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Differentiated Annuities in a Pooling Equilibrium

by

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Abstract

Regular annuities provide payment for the duration of an owner's lifetime. Period-Certain annuities provide additional payment after death to a designated beneficiary provided the insured dies within a certain period after annuitization. It has been argued that the bequest option offered by the latter is dominated by life insurance which provides non-random bequests. This is correct if competitive annuity suppliers have full information about individual longevities and price annuities accordingly. In contrast, this paper shows that when individual longevities are *private information*, a *competitive pooling equilibrium* which offers annuities at common prices to all individuals may have positive amounts of both types of annuities in addition to life insurance. In this equilibrium, individuals *self-select* the types of annuities that they purchase according to their longevity prospects. The break-even price of each type of annuity reflects the average longevity of its buyers plus expected lump-sum payouts in the case of period-certain annuities. The broad conclusion that emerges from this paper is that *adverse-selection* due to asymmetric information is reflected not only in the *amounts* of insurance purchased but, importantly, also in the choice of *insurance products* suitable for different individual characteristics. This conclusion is supported by recent empirical work about the UK annuity market (Finkelstein and Poterba (2004)).

JEL Classification: D-11, D-82

Key Words: Regular Annuities, Full Information Equilibrium, Period-Certain Annuities, Pooling Equilibrium.

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1 Introduction

Regular annuities (sometimes called *'life-annuities'*) provide payouts, fixed or variable, for the duration of the owner's lifetime. No payments are made after the death of the annuitant. There are also *period-certain* annuities which provide additional payments after death to a beneficiary in the event that the insured individual dies within a specified period after annuitization[1]. Ten-year and Twenty-year certain periods are common (see Brown, Mitchell, Poterba and Warshawsky (2001)). Of course, expected benefits during life plus expected payments after death are adjusted to make the price of period-certain annuities commensurate with the price of regular annuities.

Period-certain annuities thus provide a bequest option not offered by regular annuities. It has been argued (e.g. Davidoff, Brown and Diamond (2005)) that a superior policy for risk-averse individuals who have a bequest motive is to purchase regular annuities and a *life insurance* policy. The latter provides a certain amount upon death, while the amount provided by period-certain annuities is random, depending on the time of death.

In a competitive market for annuities with full information about longevities, annuity prices will vary with annuitants' life expectancies. Such *'separating equilibrium'* in the annuity market, together with a competitive market for life insurance ensures that any combination of period-certain annuities and life insurance is dominated by some combination of regular annuities and life-insurance.

The situation is different, however, when individual longevities are *private information* which cannot be revealed by individuals' choices and hence each type of annuities is sold at a common price available to all potential buyers. This is called a *'pooling equilibrium'*. In this case, the equilibrium price of each type of annuity is equal to the average longevity of the buyers of this type of annuity, weighted by the equilibrium amounts purchased. Consequently, these prices are higher than the average expected lifetime of the buyers, reflecting the *'adverse-selection'* caused by the larger amounts of annuities purchased by individuals with higher longevities[2].

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[1] TIAA-CREF, for example, calls these After-Tax-Retirement-Annuities (ATRA) with Death Benefits.

[2] It is assumed that the amount of annuities purchased, presumably from different firms,

When regular annuities and period-certain annuities are available in the market, self-selection by individuals tends to segment annuity purchasers into different groups. Those with relatively short expected life span and a high probability of early death after annuitization will purchase period-certain annuities (and life insurance). Those with a high life expectancy and a low probability of early death will purchase regular annuities (and life-insurance) and those with intermediate longevity prospects will hold both types of annuities.

The theoretical implications of our modelling are supported by recent empirical findings reported in Finkelstein and Poterba (2002, 2004), who studied the UK annuity market. In a pioneering paper (2004), they test two hypotheses. One, "that higher-risk individuals self-select into insurance contracts that offer features that, at a given price, are most valuable to them". The second is that "the equilibrium pricing of insurance policies reflects variation in the risk pool across different policies". They find that the UK data supports both hypotheses.

Our modelling provides a theoretical underpinning for this observation: adverse selection in insurance markets may be revealed by self-selection of *different insurance instruments*, in addition to varying *amounts* of insurance purchased.

2 First-Best Consumption and Bequests

Consider individuals on the verge of retirement who face an uncertain lifetime. They derive utility from consumption and from leaving bequests after death. For simplicity, it is assumed that utilities are separable and independent of age. Denote the instantaneous utility from consumption by $u(a)$, where a is the flow of consumption, and $v(b)$ is the utility from bequests whose level is b. The functions $u(a)$ and $v(b)$ are assumed to be strictly concave, differentiable, and satisfy $u'(0) = v'(0) = \infty$ and $u'(\infty) = v'(\infty) = 0$. These assumptions ensure that individuals will choose strictly positive levels of both a and b.

Assuming no time preference and a constant flow of consumption while alive, lifetime utility, U, is

$$U = u(a)\bar{z} + v(b) \tag{1}$$

cannot be monitored. Hence, we consider only linear price policies (e.g. no quantity constraints). See, for example, Abel (1986) and Brugiavini (1993).

where $\bar{z}$ is expected lifetime. Individuals have different longevities represented by a parameter α, $\bar{z} = \bar{z}(\alpha)$. An individual with $\bar{z}(\alpha)$ is termed 'type α'. Assume that α varies continuously, with a distribution function $G(\alpha)$ over the interval $[\underline{\alpha}, \bar{\alpha}]$, $\bar{\alpha} > \underline{\alpha}$. We take a higher α to indicate lower longevity: $\bar{z}'(\alpha) < 0$[3].

Social welfare, W, is the sum of realized individual utilities (or *ex-ante* expected utility),

$$W = \int_{\underline{\alpha}}^{\bar{\alpha}} [u(a(\alpha))\bar{z}(\alpha) + v(b(\alpha))]dG(\alpha) \tag{2}$$

where $(a(\alpha), b(\alpha))$ is consumption and bequests, respectively, of type α individuals.

Assume a zero rate of interest, so resources can be carried forward or backward in time at no cost. Hence, given total resources, R, the economy's resource constraint is

$$\int_{\underline{\alpha}}^{\bar{\alpha}} [a(\alpha)\bar{z}(\alpha) + b(\alpha)]dG(\alpha) = R \tag{3}$$

Maximization of (2) s.t. (3) yields a unique *First-Best* allocation, (a^*, b^*), independent of α, which equalizes the marginal utilities of consumption and bequests:

$$u'(a^*) = v'(b^*) \tag{4}$$

Conditions (3) and (4) jointly determine (a^*, b^*) and the corresponding optimum utility of type α individuals $U^*(\alpha) = u(a^*)\bar{z}(\alpha) + v(b^*)$. Note that while First-Best consumption and bequests are equalized across individuals with different longevities, U^* increases with longevity: $U^{*\prime}(\alpha) = u(a^*)\bar{z}'(\alpha) < 0$.

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[3]Let $F(z, \alpha)$ be probability that an individual survives to age z; $0 \leq z \leq T$, where T is maximum lifetime. $F(0, \alpha) = 1$, $\dfrac{\partial F(z,\alpha)}{\partial z} < 0$, $z \in (0, T)$, and $F(T, \alpha) = 0$, for all $\alpha \in [\underline{\alpha},\ \bar{\alpha}]$. Life expectancy of type α is $\bar{z}(\alpha) = \int_0^T F(z,\alpha)dz$. It is assumed that $\bar{z}(\alpha)$ is finite when $T = \infty$. An increase in α is taken to reduce survival probabilities, $\dfrac{\partial F(z,\alpha)}{\partial \alpha} < 0$ for all z, and hence $\bar{z}'(\alpha) < 0$.

Example: $F(z, \alpha) = \dfrac{e^{-\alpha z} - e^{-\alpha T}}{1 - e^{-\alpha T}}$, which becomes $F(z, \alpha) = e^{-\alpha z}$ when $T = \infty$.

3 Competitive Equilibrium with Regular Annuities

In a market setting, consumption is financed by annuities (for later reference these are called *'regular annuities'*) while bequests are provided by the purchase of life insurance. Each annuity pays a flow of one unit of consumption, contingent on the annuity holder's survival. Denote the price of annuities by p_a. A unit of life insurance pays upon death one unit for bequests and its price is denoted by p_b.

Each individual maximizes utility, (1), subject to the budget constraint

$$p_a a + p_b b = R \tag{5}$$

where R is a given income[4].

(a) Full Information Equilibrium

Under full information about individuals' longevities, the competitive equilibrium price of an annuity is equal to life expectancy of the purchaser: $p_a = p_a(\alpha) = \bar{z}(\alpha)$[5]. Since each unit of life insurance pays one unit with certainty, its equilibrium price is unity: $p_b = 1$. This competitive equilibrium is *efficient*, satisfying condition (4), and for a particular income distribution yields the First-Best allocation[6].

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[4]Allowing for different incomes is important for welfare analysis. The joint distribution of incomes and longevity is essential, for example, when considering tax/subsidy policies. Our focus, though, is on the possibility of pooling equilibria with different types of annuities, given *any* income distribution. For simplicity, we assume below equal incomes.

[5]The modification for a positive interest rate, $\rho > 0$, is straightforward. For example, with $F(z,\alpha) = e^{-\alpha z}$, $\bar{z}(\alpha) = \dfrac{1}{\alpha}$. The present discounted value of expected payouts is $\int\limits_0^\infty e^{-\rho z} F(z,\alpha)dz = \dfrac{1}{\alpha+\rho}$. Similarly, the price of a unit of life insurance is $\int\limits_0^\infty e^{-\rho z} f(z,\alpha)dz = \dfrac{\alpha}{\alpha+\rho}$, which is equal to 1 when $\rho = 0$.

[6]Individuals who maximize (1) s.t. the budget constraint $\bar{z}(\alpha)a + b = R(\alpha)$ will select (a^*, b^*) iff $R(\alpha) = \gamma R + (1-\gamma)b^*$, where $\gamma = \gamma(\alpha) = \dfrac{\bar{z}(\alpha)}{\int\limits_{\underline{\alpha}}^{\bar{\alpha}} \bar{z}(\alpha)dG(\alpha)} > 0$. Note that $R(\alpha)$ strictly decreases with α (increases with life expectancy).

(b) Pooling Equilibrium

Suppose that longevity is private information and hence annuities are sold at the same price, p_a, to all individuals. Life insurance is sold at the common price p_b.

Maximization of (1) s.t. (5) yields demand functions for annuities, $\hat{a}(p_a, p_b; \alpha)$, and for life insurance, $\hat{b}(p_a, p_b; \alpha)$[7]. Given our assumptions, $\frac{\partial \hat{a}}{\partial p_a} < 0$, $\frac{\partial \hat{a}}{\partial \alpha} < 0$, $\frac{\partial \hat{a}}{\partial p_b} \gtreqless 0$, $\frac{\partial \hat{b}}{\partial p_b} < 0$, $\frac{\partial \hat{b}}{\partial \alpha} > 0$, $\frac{\partial \hat{b}}{\partial p_a} \gtreqless 0$.

Total profits from the sale of annuities, π_a, and from the sale of life insurance, π_b, are:

$$\pi_a(p_a, p_b) = \int_{\underline{\alpha}}^{\overline{\alpha}} (p_a - \bar{z}(\alpha))\hat{a}(p_a, p_b; \alpha)dG(\alpha) \tag{6}$$

and

$$\pi_b(p_a, p_b) = \int_{\underline{\alpha}}^{\overline{\alpha}} (p_b - 1)\hat{b}(p_a, p_b; \alpha)dG(\alpha) \tag{7}$$

Definition 1 *A pooling equilibrium is a pair of prices $(\hat{p}_a, \hat{p}_b)$ that satisfy $\pi_a(\hat{p}_a, \hat{p}_b) = \pi_b(\hat{p}_a, \hat{p}_b) = 0$.*

Clearly, $\hat{p}_b = 1$, because marginal costs of a life insurance policy are constant and equal to 1. From (6), the zero profits condition for annuities is

$$\hat{p}_a = \frac{\int_{\underline{\alpha}}^{\overline{\alpha}} \bar{z}(\alpha)\hat{a}(\hat{p}_a, 1; \alpha)dG(\alpha)}{\int_{\underline{\alpha}}^{\overline{\alpha}} \hat{a}(\hat{p}_a, 1; \alpha)dG(\alpha)}. \tag{8}$$

The equilibrium price of annuities is seen to be an average of marginal costs (equal to life expectancy), weighted by the equilibrium amounts of annuities: $\bar{z}(\bar{\alpha}) < \hat{p}_a < \bar{z}(\underline{\alpha})$.

Furthermore, since $\hat{a}$ and $\bar{z}(\alpha)$ decrease with α, it follows from (8) that $\hat{p}_a > E(\bar{z}) = \int_{\underline{\alpha}}^{\overline{\alpha}} \bar{z}(\alpha)dG(\alpha)$. The equilibrium price of annuities is higher than

[7] The dependence on R is suppressed.

the population's average expected lifetime, reflecting the *'adverse-selection'* present in a pooling equilibrium.

Regarding price dynamics out of equilibrium, we follow the standard assumption (reflecting entry and exit of firms) that the price of each good changes in opposite direction to the sign of profits from sales of this good. It is well-known that a sufficient condition for $(\hat{p}_a, 1)$ to be unique and (locally) stable is that the matrix

$$\begin{bmatrix} \dfrac{\partial \pi_a}{\partial p_a} & \dfrac{\partial \pi_a}{\partial p_b} \\ \dfrac{\partial \pi_b}{\partial p_a} & \dfrac{\partial \pi_b}{\partial p_b} \end{bmatrix}, \tag{9}$$

be *positive definite* at $(\hat{p}_a, 1)$. Appendix A provides a sufficient condition for (9) to be positive-definite.

4 Regular and Period-Certain Annuities: First-Best and Full Information Equilibrium

We have assumed that annuities provide payouts for the duration of the owner's lifetime and no payments are made after death of the annuitant. We called these *regular annuities*. There exist also *period-certain* annuities which provide an additional payment to a designated beneficiary after death of the insured person, provided death occurs within a specified period after annuitization[8]. Ten-year and Twenty-year certain periods are common and more annuitants choose them over regular annuities (see Brown, Mitchell, Poterba and Warshawsky (2001)). Of course, benefits during life plus expected payments after death are adjusted to make the price of period-certain annuities commensurate with the price of regular annuities.

Suppose that there are regular annuities and X-year-certain annuities (in short, X-annuities) who offer a unit flow of consumption while alive and an additional lump-sum equal to the total amount that would be paid if the holder were alive until age x. Thus, if the holder dies at age z, $0 \leq z \leq x$, the payout upon death per X-annuity is equal to $x - z$. We continue to denote the amount of regular annuities by a and denote the amount of X-annuities by a_x.

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[8]TIAA-CREF, for example, calls these After-Tax-Retirement Annuities (ATRA) with death benefits.

(a) First-Best

The First-Best allocation with both types of annuities is obtained by maximization of social welfare

$$W = \int_{\underline{\alpha}}^{\bar{\alpha}} [u(a(\alpha) + a_x(\alpha))\bar{z}(\alpha) + \int_0^x v(b(\alpha) + (x-z)a_x(\alpha))f(z,\alpha)dz + \\ +v(b(\alpha))\int_x^\infty f(z,\alpha)dz]dG(\alpha) \tag{10}$$

Subject to the resource constraint

$$\int_{\underline{\alpha}}^{\bar{\alpha}} [(a(\alpha) + a_x(\alpha))\bar{z}(\alpha) + a_x(\alpha)\int_0^x (x-z)f(z,\alpha)dz + b(\alpha)]dG(\alpha) = R \tag{11}$$

where $f(z,\alpha)$ is the probability that type α dies at age z: $\int_0^x f(z,\alpha)dz + \int_x^\alpha f(z,\alpha)dz = 1$[9].

Maximization of (10) s.t. (11) yields solutions a^*, a_x^* and b^*. It is straightforward to verify that $a_x^* = 0$ for all $\underline{\alpha} \leq \alpha \leq \bar{\alpha}$, while a^* and b^* are positive, satisfying the efficiency condition (4), and are independent of α. This is an important conclusion:

The First-Best has no X-annuities: the random bequest option offered by X-annuities is dominated by regular annuities and life insurance which jointly provide for non-random consumption and bequests.

We shall now show that a full-information competitive equilibrium also has no X-annuities[10].

(b) Full-information Equilibrium

Continue to denote the price of regular annuities by p_a, and denote the price of X-annuities by p_a^x. Type α individuals maximize their expected utility,

[9]The probability of death at age z is $f(z,\alpha) = \frac{\partial}{\partial z}(1-F(z,\alpha)) = -\frac{\partial F}{\partial z}(z,\alpha)$. For example, for $F(z,\alpha) = e^{-\alpha z}$, $f(z,\alpha) = \alpha e^{-\alpha z}$, $z \geq 0$.

[10]While the competitive equilibrium is efficient, the equilibrium amounts of a and b need not be equal to a^* and b^* as they depend on the income distribution.

$U(\alpha)$,

$$U(\alpha) = u(a+a_x)\bar{z}(\alpha) + \int_0^x v(b+(x-z)a_x)f(z,\alpha)dz + +v(b)\int_x^\infty f(z,\alpha)dz \tag{12}$$

subject to the budget constraint

$$p_a a + p_a^x\ a_x + b = R. \tag{13}$$

The F.O.C. are

$$u'(a+a_x)\bar{z}(\alpha) - \lambda p_a \leq 0 \tag{14}$$

$$u'(a+a_x)\bar{z}(\alpha) + \int_0^x v'(b+(x-z)a_x)(x-z)f(z,\alpha)dz - \lambda p_a^x \leq 0 \tag{15}$$

and

$$\int_0^x v'(b+(x-z)a_x)f(z,\alpha)dz + v'(b)\int_x^\infty f(z,\alpha)dz - \lambda = 0 \tag{16}$$

with $\lambda > 0$ being the Lagrangean associated with the budget constraint (13). Denote the solution to (13) - (16) by $\hat{a}$, $\hat{a}_x$, $\hat{\lambda}$ and $\hat{b}$, all functions of p_a, p_a^x and α (dependence on x and R is supressed)[11].

Suppose that individual characteristics, $\bar{z}(\alpha)$ and $f(z,\alpha)$, are known to the sellers of annuities. Then, zero expected profits for each α entails that

$$p_a = \bar{z}(\alpha) \text{ and } p_a^x = \bar{z}(\alpha) + \int_0^x (x-z)f(z,\alpha)dz \tag{17}$$

Prices vary with individual longevities: for each α, the price of regular annuities is equal to life expectancy and that of X-annuities exceeds it by the expected lump-sum payment after death.

[11] The assumption that $v'(0) = \infty$ ensures that $\hat{b} > 0$ and hence (16) holds with equality. Note also that assumption that $u'(0) = \infty$ ensures that $\hat{a}$ and $\hat{a}_x$ cannot *both* be equal to zero.

We can now state:

Proposition 1 *Under (17),* $\hat{a}_x = 0$, $\hat{a} > 0$ *and* $\hat{b} > 0$ *for all* $\alpha \in [\underline{\alpha},\ \bar{\alpha}]$.

Proof Appendix B.

Proposition 1 has a stark conclusion: a competitive annuity market which recognizes and bases annuity prices on individual longevity characteristics has no X-annuities. In contrast, we shall show that X-annuities may be held in a pooling equilibrium in which prices do not vary with individual longevities because these are private information. Self-selection leads to a segmented market equilibrium: individuals with low longevities and high probability of early death purchase X-annuities (and life-insurance), while individuals with high longevities and low probability of early death purchase regular annuities (and life-insurance). In a range of intermediate longevities individuals hold both types of annuities.

5 Pooling Equilibrium

When α is private information, all individuals face the same prices, p_a and p_a^x. In a competitive equilibrium, these prices satisfy a zero expected profits condition for each type of annuity, based on the quantities purchased. Denote these equilibrium prices by $\hat{p}_a$ and $\hat{p}_a^x$ ($\hat{p}_b = 1$).

The zero expected profits conditions for regular and X-annuities, $\pi_a(\hat{p}_a,\ \hat{p}_a^x,\ 1) = \pi_a^x(\hat{p}_a,\ \hat{p}_a^x,\ 1) = 0$ ($\pi_b(p_a,\ p_a^x,\ 1) = 0$ for any $(p_a,\ p_a^x)$) can be written (suppressing $\hat{p}_b = 1$)

$$\hat{p}_a = \frac{\int_{\underline{\alpha}}^{\bar{\alpha}} \bar{z}(\alpha)\hat{a}(\hat{p}_a, \hat{p}_a^x; \alpha)dG(\alpha)}{\int_{\underline{\alpha}}^{\bar{\alpha}} \hat{a}(\hat{p}_a, \hat{p}_a^x; \alpha)dG(\alpha)} \tag{18}$$

and

$$\hat{p}_a^x = \frac{\int_{\underline{\alpha}}^{\bar{\alpha}} \left[\bar{z}(\alpha) + \int_0^x (x-z)f(z,\alpha)dz\right] \hat{a}_x(\hat{p}_a, \hat{p}_a^x; \alpha)dG(\alpha)}{\int_{\underline{\alpha}}^{\bar{\alpha}} \hat{a}(\hat{p}_a, \hat{p}_a^x; \alpha)dG(\alpha)} \tag{19}$$

As before, the equilibrium price of regular annuities is equal to the weighted average expected lifetime in the population, with the quantities of regular annuities purchased as weights. The equilibrium price of X-annuities is a weighted average of life expectancy in the population plus the average expected payout upon death, weights being the amounts purchased of X-annuities.

Conditions for uniqueness and stability of $\hat{p}_a$, $\hat{p}_a^x$ and $\hat{p}_b$ can be formulated along the lines in Appendix A which deals with regular annuities and life insurance[12].

We shall now explore the possible equilibrium configurations implied by (13-16):

I. $\underline{\hat{a} > 0,\ \hat{a}_x = 0}$

Condition (14) holds with equality. From (14) - (16) it now follows that:

$$p_a^x \geq p_a + \int_0^x (x-z) f(z,\alpha) dz \tag{20}$$

Risk averse individuals do not purchase X-annuities when their price exceeds the price of regular annuities plus the expected payout upon death.

We assume that

$$\frac{\partial f(z,\alpha)}{\partial \alpha} > 0, \qquad 0 \leq z \leq x \tag{21}$$

A decrease in longevity increases the probability of death at all ages between 0 and x. Suppose that there exists an $\alpha_0 \in [\underline{\alpha}, \bar{\alpha}]$, which makes (15) hold with equality: $p_a^x - p_a = \int_0^x (x-z) f(z,\alpha_0) dz$. It is seen that (19) ensures that (15) holds with strict inequality for all $\alpha \in [\underline{\alpha}, \alpha_0]$, implying that all individuals with high longevities ($\bar{z}(\alpha) \geq \bar{z}(\alpha_0)$, that is, $\alpha \leq \alpha_0$) hold only regular annuities (and life insurance).

Also, holding prices constant, $\frac{d\hat{a}}{d\alpha} < 0$ and $\frac{d\hat{b}}{d\alpha} > 0$ for $\underline{\alpha} \leq \alpha \leq \alpha_0$. The holding of annuities increases and of life insurance decreases with life expectancy.

[12] These conditions ensure that the matrix of the partial derivatives of expected profits w.r.t. p_a, p_a^x and p_b is positive definite around $\hat{p}_a, \hat{p}_a^x$ and $\hat{p}_b = 1$.

II. $\underline{\hat{a} > 0,\ \hat{a}_x > 0}$

Conditions (14) and (15) hold with equality.
From (14) - (16) we deduce that

$$p_a^x = p_a + \int_0^x \frac{v'(\hat{b} + (x-z)\hat{a}_x)}{\lambda}(x-z)f(z,\alpha)dz \tag{22}$$

The price of X-annuities exceeds the price of regular annuities by the expected payout of X-annuities upon death weighted by the marginal utility of bequests (including the payout) divided by the marginal utility of income. This implies that

$$p_a^x - p_a < \frac{v'(\hat{b})}{\lambda}\int_0^x (x-z)f(z,\alpha)dz \tag{23}$$

The difference in the price of X-annuities and regular annuities is smaller than the expected bequest via X-annuities tines the marginal utility of bequests via life insurance divided by the marginal utility of income. Inequality (23) reflects risk aversion regarding the uncertainty of bequests via X-annuities.

In Appendix C we prove that second-order conditions are satisfied in this range of α's.

III. $\underline{\hat{a} = 0,\ \hat{a}_x > 0}$

Condition (15) holds with equality. If there exists an $\alpha_1 < \bar{\alpha}$ such that $u'(\hat{a}_x)\bar{z}(\alpha_1) = \lambda p_a$, then for $\alpha \in [\underline{\alpha}_1, \bar{\alpha}]$, (14) holds (with $\hat{a} = 0$).

Again, it is shown in Appendix C that the second-order conditions hold in this range of α, and $\frac{d\hat{a}_x}{d\alpha} < 0$, $\frac{d\hat{b}}{d\alpha} > 0$.

We can now portray the generic pattern of annuity and life insurance holdings for various life expectancies (Figure 1).

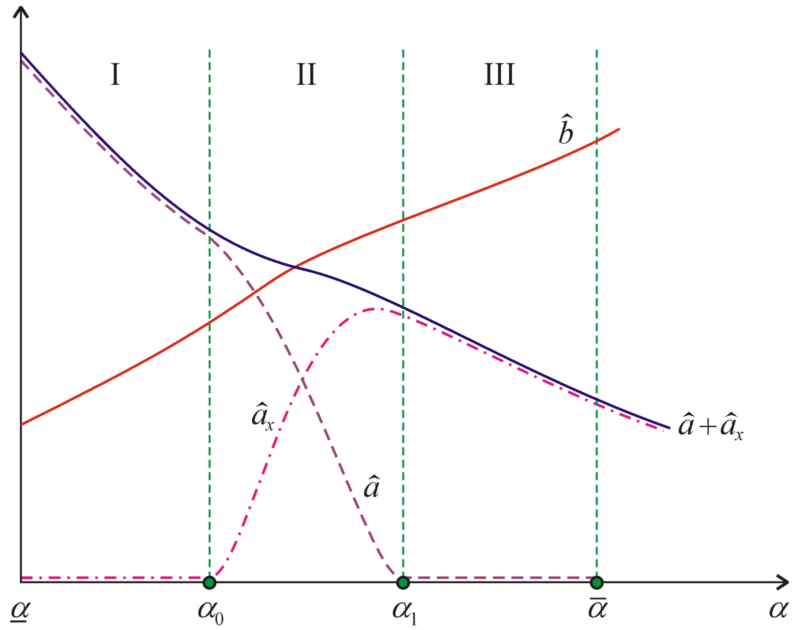

Figure 1
Optimum Annuity Holdings

6 A Simple Example

The fundamental reason why regular and X-annuities may coexist in the market is asymmetric information about individual longevities. This leads to annuity prices which yield zero expected profits given the longevity parameters of the purchasers of each type of annuities. To underscore this point consider a simple example. Suppose that each X-annuity provides a *certain* amount, $\delta > 0$, in case of 'early death'[13]. Consider two individuals with life expectancies $\bar{z}_i,\ i = 1, 2$, and let $\bar{z}_1 > \bar{z}_2$. The probabilities of early death are, correspondingly, $p_i,\ 0 \leq p_i \leq 1,\ i = 1, 2$, with $p_1 < p_2$.

Each individual maximizes expected utility, U_i,

$$U_i = u(a_i + a_x^i)\bar{z}_i + v(b_i + \delta a_x^i)p_i + v(b_i)(1 - p_i), \qquad i = 1, 2 \tag{24}$$

subject to the budget constraint

$$p_a a_i + p_a^x a_x^i + b_i = R \tag{25}$$

[13]Thus, to simplify the calculations, death within $[0, x]$ is shrunk to a point.

We look for parameter configurations, $\bar{z}_1,\ \bar{z}_2$ and δ, that lead individual 1 to purchase in equilibrium only regular annuities and individual 2 to purchase only X-annuities. Explicit solutions are obtained when u and v are logarithmic: $u(\cdot) = v(\cdot) = \ln(\cdot)$. With $a_1^x = 0$, individual 1's demands for annuities and life insurance, $(\hat{a}_1,\ \hat{b}_1)$, are

$$\hat{a}_1 = \frac{\bar{z}_1 R}{p_a(1+\bar{z}_1)}, \quad \hat{b}_1 = \frac{R}{1+\bar{z}_1} \tag{26}$$

The condition for this individual not to purchase X-annuities is that the marginal utility of one unit of an X-annuity at $(\hat{a}_1,\ \hat{b}_1)$ be lower than the marginal utility of income times p_a^x : $\dfrac{\bar{z}_1}{\hat{a}_1} + \dfrac{\delta p_1}{\hat{b}_1} + \dfrac{1-p_1}{\hat{b}_1} \leq \lambda_1 p_a^x$, where λ_1, is the marginal utility of income.

When the market is segmented, individual 1 purchasing only regular annuities and individual 2 only X-annuities, the equilibrium prices are: $\hat{p}_a = \bar{z}_1$, $\hat{p}_a^x = \bar{z}_2 + \delta p_2$. Hence, $\lambda_1 = \dfrac{1}{\hat{b}_1}$ and $\hat{a}_1 = \hat{b}_1$. Consequently, the condition for individual 1 not to purchase any X-annuities is

$$\bar{z}_1 + \delta p_1 \leq \bar{z}_2 + \delta p_2$$

or

$$\bar{z}_1 - \bar{z}_2 \leq \delta(p_2 - p_1). \tag{27}$$

When $\hat{a}_2 = 0$, the demands of individual 2 for X-annuities and life insurance at the equilibrium price $\hat{p}_a^x = \bar{z}_2 + \delta p_2$ (and $\hat{p}_b = 1$), are implicitly determined by the following conditions:

$$\frac{\bar{z}_2}{\hat{a}_x^2} + \frac{\delta p_2}{\hat{b}_2 + \delta \hat{a}_x^2} - \lambda_2(\bar{z}_2 + \delta p_2) = 0 \tag{28}$$

$$\frac{p_2}{\hat{b}_2 + \delta \hat{a}_x^2} + \frac{1-p_2}{\hat{b}_2} - \lambda_2 = 0 \tag{29}$$

and the budget constraint (25). Substituting (25) and (29) into (28), the condition that determines $\hat{a}_x^2$ can be written

$$\begin{aligned} &\frac{\bar{z}_2}{\hat{a}_x^2} + \frac{\delta p_2}{W - [z_2 - \delta(1-p_2)]\hat{a}_x^2} = \\ &\quad = (z_2 + \delta p_2)\left(\frac{p_2}{W - [z_2 - \delta(1-p_2)]\hat{a}_x^2} + \frac{1-p_2}{W - (z_2 + \delta p_2)\hat{a}_x^2}\right) \end{aligned} \tag{30}$$

It can be shown (see Figure 2) that (30) determines a unique $\hat{a}_x^2 \left(< \dfrac{R}{\bar{z}_2 + \delta p_2} \right)$.

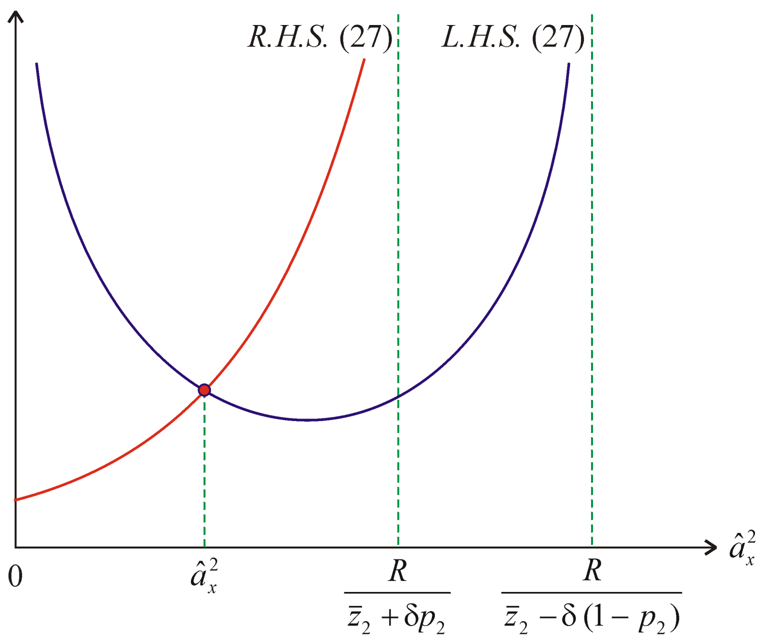

Figure 2

The condition that individual 2 does not purchase regular annuities is that, at $(\hat{a}_x^2, \hat{b}_2)$, the marginal utility of regular annuities is lower than the marginal utility of income λ_2, times $\hat{p}_a = \bar{z}_1$:

$$\frac{\bar{z}_2}{\hat{a}_x^2} - \lambda_2 \bar{z}_1 \leq 0 \tag{31}$$

It is easy to see that there are many parameter values, $\bar{z}_1,\ \bar{z}_2,\ p_1,\ p_2$ and δ which satisfy conditions (27) and (31). In particular, when $p_2 \sim 1$, then $\hat{a}_x^2 \sim \dfrac{R}{1 + \bar{z}_2}$ and $\lambda_2 \sim \dfrac{1}{R - \bar{z}_2 \hat{a}_x^2}$. Condition (27) is approximately $\bar{z}_1 - \bar{z}_2 \leq \delta(1 - p_1)$ while (31) reduces to $\bar{z}_2 < \bar{z}_1$, which holds by assumption.

7 Summary:

In efficient full-information equilibria, the holdings of any period-certain annuities and life insurance is dominated by the holdings of some combination of regular annuities and life insurance. However, when information about longevities is private, a competitive pooling equilibrium may support the coexistence of differentiated annuities and life insurance, with some individuals holding only one type of annuity and some holding both types of annuities.

Reassuringly, Finkelstein and Poterba (2004) find evidence of such self-selection in the UK annuity market. More specifically, our analysis suggests a hypothesis complementary to their observation of self-selection: those with high longevities hold regular annuities, while those with low longevities hold period-certain annuities, with mixed holdings for intermediate longevities.

Appendix A

Let $\varepsilon_{ap_a} = \dfrac{p_a}{\hat{a}(p_a, p_b; \alpha)} \dfrac{\partial \hat{a}(\hat{p}_a, p_b; \alpha)}{\partial p_a}$ be the own price elasticity of the demand for annuities. We shall prove that a monotonicity assumption about suffices for (9) to be positive-definite at $(\hat{p}_a, 1)$.

From (6) and (7), $\pi_a(\hat{p}_a, 1) = \bar{\pi}_b(\hat{p}_a, 1) = 0$, we have $\dfrac{\partial \pi_b}{\partial p_a} = 0$, $\dfrac{\partial \pi_b}{\partial p_b} = \hat{b}(\hat{p}_a, 1) > 0$, $\dfrac{\partial \pi_a}{\partial p_a} = \hat{a}(\hat{p}_a, 1) + \int_{\underline{\alpha}}^{\overline{\alpha}} (\hat{p}_a - \bar{z}(\alpha)) \dfrac{\partial \hat{a}(\hat{p}_a, 1; \alpha)}{\partial p_a} dG(\alpha) \gtreqless 0$ and $\dfrac{\partial \pi_a}{\partial p_b} = \int_{\underline{\alpha}}^{\overline{\alpha}} (\hat{p}_a - \bar{z}(\alpha)) \dfrac{\partial \hat{a}(\hat{p}_a, 1; \alpha)}{\partial p_b} dG(\alpha) \gtreqless 0$, where $\hat{a}(\hat{p}_a, 1) = \int_{\underline{\alpha}}^{\overline{\alpha}} \hat{a}(\hat{p}_a, 1; \alpha) dG(\alpha)$ and $\hat{b}(\hat{p}_a, 1) = \int_{\underline{\alpha}}^{\overline{\alpha}} \hat{b}(\hat{p}_a, 1; \alpha) dG(\alpha)$ are aggregate demands. It is seen that a sufficient condition for (9) to be positive definite at $(\hat{p}_a, 1)$ is that $\dfrac{\partial \pi_a}{\partial p_a} > 0$.

Rewriting the second term in $\dfrac{\partial \pi_a}{\partial p_a}$,

$$
\begin{aligned}
&\int_{\underline{\alpha}}^{\overline{\alpha}} (\hat{p}_a - \bar{z}(\alpha)) \hat{a}(\hat{p}_a, 1; \alpha) dG(\alpha) = \\
&= \frac{1}{\hat{p}_a} \int_{\underline{\alpha}}^{\overline{\alpha}} (\hat{p}_a - \bar{z}(\alpha)) \hat{a}(\hat{p}_a, 1; \alpha) \varepsilon_{ap_a}(\hat{p}_a, 1; \alpha) dG(\alpha)
\end{aligned}
\tag{A.1}
$$

By (6), $\hat{p}_a - \bar{z}(\alpha)$ change sign once over $[\underline{\alpha},\ \bar{\alpha}]$, say at $\tilde{\alpha}$. That is $\hat{p}_a - \bar{z}(\alpha) \lesseqgtr 0$ as $\alpha \lesseqgtr \tilde{\alpha}$.

Assume that ε_{ap_a} non-decreases in α. Since $\hat{p}_a - \bar{z}(\alpha)$ change sign once over $[\underline{\alpha},\ \bar{\alpha}]$, say at $\tilde{\alpha}$, this assumption and (8) lead to the following:

$$
\begin{aligned}
&\int_{\underline{\alpha}}^{\overline{\alpha}} (\hat{p}_a - \bar{z}(\alpha)) \hat{a}(\hat{p}_a, 1; \alpha) dG(\alpha) \geq \\
&\geq \frac{\varepsilon_{ap_a}(\hat{p}_a, 1; \tilde{\alpha})}{\hat{p}_a} \int_{\underline{\alpha}}^{\overline{\alpha}} (\hat{p}_a - \bar{z}(\alpha)) \hat{a}(\hat{p}_a, 1; \alpha) dG(\alpha) = 0
\end{aligned}
\tag{A.2}
$$

(A.2) ensures that $\dfrac{\partial \pi_a}{\partial p_a} > 0$, implying that (9) is positive-definite.

Appendix B

Proof of Proposition 1

Suppose that $\hat{a}_x > 0$, so that (15) holds with equality. If $\hat{a} > 0$, then (14) also holds with equality and, from (17) and (14) - (16), we have

$$\frac{1}{\lambda}\int_0^x v'(\hat{b} + (x-z)\hat{a}_x)(x-z)f(z,\alpha)dz = \int_0^x (x-z)f(z,\alpha)dz$$

or

$$\int_0^x \varphi(z,\alpha)(x-z)f(z,\alpha)dz = 0 \tag{B.1}$$

where $\varphi(z,\alpha) = \dfrac{v'(\hat{b} + x\hat{a}_x)}{\lambda} - 1$.

By (16), $\varphi(x,\alpha) = \dfrac{v'(\hat{b} + x\hat{a}_x)}{\lambda} - 1 < 0$, $\varphi(0,\alpha) = \dfrac{v'(\hat{b})}{\lambda} - 1 > 0$, and $\varphi(z,\alpha)$ is seen to change sign once over $[\underline{\alpha},\ \bar{\alpha}]$. Let $\varphi(\tilde{z}) = 0$. Then, $\varphi(z) \lesseqgtr 0$ as $z \lesseqgtr \tilde{z}$. Since $x - z$ decreases in z, it now follows from (B.1) that

$$\int_0^x \varphi(z,\alpha)(x-z)f(z,\alpha)dz < (x-\tilde{z})\int_0^x \varphi(z,\alpha)f(z,\alpha)dz \tag{B.2}$$

Using $\int_0^x f(z,\alpha)dz + \int_x^\alpha f(z,\alpha)dz = 1$, we have

$$\begin{aligned}\int_0^x \varphi(z,\alpha)f(z,\alpha)dz &= \frac{1}{\lambda}\int_x^\alpha f(z,\alpha)dz \left[\int_0^x v'(\hat{b} + (x-z)\hat{a}_x)f(z,\alpha)dz - \right.\\ &\left. -v'(\hat{b})\right] < 0. \end{aligned}\tag{B.3}$$

It follows from (B.2) and (B.3) that (B.1) cannot hold.

When $\hat{a} = 0$, it follows from (14) and (15) that

$$\int_0^x v'(\hat{b} + (x-z)\hat{a}_x)(x-z)f(z,\alpha)dz \geq \int_0^x (x-z)f(z,\alpha)dz \tag{B.4}$$

which, by (B.1) - (B.3) has been shown to be impossible. We conclude that $\hat{a}_x$ cannot be positive $\|$.

Appendix C

Here we prove that the second-order conditions for (13) - (16) are satisfied and derive the dependence of the demands for annuities and life insurance on α. Maximizing (12) s.t. the budget constraint (13), yields solutions $\hat{a}$, $\hat{a}_x$ and $\hat{b}$. Given our assumption that $v'(0) = \infty$, $\hat{b} > 0$ for all α and hence (16) holds with equality for all $\alpha \in [\underline{\alpha},\ \bar{\alpha}]$.

We distinguish three regions: I. $\hat{a} > 0$, $\hat{a}_x = 0$; II. $\hat{a} > 0$, $\hat{a}_x > 0$ and III. $\hat{a} = 0$, $\hat{a}_x > 0$.

I. $\underline{\hat{a} > 0,\ \hat{a}_x = 0 \quad (\underline{\alpha} < \alpha < \alpha_0)}$

The conditions that determine $\hat{a}(\hat{p}_a, \hat{p}_a^x; \alpha)$ and $\hat{b}(\hat{p}_a, \hat{p}_a^x; \alpha)$ are

$$u'(\hat{a})\bar{z}(\alpha) - \lambda p_a = 0 \tag{C.1}$$

$$v'(\hat{b}) - \lambda = 0 \tag{C.2}$$

$$W - p_a\hat{a} - \hat{b} = 0 \tag{C.3}$$

where $\lambda > 0$ is the marginal utility of income.

The second-order conditions are $u''(\hat{a})\bar{z}(\alpha) < 0,\quad v''(\hat{b}) < 0$, and

$$\Delta_1 = -(u''(\hat{a})\bar{z}(\alpha) + p_a^2 v''(\hat{b})) > 0 \tag{C.4}$$

are satisfied.

Differentiating (C.1) - (C.3) totally, holding prices constant,

$$\frac{d\hat{a}}{d\alpha} = \frac{u'(\hat{a})\bar{z}'(\alpha)}{\Delta_1} < 0, \quad \frac{d\hat{b}}{d\alpha} = \frac{p_a u'(\hat{a})\bar{z}'(\alpha)}{\Delta_1} > 0 \tag{C.5}$$

II. $\underline{\hat{a} > 0,\ \hat{a}_x > 0}$

Conditions (14) - (15) hold with equality:

$$u'(\hat{a} + \hat{a}_x)\bar{z}(\alpha) - \lambda p_a = 0 \tag{C.6}$$

$$u'(\hat{a} + \hat{a}_x)\bar{z}(\alpha) + \int_0^x v'(\hat{b} + (x - z)\hat{a}_x) f(z, \alpha) dz - \lambda p_a^x = 0 \tag{C.7}$$

$$\int_0^x v'(\hat{b}+(x-z)\hat{a}_x)f(z,\alpha)dz + v'(\hat{b})\int_x^\infty f(z,\alpha)dz - \lambda = 0 \tag{C.8}$$

$$W - p_a\hat{a} - p_a^x\hat{a}_x - \hat{b} = 0 \tag{C.9}$$

The second-order conditions are that the matrix (we ommit the terms in the functions):

$$\begin{bmatrix} u''\bar{z} & u''\bar{z} & 0 & -p_a \\ u''\bar{z} & u''\bar{z}+\int_0^x v''(x-z)^2 f dz & \int_0^x v''(x-z) f dz & -p_a^x \\ 0 & \int_0^x v''(x-z) & \int_0^x v'' f dz + v''(\hat{b})\int_x^\infty f dz & -1 \\ -p_a & -p_a^x & -1 & 0 \end{bmatrix} \tag{C.10}$$

is negative definite. The signs of the principal minors of (C.10) alternate:

$$u''\bar{z} < 0 \tag{C.11}$$

$$u''\bar{z}\int_0^x v''(x-z)^2 f dz > 0 \tag{C.12}$$

$$u''\bar{z}\left[\left(\int_0^x v''(x-z)^2 f dz\right)\left(\int_0^x v'' f dz + v''(\hat{b})\int_x^\infty f dz\right) - \left(\int_0^x v''(x-z) f dz\right)^2\right] \;<\; 0 \tag{C.13}$$

and (after some manipulations)

$$\begin{aligned}\Delta_2 = u''\bar{z}&\left[\int_0^x v''(x-z-(p_a^x-p_a))^2 f dz + (p_a^x-p_a)^2\left(\int_0^x v'' f dz + v''(\hat{b})\int_x^\infty f dz\right)\right] \\ &+p_a^2\left[\left(\int_0^x v''(x-z)\right)^2 - \int_0^x v''(x-z)^2 f dz\left(\int_0^x v'' f dz + v''(\hat{b})\int_x^\infty f dz\right)\right] < 0\end{aligned} \tag{C.14}$$

To prove (C.13), rewrite the term in square brackets,

$$\left(\int_0^x v''(x-z) f dz\right)\left(\int_0^x v'' f dz + v''(\hat{b})\int_x^\infty f dz\right)\int_0^x \varphi(z)(x-z) f dz \tag{C.15}$$

where

$$\varphi(z) = \frac{v''(x-z)}{\int\limits_0^x v''(x-z)f dz} - \frac{v''}{\int\limits_0^x v'' f dz + v''(\hat{b})\int\limits_x^\infty f dz} \tag{C.16}$$

Note that $\varphi(0) > 0$, because the first term is > 1 and the second < 1, while $\varphi(x) < 0$. Since $\varphi(z)$ changes sign once over $[0, x]$, say at $\tilde{z}$, it follows that

$$\int\limits_0^x \varphi(z)(x-z)f dz > (x-\tilde{z})\int\limits_0^x \left[\frac{v''(x-z)f}{\int\limits_0^x v''(x-z)f dz} - \frac{v''f}{\int\limits_0^x v''f dz + v''(\hat{b})\int\limits_x^\infty f dz}\right] dz > 0.$$

This proves that (C.15) is positive, it also proves, by (C.14), that $\Delta_2 < 0$.

Using the first-order conditions, one can calculate

$$\begin{aligned}\Delta_2 \frac{d\hat{b}}{d\alpha} \;=\; & p_a \left[p_a \left(\int\limits_0^x v' \frac{\partial f}{\partial \alpha} dz + v'(\hat{b}) \int\limits_x^\infty \frac{\partial f}{\partial \alpha} dz \right) \right] \left[\int\limits_0^x v''(x-z)^2 f dz - (p_a^x - p_a) \int\limits_0^x v''(x-z) \right] - \\ & - \left[\int\limits_0^x v''(x-z) \frac{\partial f}{\partial \alpha} dz - (p_a^x - p_a) \left(\int\limits_0^x v' \frac{\partial f}{\partial \alpha} dz + v'(\hat{b}) \int\limits_x^\infty \frac{\partial f}{\partial \alpha} dz \right) \right] \cdot \\ & \cdot \left[(p_a^x - p_a) u'' \bar{z} + p_a^2 \int\limits_0^x v''(x-z) f dz \right] \end{aligned} \tag{C.17}$$

In (C.17), the first term in square brackets is positive, the third and fourth are negative. We want to show that the second term is negative. Rewrite it, using (20),

$$(p_a^x - p_a)\int\limits_0^x v''(x-z)f dz \int\limits_0^x \varphi(z)(x-z)dz \tag{C.18}$$

where

$$\varphi(z) = \frac{v''(x-z)f}{\int\limits_0^x v''(x-z)f dz} - \frac{v'f}{\int\limits_0^x v'f dz + v'(\hat{b})\int\limits_x^\infty f dz}, \quad 0 \le z \le x \tag{C.19}$$

It is seen that $\varphi(0) > 0$, $\varphi(x) < 0$ and $\varphi(z)$ changes sign once over $[0, x]$, say at $\tilde{z}$. It follows that

$$\int\limits_0^x \varphi(z)(x-z)dz > (x-\tilde{z})\int\limits_0^x \varphi(z)dz = 1 - \frac{\int\limits_0^x v'f dz}{\int\limits_0^x v'f dz + v'(\hat{b})\int\limits_x^\infty f dz} > 0 \tag{C.20}$$

Since $\Delta_2 < 0$, it follows that $\dfrac{d\hat{b}}{d\alpha} > 0$.

From the budget constraint (18) it follows that $p_a \dfrac{d\hat{a}}{d\alpha} + p_a^x \dfrac{d\hat{a}_x}{d\alpha} < 0$. Sufficient conditions for $\dfrac{d\hat{a}}{d\alpha}$ and $\dfrac{d\hat{a}_x}{d\alpha}$ each to be negative can be formulated. They concern the sign of the covariance between the changes in longevity, $\dfrac{df}{d\alpha}$, and the marginal utility, $v'(x-z)$, at different ages. We skip these conditions.

III. $\underline{\hat{a} = 0,\ \hat{a}_x > 0}$

The equations that determine $\hat{a}_x(p_a^x;\alpha)$ and $\hat{b}(p_a^x;\alpha)$ are now

$$u'(\hat{a}_x)\bar{z}(\alpha) + \int_0^x v'(\hat{b} + (x-z)\hat{a}_x)(x-z)f(z,\alpha)dz - \lambda p_a^x = 0 \tag{C.21}$$

and

$$-p_a^x \hat{a}_x - \hat{b} + W = 0 \tag{C.22}$$

where

$$\int_0^x v'(\hat{b} + (x-z)\hat{a}_x)f(z,\alpha)dz + v'(\hat{b})\int_0^x f(z,\alpha)dz - \lambda = 0 \tag{C.23}$$

The individual does not purchase regular annuities when

$$u'(\hat{a}_x)\bar{z}(\alpha) - \lambda p_a \leq 0 \tag{C.24}$$

The marginal utility of X-annuities decreases as their quantity increases but so does the marginal utility of income, λ. A second-order condition for (C.21) to be a maximum is that the former decreases faster:

$$u''(\hat{a}_x)\bar{z} + \int_0^x v''(\hat{b}+(x-z)\hat{a}_x)(x-z)^2 f(z,\alpha)dz - p_a^x \int_0^x v''(\hat{b}+(x-z)\hat{a}_x)(x-z)f(z,\alpha)dz < 0 \tag{C.25}$$

The other second-order condition

$$\Delta_3 = u''(\hat{a}_x)\bar{z} + \int_0^x v''(\hat{b}+(x-z)\hat{a}_x)(x-z-p_a^x)^2 f(z,\alpha)dz + (p_a^x)^2 v'(\hat{b})\int_x^\infty f(z,\alpha)dz < 0 \tag{C.26}$$

is seen to be satisfied.

It is assumed that a decrease in life expectancy decreases the marginal utility of lifetime consumption plus the marginal utility of bequests more than the increase in the marginal utility of income.

From (C.21) - (C.23),

$$\Delta_3 \frac{d\hat{a}_x}{d\alpha} = u'(\hat{a}_x)\bar{z}'(\alpha) + \int_0^x v'(\hat{b} + (x-z)\hat{a}_x)(x-z)\frac{df(z,\alpha)}{d\alpha}dz - -p_a^x \left(\int_0^x v'(\hat{b} + (x-z)\hat{a}_x)\frac{df(z,\alpha)}{d\alpha}dz + v'(\hat{b})\int_x^\infty \frac{\partial f(z,\alpha)}{\partial\alpha}dz \right) \tag{C.27}$$

Assume that the negative effect of a decrease in life expectancy on the consumption value of X-annuities, $u'(\hat{a}_x)\bar{z}'(\alpha)$, dominates the increased value of bequests and, consequently, the rise in the marginal utility of income. This means that, in (C.27), the term in square brackets is negative and hence, $\frac{d\hat{a}_x}{d\alpha} < 0$ and $\frac{d\hat{b}}{d\alpha} > 0$.

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