Optimum Commodity Taxation in Pooling Equilibria

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Abstract

This paper extends the standard model of optimum commodity taxation (Ramsey (1927) and Diamond-Mirrlees (1971)) to a competitive economy in which markets are inefficient due to *asymmetric information*. Insurance markets are prime examples: consumers impose varying costs on suppliers but firms cannot associate costs with individual customers and consequently all are charged equal prices. In such a competitive *pooling equilibrium*, the price of each good is equal to the *average* of individual marginal costs weighted by equilibrium quantities. We derive modified *Ramsey-Boiteux Conditions* for optimum taxes in such an economy and show that, in addition to the standard formula, they include first-order effects which reflect the deviations of prices from marginal costs and the response of equilibrium quantities to the taxes levied. An explanation of the additional terms is provided. It is shown that a condition on the monotonicity of demand elasticities enables to sign the direction of the deviations from the standard case.

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I. Introduction

The setting of the standard theory of optimum commodity taxation (Ramsey (1927), Diamond and Mirrlees (1971), Salanie (2003)) is a competitive equilibrium which attains an efficient resource allocation. In the absence of lump-sum taxes, the government wishes to raise revenue by means of distortive commodity taxes and the theory develops the conditions that have to hold for these taxes to minimize the deadweight loss (the *'Ramsey-Boiteux Conditions'*). The analysis was extended in some directions to allow for an initial inefficient allocation of resources. In such circumstances, aside from the need to raise revenue, taxes/subsidies may serve as means to improve welfare due to market inefficiencies. The rules for optimum commodity taxation, therefore, mix considerations of shifting an inefficient market equilibrium in a welfare enhancing direction and the distortive effects of gaps between consumer and producer marginal valuations generated by commodity taxes.

Two major extensions of the standard model have been explored. First, the inclusion of externalities and the need to finance public goods (Sandmo (1975), Stiglitz and Dasgupta (1971), Lau, Sheshinski and Stiglitz (1978)). While specific assumptions about the form of externalities (e.g. 'atmosphere externalities') or about the form of preferences for public goods (e.g. weak separability), as well as the absence of distributional considerations, were needed to obtain transparent results, these contributions are quite general and the results seem robust. The second extension is to allow for imperfect competition (Auerbach and Hines (2001), Guesnerie and Laffont (1978), Myles (1987, 1989)). Here the results seem to depend more crucially on particular assumptions about the definition of the imperfectly competitive equilibrium, about the number of firms in oligopoly markets, about the type of taxes (specific or ad-valoren) and about the presence or absence of uncertainty (making the availability or unavailability of insurance critical). Although these papers provide valuable insight about optimum taxation in specific circumstances, no broad rules seem to emerge.

This paper goes in a different direction. Markets are assumed to be perfectly competitive but there is *asymmetric information* between firms and consumers about 'relevant' characteristics which affect the *costs* of firms, as well as consumer preferences. Leading examples are in the field of insurance. Expected costs of medical insurance depend on the health characteristics of the insured as does the value of such insurance to the purchaser. Similarly, the costs of an annuity depend on the expected payout which, in turn, depends on the holder's survival prospects. Naturally, these prospects also affect the value of the annuity to the individual's expected lifetime utility. Other examples where personal characteristics affect costs are rental contracts (e.g. cars) and fixed-fee contracts for the use of facilities (sports and other clubs).

When firms are able to identify customers' relevant characteristics (in insurance parlance, 'risk class'), competitive pressures equate prices to marginal costs for each customer type, and the competitive equilibrium is efficient. Such identification, however, may not be possible or is imperfect and costly because it requires monitoring of activities, including the amounts purchased (Rothschild-Stiglitz (1976)), and the collection of information available at a multitude of firms. In these circumstances, commodities are sold at the same prices to different types of consumers, mostly to all consumers without distinction. This is called a *pooling-equilibrium*. Zero profits in a competitive pooling equilibrium imply that the price of each good is equal to the average of individual marginal costs weighted by the equilibrium quantities purchased by all consumers.

This paper analyses the conditions for optimum commodity taxes in pooling market equilibria. The modeling of preferences and of costs is general, allowing for any finite number of markets. We obtain surprisingly simple modified Ramsey-Boiteux Conditions and explain the deviations from the standard formulas. Broadly, the additional terms that emerge reflect the fact that the initial producer price of each commodity deviates from each consumer's marginal costs, being only equal to these costs on average. Each levied specific tax affects all prices (a *general-equilibrium effect*), and, consequently, a small increase in any tax level affects the quantity-weighted gap between producer prices and individual marginal costs, the direction depending on the relation between demand elasticities and costs.

After developing general formulas (Section 3), we briefly analyze (Section 4) an example of a three-good economy and show how optimum tax rates depend on the familiar substitution/complementary with the untaxed good(s) as well as deviations of costs from prices.

II. Equilibrium with Asymmetric Information

A representative individual consumes n goods, X_i, $i = 1,2,..,n$ and a numeraire, Y. Preferences are represented by a linearly separable utility function, U

$$U = u(\mathbf{x}, \alpha) + y, \tag{1}$$

where $\mathbf{x} = (x_1, x_2,.., x_n)$, x_i is the quantity of good i and y is the quantity of the numeraire consumed by the individual. The utility function, u, is assumed to be strictly concave and differentiable in $\boldsymbol{x}$. Linear separability is assumed to conveniently eliminate income effects in the demand for $\boldsymbol{x}$[1].

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[1] As is well-known, a non-separable utility function leads (via the Slutsky conditions) to the same results as below. Non-separability becomes important, though, when dealing with income distribution effects on optimum tax rates. We have a good idea how to incorporate income heterogeneity (e.g. Salanie (2003)) and focus here on efficiency aspects.

The parameter α is a personal attribute which is singled out because it has *cost effects*. Specifically, it is assumed that the unit costs of good i consumed by individuals with a given α ('type α') is $c_i(\alpha)$. Leading examples are health and longevity insurance. The health status of an individual affects both his consumption preferences and the costs to the medical insurance provider. Similarly, the payout of annuities (e.g. retirement benefits) is contingent on survival and hence depends on the individual's relevant mortality function. Other examples are car rentals and car insurance, whose costs and value to consumers depend on driving patterns and other personal characteristics[2].

It is assumed that α is continuously distributed in the population, with a distribution function, $F(\alpha)$, over a finite interval, $\alpha \in [\underline{\alpha}, \overline{\alpha}]$, $\overline{\alpha} > \underline{\alpha}$.

The economy has a given amount of resources, $R > 0$. With unit costs of 1 (in terms of R) for the numeraire, Y, the aggregate resource constraint is written

$$\int_{\underline{\alpha}}^{\overline{\alpha}} [\mathbf{c}(\alpha)\mathbf{x}(\alpha) + y(\alpha)]dF(\alpha) = R \tag{2}$$

where $\mathbf{c}(\alpha) = (c_1(\alpha), c_2(\alpha), .., c_n(\alpha))$, $\mathbf{x}(\alpha) = (x_1(\alpha), x_2(\alpha), .., x_n(\alpha))$, $x_i(\alpha)$ being the quantity of X_i and $y(\alpha)$ the quantity of Y consumed by type α individuals.

The First-Best allocation is obtained by maximization of a utilitarian welfare function, W,

$$W = \int_{\underline{\alpha}}^{\overline{\alpha}} (u(\mathbf{x};\alpha) + y)dF(\alpha) \tag{3}$$

s.t. the resource constraint (2). The F.O.C. for an interior solution equates marginal utilities and costs for individuals of the same type[3]. That is, for each α,

$$u_i(\mathbf{x}, \alpha) - c_i(\alpha) = 0, \quad i = 1, 2, .., n \tag{4}$$

where $u_i = \dfrac{\partial u}{\partial x_i}$. The unique solution to (4), denoted $\mathbf{x}^*(\alpha) = (x_1^*(\alpha), x_2^*(\alpha), .., \ x_n^*(\alpha))$, and the optimum level of the numeraire, y^*, is determined by the resource constraint, $y^* = R - \int_{\underline{\alpha}}^{\overline{\alpha}} \mathbf{c}(\alpha)\mathbf{x}^*(\alpha)dF(\alpha)$.

The *First-Best* allocation can be supported by competitive markets with individualized prices equal to marginal costs. That is, if p_i is the price of good i, then efficiency is attained when type α individuals face the price $p_i(\alpha) = c_i(\alpha)$.

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[2] Representation of these characteristics by a single parameter is, of course, a simplification.

[3] Thus, it is assumed that R yields $y^* > 0$.

When α is *private information*, unknown to suppliers (and not verifiable by monitoring individuals' purchases), then for each good firms will charge the same price to all individuals. This is called a (Second-Best) *Pooling Equilibrium.*

Pooling Equilibrium

Good X_i is offered at a common price p_i to all individuals, $i = 1, 2, .., n$. The competitive price of the numeraire is 1. The individual maximizes utility, (1), subject to the budget constraint

$$\mathbf{px} + y = R. \tag{5}$$

It assumed that for all α, the level of *R* yields interior solutions. The F.O.C. are

$$u_i(\mathbf{x};\alpha) - p_i = 0, \quad i = 1, 2, .., n \tag{6}$$

the unique solutions to (6) are the *compensated demand functions* $\hat{\mathbf{x}}(\mathbf{p};\alpha) = (\hat{x}_1(\mathbf{p};\alpha), \hat{x}_2(\mathbf{p};\alpha), .., \hat{x}_n(\mathbf{p};\alpha))$. For each α, the optimum level of *Y*, $\hat{y}(\alpha)$, is obtained from the budget constraint (5), $\hat{y}(\mathbf{p};\alpha) = R - \mathbf{p}\hat{x}(\mathbf{p};\alpha)$, assumed to be positive for all α.

Let $\pi_i(\mathbf{p})$ be total profits in the production of good i:

$$\pi_i(\mathbf{p}) = p_i \hat{x}_i(\mathbf{p}) - \int_{\underline{\alpha}}^{\bar{\alpha}} c_i(\alpha)\hat{x}_i(\mathbf{p};\alpha)dF(\alpha), \quad i = 1, 2, .., n \tag{7}$$

where $\hat{x}_i(\mathbf{p}) = \int_{\underline{\alpha}}^{\bar{\alpha}} \hat{x}_i(\mathbf{p};\alpha)dF(\alpha)$ is the *aggregate demand* for good i.

Definition[4]. *A pooling-equilibrium is a vector of prices,* $\hat{\mathbf{p}}$, *which satisfies* $\pi_i(\hat{\mathbf{p}}) = 0$, $i = 1, 2, .., n$ *or*

$$\hat{p}_i = \frac{\int_{\underline{\alpha}}^{\bar{\alpha}} c_i(\alpha)\hat{x}_i(\hat{\mathbf{p}};\alpha)dF(\alpha)}{\int_{\underline{\alpha}}^{\bar{\alpha}} \hat{x}_i(\hat{\mathbf{p}};\alpha)dF(\alpha)}, \quad i = 1, 2, .., n. \tag{8}$$

Equilibrium prices are weighted averages of marginal costs, the weights being the equilibrium quantities purchased by the different α types. Writing (7) in matrix form:

$$\boldsymbol{\pi}(\hat{\mathbf{p}}) = \hat{\mathbf{p}}\mathbf{X}(\hat{\mathbf{p}}) - \int_{\underline{\alpha}}^{\bar{\alpha}} \mathbf{c}(\alpha)\hat{\mathbf{X}}(\hat{\mathbf{p}};\alpha)dF(\alpha) = 0, \quad i = 1, 2, .., n \tag{9}$$

[4] For general analyses of pooling equilibria see, for example, Laffont and Martimort (2002) and Salanie (1997).

where $\boldsymbol{\pi}(\hat{\mathbf{p}}) = (\pi_1(\hat{\mathbf{p}}), \pi_2(\hat{\mathbf{p}}), .., \pi_n(\hat{\mathbf{p}}))$,

$$\hat{\mathbf{X}}(\hat{\mathbf{p}};\alpha) = \begin{bmatrix} \hat{x}_1(\hat{\mathbf{p}};\alpha) & & 0 \\ & \ddots & \\ 0 & & \hat{x}_n(\hat{\mathbf{p}};\alpha) \end{bmatrix} \tag{10}$$

$\hat{\mathbf{X}}(\hat{\mathbf{p}}) = \int_{\underline{\alpha}}^{\overline{\alpha}} \mathbf{X}(\hat{\mathbf{p}};\alpha) dF(\alpha)$, $\mathbf{c}(\alpha) = (c_1(\alpha), c_2(\alpha), .., c_n(\alpha))$, and $\mathbf{0}$ is $1 \times n$ zero vector $\mathbf{0} = (0, 0, .., 0)$. Let $\hat{\mathbf{K}}(\hat{\mathbf{p}})$ be the $n \times n$ matrix with elements $\hat{k}_{ij}$,

$$\hat{k}_{ij}(\hat{\mathbf{p}}) = \int_{\underline{\alpha}}^{\overline{\alpha}} (\hat{p}_i - c_i(\alpha)) s_{ij}(\hat{\mathbf{p}};\alpha) dF(\alpha), \quad i, j = 1, 2, .., n \tag{11}$$

where $s_{ij}(\hat{\mathbf{p}};\alpha) = \dfrac{\partial \hat{x}_i(\hat{\mathbf{p}};\alpha)}{\partial p_j}$ are the substitution terms.

We can now state:

Proposition 1. *When* $\hat{\mathbf{X}}(\mathbf{p}) + \hat{\mathbf{K}}(\mathbf{p})$ *is positive-definite for any* $\mathbf{p}$, *then there exist unique and globally stable prices,* $\hat{\mathbf{p}}$, *which satisfy* (9).

Proof. Appendix A.

We shall assume throughout that the condition in Proposition 1 is satisfied. Note that when costs are independent of α, $\hat{p}_i - c_i = 0$, $i = 1, 2, .., n$, $\hat{K} = 0$ and the condition in Proposition 1 is trivially satisfied.

III. Optimum Commodity Taxation

Suppose that the government wishes to impose specific commodity taxes on X_i, $i = 1, 2, .., n$. Let the unit tax (subsidy) on X_i be t_i so that its (tax inclusive) consumer price is $q_i = p_i + t_i$, $i = 1, 2, .., n$. Consumer demands, $\hat{x}_i(\mathbf{p};\alpha)$, are now functions of these prices, $\mathbf{q} = \mathbf{p} + \mathbf{t}$, $\mathbf{t} = (t_1, t_2, .., t_n)$.

As before, equilibrium consumer prices, $\hat{\mathbf{q}}$, are determined by zero-profits conditions:

$$\hat{q}_i = \frac{\int_{\underline{\alpha}}^{\overline{\alpha}} (c_i(\alpha) + t_i)\hat{x}_i(\hat{\mathbf{p}};\alpha)dF(\alpha)}{\int_{\underline{\alpha}}^{\overline{\alpha}} \hat{x}_i(\hat{\mathbf{p}};\alpha)dF(\alpha)}, \quad i = 1,2,..,n \tag{12}$$

or, in matrix form,

$$\boldsymbol{\pi}(\hat{\mathbf{q}}) = \hat{\mathbf{q}}\hat{\mathbf{X}}(\hat{\mathbf{q}}) - \int_{\underline{\alpha}}^{\overline{\alpha}} (\mathbf{c}(\alpha) + \mathbf{t})\hat{\mathbf{X}}(\hat{\mathbf{q}};\alpha)dF(\alpha) = \mathbf{0}, \quad i = 1,2,..,n \tag{13}$$

where $\hat{\mathbf{X}}(\hat{\mathbf{q}};\alpha)$ and $\hat{\mathbf{X}}(\hat{\mathbf{q}})$ are the diagonal $n \times n$ matrices defined above, with $\hat{\mathbf{q}}$ replacing $\hat{\mathbf{p}}$.

Note that each element in $\hat{\mathbf{K}}(\hat{\mathbf{q}})$, $k_{ij}(\hat{\mathbf{q}}) = \int_{\underline{\alpha}}^{\overline{\alpha}} (\hat{p}_i - c_i(\alpha))s_{ij}(\hat{\mathbf{q}};\alpha)dF(\alpha)$, also depends on $\hat{p}_i = \hat{q}_i - t_i$ which depends on **t** directly and though **q**. It is assumed that $\hat{\mathbf{X}}(\hat{\mathbf{q}}) + \hat{\mathbf{K}}(\hat{\mathbf{q}})$ is positive definite for all **q**. Hence, given **t**, there exist unique prices, $\hat{\mathbf{q}}$ (and the corresponding $\hat{\mathbf{p}} = \hat{\mathbf{q}} \text{ - } \mathbf{t}$), which satisfy (13).

Observe that each equilibrium price, $\hat{q}_i$, depends on the whole vector of tax rates, **t**. Specifically, differentiating (13) w.r.t. the tax rates, we obtain:

$$(\hat{\mathbf{X}}(\hat{\mathbf{q}}) + \hat{\mathbf{K}}(\hat{\mathbf{q}}))\hat{\mathbf{Q}} = \hat{\mathbf{X}}(\hat{\mathbf{q}}) \tag{14}$$

where $\hat{Q}$ is the $n \times n$ matrix whose elements are $\frac{\partial \hat{q}_i}{\partial t_j}$, $i, j = 1,2,..,n$.

All principal minors of $\hat{\mathbf{X}} + \hat{\mathbf{K}}$ are positive and it has a well-defined inverse.

Hence, from (14),

$$\hat{\mathbf{Q}} = (\hat{\mathbf{X}} + \hat{\mathbf{K}})^{-1}\hat{\mathbf{X}}. \tag{15}$$

It can be deduced from (15) that equilibrium consumer prices rise w.r.t. an increase in own tax rates:

$$\frac{\partial \hat{q}_i}{\partial t_i} = \hat{x}_i(\hat{\mathbf{q}})\frac{\left|\hat{\mathbf{X}} + \hat{\mathbf{K}}\right|_{ii}}{\left|\hat{\mathbf{X}} + \hat{\mathbf{K}}\right|} \tag{16}$$

where $\left|\hat{\mathbf{X}} + \hat{\mathbf{K}}\right|$ is the determinant of $\hat{\mathbf{X}} + \hat{\mathbf{K}}$, and $\left|\hat{\mathbf{X}} + \hat{\mathbf{K}}\right|_{ii}$ is the principal minor obtained by deleting the *i*-th row and the *i*-th column. In general, the sign of cross-price effects due to tax rate increases is indeterminate, depending on substitution and complementarity terms. When all costs are independent of customer type, that is, $p_i - c_i = 0$, $i = 1,2,..,n$, then $\hat{\mathbf{K}} = 0$, and $\frac{\partial \hat{q}_i}{\partial t_i} = 1$, and $\frac{\partial \hat{q}_i}{\partial t_j} = 0$, $i \neq j$, $i, j = 1,2,..,n$.

From (1) and (3), social welfare in the pooling equilibrium is written

$$W(\mathbf{t}) = \int_{\underline{\alpha}}^{\overline{\alpha}} [u(\hat{\mathbf{x}}(\hat{\mathbf{p}};\alpha)) - \mathbf{c}(\alpha)\hat{\mathbf{x}}(\hat{\mathbf{p}};\alpha)]dF(\alpha) + R \qquad (17)$$

The problem of optimum commodity taxation can now be stated: the government wishes to raise a given amount, *T*, of tax revenue:

$$\mathbf{t}\hat{\mathbf{x}}(\hat{\mathbf{q}}) = T \qquad (18)$$

by means of taxes, **t**, that maximize $W(\mathbf{t})$.

Maximization of (17) s.t. (18) and (15) yields, after substitution of $u_i - q_i = 0$, $i = 1,2,..,n$, from individual F.O.C., that optimum tax levels, denoted $\hat{\mathbf{t}}$, satisfy:

$$(1+\lambda)\hat{\mathbf{t}}\hat{\mathbf{S}}\hat{\mathbf{Q}} + \mathbf{1}\hat{\mathbf{K}}\hat{\mathbf{Q}} = -\lambda\mathbf{1}\hat{X} \qquad (19)$$

where $\hat{\mathbf{S}}$ is the $n\times n$ aggregate substitution matrix whose elements are $s_{ij}(\hat{\mathbf{q}}) = = \int_{\underline{\alpha}}^{\overline{\alpha}} s_{ij}(\hat{\mathbf{q}};\alpha)dF(\alpha)$, $\mathbf{1}$ is the $1\times n$ unit vector, $\mathbf{1} = (1,1,..,1)$, and $\lambda > 0$ is the *Lagrange multiplier* of constraint (18).

Rewrite (19) in the more familiar form:

$$\hat{\mathbf{t}}\mathbf{S} = -\frac{1}{1+\lambda}[\mathbf{1}(\lambda\hat{\mathbf{X}} + \hat{\mathbf{K}}\hat{\mathbf{Q}})\hat{\mathbf{Q}}^{-1}]$$

substituting from (15)

$$\hat{\mathbf{t}}\mathbf{S} = -\frac{1}{1+\lambda}[\mathbf{1}(\lambda\hat{\mathbf{X}} + \hat{\mathbf{K}}\hat{\mathbf{Q}})\hat{\mathbf{Q}}^{-1}] \qquad (20)$$

Equation (20) is our fundamental result. Let us examine these optimality conditions w.r.t. a particular tax, t_i:

$$\sum_{j=1}^{n} \hat{t}_j s_{ji}(\hat{\mathbf{q}}) = -\frac{\lambda}{1+\lambda}\hat{x}_i(\hat{\mathbf{q}}) - \sum_{j=1}^{n}\hat{k}_{ji} \qquad (21)$$

Denoting aggregate demand elasticities by $\varepsilon_{ij} = \varepsilon_{ij}(\mathbf{q}) = \dfrac{q_j s_{ij}(\mathbf{q})}{\hat{x}_i(\mathbf{q})}$, $i, j = 1,2,..,n$, and using symmetry, $s_{ij}(\mathbf{q}) = s_{ji}(\mathbf{q})$ for any $\mathbf{q}$, (21) can be rewritten in elasticity form:

$$\sum_{j=1}^{n} \hat{t}'_j \varepsilon_{ji}(\hat{\mathbf{q}}) = -\theta - \sum_{j=1}^{n}\hat{k}'_{ji} \ , \quad i = 1,2,..,n \qquad (22)$$

where $\hat{t}'_j = \hat{t}_j / \hat{q}_j$, $j = 1,2,..,n$ are the optimum ratios of taxes to consumer prices, $\theta = \dfrac{\lambda}{1+\lambda}$,

$$\hat{k}_{ji}^{'} = \frac{1}{\hat{q}_i x_i(\hat{\mathbf{q}})} \int_{\underline{\alpha}}^{\overline{\alpha}} (\hat{p}_j - c_j(\alpha)) x_j(\hat{\mathbf{q}};\alpha) \varepsilon_{ji}(\hat{\mathbf{q}};\alpha) dF(\alpha) \tag{23}$$

where $\varepsilon_{ji}(\hat{\mathbf{q}};\alpha) = \dfrac{\hat{q}_i s_{ji}(\hat{\mathbf{q}};\alpha)}{x_j(\hat{\mathbf{q}};\alpha)}$, $i, j = 1,2,..,n$.

Compared to the standard case, $\hat{k}_{ji} = \hat{k}_{ji}^{'} = 0$, $i, j = 1,2,..,n$, the modified Ramsey-Boiteux Conditions have the additional terms, $\sum_{j=1}^{n} \hat{k}_{ji}$ or $\sum_{j=1}^{n} \hat{k}_{ji}^{'}$, in (21) or (22). Before discussing these terms, we want to show that their sign depends on the relation between demand elasticities and costs.

Proposition 2. (I) *Suppose that $c_j(\alpha)$ increases with α. Then $\hat{k}_{ji}^{'} < 0 (> 0)$ when ε_{ji} increases (decreases) with α.* (II) *Suppose that $c_j(\alpha)$ decreases with α. Then $\hat{k}_{ji}^{'} > 0 (< 0)$ when ε_{ji} increases (decreases) with α.*

Proof: Appendix B.

One implication of Proposition 2 is that when all demand elasticities are constant (logarithmic or power utility) then $\hat{k}_{ji} = \hat{k}_{ji}^{'} = 0$, $i, j = 1,2,..,n$, and (21) or (22) become the standard Ramsey-Boiteux Conditions, solving for the same optimum tax rates. This observation provides a clue to the interpretation of the term $\hat{k}_{ji}^{'}$.

Consider expression (23). The term $(\hat{p}_j - c_j(\alpha))\hat{x}_j(\hat{\mathbf{q}};\alpha)$ is the gap between the (before-tax) consumer price and marginal costs times the quantity purchased of good *j* by an α -type. When this gap is positive, social welfare would benefit from an increase in the quantity of good *j*. The elasticity $\varepsilon_{ji}(\hat{\mathbf{q}};\alpha)$ is the relative change in the quantity of good *j* due to a small increase in the price of good i. Hence, if its sign is positive this should make for a larger tax on good i. The opposite holds when ε_{ji} is negative. This argument is reversed when $\hat{p}_j - c_j(\alpha)$ is negative. When $c_j(\alpha)$ is monotone in α, $(\hat{p}_j - c_j(\alpha))\hat{x}_j(\hat{\mathbf{q}};\alpha)$ changes sign once and its integral is equal to zero over $[\underline{\alpha}, \overline{\alpha}]$. Hence, for $\varepsilon_{ji} > 0$, the tax on good i should be larger if the positive $p_j - c_j(\alpha)$ are multiplied by large ε_{ji} and the negative $p_j - c_j(\alpha)$ by small ε_{ji}. Proposition 2 formally states these considerations.

Following the argument above, since the L.H.S. of (22) can be shown to be equal (approximately) to the relative reduction in the quantity of good i due to the imposition of $\hat{\mathbf{t}}$, when $\sum_{j=1}^{n} \hat{k}_{ji}^{'} < 0$, this reduction, and the corresponding $\hat{t}_i$, is made smaller compared to the standard formula. A simple case is when $\varepsilon_{ji} = 0$, $j \neq i$. Since $\varepsilon_{ii} < 0$, $k_{ii}^{'} > 0$ if ε_{ii} is large (small in absolute value) when $p_j - c_j(\alpha) < 0$ and small when $p_j - c_j(\alpha) > 0$. That is, quantity reductions of good i are larger when $p_i - c_i$ is positive compared to those when $p_i - c_i$ is negative. This tends to make the optimum tax on good i smaller.

IV. A Three-Good Example

The above discussion can be further highlighted with a three good example. Let there be two goods X_i, $i = 1,2$. The untaxed numeraire, *Y*, is numbered 0. Using the identities $\varepsilon_{i0} + \varepsilon_{i1} + \varepsilon_{i2} = 0$, $i = 1,2$, (22) can be solved for $\hat{t}_1^{'} / \hat{t}_2^{'}$:

$$\frac{\hat{t}_1^{'}}{\hat{t}_2^{'}} = \frac{\theta(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{10}) + \hat{k}^2 \varepsilon_{11} + \hat{k}^1 \varepsilon_{22} + \hat{k}^2 \varepsilon_{10}}{\theta(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{20}) + \hat{k}^2 \varepsilon_{11} + \hat{k}^1 \varepsilon_{22} + \hat{k}^1 \varepsilon_{20}} \tag{24}$$

where $\hat{k}^{'} = \hat{k}_{1i}^{'} + \hat{k}_{2i}^{'}$, $i = 1,2$, and $k_{ji}^{'}$, $i, j = 1,2$, are defined in (23).

As seen in (24), the standard model $(\hat{k}^1 = \hat{k}^2 = 0)$ has a higher tax rate for the good which is more complementary with the untaxed good. This familiar conclusion has to be modified to include the interaction factors, $\hat{k}^2 \varepsilon_{10}$ and $\hat{k}^1 \varepsilon_{20}$, in a pooling equilibrium. For example, when $\varepsilon_{10} > \varepsilon_{20} > 0$ and $\hat{k}^2 > \hat{k}^1$, then the previous conclusion carries over. Clearly, though, other cases may affect $\hat{t}_1^{'} / \hat{t}_2^{'}$ in the opposite direction. In general, in a pooling equilibrium, complementarity with the untaxed good is not the only factor that determines the ratio of optimum taxes. The other factor is the effect of different tax changes on the average gap between prices and costs.

Appendix A

An interior pooling equilibrium, $\hat{\mathbf{p}}$, is defined by the system of equations

$$\boldsymbol{\pi}(\hat{\mathbf{p}}) = \hat{\mathbf{p}}\mathbf{X}(\hat{\mathbf{p}}) - \int_{\underline{\alpha}}^{\overline{\alpha}} \mathbf{c}(\alpha)\hat{\mathbf{X}}(\hat{\mathbf{p}};\alpha)dF(\alpha) = 0 \tag{A.1}$$

where $\boldsymbol{\pi}(\hat{\mathbf{p}}) = (\pi_1(\hat{\mathbf{p}}), \pi_2(\hat{\mathbf{p}}),..,\pi_n(\hat{\mathbf{p}}))$, $\hat{\mathbf{p}} = (\hat{p}_1, \hat{p}_2,.., \hat{p}_n)$, $\hat{\mathbf{X}}(\hat{\mathbf{p}})$ is the diagonal $n \times n$ matrix:

$$\hat{\mathbf{X}}(\hat{\mathbf{p}}) = \begin{bmatrix} \hat{x}_1(\hat{\mathbf{p}}) & & 0 \\ & \ddots & \\ 0 & & \hat{x}_n(\hat{\mathbf{p}}) \end{bmatrix} \tag{A.2}$$

while $\mathbf{X}(\mathbf{p};\alpha)$ is the diagonal $n \times n$ matrix:

$$\hat{\mathbf{X}}(\hat{\mathbf{p}};\alpha) = \begin{bmatrix} \hat{x}_1(\hat{\mathbf{p}};\alpha) & & 0 \\ & \ddots & \\ 0 & & \hat{x}_n(\hat{\mathbf{p}};\alpha) \end{bmatrix} \tag{A.3}$$

and $\mathbf{c}(\alpha) = (c_1(\alpha), c_2(\alpha),.., c_n(\alpha))$.

It is well known from general equilibrium theory (e.g. Arrow and Hahn (1971)) that a sufficient condition for $\hat{\mathbf{p}}$ to be unique is that the $n \times n$ matrix $\hat{\mathbf{X}}(\mathbf{p}) + \hat{\mathbf{K}}(\mathbf{p})$ be *positive definite*, where $\hat{\mathbf{K}}(\hat{\mathbf{p}})$ is the $n \times n$ matrix whose elements are $\hat{k}_{ij} = \int_{\underline{\alpha}}^{\overline{\alpha}} (\hat{p}_i - c_i(\alpha)) s_{ij}(\hat{\mathbf{p}};\alpha)dF(\alpha)$, $s_{ij}(\hat{\mathbf{p}};\alpha) = \dfrac{\partial \hat{x}_i(\hat{\mathbf{p}};\alpha)}{\partial p_j}$, $i, j = 1,2,..,n$.

Furthermore, if the price of each good is postulated to change in opposite direction to the sign of profits derived from this good, then this condition also implies that price dynamics are globally stable, converging to a unique $\hat{\mathbf{p}}$.

Intuitively, as seen from (A.1), an upward perturbation of p_1 raises π_1 iff $\hat{x}_1 + \int_{\underline{\alpha}}^{\overline{\alpha}} (\hat{p}_1 - c_1)s_{11}dF(\alpha) > 0$, leading to a decrease in p_1. A simultaneous upward perturbation of p_1 and p_2 raises π_1 and π_2 if the 2×2 upper principal minor of Δ is positive, and so on. Convexity of profit functions is the standard assumption in general equilibrium theory.

Appendix B.

Proof of Proposition 1.

Suppose that $\varepsilon_{ji}(\hat{\mathbf{q}};\alpha)=\dfrac{\hat{q}_i s_{ji}(\hat{\mathbf{q}};\alpha)}{\hat{x}_j(\hat{\mathbf{q}};\alpha)}$ ($\varepsilon_{jj}<0$, $\varepsilon_{ji}\geq \text{or} \leq 0$, $j\neq i$) increases with α. Since in equilibrium

$$\int_{\underline{\alpha}}^{\overline{\alpha}}(\hat{p}_j-c_j(\alpha))\hat{x}_j(\hat{\mathbf{q}};\alpha)dF(\alpha)=0 \tag{B.1}$$

Assume that $c_j(\alpha)$ increases with α. Then, $\hat{p}_j-c_j(\alpha)$ changes sign once over $[\underline{\alpha},\overline{\alpha}]$, say at $\tilde{\alpha}$:

$$(\hat{p}_j-c_j(\alpha))\hat{x}_j(\hat{\mathbf{q}};\alpha)\gtreqless 0, \text{as}\, \alpha\lesseqgtr\tilde{\alpha} \tag{B.2}$$

Hence,

$$(\hat{p}_j-c_j(\alpha))s_{ji}(\hat{\mathbf{q}};\alpha)<\frac{\varepsilon_{ji}(\hat{\mathbf{q}};\tilde{\alpha})}{\hat{q}_i}(\hat{p}_j-c_j(\alpha))\hat{x}_j(\hat{\mathbf{q}};\alpha) \tag{B.3}$$

for all $\alpha\varepsilon[\underline{\alpha},\overline{\alpha}]$. Integrating on both sides of (B.3), using (B.1),

$$\int_{\underline{\alpha}}^{\overline{\alpha}}(\hat{p}_j-c_j(\alpha))s_{ji}(\alpha)dF(\alpha)<\frac{\varepsilon_{ji}(\hat{\mathbf{q}};\tilde{\alpha})}{\hat{q}_i}\int_{\underline{\alpha}}^{\overline{\alpha}}(\hat{p}_j-c_j(\alpha))\hat{x}_j(\hat{\mathbf{q}};\alpha)dF(\alpha)=0 \tag{B.4}$$

Hence, $\hat{k}_{ji}$ in (21) or $\hat{k}'_{ji}$ in (23), is negative. The inequality in (B.4) is reversed when $\varepsilon_{ji}(\hat{\mathbf{q}};\alpha)$ decreases with α or when $c_j(\alpha)$ decreases with α.

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