Macroeconomic Stability with a Positively Slope IS Curve: A Further Examination

Cebula, Richard

Jacksonville University

10 June 1973

Macroeconomic Stability with a Positively Sloped IS Curve: A Further Examination

by

RICHARD J. CEBULA

Atlanta (Georgia/USA)

Introduction

In recent years, several authors have addressed themselves to the possibility of a positively sloped *IS* curve and the policy implications of such a curve[1]. But, with two apparent exceptions, *Steindl* [19] and *Cebula* and *Gallaway* [4], no recent attention has been devoted explicitly to the problem of economic stability in a system with a positively sloped *IS* curve. And here, both *Steindl* [19] and *Cebula* and *Gallaway* [4] refer simply to the traditional argument, now recognized even in the textbooks (see, for example, *Dernburg* and *MacDougall* [6]), developed by *Modigliani* [13] that the slope of the *IS* curve must be algebraically less than of the *LM* curve. None of the literature on the positively sloped *IS* curve, however, is concerned with stability in a system with a variable aggregate price level. In view of this apparent neglect (oversight), the present paper seeks to investigate the sufficiency of the stability condition formulated by *Modigliani* [13] for a system with both a positively sloped *IS* curve and a variable aggregate price level. In other words, we examine the following question: »Does the fact that the economy's *LM* curve is algebraically steeper than its *IS* curve guarantee stability in a system with an endogenous price level?«

In Section I below, we develop our basic model. In Section II, we mathematically analyze the stability of general equilibrium under conditions of a positively sloped *IS* curve and variable aggregate price level. Section III offers concluding remarks.

---

[1] Related generally to this, see [1], [2], [5], [8], [10, p. 250], and [18].

I. The Model

We begin by specifying the following economic system:

(1)	C	$= C(Yd, i)$	consumption function
(2)	I	$= I(Y, i)$	investment function
(3)	G	$= Go$	exogenous government spending
(4)	T	$= To$	exogenous tax collections
(5)	Yd	$= Y - T$	disposable income
(6)	Y	$= C + I + G$	commodity market equilibrium condition
(7)	L	$= L(Y, i)$	money demand function
(8)	Ms	$= Mo$	exogenous money supply
(9)	Mo/p	$= L(Y, i)$	money market equilibrium condition
(10)	Y	$= Y(N, K)$	production function
(11)	W/p	$= \dfrac{\partial Y}{\partial N}$	labor demand function
(12)	W	$= Wo$	exogenous money wage rate
(13)	Ns	$= Ns\,(W)$	labor supply function,

where G = real government spending
T = real tax collections
C = real consumption
Y = real income
Yd = disposable real income
i = interest rate
I = real investment
G = real government spending
T = real tax collections
L = real money demand
Ms = nominal money stock
P = aggregate price level
N = number of labor units
K = capital stock
W = money wage rate
Ns = units of labor supplied

We impose the following restrictions on the partial derivatives in our system:

$$1 > \frac{\partial C}{\partial Y} > 0, \quad \frac{\partial C}{\partial i} < 0,$$

$$(14) \qquad 1 > \frac{\partial I}{\partial Y} > 0,$$

$$\frac{\partial L}{\partial Y} > 0, \quad \frac{\partial L}{\partial i} < 0.$$

In addition, workers are subject to »money illusion«.

The slope of the IS curve in the system is given by

$$(15)\qquad \frac{1-\frac{\partial C}{\partial Y}-\frac{\partial I}{\partial Y}}{\frac{\partial C}{\partial i}+\frac{\partial I}{\partial i}}$$

If as *Modigliani* [13] argues, $\left(\frac{\partial C}{\partial i}\,\frac{\partial I}{\partial i}\right)<0$, then it follows (see *Hicks* [9] and *Modigliani* [13] (that the necessary condition for the IS curve to be positively sloped is

$$(16)\qquad \frac{\partial I}{\partial Y}>1-\frac{\partial C}{\partial Y}.$$

Alternatively, *if* we reverse the sign of $\partial C/\partial i$, a positively sloped IS curve is possible if both of the following conditions pertain:

$$(17A)\qquad \frac{\partial C}{\partial i}+\frac{\partial I}{\partial i}>0$$

$$\text{for}\qquad \frac{\partial C}{\partial i}>0 \text{ and } \frac{\partial C}{\partial i}>\frac{\partial I}{\partial i}$$

$$(17B)\qquad \frac{\partial I}{\partial Y}<1-\frac{\partial C}{\partial Y}.$$

The condition for economic stability developed by *Modigliani* [13] is given by

$$(18)\qquad \underset{IS}{\frac{\partial i}{\partial Y}}<\underset{LM,}{\frac{\partial i}{\partial Y}}$$

which for our system converts to

$$(19)\qquad \frac{1-\frac{\partial C}{\partial Y}-\frac{\partial I}{\partial Y}}{\frac{\partial C}{\partial i}-\frac{\partial I}{\partial i}}<-\frac{\frac{\partial L}{\partial Y}}{\frac{\partial L}{\partial i}}.$$

II. Mathematical Analysis

In this Section, we mathematically examine the sufficiency of the *Modigliani* condition (19). In particular, we here examine the local stability properties of an economic system which has both a positively sloped IS curve and a variable aggregate price level for a specific dynamic adjustment mechanism, i.e., a tatonnement mechanism. The stability problem is considered with the framework of a purely qualitative (+, 0, —) environment: no quantitative information is presumed available. Thus, stability, should it occur, must be *qualitative stability*, i.e., stability determined solely by the sign patterns of elements within the matrices encountered in the analysis[2].

To begin our analysis, let $Y_1, Y_2, \ldots, Ym$ be the values of m economic variables and let α be a shift parameter. Assume m functional relationships linking α and the Yi's, with the ith relationship being given by $fi(Y_1, Y_2, \ldots, Ym, \alpha)$. For a given value of α, say α^*, an equilibrium position is defined as a set of values $Y'_1, Y'_2, \ldots, Y'm$ such that

$$(20) \qquad fi(Y'_1, Y'_2, \ldots, Y'm, \alpha^*) = 0 \text{ for } i = 1, \ldots, m.$$

Let a small change in α occur. The change in the equilibrium values of the variables is given by

$$(21) \qquad \sum_{j=1}^{m} fij \frac{dY'j}{d\alpha} = -fi\alpha \text{ for } i = 1, \ldots, m.$$

where $fij = \dfrac{\partial fi}{\partial Yj}$ and $fi\alpha = \dfrac{\partial fi}{\partial \alpha}$.

Here, fij and $fi\alpha$ are evaluated at the equilibrium position $(Y'_1, Y'_2, \ldots, Y'm, \alpha^*)$. Rewrite (21) in matrix form, with $C = [fij]$; this yields

$$(22) \qquad C\left[\frac{dY'j}{d\alpha}\right] = -\left[fi\alpha\right],$$

with the terms in brackets being $m \times 1$ vectors.

Assume a dynamic adjustment process determines the time paths of the variables $Y_1, Y_2, \ldots, Ym$ whenever the system is out of equilibrium. The process may be expressed as

$$(23) \qquad \dot{Y}i = di fi\,(Y_1, Y_2, \ldots, Ym, \alpha^*), \text{ for } i = 1, \ldots, m,$$

where $\dot{Y}i = \dfrac{dYi}{dt}$, t is time, and di is a positive constant for each i, the adjustment speed for each variable. We assume that in a sufficiently small neighborhood of equilibrium, the linear terms of a Taylor series expansion approximate the adjustment process. Thus (23) becomes

$$(24) \qquad \dot{Y}i = di \sum_{j=1}^{m} fij(Yi - Y'i) \text{ for } i = 1, \ldots, m.$$

Letting $Xi = Yi - Y'i$, we express (24) in matrix notation as

$$(25) \qquad \bar{X} = DCX,$$

where $\quad \bar{X} = \dfrac{dXi}{dt}$,

[2] Related generally to the topic of qualitative economics, see [3], [7], [11], [12], [14[], [15], [16], and [17, Chapter IX].

D is a diagonal matrix, with di as the ith diagonal element, $C = [fij]$, and $X = [Xi]$.

Stability of the system (25) is a situation wherein, for arbitrary initial values of deviations from equilibrium (in a sufficiently small neighborhood of equilibrium), we have

$$(26) \qquad \lim_{t \to \infty} Xi = 0 \text{ for all } i = 1, \ldots, m$$

that is,

$$(27) \qquad \lim_{t \to \infty} Yi = Y'i \text{ for all } i = 1, \ldots, m$$

The system (25) is stable in this sense if and only if the real parts of all the characteristic roots of DC are negative[3].

We may now consider the stability of our system under conditions of a variable aggregate price level and a positively sloped IS curve. In our system, the price level, interest rate, and money wage are the dependent variables. Our system (1)–(13) may be briefly and conveniently summarized by the following three excess demand equations (see, for example, [15]):

$$(28) \qquad \begin{aligned} EY &= C(Y, d\, i) + I(Y, i) + Go - Y(N) \\ EM &= L(Y, i) - Mo/p \\ EN &= ND\,(W, p) - NS\,(w, p) \end{aligned}$$

where EY = excess demand for output
EM = excess money demand
EN = excess labor demand
ND = labor demand.

Equilibrium is a set of values for the dependent variables such that

$$(29) \qquad \begin{aligned} EY &= 0 \\ EM &= 0 \\ EN &= 0. \end{aligned}$$

Setting EY, EM, $EN = 0$, total differentiation of system (28) yields (30)–(32):

$$(30) \qquad \left(\frac{\partial Y}{\partial N} \cdot \frac{\partial Np}{\partial P}\right)\left(\frac{\partial C}{\partial Y} + \frac{\partial I}{\partial Y} - 1\right) dp + \left(\frac{\partial C}{\partial i}\,\frac{\partial I}{\partial i}\right) di + \left(\frac{\partial Y}{\partial N} \cdot \frac{\partial ND}{\partial W}\right)\left(\frac{\partial C}{\partial Y}\,\frac{\partial I}{\partial Y}\right) dw = -\,dG + dT$$

$$(31) \qquad \left(\frac{\partial Y}{\partial N} \cdot \frac{\partial N}{\partial P}\right)\left(\frac{\partial M}{\partial p2}\right) dp + \left(\frac{\partial L}{\partial i}\right) di + \left(\frac{\partial Y}{\partial N} \cdot \frac{\partial ND}{\partial W}\right)\left(\frac{\partial L}{\partial Y}\right) dw = \frac{dM}{P}$$

[3] Related to this particular discussion, see [7], [14], [15], [16], or [17, pp. 258–263].

$$\left(\frac{\partial ND}{\partial P} - \frac{\partial NS}{\partial P}\right)_{dp} + \left(\frac{\partial ND}{\partial W} - \frac{\partial NS}{\partial W}\right)_{dw} = 0 \tag{32}$$

The time paths of the dependent variables P, i, and W are determined by a tatonnement mechanism, with adjustment equations given by

$$\begin{aligned} \frac{dp}{dt} &= d_1 EY \\ \frac{di}{dt} &= d_2 EM \\ \frac{dW}{dt} &= d_3 EN \end{aligned} \tag{33}$$

Since system (33) can be approximated linearly in a sufficiently small neighborhood of equilibrium by a Taylor series, it is rewritten then as

$$\begin{aligned} \frac{dp}{dt} &= d_1 a_{11}(p-p') + d_1 a_{12}(i-i') + d_1 a_{13}(W-W') \\ \frac{di}{dt} &= d_2 a_{21}(p-p') + d_2 a_{22}(i-i') + d_2 a_{23}(W-W') \\ \frac{dW}{dt} &= d_3 a_{31}(p-p') + d_3 a_{32}(i-i') + d_3 a_{33}(W-W') \end{aligned} \tag{34}$$

p', i', and W' being the equilibrium values of p, i, and W, respectively.

The coefficients aij in (34) may be obtained directly from (30)–(32). In particular,

$$\begin{aligned} a_{11} &= \left(\frac{\partial Y}{\partial N} \cdot \frac{\partial ND}{\partial P}\right)\left(\frac{\partial C}{\partial Y} + \frac{\partial C}{\partial Y} - 1\right) \\ a_{12} &= \left(\frac{\partial C}{\partial i} + \frac{\partial I}{\partial i}\right) \\ a_{13} &= \left(\frac{\partial Y}{\partial N} \cdot \frac{\partial ND}{\partial W}\right)\left(\frac{\partial C}{\partial Y} + \frac{\partial I}{\partial Y} - 1\right) \\ a_{21} &= \left(\frac{\partial Y}{N} \cdot \frac{\partial ND}{P} + \frac{M}{p^2}\right) \\ a_{22} &= \left(\frac{\partial L}{\partial i}\right) \\ a_{23} &= \left(\frac{\partial Y}{\partial N} \cdot \frac{\partial Nd}{\partial W}\right)\left(\frac{\partial L}{\partial Y}\right) \\ a_{31} &= \left(\frac{\partial ND}{\partial p} - \frac{\partial NS}{\partial P}\right) \\ a_{32} &= 0 \\ a_{33} &= \left(\frac{\partial ND}{\partial W} - \frac{\partial NS}{\partial W}\right) \end{aligned} \tag{35}$$

We now examine the sufficiency of the *Modigliani* stability condition (19) for our system under a positively sloped *IS* curve. First, let $C = [aij]$ for $i, j = 1, 2, 3$, and let D by a 3×3 diagonal matrix with di, $i = 1, 2, 3$, the diagonal elements. Now, the sufficient conditions for a positively sloped *IS* curve are given by (17A) and (17B). (17A) requires element a_{12} in C to be positive and element a_{11} in C to be negative. Thus, under (17A), we have

$$(36) \qquad DC = \begin{bmatrix} - & + & + \\ + & - & - \\ + & 0 & - \end{bmatrix}.$$

(17B) requires element a_{12} in C to be negative and element a_{11} to be positive. This implies

$$(37) \qquad DC = \begin{bmatrix} + & - & + \\ + & - & - \\ + & 0 & - \end{bmatrix}.$$

Now, the necessary and sufficient conditions for sign stability of an indecomposable real $m \times m$ matrix E with elements a_{ij}, where the subscript i denotes the row and the subscript j denotes the column of the element in question, are given as[4]

Condition (1): $a_{ij}a_{ji} \leqq 0$ for $i \neq j$.

Condition (2): $i_1 \neq i_2 \neq \ldots \neq i_m$, $a_{i_1 i_2} \neq 0$, $a_{i_2 i_3} \neq 0$, …, $a_{i_{m-1} i_m} \neq 0$ implies $a_{i_m i_1} = 0$ for any $m > 2$.

Condition (3): $a_{ii} \leqq 0$ for all i, $a_{kk} < 0$ for some k.

Condition (4): There exists a non-zero term in the expansion of $|E|$.

Modigliani's stability condition (19) requires that $a_{11}a_{22} > a_{12}a_{22}$. Referring to DC in both (35) and (37), it is clear that neither case satisfies the conditions for sign stability, regardless of the *Modigliani* stability condition. Thus, the latter which is presumably a valid theorem for a simple model with only Y and i, is not an adequate guarantee of stability in a system with a variable aggregate price level.

III. Conclusion

The *Modigliani* stability condition states that the *LM* curve must be algebraically greater than the *IS* curve for economic stability to occur. This paper has shown that this theorem's validity does not guarantee stability in the large for the economic system if the aggregate price level is a variable. Thus, the *Modigliani* stability condition may not be generalized beyond the scope of a crude *IS-LM* model having only income and the interest rate as endogenous.

[4] See [16, p. 320] for these.

In closing, it should be noted that the *Modigliani* condition, while not guaranteeing qualitative stability (i.e., stability based solely on qualitative information), it does not necessarily preclude the possibility of a potentially stable system. In particular, let E be a 3×3 matrix with a known sign pattern. In E, let $a_{ii} < 0$ for all i. E is potentially stable if and only if one of three conditions listed below is met[5]:

(1) A has all diagonal elements negative.
(2) A has exactly two negative diagonal elements and there exists a term in the expansion of $|A|$ of negative sign.
(3) A has exactly one negative diagonal element a_{11}, and either (3a) or (3b) is satisfied.
(3a) $a_{1j}a_{j1} < 0$ for some $j = 2, 3$ and there exists a term in the expansion of $|A|$ of negative sign.
(3b) $a_{23}a_{32} < 0$ and there exists a term in the expansion of $|A|$ of positive sign.

If we relate DC in (36) and (37) to these conditions, we find that the *Modigliani* condition in fact *has* guaranteed potential stability in both cases.

References

1. *Douglas K. Adie,* An International Comparison of the Quantity and Income – Expenditures Theories, Journal of the American Statistical Association, forthcoming.
2. *Douglas K. Adie,* A Two-Stage International Cross-Section Text of the Forced Savings Theory, 1971 Business and Economic Statistics Section, Proceeding of the American Statistical Association, 291–296.
3. *Richard J. Cebula,* Economic Stability and the Commodity Trap, Schweizerische Zeitschrift für Volkswirtschaft und Statistik, Vol. 107 (December, 1971), 823–829.
4. *Richard J. Cebula* and *Lowell E. Gallaway,* Fiscal Policy Effectiveness and the Positively Sloped IS Curve, Indian Journal of Economics, forthcoming.
5. *Richard J. Cebula* and *Stephen M. Renas,* A Note on the Nature of the IS Locus, Indian Journal of Economics, forthcoming.
6. *Thomas Dernberg* and *Duncan McDougall,* Macroeconomics, 3rd edition, New York 1968.
7. *Paul Frevert,* On the Stability of Full Employment Equilibrium, Review of Economic Studies, Vol. 37 (April, 1970), 239–251.
8. *William E. Gibson* and *George G. Kaufman,* The Sensitivity of Interest Rates to Changes in Money and Income, Journal of Political Economy, Vol. 76 (May-June, 1968), 472–478.
9. *John R. Hicks,* Mr. Keynes and the 'Classics'; A suggested Interpretation, Econometrica, Vol. 5 (April, 1937), 147–159.
10. *Harry G. Johnson,* Essays in Monetary Economics, London 1967.
11. *Kelvin Lancaster,* The Solution of Qualitative Comparative Static Problems, Quarterly Journal of Economics, Vol. 80 (May, 1966), 278–295.

[5] These are given in [15, p. 301].

12. *Peter J. Lloyd,* Qualitative Calculus and Comparative Statics Analysis, Economic Record, Vol. 45 (September, 1969), 343–353.
13. *Franco Modigliani,* Liquidity Preference and the Theory of Interest and Money, Econometrica, Vol. 12 (January, 1944), 45–88.
14. *Don Patinkin,* The Limitations of Samuelson's Correspondence Principle, Metroeconomica, Vol. 4 (1952), 37–43.
15. *James P. Quirk,* The Correspondence Principle: A Macroeconomic Application, International Economic Review, Vol. 9 (October, 1968), 294–306.
16. *James P. Quirk* and *Richard Ruppert,* Qualitative Economics and the Stability of Equilibrium, Review of Economic Studies, Vol. 32 (October, 1965), 311–326.
17. *Paul A. Samuelson,* The Foundations of Economic Analysis, New York 1965.
18. *William Silber,* Monetary Policy Effectiveness: The Case of the Positively Sloped IS Curve, Journal of Finance, Vol. 26 (December, 1971), 1077–1082.
19. *Frank Steindl,* Giffen Goods, IS Curves and Macroeconomic Stability, Metroeconomica, Vol. 22 (May-August, 1970), 165–169.

Zusammenfassung

In diesem Aufsatz wird auf der Grundlage einer positiv geneigten *IS*-Kurve die Angemessenheit der makroökonomischen Stabilitätsbedingung, in der die Neigung der *LM*-Kurve größer als die der *IS*-Kurve sein soll, untersucht. Die Brauchbarkeit dieser Bedingung hat sich als äußerst begrenzt herausgestellt.