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Synergetic Interactions Between 2 Brazilian Regions:

An Application of Input-Output Linkages

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Abstract

Using a set of interregional input-output tables built by Guilhoto (1998) for the year of 1992 for 2 Brazilian regions (Northeast and rest of the economy), the methodology developed by Sonis, Hewings, and Miyazawa (1997) is applied in the construction of a series of linkages such that it is possible to examine, through the nature of the internal and external interdependencies giving by the linkages, the structure of trading relationships among the 2 regions. The methodology used in this work is based on a partitioned input-output system and exploits techniques that produce left and right matrix multipliers of the Leontief Inverse, this allows to classify the types of synergetic interactions within a preset pair-wise hierarchy of economic linkages sub-systems.

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I. Introduction

In this paper it is made use of the methodology presented by Sonis, Hewings, and Miyazawa (1997), which classifies the types of synergetic interactions and allows to examine the structure of the trading relations among the regions. This methodology is applied to a set of interregional input-output tables built by Guilhoto (1998) for 2 Brazilian regions (Northeast and rest of the economy).

In the next section the theoretical background will be presented. In the third section the theory will be applied to the Brazilian interregional tables, while in the last section some conclusions will be offered.

II. Theoretical Background[4]

Consider an input-output system represented by the following block matrix, A , of direct inputs:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{1}$$

where A_{11} and A_{22} are the quadrat matrices of direct inputs within the first and second regions, and A_{12} and A_{21} are the rectangular matrices showing the direct inputs purchased by the second region and vice versa.

The building blocks of the pair-wise hierarchies of sub-systems of intra/interregional linkages of the block-matrix Input-Output system are the four matrices $A_{11,} A_{12}, A_{21}$ and A_{22}, corresponding to four basic block-matrices:

$$A_{11} = \begin{bmatrix} A_{11} & 0 \\ 0 & 0 \end{bmatrix}, A_{12} = \begin{bmatrix} 0 & A_{12} \\ 0 & 0 \end{bmatrix}, A_{21} = \begin{bmatrix} 0 & 0 \\ A_{21} & 0 \end{bmatrix}, A_{22} = \begin{bmatrix} 0 & 0 \\ 0 & A_{22} \end{bmatrix} \tag{2}$$

This paper will usually consider the decomposition of the block-matrix (1) into the sum of two block-matrices, such that each of them is the sum of the block-matrices (2) $A_{11,} A_{12}, A_{21}$ and A_{22}. From (1) 14 types of pair-wise hierarchies of economic sub-systems can be identified by the decompositions of the matrix of the block-matrix A (see Table 2).

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[4] This section draws heavily on Sonis, Hewings, and Miyazawa (1997).

A set of inner regional multipliers, the set of inverse matrices which are the "building blocks" of the synergetic interactions between the economic sub-systems are presented in table 1. Hereafter, are presented some comments on the entries in this table (the bold numbering refers to the corresponding entries in this table).

1. The matrices $B_1 = (I - A_{11})^{-1}$ and $B_2 = (I - A_{22})^{-1}$ represent the Miyazawa internal matrix multipliers of the first and second regions showing the interindustrial propagation effects within each region, while the matrices, $A_{21}B_1$, $B_1 A_{12}$, $A_{12}B_2$, $B_2 A_{21}$ show the induced effects on output or input activities in the two regions.

2. The expressions

$$S_1 = I - A_{11} - A_{12}B_2 A_{21}, \quad S_2 = I - A_{22} - A_{21}B_1 A_{12} \tag{3}$$

are usually referred to as the Schur complements.

The inverses, D_1 and D_2 of the Schur complements (3) are referred to as the *Schur inverses* for the first and second regions. They represent the enlarged Leontief inverse for one region revealing the induced economic influence of the other region; i.e., the Schur inverses represent total propagation effects in the first and second regions.

3. Miyazawa (1966) introduced left and right external matrix multipliers of the first and second regions, $D_{11}^L, D_{11}^R, D_{22}^L, D_{22}^R$. These multipliers are incorporated in the multiplicative decompositions of the Schur inverses and they represent the total propagation effects in the first and second regions as the products of internal and external regional matrix multipliers.

4, 5. By introducing the abbreviated Schur inverses, D_{11}, D_{22}, and the left and right induced internal multipliers for the first and second regions, $B_1^L, B_1^R, B_2^L, B_2^R$, one can obtain the multiplicative decompositions of Schur inverses:

$$D_1 = B_1^L D_{11} = D_{11}B_1^R; \quad D_2 = B_2^L D_{22} = D_{22}B_2^R \tag{4}$$

and their corresponding additive representations.

Table 1

Inner regional multipliers and their properties.

1. Internal regional multipliers:

$$B_1 = (I - A_{11})^{-1}; \quad B_2 = (I - A_{22})^{-1}$$

2. Schur complements and Schur inverses:

$$S_1 = I - A_{11} - A_{12}B_2 A_{21}; \qquad S_2 = I - A_{22} - A_{21}B_1 A_{12}$$

$$D_1 = (I - A_{11} - A_{12}B_2 A_{21})^{-1}; \qquad D_2 = (I - A_{22} - A_{21}B_1 A_{12})^{-1}$$

3. Left and right Miyazawa external matrix multipliers:

$$D_{11}^L = (I - B_1 A_{12}B_2 A_{21})^{-1}; \qquad D_{22}^L = (I - B_2 A_{21}B_1 A_{12})^{-1}$$

$$D_{11}^R = (I - A_{12}B_2 A_{21}B_1)^{-1}; \qquad D_{22}^R = (I - A_{21}B_1 A_{12}B_2)^{-1}$$

Main Properties:

$$D_1 = B_1 D_{11}^R = D_{11}^L B_1; \qquad D_2 = B_2 D_{22}^R = D_{22}^L B_2$$

4. Abbreviated Schur inverses:

$$D_{11} = (I - A_{12}B_2 A_{21})^{-1}; \qquad D_{22} = (I - A_{21}B_1 A_{12})^{-1}$$

5. Left and right induced internal multipliers:

$$B_1^L = (I - D_{11}A_{11})^{-1}; \qquad B_2^L = (I - D_{22}A_{22})^{-1}$$

$$B_1^R = (I - A_{11}D_{11})^{-1}; \qquad B_2^R = (I - A_{22}D_{22})^{-1}$$

Main Properties:

$$D_1 = B_1^L D_{11} = D_{11}B_1^R; \qquad D_2 = B_2^L D_{22} = D_{22}B_2^R$$

6. Enlarged Leontief inverses:

$$D_1^* = (I - A_{11} - A_{12}A_{21})^{-1}; \qquad D_2^* = (I - A_{22} - A_{21}A_{12})^{-1}$$

7. Induced external multipliers:

$$D_{11}^* = (I - A_{12}A_{21})^{-1}; \qquad D_{22}^* = (I - A_{21}A_{12})^{-1}$$

Table 1 (Continued)

8. Left and right induced internal multipliers:

$B_1^{*L} = (I - D_{11}^{*} A_{11})^{-1}$; $B_2^{*L} = (I - D_{22}^{*} A_{22})^{-1}$

$B_1^{*R} = (I - A_{11} D_{11}^{*})^{-1}$; $B_2^{*R} = (I - A_{22} D_{22}^{*})^{-1}$

Main Properties:

$D_1^{*} = B_1^{*L} D_{11}^{*} = D_{11}^{*} B_1^{*R}$; $D_2^{*} = B_2^{*L} D_{22}^{*} = D_{22}^{*} B_2^{*R}$

9. Left and right subjoined inverses:

$D_{11}^{*L} = (I - B_1 A_{12} A_{21})^{-1}$; $D_{22}^{*L} = (I - B_2 A_{21} A_{12})^{-1}$

$D_{11}^{*R} = (I - A_{12} A_{21} B_1)^{-1}$ $D_{22}^{*R} = (I - A_{21} A_{12} B_2)^{-1}$

Main Properties:

$D_1^{*} = B_1 D_{11}^{*R} = D_{11}^{*L} B_1$; $D_2^{*} = B_2 D_{22}^{*R} = D_{22}^{*L} B_2$

10. Left and right induced subjoined inverses:

$D_{11}^{**L} = [I - D_{11}(A_{11} - A_{12} B_2 A_{22} A_{21})]^{-1}$; $D_{22}^{**L} = [I - D_{22}(A_{22} - A_{21} B_1 A_{11} A_{12})]^{-1}$;

$D_{11}^{**R} = [I - (A_{11} - A_{12} B_2 A_{22} A_{21}) D_{11}]^{-1}$; $D_{22}^{**R} = [I - (A_{22} - A_{21} B_1 A_{11} A_{12}) D_{22}]^{-1}$

Main Properties:

$D_1^{*} = D_{11}^{**L} D_{11} = D_{11} D_{11}^{**R}$; $D_2^{*} = D_{22}^{**L} D_{22} = D_{22} D_{22}^{**R}$

6-10. The formulae for this group of multipliers can be obtained by considering the block-matrices:

$$M = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & 0 \end{pmatrix}, N = \begin{pmatrix} 0 & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, S = \begin{pmatrix} 0 & A_{12} \\ A_{21} & 0 \end{pmatrix} \tag{5}$$

that represent the backward and forward linkages of the first region, the second region and the interregional relations of both regions.

The following Schur inverse

$$D_1^* = (I - A_{11} - A_{12}A_{21})^{-1} \tag{6}$$

may be referred to as the enlarged Leontief inverse, and the inverses

$$D_{11}^{*L} = (I - B_1 A_{12}A_{21})^{-1}; \quad D_{11}^{*R} = (I - A_{12}A_{21}B_1)^{-1} \tag{7}$$

are called the left and right subjoined inverse matrix multipliers.

Consider the hierarchy of Input-Output sub-systems represented by the decomposition $A = A_1 + A_2$. Introducing the Leontief block-inverse $L(A) = L = (I - A)^{-1}$ and the Leontief block-inverse $L(A_1) = L_1 = (I - A_1)^{-1}$ corresponding to the first sub-system. The outer left and right block-matrix multipliers M_L and M_R are defined by equalities:

$$L = L_1 M_R = M_L L_1 \tag{8}$$

The definition (8) implies that:

$$M_L = L(I - A_1) = (I - L_1 A_2)^{-1} \tag{9}$$

$$M_R = (I - A_1)L = (I - A_2 L_1)^{-1} \tag{10}$$

This study will apply the following form of the Leontief block-inverse:

$$L = \begin{bmatrix} D_1 & D_1 A_{12} B_2 \\ D_2 A_{21} B_1 & D_2 \end{bmatrix} \tag{11}$$

This formula can be verified by direct matrix multiplication, using definitions of the Schur inverses and their properties (see table 1, entries **1** and **2**). Further, it will presented the application of formulas (9), (10) and (11) to the derivation of a taxonomy of synergetic interactions between regions. The results are presented in the first and second levels of table 2.

Consider the hierarchy of input-output sub-systems represented by the decomposition $A = A_1 + A_2$ and their Leontief block-inverse $L(A) = L = (I - A)^{-1}$ and the Leontief block-inverse $L(A_1) = L_1 = (I - A_1)^{-1}$ corresponding to the first sub-system. The multiplicative decomposition of the Leontief inverse $L = L_1 M_R = M_L L_1$ can be converted to the sum:

$$L = L_1 + (M_L - I) L_1 = L_1 + L_1 (M_R - I) \tag{12}$$

If f is the vector of final demand and x is the vector of gross output, then the decomposition (12) generates the decomposition of gross output into two parts: $x_1 = L_1 f$ and the increment $Dx = x - x_1$. Such a decomposition is important for the empirical analysis of the structure of actual gross output. In the second level of table 2, it is presented the classification of possible additive decompositions of the Leontief block-inverse for all decompositions of input-output system into the pair-vise hierarchies.

Table 2

Taxonomy of synergetic interactions between economic sub-systems

[Each entry consists of two levels: in the first level, a description of the structure and the corresponding form of the A matrix is shown. In the second level the additive decompositions of the Leontief block-matrix are shown]

Level 1 *Description* *Form of the* A_1 *matrix*

Level 2 $L = L_1 + (M_L - I)L_1 = L_1 + L_1(M_R - I)$

I. *Hierarchy of backward linkages of first and second regions*

$$A_1 = \begin{pmatrix} A_{11} & 0 \\ A_{21} & 0 \end{pmatrix};$$

$$L = \begin{pmatrix} B_1 & 0 \\ A_{21}B_1 & I \end{pmatrix} + \begin{pmatrix} B_1 A_{12} \\ I - S_2 \end{pmatrix} B_2 \begin{bmatrix} A_{21}B_1 & I \end{bmatrix}$$

II. *Order replaced hierarchy of backward linkages*

$$A_1 = \begin{pmatrix} 0 & A_{12} \\ 0 & A_{22} \end{pmatrix};$$

$$L = \begin{pmatrix} I & A_{12}B_2 \\ 0 & B_2 \end{pmatrix} + \begin{pmatrix} I - S_1 \\ B_2 A_{21} \end{pmatrix} B_1 \begin{bmatrix} I & A_{12}B_2 \end{bmatrix}$$

III. *Hierarchy of forward linkages of first and second regions*

$$A_1 = \begin{pmatrix} A_{11} & A_{12} \\ 0 & 0 \end{pmatrix};$$

$$L = \begin{pmatrix} B_1 & B_1 A_{12} \\ 0 & I \end{pmatrix} + \begin{pmatrix} B_1 A_{12} \\ I \end{pmatrix} B_2 \begin{bmatrix} A_{21}B_1 & I - S_2 \end{bmatrix}$$

IV. *Order replaced hierarchy of forward linkages*

$$A_1 = \begin{pmatrix} 0 & 0 \\ A_{21} & A_{22} \end{pmatrix};$$

$$L = \begin{pmatrix} I & 0 \\ B_2 A_{21} & B_2 \end{pmatrix} + \begin{pmatrix} I \\ B_2 A_{21} \end{pmatrix} B_1 \begin{bmatrix} I - S_1 & A_{12}B_2 \end{bmatrix}$$

V. *Hierarchy of isolated region versus the rest of economy*

$$A_1 = \begin{pmatrix} A_{11} & 0 \\ 0 & 0 \end{pmatrix}.$$

$$L = \begin{pmatrix} B_1 & 0 \\ 0 & I \end{pmatrix} + \begin{pmatrix} B_1 A_{12} & 0 \\ 0 & I \end{pmatrix} B_2 \begin{pmatrix} I & I \\ I & I - S_2 \end{pmatrix} \begin{pmatrix} A_{21}B_1 & 0 \\ 0 & I \end{pmatrix}.$$

Table 2 (Continued)

VI. *Hierarchy of the rest of economy versus second isolated region*

$$A_1 = \begin{bmatrix} 0 & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$L = \begin{bmatrix} D_{11} & A_{12}D_2^* \\ D_2 A_{21} & D_2^* \end{bmatrix} = \begin{bmatrix} I \\ B_2 A_{21} \end{bmatrix} B_1 A_{11} D_1 \left[I \quad A_{12}B_2 \right]$$

VII. *The hierarchy of backward and forward linkages of the first region versus rest of economy*

$$A_1 = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} D_1^* & D_1^* A_{12} \\ A_{21} D_1^* & D_{22} \end{bmatrix} = \begin{bmatrix} B_1 A_{12} \\ I \end{bmatrix} D_{22} A_{22} \left[A_{21} B_1 \quad I \right]$$

VIII. *The order replaced hierarchy of backward and forward linkages of the first region versus rest of economy*

$$A_1 = \begin{bmatrix} 0 & 0 \\ 0 & A_{22} \end{bmatrix}$$

$$L = \begin{bmatrix} I & 0 \\ 0 & B_2 \end{bmatrix} + \begin{bmatrix} I & 0 \\ 0 & B_2 A_{21} \end{bmatrix} B_1 \begin{bmatrix} S_1 & I \\ I & I \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & A_{12}B_2 \end{bmatrix}$$

IX. *The hierarchy of intra- versus inter- regional relationships*

$$A_1 = \begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix}$$

$$L = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} + \begin{bmatrix} D_1 A_{12} B_2 & 0 \\ 0 & D_2 A_{21} B_1 \end{bmatrix} \begin{bmatrix} -A_{21} & I - A_{22} \\ I - A_{11} & A_{12} \end{bmatrix} \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix}$$

X. *The hierarchy of inter versus intra regional relationships*

$$A_1 = \begin{bmatrix} 0 & A_{12} \\ A_{21} & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} D_{11}^* & D_{11}^* A_{12} \\ D_{22} A_{21} & D_{22}^* \end{bmatrix} = \begin{bmatrix} I & B_1 A_{12} \\ B_2 A_{21} & I \end{bmatrix} \begin{bmatrix} D_1 A_{11} D_{11}^* & 0 \\ 0 & D_2 A_{22} D_{22}^* \end{bmatrix} \begin{bmatrix} I & A_{12} \\ A_{21} & I \end{bmatrix}$$

XI. *The hierarchy of lower triangular sub system versus interregional linkages of second region*

$$A_1 = \begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix}$$

$$L = \begin{bmatrix} B_1 & 0 \\ B_2 A_{21} B_1 & B_2 \end{bmatrix} = \begin{bmatrix} I \\ B_2 A_{21} \end{bmatrix} B_1 A_{12} B_2 \left[A_{21} B_1 \quad I \right]$$

Table 2 (Continued)

XII. *The order replaced hierarchy of interregional linkages of second region versus lower triangular sub system*

$$A_1 = \begin{bmatrix} 0 & A_{12} \\ 0 & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} I & A_{12} \\ 0 & I \end{bmatrix} \begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix} \begin{bmatrix} I - S_1 & A_{12} B_2 - S_1 A_{12} \\ A_{21} B_1 & I - S_2 \end{bmatrix}$$

XIII. *The hierarchy of upper triangular sub system versus interregional linkages of first region*

$$A_1 = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}$$

$$L = \begin{bmatrix} B_1 & B_1 A_{12} B_2 \\ 0 & B_2 \end{bmatrix} \begin{bmatrix} B_1 A_{12} \\ I \end{bmatrix} B_2 A_{21} B_1 \begin{bmatrix} I & A_{12} B_2 \end{bmatrix}$$

XIV. *The order replaced hierarchy of interregional linkages of first region versus upper triangular sub system*

$$A_1 = \begin{bmatrix} 0 & 0 \\ A_{21} & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} I & 0 \\ A_{21} & I \end{bmatrix} \begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix} \begin{bmatrix} I - S_1 & A_{12} B_2 \\ A_{21} B_1 - S_2 A_{21} & I - S_2 \end{bmatrix}$$

While 14 types of pair-wise hierarchies of economic linkages have been developed, it is possible to suggest a typology of categories into which these types may be placed. The following characterization is suggested:

1. backward linkage type (I, II): power of dispersion

2. forward linkage type (III, IV): sensitivity of dispersion

3. intra- and inter- linkages type (IX, X): internal and external dispersion

4. isolated region vs. the rest of the economy interactions style (V, VI, VII, VIII)

5. triangular sub-system vs. the interregional interactions style (XI, XII, XIII, XIV).

By viewing the system of hierarchies of linkages in this fashion, it will be possible to provide new insights into the properties of the structures that are revealed. For example, the types allocated to category 5 reflect structures that are based on order and circulation. Furthermore, these partitioned input-output systems can distinguish among the various types of dispersion (such as 1, 2 and 3) and among the various patterns of interregional interactions (such as 4 and 5). Essentially, the 5 categories and 14 types of pair-wise hierarchies of economic linkages provide the opportunity to select according the special qualities of each region's activities and for the type of problem at hand; in essence, the option exists for the basis of a typology of economy types based on hierarchical structure.

III. An Application to Brazil

Using a set of interregional input-output tables built by Guilhoto (1998) at the level of 37 sectors for the year of 1992 for 2 Brazilian regions (Northeast - Region 1 - and the rest of the economy - Region 2), the methodology presented in table 2 is applied, and the results are presented in table 3.

Table 3
Results of the Synergetic Interactions Between the 2 Brazilian Regions

Pair-Wise Hierarchy	Share (%) of x_1 in x Region 1	Share (%) of x_1 in x Region 2	Share (%) of $(x_1$-$f)$ in x Region 1	Share (%) of $(x_1$-$f)$ in x Region 2	Share (%) of f in x Region 1	Share (%) of f in x Region 2
I	89.34	60.43	23.05	0.81	66.29	59.62
II	72.14	98.05	5.85	38.44	66.29	59.62
III	94.33	59.62	28.05	0.00	66.29	59.62
IV	66.29	99.17	0.00	39.55	66.29	59.62
V	89.34	59.62	23.05	0.00	66.29	59.62
VI	72.27	99.35	5.98	39.74	66.29	59.62
VII	94.50	60.51	28.22	0.90	66.29	59.62
VIII	66.29	98.05	0.00	38.44	66.29	59.62
IX	89.34	98.05	23.05	38.44	66.29	59.62
X	69.17	60.22	2.88	0.60	66.29	59.62
XI	89.34	99.68	23.05	40.07	66.29	59.62
XII	69.11	59.62	2.82	0.00	66.29	59.62
XIII	99.66	98.05	33.37	38.44	66.29	59.62
XIV	66.29	60.17	0.00	0.55	66.29	59.62

Table 3 presents the results taking into consideration the vector f of final demand and the vector x of gross output, then the gross output is decomposed into two parts: $x_1 = L_1 f$ and the increment $Dx = x - x_1$. The values for x and x_1 are added for all sectors in regions 1 and 2 such that it is possible to estimate the contribution of each interaction to the total production in each region. As the shares of x_1 in x take also into consideration the value of the final demand it is interesting to isolate the shares of the final demand in each region such that it is possible to see how the pair-wise interaction take place in the regions.

Taking the results presented into Table 3 one can see that the value of the final demand in region 1 (Northeast) is responsible for 66.29 % of the production in this region (the remaining 33.71% are generated in the process of production) while for region 2 (the Rest of the Economy) this value is 59.62 % (40.38 % in the process of production). In a certain sense this is an indication that the rest of the economy is more developed than the Northeast region as the internal transactions in region 2 are responsible for a greater share of the total production than in region 1.

Looking at the results for the shares, excluding the final demand, for each case one has the following:

Case I (A_{11} and A_{21}): from the interactions of the inputs that the industries in region 1 buy from regions 1 and 2 its generated 23.05% of the production in this region and 0.81% of the production in region 2;

Case II (A_{12} and A_{22}): from the interactions of the inputs that the industries in region 2 buy from regions 1 and 2 it is generated 38.44% of the production in this region and 5.85% of the production in region 1, when this results are compared with the ones presented in Case II this shows a greater dependence of region 1 on the production process of region 2;

Case III (A_{11} and A_{12}): from the sales of production that the industries in region 1 sell to the production process of regions 1 and 2 one has that it is generated 28.05 % of the production in region 1 and 0.00 % in region 2 as there is no feedback among the regions;

Case IV (A_{21} and A_{22}): from the sales of production that the industries in region 2 sell to the production process of regions 1 and 2 one has that it is generated 39.55 % of the production in region 2 and 0.00 % in region 1 as there is no feedback among the regions, in this case showing a greater value of internal multipliers in region 2 than in region 1;

Case V (A_{11}): when region 1 is taking isolated this value shows how much of the internal production is due to the relations only inside the region and in this case is 23.05% which is the same as the one presented for this region in case I;

Case VI (A_{21}, A_{12} and A_{22}): taking the relations inside region 2 and the sales and purchases that it makes from region 1, it is generated 39.74% of the production in this region and 5.98% in region 1, values greater than the ones presented in case II since more transactions are now being taking into consideration;

Case VII (A_{11}, A_{21} and A_{12}): taking the relations inside region 1 and the sales and purchases that it makes from region 2, it is generated 28.22% of the production in this region and 0.90% in region 2, values greater than the ones presented in case I since more transactions are now being taking into consideration;

Case VIII (A_{22}): when region 2 is taking isolated this value shows how much of the internal production is due to the relations only inside the region and in this case is 38.44% which is the same as the one presented for this region in case II;

Case IX (A_{11} and A_{22}): when regions 1 and 2 are taking isolated, with no transactions between them, this values shows how much of the internal production is due to the relations only inside each region and in this case they are 23.05% for region 1 and 38.44% for region 2 which are the same as the ones presented in cases I and V for region 1 and in cases II and VIII for region 2;

Case X (A_{21} and A_{12}): considering only the interregional flows among regions one has that 2.88% of the production in region 1 are due to this flows while for region 2 this value is 0.60%, showing again a greater dependence of the production in region 1 in the interrelations among the regions;

Case XI (A_{11}, A_{21} and A_{22}): taking the relations inside regions 1 and 2 and the purchases that region 1 makes from region 2, it is generated 23.05% of the production in region 1 and 40.07% in region 2, showing that the purchases of region 1 from 2 practically has no impact over the production in region 1, which is confirmed by case XIV;

Case XII (A_{12}): the purchases made by the industries in region 2 from region 1 generate 2.82% of the production in region 1 and by itself without having any interaction with the other block matrices generates no production in region 2;

Case XIII (A_{11}, A_{12} and A_{22}): taking the relations inside regions 1 and 2 and the purchases that region 2 makes from region 1, it is generated 33.37% of the production in region 1 and 38.44% in region 2, showing that the purchases of region 1 from 2 practically has no impact over the production in region 1, which is confirmed by case XII;

Case XIV (A_{21}): the purchases made by the industries in region 1 from region 2 generate 0.55% of the production in region 2 and by itself without having any interaction with the other block matrices generates no production in region 1.

From the above it was possible to observe and to measure how the relations among the 2 Brazilian regions take place. The Northeast region has a greater dependence on the rest of the economy region than the rest of the economy has on the Northeast region, and at the same time the rest of the economy region seems to be more developed as it presents a more complex productive structure than the Northeast region.

IV. Conclusions

The main contribution of this paper was to show, using different synergetic interactions, that it is possible to analyze and to measure how the trading relationships among the 2 regions take place. This was done use a 2 regions interregional input-output table constructed for the Brazilian economy for the year o 1992. From the results it was possible to see that Northeast region has a greater dependence on the rest of the economy region than the rest of the economy has on the Northeast region, and at the same time the rest of the economy region seems to be more developed as it presents a more complex productive structure than the Northeast region.

V. References

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