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1 April 2013

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This version: October 22, 2013
First version: April 1, 2013

Abstract

Most of the assets on the balance sheet of typical banks are illiquid. This exposes banks to liquidity risk, which is one of the key risks for banks. Since the value of assets is determined by their risks, liquidity risk should be included in valuation. This paper develops a valuation framework for liquidity risk. An important element of the framework is the definition and derivation of an optimal admissible liquidation strategy that describes the assets a bank will liquidate in case of a liquidity stress event (LSE). The main result is that the discount rate includes a liquidity spread that is composed of three elements: 1. the probability of an LSE, 2. the severity of an LSE, and 3. the liquidation value of the asset.

The framework is illustrated by application to a stylized bank balance sheet.

1 Introduction

One of the main risks of a bank is liquidity risk. This is reflected by, for instance, the inclusion of liquidity risk measures in the Basel 3 framework [1]. The BIS paper "Principles for Sound Liquidity Risk Management and Supervision" [2], aimed at strengthening liquidity risk management in banks, stresses the importance of liquidity risk as follows: "Liquidity is the ability of a bank to fund increases in assets and meet obligations as they come due, without incurring unacceptable losses. The fundamental role of banks in the maturity transformation of short-term deposits into long-term loans makes banks inherently vulnerable to liquidity risk, both of an institution-specific nature and those affecting markets as a whole."

The research in this paper is motivated by a number of questions (relevant references are given and discussed in the next section):

1. Since liquidity risk is such an important risk for banks and the price of assets is determined by their risks, should liquidity risk affect the valuation of the assets of a bank?

2. It is well know from research in recent years that investors do expect a discount in the price for illiquid assets. But how do individual investors determine at what discount they are willing to buy or sell?

3. How are liquidity discounts of different assets related?

To address these questions this paper considers the impact of a bank's liquidity risk on the value of its assets. The purpose of this paper is to value this risk consistently across all assets on a bank's balance sheet, such as securities, mortgages, loans, and derivatives.

The approach focuses on the discounting of cashflows generated by the different assets. It is recognized that the discounting of cashflows of assets is determined by their liquidity through the possibility that the bank has to liquidate (a fraction of) the asset in the event of liquidity stress. As a consequence the discount rate includes a liquidity spread. The main result of this paper is that the liquidity spread is composed of the probability of a liquidity stress event (an event in which the bank is force to sell some of its assets), the severity of the liquidity stress event, and the liquidation value of the asset.

The outline of this paper is as follows: Firstly the relation of the research presented in this paper and existing literature is discussed. Section 3 develops a liquidity risk valuation framework and discusses some consequences. Section 4 extends the model to include credit risk and optionality. In section 5 a paradox is discussed and the value of the assets on a stylized bank balance sheet is calculated. Lastly the conclusions are summarized.

2 Related literature

2.1 Relation to Liquidity Pricing

Another line of research that is related to this paper is the extension of CAPM with liquidity risk. Two relevant papers are [3, 4], but a much larger literature exists. An important result of this line of research is that investors do require an extra return for illiquidity of an asset. In other words, the price of an asset receives a discount when the asset is illiquid. A question that is not addressed in this line of research is: how should an individual investor determine the discount he requires for illiquidity of an asset. This paper addresses this question for a specific type of investors, namely banks.

2.2 Relation to Liquidity Risk Management

Liquidity Risk Management at banks receives an increased attention since the credit crisis. An important aspect of liquidity risk management is that the risk is correctly priced. Indeed one of the principles of the BIS-paper [2] is about the pricing of liquidity risk.

Banks typically include liquidity risk in pricing by including the costs of their liquidity buffer, see e.g. [5] for a description of this method. The reasoning is that the liquid, high-quality assets in the liquidity buffer provide a lower return than the less liquid assets that a bank otherwise could invest in. This difference in return is interpreted as a cost of holding the liquidity buffer. This cost is then charged to (illiquid) assets through Funds Transfer Pricing.

Although the above approach provides a pragmatic way to price liquidity risk, there are two reasons why this cannot be considered a fundamental approach.

1. The value of the liquid assets in the liquidity buffer is given by their mark-to-market (MtM). Since these assets are by definition liquidly traded their MtM can be observed. Although from an interest rate income perspective the assets give a lower return which may be interpreted as a cost, this is questionable from a valuation perspective since these assets are valued at their MtM and do not represent a loss.

2. The lower return given by the liquid assets is a consequence of their liquidity (if this is indeed the distinction between assets in and outside the liquidity buffer). The use of this lower return to determine a cost and charge this to illiquid assets therefore is a circular reasoning[1].

[1] Besides being circular the reasoning is, strictly speaking, not consistent. E.g. a bank with a liquidity buffer of 20% of its assets which estimates a 400bp lower return on the assets in the liquidity buffer, would charge 400bp $\times$ 20%/80% = 100bp for illiquid assets. Clearly the 100bp charge is inconsistent with the assumed 400bp lower return.

This paper provides a more fundamental approach to pricing liquidity risk. It relates the probability of a liquidity stress event, the severity of the event, and the liquidation value of the asset to the liquidity spread of that asset.

2.3 Relation to research on liquidation values

This paper is further related to research on liquidation values of assets. In particular the relation between transaction size and liquidation values has been studied [6, 7]. This liquidation value of an asset is input in the liquidity risk valuation framework developed here.

An interesting suggestion made by some authors is that risk management should be based on liquidation values instead of the mark-to-market or fair value of assets. The problem with mark-to-market valuation according to Caccioli et al [8], is that it a large position when it is liquidated does not result in a cash amount equal to the mark-to-market value, since the position is sold at a discount. Therefore Caccioli et al argue that it is better to use impact-adjusted accounting, where the valuation of a position is based on its liquidation value.

This paper has a different approach, since it focuses on including liquidity risk in the fair valuation of assets. Nevertheless the result of this approach is that the fair value of an asset includes the liquidation value of that asset, such that the fair value of the asset is lower when the discount from liquidation is larger. Therefore the liquidity risk valuation framework proposed here may address some of the concerns of Caccioli et al.

3 Liquidity Risk Valuation Framework

Liquidity risk has various meanings and interpretations. This paper focuses on aspects of liquidity risk that affect the discounting of cashflows from illiquid assets. Therefore the following definition of liquidity risk is used in this paper:

Definition: Liquidity risk is the risk for an event to occur, that would force a bank to liquidate some of its assets.

Such an event can therefore be termed a liquidity stress event (LSE).

3.1 Liquidity Risk Model

In this paper LSEs are modelled as random events. The model consists of three components:

- The probability that an LSE occurs: $PL(t_1, t_2)$ will denote the probability of such an event between t_1 and t_2.

- The severity of an LSE. The severity will be indicated by the fraction of the assets that a bank needs to liquidate FL. By definition $0 \leq FL \leq 1$. For simplicity FL will be assumed to be a fixed (non-random) number.
- The dependence structure of LSEs and other events. The model assumes that LSEs are assumed to be independent from each other and from other events such as credit risk or market risk events. The independence of LSEs implies that the probability of N LSEs in the time interval t_1 to t_2 is given by $PL(t_1, t_2)^N$.

It is convenient to introduce an intensity $p(t) \geq 0$

$$PL(t_1, t_2) = \int_{t_1}^{t_2} p(t)dt, \tag{3.1}$$

where in the following the assumption is made that $p(t) = p$ is time-independent.

3.2 Valuation with liquidity risk

In an LSE a bank will liquidate some of its assets. These assets will be sold at a discount depending on the liquidity of the asset. This discount in case of an LSE may be recognized by defining an effective pay-off.

$$\text{Effective pay-off} = \begin{cases} \text{contractual pay-off} & \text{if no LSE occurs} \\ \text{stressed value} & \text{if LSE occurs} \end{cases} \tag{3.2}$$

The contractual pay-off includes all cashflows of the asset, for example optionality, cashflows in case of default, cashflows if triggers are hit etc.

The stressed value includes the discount for liquidating part of the position in the LSE. In case of a single LSE at time τ the stressed value may be expressed as

$$\text{stressed value} = fV(\tau)LV + (1 - f)V(\tau), \tag{3.3}$$

where $V(\tau)$ is the value of the asset at time τ, f is the fraction of the asset that the bank will liquidate, and LV is the liquidation value as a fraction of the value of the asset. It is assumed here that assets are divisible and any part of the assets can be liquidated.

The fraction f of the asset that the bank will liquidate will be determined by a liquidation strategy. In the next section the liquidation strategy that should be used in valuation is derived.

Definition: The value of an asset under liquidity risk is defined as the present value of the effective pay-off

$$V = PV[\text{Effective pay-off}] \tag{3.4}$$

Consider a cashflow of an illiquid asset at some future time T. In absence of default risk the value at time t of the cashflow is related to the value at time $t + dt$ through

$$V(t) = e^{-rdt}V(t+dt)(1-pdt) + e^{-rdt}[fV(t+dt)LV + (1-f)V(t+dt)]pdt \quad (3.5)$$

The first term on the r.h.s. is the contribution from the scenario that no LSE occurs between t and $t+dt$, the second term is based on (3.3) and is the contribution from the scenario that an LSE occurs. The contribution from multiple LSEs between t and $t+dt$ may be neglected as long as p is finite, since this contribution is of order $(pdt)^2$ and dt is infinitesimal small.

Equation (3.5) may be rewritten as

$$V(t) = e^{-rdt}V(t+dt)[1 - p(1-LV)fdt]\,. \quad (3.6)$$

By introducing a liquidity spread

$$l = p(1-LV)f\,, \quad (3.7)$$

this becomes

$$V(t) = e^{-rdt}V(t+dt)(1-ldt). \quad (3.8)$$

The value of a cashflow at a future time T of notional 1 in absence of default risk is derived by iterating (3.8)

$$V = e^{-(r+l)T}, \quad (3.9)$$

since $\lim_{dt\downarrow 0}(1-ldt)^{T/dt} = e^{-lT}$.

The liquidity spread (3.7) used in discounting depends on the fraction of the asset f that a bank liquidates, this fraction will be determined in the next section.

3.3 Liquidation strategy

Consider a balance sheet with a set of assets A_i with $i = 1, 2, ..., N$, where A_i denotes the market value and each asset has a unique liquidation value LV_i. Without loss of generality an ordering of the assets can be assumed: $LV_i > LV_j$ if $i < j$.

Definition: A liquidation strategy for a set of assets A_i is a set of fractions s_i of assets to sell such that

$$\sum_{i=1}^{N} s_i A_i = FL \sum_{i=1}^{N} A_i. \quad (3.10)$$

with $0 \le s_i \le 1$ and the sum over i covers all assets on the balance sheet.

Such a strategy could be, for instance, to sell the most liquid assets until sufficient assets have been liquidated to reach $FL\sum_i A_i$. Note that the strategy is allowed to depend on the order of the assets, but not on the liquidation values LV_i. The motivation is that a bank's liquidation strategy will be, more likely, of the type to liquidate assets based on their relative liquidity (e.g. most liquid assets first) instead of on their exact liquidation values.

Definition: An admissible liquidation strategy is a strategy s_i^* such that the liquidity spreads implied by the strategy

$$l_i = p(1 - LV_i)s_i^* \,, \tag{3.11}$$

satisfy the condition that for any set LV_i

$$LV_i < LV_j \Rightarrow l_i > l_j \,. \tag{3.12}$$

Definition: An optimal admissable liquidation strategy is an admissable liquidation strategy with the lowest loss in an LSE. This loss is defined as

$$\text{loss} = \sum_i s_i A_i (1 - LV_i) \,. \tag{3.13}$$

To demonstrate that the optimal admissible liquidation strategy is given by $s_i^* = s_j^*$ for all i, j, it first needs to be noted that a strategy with $s_i > s_j$ for $i < j$ is not an admissible strategy. Consider e.g. $s_1 > s_2$. Then the choice $LV_1 = LV_2 + \frac{s_1 - s_2}{2s_1}(1 - LV_2)$ implies $l_1 > l_2$. (It can be checked that this expression for LV_1 is a valid choice in the sense that $LV_1 > LV_2$ and $LV_1 < 1$.) Therefore $s_1 > s_2$ violates the requirement (3.12). Note that the same reasoning can be applied to any i, j with $i < j$, and that it is sufficient to have one choice of LV's that violates (3.12), since definition (3.12) should hold for any set LV's.

It can be concluded that the set of admissible liquidation strategies may be characterized by: $s_1 \leq s_2 \leq s_3 \leq ... \leq s_N$, where N denotes the last asset. Within this set the optimal choice is $s_1 = s_2 = s_3 = ... = s_N$, since it will lead to the lowest loss for the bank in an LSE. The conclusion is that the optimal admissible strategy is specified by $s_1 = s_2 = s_3 = ... = s_N = FL$.

The final step in the completion of the valuation framework is the determination what fraction of an asset f in (3.7) a bank will liquidate in an LSE. The optimal admissible liquidation strategy has been defined to determine this fraction. It is the natural choice for valuation of possible liquidation strategies, since it preserves the relation between liquidation values and liquidity spreads (3.12) and within this admissible set minimizes the loss of the liquidation of assets.

3.4 Summary of the model

Putting the above liquidity risk model, valuation approach and optimal admissible liquidation strategy together the result is the following.

A cashflow at time T of an asset A_i without default risk should be discounted with the discount factor

$$DF = e^{-(r+l_i)T}, \tag{3.14}$$

where the liquidity spread is given by

$$l_i = p(1 - LV_i)FL. \tag{3.15}$$

Note that the discountfactor of the cashflow depends on the liquidity of the asset that generates the cashflow through LV_i. The other two factors, the probability of an LSE p and the severity of an LSE FL, are not asset specific, but are determined by the balance sheet of the bank.

3.5 Some consequences of the model

A consequence of (3.15) is that liquidity spreads of different assets (on the same balance sheet) are related. Since in (3.15) the probability of an LSE and the fraction of assets that need to be liquidated are the same for all assets, it follows immediately that

$$\frac{l_i}{l_j} = \frac{1 - LV_i}{1 - LV_j}. \tag{3.16}$$

The liquidity spread of asset i and asset j are related through their liquidation values.

A nice feature of the model is that it allows to explain a different discount rate for a bond and a loan. Consider, for example, a zero-coupon bond and a loan with the same issuer/obligor, same maturity, notional, and seniority. The zero-coupon bond and loan therefore have exactly the same pay-off (even in case of default). Nevertheless if the zero-coupon bond is liquidly traded, a difference in valuation is expected. The model developed here, can provide an explanation for this difference. The above relation (3.16) shows that the liquidity spreads are related through the liquidation values of the zero-coupon and the loan. For example, if the probability of an LSE for a bank is estimated at 5% per year, and the severity of the event is that 20% of the assets need to be sold, and the liquidation value for the ZC-bond is estimated at 80% and for the loan at 0% (since the loan cannot be sold or securitized quickly enough) then the liquidity spreads for the bond and loan are:

$$l_{\text{bond}} = 20\text{bp}, \tag{3.17}$$

$$l_{\text{loan}} = 100\text{bp}. \tag{3.18}$$

These spreads are based on above example, and may differ significantly between banks. Nevertheless, they clarify that it is natural in this framework that a different discount rate is used for loans and bonds.

In this framework also the position size will affect the discount rate. Empirical studies find a linear relation between the size of the sale and the price impact [6, 7]. In the context of this paper this translates into a linear relation between the position size and the liquidation value:

$$LV_i = cx_i \tag{3.19}$$

where x_i is the size of position in asset i, e.g. the number of bonds, and c a constant. Consider a different position x_j in the same asset. From (3.16) it immediately follows that

$$\frac{l_i}{l_j} = \frac{x_i}{x_j}. \tag{3.20}$$

Given a linear relation between the size of a sale and the price impact, the framework derived here implies a linear relation between liquidity spread and position size.

3.6 Replication and Parameter Estimation

One of the important concepts in finance is the valuation of derivatives through determining the price of a (dynamic) replication strategy. Unfortunately, liquidity risk is a risk that cannot be replicated or hedged. In principle it is conceivable that products will be developed that guarantee a certain price for a large sale; e.g. for a certain period the buyer of the guarantee can sell N shares for a value $N \times S$, where S denotes the value of a single share. Such products would help in determining market implied liquidation values, but it is difficult to imagine that such products will be developed that apply to large parts of the balance sheet.

In any case, currently liquidity risk cannot be hedged. Nevertheless the risk should be valued. Therefore it seems appropriate to use the physical probability of an LSE and liquidation value to determine the liquidity spread in (3.15) as opposed to an imaginary risk neutral probability and liquidation value. Clearly, if it would be possible to hedge this risk then the risk neutral values implied by market prices should be used.

The physical probability of LSEs and the severity of the events are required to estimate the liquidity spread, see (3.15). These may be difficult to estimate. On the other hand, they only need to be estimated for the own institution (it would seem much easier than PD estimates for obligors of banks). Certainly after the credit crisis it is clear that the probability of an LSE for a bank is non-negligible.

4 Extensions of the model

4.1 Including Credit Risk

This section adds credit risk to the framework. Recall (3.6) with (3.7). The inclusion of default risk is straightforward under the assumption that default events are independent from LSEs. The result is

$$V(t) = e^{-rdt}V(t + dt)[1 - ldt - pd \times \mathrm{LGD}dt]\,, \tag{4.1}$$

where pd is the instantaneous probability of default and LGD the Loss Given Default. By introducing a credit spread

$$s_{\mathrm{credit}} = pd \times LGD \tag{4.2}$$

and solving (4.1) in a similar way as (3.6) gives the following value of a cashflow of nominal 1

$$V = e^{-(r+l+s_{\mathrm{credit}})T}. \tag{4.3}$$

The discount rate consists of a risk-free rate, a liquidity spread and a credit spread.

4.2 Liquidity Risk for Derivatives

Liquidity risk also affects the value of derivatives. In a Black-Scholes framework liquidity risk results in an extra term in the PDE [9]

$$\partial_t V + rS\partial_S V + \frac{1}{2}\sigma^2 S^2 \partial_S^2 V = rV + l_V \max(V, 0). \tag{4.4}$$

Here V denotes the value of the derivatives' position, S the underlying stock, σ the volatility, and l_V the liquidity spread of the derivatives' position. The last term on the r.h.s. is the extra term coming from liquidity risk and is in fact equivalent to the last term on the r.h.s. of (3.8). The max-function reflects that the value of the derivative can be both positive and negative (depending on the type of derivative) and that only positions with a positive value will be potentially liquidated in an LSE. Note that it is assumed that the underlying is perfectly liquid (in the sense that its liquidation value $LV = 1$), otherwise an additional term would occur to include the illiquidity of the underlying.

In [9] also extensions of (4.4) are discussed that include credit risk.

A remarkable feature of (4.4) is that it is similar to models that some authors have proposed for inclusion of funding costs in the valuation of derivatives. In particular the extra term $l_V \max(V, 0)$ has the exact same form as the term for inclusion of funding costs derived by e.g. [10], with funding spread replaced by liquidity spread. The model above is more complex than the model including funding costs since the liquidity spread may be dependent on, for example, position size.

5 A paradox and an example

5.1 A paradox

As discussed in section 3 the liquidity spread is determined by the loss from a forced sale of part of the assets in a liquidity stress event. The applied sell strategy is to sell the same fraction of each asset. In practice however one would sell the most liquid assets as this results in a smaller loss. Since a larger loss is accounted for in the valuation, it seems that a risk-free profit can be obtained by holding an appropriate amount of liquid assets or cash as a buffer for a liquidity stress event.

To analyze the paradox, consider a bank with a simple balance sheet, as shown below

$A = 80$	$L = 80$
$C = 20$	$E = 20$

This bank has 80 illiquid assets, 20 cash, and its funding consists of 80 liabilities, and 20 equity. It is exposed to an LSE where 20% of the funding is instantaneously removed.

If the stress event occurs the resulting balance sheet used in the valuation is

$A = 64$	$L = 60$
$C = 16$	$E = 20$

The sale of the assets will result in a loss $= (1 - LV_A)16$. This loss is born by the equity holders, who in this setup, provide the amount $(1 - LV_A)16$. This amount combined with the result from the sale of the assets $LV_A 16$, and a cash amount of 4 covers the withdrawal of funding. Note that this can be viewed as a two-step approach whereby the funding withdrawal is covered by the cash and immediately supplemented by the sale of the assets and the cash provided by the equity holders.

In practice a bank will use its cash buffer to compensate the loss of funding. In contrast to the strategy of the pro rata sale of assets used for valuation, this strategy will not lead to a loss. The resulting balance sheet is

$A = 80$	$L = 60$
$C = 0$	$E = 20$

The paradox is that the value of the assets includes the possibility of a loss (through the liquidity spread), whereas in reality this loss seems to be avoided by using the cash as a buffer.

However, the bank is now vulnerable to a next LSE, whereby 20% of its funding is withdrawn. To be able to withstand such an event a cash buffer of 16 is required. To avoid any liquidity risk this buffer should be realized immediately, which can be achieved by the same sale of assets as in the strategy for valuation, resulting in the same loss. Therefore, to avoid any liquidity risk the same loss is born by the equity holders, which resolves the paradox.

In practice the assets may be sold over a larger period of time, thereby the bank chooses to accept some liquidity risk to avoid the full loss by an immediate sale. The optimal strategy in practice is the result of risk reward considerations.

The paradox and its resolution suggests another argument that the liquidation strategy where an equal fraction of each asset is liquidated is the appropriate strategy to be used in valuation. Such a strategy keeps liquidity risk of the assets of a bank's balance sheet at the same level. After liquidation of an equal fraction of the assets the average liquidity of the assets has not increased or decreased. Therefore taking a constant level of liquidity risk as starting point would lead to the same liquidation strategy, an equal fraction of all assets, for valuation.

5.2 Example for a stylized balance sheet

Consider the following stylized balance sheet of a bank.

retail loans	10	deposits	60
corporate loans	20	wholesale funding	30
mortgages	40	equity	10
central bank eligible bonds	10		
corporate bonds$> AA^-$	10		
cash	10		

The bank has considered its vulnerability to liquidity stress events, and it concludes that in a stress event its deposits can reduce by 15 and its wholesale funding also by an amount 15 within 3 months. The probability of such an event is estimated at 5% per year. The translates into the parameters

$$p = 5\% \qquad FL = 30\% \tag{5.1}$$

The bank decides to base the liquidation values of its assets on the Basel 3 Required Stable Funding (RSF) factors. The basel document [1] states: "The RSF factors assigned to various types of assets are parameters intended to approximate the amount of a particular asset that could not be monetised through sale or use as collateral in a secured borrowing on an extended basis during a liquidity event lasting one year". Although this does not exactly match the definition of the liquidity stress event identified by the bank, since the bank's stress event only lasts 3 months, the bank chooses to identify

$$1 - LV = RSF \tag{5.2}$$

for each asset.

The result for the liquidity spread for the different assets is given in the table below.

Asset	RSF	liquidity spread (in bp)
retail loans	85%	127.5
corporate loans	65%	97.5
mortgages	65%	97.5
central bank eligible bonds	50%	75
corporate bonds> AA^-	20%	30
cash	0%	0

The above liquidity spreads are the result of the assumptions of the bank in the above example. For a specific bank the liquidity spreads depend on bank-specific features, such as the fraction of stable, less-stable, and non-stable deposits, which affect FL and p. Nevertheless the stylized bank above illustrates how different assets get different liquidity spreads.

6 Conclusions

This paper develops a liquidity risk valuation framework. It is shown that liquidity risk of a bank affects the economic value of its assets. The starting observation is that under an LSE the bank needs to liquidate some of its assets, which means these will be sold at a discount. To develop the valuation framework a liquidation strategy of the bank needs to be determined. It is shown that the optimal liquidation strategy suitable for valuation is a strategy where of each asset the same fraction is liquidated. The result is that cashflows are discounted including a liquidity spread. This liquidity spread consists of three factors: the probability of an LSE, the severity of an LSE, and the asset-specific discount in case of liquidation in an LSE.

This result has a number of consequences

- The value of a position is not independent of the rest of the balance sheet, since the balance sheet determines probability of an LSE and the severity of an LSE. In particular the same position on two different balance sheets may be valued differently.
- Two pay-offs that are exactly the same, but have a different liquidity may be valued differently. For example, a bullet loan and a zero-coupon bond of the same obligor/issuer with the exact same pay-off will have different liquidity spreads if the zero-coupon bond is liquidly traded (and the bullet loan is not).
- The size of a position affects the valuation. E.g. if a position in bonds is large compared to the turnover in an LSE, the liquidation value of the position may be lower than the liquidation value of a single bond. Therefore a large position will have a higher liquidity spread than a small position.

6.1 Future research

There are a number of areas that allow for future research.

- The observation that the PDE including liquidity risk (4.4) is the same PDE that others have derived including funding costs [10] suggests a relation between funding costs and liquidity risk. It is of interest to understand the relation between funding costs and the liquidity, even if only in some limiting cases (no taxes, assets fully diversified).
- Since the valuation of assets does not include funding costs, but does include liquidity risk, this suggests that the FTP should be based on the liquidity spread instead of funding costs. However, the impact on interest risk management would require careful investigation.
- The framework implies that the liquidity spread of two assets on the same balance sheet is related through a simple relation involving only the liquidation values of the assets (3.16). This suggests the same relation should hold for traded prices of liquid and less liquid assets (at least if a sufficient number of investors trades both assets). This allows for an empirical test of the model.
- The securitization of illiquid assets, such as loans and mortgages, into more liquid securities enhances the value of the assets. Within the liquidity risk valuation framework developed here, it is possible to estimate this value.

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