Correct or incorrect application of CAPM? Correct or incorrect decisions with CAPM?

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Abstract. This paper focuses on inconsistencies arising from the use of NPV and CAPM for capital budgeting. It shows that (i) CAPM capital budgeting decision-making based on disequilibrium NPV is deductively inferred by the Capital Asset Pricing Model, (ii) the use of the disequilibrium NPV is widespread in finance both as a decision rule and as a valuation tool, (iii) the disequilibrium NPV does not guarantee additivity nor consistency with arbitrage pricing, so that it is unreliable for valuation, (iv) Magni's (2002, 2007a, forthcoming) criticism of the NPV criterion refers to the disequilibrium NPV, and De Reyck's (2005) project valuation method, on the basis of which Magni's criticism to NPV is objected, leaves decision makers open to arbitrage losses and incorrect decisions.

Keywords. Finance, investment analysis, Net Present Value, Capital Asset Pricing Model, disequilibrium, decision, valuation, nonadditivity, arbitrage.

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Introduction

The Net Present Value (NPV) and the Capital Asset Pricing Model (CAPM) are two companion tools, widely used for capital budgeting purposes (Poterba and Summers, 1995; Ryan and Ryan, 2002; Graham and Harvey, 2002; Hammer, 2003; Brounen, de Jong, and Koedijk, 2004). They are considered norms of rationality for decision making: “Companies contemplating investments in capital projects *should* use the net present value rule” (Graham and Harvey, 2002, p. 10, italics added); “Net Present Value … *always* provides the correct financial criterion for selection between projects” (Franks and Broyles, 1979, p. 50); “It is almost impossible to overstate the importance of the net-present-value concept … most financial decisions can be viewed in terms of net present value” (Emery and Finnerty, 1997, p. 94). The NPV and the CAPM are considered by some authors two of the most important ideas in finance (see last chapter in Brealey and Myers, 2000).

In a recent article, Magni (2002) raises some doubts on the use of the NPV as a reliable capital budgeting model under uncertainty. In a reply, De Reyck (2005) fosters the idea that Magni’s application of the NPV+CAPM methodology is incorrect and that such a methodology is a theoretically sound tool for valuing projects:

> We will show in this paper that the alleged inconsistencies are a result of an incorrect valuation method described in the paper (De Reyck, 2005, p. 500)
>
> the analysis presented in the paper is flawed. In fact, the net present value rule, when applied correctly, can be used to value investment projects by comparing assets of equivalent risk. (ibidem, p. 504)

De Reyck (2005, p. 501) presents a Proposition which is used to counter Magni’s thesis:

> we have provided an alternative, and correct, valuation method for the projects and investment schemes presented in the paper. (ibidem, p. 504)

In a reply, Magni (2005, p. 1) objects that De Reyck "mistakes a project's expected rate of return for a project's cost of capital", and in a most recent article he shows that CAPM capital budgeting criterion based on disequilibrium NPV is inconsistent with arbitrage pricing (Magni, 2007a).

Using algebra and some counterexamples, and distinguishing decision from valuation, this paper shows that:

(a) the legitimacy of the disequilibrium NPV as a *decision* rule in accept/reject situations is deductively inferred from the Capital Asset Pricing Model
(b) the use of the disequilibrium NPV for both *valuation* and *decision* is widespread in corporate finance
(c) the disequilibrium NPV does not preserve additivity nor compatibility with arbitrage pricing, so it may not be used for *valuation* purposes
(d) De Reyck's proposal, which dispenses with disequilibrium NPVs, does not guarantee that absence of arbitrage will be preserved nor that decisions will be correct.

The paper is structured as follows. Section 1 introduces the CAPM capital budgeting criterion showing that if the CAPM assumptions are met, a project is profitable if and only if its disequilibrium (i.e. cost-based) NPV is positive. In other words, a disequilibrium NPV is legitimately deducted from the CAPM as a decision rule; however, this does not imply that the disequilibrium NPV is the value of the project. Section 2 provides some evidence that in corporate finance it is common to (implicitly or explicitly) use the disequilibrium NPV not only as a decision rule but also as a valuation tool. Section 3 uses some algebra and a counterexample taken from Copeland and Weston (1988) to show that the disequilibrium NPV does not guarantee additivity and is inconsistent with arbitrage pricing. This implies that the d-NPV may not be used as a valuation tool and that the d-NPV as a decision rule is unsafe. Section 4 presents De Reyck's Proposition and offers the reader a simple counterexample showing that his approach, while dispensing with the use of disequilibrium values, does not guarantee correct valuations and decisions. Some remarks conclude the paper.

Main notational conventions	
$F_j\ (\bar{F}_j)$	Asset j's end-of-period random cash flow (expected cash flow)
I_j	Cost of project j
$r_j\ (\bar{r}_j)$	Asset j's random rate of return (expected rate of return)
$V_j\ (v_j)$	CAPM-based (arbitrage-based) value of asset j
V_m	Value of the market at time 0
$V_m^1\ (\bar{V}_m^1)$	Random (expected) value of the market at time 1
$r_f\ \ (R_f)$	Risk-free rate (1+risk-free rate)
σ_m^2	Variance of the market rate of return
cov	Covariance
$\lambda := \dfrac{\bar{r}_m - r_f}{\sigma_m^2}$	Market price of risk
$P_l\ \ (P_l^\circ)$	Price of firm l's shares before (after) acceptance of the project
N_l	Number of firm l's outstanding shares
N_l°	Additional shares issued at price P_l° to finance the project
$\beta_j = \dfrac{\text{cov}(r_j, r_m)}{\sigma_m^2}$	Systematic risk
$\text{NPV}_j\ (\text{npv}_j)$	CAPM-based (arbitrage-based) Net Present Value
	$j = Z, l, m,$ 1, 2

1. The CAPM-derived capital budgeting criterion

Consider a firm facing an investment. A widely accepted capital budgeting rule is that the project should be undertaken if, after acceptance, the firm's share price increases. Abiding by this tenet and following Rubinstein (1973), we formally prove that if the market is in equilibrium and CAPM pricing holds for all assets in the security market, then a project is profitable if and only if the NPV is positive.

Proposition 1.1. Suppose all CAPM assumptions are met, and a firm *l* has the opportunity of undertaking a project that costs I_Z and generates the end-of-period payoff F_Z. Then, the project is worth undertaking if and only if

$$\text{NPV}_Z := \frac{\overline{F}_Z}{R_f + \dfrac{\lambda}{I_Z}\text{cov}(F_Z, r_m)} - I_Z > 0 \tag{1}$$

Proof: See Appendix 1.[1]

The risk-adjusted cost of capital in eq. (1) may be rewritten as a function of the project beta, so that the NPV becomes

$$\text{NPV}_Z := \frac{\overline{F}_Z}{R_f + \beta_Z(\overline{r}_m - r_f)} - I_Z > 0 \tag{1-bis}$$

where

$$\beta_Z := \frac{\text{cov}(F_Z, r_m)}{I_Z \sigma_m^2}. \tag{1-ter}$$

Note that the cost of capital depends on a (cost-based) disequilibrium beta, i.e. on a beta which is a function of the project cost. The resulting Net Present Value is therefore a (cost-based) disequilibrium NPV. Therefore, Proposition 1.1 shows that the disequilibrium NPV (henceforth often d-NPV) is legitimately deducted from the Capital Asset Pricing Model, and the CAPM capital budgeting rule is as follows: given an investment, one should undertake it if and only if its

[1] It is assumed that R_f and $R_f + (\lambda / I_Z)\text{cov}(F_Z, r_m)$ have equal sign. If this condition is not met, the thesis holds with the sign of eq. (1) reversed.

disequilibrium NPV is positive. Magni's application of the CAPM capital budgeting criterion is just grounded, implicitly (Magni, 2002) or explicitly (Magni, 2005, 2007a, forthcoming), on the use of disequilibrium values.

It is worth distinguishing between NPV as a decision rule and NPV as a valuation tool. Under certainty, a project NPV is legitimately used for both decision and valuation. Under uncertainty, we have just seen that the d-NPV as a *decision* rule for accept/reject alternatives is logically deducted from the CAPM.[2] But, strictly speaking, one cannot logically infer from eq. (1) that the d-NPV is a *valuation* tool. Actually, the CAPM capital budgeting criterion in eq. (1) tells us that an investment is profitable if its d-NPV is positive, but does not tell us that the d-NPV is the *value* of the project, because it does not measure shareholders' wealth increase (see last identity in eq. (4)). However, the shift from its application as a decision rule to its application as a valuation tool is a standard step in corporate finance, given that it is usual to consider a Net Present Value jointly a decision rule and a valuation tool: indeed, NPV is considered a decision criterion just because NPV is considered the project value (see Solomon and Pringle, 1980, p. 451, on equivalence of value and decision criterion). This means that, whenever the NPV is presented as a decision rule, it is implied that it serves a valuation purpose as well. This is the reason why, when encountering authors that favour the use of the d-NPV, one should consider their approval as implicitly referred to decision and valuation as well (after all, those scholars that favour the d-NPV would not use the term Net Present *Value* if they would not regard it as the very value of the project). In other words, even in uncertain contexts it is usual to think of decision and valuation as two sides of the same medal.

The issue is studied in some detail in the following section, where we present distinctive (and even explicit) signs suggesting that many authors favour the use of the d-NPV in both senses.

2. The disequilibrium NPV in corporate finance

This section gives some evidence that the use of the d-NPV in corporate finance (as a decision rule and) as a valuation tool is widespread in finance, implicitly or explicitly.

The evidence is threefold:

A. <u>Explicit stances</u>. Some authors *explicitly* endorse the use of the d-NPV as the correct one to value projects; some others *explicitly* warn against it, just because they consider its use extensive in finance.

[2] The use of the disequilibrium NPV for *ranking* projects is not deductable from eq. (1).

B. Implicit stances in textbooks. The approach to NPV adopted by most corporate finance textbooks ingenerates the idea that the d-NPV is correct (both as a decision rule and as a valuation tool).
C. Implicit stances in classical papers. Some of the classical contributions that introduced and developed the use of the CAPM for capital budgeting purposes in the financial community implicitly convey the idea that the d-NPV should be used as a decision rule and as a valuation tool.

Let us begin with A. One can actually find, scattered in academic papers and corporate finance textbooks, explicit claims or signs that the d-NPV is the correct NPV for valuing and ranking projects. Bossaerts and Ødegaard (2001) endorse the use of cost-based betas and d-NPVs "to value a risky cash flow" (p. 60); an unambiguous numerical example follows their recommendation. Copeland and Weston explain the use of the CAPM for capital budgeting in various occasions. While they mostly take it for granted that an investor should use the d-NPV for decision and valuation as well, they often remind the reader the valuation role of the d-NPV: in Weston and Copeland (1988) they write that "the present value, PV, of a one-period project can be found by discounting its expected cash flows $E(CF)$, at a risk-adjusted rate, $E(r_j)$" (p. 375); in the same book they explicitly give the formula of the d-NPV referring to it as "the risk-adjusted method for evaluating projects" (p. 381), and the same expression with the same formula is found in Copeland and Weston (1983, p. 135). Also, their expression "appropriate discount rates" (Copeland and Weston, 1988, p. 417) implicitly refers to the valuation role of the d-NPV. These authors provide several numerical examples to illustrate the implementation of the d-NPV (e.g. Copeland and Weston, 1983, p. 135; Weston and Copeland, 1988, pp. 372-375 and pp. 379-381; Copeland and Weston, 1988, pp. 415-418). Rendleman (1978) explicitly claims that the disequilibrium covariance term is the correct covariance term, and shows that some ranking errors arise if one uses a market-determined (i.e. equilibrium) covariance term.[3] Jones and Dudley (1978, p. 378) compute the required rate of return of a mispriced asset by discounting cash flows with a cost-based discount rate, i.e. by using the d-NPV (see their Tables 18.2 and 18.3).[4] Lewellen (1977) uses the d-NPV approach for valuation. In particular, he computes a CAPM-derived risk-adjusted discount rate (see his example at pp. 1333-1335) which is used for valuing projects. There are various occurrences of

[3] Admittedly, while explicitly referring to the disequilibrium covariance term as the correct one, Rendleman's paper is not unambiguous, because he deals with the notion of excess return as well, but does not make explicit the subtle relations existing among the notions of cost of capital, NPV and excess return in uncertain contexts. On the same topic see also Weston and Chen (1980).
[4] In the example the rates of return in the various states are taken as given. As the asset is, by assumption, *mispriced*, the rates of return are evidently based on the disequilibrium price (cost) and the corresponding beta is a disequilibrium beta. As a result the discount rate is a disequilibrium (cost-based) discount rate.

the terms "value" and "worth": for example, "total-project present worth" (p. 1335), "present value of project B" (p. 1335), "Which of the two proposals, then, is more valuable?" (p. 1334); "Project B, therefore, is the more desirable one because .. its net cash flows prospects …be worth more in present value terms" (p. 1334), and the like. There are even explicit references, such as the use of symbol $V_0(t)$ to indicate "net current worth" (p. 1332), or symbols $V_0(A)$ and $V_0(B)$ to indicate the "(market) values" (p. 1334) of projects A and B, as well as unambiguous expressions such as "risk-adjusted rates … as approximations to value measurement" (p. 1331), "risk-adjusted discount rate $\overline{R}$ to evaluate that period's prospects" (p. 1332), "risk-adjusted-discount-rate capital budgeting format … present value computation which uses that approach" (p. 1335).[5]

Other authors indirectly testify with their papers that the use of the d-NPV is widespread. For example, Ekern (2006, p. 1) refers to the use of the d-NPV as a "common pitfall" and devotes his paper to providing other valuation methods that avoid the use of the d-NPV. Grinblatt and Titman (1998) devote several pages to warn against the d-NPV (pp. 384-388), which attests that they are aware that the use of the d-NPV for valuing projects is extensive in finance. Ang and Lewellen (1982) warn against the d-NPV as well and are unequivocally explicit in writing that the d-NPV is the "standard discounting approach" (p. 9) in finance. That such a procedure is actually used as a valuation tool is obvious to the authors, who write of "the standard use of CAPM-derived discount rates to *value* cash flows" (p. 5, italics added).

Let us now turn to implicit stances in textbooks. The assertion in B is just a consequence of three basic statements that, taken separately, are routine in finance:

- B.1 the NPV is the sum of the discounted expected cash flows of the project, with the cost of capital as the discount rate
- B.2 the NPV is a value
- B.3 the cost of capital derived from the CAPM is cost-based.

Assertion B.1 is obviously beyond question, as this is just the universally accepted definition of NPV, available in any textbook (e.g. Benninga, 2006, p. 239). As for assertion B.2, any corporate finance textbook introduces the basic notion that the NPV is the value of the project, net of the initial cost: "If you want to be sure that you are appraising investment opportunities properly there is no short-cut – you must use the NPV approach" (Ogier, Rugman, and Spicer, 2004, p. 187); "when we calculate a project's NPV, we are asking whether the project is worth more than it costs. We are estimating its value by calculating what its cash flows would be worth if a claim on them

[5] The values are computed through the cost-based discount rates that are drawn from the second Table of p. 1334 (where projects' rates of return are cost-based).

were offered separately to investors and traded in the capital markets" (Brealey and Myers, p. 1006). After all, the expression Net Present Value self-contains the term "value", which just means that it measures the "net addition to shareholders' wealth" (Brealey and Myers, 2000, p. 305). Entire chapters, sections, subsections of books explicitly or implicitly remind this universally shared idea: "the NPV is important because it gives a direct measure of the dollar benefit (on a present value basis) of the project to the firm's shareholders" (Brigham and Gapensky, 1994, p. 409); "the project's NPV is its estimated contribution to the wealth of the firm's security" (Fama, 1996, p. 415); "discounted cash flow (DCF) techniques should be used to evaluate capital expenditure proposals. Further, the net present value (NPV) method is the best of the DCF criteria" (Brigham, 1975, p. 17). Implicitly or explicitly, the equation NPV=value is considered legitimate even under uncertainty: for example, Brigham and Gapenski (1994) refer to the NPV naming it "the risk-adjusted discount rate method, in which riskier projects are *evaluated* using a higher discount rate" (p. 488, italics added), while Haley and Schall (1978) highlight that "financial theorists have devoted much attention to the appropriate discount rate to be used to *evaluate* a risky investment" (p. 853, italics added). To compute the NPV, one must therefore find the correct cost of capital and uses it as the discount rate: "the cost of capital is used … as a discount rate for computing present values (Haley and Schall, 1978, p. 852);[6] "The net present value states in a single figure the value of a stream of cash flows from a business or venture, taking into account the required return to investors…Cost of capital theory tells us how to calculate the appropriate discount rate" (Ogier, Rugman, and Spicer, 2004, p. 169). As "the discount rate used to calculate a project's NPV should reflect the project's risk" (Brigham, 1975, p. 17) a model must be selected to determine such a risk. The CAPM is a usual suggestion: "Textbooks in corporate finance typically … prescribe discounting expected payoffs with costs of capital (usually CAPM) expected 1-period simple returns" (Fama, 1996, p. 426). If the evaluator uses the CAPM, the resulting discount rate is the CAPM-derived cost of capital in eq. (1): "The CAPM provides a framework for estimating the appropriate opportunity cost to be used in *evaluating* an investment" (Rao, 1992, p. 336, italics added). Many contributions in the 60s and 70s present the CAPM capital budgeting criterion, some of which explicitly deal with the notion of risk-adjusted cost of capital (Tuttle and Litzenberger, 1968; Hamada, 1969; Rubinstein, 1973). These papers often explicitly highlight the role of the CAPM-derived cost of capital, and, as assertion B.3 reminds, they formally prove that it depends on *cost*, which just means that the CAPM-derived cost of capital is a *disequilibrium* rate of return (see a survey in Magni, 2007b).

[6] Whenever one writes of "discount rate", one refers (implicitly or explicitly) to the appropriate cost of capital used for *valuing* projects (see also the previous quotation by Haley and Schall, 1978).

Considered together, the somewhat trivial assertions B.1-B.3 give an indirect (but strong) support of assertion B. The implicit argument a reader is induced to draw from them goes as follows: a project's value is given by its NPV (B.2). The NPV is calculated by discounting cash flows with an appropriate discount rate, which is the project's cost of capital (B.1); if the CAPM is used to compute it, the discount rate is cost-based (B.3). Therefore: if the CAPM is used, the project's value is cost-based, i.e. it is given by the d-NPV (B). As a result, not only "most textbooks in finance do not warn against a common pitfall in discounting expected cash flows by risk adjusted discount rates" (Ekern, 2006, p. 1), but they even seem to implicitly uphold the use of the disequilibrium NPV model, which "may well be the one selected by analysts, practitioners and other decision makers having had some exposure to finance as reflected in popular textbooks" (Ekern, 2006, pp. 1-2).

Finally, we have assertion in C. In the late '60s and in the '70s foremost authorities were concerned with drawing a capital budgeting rule from the CAPM. Among these authors, we find Tuttle and Litzenberger (1968), Mossin (1969), Hamada (1969), Litzenberger and Budd (1970), Rubinstein (1973), Bierman and Hass (1973, 1974), Stapleton (1971, 1974), Bogue and Roll (1974).[7] Although these authors are never explicit about this issue, an analysis of some of their papers seems to be supportive of the disequilibrium NPV.

For example, Tuttle and Litzenberger (1968), Hamada (1969), Litzenberger and Budd (1970), Rubinstein (1973) directly and explicitly focus on the notion of cost of capital. They all prove that a project is worth undertaking if its expected rate of return exceeds the risk-adjusted cost of capital (which depends on the project cost, not on the project's equilibrium value). Litzenberger and Budd (1970, p. 399) acknowledge the coincidence between the cost of capital introduced in Tuttle and Litzenberger (1968) and the cost of capital presented in Hamada (1969). The latter coincides with Rubinstein's (1973) cost of capital, which plays a fundamental role in Rubinstein's survey.[8]

The overwhelming insistence of these (and other) authors on the notion of (risk-adjusted) cost of capital (which is shown to be cost-based) and the attention drawn on the connection between cost of capital and shareholders' wealth increase, induce readers to consider the risk-adjusted cost of capital as the correct discount rate for valuing projects. The authors themselves seem to support these thesis. Tuttle and Litzenberger (1968) aim at "screening and ranking opportunities under condition of uncertainty" (p. 427) and cite the "standard present value method" (p. 428). The latter method boils down to accepting projects having "positive net present values with the average cost

[7] The CAPM capital budgeting rules introduced by all these authors are equivalent (see Senbet and Thompson, 1978, and Magni, 2007b).

[8] Bierman and Hass (1973) present a criterion based on the cost of capital as well, although implicitly. Their equation (10) is just Rubinstein's (1973, p. 171) equation divided by $(1+r_f)$ (see Magni, 2007b).

of capital as the discount rate" (p. 427). In their paper they just show that the use of an average cost of capital is not correct and that a project-specific cost of capital should replace it. Such a project-specific cost of capital "is used as the discount rate for the particular project under consideration". As above reminded, their cost of capital is just the same as that in our eq. (1), so their remarks seem to be supportive of the d-NPV as a valuation tool (just remember that in finance the *standard present value* method the authors refer to is a valuation method, and that a *discount rate* is used for valuing projects). Rubinstein (1973) seems to be even more inclined to the d-NPV. In the last sentence of the second paragraph at p. 174 of his paper the author writes that the (risk-adjusted) cost of capital is the "appropriate discount rate for the project". Some pages earlier he makes the same claim, when he writes of "risk-adjusted discount rate for the project" and repeats the expression "discount rate" just after the sentence (p. 172). If Rubinstein thinks the (disequilibrium) cost of capital is the appropriate discount rate for the project, then he ingenerates the idea that the disequilibrium NPV is the correct NPV.[9] Also, in footnote 14 at p. 174 he writes, referring to mutually exclusive investments: "This result follows immediately from equation (c) of footnote 10 and is equivalent to accepting the project with the highest net present value". He is then claiming that the (disequilibrium) NPV is a correct tool for ranking projects (see our footnote 2 above). Also, in footnote 8 he uses the expression "present value risk-adjusted discount rate … form": this expression means that his rule may be shaped in the form of a present value obtained by discounting expected cash flows with the cost of capital just introduced. That is, he refers to the (disequilibrium) NPV and uses the term "value" to mean the result of the (cost-based) discounting process. Then, no wonder if a reader draws the idea that the disequilibrium NPV is a "value", and that such an NPV may be used for valuation purposes.

As a last remark, just think that if these authors considered the disequilibrium NPV to be incorrect, they would not indulge in the notion of a project's cost of capital, and would not underline that it is the *discount rate* for the project. They would probably underline the fact that the cost of capital they find may not be used for discounting cash flows, because the result of this discounting process is not the value of the project. Or they would warn the reader that the d-NPV may be used for accept/reject decisions but not for valuations. And they would not provide the reader with proofs where a *cost-based* required rate of return is involved: they could alternatively provide an equivalent proof where an equilibrium required rate of return or an equilibrium project value is shown (as in Bogue and Roll, 1974, p. 606). In other words, we have authors that present a rule in a rate-of-return-exceeding-cost-of-capital form and claim that the cost of capital is a discount

[9] Whenever an author writes that the cost of capital is the appropriate discount rate for the project, he is giving the idea that the resulting NPV is the value of the project (see also footnote 6 above). And Rubinstein's NPV is a disequilibrium NPV, due to its cost-based beta.

rate for the project. This ingenerates the idea that the resulting NPV is the project's value, as the "standard present value method" cited by Tuttle and Litzenberger presupposes.[10] The idea is reinforced by the fact that these papers do not unambiguously distinguish between decision and valuation. A reader is then left with the idea that the authors simply take for granted that the disequilibrium NPV is correct, and that its use is legitimate not only for decision but also for valuation.

As a result, we may say that the use of disequilibrium NPV is widespread in finance, sometimes explicitly, sometimes implicitly. Not many are the authors that warn against it (Ang and Lewellen, 1982, Magni, 2002, 2007a, forthcoming) and very few are so rigorous as to clarify the distinction between decision and valuation (Grinblatt and Titman, 1998, Ekern, 2006). Actually, no debate has shed sufficient light yet on the distinction between decision and valuation and on the relation between disequilibrium values and equilibrium values: "The topic is mostly absent from most popular textbooks" (Ekern, 2006, p. 5).

To sum up the first two sections:

- the use of the d-NPV as a decision rule in accept/reject situations is logically implied by the CAPM. Therefore, given an investment opportunity, its use is legitimate.
- the use of the d-NPV as a valuation tool is, implicitly or explicitly, widespread in corporate finance, although it is not logically implied by the CAPM.

Next section focuses on nonadditivity of the d-NPV. From the point of view of valuation, one finds that the d-NPV leads to incorrect values; from the point of view of decision-making one finds that, despite the logical connection reminded above, the d-NPV is unsafe, for it may lead to different choices depending on the way the course of action is represented.

3. Nonadditivity

Although the use of the d-NPV as a valuation tool may not be inferred from eq. (1), one cannot dismiss it unless a direct proof is presented of its unreliability. In finance, additivity and arbitrage pricing are the benchmarks for rational valuation: the following Propositions show that the d-NPV is nonadditive and inconsistent with arbitrage theory.

Proposition 3.1. The disequilibrium NPV is nonadditive.

[10] The definition of cost of capital as that discount rate that leads to the value of a project is basic in finance (as already underlined) and dates back earlier than the CAPM. For example, Solomon (1963, p. 27) makes explicit that *value* is obtained by discounting cash flows at the cost of capital and that this is just the reason why the value-exceeds-cost rule is logically equivalent to the rate-of-return-exceeding-cost-of-capital rule.

Proof: See Appendix 2.

The following Proposition shows that the disequilibrium NPV does not guarantee consistency with arbitrage pricing. Dealing with arbitrage pricing, we now implicitly assume that the market is complete (if market is not complete, there is no such a notion of arbitrage-based value of a project; however, CAPM-based valuation can be applied in incomplete markets as well).

Proposition 3.2. Consider a project Z and a security t lying on the Security Market Line, such that $F_Z = \theta F_t$ for some θ. Suppose project Z does not lie on the SML. Then, its arbitrage-based npv differs from its CAPM-based (disequilibrium) NPV.

Proof: See Appendix 3.

Corollary 3.1. The disequilibrium NPV is not the value of the project.

Proof: Obvious conclusion from Propositions 3.1 and 3.2.

Therefore: the use of d-NPV for decision is legitimate (Proposition 1.1), but its use for valuation is illegitimate (Corollary 3.1). We now give an illustration of the nonadditivity of the d-NPV just considering one of the examples employed by Copeland and Weston to illustrate the use of the NPV under uncertainty. In particular, let us consider the example in Copeland and Weston (1988, pp. 414-418). Tables 1 and 2 present the two projects the authors are concerned with and the security market. The authors first compute the betas using the formula we have reminded in equation (1-ter), and then calculate the two risk-adjusted costs of capital: –9.33% for project 1 and 14% for project 2.

Table 1

Copeland and Weston's example –cash flows and rates of return

		Market		Project 1 ($I_1 = 100$)		Project 2 ($I_2 = 100$)	
	Probability	r_m (%)	r_f (%)	F_1	r_1 (%)	F_2	r_2 (%)
State 1	0.333	26	4	105	5	107.5	7.5
State 2	0.333	14	4	115	15	100	0
State 3	0.333	20	4	95	–5	102.5	2.5

Table 2				
Copeland and Weston's example –relevant statistics and values				
	$\bar{r}_j$ (%)	$\mathrm{cov}(r_j, r_m)$	β_j	cost of capital (%)
Project 1	5.00	−0.002	−0.833	−9.33
Project 2	3.33	0.0015	0.625	14.00
Market	20.00	0.0024	1.000	

Computing the NPVs of the projects, one finds

$$\mathrm{NPV}_1 = -100 + \frac{(105+115+95)/3}{1+(-0.0933)} = 15.808$$

and

$$\mathrm{NPV}_2 = -100 + \frac{(107.5+100+102.5)/3}{1+0.14} = -9.356\,.$$

Consider now the project obtained by summing the cash flows of projects 1 and 2 (Tables 3-4).

Table 3			
Project (1+2) –cash flow and rates of return			
		Project (1+2) ($I_{1+2} = 200$)	
	Probability	F_{1+2}	r_{1+2}
State 1	0.333	212.5	6.25
State 2	0.333	215	7.50
State 3	0.333	197.5	−1.25

Table 4				
Project (1+2) –relevant statistics and values				
	$\bar{r}_{1+2}$ (%)	$\mathrm{cov}(r_{1+2}, r_m)$	β_{1+2}	Cost of capital (%)
Project (1+2)	4.166	−0.00025	−0.104	2.33

The NPV of this portfolio is

$$\mathrm{NPV}_{1+2} = -200 + \frac{(212.5 + 215 + 197.5)/3}{1 + 0.0233} = 3.583 .$$

However,

$$\mathrm{NPV}_1 + \mathrm{NPV}_2 = 15.808 - 9.356 = 6.452 \neq 3.583 = \mathrm{NPV}_{1+2} ,$$

which means that valuation is nonadditive. This boils down to saying that the d-NPV may not be a project's value.

Although the use of the d-NPV as a decision rule is validated by the CAPM-derived eq. (1), its nonadditivity gives some problems even in the context of decision-making. A simple counterexample is found by modifying the previous example and considering, other things unvaried, a cost of 104.6 for project 1. Making the easy calculations again, one finds that

$$\mathrm{NPV}_{1+2} = -1.091 < 0 < 1.108 = \mathrm{NPV}_1 + \mathrm{NPV}_2 .$$

The portfolio is either profitable or not depending on whether one represents it as a unique project or as a sum of projects. Therefore, the use of d-NPV as a decision rule is unsafe, because any course of action taken by a decision maker may be equivalently represented in different ways.[11] To use a decision tool that leads to different choices when framing is changed means that decision makers may fall prey to framing effects. In other words, CAPM-NPV-minded decision makers' choice behaviour may violate the rational principle of description invariance, according to which decision should not change if the problem at hand is differently framed, as long as the descriptions are logically equivalent (Kahneman and Tversky, 1979, 1984; Tversky and Kahneman, 1981). In a nutshell: eq. (1) does legitimate the d-NPV as a decision tool for a *given* investment, but

[11] An "amount of 100 euros" is also "two amounts of 50" or "three amounts of 30, 50, 20" or any other collection of amounts (the decision maker has, so to say, sovereign descriptive power). In particular, facing two projects (to be both accepted or rejected), the decision maker may calculate NPV as a single NPV or as a sum of several NPVs (see Magni, 2002, p. 211).

nonadditivity renders it unsafe, for decision makers may be tempted to compute the NPV in different ways (depending on various economic and psychological factors. See Magni, 2002, p. 211).

4. De Reyck's Proposition

De Reyck (2005) claims that Magni's (2002) approach to the CAPM+NPV methodology is mistaken. As seen in section 1, Magni's approach to the d-NPV is correct, and his criticism of the d-NPV is warranted by Proposition 3.1 and 3.2. It seems that De Reyck attributes Magni an incorrect application of a correct measure, rather than a correct application of an incorrect measure. This misunderstanding, redolent of an ambiguity existing in the literature about the standard application of the CAPM capital budgeting criterion, conceals what these authors have in common: both consider the d-NPV an incorrect measure for *valuing* projects.[12]

De Reyck's defence of the NPV+CAPM methodology is based on the implicit idea that equilibrium values should replace disequilibrium values. Formally, De Reyck's (implicit) net present value is

$$\text{e-NPV}=V_Z^e - I_Z = \frac{\overline{F}_Z}{\left(R_f + \lambda \operatorname{cov}(r_Z^e, r_m)\right)} - I_Z\,,$$

where $r_Z^e = F_Z / V_Z^e - 1$ is the project's equilibrium rate of return and V_Z^e is the project's equilibrium value (the symbol e-NPV stands for "equilibrium NPV"). As known, the solution of the former equation is

$$\text{e-NPV}=\frac{\overline{F}_Z - \lambda \operatorname{cov}(F_Z, r_m)}{R_f} - I_Z\,.$$

The latter is just the (explicit) starting point of De Reyck's defence (see his eq. (2.5)). He rests on the following Proposition:

De Reyck's Proposition. Let project *X* be a one-period project resulting in cash flows $\widetilde{x} = (x_1, x_2)$ with probabilities *p* and 1−*p*, respectively, and let *i* be project *X*'s cost of capital, obtained through the market valuation of a security or project with the exactly the same payoff pattern. Project *Y* with $\widetilde{y} = (y_1, y_2)$ with probabilities *p* and 1−*p* can then be valued as follows:

[12] The source of the misunderstanding is that Magni (2002) (and Magni, 2005, 2007a, forthcoming as well) considers the *disequilibrium* NPV as the standard CAPM-based Net Present Value in finance, whereas to De Reyck (2005) the standard CAPM-based Net Present Value is the *equilibrium* NPV. See also the concluding remarks of this paper.

$$V_Y = \frac{E(\tilde{y}) - \rho(\tilde{y},\tilde{x})(i - r_f)V_X \dfrac{\sigma(\tilde{y})}{\sigma(\tilde{x})}}{1 + r_f} \tag{12}$$

with $\rho(\tilde{y},\tilde{x}) \in \{-1, 1\}$.[13]

We now show a counterexample where De Reyck's method leads to incorrect valuation and decision (incompatible with the no-arbitrage principle). Consider two projects X and Y: project X's end-of-period payoff is 100 euros, 0 euros, 220 euros in state 1, 2, 3 respectively; project Y's end-of-period payoff is 130 euros, 30 euros, 250 euros in state 1, 2, 3 respectively. The probability of the states are 0.3, 0.1, 0.6 respectively and the cost of Y is assumed to be I_Y=20.

It is worth noting that $\tilde{y} = \tilde{x} + 30$ so that

$$\rho(\tilde{y},\tilde{x})\frac{\sigma(\tilde{y})}{\sigma(\tilde{x})} = \frac{\text{cov}(\tilde{x},\tilde{y})}{\sigma^2(\tilde{x})} = \frac{\text{cov}(\tilde{x},\tilde{x}+30)}{\sigma^2(\tilde{x})} = \frac{\text{cov}(\tilde{x},\tilde{x})}{\sigma^2(\tilde{x})} = 1\,. \tag{13}$$

Owing to eq. (13), if we are to apply De Reyck's valuation method, eq. (12) becomes

$$V_Y = \frac{E(\tilde{y}) - (i - r_f)V_X}{1 + r_f}\ . \tag{14}$$

As anticipated, De Reyck uses the certainty-equivalent version of the CAPM to value a project (see De Reyck, 2005, eq. (2.5) at p. 501), so that $V_X = \dfrac{E(\tilde{x}) - \lambda\,\text{cov}(\tilde{x}, r_m)}{1 + r_f}$. Therefore, eq. (14) becomes

$$V_Y = \frac{E(\tilde{y}) - E(\tilde{x}) + (1 + r_f)\dfrac{E(\tilde{x}) - \lambda\,\text{cov}(\tilde{x}, r_m)}{R_f}}{R_f} \tag{15}$$

[13]The author writes of the "project *X*'s cost of capital, obtained through the market valuation of a security or project with *exactly the same payoff pattern*" (italics added), which possibly means that the author assumes that project *X* is replicable by some portfolio in the security market. Therefore, either the Proposition does not apply in incomplete markets or the replicability assumption is redundant. As De Reyck does not use this assumption in the proof, the latter holds.

Further, some ambiguity arises: in the proof the author writes of "returns of the project in each project state" (p. 501) and "project returns in the different scenarios" (p. 502), but, instead, he uses the *equilibrium* rates of return of the project, not the *project* returns (see also Magni, 2005, Table 1). In his subsequent "Clarifying example" he writes again of "returns generated by $\tilde{x}$ ", "returns generated by $\tilde{z}$ " (De Reyck, 2005, p. 504) but he does not provide the reader the cost of the project; he uses the equilibrium value (not the cost) of the project to compute the projects' returns. So doing, he obtains the *equilibrium* rates of return of the projects (not the actual projects' returns).

where we have used the obvious relation $i = \frac{E(\tilde{x})}{V_X} - 1$.[14] Suppose now a decision maker must decide whether to accept or reject this project; if he uses De Reyck's approach, he will undertake the project if and only if

$$\frac{E(\tilde{y}) - E(\tilde{x}) - (1 + r_f)\dfrac{E(\tilde{x}) - \lambda \operatorname{cov}(\tilde{x}, r_m)}{R_f}}{R_f} = V_Y > I_Y .$$

Suppose the security market is as shown in Table 5, where one risky security is traded and the risk-free rate is 33.33%. It is worth noting that the market is not complete and project X is not replicable, so that either De Reyck's Proposition does not apply, or the replicability assumption is redundant. If one accepts the latter, then De Reyck's approach applies. Using the data of Table 5, we have $\lambda = 18.11$, $\operatorname{cov}(\tilde{x}, r_m) = 9.388$, $R_f = 1.333$. Therefore,

$$V_Y = \frac{192 - 162 + 1.333\dfrac{162 - 18.11(9.388)}{1.333}}{1.333} = 16.47 < 20 = I_Y$$

and the project is not undertaken. This decision is incorrect and the valuation is therefore unreliable. Indeed, the investor may take a short position on 0.25 units of the riskless asset and use the proceeds to undertake project Y, so realizing a strong arbitrage (see Table 6).

Also, this valuation leaves the investor open to arbitrage losses. In fact, suppose an arbitrageur offers the investor a contract whereby they exchange project Y's cash flows, with the arbitrageur taking a long position and the investor taking a short position. In other words, the arbitrageur pays the investor an amount equal to project Y's cost while the investor pays back the

---

[14] Note that Eq. (15) gives us a short way to obtain De Recyk's thesis: reminding that $\rho(\tilde{y}, \tilde{x}) \in \{-1, 1\}$ means $\tilde{y} = \alpha\tilde{x} + k$, with $\alpha \in R$, and considering that the latter implies $\rho(\tilde{y}, \tilde{x})\sigma(\tilde{y}) / \sigma(\tilde{x}) = \alpha$, we have

$$V_Y = \frac{E(\tilde{y}) - \lambda \operatorname{cov}(\alpha\tilde{x} + k, r_m)}{R_f} = \frac{E(\tilde{y}) - \alpha E(\tilde{x}) + R_f \dfrac{\alpha E(\tilde{x}) - \lambda \operatorname{cov}(\alpha\tilde{x}, r_m)}{R_f}}{R_f}$$
$$= \frac{E(\tilde{y}) - \left(E(\tilde{x}) / V_x - R_f\right)\alpha V_X}{1 + r_f} = \frac{E(\tilde{y}) - \rho(\tilde{y}, \tilde{x})(i - r_f)(\sigma(\tilde{y}) / \sigma(\tilde{x}))V_X}{1 + r_f}.$$

Q.E.D.

arbitrageur the end-of-period random amount $\tilde{y}$. The investor accepts, because to him the net present value is $-(V_Y - 20) = -16.47 + 20 > 0$. The arbitrageur then offers the investor a second contract according to which the investor pays 22 euros to the arbitrageur, who will in turn reimburse a nonrandom amount of 30 euros at the end of the period. The investor accepts again, because the net present value is positive: $\frac{30}{1.333} - 20 = 22.5 - 22 > 0$. So doing, he is trapped in an arbitrage loss, as Table 7 shows.

Table 5. The security market

	Risky security (3000 shares)	Risk-free security	Market (000)[15]	Probability
Cash Flow	$\begin{cases} 98 \\ 71 \\ 100 \end{cases}$	$\begin{cases} 120 \\ 120 \\ 120 \end{cases}$	$\begin{cases} 294 \\ 213 \\ 300 \end{cases}$	$\begin{cases} 0.3 \\ 0.1 \\ 0.6 \end{cases}$
Price	54	90	162	
Rate of return (%)	$\begin{cases} 81.48 \\ 31.48 \\ 85.18 \end{cases}$	$\begin{cases} 33.33 \\ 33.33 \\ 33.33 \end{cases}$	$\begin{cases} 81.48 \\ 31.48 \\ 85.18 \end{cases}$	$\begin{cases} 0.3 \\ 0.1 \\ 0.6 \end{cases}$
Expected rate of return (%)	78.7	33.33	78.7	
Variance of the market rate of return			0.02505	

In other words, project valuation as suggested by De Reyck (i) does not guarantee absence of arbitrage, (ii) may lead to reject projects that are worth undertaking, and (iii) induces decision makers to possibly incur arbitrage losses. This result seems intriguing because De Reyck does not use the d-NPV. This suggests that although the d-NPV is incorrect for valuation, moving from disequilibrium values to equilibrium values does not guarantee that correct results are reached in valuation and decision.

[15] In the Table the term "market" refers to the market of risky securities. Since we have only one basic security, the market rate of return coincides with the risky security rate of return.

Table 6. (Strong) arbitrage opportunity		
	Cash flow at time 0	Cash flow at time 1
Financing project with risk-free securities	0.25(90)	$-0.25(120)$
Undertaking project Y	-20	$\tilde{y}=\tilde{x}+30$
Net flows[16]	2.5	$\tilde{x}$

Table 7. Arbitrage loss		
	Cash flow at time 0	Cash flow at time 1
First contract	20	$-\tilde{y}=-\tilde{x}-30$
Second contract	-22	30
Net flows	-2	$-\tilde{x}$

Concluding remarks

CAPM and capital budgeting find a common ground in the notions of (risk-adjusted) cost of capital and net present value. A project is worth undertaking if its NPV is positive, with the CAPM-derived cost of capital as the discount rate. The problem of whether a disequilibrium NPV is the correct one or not has been so far highly neglected in the literature. It seems that many authors explicitly or implicitly incline to the disequilibrium NPV as the right tool for making decisions and

[16] Note that $\tilde{x}$ assumes nonnegative values in all states.

valuing projects. Other scholars claim that it is incorrect, but admit that such an NPV is just the standard NPV used in finance, or that its use is common among scholars and practitioners. While the use of the disequilibrium NPV as an accept/reject decision rule is consistent with the CAPM, one cannot infer its use as a valuation tool (i.e. as a measure of wealth increase). However, due to d-NPV nonadditivity, both valuation and decision are affected. As a valuation tool, the d-NPV is illegitimate, because inconsistent valuations are drawn from different descriptions of the same course of action. As a decision rule the d-NPV is (legitimate but) unsafe, because opposite decisions may be taken depending on the way the course of action is represented (and therefore depending on the way the NPV is calculated). De Reyck's (2005) Proposition, aimed at defending the NPV+CAPM methodology, dispenses with the disequilibrium NPV which Magni (2002, 2005, 2007a, forthcoming) criticizes. De Reyck's starting point is an equilibrium NPV. However, even in this case, valuation and decision may turn to be incorrect.

Hence, the use of the NPV+CAPM procedure deserves further attention and deeper investigation. To this end, a detailed analysis of the different notions of NPV (equilibrium or disequilibrium) should be conducted and precise rules should identify those case where either the d-NPV or the e-NPV may or may not be used, specifying whether the use is made for valuation or for decision. The role of additivity in validating the NPV method should deserve supplementary study and relations between the notions of NPV and excess return should be also scrutinized. Finally, in terms of sociology of science, it is worth investigating whether the use of the disequilibrium is actually the standard use in corporate finance, as some authors explicitly claim, or is just an incorrect interpretation of the NPV many authors and practitioners incur (or seem to incur). Above all, it would be interesting to know how and why the d-NPV has developed, and understand why authors concerned with presenting a correct definition of cost of capital have neglected to warn readers against the use of that cost of capital for valuing projects.

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Appendix 1

Consider firm l. Before acceptance of the project, we have

$$\bar{r}_l = r_f + \lambda \operatorname{cov}(r_l, r_m).$$

Using the definition of rate of return and reminding that $R_f = 1 + r_f$, we have

$$\frac{\bar{F}_l}{V_l} = R_f + \lambda \operatorname{cov}(r_l, r_m)$$

and, multiplying by the firm value V_l, we obtain

$$\bar{F}_l = R_f V_l + \lambda \operatorname{cov}(F_l, r_m) = R_f N_l P_l + \lambda \operatorname{cov}(F_l, r_m). \tag{2}$$

After acceptance of the project, the new value is set as

$$V_l^\circ = \frac{\bar{F}_l + \bar{F}_Z - \lambda \operatorname{cov}(F_l + F_Z, r_m)}{R_f}.$$

The existing shares are N_l, so the new resulting price P_l° is such that $V_l^\circ = N_l P_l^\circ + I_Z$, which determines $P_l^\circ = \dfrac{V_l^\circ - I_Z}{N_l}$. To actually make the investment the firm shall issue $N_l^\circ = \dfrac{I_Z}{P_l^\circ}$ shares at the price P_l°. The Security Market Line is now such that

$$\frac{\bar{F}_l + \bar{F}_Z}{V_l^\circ} = R_f + \lambda \operatorname{cov}\left(\frac{F_l + F_Z}{V_l^\circ}, r_m\right)$$

whence

$$\bar{F}_l + \bar{F}_Z = R_f V_l^\circ + \lambda \operatorname{cov}(F_l + F_Z, r_m).$$

Having determined the new price and the number of stocks issued, the latter equality boils down

$$\bar{F}_l + \bar{F}_Z = R_f (N_l + N_l^\circ) P_l^\circ + \lambda \operatorname{cov}(F_l + F_Z, r_m). \tag{3}$$

Subtracting (3) from (2), and using the fact that $N_l^\circ P_l^\circ = I_Z$ we get to

$$
\begin{aligned}
& -\overline{F}_Z = R_f N_l P_l + \lambda \operatorname{cov}(F_l, r_m) - R_f (N_l + N_l^\circ) P_l^\circ - \lambda \operatorname{cov}(F_l + F_Z, r_m) \\
\Leftrightarrow & -\overline{F}_Z = R_f N_l (P_l - P_l^\circ) - R_f I_Z - \lambda \operatorname{cov}(F_Z, r_m). \\
\Leftrightarrow & \ \overline{F}_Z - I_Z \left[R_f + \lambda \operatorname{cov}\left(\frac{F_Z}{I_Z}, r_m \right) \right] = R_f N_l (P_l^\circ - P_l) \\
\Leftrightarrow & \ \frac{\overline{F}_Z}{R_f + \lambda \operatorname{cov}\left(\frac{F_Z}{I_Z}, r_m \right)} - I_Z = \frac{R_f N_l (P_l^\circ - P_l)}{R_f + \lambda \operatorname{cov}\left(\frac{F_Z}{I_Z}, r_m \right)}.
\end{aligned} \tag{4}
$$

We have then

$$
P_l^\circ - P_l > 0 \Leftrightarrow V_Z - I_Z = \mathrm{NPV}_Z > 0, \tag{5}
$$

with

$$
\mathrm{NPV}_Z := \frac{\overline{F}_Z}{R_f + \lambda \operatorname{cov}\left(\frac{F_Z}{I_Z}, r_m \right)} - I_Z
$$

Q.E.D. [17]

Remark. It is worth noting that the last equality in eq. (4) is not legitimate if $R_f + (\lambda / I_Z)\operatorname{cov}(F_Z, r_m) = 0$, which means $I_Z = -(\lambda / R_f)\operatorname{cov}(F_Z, r_m)$ or, equivalently, $\beta_Z = R_f / (r_f - \bar{r}_m)$.[18] In this case, the notion of disequilibrium NPV vanishes. If, under this condition, one computes the project's *equilibrium* NPV (see section 4), one finds that it is equal to e-NPV=$\overline{F}_Z / R_f$, which coincides with the (gross) present value of a risk-free bond with face value $\overline{F}_Z$ (this bond's *net* present value is obviously zero, because it lies on the SML). Given that the third equality in eq. (4) becomes $\overline{F}_Z / R_f = N_l (P_l^\circ - P)$, the present value of the risk-free bond measures shareholders' wealth increase in case of project acceptance.

[17] Consistently with the relevant financial literature, this proof assumes that effects on R_m, and λ are negligible. On these assumptions, see Hamada (1969, p. 23), Rubinstein (1973, footnote 10), Bogue and Roll (1974, footnote 13).
[18] Note that, in this case, the beta is negative.

Appendix 2

The Net Present Value is additive if $\mathrm{NPV}_1 + \mathrm{NPV}_2 = \mathrm{NPV}_{1+2}$ for *any* project 1 and 2. Considering a generic project *j*, there always exists an $a_j \in R$ such that

$$\overline{F}_j - a_j = I_j \left[R_f + \lambda \operatorname{cov}\left(\frac{F_j}{I_j}, r_m\right) \right] \tag{6}$$

(if the project lies on the SML, then $a_j = 0$ and I_j is the equilibrium value of the project). The disequilibrium Net Present Value of project *j* may be written as

$$\mathrm{NPV}_j = \frac{\overline{F}_j I_j}{I_j R_f + \lambda \operatorname{cov}(F_j, r_m)} - I_j .$$

Owing to eq. (6), the latter becomes

$$\mathrm{NPV}_j = \frac{\overline{F}_i I_j}{\overline{F}_j - a_j} - I_j . \tag{7}$$

Using eq. (7), we have

$$\mathrm{NPV}_1 + \mathrm{NPV}_2 = \frac{\overline{F}_1 I_1}{\overline{F}_1 - a_1} + \frac{\overline{F}_2 I_2}{\overline{F}_2 - a_2} - (I_1 + I_2) \tag{8}$$

Letting *j*=1+2, the NPV of the portfolio is

$$\mathrm{NPV}_{1+2} = \frac{(\overline{F}_1 + \overline{F}_2)(I_1 + I_2)}{(I_1 + I_2) R_f + \lambda \operatorname{cov}(F_1 + F_2, r_m)} - (I_1 + I_2) .$$

From eq. (6), the denominator of NPV_{1+2} is equal to $(\overline{F}_1 - a_1) + (\overline{F}_2 - a_2)$. Hence, we find

$$\mathrm{NPV}_{1+2} = \frac{(\overline{F}_1 + \overline{F}_2)(I_1 + I_2)}{(\overline{F}_1 + \overline{F}_2) - a_1 - a_2} - (I_1 + I_2) . \tag{9}$$

Equations (8) and (9) coincide for $a_1 = a_2 = 0$, but they do not coincide for *any* a_1, $a_2 \in R$, as additivity requires.

Q.E.D.

Appendix 3

As security t lies on the SML, its value satisfies

$$V_t = \frac{\overline{F}_t}{R_f + \lambda \operatorname{cov}\left(\frac{F_t}{V_t}, r_m\right)}. \tag{10}$$

The assumption $F_Z = \theta F_t$ and the no-arbitrage condition implies that the arbitrage-based value of the project is

$$v_Z = \theta V_t = \frac{\theta \overline{F}_t}{R_f + \lambda \operatorname{cov}\left(\frac{F_t}{V_t}, r_m\right)}. \tag{11}$$

Suppose the thesis of the Proposition is not true. Then, the equality $\mathrm{NPV}_Z = \mathrm{npv}_Z$ implies $V_Z = v_z$, which boils down to

$$\frac{\overline{F}_Z}{R_f + \lambda \operatorname{cov}\left(\frac{F_Z}{I_Z}, r_m\right)} = \frac{\theta \overline{F}_t}{R_f + \lambda \operatorname{cov}\left(\frac{F_t}{V_t}, r_m\right)},$$

whence $\operatorname{cov}\left(\frac{F_Z}{I_Z}, r_m\right) = \operatorname{cov}\left(\frac{F_t}{V_t}, r_m\right)$, which reduces to $\operatorname{cov}\left(\frac{\theta F_t}{I_Z}, r_m\right) = \operatorname{cov}\left(\frac{F_t}{V_t}, r_m\right)$, which in turn implies $I_Z = \theta V_t$. This equality, the first equality in eq. (11) and the above equality $V_Z = v_Z$ imply $V_Z = I_Z$ (NPV_Z=0); the latter identity is impossible, given that we have assumed that the project does not lie on the SML.

Q.E.D.

Remark. It is worthwhile noting that if one removes from Proposition 3.2 the assumption that the project does not lie on the SML, the above proof just leads to the conclusion that absence of arbitrage implies $V_Z = I_Z$.