Economic Growth and Patent Policy: Quantifying the Effects of Patent Length on R&D and Consumption

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Abstract

Is the patent length an effective policy instrument in stimulating R&D? This paper develops a generalized variety-expanding growth model and then calibrates the model to the aggregate data of the US economy to analyze the effects of extending the patent length. The numerical exercise suggests that at the empirical range of patent-value depreciation rates, extending the patent length beyond 20 years leads to only a very small increase in R&D despite R&D underinvestment in the market economy. On the other hand, shortening the patent length can lead to a significant reduction in R&D and consumption. This paper also makes use of the dynamic general-equilibrium framework to examine the fraction of total factor productivity (TFP) growth that is driven by R&D, and the calibration exercise suggests that about 35% to 45% of the long-run TFP growth in the US is driven by R&D. Finally, this paper identifies and analytically derives a dynamic distortion of the patent length on saving and investment in physical capital that has been neglected by previous studies, which consequently underestimate the distortionary effects of patent protection.

Keywords**:** endogenous growth, intellectual property rights, patent length, R&D

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1. Introduction

Is the patent length an effective policy instrument in stimulating R&D? The statutory term of patent in the United States (US) was 17 years from 1861 to 1995 and then extended to 20 years as a result of the TRIPS agreement. Suppose that there is underinvestment in R&D in the market economy as suggested by Jones and Williams (1998) and (2000), why hasn't the term of patent been lengthened to stimulate R&D?[1] Especially, Kwan and Lai (2003) show in a variety-expanding growth model that extending the effective lifetime of patents would lead to a substantial increase in R&D and consequently welfare gains.

This paper attempts to provide an answer to the above questions by developing a generalized variety-expanding growth model and calibrating the model to the aggregate data of the US economy to analyze the effects of extending the patent length. It turns out that whether an extension in the patent length would lead to a significant increase in R&D depends crucially on whether the model is calibrated properly to match the empirical patent-value depreciation rate. The numerical exercise suggests that at the empirical range of patent-value depreciation rates, extending the patent length beyond 20 years leads to only a very small increase in R&D despite R&D underinvestment in the market economy. On the other hand, shortening the patent length can lead to a significant reduction in R&D and consumption. In other words, the patent length loses its effectiveness in stimulating R&D at around 20 years. This paper also makes use of the dynamic general-equilibrium (DGE) framework to examine the fraction of total factor productivity (TFP) growth that is driven by R&D. The calibration exercise suggests that about 35% to 45% of the long-run TFP growth in the US is driven by R&D. Finally, this paper identifies and analytically derives a dynamic distortion of the patent length on saving and investment in physical capital that has been neglected by previous studies, which consequently underestimate the distortionary effects of patent protection. The dynamic distortion arises because when the patent length increases, the fraction of monopolistic industries goes up. The resulting higher aggregate markup causes the wedge between the

[1] The WTO's Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS), initiated in the 1986-94 Uruguay Round, extends the statutory term of patent in the US from 17 years (counting from the issue date when a patent is granted) to 20 years (counting from the earliest claimed filing date) to conform with the international standard. Because of the difference in the starting date, the effective extension of patent length was much shorter than 3 years.

marginal product of capital and the rental price to increase. As a result, the market equilibrium rate of investment in physical capital decreases and deviates further from the social optimum.

This paper relates to a number of studies on R&D underinvestment. In a companion paper, Chu (2007) numerically evaluates the effect of blocking patents on R&D in a generalized quality-ladder growth model with overlapping intellectual property rights, and he finds that eliminating blocking patents can be very effective in stimulating R&D. In performing a similar quantitative analysis, Chu (2007) and the current paper together provide a quantitative assessment on the relative effectiveness of extending the patent length and eliminating blocking patents in solving the R&D-underinvestment problem suggested by Jones and Williams (1998) and (2000). The crucial difference arises because extending the patent length affects future monopolistic profits while eliminating blocking patents affects current monopolistic profits. Furthermore, the calibration exercise takes into consideration Comin's (2004) critique. Comin (2004) argues two points: (a) Jones and Williams' (2000) finding of R&D underinvestment is based on the assumption in their calibration that the long-run TFP growth is solely driven by R&D; and (b) the level of R&D spending in the data may be optimal if R&D only drives a small fraction of the long-run TFP growth. The current paper contributes to this debate by bringing in an additional moment that is the patent-value depreciation rate in order to calibrate the fraction of long-run TFP growth driven by R&D, and the details will be discussed in Section 2.9.

This paper also complements the theoretical studies in the patent-design literature that is mostly based on a partial-equilibrium setting by providing a quantitative DGE analysis on patent policy. The seminal work on patent length is Nordhaus (1969), and he concludes that the optimal patent length should balance between the static distortionary effects of markup pricing and the gains from enhanced innovations. Gilbert and Shapiro (1990) argue that given the choices of patent length and patent breadth as policy instruments, the socially optimal policy combination is an infinite patent length and a minimum degree of patent breadth.[2] In a DGE setting, Judd (1985) also concludes that the optimal patent length is

[2] Some other studies on optimal patent design include Tandon (1982), Klemperer (1990), Green and Scotchmer (1995), O'Donoghue (1998), O'Donoghue, Schotchmer and Thisse (1998), Hunt (1999) and Schotchmer (2004).

infinite.[3] On the other hand, Futagami and Iwaisako (2003) show that the optimal patent length may be finite when there is no underinvestment in R&D. However, the above studies do not feature endogenous capital accumulation so that the dynamic distortion on capital accumulation is absent.

In terms of quantitative analysis, the most closely related work is Kwan and Lai (2003), and they find substantial welfare gains from extending the effective lifetime of patent. There is an important reason for the contradicting results between Kwan and Lai (2003) and the current paper. By using the same final-goods production function as in Romer (1990), Kwan and Lai (2003) necessarily restrict the size of the markup to the inverse of the capital share. This setup restricts the balanced-growth rate of monopolistic profits captured by each patent to equal the population growth rate that is nonnegative. Relaxing this parameter restriction indicates that at the empirical range of patent-value depreciation rates estimated by previous studies, the implied growth rates of the number of varieties (that are no longer the same as the TFP growth rate) are very high; consequently, the share of monopolistic profits captured by each patent declines sharply overtime rendering patent extension ineffective in stimulating R&D. In other words, the potentially rapid decline in the market share captured by each patent due to the introduction of new varieties enables the model to feature the empirically observed depreciation in the market value of patents. As a result, extending the patent length has limited effects on R&D.

Before closing the introduction, I briefly survey the empirical literature on estimating the market value of patents using patent renewal data to gather some information about the magnitude of the rate at which a patent's value declines overtime.[4] The pioneering study that estimates a deterministic patent renewal model is Pakes and Schankerman (1984), and they find that the market value of patents depreciates at a rate of 25% per year with a 95 percent confidence interval of 18%-36%. Schankerman and Pakes (1986) provide more recent data on a number of European countries, in which about half of all patents are not renewed within 10 years and only 10% of them are renewed until the end of the statutory

[3] Some other DGE studies on patent policy include Li (2001), Goh and Olivier (2002), O'Donoghue and Zweimuller (2004) and Grossman and Lai (2004). However, all of these studies neglect the dynamic distortion on capital accumulation and are qualitatively oriented. Chu (2007) provides a more detailed discussion on these studies.

[4] For a more detailed survey on early studies, see e.g. Griliches (1990).

term.[5] Pakes (1986) develops a stochastic renewal model to capture the effect of learning about a patent's value in the initial years, and he also finds high rates of depreciation ranging from 11.4% in Germany to 19.0% in the United Kingdom. Lanjouw (1998) uses a more general stochastic renewal model to estimate the value of patents in a number of industries. In addition to the rates of depreciation, her model also estimates the annual probability that a patent becomes obsolete (i.e. complete depreciation), and it ranges from 7% for computer patents to 12% for engine patents. Although the empirical estimates tend to vary across studies, across countries and across industries, there seems to be suggestive evidence that the rates of depreciation and obsolescence are quite high for patents.

The rest of this paper is organized as follows. Section 2 describes the model and derives the dynamic distortionary effect of the patent length. Section 3 calibrates the model to the data, and the final section concludes with some important caveats. All proofs are contained in Appendix I.

2. The Model

The variety-expanding model is a generalized version of Romer (1990). The basic framework is modified to introduce a finite patent length denoted by T for each invented variety of intermediate goods. The final goods are produced with labor and a composite of intermediate goods. The intermediate-goods industries are monopolistic for the producers owning a valid patent and become competitive once the patent expires. The relative price between the monopolistic and competitive goods leads to the usual *static* distortionary effect that reduces the output of final goods. The markup in the monopolistic industries drives a wedge between the marginal product of capital and the rental price; consequently, it leads to an additional *dynamic* distortionary effect that causes the market equilibrium rate of investment in physical capital to deviate from the social optimum. To prevent the model from overestimating the social benefits of R&D and hence the extent of R&D underinvestment, the long-run TFP growth is assumed to be driven by R&D as well as an exogenous process as in Comin (2004). In addition, this class of first-generation R&D-

[5] All the studies cited here are based on European data. In the US, patent maintenance fees were not initiated until 1982, and the fees are due 3.5 years ($900), 7.5 years ($2300) and 11.5 years ($3800) after a patent is granted, rather than annually as in some European countries.

driven endogenous growth models exhibits scale effects and is inconsistent with the empirical evidence in Jones (1995a).[6] In the present model, scale effects are eliminated as in Jones (1995b). After eliminating scale effects, the resulting model becomes a semi-endogenous growth model, in which the balanced-growth rate is proportional to the exogenous population growth rate.

The various components of the model are presented in Sections 2.1–2.8, and the competitive equilibrium is defined in Section 2.9. Section 2.10 derives the socially optimal allocations, and Section 2.11 derives the dynamic distortionary effect of the patent length on capital accumulation. The analysis focuses on the balanced-growth path.

2.1. Representative Household

There is a representative household whose lifetime utility is given by

$$U = \int_0^{\infty} e^{-(\rho - n)t} \frac{c_t^{1-\sigma}}{1-\sigma} dt , \tag{1}$$

where $\sigma \geq 1$ is the inverse of the elasticity of intertemporal substitution and ρ is the exogenous subjective discount rate. The household has $L_t = e^{n.t}$ members at time *t*, and $n > 0$ is the constant exogenous population growth rate. ρ is assumed to be greater than n to ensure that utility is bounded. c_t is the per capita consumption of final goods (the numeraire). The household maximizes utility subject to a sequence of budget constraints given by

$$\dot{a}_t = (r_t - n)a_t + w_t - c_t \ . \tag{2}$$

Each member of the household inelastically supplies one unit of homogenous labor in each period to earn a wage income w_t. a_t is the amount of financial assets, which consist of physical capital and patents, owned by each member of the household, and r_t is the real rate of return on these financial assets. From the household's intertemporal optimization, the familiar Euler equation is

[6] See Jones (1999) for an excellent theoretical analysis on scale effects.

(3) $$\dot{c}_t / c_t = (r_t - \rho) / \sigma .$$

Along the balanced growth path, c_t increases at a constant rate g_c. Therefore, the equilibrium real interest rate along the balanced-growth path is

(4) $$r = \rho + g_c \sigma .$$

2.2. Final Goods

The sector producing the final goods is characterized by perfect competition, and the producers take both the output price and input prices as given. In particular, the final-goods production function is

(5) $$Y_t = Z_t^{1-\alpha} L_{y,t}^{1-\alpha} \left(\int_0^{V_t} X_t^{\alpha\eta}(j) dj \right)^{1/\eta}$$

for $\eta \in (0, 1/\alpha)$. Y_t is the amount of final goods produced. $Z_t = Z_0 \exp(g_Z t)$ represents an exogenous process of productivity improvement that is freely available to all final-goods producers. $L_{y,t}$ is the number of production workers. $X_t(j)$ is the amount of intermediate goods of variety $j \in [0, V_t]$, in which V_t is the number of varieties that has been invented as of time *t*. The production function in (5) nests Romer (1990) as a special case with $\eta = 1$ and $Z_t = 1$ for all *t*. For $\eta = 1$, the monopoly markup μ is restricted to be $1/\alpha$ (i.e. roughly the inverse of the capital share); therefore, Jones and Williams (2000) propose a more realistic specification that allows η to differ from one so that the markup is given by $\mu = 1/(\alpha\eta)$ in order to relax the parameter restriction between the markup and the capital share.

The final-goods producers take $Z_t^{1-\alpha}$ as given. The current paper includes this exogenous TFP process for two reasons: (a) to avoids the mistake in assuming that the long-run TFP growth in the data is solely driven by R&D; and (b) to relax the restriction between the patent-value depreciation rate and the long-run TFP growth rate denoted by g_{TFP}. This restriction will be discussed in details in Section 2.9. In short, by setting $g_z = 0$ (i.e. by assuming that the long-run TFP growth is solely driven by R&D), the balanced-growth rate of the number of varieties g_V is pinned down by the TFP growth rate g_{TFP}, the capital share α, and the markup $1/(\alpha\eta)$. Once g_V is determined, the patent-value depreciation rate is also uniquely pinned down, and the calibration results suggest that this implied patent-value depreciation rate differs substantially from the previous empirical estimates based on patent renewal data. Therefore, the current paper adopts the specification in (5) that allows g_z to differ from zero in order to bring in the empirical estimates for the patent-value depreciation rate and perform a more realistic calibration.

Profit maximization yields the first-order conditions for the wage rate and the price of intermediate-goods $P_t(j)$ for $j \in [0, V_t]$ given by

(6) $$w_t = (1-\alpha)Y_t / L_{y,t},$$

(7) $$P_t(j) = \alpha Z_t^{1-\alpha} L_{y,t}^{1-\alpha} \left(\int_0^{V_t} X_t^{\alpha\eta}(j)dj \right)^{(1-\eta)/\eta} X_t^{\alpha\eta-1}(j) \ .$$

2.3. Intermediate Goods

There is a continuum of industries, indexed by $j \in [0, V_t]$, producing the differentiated intermediate goods $X_t(j)$. Once a variety has been invented, the production function in industry j is

(8) $$X_t(j) = K_{y,t}(j) .$$

$K_{y,t}(j)$ is the amount of capital employed by industry j. The profit function facing the producer(s) of variety j is

(9) $$\pi_t(j) = P_t(j) X_t(j) - R_t K_{y,t}(j) .$$

R_t is the rental price of capital. Denote the steady-state fraction of monopolistic industries by ω, which is endogenously determined by the patent length *T*. Without loss of generality, the industries are ordered such that industries $j \in [0, \omega V_t]$ are protected by patents and industries $j' \in (\omega V_t, V_t]$ are not protected by patents. Then, the first-order conditions are

(10) $$P_t(j) = R_t / \alpha\eta$$

for $j \in [0, \omega V_t]$, and

(11) $$P_t(j') = R_t$$

for $j' \in (\omega V_t, V_t]$.

2.4. Aggregate Production Function and Static Distortion

The total amount of capital employed by the intermediate-goods sector at time t is

(12) $$K_{y,t} = \int_0^{V_t} X_t(j)dj = V_t(\omega X_t(j) + (1-\omega)X_t(j')).$$

Lemma 1: *The aggregate production function for the final goods is*

(13) $$Y_t = \tilde{\omega} A_t^{1-\alpha} Z_t^{1-\alpha} L_{y,t}^{1-\alpha} K_{y,t}^{\alpha},$$

where A_t *is the level of R&D-driven TFP and is defined as*

(14) $$A_t^{1-\alpha} \equiv V_t^{(1-\alpha\eta)/\eta},$$

and $\tilde{\omega}$ *is defined as*

(15) $$\tilde{\omega} \equiv \frac{(\omega(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)} + 1 - \omega)^{1/\eta}}{(\omega(\alpha\eta)^{1/(1-\alpha\eta)} + 1 - \omega)^{\alpha}}.$$

$\tilde{\omega}$ *is strictly less than one for* $\omega \in (0,1)$ *and equals one for* $\omega \in \{0,1\}$. *In addition,* $\partial\tilde{\omega}/\partial\omega < 0$ *when*

$$\omega < \overline{\omega} \equiv \frac{1}{1-\alpha\eta}\left(\frac{1}{1-(\alpha\eta)^{1/(1-\alpha\eta)}} - \frac{\alpha\eta}{1-(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)}}\right) < 1.$$

Proof: See Appendix I.

$\tilde{\omega}$ captures the usual static distortionary effect of patent protection in creating a monopolistic markup in the patent-protected industries. In other words, the markup in the monopolistic industries distorts production towards the competitive industries and thus reduces the total output of the final goods. Increasing the fraction of monopolistic industries worsens this static distortionary effect when $\omega < \overline{\omega}$. This static distortionary effect is not monotonic in the patent length because at an infinite patent length, all industries are monopolistic and the relative-price distortion disappears.

2.5. National Income Account Identities

The market-clearing condition for the final goods is

(16) $$Y_t = C_t + I_t .$$

$C_t = L_t c_t$ is aggregate consumption, and I_t is investment in physical capital. The correct value of gross domestic product (GDP) should include the amount of investment in R&D such that

(17) $$GDP_t = Y_t + w_t L_{r,t} + R_t K_{r,t} .$$ [7]

$L_{r,t}$ and $K_{r,t}$ are respectively the number of workers and the amount of capital in the R&D sector that invents new varieties. The amount of monopolistic profits, the factor payments for production workers and capital in the intermediate-goods sector are given by

(18) $$w_t L_{y,t} = (1-\alpha) Y_t ,$$

(19) $$R_t K_{y,t} = \alpha Y_t \hat{\omega} ,$$

(20) $$\pi_t V_t \omega = \alpha Y_t (1-\hat{\omega}) ,$$

where $\hat{\omega} \in [\alpha\eta, 1]$ is determined by the fraction of monopolistic industries ω and is defined as

(21) $$\hat{\omega} \equiv \frac{\omega(\alpha\eta)^{1/(1-\alpha\eta)} + 1 - \omega}{\omega(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)} + 1 - \omega} .$$

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[7] In the national income account, private spending in R&D is treated as an expenditure on intermediate goods. Therefore, the values of capital investment and GDP in the data are I_t and Y_t respectively. The Bureau of Economic Analysis and the National Science Foundation's R&D satellite account provides preliminary estimates on the effects of including R&D as an intangible asset in the national income accounts.

A rise in the fraction of monopolistic industries ω leads to a decrease in $\hat{\omega}$ and consequently, increases the wedge between the marginal product of capital and the rental price. As will be shown below, this decrease in $\hat{\omega}$ also leads to a lower rate of investment in physical capital. Therefore, $\hat{\omega}$ captures the dynamic distortionary effect of the patent length on capital accumulation.

2.6. Capital Accumulation

The market-clearing condition for physical capital is

(22) $$K_t = K_{y,t} + K_{r,t}.$$

K_t is the total amount of capital available in the economy at time *t*. The law of motion for capital is

(23) $$\dot{K}_t = I_t - K_t \delta$$

δ is the rate of depreciation. Denote the balanced-growth rate of capital by g_K, the endogenous steady-state investment rate is

(24) $$i = (g_K + \delta) K_t / Y_t$$

for all *t*. The no-arbitrage condition $r_t = R_t - \delta$ implies that the steady-state capital-output ratio is

(25) $$\frac{K_t}{Y_t} = \frac{\alpha \hat{\omega}}{(1 - s_K)(\rho + g_c \sigma + \delta)}.$$

s_K is the endogenous steady-state R&D share of capital. Substituting (25) into (24) yields

(26) $$i = \frac{\alpha \hat{\omega}}{1 - s_K} \left(\frac{g_K + \delta}{\rho + g_c \sigma + \delta} \right).$$

In the Romer model, (skilled) labor is the only input for R&D (i.e. $s_K = 0$); therefore, the distortionary effect of patent length on the rate of investment in capital is unambiguously negative (i.e. $\partial i / \partial T < 0$). In the current model with $s_K \geq 0$, there is an opposing positive effect operating through s_K. Intuitively, an increase in the patent length raises the private return on R&D and hence the share of capital employed in the R&D sector. Proposition 1 in Section 2.11 shows that the negative effect still dominates.

2.7. R&D

The no-arbitrage value of a patent $P_{r,t}$ for a new variety invented at time *t* is the expected present value of the stream of monopolistic profits earned by an R&D entrepreneur until the patent expires given by

$$P_{r,t} = \int_{t}^{t+T} e^{-r(\tau - t)} \pi_\tau d\tau = \Omega(T)\pi_t . \tag{27}$$

$\Omega(T) \equiv \dfrac{1 - e^{-(\rho + g_c \sigma - g_\pi)T}}{\rho + g_c \sigma - g_\pi}$ is the present-value discount factor after substituting in (4) for the steady-state real interest rate and defining g_π as the balanced-growth rate of monopolistic profits. When $g_\pi = 0$, the market value of a patent depreciates approximately at an annual rate of 1/*T*.[8] For example, when the patent length is 20 years and $g_\pi = 0$, the market value of a patent depreciates at roughly 5% per year. However, the empirical estimates from the patent renewal data suggest that a reasonable range for the patent-value depreciation rates is between 15% and 25%; therefore, $g_\pi \in [-0.2, -0.1]$. In other words, an invention loses about 10% to 20% of its market share per year on average. The marginal effect of patent length on $\Omega(T)$ given by $\Omega'(T) = e^{-(\rho + g_c \sigma - g_\pi)T}$ is positive, and this marginal effect depends positively on the profit growth rate g_π. Therefore, a highly negative profit growth rate (i.e. a high patent-value depreciation rate) would render patent extension ineffective in raising the market value of patents.

The instantaneous probability $\lambda_t(k)$ of an innovation success for R&D entrepreneur $k \in [0,1]$ is

$$\lambda_t(k) = \overline{\varphi}_t L_{r,t}^{1-\beta}(k) K_{r,t}^{\beta}(k) , \tag{28}$$

where $\overline{\varphi}_t$ is the productivity parameter of R&D inputs that the entrepreneurs take as given. This specification nests the "knowledge-driven" specification in Romer (1990) as a special case with $\beta = 0$ and the "lab equipment" specification in Rivera-Batiz and Romer (1991) and Jones and Williams (2000)

[8] This approximation is exact when the interest rate is zero.

as a special case with $\beta = \alpha$. The R&D sector is characterized by constant returns to scale and perfect competition. The amount of expected profit of investing in R&D is

$$\pi_{r,t}(k) = P_{r,t}\lambda_t(k) - w_t L_{r,t}(k) - R_t K_{r,t}(k). \tag{29}$$

The first-order conditions for R&D entrepreneur *k* are

$$(1-\beta)P_{r,t}\overline{\varphi}_t(K_{r,t}(k)/L_{r,t}(k))^{\beta} = w_t, \tag{30}$$

$$\beta P_{r,t}\overline{\varphi}_t(K_{r,t}(k)/L_{r,t}(k))^{\beta-1} = R_t. \tag{31}$$

(30) and (31) together with (18) and (19) determine the resource allocation between production and R&D.

2.8. Law of Motion for the Number of Varieties

To eliminate scale effects and to ensure the existence of a balanced-growth path in the presence of population growth, I follow Jones and Williams (2000) to assume that the R&D productivity parameter $\overline{\varphi}_t$ is a function of V_t given by

$$\overline{\varphi}_t = \varphi V_t^{\phi}(K_{r,t}^{\beta}L_{r,t}^{1-\beta})^{\gamma-1}. \tag{32}$$

$\phi \in (-\infty,1)$ captures the externality in intertemporal knowledge spillovers,[9] and $\gamma \in (0,1]$ captures the negative externality in intratemporal duplication or the so-called "stepping-on-toes" effects. The law of motion for the number of varieties is

$$\dot{V}_t = \int_0^1 \lambda_t(k)dk = \overline{\varphi}_t K_{r,t}^{\beta}L_{r,t}^{1-\beta} = \varphi V_t^{\phi}(K_{r,t}^{\beta}L_{r,t}^{1-\beta})^{\gamma}. \tag{33}$$

Along the balanced-growth path, $K_{r,t} = \int_0^1 K_{r,t}(k)dk$ increases at g_K, and $L_{r,t} = \int_0^1 L_{r,t}(k)dk$ increases at the exogenous population growth rate n. Therefore, the balanced-growth rate of V_t denoted by g_V is

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[9] As discussed in Jones (1995b), $\phi \in (0,1)$ corresponds to the "standing-on-shoulder" effect, in which R&D productivity increases as V_t increases, and $\phi \in (-\infty,0)$ refers to the "fishing-out" effect, in which R&D productivity decreases as V_t increases.

$$(34) \qquad g_V \equiv \frac{\dot{V}_t}{V_t} = \varphi \frac{(K_{r,t}^{\beta} L_{r,t}^{1-\beta})^{\gamma}}{V_t^{1-\phi}} = \left(\frac{\gamma\beta}{1-\phi}\right) g_K + \left(\frac{\gamma(1-\beta)}{1-\phi}\right) n .$$

Finally, the steady-state fraction of patented varieties ω is given by

$$(35) \qquad \omega \equiv \frac{\tilde{V}_t}{V_t} = \frac{\dot{V}_t - \dot{V}_{t-T}}{\dot{V}_t} = 1 - e^{-g_V T} ,$$

where $\tilde{V}_t$ is the number of patented varieties at time *t* and $\dot{V}_t - \dot{V}_{t-T}$ is the net increase in the number of patented varieties at time *t*.

2.9. Decentralized Equilibrium and Balanced-Growth Path

The analysis starts at $t = 0$ when the economy has reached the balanced-growth path corresponding to the patent length *T*. The equilibrium is a sequence of prices $\{w_t, r_t, R_t, P_t(j), P_{r,t}\}_{t=0}^{\infty}$ and a sequence of allocations $\{c_t, a_t, X_t(j), Y_t, I_t, K_{y,t}, L_{y,t}, K_{r,t}, L_{r,t}, K_t, L_t\}_{t=0}^{\infty}$ that are consistent with initial conditions $\{K_0, L_0, Z_0, V_0, A_0, \overline{\varphi}_0\}$ and their subsequent laws of motions. Also, in each period,

(a) the representative household chooses $\{c_t, a_t\}$ to maximize utility taking $\{w_t, r_t\}$ as given;

(b) the competitive firms in the final-goods sector choose $\{X_t(j), L_{y,t}\}$ to maximize profits according to the production function taking $\{P_t(j), w_t\}$ as given;

(c) the monopolistic firms $j \in [0, \omega V_t]$ in the intermediate-goods sector choose $\{P_t(j), K_{y,t}(j)\}$ to maximize profits according to the demand curve from the final-goods sector and the production function taking $\{R_t\}$ as given;

(d) the competitive firms $j' \in (\omega, 1]$ in the intermediate-goods sector choose $\{K_{y,t}(j')\}$ to maximize profits according to the production function taking $\{P_t(j'), R_t\}$ as given;

(e) the entrepreneurs $k \in [0,1]$ in the R&D sector choose $\{L_{r,t}(k), K_{r,t}(k)\}$ to maximize profits according to the production function taking $\{w_t, R_t, P_{r,t}, \overline{\varphi}_t\}$ as given;

(f) the market for the final goods clears such that $Y_t = C_t + I_t$;

(g) the full employment of capital such that $K_t = K_{y,t} + K_{r,t}$; and

(h) the full employment of labors such that $L_t = L_{y,t} + L_{r,t}$.

Equating the first-order conditions (18) and (30) and imposing the balanced-growth condition

(36) $$V_t = \frac{\overline{\varphi}_t K_{r,t}^{\beta} L_{r,t}^{1-\beta}}{g_V}$$

yield the steady-state shares of labor inputs given by

(37) $$\frac{s_L}{1-s_L} = \frac{1-\beta}{1-\alpha}\left(\frac{(1-e^{-(\rho+g_c\sigma-g_\pi)T})g_V}{\rho+g_c\sigma-g_\pi}\right)\frac{\alpha(1-\hat{\omega})}{\omega}.$$

Similarly, equating (19) and (31) and imposing (36) yield the steady-state shares of capital inputs as

(38) $$\frac{s_K}{1-s_K} = \frac{\beta}{\alpha}\left(\frac{(1-e^{-(\rho+g_c\sigma-g_\pi)T})g_V}{\rho+g_c\sigma-g_\pi}\right)\frac{\alpha(1-\hat{\omega})}{\omega}.$$

The balanced-growth rates of various variables are given as follows. From the aggregate production function in (13) and the steady-state investment rate in (26),

(39) $$g_Y = g_K = g_I = g_C = g_A + g_Z + n.$$

Therefore, $g_c = g_A + g_Z$. From the definition of R&D-driven TFP in (14),

(40) $$g_A = \frac{1}{1-\alpha}\left(\frac{1-\alpha\eta}{\eta}\right)g_V.$$

Finally, from (20), the balanced-growth rate of monopolistic profits is

(41) $$g_\pi = g_Y - g_V = g_A + g_Z + n - g_V.$$

Note that g_π is restricted to equal $g_Z + n > 0$ when $\eta = 1$ because $g_A = g_V$. However, when η is large (i.e. a low markup), g_V becomes large relative to g_A. Therefore, holding g_A constant, an increase in η leads to a decrease in g_π. Eventually, g_π becomes negative for a low enough markup.

Denote the fraction of the long-run TFP driven by R&D by ξ such that $g_A = \xi g_{TFP} /(1-\alpha)$ and $g_Z = (1-\xi) g_{TFP} /(1-\alpha)$. If $\xi = 1$, then the value of g_π is pinned down by g_A, α, η and n according to (40) and (41). The resulting implied value of g_π could be seriously biased due to the misspecification of the model. Therefore, I allow ξ to differ from one and calibrate this parameter from the data. Firstly, I make use of the previous empirical estimates for the patent-value depreciation rate to determines g_π. Note that once g_π is given, (41) pins down a unique value for g_V for given values of g_{TFP}, α and n. Then, given g_V, (40) pins down a unique value for g_A for given values of α and η.

Finally, from (34) and using (39) and (40), the balanced-growth condition that determines the externality parameters γ and ϕ is given by

(42) $$g_V = \left(\frac{1-\phi}{\gamma} - \frac{\beta}{1-\alpha} \left(\frac{1-\alpha\eta}{\eta} \right) \right)^{-1} (\beta g_Z + n).$$

2.10. Socially Optimal Allocations

To derive the socially optimal rate of capital investment i^* and R&D shares of labor s_L^* and capital s_K^*, the social planner maximizes

(43) $$U = \int_0^\infty e^{-(\rho-n)t} \frac{((1-i_t)Y_t / L_t)^{1-\sigma}}{1-\sigma} dt$$

subject to: (a) the aggregate production function expressed in terms of $s_{L,t}$ and $s_{K,t}$ given by

(44) $$Y_t = V_t^{(1-\alpha\eta)/\eta} Z_t^{1-\alpha} (1-s_{K,t})^\alpha (1-s_{L,t})^{1-\alpha} K_t^\alpha L_t^{1-\alpha};$$

(b) the law of motion for capital expressed in terms of i_t given by

(45) $$\dot{K}_t = i_t Y_t - K_t \delta ;$$

and (c) the law of motion for the number of varieties expressed in terms of $s_{L,t}$ and $s_{K,t}$ given by

(46) $$\dot{V}_t = V_t^{\phi} (s_{K,t})^{\beta\gamma} (s_{L,t})^{(1-\beta)\gamma} K_t^{\beta\gamma} L_t^{(1-\beta)\gamma} \varphi .$$

After deriving the first-order conditions, the social planner solves for i^*, s_L^* and s_K^* on the balanced-growth path.

Lemma 2: *The socially optimal rate of capital investment* i^* *is*

(47) $$i^* = \left(\alpha + \beta \left(\frac{1-\alpha\eta}{\eta} \right) \frac{\gamma g_V}{\rho - n + (\sigma - 1) g_c + (1-\phi) g_V} \right) \frac{g_K + \delta}{\rho + g_c \sigma + \delta} ,$$

and the socially optimal R&D shares of labor s_L^* *and capital* s_K^* *are respectively*

(48) $$\begin{aligned} \frac{s_L^*}{1-s_L^*} &= \frac{1-\beta}{1-\alpha} \left(\frac{1-\alpha\eta}{\eta} \right) \frac{\gamma g_V}{\rho - n + (\sigma-1) g_c + (1-\phi) g_V} \\ \neq \frac{s_L}{1-s_L} &= \frac{1-\beta}{1-\alpha} \left(\frac{\alpha(1-\hat{\omega})}{\omega} \right) \frac{(1-e^{-(\rho-n+(\sigma-1)g_c+g_V)T}) g_V}{\rho - n + (\sigma-1) g_c + g_V} \end{aligned} ,$$

(49) $$\begin{aligned} \frac{s_K^*}{1-s_K^*} &= \frac{\beta}{\alpha} \left(\frac{1-\alpha\eta}{\eta} \right) \frac{\gamma g_V}{\rho - n + (\sigma-1) g_c + (1-\phi) g_V} \\ \neq \frac{s_K}{1-s_K} &= \frac{\beta}{\alpha} \left(\frac{\alpha(1-\hat{\omega})}{\omega} \right) \frac{(1-e^{-(\rho-n+(\sigma-1)g_c+g_V)T}) g_V}{\rho - n + (\sigma-1) g_c + g_V} \end{aligned} .$$

Proof: See Appendix I.

(48) and (49) indicate the various R&D externalities: (a) the negative externality in intratemporal duplication $\gamma \in (0,1]$; (b) the positive or negative externality in intertemporal knowledge spillovers $\phi \in (-\infty, 1)$; (c) the positive externality from the dynamic surplus-appropriability problem due to a finite patent length given by $(1-e^{-(\rho-n+(\sigma-1)g_c+g_V)T}) < 1$; and finally, (d) the positive externality from the static surplus-appropriability problem given by $(1-\alpha\eta)/\eta > \alpha(1-\hat{\omega})/\omega$ for all *T*. Given the existence of

positive and negative externalities, it requires a careful calibration that will be performed in Section 3 to determine whether the market economy over- or under-invests in R&D.

2.11. Dynamic Distortion

If the market economy underinvests in R&D, the government may want to increase the patent length to reduce the extent of this market failure. However, an increase in T would worsen the dynamic distortionary effect on capital accumulation. Therefore, the government needs to trade off the gains from the increase in R&D and the losses caused by the dynamic distortion and potentially the static distortion. Proposition 1 provides the condition under which the markup-pricing distortion moves the market equilibrium rate of capital investment i away from the social optimum i^*.

Proposition 1: *The decentralized equilibrium capital investment rate* i *is below the socially optimal investment rate* i^* *if either there is underinvestment in R&D or labor is the only factor input for R&D. In addition, an increase in the patent length always reduces the equilibrium capital investment rate* i.

Proof: See Appendix I.

The second part of the proposition is quite intuitive. When the patent length increases, the fraction of monopolistic industries rises. The resulting higher aggregate markup drives a bigger wedge between the marginal product of capital and the rental price. Therefore, the market equilibrium rate of investment in physical capital decreases. As for the first part of the proposition, the discrepancy between the market equilibrium rate of investment in physical capital and the social optimum arises because of: (a) the aggregate markup; and (b) the discrepancy between the market equilibrium R&D capital share s_K and the socially optimal R&D capital share s_K^*. Since the market equilibrium capital investment rate i is an increasing function of s_K, the underinvestment in R&D in the market equilibrium is sufficient for $i < i^*$. On the other hand, when there is overinvestment in R&D in the market equilibrium, whether i is below or above i^* depends on whether the effect of the aggregate markup or the effect of R&D overinvestment dominates. For the case in which labor is the only factor input for R&D, $s_K = 0$; therefore, only the effect of the aggregate markup is present.

3. Calibration

This section firstly calibrates the structural and externality parameters using long-run aggregate data of the US economy and then computes the changes in R&D and consumption from varying the patent length. After that, the dynamic distortionary effects are also examined.

3.1. Structural Parameters

The statutory patent length T in the US is 20 years, and the average annual labor-force growth rate n is 1.66%.[10] The annual discount rate ρ and the annual rate of depreciation δ for physical capital are set to conventional values of 0.04 and 0.08 respectively. β is set equal to α corresponding to the lab-equipment specification in Rivera-Batiz and Romer (1991) and Jones and Williams (2000).[11] Once the above parameters are determined, the model provides five steady-state conditions (summarized in (50)-(54) below) to match the following five moments in the data and to determine the remaining five structural parameters $\{\sigma, \alpha, \eta, g_V, \xi\}$. The ratio of private investment to GDP is 20.21%,[12] and the labor share of total income is set to a conventional value of 0.7. The ratio of private spending on R&D to GDP is 1.49%.[13], and the average annual TFP growth rate $g_{TFP} = (1-\alpha)(g_A + g_Z)$ is 1.02%.[14] As discussed

[10] This number is calculated using data between 1956 and 2006 from the Bureau of Labor Statistics.

[11] I have considered different plausible values for $\beta \in \{0, \alpha, 2\alpha, 3\alpha\}$ as a sensitivity analysis, and the results are robust to these parameter changes.

[12] This number is calculated using data between 1956 and 2006 from the Bureau of Economic Analysis, and GDP is net of government spending.

[13] This number is calculated using data between 1956 and 2004 from the Bureau of Economic Analysis and the National Science Foundation. R&D is net of federal spending, and GDP is net of government spending.

[14] Multifactor productivity for private non-farm business sector from the Bureau of Labor Statistics is available from 1956 to 2002.

in Section 2.7, the empirical estimates based on the patent renewal data suggest that a reasonable range for the patent-value depreciation rates is between 15% and 25%; therefore, $g_\pi \in [-0.2, -0.1]$.

(50) $$\frac{I}{Y} = \frac{\alpha\hat{\omega}}{1 - s_K}\left(\frac{n + g_c + \delta}{\rho + g_c\sigma + \delta}\right),$$

(51) $$\frac{wL}{Y} = \frac{1-\alpha}{1 - s_L},$$

(52) $$\frac{wL_r + RK_r}{Y} = (1-\alpha)\frac{s_L}{1-s_L} + \hat{\omega}\alpha\frac{s_K}{1-s_K},$$

(53) $$g_{TFP} = \frac{1}{\xi}\left(\frac{1-\alpha\eta}{\eta}\right)g_V.$$

(54) $$g_\pi = g_{TFP}/(1-\alpha) + n - g_V.$$

For $\alpha = \beta$, the R&D share of labor and capital is $\frac{s_r}{1-s_r} = \left(\frac{(1 - e^{-(\rho - n + (\sigma-1)g_c + g_V)T})g_V}{\rho - n + (\sigma - 1)g_c + g_V}\right)\frac{\alpha(1-\hat{\omega})}{\omega}$.

Table 1 lists the calibrated structural parameters along with the implied markup $\mu = 1/(\alpha\eta)$ and the implied real interest rate $r = \rho + g_A\sigma$.

Table 1: Calibrated Structural Parameters

g_π	σ	α	η	ξ	g_V	μ	r
-0.10	2.872	0.310	2.970	0.338	0.131	1.085	0.082
-0.15	2.907	0.310	3.008	0.391	0.181	1.071	0.083
-0.20	2.933	0.310	3.024	0.458	0.231	1.065	0.083

The implied markup is within the empirically plausible range. For example, Laitner and Stolyarov's (2004) estimated markup is 1.09-1.11, and Basu and Fernald (1997) estimate that the aggregate profit share in the US is about 3%.[15] Also, the implied real interest rate is closed to the historical rate of return in the US stock market. The calibrated values for ξ suggest that roughly 35% to 45% of the long-run TFP growth in the US is driven by R&D.

[15] Assuming cost minimization, the return to scale = markup x (1 - the profit share). Basu and Fernald's (1997) estimates also suggest that "a typical industry has roughly constant returns to scale." (p. 250)

3.2. Externality Parameters

For each set of the calibrated parameters, the balanced-growth condition in (42) determines a unique value for $\gamma/(1-\phi)$, which is sufficient to determine the effect of R&D on the balanced-growth level of consumption. However, holding $\gamma/(1-\phi)$ constant, a larger γ implies a faster rate of convergence to the balanced-growth path; therefore, it is important to consider different values for $\gamma \in [0.1, 1.0]$ that is the parameter for the negative externality in intratemporal duplication. The calibrated values for ϕ that is the parameter for the externality in intertemporal knowledge spillovers are listed in Table 2, and the positive values suggest positive knowledge spillovers (i.e. the standing-on-shoulder effect).

Table 2: The Implied Values for ϕ

g_π / γ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
-0.10	0.984	0.968	0.952	0.935	0.919	0.903	0.887	0.871	0.855	0.839
-0.15	0.988	0.977	0.965	0.953	0.942	0.930	0.918	0.907	0.895	0.883
-0.20	0.991	0.982	0.973	0.963	0.954	0.945	0.936	0.927	0.918	0.908

3.3. Socially Optimal R&D

This section computes the socially optimal level of R&D share $(1-\alpha)s_L^*/(1-s_L^*)+\alpha s_K^*/(1-s_K^*)$.

[insert Figure 1 here]

Figure 1 shows that there is underinvestment in R&D unless γ is very small. To reduce the plausible parameter space for γ, I make use of the empirical estimates for the social rate of return to R&D. Following Jones and Williams' (1998) derivation, Appendix II shows that the net social rate of return $\tilde{r}$ can be expressed as

(55) $$\tilde{r}=\frac{1+g_Y}{1+g_V}\left(1+g_V\left(\frac{1-\alpha\eta}{\eta}\frac{\gamma}{s_r}+\phi\right)\right)-1.$$

Holding other things constant, $\tilde{r}$ is increasing in γ. Table 3 shows that for the range of values for $\gamma \le 0.2$ that exhibits R&D underinvestment, the implied social rates of return $\tilde{r}$ are less than 8%, which are far below the empirical estimates summarized in Jones and Williams (1998).

Table 3: The Implied Social Rates of Return to R&D										
g_π / γ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
-0.10	5.0%	7.0%	8.9%	10.8%	12.7%	14.6%	16.5%	18.4%	20.3%	22.2%
-0.15	5.3%	7.4%	9.6%	11.7%	13.9%	16.0%	18.1%	20.3%	22.4%	24.6%
-0.20	5.6%	8.0%	10.5%	12.9%	15.3%	17.8%	20.2%	22.7%	25.1%	27.5%

Since the empirical estimates for the social rate of return to R&D vary across studies, I will leave it to the readers to decide on their preferred values. For the relevant range of γ, there is underinvestment in R&D. This finding of R&D underinvestment is due to the calibration result that a non-negligible fraction of long-run TFP growth is driven by R&D. In other words, if the calibrated values for ξ were smaller, then the socially optimal levels of R&D spending would be lower. This is because a lower calibrated value for ξ implies a lower calibrated value for $\gamma/(1-\phi)$, which in turn implies that R&D would have a smaller effect on consumption.

3.4. Patent Extension

Given the above finding of R&D underinvestment, a natural question to ask is whether extending the patent length can effectively mitigate this problem. Figure 2 shows that the magnitude of the increases in R&D from extending the patent length beyond 20 years depends on the patent-value depreciation rate.

[insert Figure 2 here]

At a high patent-value depreciation rate, the stimulating effect of the patent length on R&D is almost negligible. At a low patent-value depreciation rate, extending the patent length from 20 to 50 years increases R&D slightly by 0.2 percentage points, and the resulting level of R&D is still far below the social optimum. Therefore, this numerical exercise suggests that patent extension is not an effective method in stimulating R&D confirming the intuition discussed in Section 2.7. On the other hand, shortening the patent length can reduce R&D spending significantly.

The next exercise computes the percentage changes in long-run consumption when the patent length varies from 20 years.

Lemma 3: For $\alpha = \beta$, the expression for the endogenous parts of consumption on the balanced-growth path is

$$c(T) = \begin{pmatrix} \tilde{\omega}(T)^{\frac{\eta(1-\phi)}{\eta(1-\phi)(1-\alpha)-\alpha\gamma(1-\alpha\eta)}} i(T)^{\frac{\alpha\eta(1-\phi)+\alpha\gamma(1-\alpha\eta)}{\eta(1-\phi)(1-\alpha)-\alpha\gamma(1-\alpha\eta)}} (1-i(T)) \\ s_r(T)^{\frac{\gamma(1-\alpha\eta)}{\eta(1-\phi)(1-\alpha)-\alpha\gamma(1-\alpha\eta)}} (1-s_r(T))^{\frac{\eta(1-\phi)}{\eta(1-\phi)(1-\alpha)-\alpha\gamma(1-\alpha\eta)}} \end{pmatrix}. \tag{56}$$

Proof: See Appendix I.

Figure 3 plots the percentage changes in long-run consumption.

[insert Figure 3 here]

Given the small increases in R&D from patent extension, the positive effect on long-run consumption is no more than 3% at the lower bound of the patent-value depreciation rates and is as small as 0.61% at the upper bound. A back-of-the-envelope calculation shows that the increase in consumption mostly comes from $\frac{\gamma(1-\alpha\eta)}{\eta(1-\phi)(1-\alpha)-\alpha\gamma(1-\alpha\eta)}\Delta \ln s_r(T)$ that is the direct effect of increasing R&D spending on consumption. In other words, the other general-equilibrium and distortionary effects only have secondary impacts.

3.5. Dynamic Distortion

Proposition 1 derives the sufficient condition under which the market equilibrium rate of investment in physical capital is below the socially optimal level in (47). The next numerical exercise quantifies this discrepancy. Figure 4 presents the socially optimal rates of investment in physical capital along with the US's long-run investment rate, and the difference is about 1.7% on average.

[insert Figure 4 here]

The equilibrium rate of investment in physical capital is decreasing in the aggregate markup; therefore, extending the patent length decreases the capital investment rate and causes it to deviate further from the social optimum. Figure 5 presents the equilibrium rates of capital investment at different patent length.

[insert Figure 5 here]

Extending the patent length from 20 to 50 years would cause the steady-state equilibrium rate of investment in physical capital to decrease slightly by at most 0.24%.

4. Conclusion

This paper provides a quantitative framework to evaluate the effects of extending the patent length. At the empirical range of patent-value depreciation rates, extending the patent length beyond 20 years leads to only a very small increase in R&D. Therefore, the policy implication is that the patent length is not an effective policy instrument in solving the R&D-underinvestment problem. Although the analysis focuses on the balanced-growth path, taking into consideration the transition dynamics should not alter this policy implication. This is because if the long-run effects on consumption are so small, accounting for the potential short-run consumption losses would make the overall welfare gains even more negligible. The calibration exercise also suggests that about 35% to 45% of the long-run TFP growth in the US is driven by R&D and extending the patent length would worsen the dynamic distortionary effect slightly by reducing the steady-state rate of investment in physical capital by at most 0.24%.

The readers are advised to interpret the numerical results with some important caveats in mind. Although the variety-expanding growth model has been generalized to capture more realistic features of the US economy, it is still an oversimplification of the real world. Furthermore, the finding of R&D underinvestment is based on the assumptions that the empirical estimates for the social rate of return to R&D and the data on private R&D spending are not incorrectly measured by an order of magnitude. The validity of these assumptions remains as an empirical question. Therefore, the numerical results should be viewed as illustrative at best.

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Appendix I: Proofs

Lemma 1: *The aggregate production function for the final goods is*

(a1)
$$Y_t = \tilde{\omega} A_t^{1-\alpha} Z_t^{1-\alpha} L_{y,t}^{1-\alpha} K_{y,t}^{\alpha},$$

where A_t *is the level of R&D-driven TFP and is defined as*

(a2)
$$A_t^{1-\alpha} \equiv V_t^{(1-\alpha\eta)/\eta},$$

and $\tilde{\omega}$ *is defined as*

(a3)
$$\tilde{\omega} \equiv \frac{(\omega(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)} + 1 - \omega)^{1/\eta}}{(\omega(\alpha\eta)^{1/(1-\alpha\eta)} + 1 - \omega)^{\alpha}}.$$

$\tilde{\omega}$ *is strictly less than one for* $\omega \in (0,1)$ *and equals one for* $\omega \in \{0,1\}$. *In addition,* $\partial\tilde{\omega}/\partial\omega < 0$ *when*

$$\omega < \overline{\omega} \equiv \frac{1}{1-\alpha\eta}\left(\frac{1}{1-(\alpha\eta)^{1/(1-\alpha\eta)}} - \frac{\alpha\eta}{1-(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)}}\right) < 1.$$

Proof: (7), (10) and (11) imply that $X_t(j) = (\alpha\eta)^{1/(1-\alpha\eta)} X_t(j')$. Substituting this condition into (12) and then the resulting condition into (5) yields the aggregate production function. For $\omega \in \{0,1\}$, $\tilde{\omega}$ equals one. Simple differentiation yields

(a4)
$$\frac{\partial \ln \tilde{\omega}}{\partial \omega} = \frac{1}{\eta}\frac{(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)} - 1}{\omega(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)} + 1 - \omega} - \alpha\frac{(\alpha\eta)^{1/(1-\alpha\eta)} - 1}{\omega(\alpha\eta)^{1/(1-\alpha\eta)} + 1 - \omega}.$$

At $\omega = \overline{\omega}$, $\partial \ln \tilde{\omega}/\partial\omega = 0$ and

(a5)
$$\left.\frac{\partial^2 \ln \tilde{\omega}}{\partial \omega^2}\right|_{\omega=\overline{\omega}} = -\left(\frac{1}{\eta}\left(\frac{(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)} - 1}{\omega(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)} + 1 - \omega}\right)^2 - \alpha\left(\frac{(\alpha\eta)^{1/(1-\alpha\eta)} - 1}{\omega(\alpha\eta)^{1/(1-\alpha\eta)} + 1 - \omega}\right)^2\right) > 0. \blacksquare$$

Lemma 2: *The socially optimal rate of capital investment* i^* *is*

(b1) $$i^* = \left(\alpha + \beta \left(\frac{1-\alpha\eta}{\eta} \right) \frac{\gamma g_V}{\rho - n + (\sigma - 1) g_c + (1-\phi) g_V} \right) \frac{g_K + \delta}{\rho + g_c \sigma + \delta},$$

and the socially optimal R&D shares of labor s_L^* *and capital* s_K^* *are respectively*

(b2) $$\frac{s_L^*}{1-s_L^*} = \frac{1-\beta}{1-\alpha} \left(\frac{1-\alpha\eta}{\eta} \right) \frac{\gamma g_V}{\rho - n + (\sigma - 1) g_c + (1-\phi) g_V},$$

(b3) $$\frac{s_K^*}{1-s_K^*} = \frac{\beta}{\alpha} \left(\frac{1-\alpha\eta}{\eta} \right) \frac{\gamma g_V}{\rho - n + (\sigma - 1) g_c + (1-\phi) g_V}.$$

Proof: To derive the socially optimal rate of capital investment and R&D shares of labor and capital, the social planner chooses i_t, $s_{L,t}$ and $s_{K,t}$ to maximize

(b4) $$U = \int_0^\infty e^{-(\rho-n)t} \frac{((1-i_t)Y_t / L_t)^{1-\sigma}}{1-\sigma} dt$$

subject to: (a) the aggregate production function expressed in terms of $s_{L,t}$ and $s_{K,t}$ given by

(b5) $$Y_t = V_t^{(1-\alpha\eta)/\eta} Z_t^{1-\alpha} (1-s_{K,t})^{\alpha} (1-s_{L,t})^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha};$$

(b) the law of motion for capital expressed in terms of i_t given by

(b6) $$\dot{K}_t = i_t Y_t - K_t \delta;$$

and (c) the law of motion for R&D technology expressed in terms of $s_{L,t}$ and $s_{K,t}$ given by

(b7) $$\dot{V}_t = V_t^{\phi} (s_{K,t})^{\beta\gamma} (s_{L,t})^{(1-\beta)\gamma} K_t^{\beta\gamma} L_t^{(1-\beta)\gamma} \varphi.$$

The current-value Hamiltonian H is

$$H=(1-\sigma)^{-1}\left(\frac{(1-i_t)V_t^{(1-\alpha\eta)/\eta}Z_t^{1-\alpha}(1-s_{K,t})^{\alpha}(1-s_{L,t})^{1-\alpha}K_t^{\alpha}L_t^{1-\alpha}}{L_t}\right)^{1-\sigma}$$

(b8)
$$+\upsilon_{K,t}(i_tV_t^{(1-\alpha\eta)/\eta}Z_t^{1-\alpha}(1-s_{K,t})^{\alpha}(1-s_{L,t})^{1-\alpha}K_t^{\alpha}L_t^{1-\alpha}-K_t\delta)$$
$$+\upsilon_{V,t}V_t^{\phi}(s_{K,t})^{\beta\gamma}(s_{L,t})^{(1-\beta)\gamma}K_t^{\beta\gamma}L_t^{(1-\beta)\gamma}\varphi \quad .$$

The first-order conditions are

(b9)
$$H_i=-\frac{1}{(1-i_t)}\left(\frac{(1-i_t)Y_t}{L_t}\right)^{1-\sigma}+\upsilon_{K,t}Y_t=0,$$

(b10)
$$H_{s_L}=-\left(\frac{1-\alpha}{1-s_{L,t}}\right)\left(\frac{(1-i_t)Y_t}{L_t}\right)^{1-\sigma}-\upsilon_{K,t}\left(\frac{1-\alpha}{1-s_{L,t}}\right)i_tY_t+\upsilon_{V,t}\left(\frac{(1-\beta)\gamma}{s_{L,t}}\right)\dot{V}_t=0,$$

(b11)
$$H_{s_K}=-\left(\frac{\alpha}{1-s_{K,t}}\right)\left(\frac{(1-i_t)Y_t}{L_t}\right)^{1-\sigma}-\upsilon_{K,t}\left(\frac{\alpha}{1-s_{K,t}}\right)i_tY_t+\upsilon_{V,t}\left(\frac{\beta\gamma}{s_{K,t}}\right)\dot{V}_t=0,$$

(b12)
$$H_K=\frac{\alpha}{K_t}\left(\frac{(1-i_t)Y_t}{L_t}\right)^{1-\sigma}+\upsilon_{K,t}\left(\alpha\frac{i_tY_t}{K_t}-\delta\right)+\upsilon_{V,t}\left(\beta\gamma\frac{\dot{V}_t}{K_t}\right)=(\rho-n)\upsilon_{K,t}-\dot{\upsilon}_{K,t},$$

(b13)
$$H_V=\frac{1-\alpha\eta}{V_t\eta}\left(\frac{(1-i_t)Y_t}{L_t}\right)^{1-\sigma}+\upsilon_{K,t}\left(\frac{(1-\alpha\eta)i_tY_t}{V_t\eta}\right)+\upsilon_{V,t}\left(\phi\frac{\dot{V}_t}{V_t}\right)=(\rho-n)\upsilon_{V,t}-\dot{\upsilon}_{V,t}.$$

Note that the first-order conditions with respect to the co-state variables $\upsilon_{K,t}$ and $\upsilon_{V,t}$ yield the laws of motion for capital and the number of varieties. Then, imposing the balanced-growth conditions yields

(b14)
$$H_i:\left(\frac{(1-i)Y_t}{L_t}\right)^{1-\sigma}=(1-i)\upsilon_{K,t}Y_t,$$

(b15)
$$H_{s_L}:\gamma g_VV_t\upsilon_{V,t}\left(\frac{1-s_L}{s_L}\right)=\frac{1-\alpha}{1-\beta}\left(\left(\frac{(1-i)Y_t}{L_t}\right)^{1-\sigma}+\upsilon_{K,t}iY_t\right),$$

(b16)
$$H_{s_K}:\gamma g_VV_t\upsilon_{V,t}\left(\frac{1-s_K}{s_K}\right)=\frac{\alpha}{\beta}\left(\left(\frac{(1-i)Y_t}{L_t}\right)^{1-\sigma}+\upsilon_KiY_t\right),$$

(b17) $$H_K : \alpha\left(\left(\frac{(1-i)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_{K,t} iY_t\right) + \beta\gamma g_V V_t \upsilon_{V,t} = (\rho + g_c\sigma + \delta)K_t\upsilon_{K,t},$$

(b18) $$H_V : \frac{1-\alpha\eta}{\eta}\left(\left(\frac{(1-i)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_{K,t} iY_t\right) = (\rho - n + (\sigma-1)g_c + (1-\phi)g_V)V_t\upsilon_{V,t}.$$

Finally, solving (b14)-(b18) yields (b1)-(b3).■

Proposition 1: *The decentralized equilibrium capital investment rate* i *is below the socially optimal investment rate* i^* *if either there is underinvestment in R&D or labor is the only factor input for R&D. In addition, an increase in the patent length always reduces the equilibrium capital investment rate* i.

Proof for $i < i^*$**:** The socially optimal investment rate i^* is

$$i^* = \left(\alpha + \beta\left(\frac{1-\alpha\eta}{\eta}\right)\frac{\gamma g_V}{\rho - n + (\sigma-1)g_c + (1-\phi)g_V}\right)\frac{g_K + \delta}{\rho + g_c\sigma + \delta}.$$

The market equilibrium rate of investment i is

$$i = \hat{\omega}\left(\alpha + \beta\left(\frac{\alpha(1-\hat{\omega})}{\omega}\right)\frac{(1-e^{-(\rho-n+(\sigma-1)g_c+g_V)T})g_V}{\rho - n + (\sigma-1)g_c + g_V}\right)\frac{g_K + \delta}{\rho + g_c\sigma + \delta}.$$

Thus, either (i) labor is the only factor input for R&D such that $\beta = 0$, or (ii) there is underinvestment in R&D such that $\left(\frac{1-\alpha\eta}{\eta}\right)\frac{\gamma g_V}{\rho - n + (\sigma-1)g_c + (1-\phi)g_V} > \left(\frac{\alpha(1-\hat{\omega})}{\omega}\right)\frac{(1-e^{-(\rho-n+(\sigma-1)g_c+g_V)T})g_V}{\rho - n + (\sigma-1)g_c + g_V}$ is a *sufficient* condition for $i < i^*$ because $\hat{\omega} < 1$.■

Proof for $\partial i / \partial T < 0$**:** Recall that $\hat{\omega}(T)$ is a function of *T* and is given by

$$\hat{\omega}(T) = \frac{\omega(T)(\alpha\eta)^{1/(1-\alpha\eta)} + 1 - \omega(T)}{\omega(T)(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)} + 1 - \omega(T)},$$

where $\omega(T) = (1 - e^{-g_V T})$. Differentiating $\hat{\omega}$ with respect to T yields

$$\frac{\partial \hat{\omega}(T)}{\partial T} = -\hat{\omega}(T)\left(\frac{(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)} - 1}{\omega(T)((\alpha\eta)^{\alpha\eta/(1-\alpha\eta)} - 1) + 1} - \frac{(\alpha\eta)^{1/(1-\alpha\eta)} - 1}{\omega(T)((\alpha\eta)^{1/(1-\alpha\eta)} - 1) + 1} \right) \frac{\partial \omega(T)}{\partial T} < 0$$

for $T \in (0, \infty)$ because $\dfrac{(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)} - 1}{\omega(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)} + 1 - \omega} > \dfrac{(\alpha\eta)^{1/(1-\alpha\eta)} - 1}{\omega(\alpha\eta)^{1/(1-\alpha\eta)} + 1 - \omega}$. Recall that the equilibrium rate of capital investment is $i(T) = \dfrac{\alpha\hat{\omega}(T)}{1 - s_K(T)} \left(\dfrac{g_K + \delta}{\rho + g_c\sigma + \delta} \right)$. Differentiating $i(T)$ with respect to T yields

$$\frac{\partial i(T)}{\partial T} = i(T)\left(\frac{1}{\hat{\omega}(T)} \frac{\partial \hat{\omega}(T)}{\partial T} + \frac{1}{1 - s_K(T)} \frac{\partial s_K(T)}{\partial T} \right),$$

where

$$\frac{\partial s_K(T)}{\partial T} = s_K(T)(1 - s_K(T))\left(\frac{(\rho - n + (\sigma - 1)g_c + g_V)e^{-(\rho - n + (\sigma - 1)g_c + g_V)T}}{1 - e^{-(\rho - n + (\sigma - 1)g_c + g_V)T}} - \frac{g_V e^{-g_V T}}{1 - e^{-g_V T}} - \frac{\partial \hat{\omega}(T)/\partial T}{1 - \hat{\omega}(T)} \right).$$

Finally,

$$\frac{\partial i(T)}{\partial T} = i(T)\left(\begin{array}{l} \left(\dfrac{1 - \hat{\omega}(T)(1 - s_K(T))}{\hat{\omega}(T)(1 - \hat{\omega}(T))} \right) \dfrac{\partial \hat{\omega}(T)}{\partial T} \\ + s_K(T)\left(\dfrac{(\rho - n + (\sigma - 1)g_c + g_V)e^{-(\rho - n + (\sigma - 1)g_c + g_V)T}}{1 - e^{-(\rho - n + (\sigma - 1)g_c + g_V)T}} - \dfrac{g_V e^{-g_V T}}{1 - e^{-g_V T}} \right) \end{array} \right) < 0$$

because $\dfrac{(\rho - n + (\sigma - 1)g_c + g_V)e^{-(\rho - n + (\sigma - 1)g_c + g_V)T}}{1 - e^{-(\rho - n + (\sigma - 1)g_c + g_V)T}} < \dfrac{g_V e^{-g_V T}}{1 - e^{-g_V T}}$ for $T \in (0, \infty)$. ■

Lemma 3: *For $\alpha = \beta$, the expression for the endogenous parts of consumption on the balanced-growth path is*

(c1) $$c(T) = \left(\begin{array}{l} \tilde{\omega}(T)^{\frac{\eta(1-\phi)}{\eta(1-\phi)(1-\alpha) - \alpha\gamma(1-\alpha\eta)}} \, i(T)^{\frac{\alpha\eta(1-\phi) + \alpha\gamma(1-\alpha\eta)}{\eta(1-\phi)(1-\alpha) - \alpha\gamma(1-\alpha\eta)}} (1 - i(T)) \\ s_r(T)^{\frac{\gamma(1-\alpha\eta)}{\eta(1-\phi)(1-\alpha) - \alpha\gamma(1-\alpha\eta)}} (1 - s_r(T))^{\frac{\eta(1-\phi)}{\eta(1-\phi)(1-\alpha) - \alpha\gamma(1-\alpha\eta)}} \end{array} \right).$$

Proof: The following derivation applies to the more general case in which α and β can be different. Along the balanced growth path, per capita consumption is

$$c_t = (1-i)\left(\frac{Y_t}{L_t}\right) = \tilde{\omega}(1-i)(1-s_L)\left(\frac{K_{y,t}}{L_{y,t}}\right)^{\alpha} A_t^{1-\alpha} Z_t^{1-\alpha}. \tag{c2}$$

The capital-labor ratio $K_{y,t}/L_{y,t}$ in the intermediate-goods sector is

$$\frac{K_{y,t}}{L_{y,t}} = A_t Z_t \left(\frac{i\tilde{\omega}(1-s_K)}{g_A+n+\delta}\right)^{1/(1-\alpha)}. \tag{c3}$$

The balanced-growth path of V_t from (46) is

$$V_t = \left(A_t^{\beta\gamma} Z_t^{\beta\gamma}(s_K)^{\beta\gamma}(s_L)^{(1-\beta)\gamma}\left(\frac{1-s_L}{1-s_K}\right)^{\beta\gamma}\left(\frac{i\tilde{\omega}(1-s_K)}{g_A+n+\delta}\right)^{\beta\gamma/(1-\alpha)}\frac{\varphi}{g_V}L_t^{\gamma}\right)^{1/(1-\phi)}. \tag{c4}$$

Therefore, the balanced-growth path of A_t from (c4) and (14) is

$$A_t = \left(\frac{(i\tilde{\omega})^{\beta\gamma/(1-\alpha)}(s_K)^{\beta\gamma}(s_L)^{(1-\beta)\gamma}(1-s_L)^{\beta\gamma}(1-s_K)^{\beta\gamma\alpha/(1-\alpha)}}{(g_A+n+\delta)^{\beta\gamma/(1-\alpha)}}\frac{\varphi}{g_V}L_t^{\gamma}Z_t^{\beta\gamma}\right)^{\frac{1-\alpha\eta}{\eta(1-\phi)(1-\alpha)-\beta\gamma(1-\alpha\eta)}}. \tag{c5}$$

Finally, substituting (c3) and (c5) into (c2) and dropping the exogenous terms yield the expression for the balanced-growth level of per capita consumption corresponding to the patent length T given by

$$c(T) = \left(\begin{array}{l} \tilde{\omega}(T)^{\frac{\eta(1-\phi)}{\eta(1-\phi)(1-\alpha)-\beta\gamma(1-\alpha\eta)}}\, i(T)^{\frac{\alpha\eta(1-\phi)+\beta\gamma(1-\alpha\eta)}{\eta(1-\phi)(1-\alpha)-\beta\gamma(1-\alpha\eta)}}(1-i(T)) \\ s_K(T)^{\frac{\beta\gamma(1-\alpha\eta)}{\eta(1-\phi)(1-\alpha)-\beta\gamma(1-\alpha\eta)}}(1-s_K(T))^{\frac{\alpha\eta(1-\phi)}{\eta(1-\phi)(1-\alpha)-\beta\gamma(1-\alpha\eta)}} \\ s_L(T)^{\frac{(1-\beta)\gamma(1-\alpha\eta)}{\eta(1-\phi)(1-\alpha)-\beta\gamma(1-\alpha\eta)}}(1-s_L(T))^{\frac{(1-\alpha)\eta(1-\phi)}{\eta(1-\phi)(1-\alpha)-\beta\gamma(1-\alpha\eta)}} \end{array}\right). \tag{c6}$$

Finally, after setting $\alpha=\beta$, (c6) becomes (c1).

Appendix II: The Social Rate of Return to R&D

Jones and Williams (1998) define the social rate of return as the sum of the additional output produced and the reduction in R&D that is made possible by reallocating one unit of output from consumption to R&D in the current period and then reducing R&D in the next period to leave the subsequent path of technology unchanged. I rewrite the law of motion for R&D technology as

(d1) $$\dot{V}_t = G(V_t, R_t) \equiv \varphi V_t^{\phi} R_t^{\gamma},$$

where $R_t \equiv K_{r,t}^{\alpha} L_{r,t}^{1-\alpha}$. The aggregate production function is rewritten as

(d2) $$Y_t = F(V_t, Z_t, L_{y,t}, K_{y,t}) \equiv \tilde{\omega} V_t^{(1-\alpha\eta)/\eta} Z_t^{1-\alpha} L_{y,t}^{1-\alpha} K_{y,t}^{\alpha}.$$

Using the above definition, Jones and Williams (1998) show that the gross social return is

(d3) $$1+\tilde{r} = \left(\frac{\partial G}{\partial R}\right)_t \left(\frac{\partial F}{\partial V}\right)_{t+1} + \frac{(\partial G/\partial R)_t}{(\partial G/\partial R)_{t+1}} \left(1 + \left(\frac{\partial G}{\partial V}\right)_{t+1}\right).$$

After imposing the balanced-growth conditions, the net social return becomes

(d4) $$\tilde{r} = \frac{1+g_Y}{1+g_V}\left(1 + g_V\left(\frac{1-\alpha\eta}{\eta}\frac{\gamma}{s_r} + \phi\right)\right) - 1.$$

Appendix III: Figures

Figure 1: Socially Optimal R&D Shares

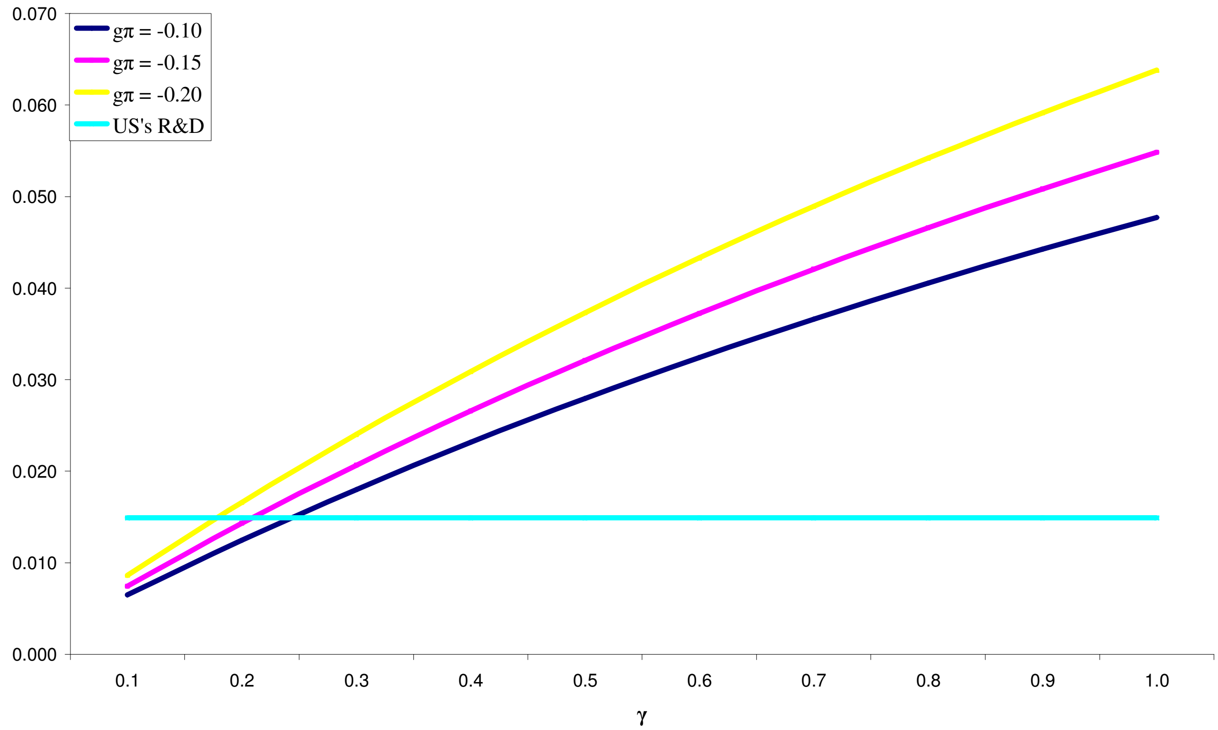

Figure 2: R&D Shares at Different Patent Length

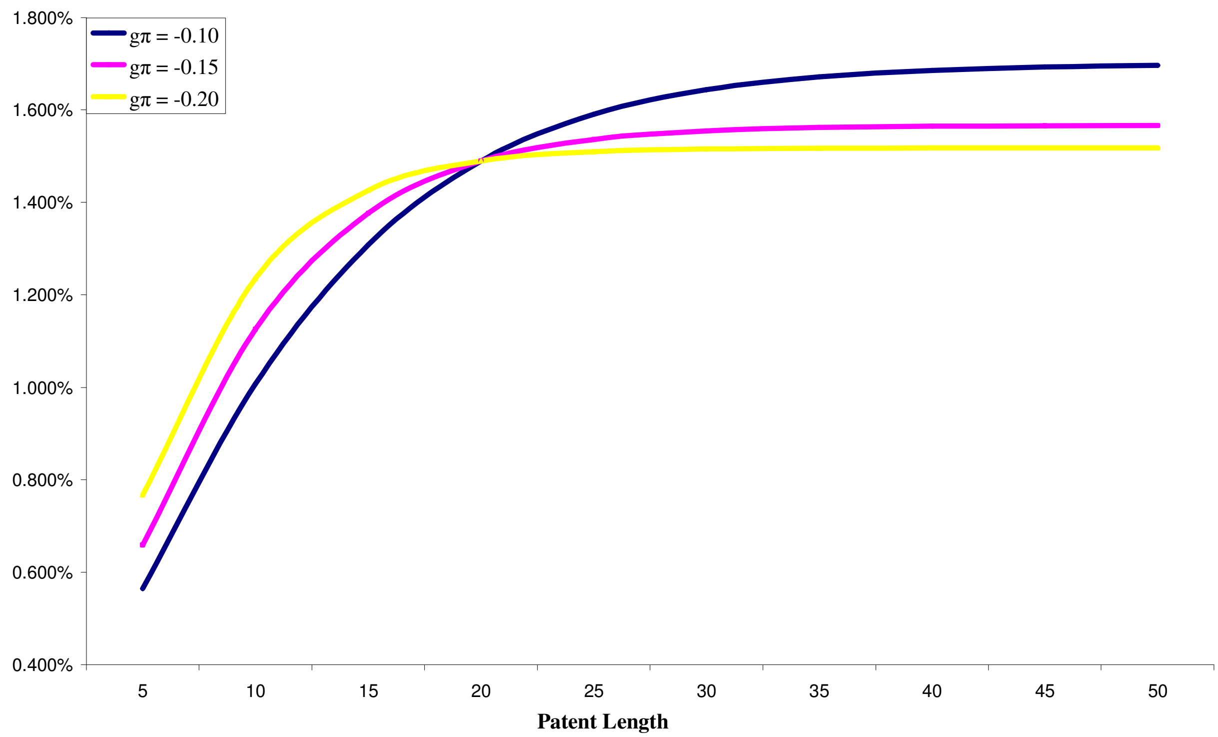

Figure 3: Percentage Changes in Long-Run Consumption

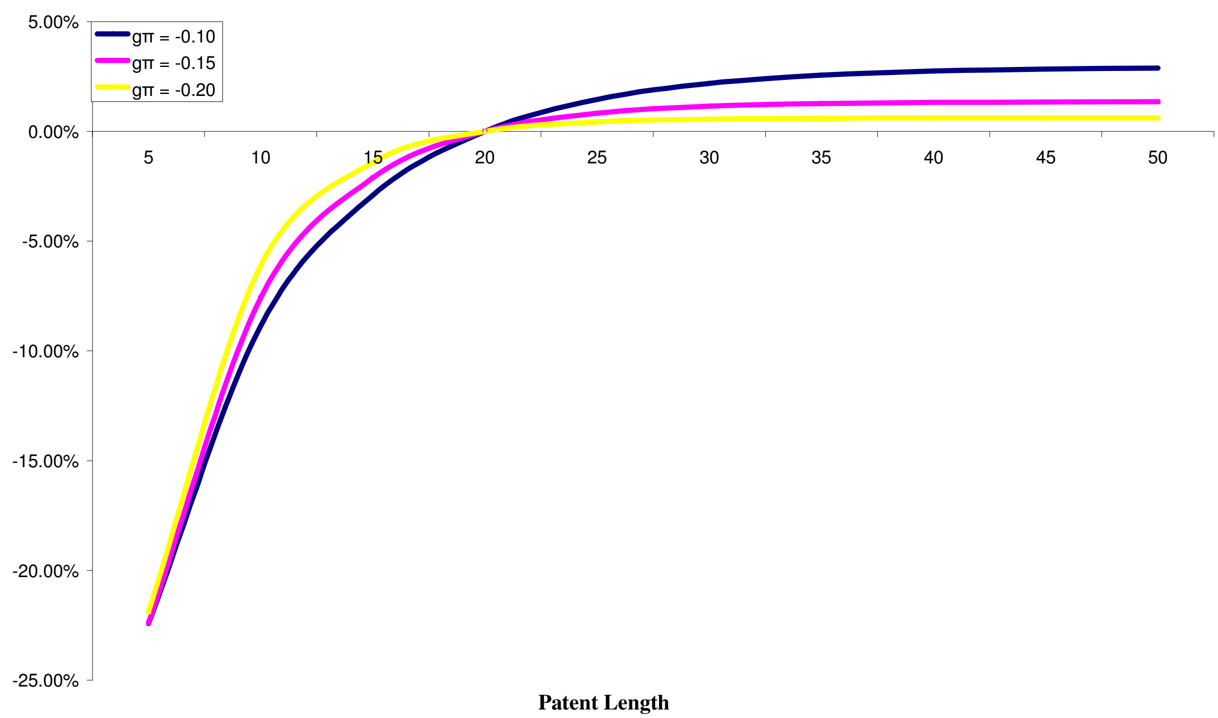

Figure 4: Socially Optimal Rates of Investment in Physical Capital

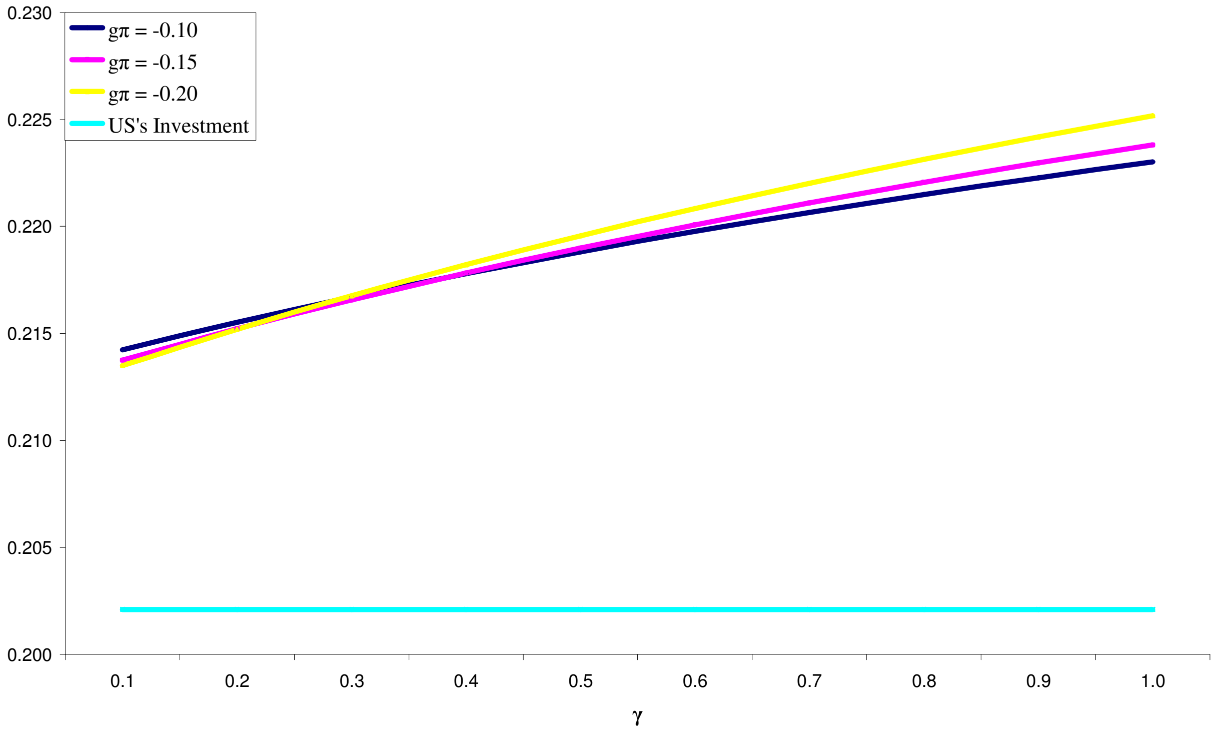

Figure 5: Equilibrium Rates of Investment in Physical Capital at Different Patent Length