Okun’s law: evidence for the Brazilian economy

Guilherme Alexandre Tombolo and Marcos Minoru Hasegawa

Federal University of Parana - Graduation Program in Economic Development (PPGDE/UFPR), Federal University of Parana - Graduation Program in Economic Development (PPGDE/UFPR)

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Guilherme Alexandre Tombolo †
Marcos Minoru Hasegawa ‡

Abstract
In this article we seek to estimate the Brazilian “Okun’s law” with quarterly data ranging from 1980Q1 until 2013Q3. Considering the typical Okun’s relationship, \( \Delta u = \alpha - \beta Y_t \), where \( \beta \) is the Okun coefficient, we have obtained estimates of \( \beta \) between -0.1878 and -0.2055, such values are in general lower than the values obtained to other countries in similar studies.

JEL Classification: E23, E24, E27
Keywords: Output, Unemployment, Okun’s Law

1. Introduction

Okun’s law implies an empirical and negative relationship between the change in the unemployment rate and the rate of output growth. Proposed by Okun (1962), this relationship has proved robust over time and across countries. Okun’s law is a basic building block of traditional macroeconomics models, where the combination between Okun’s law and the Phillips Curve is used to derive the aggregate supply relationship (Cuaresma, 2008). Okun was concerned with the potential output concept and the social costs of the unemployment in terms of lost output.

Okun’s law should be seen as a rule of thumb rather than a 'scientific’ law because it is not based on a theory. However such rule is useful as a shortcut to more complicated macroeconomic models. As far as we know there are no studies specifically concerned in getting Okun coefficient values for Brazil. As the ‘Okun’s law” is a rule of thumb often used in practical analyzes and undergraduate textbooks, Our objective in this paper is to estimate the Okun coefficient for the Brazilian economy for comparison with the values obtained in other countries.

To achieve our goal this paper is divided into five sections besides this introduction: the second section describes the Okun’s law; the third section presents estimates of Okun's law to other countries by different methods; in the fourth section we estimate the Okun’s law to Brazil by different methods, and the fifth section concludes.

† PhD student in the Graduation Program in Economic Development of Federal University of Paraná – Brazil (PPGDE / UFPR). E-mail: guilhermetombolo@ufpr.br
‡ PhD in Applied Economics by the University of São Paulo (USP) (2003), professor of economics in Federal University of Paraná and PPGDE/UFPR - Brazil. E-mail: hasegawa@ufpr.br
2. The Okun’s Law

The question that Okun wanted to answer in his 1962 paper “Potential GNP: Its measurement and significance”, was: “how much output the economy can produce at full employment?” And the answer, according to Okun, was the concept of potential output and its measurement.

Potential output for Okun is:

Potential GNP is a supply concept, a measure of productive capacity. But it is not a measure of how much output could be generated by unlimited amounts of aggregate demand. The nation would probably be most productive in the short-run with inflationary pressure pushing the economy. But the social target of maximum production and employment is constrained by a social desire for price stability and free markets. The full employment goal must be understood as striving for maximum production without inflationary pressure... [Okun (1962, p.1]

In this passage we note that Okun takes account of price stability as a necessary condition for calculation of potential output. This idea anticipates, in a sense, the idea of a NAIRU (non-accelerating inflation rate of unemployment), due to Phelps (1967) and Friedman (1968) as noted by Prachowny (1993, p. 331).

Okun’s equation is usually written as:

$$\Delta u_t = -\beta (g_y - \bar{g}_y)$$

The $\beta$ coefficient, sometimes called Okun coefficient, measures how much the unemployment rate varies when the output growth rate ($g_y$) is different from the potential output growth rate ($\bar{g}_y$). In their calculations for the U.S., Okun found $\beta \approx 0.30$. But in fact, Okun was interested in the inverse of $\beta$, i.e., $1/\beta$, because he wanted to know how much output is "lost" when unemployment is above a certain amount. As $1/\beta \approx 3$, a percentage increase (decrease) in unemployment should result in three point percentage of decrease (increase) in output growth.

The coefficient $\beta$ in the above equation is affected by variables involved in labor market and production process like labor productivity, hours worked, labor’s laws, unions-entrepreneurs relationship, level of firms’ capacity utilization, among others things. By the way, Okun’s arguments suggest that the empirical link between unemployment and output should not be interpreted in a ceteris paribus way, but rather as capturing a simultaneous effect among labor force, hours worked and labor productivity (Cuaresma, 2008). The potential output growth rate, in turn, is an unobserved variable; such variable depends on the labor force growth rate, technical progress, productivity, among other things (Stock and Vogler-Ludwig, 2010).

However, Barreto and Howland (1993) strongly criticize the Okun procedure of use the reciprocal of $\beta$ to predict the output growth rate associated with a given unemployment rate.¹

¹ Plosser and Schwert (1979) also raised this issue.
According to the authors, the reciprocal of $\beta$ is a biased\(^2\) estimate for the effect of unemployment on the output growth rate. The question is related to what we want to predict. If the intention is to predict the output given the unemployment, so one regresses output on unemployment; if the intent is to predict unemployment given the output, so one regresses the unemployment on output (Barreto and Howland, 1993).

As now there are more sophisticated procedures for calculating potential output, and in view of the Barreto and Howland (1993), our goal in this paper is focused on regressions of unemployment rate on the output, i.e., how much the unemployment rate varies given the variations in the output growth rate. Of course, ideally the relationship between unemployment and output must be estimated within a system of simultaneous equations involving other theoretically relevant economic variables. But as the 'Okun's law' is a rule of thumb often used in practical analyzes and undergraduate textbooks, we estimate the relationship to Brazil for comparison with the values obtained in other countries.

3. The Okun coefficient in different countries


$$u_t^c = \alpha + \beta u_{t-1}^c + \gamma y_t^c + \varepsilon_t$$

where $u_t^c$ is the unemployment gap, $y_t^c$ is the output gap and $\varepsilon_t$ is a random error. The 'short term' Okun coefficient in this model is $\gamma$, while the 'long-term' coefficient is $\theta = \gamma/(1 - \beta)$. The output and unemployment gaps were calculated using state-space models as developed by Harvey (1985, 1989).

Lee (2000) estimated the Okun's coefficient for 16 OECD countries using postwar annual data. He estimated two types of models with the output as the independent variable and unemployment as the dependent variable. The first model is estimated in first differences:

$$\Delta y_t = \beta_0 - \beta_1 \Delta u_t + \varepsilon_t$$

where $\Delta y_t$ is the difference of the output logs, $\Delta u_t$ is the variation in the unemployment rate and $\varepsilon_t$ is a random error. The second model, in turn, is estimated using the variables gaps:

$$y_t^c = -\beta_2 u_t^c + \varepsilon_t$$

where $y_t^c$ is the gap of the output log, $u_t^c$ is the unemployment rate gap and $\varepsilon_t$ is a random error. The gaps were calculated using the Hodrick-Prescott filter, the Beveridge-Nelson filter and the Kalman filter.

Sogner and Stiassny (2002) estimated the Okun relationship for 15 OECD countries using 1960-1999 annual data. The estimated model used a lag in the rate of output growth to capture lags in the employment reaction:

\(^2\) Generally an over-estimate.
\[\Delta u_t = \alpha_0 + \alpha_1 \Delta y_t + \alpha_2 \Delta y_{t-1} + v_t\]

\[\Delta u_t = \alpha_0 + \alpha_1 \Delta y_t + \alpha_2 \Delta y_{t-1} + v_t\]

where \(\alpha_3\) is the impact effect of output growth rate on the change in unemployment rate, and \((\alpha_1 + \alpha_2)\) is the long-term effect, i.e., the Okun coefficient. \(\Delta u_t\) and \(\Delta y_t\) are respectively the unemployment rate variation and the output log first differences. \(v_t\) is a error that can follow an AR (1) process \(v_t = \rho v_{t-1} + \varepsilon_t\).

Table 1 shows the estimates obtained for some countries in the studies described above. Japan and Italy are the countries with the lowest estimated coefficients, possibly demonstrating a greater rigidity in labor market in these countries. Germany, France and UK have more similar coefficients with the United States. It is also interesting to compare the estimates of Lee (2000) with those of Moosa (1997) and Sogner and Stiassny (2002). Lee, unlike the other two authors, estimated an output versus unemployment model and therefore estimated a reversed Okun coefficient. Recalling the Barreto and Howland (1993) criticism on the bias involved in using the reciprocal of the Okun coefficient to predict the output growth rate, one notes that the coefficients obtained by Lee are higher than those obtained by the two other authors, mainly to Japan and Italy. Again the question is what one wants to predict. If the intention is to predict the output given the unemployment, then the Lee specification is the correct one, on the other hand, if the intent is to predict unemployment given the output, then the specification of Moosa (1997) and Sogner and Stiassny (2002) is the correct one.

<table>
<thead>
<tr>
<th>Study</th>
<th>Japan</th>
<th>Italy</th>
<th>Germany</th>
<th>France</th>
<th>USA</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moosa (1997)</td>
<td>-0.088</td>
<td>-0.184</td>
<td>-0.407</td>
<td>-0.363</td>
<td>-0.456</td>
<td>-0.372</td>
</tr>
<tr>
<td>Lee (2000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>First Differences</td>
<td>-0.227</td>
<td>-0.917</td>
<td>-0.405</td>
<td>-0.344</td>
<td>-0.543</td>
<td>-0.719</td>
</tr>
<tr>
<td>Kalman Filter</td>
<td>-0.079</td>
<td></td>
<td>-0.581</td>
<td>-0.400</td>
<td>-0.532</td>
<td>-0.671</td>
</tr>
<tr>
<td>Hodrick-Prescott</td>
<td>0.153</td>
<td>-1.754</td>
<td>-0.459</td>
<td>-0.455</td>
<td>-0.478</td>
<td>-0.709</td>
</tr>
<tr>
<td>Beveridge-Nelson</td>
<td>-0.182</td>
<td>-0.415</td>
<td>-0.565</td>
<td>-0.344</td>
<td>-0.493</td>
<td>-0.662</td>
</tr>
<tr>
<td>Sogner e Stiassny (2002)</td>
<td>-0.12</td>
<td>-0.21</td>
<td>-0.38</td>
<td>-0.43</td>
<td>-0.52</td>
<td>-0.58</td>
</tr>
</tbody>
</table>

**Source:** Stock and Vogler-Ludwig (2010, p. 27).

### 4 Estimates for Brazil

The GDP data used in our estimates covers the 1980Q1-2013Q4 period and is provided by the Brazilian Institute of Geography and Statistics (IBGE). The quarterly GDP is seasonally adjusted by the package X12-ARIMA version 0.3 installed on econometric software Gretl 1.9.10. The graph of GDP is plotted in Figure 1.

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3 Cottrell and Lucchetti (2014)
The unemployment rate used comes from the Monthly Employment Survey (PME) of IBGE. The PME began to be held in 1980 and suffered a methodological change in March 2002. Among other changes, the major one was the minimum age of the working age population. Until February 2002, it was considered persons aged 10 or older, from March 2002 the survey has considered only those with 15 years or more of age. The two surveys were collected by IBGE between March 2002 and December 2002 in order to compare the results.

In the old methodology, the unemployment rate in March 2002 was 7.76%, in the same month this rate was 12.93% by the new methodology, 5.17 percentage points higher. In order to chain the two series, we add to every unemployment rate prior to March 2002, 5.17 percentage points. Thus the rate of February 2002 increased from 7.79% to 12.96%, the rate in January 2002 rose from 7.51% to 12.68% and so on until January 1980. The aggregation of the monthly unemployment rates in quarterly rates was done by a weighted average of monthly rates according to the formula:

$$u_t^Q = \frac{\sum_{i=1}^{3} (u_{iq}^M)^2}{\sum_{i=1}^{3} u_{iq}^M}$$

where $u_t^Q$ is the quarterly unemployment rate in quarter q, and $u_{iq}^M$ is the monthly unemployment rate in month i of quarter Q. For example, in January, February and March 1980 unemployment rates were, respectively, 12.48, 12.35 and 12.41%; so the unemployment rate in the first quarter of 1980 was:

$$u_1 = \frac{(12.48)^2 + (12.35)^2 + (12.41)^2}{12.48 + 12.35 + 12.41} \approx 12.41$$

and so it was done for the other quarters. These unemployment rates were seasonally adjusted by the X12-ARIMA version 0.3 package installed on econometric software Gretl 1.9.10. The graph of unemployment is plotted in Figure 2.
Using data from the quarterly GDP and the quarterly rate of unemployment, both properly seasonally adjusted, we proceeded to estimate the Okun relationship to Brazil. Since potential output $\bar{q}_y$ is an unobserved variable, the empirical Okun relationship is obtained running the following ordinary least squares (OLS) regression:

$$\Delta u_t = -\beta (g_{yt} - \bar{g}_y)$$
$$\Delta u_t = \alpha - \beta g_{yt} + \varepsilon_t$$

(1)

where $\Delta u_t$ is the difference in the unemployment rate, $\alpha = \beta \bar{g}_y$, $g_{yt}$ is the GDP growth rate in %, and $\varepsilon_t$ is a random error. The results of the model (1) are shown in Table 2. The estimated $\beta$ coefficient around -0.116514 is low by international standards. The intercept is statistically insignificant. However, the residuals diagnostics indicates the presence of residual autocorrelation. The maximum likelihood estimate shown in Table 3 produces a $\beta$ coefficient around -0.091595, somewhat smaller than the one estimated in Table 2. In Table 3, the intercept is also statistically insignificant.

Table 2: OLS Estimates of Okun’s Law for Brazil (Model 1) – 1980Q2-2013Q3

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Standard-Error$^1$</th>
<th>t-ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant $\alpha$</td>
<td>0.0265338</td>
<td>0.0416056</td>
<td>0.6377</td>
</tr>
<tr>
<td>$gy$ $\beta$</td>
<td>-0.1165140</td>
<td>0.0206797</td>
<td>-5.6340</td>
</tr>
</tbody>
</table>

Obs. 134 $R^2 = 0.2014$ $F(1,132) = 31.74$ rho = 0.1393 Durbin-Watson = 1.72

Residuals and Model Diagnostics

<table>
<thead>
<tr>
<th>Test Object</th>
<th>Test</th>
<th>Null Hypothesis</th>
<th>Test statistic</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normality</td>
<td>Jarque-Bera: Qui$^2$ (2)</td>
<td>normal residuals</td>
<td>5.3206</td>
<td>0.069</td>
</tr>
</tbody>
</table>

$^1$ The models estimated in this paper have used the econometric software GNU Gretl 1.9.10.
One reason for the low coefficient \( \beta \) obtained can be the fact that the relationship between changes in the unemployment rate and the GDP growth rate involves some sort of lag. As shown above, Sogner and Stiassny (2002) proposed the following specification:

\[
\Delta u_t = \alpha_0 + \alpha_1 g_{yt} + \alpha_2 g_{yt-1} + v_t \\
\Delta u_t = \alpha_0 + \alpha_1 \Delta g_{yt} + (\alpha_1 + \alpha_2) g_{yt-1} + v_t
\]

The coefficient \( \alpha_1 \) is the impact coefficient, and \( (\alpha_1 + \alpha_2) \) is the effect coefficient, i.e., the Okun coefficient itself. The estimative by Cochrane-Orcutt Method of the model 2 is shown in Table 4. The value estimated to coefficient \( \alpha_1 \) was -0.0971064, a value close to the Okun’s coefficient estimated in Model 1. The value of \( (\alpha_1 + \alpha_2) \) estimated in model 2 was -0.18777, such number is closer to the value of international estimates. If we look at Table 2, we see that this value (-0.18777) is similar to the value obtained for Italy (-0.21) by Sogner and Stiassny (2002) in a model with the same specification of equation (2) above.
Table 4: Estimate of Okun’s Law to Brazil using the specification of Sogner and Stiassny (2002) and applying the Cochrane-Orcutt Method (Model 2) – 1980Q3-2013Q3

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Standard-Error</th>
<th>t-ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.073551</td>
<td>0.0389697</td>
<td>1.887</td>
</tr>
<tr>
<td>Δg_t</td>
<td>-0.0971064</td>
<td>0.0170404</td>
<td>-5.699</td>
</tr>
<tr>
<td>Δg_{t-1}</td>
<td>-0.187770</td>
<td>0.0244486</td>
<td>-7.68</td>
</tr>
<tr>
<td>Dummy 1991 Q3</td>
<td>-1.426380</td>
<td>0.3348050</td>
<td>-4.26</td>
</tr>
<tr>
<td>Dummy 1998 Q1</td>
<td>1.344270</td>
<td>0.3371870</td>
<td>25.00</td>
</tr>
</tbody>
</table>

Obs. 133          R² = 0.4715         F(4,128) = 24.29      rho = -0.01            Durbin-Watson = 2.02

Residuals and Model Diagnostics

<table>
<thead>
<tr>
<th>Test Object</th>
<th>Test</th>
<th>Null Hypothesis</th>
<th>Test statistic</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normality</td>
<td>Jarque-Bera: Qui² (2)</td>
<td>normal residuals</td>
<td>0.0469</td>
<td>0.9768</td>
</tr>
<tr>
<td>4th Order ARCH</td>
<td>LM</td>
<td>no ARCH effect</td>
<td>1.0892</td>
<td>0.8960</td>
</tr>
</tbody>
</table>

Source: Prepared by authors.

A second method used to estimate the long-term version of Okun’s coefficient was proposed by Gordon (1984). This method consists of the following distributed lag model:

\[ \Delta u_t = \gamma_0 + \sum_{i=1}^k \gamma_{1i} \Delta u_{t-i} + \sum_{i=0}^k \gamma_{2i} g_{g_{t-i}} + \varepsilon_t \] (3)

The estimated long-term version of Okun’s coefficient is:

\[ u_t = \theta_0 + \theta_1 g_{u_t} + \vartheta_t \] (3.1)

where \( \theta_0 = \Delta \bar{u}_t - \theta_1 \bar{g}_{y_t} \), \( \theta_1 = \sum_{i=0}^k \gamma_{2i} / (1 - \sum_{i=1}^k \gamma_{1i}) \) and \( \vartheta_t = \Delta u_t - \theta_0 - \theta_1 g_{y_t} \). The bar above the variable indicates the mean of the variable.

The estimated model 3 is shown in Table 5.

Table 5: OLS estimate of Okun’s Law to Brazil using the specification of Gordon (1984) (Model 3) – 1980Q3-2013Q3

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Standard-Error</th>
<th>t-ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.0738787</td>
<td>0.0361453</td>
<td>2.08290</td>
</tr>
<tr>
<td>Δu_{t-i}</td>
<td>0.1279210</td>
<td>0.0671094</td>
<td>1.90600</td>
</tr>
<tr>
<td>g_{y_t}</td>
<td>-0.0993935</td>
<td>0.0177955</td>
<td>-6.10450</td>
</tr>
<tr>
<td>g_{y_{t-1}}</td>
<td>-0.0797903</td>
<td>0.0178404</td>
<td>-4.38790</td>
</tr>
<tr>
<td>Dummy 1991 T3</td>
<td>-1.2851000</td>
<td>0.0374830</td>
<td>-31.4629</td>
</tr>
<tr>
<td>Dummy 1998 T1</td>
<td>1.3783800</td>
<td>0.0545830</td>
<td>26.7440</td>
</tr>
</tbody>
</table>

Obs. 133          R² = 0.4705         F(5,127) = 3.40e+19      rho = -0.01            Durbin-Watson = 2.02

Residuals and Model Diagnostics

<table>
<thead>
<tr>
<th>Test Object</th>
<th>Test</th>
<th>Null Hypothesis</th>
<th>Test statistic</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normality</td>
<td>Jarque-Bera: Qui² (2)</td>
<td>normal residuals</td>
<td>0.03927</td>
<td>0.9768</td>
</tr>
<tr>
<td>1st Order Autocorrelation</td>
<td>Durbin-Watson</td>
<td>no autocorrelation</td>
<td>1.6623</td>
<td>0.3924</td>
</tr>
<tr>
<td>4th Order Autocorrelation</td>
<td>LM de Breusch- Godfrey</td>
<td>no autocorrelation</td>
<td>0.8292</td>
<td>0.5090</td>
</tr>
<tr>
<td>Heteroskedasticity</td>
<td>LM de White: Qui² (2)</td>
<td>no heteroskedasticity</td>
<td>7.9857</td>
<td>0.7146</td>
</tr>
</tbody>
</table>
Based on the estimated model 3 we can calculate the long-term version of Okun’s coefficient described in equation 3.1:

\[
\Delta u_t = 0.0819 - 1.4736 \text{d1991Q3} + 1.5806 \text{d1998Q1} - 0.2055 \text{g}_{yt} \\
\]

\[
t = (2.12) \quad (-3.53) \quad (3.76) \quad (-9.90) \\
p = 0.0115 \quad 0.0242 \quad 0.0198 \quad 0.0006 \\
pb = 0.1873 \quad 0.0000 \quad 0.0000 \quad 0.0000 \\
R^2 = 0.8079 \quad obs. = 134 (1980: 2 \rightarrow 2013: 3)
\]

where \( t \) is the t-ratio, \( p \) is the p-value and \( pb \) is the p-value calculated by bootstrap with 10,000 replications.

Again the Okun’s coefficient estimated around 0.2055 is plausible and in accordance with estimates to other countries. A look at Table 1 shows that this value is similar to estimates for Italy and Japan, but this value is about half of the estimates to other countries like USA, UK and Germany. This fact may indicate that the Brazilian labor market, as well as Japanese and Italian, is more rigid than the American, German, etc.

### 5 Final Remarks

In this article our objective was to estimate the "Okun's law" to Brazil with 1980Q1-2013Q3 quarterly data. Whereas the typical specification of Okun's law is \( \Delta u_t = \alpha - \beta g_y \), where \( \beta \) is the coefficient of Okun, we obtained estimates of \( \beta \) between -0.1878 and -0.2055, such values are similar to those of Italy (-0.21) and greater than the value of Japan (-0.12), but lower than those in countries like the UK (-0.58), USA (-0.52), France (-0.43) and Germany (-0.38). However, we must carefully consider these estimates because of the nature of the data used. We use quarterly GDP and unemployment rate for 1980Q1-2013Q3 period, but the unemployment rate refers only to some regions of the country.

The unemployment rate used was published by Monthly Employment Survey (PME) by Brazilian Institute of Geography and Statistics (IBGE). PME is conducted monthly in six metropolitan areas (Recife, Salvador, Belo Horizonte, of Rio de Janeiro, São Paulo and Porto Alegre), the sum of the economically active population (EAP) in these regions is about 25% of the Brazilian EAP. The GDP share of these metropolitan regions on Brazilian GDP was about 33% at the end of the 2000s, i.e., these metropolitan regions produce about one-third of Brazil's GDP and employ about a quarter of Brazilian EAP.

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Estimates of Sogner and Stiaassny (2002).
References


