II. *The order replaced hierarchy of interregional linkages of second region versus lower triangular sub system*	$A_1 = \begin{bmatrix} 0 & A_{12} \\ 0 & 0 \end{bmatrix}$
III. *The order replaced hierarchy of interregional linkages of first region versus upper triangular sub system*	$A_1 = \begin{bmatrix} 0 & 0 \\ A_{21} & 0 \end{bmatrix}$
IV. *The order replaced hierarchy of backward and forward linkages of the first region versus rest of economy*	$A_1 = \begin{bmatrix} 0 & 0 \\ 0 & A_{22} \end{bmatrix}$
V. *Hierarchy of forward linkages of first and second regions*	$A_1 = \begin{bmatrix} A_{11} & A_{12} \\ 0 & 0 \end{bmatrix}$
VI. *Hierarchy of backward linkages of first and second regions*	$A_1 = \begin{bmatrix} A_{11} & 0 \\ A_{21} & 0 \end{bmatrix}$
VII. *The hierarchy of intra-versus inter-regional relationships*	$A_1 = \begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix}$
VIII. *The hierarchy of inter versus intra regional relationships*	$A_1 = \begin{bmatrix} 0 & A_{12} \\ A_{21} & 0 \end{bmatrix}$
IX. *Order replaced hierarchy of backward linkages*	$A_1 = \begin{bmatrix} 0 & A_{12} \\ 0 & A_{22} \end{bmatrix}$
X. *Order replaced hierarchy of forward linkages*	$A_1 = \begin{bmatrix} 0 & 0 \\ A_{21} & A_{22} \end{bmatrix}$
XI. *The hierarchy of backward and forward linkages of the first region versus rest of economy*	$A_1 = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & 0 \end{bmatrix}$
XII. *The hierarchy of upper triangular sub system versus interregional linkages of first region*	$A_1 = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}$
XIII. *The hierarchy of lower triangular sub system versus interregional linkages of second region*	$A_1 = \begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix}$
XIV. *Hierarchy of the rest of economy versus second isolated region*	$A_1 = \begin{bmatrix} 0 & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

By viewing the system of hierarchies of linkages in this fashion, it will be possible to provide new insights into the properties of the structures that are revealed. For example, the types allocated to category 5 reflect structures that are based on order and circulation. Furthermore, these partitioned input-output systems can distinguish among the various types of dispersion (such as 1, 2 and 3) and among the various patterns of interregional interactions (such as 4 and 5). Essentially, the 5 categories and 14 types of pair-wise hierarchies of economic linkages provide the opportunity to select according to the special qualities of each region's activities and for the type of problem at hand; in essence, the option exists for the basis of a typology of economy types based on hierarchical structure. The use of different synergetic interactions allows one to analyze and to measure how the transactions do occur among the regions, being possible to verify how much the relation of production on a given region do affect the production in another region.

2.7.2. The n Regions Case

For the *n* regions case the number of decompositions increases dramatically as one increases the number of regions, such that from the 15 decompositions (including the whole system) for the 2 regions case, one goes to: a) 511 decompositions for the three regions case; b) 65,535 decompositions for the 4 regions; c) 33,554,431 decompositions for the 5 regions; and so on. In this way, the equation representation of the system for the *n* regions case becomes very complex, so what is presented here is a general idea of how the system works, as can be seen in a schematic way for the 5 regions case, as it is presented in Figure 2. From this figure one can see that in the 5 regions case one has 25 matrices. At first, one has to consider each matrix isolated, the next step is to consider the 25 matrices combined 2 at time, then 3 at time, and so forth, until one gets to the whole system. To measure the net contribution of each combination for the production in the productive process one has to subtract from the result of the combination of *k* matrices all the possible lower level combinations of these matrices, e.g., the result of a set of 5 matrices must be subtracted from the results of all the possible combination of these five matrices at the level of 4, 3, 2, and 1 matrices.

Some works have already being developed for Brazil using the methodology proposed by por Sonis, Hewings, and Miyazawa (1997b). For the two regions case one has the work of Guilhoto, Hewings and Sonis (1999), while Moretto (2000) and Silveira (2000) explore the methodology for the 4 regions case. The two regions used in Guilhoto, Hewings and Sonis (1999) are the Northeast and the Rest of Brazil regions. Moretto (2000) works with a four regions interregional input-output output system construct for the state of Paraná. The work of Silveira (2000) uses an interregional system that includes the Brazilian states of Minas Gerais, Bahia, Pernambuco, and the Rest of Brazil economy.

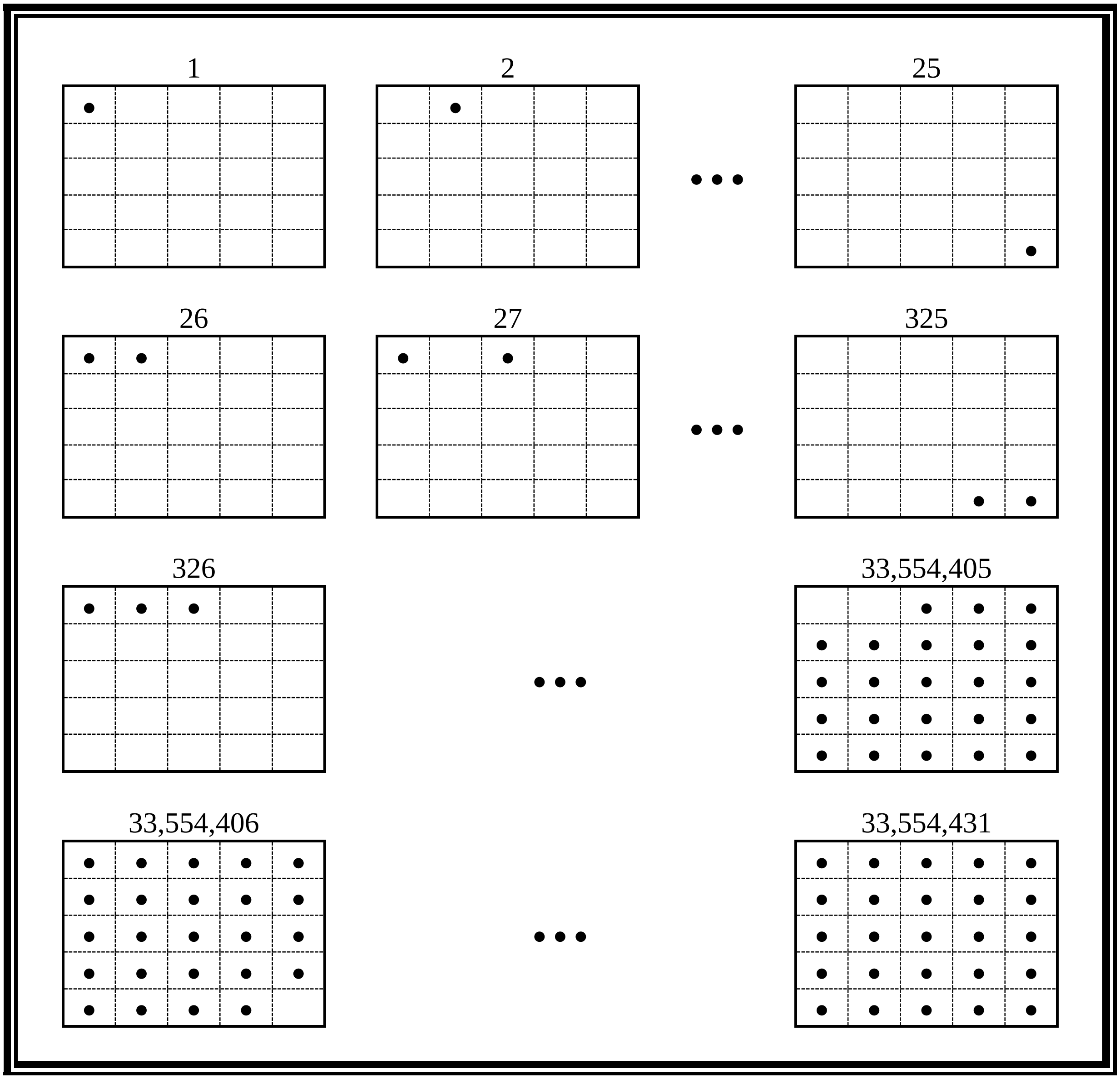

Figure 2. Schematic Representation of the Possible Forms of the A_1 Matrix – 5 Regions Case.

2.8. Decomposition, Source, and Evolution of the Output Change[5]

The analysis draws on some recent work by Feldman, McClain, and Palmer (FMP) (1987) and Sonis, Hewings and Guo (SHG) (1995). FMP examined the degree to which changes in final demand and changes in the input coefficients contributed to changes in output in the United States economy over the period 1963 to 1978. SHG proposed an alternative decomposition approach which addressed explicitly the contributions of changes in terms of their system-wide impact. This decomposition separates the pure effects of changes in technology and in final demand from those caused by the synergetic interaction between these two components. Further, each component of the change in gross output in each sector can be divided into two parts, the s*elf*- and *non-self-generated changes*; in the former case, the change in output can be traced to changes in the sector itself (i.e., a final demand, technological, or synergetic change) while in the latter case, the change occurs in another sectors.

2.8.1. Analysis of the FMP Approach

In their paper, FMP proposed the following decomposition for the analysis of the influence of changes in the input coefficients and in the components of final demand on output levels. Let X_0 and X_t be the gross output vectors for the two time periods $\boldsymbol{0}$ and $\boldsymbol{t}$; similarly, let B_0 and B_t be the Leontief inverses and f_0 and f_t the vectors of final demand. Define:

$$\begin{aligned} \Delta X &= X_t - X_0 \\ \Delta B &= B_t - B_0 \\ \Delta f &= f_t - f_0 \end{aligned} \tag{64}$$

Assume, further, that the matrix, $\boldsymbol{A}$, of direct input coefficients is ($\boldsymbol{nxn}$) and that the vectors are of dimension ($\boldsymbol{nx1}$). Consider the following representation of change in gross output:

$$\Delta X = X_t - X_0 = B_t f_t - B_0 f_0 \tag{65}$$

From equation (65) it is possible to arrive at:

$$\Delta X = 1/2\left[B_t\left(f_t - f_0\right) + B_0\left(f_t - f_0\right)\right] + 1/2\left[\left(B_t - B_0\right)f_0 + \left(B_t - B_0\right)f_t\right] \tag{66}$$

[5] This section draws on Sonis, Hewings and Guo (1995)

where the first term on the right hand side o equation (66) represents the contribution of changes in final demand to output changes and the second term accounts for the contribution of changes in input coefficients to changes in output.

The first and second right hand terms of equation (66) can be, respectively, presented as:

$$B_0\Delta f + 1/2\,\Delta B\Delta f \text{ , and}$$

$$\Delta B f_0 + 1/2\,\Delta B\Delta f \qquad (67)$$

Therefore, it is evident that the proposed decomposition of the changes into components (67) cannot entirely separate the effects of coefficient change from those in final demand. The presence of the term, $\Delta B\Delta f$, creates a problem of how to assign the synergetic effects of coefficient change and final demand change especially if this component turns out to account for a large percentage of the change in output. FMP noted, in their paper (Footnote. 7 p. 505) that their method ascribed half of this interaction term to each component. What is needed is a more flexible approach, and this is presented in the next section.

2.8.2. Triple Decomposition of the Output Change

Instead of using a decomposition o output change in only two components, the FMP approach, one can use a Paache-type decomposition of the change described in equation (65), such that a triple decomposition is obtained, i.e.:

$$\begin{aligned}\Delta X &= (B_0 + \Delta B)(f_0 + \Delta f) - B_0 f_0 = \\ &= B_0\Delta f + \Delta B f_0 + \Delta B\Delta f\end{aligned} \qquad (68)$$

In this way the change in output is divided into changes in final demand, technology, and the synergetic interaction between final demand and technology.[6]

For sector *i*, equation (68) can be represented in the following way:

$$\Delta X_i = \Delta X_i^f + \Delta X_i^B + \Delta X_i^{Bf} \qquad (69)$$

where the superscripts refer to changes associated with final demand (***f***), technology (***B***) and their synergetic interaction (***Bf***).

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[6]Previous studies of the sources of structural change in interpreting sectoral output or price variations can be found in Chenery and Watanabe (1958), Syrquin (1976), Bezdek and Wendling (1976), Chenery and Syrquin (1979), Kubo and Robinson (1984), Fossell (1989), and Skolka (1989).

The first component, ΔX_i^f , identifies the impact on sectoral output if only the structure of final demand would change, keeping the technology constant. One would expect that through time, given that the level of final demand has a tendency to increase, positive results would be obtained for this component.

The second component, ΔX_i^B, will give the impact of change in technology, given the same level of final demand, on the sectoral output. Positive values for this component represents that a greater level of total production is needed to supply the same level of final demand, while negative values reflect a lower level of total production. A negative sign for this component can mean a combination of the following: the firms are getting more efficient in the productive process, using less material inputs; the share of value added is increasing; firms are reducing the use of national inputs and increasing the use of imported ones. A positive sign can be an indication that: there is an increase in complexity of the economy, i.e., to produce a given good the industries now need to buy inputs from more sources than before, increasing in this way the multiplier effect of this sector over the economy; or that the share of the value added is decreasing; or that firms are increasing the use of domestic sources of inputs; or, it could also imply that firms are becoming less efficient in the productive process.

The third component, ΔX_i^{Bf} , is the result of the synergetic interaction between the changes in the final demand and changes in the technology, i.e., given the changes in final demand and technology, how much the total production has to change to satisfy both changes. The sign of this component can be either positive or negative.

Instead of working with changes in the components shown in (69), an alternative proposal is presented whereby the analysis is conducted with growth rates. In this way, it is easier to make comparisons, and to identify how the sectors are growing in the economy. Hence, (69) can be simply transformed by dividing through by X_{0i} and multiplying by 100, as follows:

$$\frac{\Delta X_i}{X_{0i}}.100 = \frac{\Delta X_i^f}{X_{0i}}.100 + \frac{\Delta X_i^B}{X_{0i}}.100 + \frac{\Delta X_i^{Bf}}{X_{0i}}.100 \tag{70}$$

Or, using lower letters to represent growth rates, equation (70) can be represented as:

$$\boxed{x_i = x_i^f + x_i^B + x_i^{Bf}} \tag{71}$$

So, for example if the level of production in sector *i* increases in 10% (x_i) this can be accomplished by an increase of 17% on the component of final demand (x_i^f), compensated, in part, by a decrease of 5% in the technological factor (x_i^B), i.e., a more efficient way of producing goods, and a 2% decrease due to the synergetic interaction between the variation in the final demand and in the technology (x_i^{Bf}).

2.8.3. Changes Generated Inside and Outside the Sector

In addition to the decomposition into three components presented in the previous section, each one of the changes in these components can be traced to its source, i.e., if its originated in the sector itself or in other sectors of the economy. These further decompositions are referred to, respectively, as the s*elf*- and *non-self-generated changes*. Empirical evidence suggests that the allocation between these two components can be rather varied across sectors.

The definition of the parts, where *s* refers to self-generated and *ns* as non-self-generated, is as follows:

$$\begin{aligned} &s\Delta X_i^f = b_{ii}\Delta f_i \ ; \quad ns\Delta X_i^f = \Delta X_i^f - s\Delta X_i^f \\ &s\Delta X_i^B = \Delta b_{ii} f_i \ ; \quad ns\Delta X_i^B = \Delta X_i^B - s\Delta X_i^B \\ &s\Delta X_i^{Bf} = \Delta b_{ii}\Delta f_i \ ; \quad ns\Delta X_i^{Bf} = \Delta X_i^{Bf} - s\Delta X_i^{Bf} \end{aligned} \tag{72}$$

The *self-generated* changes are obtained by using b_{ii}, f_i, and their changes through time, i.e., what we are trying to measure here are the changes in total production of sector *i* that are linked with the final demand of only sector *i*. By the *non-self-generated* changes we mean changes in the total production of sector *i* that are linked with the final demand of the other sectors in the economy, and they are obtained by subtracting the *self-generated* changes from the total changes.[7]

Further, consider, respectively, the global self and non-self output change as:

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[7] Despite the fact that we are using b_{ii} to measure the *self-generated* changes, and the value of b_{ii} is related to all the others direct technical coefficients, a_{ij}, our real interest here is in measure the direct and indirect production of sector *i* needed to fulfill the final demand needs of sector *i* alone.

$$s\Delta X_i = s\Delta X_i^f + s\Delta X_i^B + s\Delta X_i^{Bf} \tag{73}$$

and

$$ns\Delta X_i = ns\Delta X_i^f + ns\Delta X_i^B + ns\Delta X_i^{Bf} \tag{74}$$

Dividing equations (73) and (74) by X_{0i} and multiplying each one by 100 give us the same procedure as the one did in equations (70) and (71) where growth rates were obtained for the change in output, and its components. In that way one has from equations (73) and (74) that:

$$\boxed{x_i^s = x_i^{sf} + x_i^{sB} + x_i^{sBf}} \tag{75}$$

$$\boxed{x_i^{ns} = x_i^{nsf} + x_i^{nsB} + x_i^{nsBf}} \tag{76}$$

Where the variables in equations (71), (75), and (76) can be related in the following way:

$$\boxed{x_i = x_i^s + x_i^{ns}} \tag{77}$$

$$\boxed{\left\{\begin{array}{l} x_i = x_i^{sf} + x_i^{nsf} \\ x_i = x_i^{sB} + x_i^{nsB} \\ x_i = x_i^{sBf} + x_i^{nsBf} \end{array}\right.} \tag{78}$$

Through an analysis of the components x_i^s and x_i^{ns}, it is possible to determine whether the main source of growth in sector *i* is due either to s*elf*- or to *non-self-generated changes*. In addition, by using the same kind of analysis that was accomplished with equation (71), analysis of equations (75) and (76) can reveal what the major sources were for the s*elf*- and *non-self-generated changes*, i.e., final demand, technology, or synergetic interaction between final demand and technology.

2.8.4. Evolution of Changes

With more than two time periods it is possible to see how the importance of the three components (final demand, technology, and synergetic interaction) have evolved in the determination of output change. This is accomplished by considering the importance of the component in the total impact on the output change, where the total impact is defined as follows:

$$\Delta T_i = abs\left(\Delta X_i^f\right) + abs\left(\Delta X_i^B\right) + abs\left(\Delta X_i^{Bf}\right) \tag{79}$$

where ΔT_i is the total impact in sector *i*, and $abs\left(\Delta X_i^f\right)$, $abs\left(\Delta X_i^B\right)$, and $abs\left(\Delta X_i^{Bf}\right)$ are the absolute values of the final demand, technology, and synergetic components.

Note that total impact is defined in a different way of output change, i.e., while output change takes into consideration the sign of its components, the total impact does not do so. The difference is mainly due to the fact that when output changes are measured, the focus of attention is on the *net effect*, while with total impact, the interest is in the magnitude of the components, independent of the fact that they might have a negative or a positive influence over the sectoral output change.

Dividing equation (79) by ΔT_i and multiplying it by 100, one has:

$$100 = \frac{abs\left(\Delta X_i^f\right)}{\Delta T_i}.100 + \frac{abs\left(\Delta X_i^B\right)}{\Delta T_i}.100 + \frac{abs\left(\Delta X_i^{Bf}\right)}{\Delta T_i}.100 \tag{80}$$

Or in shares:

$$100 = Z_i^f + Z_i^B + Z_i^{Bf} \tag{81}$$

where Z_i^f, Z_i^B, and Z_i^{Bf} in equation (81) represent the shares of final demand, technology and synergetic interaction on the total impact over sector *i* for a given time period. The evolution of changes through different time periods is obtained by estimating the difference of the shares of the three components from (81) for two time periods, i.e.,

$$\boxed{\left\{\begin{array}{l} \Delta Z_i^f = Z_{it}^f - Z_{i0}^f \\ \Delta Z_i^B = Z_{it}^B - Z_{i0}^B \\ \Delta Z_i^{Bf} = Z_{it}^{Bf} - Z_{i0}^{Bf} \end{array}\right.} \tag{82}$$

where positive values for anyone of the components, ΔZ_i^f, ΔZ_i^B, and ΔZ_i^{Bf}, implies an increase, over time, of the importance of final demand, technology, or synergetic interaction, in determining the output change in sector *i*. Concomitantly, negative values means a decrease in importance.

In the same way that (82) was used to measured the evolution of changes in total output of sector *i*, it can also be used to measure the evolution of changes in the self and non-self generated changes of the total output. In this way, applying the procedure presented in (79) through (82) to self and non-self generated changes, one has:

$$\boxed{\left\{\begin{array}{l} \Delta Z_i^{sf} = Z_{it}^{sf} - Z_{i0}^{sf} \\ \Delta Z_i^{sB} = Z_{it}^{sB} - Z_{i0}^{sB} \\ \Delta Z_i^{sBf} = Z_{it}^{sBf} - Z_{i0}^{sBf} \end{array}\right.} \tag{83}$$

$$\boxed{\left\{\begin{array}{l} \Delta Z_i^{nsf} = Z_{it}^{nsf} - Z_{i0}^{nsf} \\ \Delta Z_i^{nsB} = Z_{it}^{nsB} - Z_{i0}^{nsB} \\ \Delta Z_i^{nsBf} = Z_{it}^{nsBf} - Z_{i0}^{nsBf} \end{array}\right.} \tag{84}$$

where the interpretation of equations (83) and (84) is identical to the one made for equation (81), except for the fact that the *s* refers to self and *ns* refers to non-self generated changes.

3. Some Applications for the Brazilian Economy

Below it is presented some of the applications of the above theories that were made to the Brazilian economy.

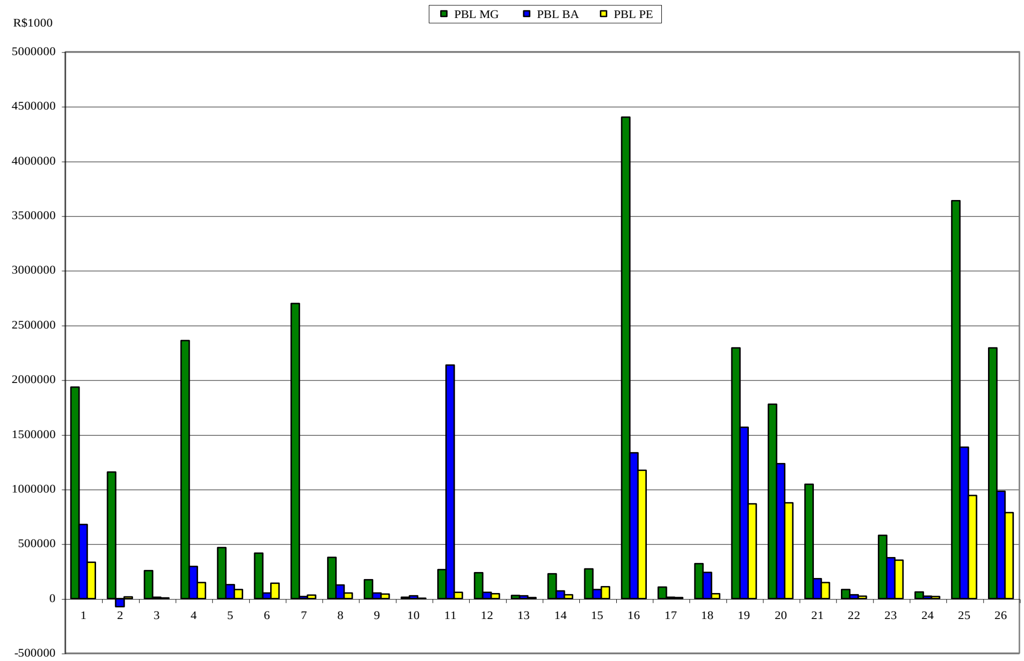
The diversity of the data and the authors clearly shows the multiplier effect that the Sonis and Hewings theories are having in the Brazilian Economy.

3.1. The Pure Linkage Approach

Table 2 - Pure Linkages of the São Francisco Interregional System, 1995.

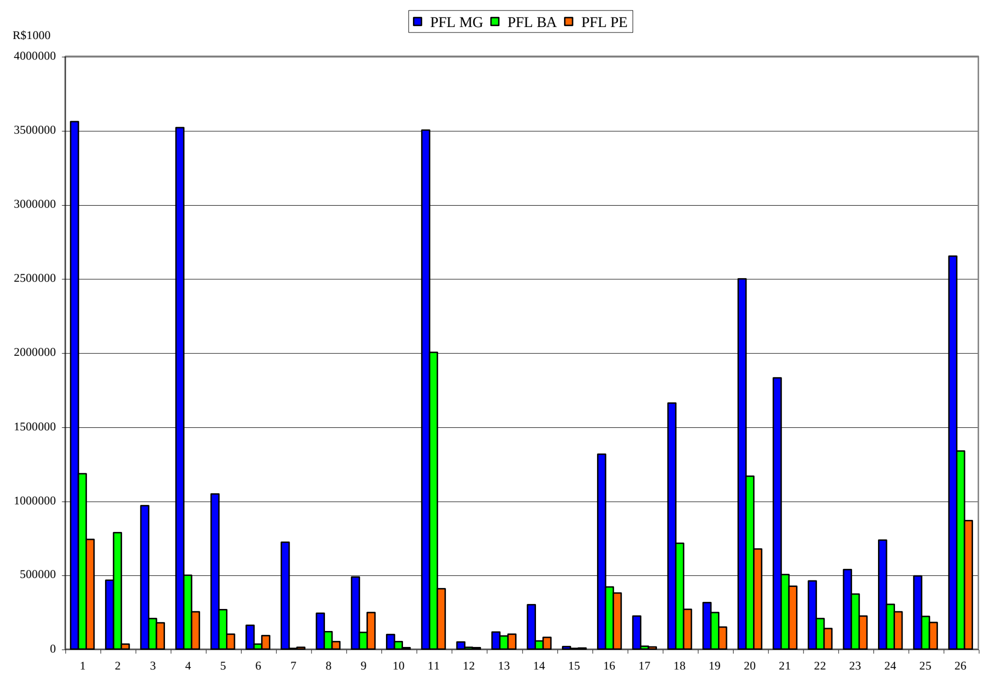
	Sector	Minas Gerais		Bahia		Pernambuco		Minas Gerais		Bahia		Pernambuco		Minas Gerais		Bahia		Pernambuco	
		PBL	RPBL	PBL	RPBL	PBL	RPBL	PFL	RPFL	PFL	RPFL	PFL	RPFL	PTL	RPTL	PTL	RPTL	PTL	RPTL
1	Agriculture	1933431	7	676169	21	331729	28	3559473	0	1182861	10	739602	16	5492904	2	1859030	14	1071331	26
2	Mining	1155454	14	-73775	78	14964	69	465038	26	784541	15	32875	67	1620492	18	710767	32	47839	72
3	Nonmetallic Minerals	254705	33	12265	70	5924	75	967890	13	205844	47	176171	49	1222595	23	218109	54	182095	56
4	Metal Products	2357573	3	294228	30	147051	39	3519383	1	497464	23	252182	38	**5876956**	0	791693	30	399233	42
5	Machinery	465662	23	127726	42	81621	46	1045536	12	266606	37	101691	56	1511198	22	394333	43	183313	55
6	Electrical Equipament	412981	24	49066	55	138795	41	161250	50	32880	66	90316	59	574230	36	81945	68	229111	53
7	Transport Equipament	2697698	2	19053	68	31162	61	719878	18	5509	78	13072	71	3417577	8	24562	75	44234	73
8	Wood and Wood products	374597	25	124854	43	50299	53	242420	42	118519	53	51335	63	617017	35	243373	51	101634	66
9	Paper and Printing	172813	38	49627	54	40325	58	486222	25	112627	55	245692	41	659035	34	162253	57	286017	48
10	Rubber Products	10924	72	23244	64	2841	76	98827	58	49684	64	10090	74	109751	64	72928	69	12932	78
11	Chemical Products	265084	32	2133447	6	56662	51	3502079	2	2002617	5	408309	30	3767163	7	**4136065**	5	464971	41
12	Pharmaceuticals and Cosmetics	233868	35	55411	52	43636	56	47102	65	11903	72	10406	73	280970	49	67314	70	54042	71
13	Plastics	26230	62	25641	63	9306	73	116116	54	89659	60	99688	57	142346	59	115299	61	108994	65
14	Textiles	225372	36	68392	49	35451	59	300458	35	55679	62	78289	61	525829	40	124071	60	113740	63
15	Clothing and Footwear	272042	31	81074	48	107763	44	16902	69	5835	76	6445	75	288943	47	86909	67	114209	62
16	Food and Kindred Products	4403530	0	1333803	11	1171934	13	1314318	9	418393	29	377820	31	5717848	1	1752196	16	1549753	20
17	Other Industrial Products	105746	45	11680	71	7312	74	222212	44	19806	68	14172	70	327958	44	31487	74	21485	76
18	Public Utilities	318825	29	237683	34	42508	57	1659137	7	714033	19	268463	36	1977962	13	951716	28	310971	46
19	Construction	2290149	5	1566671	9	866300	19	314232	33	245938	40	149280	51	2604381	10	1812610	15	1015579	27
20	Trade	1775268	8	1233544	12	874306	18	2499323	4	1167355	11	674477	20	4274591	4	2400899	11	1548783	21
21	Transport	1045591	15	180844	37	145411	40	1829131	6	502145	22	423293	28	2874721	9	682989	33	568705	38
22	Communication	81360	47	34377	60	22627	65	459788	27	207141	46	139264	52	541148	39	241518	52	161890	58
23	Financial Institutions	578784	22	374305	26	350460	27	536940	21	371889	32	222906	43	1115724	25	746194	31	573365	37
24	Public Administration	59280	50	22004	66	19428	67	734322	17	302284	34	251195	39	793603	29	324288	45	270623	50
25	Realty Services	3636837	1	1383112	10	943540	17	494387	24	221127	45	180553	48	4131224	6	1604238	19	1124092	24
26	Other Services	2291998	4	979073	16	785321	20	2652520	3	1337427	8	867039	14	4944518	3	2316500	12	**1652359**	17
	Average	1055608		423981		243334		1075572		420376		226332		2131180		844357.2		469665.4	

Source: Silveira (2000)



Source: Silveira (2000)

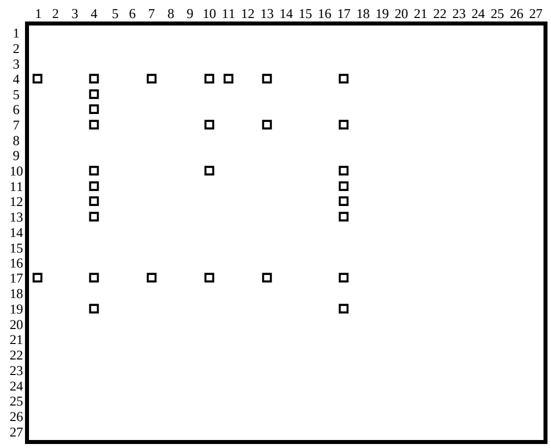
Figure 3 - Pure Backward Linkage of the São Francisco Interregional System.



Source: Silveira (2000)

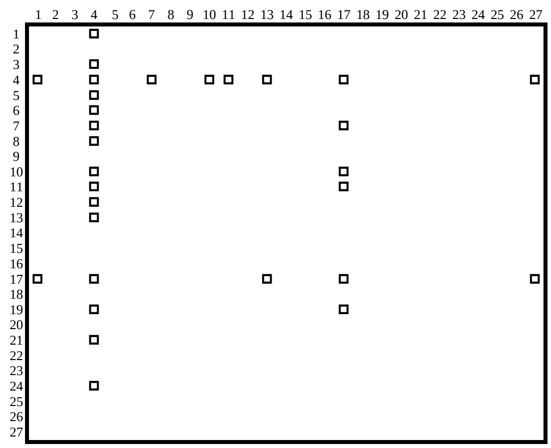
Figure 4 - Pure forward linkage of the São Francisco Interregional System.

3.2. The Fields of Influence



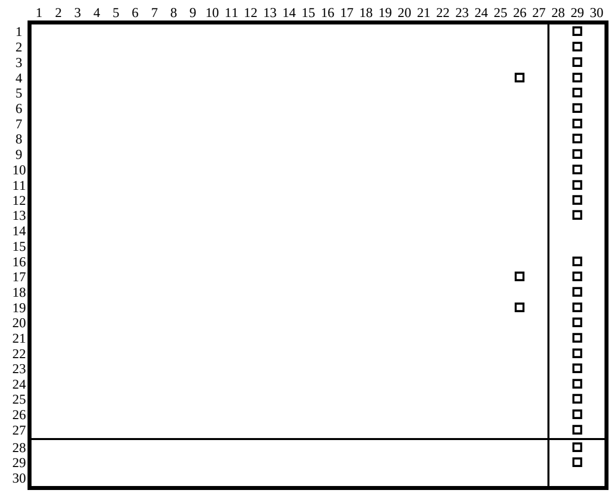
Source: Hewings et al (1989)

Figure 5 - Coefficients with the Largest Field of Influence, Brazil - 1975



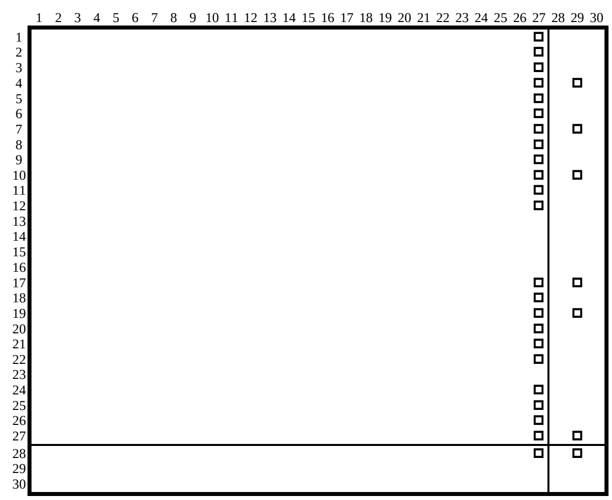
Fonte: Guilhoto (1992)

Figure 6 - Coefficients with the Largest Field of Influence, Brazil - 1980



Fonte: Hewings et al. (1989)

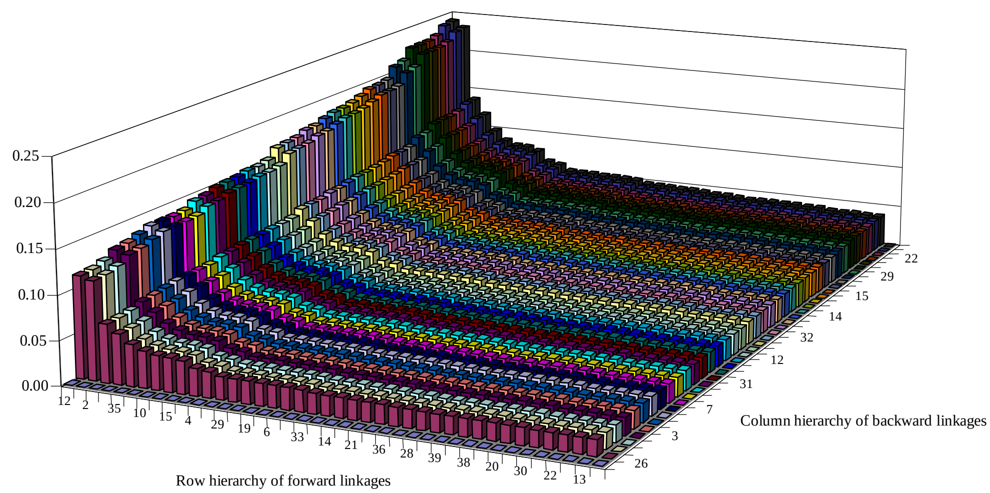
Figure 7 - Coefficients with the Largest Field of Influence, SAM Brazil - 1975



Fonte: Guilhoto et al. (1996)

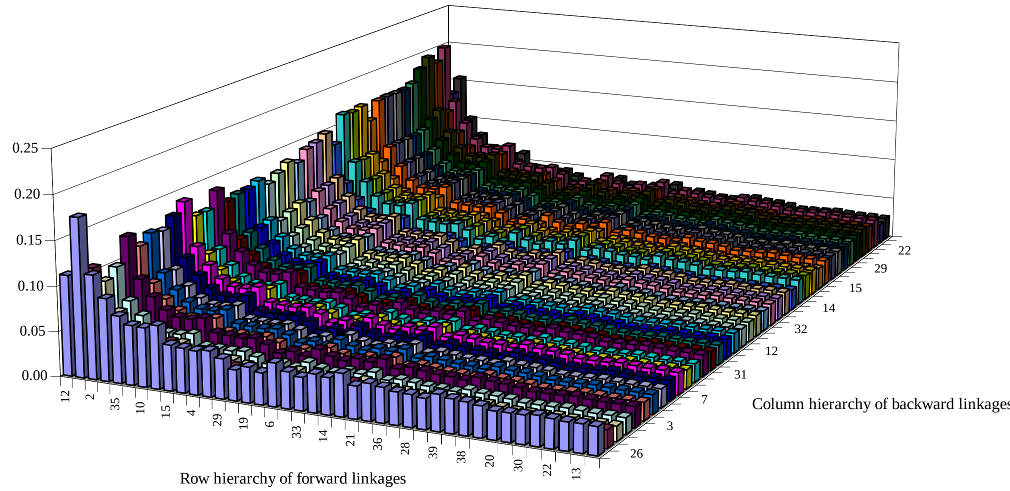
Figure 8 - Coefficients with the Largest Field of Influence, SAM Brazil – 1980

3.3. The Structure of Production: Economic Landscapes



Source: Rodrigues (2000).

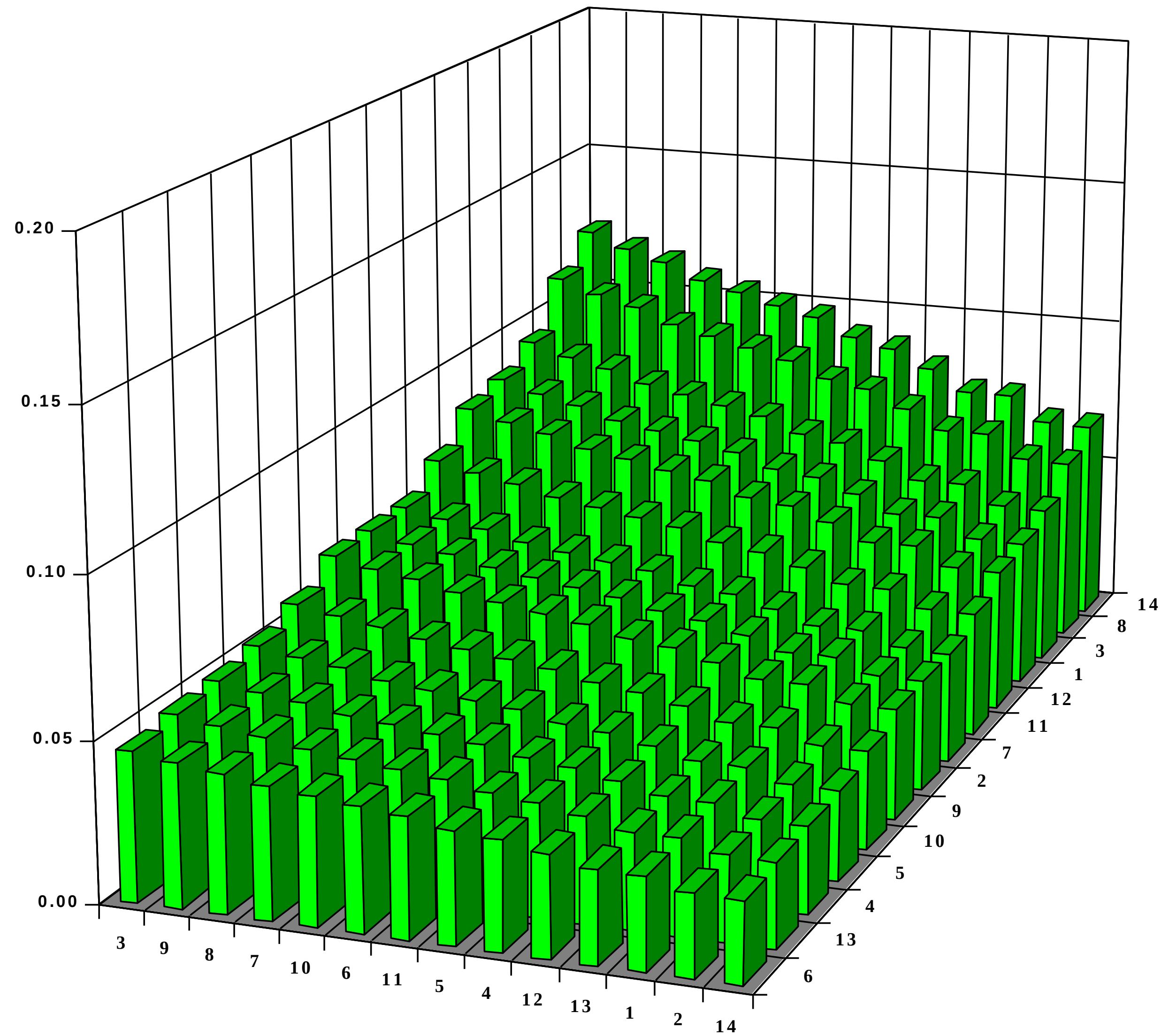
Figure 9. Paraná 1985: Cross-structure "Landscape" for First Order Intensity Field

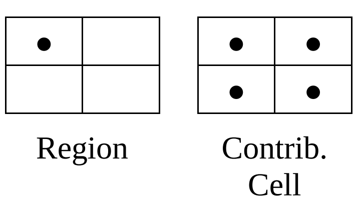


Source: Rodrigues (2000).

Figure 10. Paraná 1995: Cross-structure “Landscape” Using Paraná 1985 Imposed Hierarchy

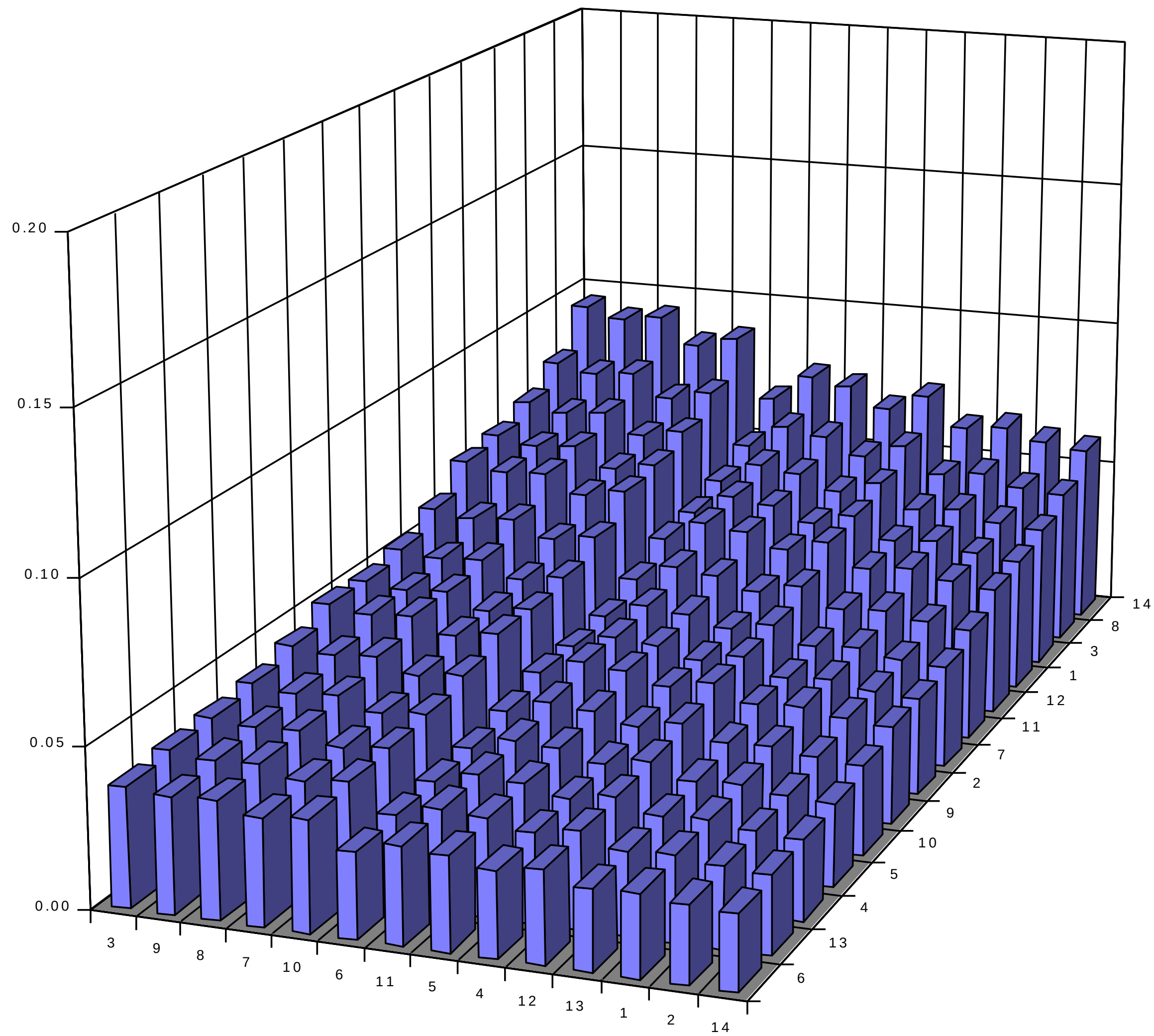
3.4. Hierarchical Inclusion of Economic Landscapes

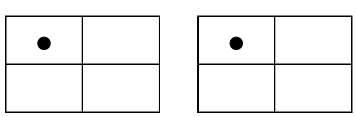




Source: Guilhoto et al (1999)

Figure 11 - Landscape of the Northeast Region





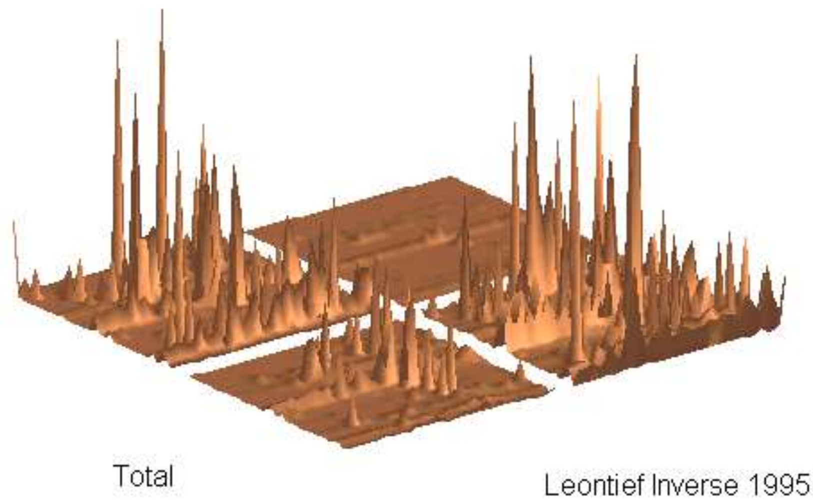
Region

Contrib.
Cell

Source: Guilhoto et al (1999)

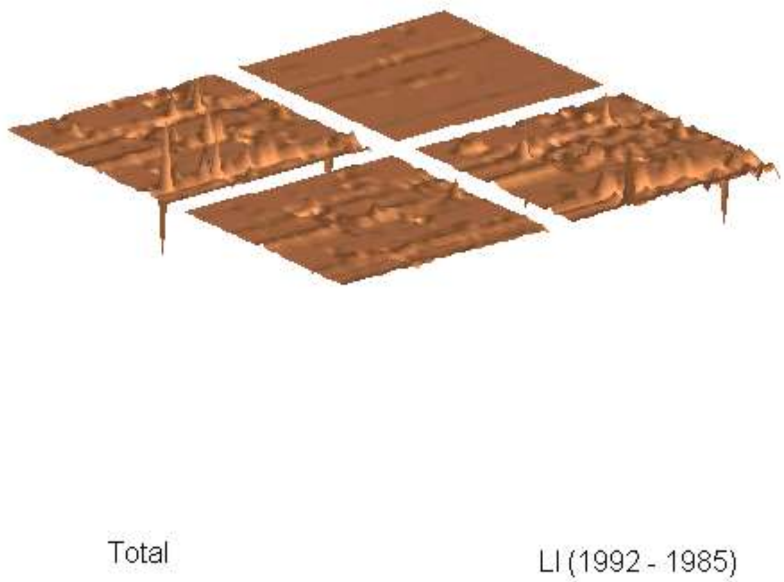
Figure 12 - Contribution of Inputs Within the Northeast Region to the Northeast Region Landscape

3.5. Economic Landscapes – Version 2.0



Source: Guilhoto et al (2000)

**Figure 13 – Economic Landscape of the Leontief Inverse
Northeast and Rest of Brazil Regions: 1995**



Source: Guilhoto et al (2000)

Figure 14 – Economic Landscape of the Changes in the Leontief Inverse Northeast and Rest of Brazil Regions: 1992 Less 1985

3.6. Synergetic Interactions Among Regions

North

	N	NE	CW	SE	S	
N	64.27	0.49	1.68	17.60	7.01	91.05
NE	0.01	0.18	0.00	0.04	0.01	0.24
CW	0.00	0.01	0.34	0.12	0.02	0.49
SE	0.19	0.21	0.15	4.97	0.47	5.99
S	0.03	0.06	0.03	0.44	1.64	2.20
	64.50	0.95	2.20	23.17	9.15	99.97

Northeast

	N	NE	CW	SE	S	
N	0.13	0.00	0.00	0.05	0.01	0.19
NE	0.81	73.03	0.98	12.76	4.03	91.61
CW	0.00	0.01	0.29	0.08	0.02	0.40
SE	0.12	0.28	0.19	4.91	0.48	5.98
S	0.02	0.06	0.03	0.24	1.41	1.76
	1.08	73.38	1.49	18.04	5.95	99.94

Central West

	N	NE	CW	SE	S	
N	0.06	0.00	0.00	0.01	0.01	0.08
NE	0.00	0.17	0.00	0.04	0.01	0.22
CW	0.32	0.83	68.41	20.42	3.46	93.44
SE	0.06	0.18	0.09	4.65	0.28	5.26
S	0.02	0.02	0.01	0.13	0.79	0.97
	0.46	1.20	68.51	25.25	4.55	99.97

Southeast

	N	NE	CW	SE	S	
N	0.20	0.01	0.00	0.06	0.01	0.28
NE	0.01	0.51	0.01	0.14	0.04	0.71
CW	0.01	0.00	0.40	0.13	0.02	0.56
SE	1.67	2.53	1.89	84.49	6.02	96.60
S	0.02	0.05	0.04	0.24	1.49	1.84
	1.91	3.10	2.34	85.06	7.58	99.99

South

	N	NE	CW	SE	S	
N	0.12	0.00	0.00	0.03	0.01	0.16
NE	0.01	0.32	0.00	0.07	0.02	0.42
CW	0.00	0.01	0.25	0.11	0.01	0.38
SE	0.05	0.10	0.07	3.39	0.22	3.83
S	0.86	1.95	1.16	14.41	76.82	95.20
	1.04	2.38	1.48	18.01	77.08	99.99

Shares of Main Relations

	N	NE	CW	SE	S
N. of Matrices	6	6	4	6	5
% Prod.	97.17	97.12	96.94	98.09	97.73

Source: Guilhoto et al (2001).

Figure 15. Contribution (%) of Each Block Matrix to the Total Share of $(x_1\text{-}f)$ in x to the regions North, Northeast, Central West, Southeast, and South.

3.7. Decomposition, Source, and Evolution of the Output Change

Sector	Period	Total	Total		Total			Self			Non-Self		
			Self	Non	Dem	Tech	Syn	Dem	Tech	Syn	Dem	Tech	Syn
1. Agriculture	59-70	+	-	+	+	-	-	-	+	-	+	-	-
	70-75	+	+	+	+	-	-	+	-	-	+	-	-
	75-80	+	-	+	+	-	-	-	+	-	+	-	-
2. Mining	59-70	+	+	+	+	-	-	+	-	-	+	-	-
	70-75	+	+	+	+	-	-	+	+	+	+	-	-
	75-80	+	+	+	+	+	+	+	-	-	+	+	+
3. Construction	59-70	+	+	-	+	-	-	+	-	-	+	-	-
	70-75	+	+	-	+	-	-	+	-	-	+	-	-
	75-80	+	+	+	+	+	+	+	+	+	+	+	+
4. Manufacturing	59-70	+	+	+	+	-	-	+	-	-	+	-	-
	70-75	+	+	+	+	+	+	+	+	+	+	+	+
	75-80	+	+	+	+	+	+	+	+	+	+	+	+
5. Trade and Transp.	59-70	+	+	+	+	-	-	+	-	-	+	+	-
	70-75	+	+	+	+	+	+	+	+	+	+	+	+
	75-80	+	+	+	+	-	+	+	+	+	+	-	+
6. Services	59-70	+	+	-	+	-	-	+	-	-	+	-	-
	70-75	+	+	+	+	+	+	+	+	+	+	+	+
	75-80	+	+	+	+	+	+	+	+	+	+	+	+

Source: Guilhoto et al. (2001)

Figure 16
Signs of the Growth Rates of Output and of Its Components - Brazil

Sector	Period	Total			Self			Non-Self		
		Dem	Tech	Syn	Dem	Tech	Syn	Dem	Tech	Syn
1. Agriculture	59/70 - 70/75	+	-	-	+	-	-	+	+	-
	70/75 - 75/80	-	+	-	-	+	+	-	+	-
2. Mining	59/70 - 70/75	+	-	-	-	+	-	+	-	-
	70/75 - 75/80	-	+	+	+	-	-	-	+	+
3. Construction	59/70 - 70/75	+	-	-	+	-	-	-	+	-
	70/75 - 75/80	-	+	+	-	+	+	-	-	+
4. Manufacturing	59/70 - 70/75	-	+	+	-	+	+	-	+	+
	70/75 - 75/80	-	+	+	-	+	-	-	+	-
5. Trade and Transp.	59/70 - 70/75	-	+	+	+	-	-	-	+	+
	70/75 - 75/80	+	-	-	-	+	+	+	+	-
6. Services	59/70 - 70/75	+	-	-	+	-	+	+	-	-
	70/75 - 75/80	-	+	+	-	+	+	-	+	+

Source: Guilhoto et al. (2001)

Figure 17
Signs of the Evolution of Changes - Brazil

4. Final Comments

In this paper it was made a review of some of the contributions of Sonis and Hewings to the economic theory with some of the applications made to the Brazilian economy.

Everyone knows that one review is never complete and that some important work might be missing in it. As it was left clear before, the goal of this paper was not to cover every detail of the work done by Hewings and Sonis but just to give an overview of it and its importance to a better understanding of the Brazilian economy.

While this paper concentrates on the theory of input-output analysis, the contributions of Hewings and Sonis go beyond this theory, entering into the fields of spatial econometrics, applied general equilibrium models, dynamic models, history of economic thought, environmental studies, econometric input-output models, etc.

Of the various Brazilian scholars that, in a way or another, have worked directly with Sonis & Hewings can be mentioned, besides myself, and among others, Manuel Fonseca, Eduardo Martins, Ricardo Gazel, Eduardo Haddad, André Magalhães, Paulo Resende, Edson

Domingues, Flávia Bliska, Monica Haddad, Carlos Eduardo Lobo e Silva, Carlos Azzoni, Décio Kadota, etc. Indirectly, the above scholars have introduced a whole new generation of Brazilian students to the theories of Sonis and Hewings, and they, by their turn to more students, and things go on in a type n multiplier, $n \to \infty$, where the multiplier now becomes exponential.

From the above, what is clear is that despite the importance of the theoretical contributions made by Sonis & Hewings, it should never be forgotten that these outstanding researchers, who are also extraordinary people, always give support and stimulus for people to investigate new ideas and to put them in practice. From the combination of their intellectual capabilities with their personalities is that comes their success as scholars and as individuals.

5. References

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