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Monetary Policy and Growth with Trend Inflation and Financial Frictions*

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Abstract

This paper studies the effects that conventional and unconventional monetary policies generate when endogenous growth, trend inflation and financial frictions are considered in a New Keynesian macroeconomic model. Financial variables play a key role in the determination of the steady state growth rate, given the value of the trend inflation. Calibrating the model following Gertler and Karadi (2011), long-run growth rate, welfare, normalized investment and financial wealth are maximized when trend inflation is 1.7% while leverage, external finance premium and marginal gain of the financial intermediaries are minimized. Finally, unconventional policies could extend their impact to the long run.

JEL code: E31, E44, E58, O42

Keywords: New Keynesian DSGE models, endogenous growth, financial frictions, trend inflation, unconventional monetary policy.

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1 Introduction

The recent economic crisis, triggered in mid-2007, has highlighted the severe and long-lasting consequences that financial frictions have on the economies. After the outbreak of the crisis, the power of the credit markets to provide liquidity to the economies has been weakened, which is delaying economic recovery in most developed countries. Since the natural mechanisms of the credit markets are not operating flexibly and, as a result, the countries’ output is being seriously affected, central banks have adopted a two-pronged strategy. On the one hand, monetary authorities have implemented a very expansive monetary policy through near zero nominal interest rates and, on the other, they have provided direct lending to private markets in order to ensure the necessary liquidity to satisfy the real demand. However, neither the lax monetary policy nor the direct liquidity injections appear to be appeasing the short-term negative effects of the crisis. This fact leads us to think that the recovery may take some time because the effects of the financial crisis could be reaching the long-term dynamics. Hence, we need economic models of the highest accuracy that connect the short and long term in order to provide a deeper perspective. Further study of the key macroeconomic roles of the financial and monetary markets is crucial not only to explain business cycles, but also to better understand the long-run path of the economies.

So far, the literature has not considered endogenous economic growth and trend inflation together in models with sticky prices and frictions in the financial markets. The analysis of the factors that explain economic growth is a well-known field of study, and the inclusion of financial rigidities in macro models is not unusual nowadays, especially since the outbreak of the financial crisis. Nevertheless, to the best of our knowledge, the two analyses have never been merged. It is uncommon to combine economic growth with monetary issues, even less so with models with financial frictions that usually address short-term relationships. Economic growth is considered a long-term phenomenon that should not be taken into account when analyzing the short-term behavior. However, dynamic macroeconomic models in the short run are built around a trend which must be consistent with economic growth. If none of the short-term elements affects the steady state, so that the latter acts only as a trend reference, this distinction would not be ignoring anything important.

If any feature of the short-term behavior alters the steady state, then the situation is quite different. Disregarding long and short-term interactions would then cause a misunderstanding of the macroeconomic behavior, not only in the short but also in the long run. This paper widens the macroeconomic benchmark model by considering a range of issues that have not yet been addressed simultaneously in order to derive some key implications for the monetary policy performance. We design a theoretical model which combines elements that may be critical for the interactions between the short and long term by modifying the DSGE New Keynesian framework.

To adopt the long-term perspective, we include endogenous economic growth,
driven by the stock of knowledge generated by capital accumulation. In addition, we consider financial frictions in the credit markets. Furthermore, we allow the central bank to implement unconventional monetary policy in crisis periods by direct lending to non-financial firms. Finally, we remove the assumption of zero inflation in the steady state. With all these elements, we focus on the relationship between real and financial variables and the performance of monetary policy in the short and the long run.

Macroeconomic research has developed accurate models to evaluate monetary policy including nominal rigidities, among them, Christiano et al. (2005) and Smets and Wouters (2007). However, the assumption that financial markets are completely flexible has been prevalent in the academic literature. Bearing in mind the facts discussed above, it seems that the rigidities in the credit markets, which are strategic for the economic system, have amplified and extended the impact of the crisis. The critical importance of these markets for the performance of the monetary policy has attracted interest since Bernanke, Gertler and Gilchrist published their seminal paper in 1999 in which they introduce the so-called financial accelerator. The basic idea of this approach is the amplifying effects that credit markets have on business cycles due to the rigidities in this sector which are based on the limitation of non-financial firms in obtaining investment funds as a result of their balance sheet constraints. Other works that highlight the importance of financial rigidities are Kiyotaki and Moore (1997), Holmstrom and Tirole (1997) and Carlstrom and Fuerst (1997). Recently, Kiyotaki and Moore (2008), Christensen and Dib (2008), Brunnermeier et al. (2009) and Christiano, Motto and Rostagno (2010) have examined several topics related to this issue from different angles. An excellent survey of past and recent work is Brunnermeier et al. (2012). These studies conclude that the effects of monetary policy shocks are larger and more persistent when taking financial frictions into account.

Given the altered flow of credit and the resulting slowdown in the economic recovery, as well as the reaching of the zero lower bound of nominal interest rates, one of the various proceedings taken by central banks to solve the financial distress has been to implement unconventional monetary policy measures by issuing direct loans or by injecting immediate liquidity into financial institutions. Such policies are designed to soften the negative effects of disruptions related to the valuation of assets, but the study of their consequences is still at an early stage, as noted by Joyce et al. (2012). These authors also point out the lack of effectiveness and accuracy of this type of policy. A deeper study of credit policy and unconventional monetary policy from a macroeconomic point of view is, therefore, worth considering. Curdia and Woodford (2011) compute the effects of this credit policy examining several instruments, heterogeneity across agents and different monetary policy rules. In Gertler and Karadi (2011) and Gertler and Kiyotaki (2012), the response under different intensities of credit policy is studied. However, unlike these business cycles studies, we will try to identify the consequences of this kind of unconventional policies in the short and long-term by adapting the structure of financial intermediaries proposed by Gertler
and Karadi (2011). In this work, an agency problem is introduced whereby financial intermediaries are restricted in their leverage position to providing funds. What we obtain when we consider economic growth in this framework is that the effect of these measures is not restricted to the business cycles, but also affects the long-run scenario.

We introduce endogenous growth into the New Keynesian framework. There are few precedents in this field, among them, Hiroki (2009), Rannenberg (2009), Amano et al. (2009, 2012), Inoue and Tsuzuki (2010), Anicchiarico, Pelloni and Rossi (2011) and Vaona (2012). However, none of these studies consider the financial sector. Some works combining endogenous economic growth and the financial sector analyze the effects of the degree of financial development in neoclassical growth models, as in Levine (1997), but do not include nominal rigidities or monetary policy rules.

Interest rate has been revealed as an effective and quick instrument of monetary policy, but also quite sensitive. As a consequence, the probability of taking a non-optimal decision is high. The choice of variables to be considered when setting the objectives and instruments of the decision-making process and their correct definition is of great importance, both in the short and in the long term. For this reason, one element that we consider essential to address the long-term analysis is the non-null inflation rate in the steady state, that is, trend inflation (hereafter TI). Mainstream monetary policy literature supposes that TI is zero, arguing analytical convenience or that it represents an optimal solution in cashless models. However, evidence shows that this is not a realistic assumption, since most central banks set positive targets on inflation rate in the medium-term, and the estimations of the long-run inflation rate are always greater than zero. Several contributions, including Ascari (2004), Hornstein and Wolman (2005), Sahuc (2006), Amano et al. (2007), Kiley (2007), Bakhshi et al. (2007), Cogley and Sbordone (2008), Ascari and Ropele (2007, 2009) and Coibion and Gorodnichenko (2011), have examined this issue by focusing on the validity of the conclusions about monetary policy in the New Keynesian models when modifying this assumption. They find that its relaxation changes the short and medium-term properties of the models. Among other outcomes, these studies conclude that the results obtained when including zero TI are not, in general, robust. In particular, the slope of the New Keynesian Phillips curve decreases with the TI and the output gap has less influence in determining the inflation rate. The only works that have combined TI and long-term growth have been Amano et al. (2009, 2012), who find a great loss in the steady state output arising from the presence of TI, and Vaona (2012), who finds a nonlinear relationship between inflation and economic growth in a context in which monetary policy is not established by setting the nominal interest rate but through the quantity of money. The presence of financial frictions is omitted in all these papers.

Once trend inflation, financial frictions and economic growth are simultaneously considered in a model that extends the one used in Gertler and Karadi (2011), our results show that the current monetary policy generates interactions between real and
financial variables that are reflected in a direct link between the long-term economic growth rate and the marginal gain from expanding assets in the financial system, given the value of TI. As this value is given by the monetary policy rule, where the nominal short-term interest rate is decided, monetary policy disrupts the connexion between economic growth and the profitability of the financial sector not only in the cycle, but also in the trend. In this context, both financial and real variables are sensitive to the level of TI in the long run and there is a set of interesting non-linear relationships between them in the steady state. In a calibrated model based on the one developed in Gertler and Karadi (2011), the main finding is that the long-run growth rate, welfare, normalized investment and financial wealth are maximized at a trend inflation of 1.7%, whilst leverage, the external finance premium and the marginal gain of financial intermediaries are minimized.

The long-run relevance of TI disappears whenever there are not price rigidities but the connection between the growth rate and the financial variables is not affected. Alternatively this last connection disappears without financial frictions, remaining the non-linear relationship between the growth rate and TI. Consequently, the growth rate is independent of TI and financial variables without price rigidities and financial frictions.

After the steady state is studied, we analyze the response of the model to a monetary and technology shock, showing that the level of TI affects the amplitude of the reactions. Moreover, if we trigger a simulated crisis, the unconventional monetary policy alters the magnitude and persistence of the effects of the shocks, extending their effects up to the long term. As noted before, some features of our macroeconomic model have been studied previously. However, the fundamental result we find is innovative as it combines economic growth, TI and financial frictions in a New Keynesian model. The main finding is the influence of the TI, set by the monetary policy rule, in determining the rate of economic growth through the marginal gain of the financial system. These three variables, which ultimately synthesize the long-term economic performance and are also important for the short term, are closely linked in both time perspectives.

The rest of the paper is organized as follows. In Section 2, the theoretical model is developed, as well as the study of the main relationships in the steady state. Section 3 is devoted to calibrating the model, to analyzing the steady state numerically and to subjecting the model to monetary policy and technology shocks in different scenarios depending on the level of TI considered. In Section 4, we analyze the response of the variables to a capital quality shock under different intensities of credit policy. Finally, Section 5 summarizes the main findings.
2 The model

The model proposed is a modification of the standard DSGE New Keynesian macroeconomic model which combines price rigidities, endogenous capital accumulation and spill-over effects à la Romer (1986) as the source of economic growth. Financial frictions have been included following Gertler and Karadi (2011), henceforth GK, and non-zero trend inflation is allowed. The model considers the presence of six types of agents in the economy: households, intermediate good firms, capital producers, retail firms, financial intermediaries and the central bank. The agents are characterized as follows:

1. Household members work, consume and save, holding their deposits in financial intermediaries.

2. Intermediate goods firms operate in perfect competition markets. These firms buy capital and rent labor force in order to produce their goods. They are financed with their own funds, but the purchase of capital is funded through bank loans.

3. Capital producers, whose behavior is characterized by an investment function that includes adjustment costs, sell their production to intermediate goods firms.

4. Retail firms acquire and differentiate intermediate goods and sell them to households, setting their prices à la Calvo.

5. Financial intermediaries’ liabilities are the households’ deposits, whilst net wealth and loans granted to intermediate goods producers are their assets. The granting of loans has an upper limit which depends on the intermediaries’ leverage.

6. The central bank implements monetary policy both conventional, through the modification of the short-term nominal interest rate following a Taylor rule, and non-conventional, by direct lending to intermediate goods firms.

2.1 Agents

2.1.1 Households

Households are composed of infinite horizon individuals uniformly distributed in a continuum \([0, 1]\). Each household has a fraction \(\sigma\) of its members bankers and a fraction \((1 - \sigma)\) workers. Each banker manages a financial institution and transfers the profits to his household. The workers produce goods earning the competitive
wage. Households consume and allocate their savings as bonds and deposits in the financial intermediaries. Their expected utility is defined as follows:

\[ E_t \sum_{i=0}^{\infty} \beta^i \left[ \log C_t + N_t^{1+\varphi} \right] \quad (1) \]

where \( \beta \in (0, 1) \) is the subjective discount factor, \( C_t \) is the consumption, \( N_t \) is the labor supply, \( \chi > 0 \) is the relative utility weight on labor and \( \varphi > 0 \) determines the intertemporal elasticity of the labor supply (inverse of Frisch elasticity).

Additionally, households must fulfill the budget constraint, which does not allow the present value of the expenditures to exceed the sum of the income and the value of the initial assets:

\[ C_t + \frac{D_t}{R_t} = D_{t-1} + \Gamma_t + W_t N_t - T_t \quad (2) \]

where \( D_t \) are real one-period life deposits and nominally riskless discount bonds that households hold in their portfolios, \( R_t \) is the real gross interest rate, \( \Gamma_t \) are real firms profits and payouts from firms and financial intermediaries, \( W_t \) is the real wage and \( T_t \) are the lump sum taxes.

Moreover, we add the following restriction to avoid Ponzi schemes (Galí, 2008):

\[ \lim_{T \to -\infty} E_t \{ D_T \} \geq 0 \quad (3) \]

Solving the households’ utility maximization problem, we obtain the labor supply optimality condition and the Euler equation:

\[ W_t = C_t \chi N_t^\varphi \]

\[ E_t \Lambda_{t+1} R_t = 1 \quad (4) \]

with

\[ \Lambda_{t,T} = \beta^{T-t} \frac{C_t}{C_T} \quad \text{where} \quad T = t + 1 \quad (5) \]

### 2.1.2 Intermediate goods firms

Each intermediate goods producer is indexed by \( j \in [0, 1] \) and obtains the production at time \( t \) by incorporating the capital acquired at the end of period \( t - 1 \) and by renting labor force to the households. The markets of both productive factors are competitive. The firms have a Cobb-Douglas production function, common to all of them, that generates economic growth à la Romer (1986):

\[ Y_{jt} = e^{\xi_t} (e^{\xi_t} K_{jt})^\alpha (K_t N_{jt})^{1-\alpha} \quad \text{where} \quad 0 < \alpha < 1 \]
\( Y_{jt}^i \) is the production obtained by firm \( j \) with a capital stock \( K_{jt} \) and labor \( N_{jt} \). The index \( K_t = \int_0^1 K_{jt} \, dj \) is the stock of knowledge generated by capital accumulation, which firms take as given, and will be our source of economic growth driving the total factor productivity. \( \xi_t \) and \( z_t \) are shocks common to all firms. The first is the capital quality shock and the second the aggregate productivity shock, both following first order autoregressive processes of the type:

\[
\begin{align*}
  z_t &= \rho_z z_{t-1} + u_t^z \\
  \xi_t &= \rho_\xi \xi_{t-1} + u_t^\xi
\end{align*}
\]

where \( \rho_z, \rho_\xi \in [0, 1) \) measure the degree of persistence of the shocks and \( u_t^z, u_t^\xi \) are random errors. By aggregating the production functions of the firms, assuming that they are identical and the capital-labor ratio is common across them, we have:

\[
Y_t^i = e^{zt \left( e^{\xi_t} \right)^\alpha} K_t N_t^{1-\alpha}
\]

Moreover, these firms fund capital purchases by issuing financial claims in period \( t \) \( (S_t) \), whose relative prices will be the capital price \((Q_t)\):

\[
Q_t S_t = Q_t K_{t+1}
\]

The real wage (11) can be obtained by minimizing costs. Moreover, given that intermediate good producers do not obtain profits and return the used capital to capital producers with a relative price equal to unity, the price of the funds is equivalent to the expected capital return \((R_t^i)\):

\[
W_t = P_t^i (1 - \alpha) \frac{Y_t^i}{N_t}
\]

\[
E_t \{ R_{t+1}^i \} = \frac{P_t^i \alpha Y_{t+1}^i}{e^{\epsilon_{t+1} K_{t+1}}} + Q_{t+1} - \delta e^{\xi_{t+1}}
\]

where \( P_t^i \) is the relative price of intermediate goods and \( 0 < \delta < 1 \) the depreciation rate.

### 2.1.3 Capital producers

The physical capital stock, whose net investment is produced with adjustment costs, is defined as follows\(^1\):

\(^1\)We have not included the shock that affects the quality of capital in these equalities, as we maintain that it should only be included in the equations relating to the productive sector of the economy.
\[ K_{t+1} = K_t + I_t^n \] (13)

\[ I_t^n = I_t - \delta K_t \] (14)

\( I_t^n \) being the net investment and \( I_t \) the gross investment. At the beginning of each period, capital producers convert the used capital, which has been acquired from intermediate goods producers, into new capital and resell it to them, along with the newly created capital. The refurbished capital does not entail adjustment costs, but only the net investment. If we formulate the investment decision problem, which is common to all capital producers, we can obtain the capital price. This problem is the following:

\[
\max E_t \sum_{T=t}^{\infty} \Lambda_{t,T} \left\{ Q_T I_T^{n,k} - \left[ I_T^{n,k} + f \left( \frac{I_T^{n,k} + I_k}{I_T^{n,k} + I_k} \right) \left( I_T^{n,k} + I_k \right) \right] \right\}
\]

where \( I_T^{n,k} = \frac{I_T^n}{K_t} \) and \( I_k = \frac{I_K}{K} \) is the value of the gross investment-capital ratio in the steady state. The functional form of the adjustment costs is:

\[
f \left( \frac{I_T^{n,k} + I_k}{I_T^{n,k} + I_k} \right) = \frac{\zeta}{2} \left( \frac{I_T^{n,k} + I_k}{I_T^{n,k} + I_k} - 1 \right)^2 \] (15)

\( \zeta > 0, f(1) = f'(1) = 0 \) and \( f''(1) > 0 \)

The price of the new capital is obtained from the first order condition:

\[ Q_t = 1 + f + \frac{I_t^{n,k} + I_k}{I_t^{n,k} + I_k} \frac{f'}{E_t \Lambda_{t,t+1} \left( \frac{I_t^{n,k} + I_k}{I_t^{n,k} + I_k} \right)^2} \] (16)

2.1.4 Retail firms

Each retailer differentiates a unit of intermediate good by re-packaging it. The final output \( Y_t \) is composed of a continuum of retail final goods:

\[ Y_t = \int_0^1 Y_{st}^{(\epsilon-1)/\epsilon} ds \]

where \( Y_{st} \) is the output of retailer \( s \). If users of the final output minimize costs:

\[ Y_{st} = \left( \frac{P_{st}}{P_t} \right)^{-\epsilon} Y_t \] (17)

\[ P_t = \left[ \int_0^1 P_{st}^{1-\epsilon} ds \right]^{1/\epsilon} \] (18)
where $P_{st}$ is the price of $Y_{st}$ and $P_t$ is the price index of the final output.

In order to include nominal rigidities, we follow the model of Calvo (1983). Consequently, we assume that retailers will adjust their prices each period with an exogenous probability $(1 - \theta)$, which is constant and common to all of them. Thus, the maximization problem of the firms, assuming zero TI, can be stated as follows:

$$
\max_{P_t^*} \sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} E_t \left\{ Y_{st+i} \left( \frac{P_{t}^*}{P_{t+i}} - P_{t+i}^i \right) \right\} \tag{19}
$$

s.t. \quad Y_{st+i} = \left( \frac{P_{t}^*}{P_{t+i}} \right)^{-\varepsilon} Y_{t+i}

where $P_t^*$ is the price set by those firms which change it at time $t$. Expression (19) can be rewritten by substituting the demand curve into the objective function in order to eliminate $Y_{st+i}$. This leads us to obtain the following first order condition:

$$
E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} E_t \left\{ (1 - \varepsilon) \left( \frac{P_t^*}{P_{t+i}} \right)^{-\varepsilon} Y_{t+i} + \varepsilon P_{t+i}^i \left( \frac{P_t^*}{P_{t+i}} \right)^{-\varepsilon} Y_{t+i} \right\} = 0 \tag{20}
$$

We can solve this equation for $P_t^*$ and, after some rearrangements, arrive at the expression of the optimal price which is set by all firms:

$$
P_t^* = \mu \frac{E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} (P_{t+i})^\varepsilon Y_{t+i} P_{t+i}^i}{E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} (P_{t+i})^{\varepsilon-1} Y_{t+i}} \tag{21}
$$

where $\mu = \frac{\varepsilon}{\varepsilon-1}$. Additionally, the general price level follows this path:

$$
P_t = \left[ (1 - \theta) (P_t^*)^{1-\varepsilon} + \theta (P_{t-1})^{1-\varepsilon} \right]^{\varepsilon-1} \tag{22}
$$

This is the standard derivation of the optimal price and the general price level without trend inflation. If we now abandon the $\Pi = 1$ assumption, $\Pi$ being the gross inflation rate in the steady state, the price equations (21) and (22) should be modified accordingly. We can now define $X_t = \frac{P_t^*}{P_t}$ and $\frac{P_t}{P_{t+i}^i} = \frac{1}{\prod_{k=1}^{i} \Pi_{t+k}}$ and obtain the expressions:

$$
X_t = \mu \frac{E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} \left( \prod_{k=1}^{i} \Pi_{t+k} \right)^\varepsilon P_{t+i}^i Y_{t+i}}{E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} \left( \prod_{k=1}^{i} \Pi_{t+k} \right)^{\varepsilon-1} Y_{t+i}} \tag{23}
$$

$$
X_t = \left[ \frac{1 - \theta}{1 - \theta \Pi_t} \right]^{\varepsilon-1} \tag{24}
$$
2.1.5 Financial intermediaries

The structure of our financial sector is based on the one developed by GK, who modify the original idea of Bernanke, Gertler and Gilchrist (1999). Since this section is a replication of the GK model, we will only make a brief presentation in order to state notation and describe the main relationships.

Financial intermediaries obtain funds from households and lend them to intermediate goods firms. Thus, financial intermediaries are the link between savers and investors. The balance of each intermediary $f$ is represented as:

$$Q_t S_{ft} = F_{ft} + D_{ft}$$  \hspace{1cm} (25)

$F_{ft}$ is the net wealth held by the intermediary at the end of period $t$, $D_{ft}$ are the deposits created by households which are remunerated at an interest rate $R_{t+1}$, $S_{ft}$ is the number of financial claims issued by goods producers that the intermediary has in its portfolio and $Q_t$ is the price of each claim. $R_{t+1}^q$ is the yield obtained by the financial intermediary derived from these claims. Therefore, the evolution of the bank’s wealth depends on the external finance premium $(R_{t+1}^q - R_{t+1})$:

$$F_{ft+1} = (R_{t+1}^q - R_{t+1}) Q_t S_{ft} + R_{t+1} F_{ft}$$  \hspace{1cm} (26)

Financial intermediaries provide funds if and only if they do not obtain losses with their operations, that is, if the external finance premium is equal to or greater than zero:

$$E_t \Lambda_{t,t+1+i} (R_{t+1+i}^q - R_{t+1+i}) \geq 0 \hspace{1cm} i \geq 0$$  \hspace{1cm} (27)

This condition always holds with equality under the assumption of frictionless financial markets. However, if these markets are imperfect, this relationship could be positive. In this way, the bankers maximize their expected wealth:

$$V_{ft} = \max E_t \sum_{i=0}^{\infty} (1 - \gamma)^i \Lambda_{t,t+1+i} \left( (R_{t+1+i}^q - R_{t+1+i}) Q_{t+i} S_{ft+i} + R_{t+1+i} F_{ft+i} \right)$$  \hspace{1cm} (28)

where $\gamma$ is the probability of survival of the bankers. Furthermore, we introduce the GK agency problem in order to limit the expansion of assets, which would occur if the inequality (27) is positive. The bankers have the opportunity to divert a proportion $\lambda$ of the available funds towards their households at the beginning of each period. However, if this occurs, the depositors can force bankruptcy and recover the proportion $(1 - \lambda)$ of available funds. Therefore, the depositors are willing to lend their funds to the bankers whenever the following equation holds:

$$V_{ft} \geq \lambda Q_t S_{ft}$$  \hspace{1cm} (29)
meaning that the gain from diverting a fraction $\lambda$ of assets is lower than the loss of doing it. We can write:

$$V_{ft} = v_t Q_t S_{ft} + h_t F_{ft}$$

where $v_t$ is the marginal gain of the banks derived from expanding their assets, $Q_t S_{ft}$, maintaining their net wealth fixed. It can be expressed as:

$$v_t = E_t \{(1 - \gamma) A_{t,t+1} (R_{t+1}^q - R_{t+1}) + A_{t,t+1} \gamma x_{t,t+1} v_{t+1}\}$$

$h_t$ is the expected value of having an additional unit of $F_{ft}$, assuming that $S_{ft}$ remains constant, with the following definition:

$$h_t = E_t \{(1 - \gamma) + \Lambda_{t,t+1} \gamma x_{t,t+1} h_{t+1}\}$$

$x_{t,t+i} = \frac{Q_{t+i} S_{ft+i}}{Q_t S_{ft}}$ is the growth rate of assets and $t_{t,t+i} = \frac{F_{ft+i}}{F_{ft}}$ is the growth rate of wealth. Constraint (29) can also be expressed as follows:

$$h_t F_{ft} + v_t Q_t S_{ft} \geq \lambda Q_t S_{ft}$$

When this constraint binds we have an equality. With $F_{ft} > 0$ and $v_t > 0$ the bankers obtain profits by expanding their assets and we have $v_t < \lambda$. Thus, the maximum amount of funds that the intermediaries can raise depends on their wealth, which can be stated as:

$$Q_t S_{ft} = \frac{h_t}{\lambda - v_t} F_{ft} = \phi_t^p F_{ft}$$

where $\phi_t^p$ can be interpreted as the private leverage ratio. This ratio increases with $v_t$ and its limit is the point at which the gain of diverting funds is offset by its cost, since the increase of this variable in turn augments the opportunity cost of being forced into bankruptcy. We can now rewrite (26) as follows:

$$F_{ft+1} = \left[ (R_{t+1}^q - R_{t+1}) \phi_t^p + R_{t+1} \right] F_{ft}$$

If we now redefine the variables $t_{t,t+1}$ and $x_{t,t+1}$, we obtain that:

$$t_{t,t+1} = \frac{F_{ft+1}}{F_{ft}} = \left( R_{t+1}^q - R_{t+1} \right) \phi_t^p + R_{t+1}$$

$$x_{t,t+1} = \frac{Q_{t+1} S_{ft+2}}{Q_t S_{ft+1}} = \frac{\phi_t^p F_{ft+1}}{\phi_t^p F_{ft}} = \frac{\phi_t^p}{\phi_t^p} t_{t,t+1}$$

Given that the banks’ total demand does not depend on firm-specific factors, we can aggregate it, which leads us to the following equation:

$$Q_t S_t = \phi_t^p F_t$$
If we now distinguish between the wealth of the new \((F_t^n)\) and the old \((F_t^o)\) bankers, the total wealth can be stated as:

\[
F_t = F_t^o + F_t^n
\]  

(39)

where

\[
F_t^o = \gamma \left[ (R_t^q - R_t) \phi_{t-1}^o + R_t \right] F_{t-1}
\]  

(40)

Finally, the initial funds\(^3\) of the new bankers are defined as the ratio \(\omega\) of the old bankers’ wealth, which corresponds to \((1 - \gamma) Q_t S_{t-1}\). This leads us to the following expression:

\[
F_t^n = \omega Q_t S_{t-1}
\]  

(41)

Combining (40) and (41), we obtain the evolution of the net wealth:

\[
F_t = \gamma \left[ (R_t^q - R_t) \phi_{t-1}^o + R_t \right] F_{t-1} + \omega Q_t S_{t-1}
\]  

(42)

2.1.6 Central Bank

The central bank is responsible for implementing monetary policy. It takes decisions about the short-term nominal interest rate \((R_t^n)\) in each period following a Taylor rule that is specified below:

\[
R_t^n = R\Pi \left( \frac{\Pi_t}{\Pi} \right) \phi_{\pi} \left( \frac{Y_t}{Y} \right) \phi_{\gamma} e^{\eta_t}
\]  

(43)

where \(R\) is the intercept reflecting the structural factors in the reaction function of the central bank (which can also be interpreted as the natural interest rate), \(\Pi\) is the steady state gross inflation or target, \(Y\) is the steady state level of output consistent with \(\Pi\), \(\phi_{\pi}, \phi_{\gamma}\) are, respectively, the parameters that measure the central bank’s reaction to inflation and output deviations from its steady state levels, and the monetary policy shock \(\eta_t\) is defined as an AR(1) process:

\[
\eta_t = \rho_\eta \eta_{t-1} + u_t^\eta
\]  

(44)

where \(\rho_\eta \in [0,1)\) and \(u_t^\eta\) is the random error. Finally, the relationship between the real and nominal interest rate is set by the Fisher equation:

\[
R_t^n = R_t E_t \Pi_{t+1}
\]  

(45)

During specific periods in which the private financial system is unable to provide the necessary liquidity to firms due to balance sheets constraints, the central bank can act as a direct lender to the non-financial firms. \(S_{t}^{cb}\) denotes the amount of loans

\(^3\)These funds are transferred by the households.
issued by the central bank, assessed at the price of capital, and $S_p$ the financial claims intermediated by financial intermediaries, so the total amount of loans can be stated as follows:

$$Q_t S_t = Q_t S_p^t + Q_t S_c^t$$ (46)

To support these actions, the central bank issues riskless debt $D_c^t$, purchased by households, and pays the market lending rate $R_t + 1$. Although these operations are not restricted by firms’ balance sheets, they do have associated efficiency costs $\tau$. The central bank is willing to pay a proportion $\psi_t$ of all claims, obtaining a profit of $D_c^t (R_t^q - R_t^q - R_t^q)$:

$$D_c^t = Q_t S_c^t = \psi_t Q_t S_t$$ (47)

Therefore, we can rewrite equation (46) to incorporate this lending mechanism:

$$Q_t S_t = \phi^T_t F_t + \psi_t Q_t S_t = \phi^T_t F_t$$ (48)

where $\phi^T_t$ is the leverage ratio of total intermediated funds:

$$\phi^T_t = \frac{1}{1 - \psi_t} \phi^p_t$$ (49)

The cost of intervention is funded with taxes and profits from financial intermediation. This is reflected in the following equation:

$$\tau \psi_t Q_t K_{t+1} = T_t + D_{t-1}^b (R_t^q - R_t)$$

We analyze the effects of this kind of monetary policy specifically in Section 4.

### 2.2 Equilibrium Conditions

The aggregate equilibrium of the economy is defined as follows:

$$Y_t = C_t + I_t + f \left( \frac{I_{t-1}^{n,k} + I_k}{I_{t-1}^{n,k} + I_k} \right) (I_{t-1}^a + I) + \tau \psi_t Q_t K_{t+1}$$ (50)

For the sake of simplicity, we assume that there are no other public expenditures than those derived from the efficiency costs of the unconventional monetary policy. The total output of the economy weighted by the price dispersion $\Delta_t = \int_0^1 \left( \frac{P_s}{P_s} \right)^{-\varepsilon} ds$ is equivalent to the intermediate goods firms’ output.

$$Y_t^i = \Delta_t Y_t$$ (51)
Assuming that the price distribution between firms that do not change their price is the same as the full price distribution in period $t$, we can obtain that:

$$\Delta_{t+1} = \theta \Pi_{t+1} \Delta_t + (1 - \theta) X_{t+1}$$

(52)

Including these three equations, the model is closed.

2.3 Steady State Equilibrium

Since our aim is to analyze the long-term behavior of the model, it is necessary to precisely define the steady state and the key relationships that emerge in this situation. Because our model incorporates economic growth, some of the variables grow in the steady state, so the model must be normalized to show constant steady values. In order to do this, the normalization of the growing variables $(w_t, C_t, K_t, I_t, F_t, Y_t, Y^*), F_t, F^n, F^n)$ by the capital is needed. Economic growth is represented by the gross growth rate $G_t = \frac{K_t}{K_{t-1}}$. We detail the normalized model in Appendix A1, where normalized variables are denoted with a superscript $k$. The equations of the normalized model evaluated in the steady state are reported in Appendix A2, where the variables without a time subscript are the steady state values. The solution of the system is characterized by the pair of values of $G$ and $v$ that, given $\Pi$, satisfy the following two relationships:

$$G = \frac{(1 - \gamma)^2 \omega}{(1 - \psi) [\lambda (1 - \gamma) - v] \left[ 1 - \frac{\gamma G}{\beta} \frac{\lambda}{\lambda - v} \right]}$$

(53)

$$G = \frac{1}{1 + \frac{\tau \psi}{\alpha X \Psi}} \left\{ \mu Y \Theta \left[ \frac{1}{\Delta} - \frac{(1 - \alpha) X \Psi}{\chi \mu Y} \left( \frac{\mu Y \Theta}{\alpha \Psi X} \right)^{(1+\psi)/(1-\alpha)} \right] + (1 - \delta) \right\}$$

(54)

where $\Theta = \left[ 1 + \frac{(1 - \gamma) - v}{\lambda (1 - \gamma)} \right] G / \beta - (1 - \delta)$, $\Psi = (1 - \theta \Pi^\psi)$ and $X = \left[ \frac{1 - \theta}{1 - \theta \Pi^\psi} \right]^{1/(1-\psi)}$.

It is clear that $Y = \Psi = X = 1$ whenever $\theta = 0$. Then, $G$ is independent of TI without price rigidities but is affected by $v$. If there is no financial frictions $R^q = R$ and, hence, $v = 0$. Thus, (53) is not relevant and $G$ in (54) is independent of the financial variables, provided $\tau = \psi = 0$, but the dependence on TI remains. Consequently, $G$ is independent of TI and the financial variables without price rigidities and financial frictions.

Equations (53) and (54) are obtained as equations (A2.30) and (A2.33) in Appendix A2. The first contains the pair of values $(G, v)$ compatible with equilibrium in the financial sector and the second in the goods market, given the value of TI. They determine the steady state equilibrium in the plane $(G, v)$ given the value of $\Pi$ targeted by the monetary policy. We should note that, unlike GK, we have endogeneized all
the steady state values, including the private leverage ratio. The existence of these two relationships between the economic growth rate and the marginal gain of the financial intermediaries for expanding their assets (which determines key factors such as the private leverage ratio), indicates that financial variables affect economic growth in the long term. This is a key result of our analysis. The variable \( v \) which, in turn, depends positively on the dynamics of the stock of credit accumulated and on the discounted expected external finance premium (the two components of the expected profits of the financial intermediaries) is the essential link through which the financial system affects the growth rate in the long run. Thus, the credit available in the economic system and its expected returns will determine the pace and the trend of the real activity. Moreover, the fact that the relationship between \( G \) and \( v \) in the steady state depends on TI establishes a leading role for the monetary policy through the influence of the financial activity in the growth process and its dependence on the external finance premium, not only in the performance of the financial system but also in the growth of the economy. We will study this mechanism more deeply in the calibration section.

3 Calibration, Steady State and Shocks

In this section we evaluate the model numerically in order to ascribe the values of the variables in the steady state and their responses to some selected shocks. Table 1 shows the values assigned to the parameters.
The values of the parameters $\beta, \alpha, \delta$ are standard in the literature. The remaining parameter values like $\epsilon, \varphi, \theta, \varsigma, \phi_\pi, \phi_y$ and those related to financial variables such as $\lambda, \gamma$ and $\omega$ are taken from GK. The exception is the parameter that reflects the relative utility weight on labor, calibrated to get an annual growth of 2.5% in our benchmark scenario where annual TI is also 2.5%. This calibration ensures a stable steady state and reasonable rates of economic growth, interest rate spread and private leverage at any level of trend inflation without reducing the consistency of the model.

With these values as a reference, the steady state is analyzed in a first subsection and the effects of two types of shocks are described in a second one.

### 3.1 Steady State Analysis

Given the values of the parameters, we can find the steady state solution in the plane $\{v, G\}$. In the benchmark scenario, this point is $\{v = 0.0052, G = 1.0062\}$. Accordingly, when the incentive constraint (33) binds with $F^k > 0$ we have $0 < v < \lambda$. The two main equations which determine the steady state are plotted in Figure 1, where the red line corresponds to equation (53) and the blue line to equation (54). The crossing point determines the values of $G$ and $v$ that make the equilibrium in the financial sector and in the goods market simultaneously possible.
We see that, while the function containing the equilibrium points in the financial sector is not monotonous, the equilibrium points in the goods market show an inverse relationship between $G$ and $v$ in the relevant range of values ($0 < v < \lambda$). This U-shaped relationship between $G$ and $v$ in the financial sector is an interesting structural characteristic of the model that provides remarkable subsequent results related to the TI. These outcomes follow from the fact that, while the U-shaped relationship does not depend on TI, the other function does.

[Figure 1 about here]

Taking the pair of values mentioned above as our point of departure, from the equations defined in Appendix A2 we can obtain the values of the remaining variables. The private leverage ratio is 4.85, a level consistent with the results of GK, and the external finance premium is 117 basis points, a reasonable value given the TI considered.

However, these results depend on the value targeted for TI. If we consider the scenarios within a wide range $[-5\%, 8\%]$ of the annualized rate of TI, one of the most interesting results obtained concerns the non-linear relationship between the long-run growth rate and TI, as we show in the first plot of Figure 2. It can be seen how the long-term growth rate and trend inflation are positively related up to a maximum value located around $1.7\%$ of the annualized TI, after which they decline faster. Normalized investment and normalized financial wealth also peaks at $TI = 1.7\%$, whilst the marginal gain of the financial intermediaries, the external finance premium and the private leverage reach their minimum. Regarding other variables, normalized final output, normalized consumption and real wage are highest when $TI=0.5\%$, while the price of intermediate goods reaches its maximum at $TI = 1\%$. Although normalized variables reach their respective maximums for different levels of trend inflation, Figure 2 shows in the last row that, at a point in time far enough away, which would be equivalent to the steady state, the households’ welfare specified in equation (1) is maximized when $TI = 1.7\%$. The last plot displays the deviations from the values when $TI=1.7\%$ for the main variables.

These findings suggest that there is an optimal level of trend inflation that maximizes growth and welfare, as is also concluded in Amano et al. (2009). The difference in our conclusion is that the optimal value of TI does not imply deflation. Coibion, Gorodnichenko, and Wieland (2010) do not include endogenous growth in their model, but also find an optimal rate for positive but low levels of trend inflation. Thus, a shift in TI alters the steady state values of the real variables. The relationship between TI and growth suggests that the monetary policy goal of stabilizing inflation around a certain level could be conditioning economic growth and output

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4The exact value is $1.72\%$.  
5Growth is plotted as percentage points differential.
level in the long term. Therefore, given that monetary policy affects TI through the specification and targets of the monetary policy rules and the way those rules react to deviations in the inflation rate, monetary policy not only affects business cycles, but also modifies the long run equilibrium. This result highlights that monetary policy is not neutral in the long term, confirming conclusions recently reached by, among others, Amano et al. (2009) and Vaona (2012) in different and more restricted contexts as they do not consider the role played by the financial sector.

To understand the mechanism described above is crucial in order to examine the behavior of the variables pertaining to the financial sector. Figure 2 shows the relationship between TI and some essential financial variables. The relationship between TI and the external finance premium is non-linear since, for low rates, it decreases but, from the level of TI that maximizes the growth rate, it increases. After that level, the shortening of the capital return is lower than the reduction of the real interest rate on assets, so the marginal gain for the financial intermediaries to expand their assets goes up and, accordingly, the private leverage ratio increases. The increase in this ratio lowers the net wealth held by the intermediaries and, consequently, capital growth falls. The coincidence of the TI level that maximizes the growth rate with the lowest values of the marginal gain of the financial intermediaries and the external finance premium is an outstanding result in our model. In fact, the real interest rate and the capital return are also maximum with this value of TI. This is a consistent set of symmetrical results that strengthen our approach.

We now carry out a sensitivity analysis in order to check the robustness of our results as well as to determine the impact on the long-run equilibrium of changes in the parameters that reflect the structure of the financial system. Figure 3 displays the differential in percentage points related to the benchmark steady state of our two key variables, \( v \) and \( G \), for a wide range of values of the financial sector parameters \( \lambda, \gamma, \omega, \tau \) and \( \psi \). First, we should note that the effects of the parameters are opposite on the marginal gain of the financial intermediaries and on the economic growth rate, whilst the level of trend inflation barely affects the equilibrium for different parameter values. Marginal gain rises whilst economic growth diminishes with the fraction of capital that bankers can divert. This means that the proportion of the potential unjustified economic losses suffered by the financial system affects the financial marginal gain and the real activity negatively in the long run. Comparing the two extreme cases considered, the marginal gain rate increases by 6 percentage points and growth decreases by 2. As regards the survival rate of the bankers, the parameter that could represent the dynamic process of entry and exit of intermediaries in the financial

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6The relationship is due to price rigidities. All the conclusions are unchanged for reasonable values of \( \theta \).
system, economic growth increases with its value. For example, if \( \gamma = 0.91 \) instead of 0.97, which would be equivalent to an easier entry, the steady state growth rate would decrease one percentage point. This suggests that mobility in the composition of the financial system discou...
Thus, inflation depends on the inflation expectations, the marginal costs of retailers and the output gap growth rate. Note that, if $\Pi = 1$, this expression is reduced to the standard New Keynesian Phillips curve.

After linearization, to evaluate the model response we analyze two standard types of shocks: a monetary policy shock and a technology shock. For each possible scenario, Figures 4 and 5 show the deviations from each steady state value of the main variables expressed in percentage points across 80 quarters. The darker lines represent higher TI and the lighter ones lower TI. We have reduced the considered range of TI to $[0\% - 5\%]$ in order to focus on the most probable scenarios as well as to appreciate the behavior above and below 1.7%, the value that maximizes the growth rate in the steady state.

Firstly, to assess the impact and persistence of monetary policy on the dynamics of the variables, we will submit the model to a standard monetary shock defined in equation (44). Regarding the size of the shock, we will force an increase of 25 basis points in the quarterly short-term nominal interest rate, which corresponds to an annual rate of 1% not explained by the variables included in the monetary policy rule.

Figure 4 shows the responses of both financial and real variables for the different values of TI. The growth rate decreases with more intensity when the steady state inflation rate is higher and, even though the effect is slight, the difference between extreme scenarios is wide. The effect diminishes gradually but remains for up to 20 years after the shock, which is longer than in the standard New Keynesian model due to the financial accelerator mechanism. Normalized net investment response and its relationship with TI is similar, though broader, to the growth rate response. Normalized output is less dependent on the level of TI, although the initial impact of the shock produces a response 9% higher when TI=5% than when TI=0%.

So far as the financial variables are concerned, the effects on the marginal gain for expanding financial assets, external finance premium, private leverage and the net wealth of financial intermediaries are larger when TI is higher. The sequence of the events is as follows. The rise of the nominal interest rate results in an increase of the interest rate differential. This spread pushes up the marginal gain to expand the assets for the intermediaries. Therefore, the private leverage ratio grows nearly 3% in the benchmark scenario. The result of these movements is a decrease of the intermediary normalized net wealth of 4% which finally causes a decline in the capital growth. The inflation rate falls as expected, without a clear effect of TI.

Therefore, it has been shown that an exogenous shock in the monetary policy rule causes effects on the real and financial variables that depend on the TI level. These
effects are greater for all variables the higher the TI level (except the inflation rate, which is not affected by TI in terms of deviations from the steady state level).

Secondly, we will study the effects of a technology shock in (7). The disturbance is a positive deviation of 1% in the total factor productivity included in the production function. Figure 5 displays the impulse response functions of the variables.

As in the monetary shock experiment, the response of the variables to a technology shock also depends on the TI considered, but in this case the differences are mild. At the beginning, the technology shock slightly increases the growth rate without differences but the increase is greater later the lower is TI. Normalized investment increases up to 0.9% and these effects remain for 20 years after the shock. Normalized output increases around 0.8%, and more with high TI, and the response remains after 7 years. The effects on the financial variables are similar for any TI. As a result of the rise in the external finance premium, asset prices increase and the financial intermediaries balance sheets improve. These movements lead to a lowering of the marginal gain of the intermediaries and, therefore, to a deleveraging process up to 10%. The 12% increase of the net wealth derived from the previous shifts increases the growth rate. The inflation rate drops more with high TI.

So far, we have analyzed conventional shocks, concluding that the magnitude and persistence of the effects of the shocks have been amplified by the financial accelerator, but also depend on the TI level. These effects are greater the higher is the TI value with a monetary shock, except for the inflation rate which is not affected by the TI. In the case of the technology shock the effect tend to be greater on normalized output and inflation rate the higher is the TI. The opposite occurs on growth rate and normalized investment whilst the effects on the financial variables do not depend on the TI level. Finally, we should note that none of the proposed shocks reaches the zero lower bound of the nominal interest rate.

In the next section, we will force a crisis in the economy in order to study the unconventional monetary policy.

4 Unconventional Monetary Policy

Hitherto we have assumed that the central bank only implements monetary policy by setting the short-term nominal interest rate, that is, through conventional monetary policy. Now let us assume that the central bank can also implement unconventional monetary policy by acting as a direct lender. Although the central bank has to bear efficiency costs in these interventions and, thus, is less efficient than private intermediaries, it does not face any restriction. The central bank has the power to
act as a financial intermediary, but we assume, as GK do, that it only uses this power, to any extent, during crisis periods. The crisis is characterized by a sudden drop in the quality of capital, causing a rise in the external finance premium. These movements are consistent with what has been seen during the financial crisis. The evolution of the unconventional monetary policy responds to the following pattern:

$$\psi_t = \psi + b \left[ E_t \left( R^d_{t+1} - R^q_{t+1} \right) - (R^d - R) \right]$$  \hspace{1cm} (60)

where $b$ is the policy parameter that represents the degree of central bank reaction to deviations in the external finance premium from its steady state value and $\psi$ the steady state level of the unconventional monetary policy. We assume that $\psi = 0.001$, a positive value but close to zero. The shock, defined in (8), is a 5% decline in the capital quality with a persistence degree parameter of 0.66. The responses of the main variables in percentage deviations from the steady-state level along 80 quarters are shown in Figure 6. We distinguish four degrees of intervention represented by parameter $b$ ($b = 0$ or no intervention, $b = 10$ or low intervention, $b = 50$ or medium intervention and $b = 100$ or high intervention).

The consequences of this disturbance for the optimal TI and without the credit policy are represented by the black line in Figure 6 and are characterized as follows. The reduction of the effective level of capital generates a decline in the total assets value. In addition, the deterioration of the position of the financial intermediaries’ balance sheets diminishes the demand for capital and, therefore, its price. Thus, the decline in investment and its price causes a further deterioration in the balance sheets that is amplified by the private leverage. Due to the fall in effective capital, the interest rate spread increases and, therefore, there is an increase in the marginal gain and drops in both the normalized output and the normalized net investment. Inflation increases initially but, after five quarters, it falls below the steady-state level.

We now focus on the differences in the model response to the intensity of credit policy. As can be seen in Figure 6, the response of the variables to this kind of shock depends on the degree of the central bank intervention. Real variables, such as growth rate or normalized output, normalized investment and its price, go down, as expected. Inflation and labor increase in the very initial periods to drop afterwards. The responses are clearly dampened by central bank policies.

As regards financial variables, the marginal gain of the financial intermediaries depends largely on the unconventional actions of the central bank. This variable increases by 150% if the monetary authorities do not interfere, and only by 45% if

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8We only present the results for the optimal level of trend inflation, TI=1.7% for the sake of clarity and because there are slight variations between scenarios for this type of shock.
they do so with a high degree of intensity. In response to the shift of the marginal gain, private leverage will also increase, in this case by more than 130% without intervention and only by 40% with a high intervention. This movement is contrary to the predictions made by GK, who describe a deleveraging process. In our model the private leverage ratio increases considerably, as seen in the U.S. data\(^9\), where it can be observed how this ratio has increased about 25% from 2008 to 2013 despite the substantial liquidity injections made by the Federal Reserve. Finally, the normalized financial wealth of intermediaries also depends on the intensity of the policy. As in the previous section, nominal interest rate does not reach its zero lower bound.

Another topic of interest is the persistence of the shock in terms of the intensity of the unconventional monetary policy. As can be seen in Figure 6, this type of policies mainly reduces the initial impact of the shock on the financial variables. However, the more intense the intervention, the greater the persistence of the effects on all variables. For example, in the case of intermediaries’ wealth, if the central bank does not interfere, the effects disappear within 10 years of the shock. Conversely, if the degree of intervention is medium or high, the effects endure for more than 20 years after the shock. Hence, the general conclusion to be drawn is that the greater the intensity of the credit policy, the lower the negative initial impact of the crisis but the longer the response of the variables to a decrease in the value of capital.

Therefore, monetary authorities should adopt a long-term perspective weighting the magnitude and persistence of the effects on real and financial variables in order to choose the intensity of the unconventional monetary policy. The consideration of this perspective could increase the accuracy of the policies that central banks implement in crisis periods.

### 5 Conclusions

This article extends the standard New Keynesian DSGE model with endogenous economic growth, financial frictions and trend inflation in order to get a suitable framework to analyze the long-lasting effects of monetary policy on real and financial variables. In the analysis of the steady state and the dynamics of the model, we find that both the financial accelerator and trend inflation affect economic growth in the short and the long term.

After calibrating the model, firstly, we have studied the long-run behavior. Our model shows the existence of a key link in the long run between the growth rate and the marginal gain for the expansion of the financial assets, given the value of the trend inflation. A non-linear relationship between long-term growth rate and trend inflation is one of our outstanding results where, in a model based on the one used in Gertler and Karadi (2011), growth rate, normalized investment and normalized

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\(^9\)Source: SNL Financial. Core Capital as a percent of average total assets minus ineligible intangibles.
financial wealth reach a maximum when annualized trend inflation is 1.7%, whilst leverage, the external finance premium and the marginal gain of the financial intermediaries reach a minimum. This is a symmetric and coherent set of results that show the strength of the long-run connections between economic growth, the profitability of the financial sector and monetary policy.

The long-run relevance of TI disappears whenever there is not price rigidities, but the connection between the growth rate and the financial variables is not affected. Alternatively, this last connection disappears without financial frictions, remaining the non-linear relationship between the growth rate and TI. Consequently, the growth rate is independent of TI and the financial variables without price rigidities and financial frictions.

So, financial variables affect the steady state equilibrium growth rate, especially the marginal gain for expanding the financial assets, given the level of trend inflation. Through the sensitivity analysis of the financial parameters, we have found that the structure of the financial sector has a well established set of implications for the long-run equilibrium, keeping the trend inflation constant. On the one hand, the rate of growth decreases with the fraction of capital the bankers can divert and the rate of survival of the bankers. On the other hand, growth rate increases with the initial capital of the new bankers and the intensity of the unconventional monetary policy. The effect on the marginal gain is the opposite.

Then, we have focused on the dynamics of the model following the emergence of different disturbances. The response of the main variables to a monetary policy shock is higher when the trend inflation is high. In the case of a technology shock, the dynamics of the real and financial variables are less dependent on trend inflation.

Finally, with the aim of assessing the effects of an unconventional monetary policy, we have simulated a crisis by forcing a decline in the quality of capital. It has been shown that real variables such as economic growth rate, output and investment are affected by the credit policy. Likewise, the response of financial variables such as the marginal gain, the leverage ratio or the financial wealth of intermediaries also depends on the intensity of the credit policy. Furthermore, when the credit policy is of higher intensity, although the negative effects of this shock are smaller at first, mainly on the financial variables, their persistence is higher, lasting at least 20 years after the disturbance. Thus, we demonstrate that more aggressive credit policies may lead to a prolongation of the effects of crises beyond the medium term.

In sum, it has been shown that, if the monetary authorities wish to encourage long-term economic growth, they should take care of the long-run nonlinear connections between economic growth, the profitability of the financial sector and the main target of the monetary policy.
References


28


A 1. Normalized model

\[ w_t^k = \chi C_t^k N_t^{\nu} \quad (A1.1) \]

\[ E_t \Lambda_{t,t+1} R_t = 1 \quad (A1.2) \]

\[ \Lambda_{t,t+1} = \beta \frac{C_t^k}{C_{t+1}^k G_{t+1}} \quad (A1.3) \]

\[ w_t^k = (1 - \alpha) P_t Y_t^i \quad (A1.4) \]

\[ E_t \{ R_{t+1}^q \} = \frac{P_t^i \gamma_t^{i,k} + Q_{t+1} - \delta}{Q_t} \quad (A1.5) \]

\[ Y_t^{i,k} = e^{\gamma_t} (e^{\xi_t})^\alpha N_t^{1-\alpha} \quad (A1.6) \]

\[ I_t^{n,k} = I_t^k - \delta \quad (A1.7) \]

\[ G_{t+1} = 1 + I_t^{n,k} \quad (A1.8) \]

\[ Q_t = 1 + f + \frac{I_t^{n,k} + I_t^k}{I_t^{n,k} + I_t^k} f' - E_t \Lambda_{t,t+1} \left( \frac{I_{t+1}^{n,k} + I_{t+1}^k}{I_t^{n,k} + I_t^k} \right)^2 f' \quad (A1.9) \]

\[ Y_t^{k} = C_t^k + I_t^k + f \left( \frac{I_t^{n,k} + I_t^k}{I_t^{n,k} + I_t^k} \right) \left( I_t^{n,k} + I_t^k \right) + \tau \psi_t Q_t G_{t+1} \quad (A1.10) \]

\[ Y_t^{k} \Delta_t = Y_t^{i,k} \quad (A1.11) \]

\[ v_t = E_t \left\{ (1 - \gamma) \Lambda_{t,t+1} (R_{t+1}^q - R_{t+1}) + \gamma \Lambda_{t,t+1} x_{t,t+1} v_{t+1} \right\} \quad (A1.12) \]

\[ h_t = E_t \left\{ (1 - \gamma) + \gamma \Lambda_{t,t+1} t_{t,t+1} h_{t+1} \right\} \quad (A1.13) \]

\[ \phi_t^p = \frac{h_t}{\lambda - v_t} \quad (A1.14) \]

\[ t_{t,t+1} = (R_{t+1}^d - R_{t+1}) \phi_t^p + R_{t+1} \quad (A1.15) \]
\[ x_{t,t+1} = \frac{\phi_{t+1}^p}{\phi_t^p} x_{t,t+1} \]  
(A1.16)

\[ Q_t G_{t+1} = \phi_t^T F_t^k \]  
(A1.17)

\[ \phi_t^T = \frac{1}{1 - \psi_t^p} \phi_t^p \]  
(A1.18)

\[ \psi_t = v \left[ E_t \left( R_t^q - R_t \right) - (R_t^q - R) \right] \]  
(A1.19)

\[ F_t^k = F_t^{o,k} + F_t^{n,k} \]  
(A1.20)

\[ F_t^{o,k} = \gamma \left[ (R_t^q - R_t) \phi_{t-1}^p + R_t \right] F_{t-1}^k \]  
(A1.21)

\[ F_t^{n,k} = (1 - \gamma) \omega Q_t \xi_t \]  
(A1.22)

\[ X_t = \mu_0 \sum_{i=0}^{\infty} \theta^i \Lambda_{t+i} \left( \prod_{k=1}^{i} \Pi_{t+k} \right)^{\epsilon} P_{t+i}^{i} Y_{t+i}^{k} \]  
\[ E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t+i} \left( \prod_{k=1}^{i} \Pi_{t+k} \right)^{\epsilon-1} Y_{t+i}^{k} \]  
(A1.23)

\[ X_t = \left[ \frac{1 - \theta}{1 - \theta \Pi_t^{-1}} \right]^{-1} \]  
(A1.24)

\[ \Delta_{t+1} = \theta \Pi_{t+1} \Delta_t + (1 - \theta) X_{t+1}^{- \epsilon} \]  
(A1.25)

\[ R_t^n = R \Pi_t \left( \frac{\Pi_t}{\Pi} \phi_t^o \left( \frac{Y_t^k}{Y_t^k \xi_t} \right) \phi_t^g \right) e^n \]  
(A1.26)

\[ R_t^n = R_t E_t \Pi_{t+1} \]  
(A1.27)

### A 2. Steady State Equations

\[ w^k = \chi C^k N^p \]  
(A2.1)

\[ \Lambda R = 1 \]  
(A2.2)

\[ \Lambda = \frac{\beta}{G} \]  
(A2.3)
\[w^k = (1 - \alpha) P^i \frac{Y_{i,k}}{N}\]  
(A2.4)

\[R^q = \frac{P^i \alpha Y_{i,k} + Q - \delta}{Q}\]  
(A2.5)

\[Y_{i,k} = N^{1-\alpha}\]  
(A2.6)

\[Q = 1\]  
(A2.7)

\[G = (1 - \delta) + I^k\]  
(A2.8)

\[Y^k = C^k + I^k + \tau \psi QG\]  
(A2.9)

\[Y^k \Delta = Y_{i,k}\]  
(A2.10)

\[v = (1 - \gamma) \Lambda (R^q - R) + \gamma \Lambda xv\]  
(A2.11)

\[h = (1 - \gamma) + \gamma \Lambda th\]  
(A2.12)

\[\phi^p = \frac{h}{\lambda - v}\]  
(A2.13)

\[t = (R^q - R) \phi^p + R\]  
(A2.14)

\[x = t\]  
(A2.15)

\[QG = \phi^T F^k\]  
(A2.16)

\[F^k = F^{o,k} + F^{n,k}\]  
(A2.17)

\[F^{o,k} = \gamma [(R^q - R) \phi^p + R] F^k\]  
(A2.18)

\[F^{n,k} = (1 - \gamma) \omega Q\]  
(A2.19)

\[X = P^i \mu \frac{1 - \theta \beta \Pi^{k-1}}{1 - \theta \beta \Pi^e}\]  
(A2.20)
These are the normalized equations evaluated in the steady state. We must now define the reduced system in order to find the endogenous equilibrium. If we equalize (A2.1) and (A2.4) and we additionally use (A2.6), (A2.8), (A2.9), (A2.20) and (A2.21), we can solve for $G$:

$$G = \frac{1}{1 + \tau \psi} \left[ \frac{N^{1-\alpha}}{\Delta} - \frac{X (1 - \alpha) \Psi}{\chi \mu \bar{\gamma}} N^{-\varphi-\alpha} + (1 - \delta) \right]$$  \hspace{1cm} (A2.26)

where $\bar{\gamma} = (1 - \theta \beta \Pi^{\epsilon-1})$, $\Psi = (1 - \theta \beta \Pi^{\epsilon})$. Furthermore, from (A2.11)-(A2.15) we obtain that:

$$(R^q - R) = \frac{\lambda (1 - \gamma - v)}{\lambda - v} G \frac{\lambda}{\beta (1 - \gamma)}$$  \hspace{1cm} (A2.28)

Now, from (A2.2), (A2.3), (A2.7), (A2.13), and (A2.16)-(A2.19):

$$1 = \frac{(1 - \gamma) \omega}{(1 - \psi) \beta (R^q - R) (\lambda - v) \left[ 1 - \gamma G \frac{\lambda}{\beta (1 - v)} \right]}$$  \hspace{1cm} (A2.29)

Replacing (A2.28) in (A2.29):

$$G = \frac{(1 - \gamma)^2 \omega}{(1 - \psi) \left[ \lambda (1 - \gamma - v) \left[ 1 - \gamma G \frac{\lambda}{\beta (1 - v)} \right] \right]}$$  \hspace{1cm} (A2.30)

From (A2.2), (A2.3), (A2.5), (A2.6), (A2.7) and (A2.20):

$$(R^q - R) = \frac{\alpha X \Psi}{\mu \bar{\gamma}} N^{1-\alpha} + (1 - \delta) - \frac{G}{\beta}$$  \hspace{1cm} (A2.31)

Equating (A2.28) and (A2.31):

$$N = \left\{ \frac{\mu \bar{\gamma}}{\alpha X \Psi} \left[ \left( 1 + \frac{[\lambda (1 - \gamma - v) v]}{(\lambda - v)} \right) \frac{G}{\beta} - (1 - \delta) \right] \right\}^{\frac{1}{1-\alpha}}$$  \hspace{1cm} (A2.32)
Introducing the last expression in (A2.26):

\[
G = \frac{1}{1 + \tau^\psi} \left\{ \frac{\mu \gamma \Theta}{\alpha X \Psi} \left[ \frac{1}{\Delta} - (1 - \alpha) \frac{X \Psi}{\chi \mu Y} \left( \frac{\mu \gamma \Theta}{\alpha X \Psi} \right)^{-(1+\alpha)} \right] + (1 - \delta) \right\} 
\]

(A2.33)

where \( \Theta = \left[ \left( 1 + \frac{\lambda(1-\gamma)-\psi \psi}{(1-\gamma) \lambda - \psi} \right) \frac{G}{\beta} - (1 - \delta) \right] \)

\[A\]

3. Log-linearized Model

The accented variables refer to the logarithmic deviation with respect to its steady state value.

\[
\tilde{\omega}^k_t = \tilde{c}^k_t + \varphi \tilde{n}_t 
\]

(A3.1)

\[
\tilde{\Lambda}_{t,t+1} + \tilde{r}_t = 0 
\]

(A3.2)

\[
\tilde{\Lambda}_{t,t+1} = \tilde{c}^k_t - \tilde{c}^k_{t+1} - \tilde{g}_{t+1} 
\]

(A3.3)

\[
\tilde{\omega}^k_t = \tilde{p}^i_t + \tilde{y}^{i,k}_t - \tilde{n}_t 
\]

(A3.4)

\[
R^q \tilde{r}^q_{t+1} = \alpha Y^{q,k} P^i \left( \tilde{p}^i_{t+1} + \tilde{y}^{i,k}_{t+1} \right) + \tilde{q}_{t+1} - R^q \tilde{g}_t + (1 - \delta) \tilde{\xi}_{t+1} 
\]

(A3.5)

\[
\tilde{y}^{i,k}_t = \tilde{z}_t + \alpha \tilde{\xi}_t + (1 - \alpha) \tilde{n}_t 
\]

(A3.6)

\[
\tilde{r}^{n,k}_t = \frac{I^k}{I^{n,k}} \tilde{r}^{k}_t 
\]

(A3.7)

\[
\tilde{g}_{t+1} = \frac{I^{n,k} \tilde{r}^{n,k}_t}{G} 
\]

(A3.8)

\[
\tilde{q}_t = \frac{\varsigma}{I^{n,k}} \left[ \left( \tilde{r}^{n,k}_t - \tilde{r}^{n,k}_{t-1} \right) - \tilde{\Lambda}_{t,t+1} \left( \tilde{r}^{n,k}_{t+1} - \tilde{r}^{n,k}_t \right) \right] 
\]

(A3.9)

\[
\tilde{y}^k_t = \frac{C^k}{Y^k} \tilde{c}^k_t + \frac{I^k}{Y^k} \tilde{r}^k_t 
\]

(A3.10)

\[
\tilde{y}^{i,k}_t = \tilde{\Delta}_t + \tilde{y}^k_t 
\]

(A3.11)
\[ \hat{v}_t = \tilde{\Lambda}_{t,t+1} + \frac{(1 - \gamma) \Lambda}{v} (R^q \tilde{r}_t - R \tilde{r}_{t+1}) + \gamma \Lambda x (\tilde{x}_{t,t+1} + \hat{v}_{t+1}) \]  

(A3.12)

\[ \hat{h}_t = \gamma t \Lambda \left( \tilde{\Lambda}_{t,t+1} + \tilde{t}_{t,t+1} + \tilde{h}_{t+1} \right) \]  

(A3.13)

\[ \tilde{\phi}_t = \frac{\hat{h}_t}{\lambda - v} v \]  

(A3.14)

\[ \tilde{t}_{t,t+1} = R^p \phi^p \tilde{r}_{t+1} + R (1 - \phi^p) \tilde{t}_t + \phi^p (R^q - R) \tilde{v}_t \]  

(A3.15)

\[ \tilde{x}_{t,t+1} = \tilde{\phi}_{t+1} - \tilde{\phi}_t + \tilde{t}_{t,t+1} \]  

(A3.16)

\[ \tilde{q}_t + \tilde{g}_{t+1} = \tilde{\phi}_t^T + \tilde{F}_t^k \]  

(A3.17)

\[ \tilde{\phi}_t = \tilde{\phi}_t^p \]  

(A3.18)

\[ \tilde{\psi}_t = 0 \]  

(A3.19)

\[ \tilde{F}_t^k = \frac{F_{o,k}}{F_k} \tilde{F}_{t-o,k} + \frac{F_{n,k}}{F_k} \tilde{F}_{t-n,k} \]  

(A3.20)

\[ \tilde{F}_{t-o,k} F_{o,k} = \gamma F^k \left[ \phi^p R^q \tilde{r}_t^q + R (1 - \phi^p) \tilde{r}_t + \phi^p (R^q - R) \tilde{v}_t \right] + F_{o,k} \tilde{F}_{t-o,k} \]  

(A3.21)

\[ \tilde{F}_{t-n,k} = \tilde{q}_t + \xi_t \]  

(A3.22)

\[ \tilde{\pi}_t = \beta \left[ (1 - \theta \Pi^e) (1 - \Pi - 1) + \theta \Pi^e (1 + \Pi^e - 1) \right] E_t \tilde{\pi}_{t+1} \]  

(A3.23)

\[ -\theta \beta^2 \Pi^e E_t \tilde{\pi}_{t+2} + \frac{(1 - \theta \beta \Pi^e) (1 - \theta \Pi^e)}{\theta \Pi^e - 1} \tilde{\pi}_t \]  

\[ -\beta (1 - \theta \Pi^e - 1) (1 - \theta \beta \Pi^e) E_t \tilde{p}_t^j + \beta (\Pi - 1) (1 - \theta \Pi^e - 1) E_t \tilde{g}_t^y \]  

(A3.24)

\[ \tilde{\Delta}_{t+1} = \theta \Pi^e \left( \epsilon \tilde{\pi}_{t+1} + \Delta_t \right) - \frac{(1 - \theta) \epsilon X_{-e}}{\Delta} \tilde{X}_{t+1} \]  

(A3.25)

\[ \tau^n_t = r + \pi x + \phi_y \tilde{\pi}_t + \phi_y \tilde{g}_t^k + \xi_t \]  

(A3.26)
Figure 1: Steady State Equilibrium for II=2.5%
Figure 2: Steady State Values

- **Annual Growth (%):**
  - Y-axis: 2.2 to 2.55
  - X-axis: Annual Rate of Trend Inflation (%)

- **Private Leverage:**
  - Y-axis: 4.8 to 5.05
  - X-axis: Annual Rate of Trend Inflation (%)

- **Marginal Gain Annual Rate (%):**
  - Y-axis: 2.1 to 2.2
  - X-axis: Annual Rate of Trend Inflation (%)

- **Annualized External Finance Premium:**
  - Y-axis: 0.195 to 0.21
  - X-axis: Annual Rate of Trend Inflation (%)

- **Normalized Financial Wealth:**
  - Y-axis: 0.195 to 0.21
  - X-axis: Annual Rate of Trend Inflation (%)

- **Normalized Investment:**
  - Y-axis: 0.0355 to 0.0363
  - X-axis: Annual Rate of Trend Inflation (%)

- **Welfare Costs:**
  - Y-axis: 0 to 10
  - X-axis: Annual Rate of Trend Inflation (%)

- **% Deviation from TI=1.7%:**
  - Y-axis: -2 to 4
  - X-axis: Annual Rate of Trend Inflation (%)

Legend:
- Red: Growth
- Red: Investment
- Black: Leverage
- Black: Financial wealth
- Gray: EFP
- Gray: Marginal gain
Figure 3: Sensitivity Analysis

\[ \lambda_v \]

\[ \gamma_v \]

\[ \omega_v \]

\[ \tau_v \]

\[ \psi_v \]
Figure 4: Responses to a Monetary Policy Shock. % Deviation from SS
Figure 5: Responses to a Technology Shock. % Deviation from SS

- Growth
- Normalized Output
- Normalized Investment
- External Finance Premium
- Marginal gain of financial intermediaries
- Private Leverage
- Normalized Financial Wealth
- Inflation Rate

Legend:
- 0%
- 1%
- 2%
- 3%
- 4%
- 5%
Figure 6: Responses to a Capital Quality Shock and Credit Policy if π=1.7%. % Deviation from SS.

<table>
<thead>
<tr>
<th>Growth</th>
<th>Normalized Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalized Investment</td>
<td>Investment Price</td>
</tr>
<tr>
<td>Labor</td>
<td>External Finance Premium</td>
</tr>
<tr>
<td>Marginal Gain of Banks</td>
<td>Private Leverage</td>
</tr>
<tr>
<td>Normalized Financial Wealth</td>
<td>Annualized Inflation Rate</td>
</tr>
</tbody>
</table>

Legend:
- b=0
- b=10
- b=50
- b=100