Interest Rates Rigidities and the Fisher Equation

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Abstract

The literature on nominal interest rates rigidity does not fully address its macroeconomic implications. How nominal interest rates rigidity would interact with the Fisher equation is simple, yet the implications are surprising. If nominal rates cannot catch up to real rates, the Fisher effect becomes inverted in the short term: big enough credit crunches bring deflation and central banks must lower interest rates to stimulate inflation. The paper shows that nominal interest rates rigidity is sufficient to characterize the little we know about inflation. It also shows that, unlike for other products, the pricing of loans is influenced by past negotiated loans, generating rigidity.

Keywords: Interest Rate Rigidity, Inflation, Monetary Policy.

JEL: E31, E43, E52.
1 Introduction

Posing nominal rigidities explains stylized facts about inflation more easily when applied to nominal interest rates than to prices. The inverse relation between interest rates and inflation in the conduct of monetary policy and the procyclical nature of inflation are the two main stylized facts of interest about inflation. Central banks do raise interest rates to lower inflation and lower them to raise inflation. This was recognized well before Taylor (1993); in fact, the Federal Reserve under Volker made the point quite forcefully. As for the procyclical nature of inflation, the data has established it thoroughly. Furthermore, the current popular model, the New Keynesian Phillips Curve (NKPC), lacks parsimony both in its derivation and in the way it would explain those two main stylized facts.\(^1\) To visualize nominal interest rates rigidities, imagine if nominal interest rates were perfectly constant for some reason: when the real interest rate goes up, inflation goes down if the nominal interest rate can’t move.

Research about interest rate setting points to a resistance in nominal interest rates both in the pass-through from central banks target rate to market rates (interest rate pass-through) and from market rates to lending rates. Kobayashi (2008, Section 2) surveys literature on interest rate pass-through and Dinger (forthcoming) and de Bondt, Mojon and Valla (2005), on bank rate rigidities; see also Illes and Lombardi (2013) for recent issues related to the financial crisis. This literature indicates that imperfect pass-through correlates with the financial soundness of individual banks, the predictability of monetary policy, a level of income smoothing from banks and, as de Bondt (2005) found, the long term interest rate is the key variable explaining the imperfect pass-through of monetary policy. Still, most of this research focuses on empirical aspects of nominal interest rate rigidities or theoretical justifications for them, little research, however, has been conducted on the macroeconomic effects of those rigidities. Macroeconomists may have ignored interest rate rigidities because the control of nominal rates by monetary authorities seems perfect as indicated by Figure 1.1 and 1.2 of Woodford (2003). Now, perfect control does not mean direct control, and nominal rigidities may imply that market operations need to disproportionately affect real rates.

\(^1\)See Woodford (2003) for a discussion and derivation of the the NKPC.
Relationship banking can explain interest rate smoothing by banks.\footnote{See Berlin and Mester (1999), Boot (2000), Freixas (2005) on relationship banking.} The relationship between banks and their clients enable a form of intertemporal smoothing of interest rate shocks as proposed by Berger and Udell (1992) after they found that bank rate rigidities may not come, as previously though, from rationing. I will take a parallel road and not implicitly model a bank, though the notion of risk sharing is implicit in the model presented in Section 2.

Carmichael and Stebbing (1983) also tackled nominal interest rate rigidities. They argue that the source of rigidities was competition from money which has a constant (zero) nominal yield. In their view, the actual after-tax nominal interest rates represents only “a real risk premium on financial assets” and inflation seems to be an exogenous variable to interest rate setting. Their approach leads to what they call the Inverted Fisher effect. This paper also has an inverted Fisher effect, but the channels go from real interest rates and monetary policy to inflation instead of inflation to real interest rates.

Other authors like Ravenna and Walsh (2006) and Kobayashi (2008) investigates the implications of rate rigidities by incorporating them to standard models with price rigidities (DSGE’s). It results in a modified NKPC with interest rates added to real marginal cost, or to the output gap, as drivers for inflation. Using interest rate rigidities inside a price-rigidity framework avoids coherence problems due to the incompatibility between interest rates and price rigidities. But the central bank still has to raise interest rates to create inflation. Because of this, their framework is not a halfway solution to the one proposed here.

I take the view that we should ignore price rigidities, at least for now. First, modeling both price and nominal interest rate rigidities makes it hard to discern which has what effect. Second, and most important for the sake of parsimony, the model presented here does not need to be built on top of both the NKPC postulates and the ones presented in Section 2.

Section 2 presents a model that yields rigidities through negotiations of nominal interest rates, once the real ones are chosen or imposed, between lenders and borrowers. Marginal behavior and perfect competition result in the minimization of average wealth transfers because of the influence nominal interest rates has on already-issued debts when real interest rates are not at play. The model performs this minimization while accounting for monetary authorities interventions.
Section 3 presents a numerical illustration in the form of impulse respond functions and simulations. The impulse respond functions show how inflation targeting reduces real interest rates volatility. They also uncovers a price puzzle à la Sims in inflation dynamics from the preemptive nature of nominal adjustments. For their part, the simulations show how different episode of inflation and interest rate movements in the last hundred years can be modeled simply through monetary policy shocks while keeping other shocks the same. Simulations suggest inflation targeting reduces real interest rates variability, while a gold standard increases real interest rates volatility, giving some sense for the causes of the Great Moderation.

2 Model

Subsection 2.1 of this section contains a general discussion on rigidities and inflation. The goal is to motivate the model built in Subsection 2.2.

2.1 General discussion

A reading of the history of macroeconomic theories (Blanchard (2000) or textbook) leads one to conclude that an understanding of inflation has to come from some sort of rigidity and that choice must be made on which rigidity, prices, interest rates or velocity of money, is best. The quantity theory and monetarist tradition relied on the rigidity of velocity, but when it comes to velocity, this paper takes the view that finding a credible theory of velocity is a dead end, because the velocity term in the equation of exchange is endogenous to the other terms and because the very definition of money makes it hard to model (Sims (2013)). New Keynesians rely on the rigidity of prices themselves. In both the Fisher equation \(i \approx r + \pi\) and the equation of exchange \(MV = PY\), there are two free variables. A use of the Fisher equation forces one to posit either price or nominal interest rate rigidities for identification purposes.

Then whether stickiness is a solution at all is problematic not only because we have to postulate a possibly unattractive form for it, but also because we have to choose which variable is sticky or stickier. This paper examines rigidities in nominal interest rates to determine whether they create a simple and credible narrative for the behavior of inflation. The choice comes first from the argument that even if price rigidities were more probable, there
would still be a need to investigate other possibilities. Second, the narrative for price rigidities is more complex than that for interest rates. Third, if both prices and nominal interest rates were rigid, interest rates rigidities may dominate as interest rates affect profits more than do prices.

The narrative for price rigidities is more complex than that for nominal interest rate rigidities. Though the approach has an intuitive explanation of procyclical inflation (Rotemberg and Woodford (1999)), it is less fluid for the conduct of monetary policy. If the nominal interest rate goes up, inflation can’t follow when prices are sticky and so real interest rate goes up. That would result in inflationary pressures unless the side effects of the rise in real interest rate is to lower marginal costs. In other words, the narrative goes against experience unless we tweak the model so higher real rates lower marginal costs. In terms of derivation, the NKPC poses monopolistic competition between symmetric firms and Calvo contracts, making the model less parsimonious than the one I propose.

Using nominal interest rates also has the advantage over prices that their volatility in micro data is much lower, as shown by Craig and Dinger (forthcoming). Furthermore, if we take the view that both prices and interest rates are sticky, interest rates rigidities may dominate. Interest rates have a disproportionate effect on profits compared to prices or even wages since future profits are discounted using interest rates while only relative prices and real wages affect profits. Then, if interest rates rigidities dominate, the mechanics still pushes the firms able to update their prices in a way that compensates for the firms that cannot.

From a new classical point of view (of money neutrality), only interest rates determine inflation. Interest rates bind statics and dynamics to a model, and if the Quantity Theory does not mean much, then the Fisher equation has to be the only determinant of inflation. For example, an RBC model would put everything in real terms letting inflation and nominal interest rates be thrown outside the model, making nominal interest rates undefined. If we define interest rate rigidities within an otherwise new classical context, inflation would still be procyclical, but central banks would have to be postulated to have a rôle. As is shown in the discussion for equation (5), the existence of a Taylor rule, or the possibility that a central bank raises interest rates when it thinks inflation is too high, supposes that it is not a neutral institution.
2.2 Implementation

Three equations determine inflation, real and nominal interest rates. The Fisher equation translates real and nominal interest rates into inflation, the rigidity equation translates real interest rates into nominal interest rates and the real interest rates equation comes from the real economy, but with feedback from monetary policy. A larger, more complete, model would be superfluous; there is no point in defending the postulates necessary to build it and make this paper overly long.

The first equation,

\[ 1 + i_t = E_t [(1 + r_{t+1})(1 + \pi_{t+1})], \]

yields the ex-ante definition of real interest rates, or Fisher’s equation. The notation is standard, \( i \) is nominal interest rates, \( r \) is real interest rates and \( \pi \) is inflation.

At question here is whether the equation’s interest rates represent the interest a government or a strong-credit entity pays or a representative agent’s interest rate, since headline inflation does not represent what governments or strong-credit entities buy, but what everyone buys. I ignore this issue as this is not an empirical paper. Subsection 3.3 discusses this and other empirical issues.

The second equation of the model implements the rigidity in nominal interest rates. The form it should take is not obvious. The rigidity equation is the one that defines the model. It comes from negotiations between lenders and borrowers. Negotiations mean nominal rates are set by minimizing average wealth transfers unlike real rates that have to contend with the real economy. This minimization leads to a “Compensation Mechanism” that translates changes in real interest rates in part to changes in nominal interest rates and in part to changes in inflation. Monetary policy keeps agents from deferring the adjustment and leads to the “Relent Mechanism” that forces agents to relent to the central bank’s inflation target in order for future real interest rates forecast not to be biased. The rigidity equation becomes a weighted average of both mechanisms.

The minimizing average of wealth transfers determine nominal interest rates in the model as both lenders and borrowers act in perfect competition, setting rates to yield zero expected profits. Loans are different from other products in that the past affects present pricing because nominal interest rates affect the wealth of agents to a larger extent than does the loans being
negotiated at any point in time. Therefore, price setting in a competitive environment means the best nominal rates each side can offer is the one that minimizes average wealth transfers.

The real rates will not be set in this way because borrowers and lenders have to contend with competition from the real economy. Borrowers and lenders turn into price takers when confronted by intertemporal preferences and the marginal productivity of capital.

Since exogenous changes in real interest rates can either be achieved through nominal interest rates or inflation, what these two options means to lenders and borrowers will determine the amount of pass-through from real to nominal rates. Changes in nominal interest rates affect only the wealth of people who don’t fully expect to keep to the maturity date, while people who will not hold are equally affected by unexpected changes in both inflation and nominal interest rates. Because reinvestment neutralizes the effects of changes in nominal interest rates, people who intend to hold a debt or bond until maturity are only affected by unexpected inflation. Therefore, in the short term, a “Compensation Mechanism” can be constructed. If this mechanism was the only one in play, the equation would take the form of

\[ i_t - E_{t-1} [i_t] = \sum_{s=0}^{\infty} \beta^s \lambda_{t,t+s} \left( 1 - \frac{\eta_{t,t+s}}{2} \right) (E_t [r_{t+1+s}] - E_{t-1} [r_{t+1+s}]), \]  \(2\)

where \(\beta\) is the discount factor, \(\lambda_{t,t+s}\) is the fraction of debts at time \(t\) that have maturities at or beyond \(t + s\) and \(\eta_{t,t+s}\) is the expected fraction of debts in \(\lambda_{t,t+s}\) that are expected to be sold or reimbursed at time \(t + s\).

Since the model will get a little more complicated, let's define \(x\), for readability, as

\[ x_t \equiv \sum_{s=0}^{\infty} \beta^s \lambda_{t,t+s} \left( 1 - \frac{\eta_{t,t+s}}{2} \right) (E_t [r_{t+1+s}] - E_{t-1} [r_{t+1+s}]). \]

Note that agents immediately have to account for future expected changes in real interest rates. As such, this leads to the preempted aspect of interest changes and generates the price puzzle described by Sims (1992). Agents cannot defer the adjustment because the inflation targets of monetary authorities will not permit trade-offs between nominal interest rates and inflation when they are known early in advance. This leads to the “Relent Mechanism”.

The Relent Mechanism determines nominal interest rates in the long term. Agents have to relent to monetary authorities in order for expected real
interest rates to equal the ones they are negotiating. Monetary authorities’ actions to achieve their target inflation rate has a predictable effect on real interest rates if the nominal interest rate is different from the sum of the expected real interest rate and the target inflation rate. Because agents need their expected real rate to be unbiased, for the longer term, we have the Relent Mechanism. For a large enough $v$, the equation,

$$E_t [i_{t+v}] = E_t \left[ \pi_{t+1+v}^* + r_{t+1+v} \right],$$

simply states that agents negotiate nominal and real interest rates at longer maturity by giving in to monetary authorities. This is done to keep the central bank from messing up their negotiated prices. $\pi_{t+1+v}^*$ is the central bank inflation target for time $t + 1 + v$.

The rigidity equation will come from a weighted average of both equation (2) and equation (3) using a parameter $\omega_s$. It should be noted that the Relent Mechanism still uses expected real rates, not the rates that would be in use without monetary authority intervention. This way, agents weigh in interventions they already know will happen.

At time $t + s$, with $s < v$, we have a mix of both mechanism that takes the form

$$E_t [i_{t+s}] = (1 - \omega_{s+1}) E_t \left[ \pi_{t+1+s}^* + r_{t+1+s} \right] + \omega_{s+1} (x_t + E_{t-1} [i_{t+s}]).$$

When taken back in time, the equation can be written as

$$E_{t-s} [i_t] = (1 - \omega_{s+1}) E_{t-s} \left[ \pi_{t+1}^* + r_{t+1} \right] + \omega_{s+1} (x_{t-s} + E_{t-1-s} [i_t]),$$

which solves recursively to yield

$$i_t = \sum_{s=0}^{\infty} \left( (1 - \omega_{s+1}) E_{t-s} \left[ \pi_{t+1}^* + r_{t+1} \right] + \omega_{s+1} x_{t-s} \right) \prod_{z=0}^{s} \omega_z$$  \hspace{1cm} (4)

by posing $\omega_s = 0$ for a large $s$ and $\omega_0 = 1$ to shorten the equation.

While equation (1) and the latter equation (5) are conventional, this rigidity equation, equation (4), defines the model.

Finally, the third equation implements the behavior of real interest rates. Real interest rates will be only partially exogenous. The reason is the existence of a Taylor rule. If $r$ were independent of the movement of $i$, there would be no Taylor rule. The reason is simple and the mathematics, straightforward: imagine an unexpected rise in $\pi$; accordingly, the central bank raises
according to the Fisher equation, \( \pi \) goes up again so the central bank keep raising \( i \) endlessly. All solution paths are explosive. If there is a possibility of a Taylor rule, and practice and estimations (Taylor (1993), Clarida, Galí and Gertler (1998)) say there is, the central bank cannot alter \( i \) without affecting \( r \). In fact, the central bank’s action has to affect \( r \) more than \( i \) for it to have the inverse effect on \( \pi \). The central bank, if it has that effect on inflation, must also affect real rates of interest. As such, the institution is not neutral.\(^3\)

So the final equation has an exogenous part and a monetary policy part. The monetary policy part acts as an error correction term between inflation and its target,

\[
r_t = r_t^e + \phi \left( E_r \left[ \sum_{s=1}^S \pi_{t+s} \right] - \pi_t^* \right),
\]

where \( \phi \) is positive and the length, \( S \), represents, together with the \( \omega \)’s from before, the tightness of monetary policy. The shock term, \( r^e \), represents equilibrium real interest rates in the absence of monetary policy intervention. In other words, it represents the real economy.

When part of a more complete model, the exogenous part, \( r^e \), would instead come from the model itself. This would involve a central bank inserting itself in the equilibrium by buying or selling bonds. But, as stated earlier, such an approach lays well beyond the scope of this paper.

Some restrictions on the model will be needed to make it useful, the next section proposes some and presents results.

3 Implications

This section builds on the model by making assumptions on parameters and postulating stochastic processes for real interest rates. Precisely, a linear form is imposed on \( \omega \), an exponential form is imposed on the equation for \( x \) and \( r^e \) is expressed as the sum of two exogenous shocks. Parameters are calibrated for illustration purposes to yield reasonable results. These assumptions make

\(^3\)Expressed in deviation from the steady state, the Taylor rule is \( \dot{i}_t = \alpha_p \dot{\pi}_t + \alpha_y \dot{y}_t \) where \( \dot{y} \) is the output gap. Inserted into Fisher’s equation, it yields \( \ddot{\pi}_{t+1} + \dot{r}_{t+1} = \alpha_p \pi_t + \alpha_y \dot{y}_t \) which is explosive if both \( r \) and \( y \) are exogenous since \( \alpha_p > 1 \). If instead we write the Taylor rule as \( \dot{i}_t = \alpha_p \pi_t + \alpha_y \dot{y}_t \), insertion in the Fisher equation will yield \( (\alpha_p - 1) \ddot{\pi}_{t+1} = \dot{r}_{t+1} - \alpha_y \dot{y}_t \) which is not explosive, but in which reactions of inflation to both real interest rates and the output gap are counterintuitive.
it possible to derive impulse response functions and conduct simulations.\footnote{Impulse response functions, simulations and figures were obtained with Dynare on Matlab. The default seed was used so not to influence results.} The next two subsections present the results.

For illustration purposes, $r^e_t$ of equation (5) takes the form of two exogenous shocks,

$$r^e_t = \bar{r} + r^a_{t-4} + r^s_t,$$

where $r^a$ is an anticipated shock (four periods in advance) and $r^s$ is a surprise shock. They are both defined as autoregressive process of order 1.

Furthermore, the rigidity equation, equation (4), has to be characterized. Because an exponential function would be intractable, we pose a linear function for $\omega_s$ with a length of $n$, or,

$$\omega_s = \begin{cases} \frac{n-s}{n} & \text{for } 0 \leq s \leq n \\ 0 & \text{otherwise} \end{cases}.$$  

A linear function has the advantage of ending at some point. This is important because a Bellman equation would need an exponential function. The linear function yields a tractable version of equation (4), which becomes

$$i_t = \sum_{s=1}^{n} \frac{n!}{n^s(n-s)!} \left( \frac{n}{s} \pi^{*}_{t+1} + \frac{n-s}{n} x_{t-s+1} \right).$$

Variable $x$ is simplified by posing two time invariant parameters $\gamma$ and $\theta$, such that $\theta \gamma^s = \beta^s \lambda_{t,t+s}(1 - \eta_{t,t+s}/2)$ for all $t$. So $x_t$ becomes

$$x_t = \sum_{s=0}^{\infty} \theta \gamma^s (E_t [r_{t+1+s}] - E_{t-1} [r_{t+1+s}]).$$

These equations form the basis of the simulations that follow. Both impulse response functions and simulations were done with the model using parameters chosen for illustration purposes. Monetary policy is such that the inflation target, $\pi^*$, is at 0.5 percent per quarter and the horizon, $S$, is calibrated at 4 for a forward horizon of one year. One year corresponds to the usual time frame used by central banks and 2 percent per year is a common inflation target. Furthermore, the parameter $\phi$ is used to show what happens when the central bank reacts ($\phi = 1.5$) or not ($\phi = 0$) to shocks. For impulse response functions, the persistence parameters for the shocks are set to $\rho = 0.75$ for temporary shocks and $\rho = 0.999999$ for permanent shocks (not exactly 1 to secure a numerical solution). Other parameters are such that $\gamma = 0.99$ and $\theta = 0.0025$. Table 1 lists the parameter values.
Table 1: Parameter values

<table>
<thead>
<tr>
<th></th>
<th>symbol</th>
<th>value</th>
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</thead>
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<tr>
<td>Monetary policy horizon</td>
<td>$S$</td>
<td>4</td>
</tr>
<tr>
<td>Inflation target</td>
<td>$\pi^*$</td>
<td>0.005</td>
</tr>
<tr>
<td>Monetary policy tightness</td>
<td>$\phi$</td>
<td>1.5</td>
</tr>
<tr>
<td>Effective discount factor</td>
<td>$\gamma$</td>
<td>0.99</td>
</tr>
<tr>
<td>Impact parameter</td>
<td>$\theta$</td>
<td>0.0025</td>
</tr>
<tr>
<td>Length of linear function</td>
<td>$n$</td>
<td>16</td>
</tr>
</tbody>
</table>

3.1 Impulse response functions

This subsection shows what the model implies in terms of impact analysis. Results from impulse response functions include: (1) a constant inflation target lowers real interest rate volatility compared to no monetary policy intervention, (2) a moving inflation target raises volatility even more, (3) persistent shocks have more impact on nominal interest rates while transitory shocks have more impact on inflation and (4) future real interest rate shocks lead to immediate response thereby simulating what has been called the price puzzle. Furthermore, the results reinforce the intuition behind nominal interest rate rigidities as central banks need to move interest rates in the opposite direction of where it wants to move inflation and credit conditions causes inflation to be procyclical.

Figure 1 and Figure 2 show three possible outcome for both temporary and permanent positive one percent shocks. The shaded area represents inflation implied by the difference between nominal and real interest rates. These shocks are expressed in deviations from control levels. In both figures shown, we set $\phi = 1.5$, since both $\phi = 1.5$ and $\phi = 0$ yield similar looking responses but with different size of deviations. This is the first result, it basically means that a monetary policy of inflation targeting lowers variations in real interest rates. It also means that real interest rates would increase in volatility if a central bank was to adopt an inflation target that varied through time. The monetary policy influence on real interest rates volatility will be investigated in next subsection.

The figures indicate two additional results. First, a permanent shock has a larger influence on nominal interest rates. More persistent shocks have more effect on nominal interest rates, while temporary shocks affect inflation more.
Second, anticipated shocks simulate what has been called a price puzzle. As we can see in Figure 2, rises in the nominal interest rate and the inflation rate appear simultaneously some time before deflation sets in.

Those results reinforce the intuition behind nominal interest rate rigidities. First, it shows that, in the presence of rate rigidities, the Fisher effect is inverted in the short term, so central banks can use interest to generate inflation. The argument is a simple application of Fisher’s equation \( i \approx r + \pi \).

A rise in real interest rates can either be performed through a one to one rise in nominal interest rates, an inverse one to one drop in inflation or a mix of the two. In the presence of rigidities, a rise in real interest rates could

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5The price puzzle was noticed by Sims (1992) who described it as an empirical anomaly. It states that an increase (decrease) in interest rates will increase (decrease) inflation in the short term before having the expected effect some quarters later. The name, price puzzle, was quoted by Eichenbaum (1992).
not be followed by a proportional movement from nominal interest rates and therefore would lead to disinflation. Therefore, the central bank’s operations does not directly lower nominal interest rates, but, in reality, *yanks* down the real ones. In doing so, it creates inflation.

Second, the results imply that the relationship between economic fluctuations and inflation needs to be supported by a relationship between credit conditions and fluctuations. The relationship between economic fluctuations and prices, between inflation and the output gap or the presence of deflationary pressures in (lets call them) ordinary recessions needs to come from an inverse relation between real interest rates and production within the cyclical time frame.

The existence of a link between credit conditions, as a fuller characterization of recessions, and the 2008 recession as been widely discussed. Such is the case in other periods as shown, for example, by Ng and Wright.
(2013), and Eckstein and Sinai (1986). To quote Bernanke, Gertler and Gilchrist (1999, p. 1343): “First, it appears that introducing credit-market frictions into the standard models can help improve their ability to explain even ‘garden-variety’ cyclical fluctuations.” The predicting power of an inversion of the yield curve on recessions (Stock and Watson (1989), Adrian, Estrella and Hyun (2010) and Rudebusch and Williams (2009)) also suggest that a credit crunch is an important part of the business cycle. The yield curve inverts when credit is expected to be temporarily tight in the very near future.

Of course, such a link does not predict the causal relationship, whether from production to credit conditions in a financial accelerator type of argument, as in Bernanke, Gertler and Gilchrist (1996, 1999) or Kiyotaki and Moore (1997), or from credit conditions to production in a credit cycle type of argument. Without clarifying the issue, this paper makes a strong case for future research into the relationship between fluctuations and credit conditions. In fact, literature pertaining to credit conditions, whether of financial accelerator or credit cycles flavors, is developing at a healthy pace.

3.2 Monetary regime simulations

Four different monetary regimes approximate interest rates data: a gold standard, a liquidity trap, a non-target fiat and an inflation target regime. A moving inflation target simulates the gold standard and replicates its high volatility in ex post real interest rates. I skipped the liquidity trap. A very persistent inflation target process simulates the non-target fiat regime, replicating the non stationarity seen in nominal interest rates and the more persistent ex post real rates. For the inflation target regime, the target obviously does not move. All in all, simulations show there exists inflation target schemes that can simulate how most real and nominal interest rates behavior has looked.

The first part of Figure 3 shows data from Shiller (1989, Chapter 26) for one year nominal and ex post real interest rates. The other parts of Figure 3 show simulations of the different monetary regimes. In order to simulate the averaging effect that yearly data produces, the simulation output, programmed as quarterly, is annualized. Figure 4 shows the original quarterly results. For all simulations, real interest rate processes have persistences of 0.9 and standard deviations of 0.0015 and 0.003 for surprise and anticipated shocks respectively. Table 2 lists the shock parameter values applied to the
Figure 3: Observed and simulated annualized real and nominal interest rates for different monetary regimes

Note: in percent.

The interest rates data are divided into four different monetary regimes. Table 3 compares the standard errors and correlation of nominal and real interest rates for the data and the three regimes simulated. Is is the restrictions imposed on the model that explain the differences between data and simulations. Consequently, the characterization of the rigidity equation
Figure 4: Simulated quarterly real and nominal interest rates for different monetary regimes

Note: in percent.

needs more work. This is left for future research.

The first regime, before the Great Depression, corresponds to gold standard periods. It is one regime, though the gold standard was implemented in different ways in practice, since different implementations are not apparent in rates data. That regime is characterized by a high volatility in ex post real interest rates and lower volatility in nominal interest rates; inflation volatility is also large. The focus of the central bank on inflation, that started much later, may explain that recent inflation persistence is visibly absent in this period, although it is a stylized fact of the post-war economy. A moving inflation target simulates the regime. This inflation target process is autoregressive of order 1. It had to have a large variance to make the graphs look right. Consequently, the process has a standard deviation of 0.015 and a persistence of 0.5. This supposes a large variance in gold prices, coherent
with gold prices still seen today.

Inflation target volatility has an interesting effect. This volatility greatly increases real interest rate volatility and puts the Great Moderation in perspective.\(^6\) The standard deviation of real interest rates under the gold standards is predicted by this model to be nearly four times that of real interest rates under the inflation target regime.

The second regime, from the Great Depression to before the Bretton Woods system, has seen the economy going in and out of either a liquidity trap or severe financial repression. That regime is characterized by often negative ex post real interest rates and nominal interest rates near the zero bound. The regime was not simulated. The model is not equipped to deal with asymmetries caused by the zero lower bound of nominal interest rates.

\(^6\)The Great Moderation was uncovered by Kim and Nelson (1999) and McConnell and Perez-Quiros (2000) and quoted by Stock and Watson (2003) to describe how, since the mid-eighties, the volatility of many macroeconomic variables had diminished.

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### Table 2: Shock parameter values

<table>
<thead>
<tr>
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<th>standard deviation</th>
<th>persistence</th>
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<tbody>
<tr>
<td>Surprise real interest rate shock</td>
<td>0.0015</td>
<td>0.9</td>
</tr>
<tr>
<td>Anticipated real interest rate shock</td>
<td>0.003</td>
<td>0.9</td>
</tr>
<tr>
<td>Inflation target shocks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>... under gold standard regime</td>
<td>0.015</td>
<td>0.5</td>
</tr>
<tr>
<td>... under non-target fiat regime</td>
<td>0.002</td>
<td>0.99</td>
</tr>
<tr>
<td>... under inflation target regime</td>
<td>0</td>
<td>—</td>
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### Table 3: Standard errors and correlation

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<th>Data</th>
<th>gold standard</th>
<th>non-target fiat</th>
<th>inflation target</th>
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<tbody>
<tr>
<td>std((r))</td>
<td></td>
<td>8.33</td>
<td>5.23</td>
<td>2.18</td>
</tr>
<tr>
<td>std((i))</td>
<td></td>
<td>1.27</td>
<td>3.60</td>
<td>2.12</td>
</tr>
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<table>
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<th>Simulations</th>
<th>gold standard</th>
<th>non-target fiat</th>
<th>inflation target</th>
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<td>0.30</td>
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</table>
With bonds having the same yield as money, there may also be a few additional issues relating to the buying and selling of bonds by the government that are posited in the model. Consequently, the issues surrounding zero interest rates will not be addressed in this paper.

The third regime, the non-target fiat, starts with the Bretton Woods Agreement and ends sometimes in the late eighties or early nineties. Though Bretton Woods is officially a gold standard regime, it acts more like a fiat regime; fixed exchange rates meant that the inflation target may have simply been managed globally. This regime seems plagued by non-stationarity. Unlike under the other regimes, sometimes real and nominal interest rates move in visibly opposite directions. From the model’s perspective, these opposite moves signal when changes in real rates are prompted by changes in the inflation target. For the postwar period until around the eighties, I determined that a very persistent inflation target process, almost a unit root, would best describe what was happening as inflation went gradually higher until Volker, then went gradually down until the early nineties. The apparent non-stationarity in the data pleads for the use of highly persistent shocks. Thus, the inflation target process is autoregressive of order 1 with a standard deviation of 0.002 and a persistence of 0.99.

The results from simulation of this regime show how almost-non-stationarity in inflation leads to almost-non-stationarity in nominal rates. Real rate are still stationary, but with more persistence than in other regimes. Furthermore, we have real and nominal interest rates moving sometimes in opposite directions as found in the data.

The last regime saw some form of inflation targeting scheme. Consistent with a highly persistent inflation rate, both real and nominal interest rates seem to want to move in tandem. In this regime, the inflation target does not change. The inflation target process could have included a small non-persistent error term to account for errors in predictions made by the central bank; this would be useful in a projection model where curve fitting is necessary, not here. The results are as expected as the annualized nominal and real rates tend to move in tandem except for when the real rate moves in a spike. In such a spike move, the nominal rate will not completely follow.

To summarize, all three regimes can be described by different inflation targets: the gold standards by a volatile inflation target, the non-target fiat regime by a variable but persistent target and the more recent regime by a fixed target. In terms of results, the variable inflation targets increase real interest rate volatility, non-stationary inflation targets lead to non-stationary
nominal interest rates and more persistence of real interest rates, while constant targets lead to more stable real interest rates.

We can conclude that there exists an inflation target schemes that can simulate how real and nominal interest rates behavior looks. For lack of a thorough empirical investigation, we can say, at the very least, the model looks promising: the interest rate processes that emerges from simulation are realistic.

3.3 Digression on estimation

Estimation of the Fisher equation dates from Fisher himself. Of course, it is always a restriction on the equation that is being estimated since the equation itself is more or less a definition. Most involve real interest rate persistence, an extensive recent review of which is provided by Neely and Rapach (2008). In developing an estimation strategy, we are faced with the problem of determining the restrictions on the economy that we want to test.

Use of data is also problematic since the representative agent’s interest rate may be difficult to find or even to define. The concept of a representative agent’s interest rate will complicate empirical validation, but this concept will also legitimize the point made about tight credit in recessions as spreads are procyclical and may not entirely reflect the movement of the probabilities of default, but other aspects as well, among them liquidity preferences. The ideal may be an interest rate of lower investment grade, say BBB, without the risk premium.

Furthermore, precedent studies are difficult to apply here for two reasons. First, as stated earlier, studies of interest rates have often taken nominal interest rates to be exogenous, as they are believed to be directly controlled by monetary authorities. The present framework does allow complete control of nominal interest rates by said authorities, but only indirectly. Second, calculations of ex ante real interest rates are done using inflation expectations which, even when relying on polls, come from smoothed forms of realized inflation. As such, if nominal interest rates are believed to be rigid, so will the resultant real interest rates. From this model’s perspective, oversmoothing of real interest rates will create a bias.

One option would be to develop a complete structural model that would generate a credible link between real interest rates and production. Since standard RBC’s or even DSGE’s do not always have pro-cyclical interest rates, this model would have to have a credit channel à la Bernanke, Gertler
and Gilchrist (1996, 1999), Kiyotaki and Moore (1997) or more recent variants. The choices involved would greatly expand the scope of this paper without making it more relevant. The work of evaluating this theory of inflation within complete models is thus left to future research as it deserves its own paper or papers.

Another option involves testing the persistence of nominal interest rate with restrictions imposed by inflation data and the present theoretical framework as to how the rigidities take shape. Unfortunately, preliminary investigations have shown that complex questions arise from the form of the equations to estimate especially in terms of identification. It is, for example, possible to characterize nominal interest rates and inflation with the sole use of of a variable inflation target while leaving the equilibrium real interest almost unchanged over the whole sample. These questions need specific answers. Again, the sheer size of this article would grow uncomfortably bigger. Therefore, it has been decided to keep the estimation for later research.

4 Conclusion

A vast literature supports the central postulate that interest rate are rigid, as well as the procyclicality of inflation and how we believe monetary authorities use interest rates to manipulate inflation. This paper looked at the impact of rigidities in nominal interest rates. It offers a model to explain rigidities in nominal interest rates based on rational decisions in perfect competition without relying on commitments as commitments are unrealistic when faced with real interest rates and inflation.

In terms of results, the paper shows that an interest rate model of inflation could be a satisfactory implementation of price evolution inside a larger macroeconomic model. It explains the two major stylized facts about inflation: how central banks move nominal interest rates in opposite direction to control inflation and why credit conditions around recessions lead to deflationary pressures. Side contributions include a possible explanation of the price puzzle, from the preemptive aspect of nominal interest rate setting, and of the Great Moderation, from the destabilizing effect of variable inflation target policies prior to the mid-eighties.

One improvement would benefit the model: improving the rigidity equation. As we have seen, the parametrization of the rigidity equation was constructed for exposition purposes and might not do in a workhorse model.
Furthermore, although simulation results are realistic, no estimation of the model’s parameters or test of the theory has been offered. They are left for future research.

References


