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Justifiable Choice

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Abstract

Most existing decision-making models assume that choice behavior is based on preference maximization even when the preferences are incomplete. In this paper we study an alternative approach - "justifiable choice": each agent has several preference relations ("justifications"), and she can use each justification in every choice problem. We present a new behavioral property that requires an alternative to be chosen if it is not inferior to all mixtures of chosen alternatives, and show that this property characterizes justifiable choice. The main application of this property yields a multiple-utility representation, which substantially differs from existing related representations. In addition, we obtain a multiple-prior representation, and study the notions of indecisiveness and being more decisive.

Key words: menu effects, incomplete preferences, multiple utilities, multiple priors, indecisiveness, non-binary choice, tradeoff contrast effect.
JEL classification: D81

1 Introduction

In several disciplines, there has been significant interest in decision-making models in which one's preferences are allowed to be incomplete, thereby letting the decision maker remain indecisive on occasion (see, e.g., Roemer, 1999; Rigotti and Shannon, 2005; Mandler, 2005; Manzini and Mariotti, 2007; Salant and Rubinstein, 2008; Bernheim and Rangel, 2009). Most such models assume that the decision maker maximizes an incomplete preference relation (see, e.g., Aumann, 1962; Bewley, 2002; Dubra, Maccheroni and Ok, 2004;

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Eliaz and Ok, 2006).[2]

However, upon relaxation of the completeness axiom, it is not clear why preference maximization should be deemed as the proper characterization of rational behavior. An alternative approach for choice with incomplete preferences is *justifiable choice* (see, Lehrer and Teper, 2011). According to this approach, the decision maker has several complete preference relations called *justifications* (or rationales). Additional payoff-irrelevant information that is available during the choice process determines which justification is used,[3] and the decision maker selects the best element according to this rationale. We assume that each justification can be used in every choice problem. Justifiable choice differs from preference maximization in two key aspects: (1) it allows the revealed preferences to depend on the menu, and (2) it does not allow the simultaneous use of conflicting rationales.

In this paper we present a new behavioral property: *convex axiom of revealed non-inferiority* (henceforth, *CARNI*), and show that it characterizes justifiable choice.

1.1 New Behavioral Property (CARNI)

In our framework, choice behavior is described by a choice correspondence C which selects, in each closed set of alternatives, a non-empty subset of choosable alternatives. We say that alternative x is revealed *inferior* to alternative y, if x is never chosen when y is a mixture of alternatives in the choice set. The new axiom we propose, *convex axiom of revealed non-inferiority* (CARNI), requires that an alternative be chosen if it is revealed not to be inferior to all the mixtures of the chosen alternatives. In Subsection 2.2 we study the relationships between CARNI and existing axioms.

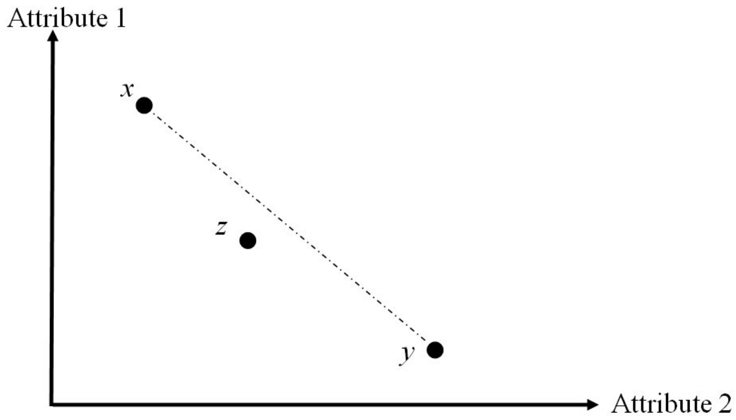
We present two motivations for CARNI: one normative and one descriptive. Our main model uses the framework of von Neumann and Morgenstern (1944), where each choice set includes lotteries over a finite set of consequences. In some situations, decision makers may use internal randomization devices. In such situations, when a decision maker has to select one of the elements in A, she may base her choice on a private lottery (i.e., tossing a coin), and by doing this, she can induce compound lotteries. If the decision maker satisfies *reduction of compound lotteries*, then these compound lotteries are equivalent to mixtures of alternatives in A, and CARNI has a normative appeal.

A descriptive appeal of CARNI is that it captures one aspect of the *tradeoff contrast effect* of Simonson and Tversky (1992). According to this effect, the tendency to choose an alternative is hindered (or enhanced) if the tradeoffs within the set under consideration are unfavorable (or favorable) to that option. One aspect of this bias is described in Figure 1 (where each axis represents a positive attribute). Because the contrast between the x-y tradeoff and the x-z and y-z tradeoffs is unfavorable to object z, it is expected to fare worse (to be chosen less often) in the triple than in the pairs. That is, z fares worse

[2] Eliaz and Ok (2006) assume it explicitly. The other papers use an incomplete preference relation as the primitive of the model, and by that, implicitly assume that choice behavior is determined by maximizing this relation.

[3] We do not explicitly model the process in which payoff-irrelevant information determines the justification. Some examples for such processes are: framing effect (Tversky and Kahneman, 1981), availability heuristics, and anchoring (Tversky and Kahneman, 1974).

Figure 1. Contrast Tradeoff Detraction



because it is inferior to a mixture of other alternatives in the choice set. CARNI captures a binary version of this aspect: an element is chosen if it is not inferior to mixtures of alternatives in the choice set.

We note, that this descriptive appeal of CARNI is somewhat limited because: (1) CARNI does not capture other aspects of the tradeoff contrast effect, such as the *attraction effect* and the *compromise effect* (see de-Clippel and Eliaz, 2011 for an axiomatic model that captures these aspects); (2) the experimental evidence for the tradeoff contrast effect is for choice between multi-attribute products (Simonson and Tversky, 1992, 1993), while we apply it to choice between lotteries; in a separate note (Heller, 2010) we present some experimental evidence that suggests the existence of a moderate degree of this effect also for choice between lotteries.

Our main result shows that satisfying three standard axioms (non-triviality, continuity, and independence) and CARNI is equivalent to the following multiple-utility representation: There exists a unique convex and compact set of vN-M utility functions, such that a lottery is selected if it is best with respect to one of these utilities. [4]

1.2 Comparison With Existing Multiple-Utility Representation

Eliaz and Ok (2006) presented a closely related preference maximization model. Their key axiom, *weak axiom of revealed non-inferiority* (WARNI), is similar to CARNI except that it does not relate to mixtures. It requires that an alternative be chosen if it is revealed not to be inferior to all of the chosen alternatives. This yields the following representation: There exists a convex set of vN-M utilities, such that lottery q is chosen if no lottery in the choice set is strictly better than q with respect to all of these utilities. In the following paragraphs we detail the two key aspects in which our model differs from theirs: menu effects and conflicting rationales.

Recently, Manzini and Mariotti (2010) experimentally tested how people violate the weak axiom of revealed preference (WARP). Specifically, they divide the possible violations of WARP into two groups: 1) *pairwise inconsistency* - choices over pairs of alternatives are not transitive; and 2) *menu effects* - choices over two-element sets do not induce choices

[4] A similar representation was presented non-axiomatically in Levi (1974).

over larger sets. Manzini and Mariotti show that menu effects are largely responsible for failures of WARP, and they conclude that on the basis of their data, "*any procedure that fails to account for menu effects will not make a significant improvement of the standard maximization model.*" WARNI implies that choices must be consistent with preference maximization,[5] and thus it cannot account for menu effects. *CARNI presents a small deviation from WARP that is able to accommodate an interesting menu effect, while retaining a normative appeal.*

In Eliaz and Ok's representation an alternative can be chosen based on the simultaneous use of conflicting rationales: lottery q can be chosen in the triple $\{q, r, r'\}$ if it is better than r according to one utility, and better than r' according to a different utility, even though q does not maximize any utility. In our model, an element can be chosen only if it maximizes one of the utilities. This seems more natural in many choice situations. One example for such a situation, which is described in Lehrer and Teper (2011), is decisions in large-scale organizations, where responsibility for different choices is delegated to different employees, each employee has a different rationale, and all rationales are consistent with the organization's common information and policy. Another example for such a situation is the following.

Example 1 There are four consequences: *bn*="beef near", *bf*="beef far", *cn*="chicken near", *cf*="chicken far". Let q be a 50:50 lottery with prizes bf and cf. Assume that the decision maker may like either chicken or beef (two justifications) and also dislikes eating too far from home. Then q may beat bn based on the "chicken" justification (that is, $\{bn, q\} = C(\{bn, q\})$); similarly, q may beat cn based on the "beef" rationale ($\{cn, q\} = C(\{cn, q\})$). But intuitively, if both bn and cn are available, q should not be chosen ($\{bn, cn\} = C(\{bn, cn, q\})$): the decision maker can get her favorite meal at a nearby restaurant, regardless of whether she wants beef or chicken. Observe, that this choice behavior is consistent with CARNI (q is not chosen in the triple because it is inferior to the mixture of bn and cn), and is inconsistent with WARNI (as WARNI implies: $q \in C(\{bn, q\})$ and $q \in C(\{cn, q\}) \Rightarrow q \in C(\{bn, cn, q\})$).

1.3 Other Related Literature

Our paper is related and inspired by two strands of literature. The first is the literature studying choice with incomplete preferences and multiple rationales (e.g., Nehring, 1997; Kalai, Rubinstein and Spiegler, 2002; Mandler, 2005; Manzini and Mariotti, 2007; Salant and Rubinstein, 2008; and Cherepanov, Feddersen and Sandroni, 2010). In most of these papers, the model only describes choices from subsets of an arbitrary finite set of outcomes, and little structure is imposed on the different rationales. In this paper, we assume that there is also data about the choices from lotteries over outcomes, and this allows us to impose more structure on the justifications: the set of justifications is convex and closed, and each justification is a complete and affine preorder.

The second strand of literature generalizes expected utility and subjective expected utility by weakening some of its assumptions (e.g., Machina, 1982; Gilboa and Schmeidler, 1989;

[5] That is, alternative q is chosen in menu A if and only if it is chosen in any couple $\{q, r\}$ for each element r in A.

Schmeidler, 1989; Ghirardato, Maccheroni and Marinacci 2004; Klibanoff, Marinacci, and Mukerji, 2005; Maccheroni, Marinacci, and Rustichini, 2006; Gilboa et al., 2010; Ok, Ortoleva, Riella, 2012). Most of this literature weakens the independence axiom, and keep the weak axiom of revealed preference. In this paper we do the opposite (a similar approach is used in Seidenfeld, Schervish and Kadane, 2010).

1.4 Structure

Section 2 presents the models and the results: the main result described above, and an analogous multiple-prior representation in the framework of Anscombe and Aumann (1963). Section 3 studies the notions of indecisiveness and being more decisive in our models. All the proofs are presented in Section 4.

2 Models and Results

2.1 Risk (von Neumann-Morgenstern Framework)

2.1.1 Preliminaries

Let X be a finite set of consequences (certain prizes).[6] Let $Y = \triangle(X)$ be the set of lotteries over X. Let $\mathcal{Y}$ be the set of non-empty closed sets in Y. The mixture (convex combination) of two lotteries is defined as follows: $(\alpha q + (1-\alpha) r)(x) = \alpha q(x) + (1-\alpha) r(x)$ (where $\alpha \in [0,1]$, $q, r \in Y$ and $x \in X$). Similarly, given $A \in \mathcal{Y}$, let $\alpha q + (1-\alpha) A$ denote the set of lotteries that include all convex combinations of q with lottery r in A, with weights α and $1-\alpha$ respectively: $(\alpha q + (1-\alpha) A) = \{\alpha q + (1-\alpha) r | r \in A\}$.

The primitive of the model is a choice correspondence C over $\mathcal{Y}$.[7] For each such set $A \in \mathcal{Y}$, $C(A)$ is a non-empty subset of A. The interpretation of C is the following: when a decision maker faces a choice from menu A, she selects one of the alternatives in $C(A)$, and any alternative in $C(A)$ may be chosen. That is, the decision maker considers all the elements in $C(A)$, and only them, as choosable alternatives. The selection of a specific element in $C(A)$ is not explicitly modeled. When $q \in C(A)$ we say that q is (sometimes) chosen (or selected) from A; similarly, when $q \notin C(A)$ we say that q is not chosen from A. Given $A \in \mathcal{Y}$, *conv* (A) denotes the convex hull of A (the smallest convex set that contains A).

The following three standard axioms (assumptions) are imposed on C:

A1 *Non-triviality.* $\exists A \in \mathcal{Y}$ and $\exists q \in A$, such that $q \notin C(A)$.
A2 *Continuity.* For any lottery $q \in Y$, the set $\{r \in Y | r \in C(\{q, r\})\}$ is closed, and the set $\{r \in Y | \{r\} = C(\{q, r\})\}$ is open.

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[6] We define X to be finite for simplicity of presentation. Both models can be extended to a compact metric space of outcomes (see, Evren, 2010; Gilboa et al., 2010).
[7] We define C only on closed sets because in non-closed sets the Pareto frontier might be an empty set. Our results remain the same if C is defined only on finite (non-empty) sets.

A3 *Independence.* Let $q \in A \in \mathcal{Y}$, $r \in Y$ and $\alpha \in (0,1)$. $q \in C(A) \Leftrightarrow \alpha r + (1-\alpha) q \in C(\alpha r + (1-\alpha) A)$.

Axioms A1-A3 are standard. Axiom A1 requires that C be non-trivial (there is a choice set with at least one unchoosable alternative). Axiom A2 (continuity) is equivalent to the requirement that for any lottery $q \in A$, the sets $\{r | r \succeq q\}$ and $\{r | r \preceq q\}$ are closed, where $\succeq$ is the preference relation that is revealed from binary choices: $r \succeq q \Leftrightarrow r \in C(\{q,r\})$.[8] Assume that the decision maker is going to select lottery q in A, when she finds out that there is probability α that she will be obliged to take lottery r. Axiom A3 (independence) requires the decision maker to choose the mixture of q and r in the new choice problem (the mixture of A and r).

2.1.2 Convex Axiom of Revealed Non-Inferiority (CARNI)

It is well known that a choice correspondence is consistent with (complete) preference maximization if and only if it satisfies WARP:

WARP *(Weak Axiom of Revealed Preference)* - Let $A, B \in \mathcal{Y}$ and $q, r \in A \cap B$. $q \in C(A)$ and $r \in C(B)$ implies $q \in C(B)$.

That is, if q and r are elements in the intersection of two sets, q is chosen in the first set, and r is chosen in the second set, then both alternatives should be chosen in both sets. Von Neumann and Morgenstern (1944) show that Axioms A1-A3 and WARP are equivalent to expected utility representation: There exists a unique vN-M utility function u, such that the chosen lotteries are best according to u. That is, for every set $A \in \mathcal{Y}$ and every lottery $q \in A$: $q \in C(A) \Leftrightarrow u(q) \geq u(r) \ \forall r \in A$.

With an eye to our relaxation of WARP, we formulate it slightly differently:

WARP' *(equivalent formulation* to WARP) - Let $q \in A \in \mathcal{Y}$. If there exists $r \in C(A)$ and $B \in \mathcal{Y}$ such that $q \in C(B)$ and $r \in B$, then $q \in C(A)$.

WARP is appropriate when the psychological preferences of the decision maker are complete. In such cases, if q is selected from a menu that includes r then it implies that q is revealed to be as good as r. Thus if r is chosen from A so is q.

When the psychological preferences are incomplete, there is a difference between something being superior and it being non-inferior for a decision maker. Eliaz and Ok (2006) propose the following axiom to deal with choice that is induced from incomplete preferences:

WARNI (Weak Axiom of Revealed Non-Inferiority) - Let $q \in A \in \mathcal{Y}$. If for every $r \in C(A)$ there exists $B \in \mathcal{Y}$ such that $q \in C(B)$ and $r \in B$, then $q \in C(A)$.

According to Eliaz and Ok (2006)'s definition, element q is revealed not to be inferior to r, if q is chosen from a set and r is an element in that set. WARNI requires that if q is revealed not to be inferior to all of the alternatives chosen from A, then it must be chosen from A as well. Following Eliaz and Ok (2006) one can show that axioms A1-A3 and WARNI are equivalent to the following multiple-utility representation: There exists a

[8] Alternatively, A2 is equivalent to the requirement that sets $\{r | r \succ q\}$ and $\{r | r \prec q\}$ are open, where $\succ$ is the revealed strict preference relation ($r \succ q \Leftrightarrow \{r\} = C(\{q,r\})$).

convex and compact set U of vN-M utility functions (unique up to linear transformations), such that for every $A \in Y$ and every lottery $q \in A$: [9]

$$q \in C(A) \Leftrightarrow \forall r \in A, \exists u_r \in U, \text{ s.t. } u_r(q) \geq u_r(r). \tag{1}$$

As discussed in the introduction, in some choice situations, it seems more appropriate to require a convex variation of WARNI. This requirement is captured by CARNI:

A4 *Convex Axiom of Revealed Non-Inferiority (CARNI).* Let $q \in A \in \mathcal{Y}$. If $\forall r \in conv(C(A))$ there exists $B \in \mathcal{Y}$ such that $q \in C(B)$ and $r \in conv(B)$, then $q \in C(A)$.

We say that element q is revealed not to be inferior to r, if q is selected from a set and r is a mixture of elements in that set. CARNI requires that if q is revealed not to be inferior to all the mixtures of the elements chosen from A, then it must be chosen from A as well. [10] The relationships between CARNI and related existing axioms are discussed in Subsection 2.2.

2.1.3 Representation Theorem

The standard axioms A1-A3 (non-triviality, continuity, independence) and CARNI (A4) yield a multiple-utility representation in which a lottery is chosen if and only if it is best with respect to one of the utilities. Formally:

Theorem 1 *Let C be a choice correspondence over $\mathcal{Y}$. The following are equivalent:*

(1) C satisfies axioms A1-A4.
(2) There exists a convex compact set U of of vN-M utility functions, such that:
  (a) for every $A \in \mathcal{Y}$ and every lottery $q \in A$:

$$q \in C(A) \Leftrightarrow \exists u \in U, \ s.t. \forall r \in A, \ u(q) \geq u(r). \tag{2}$$

  (b) There are two lotteries $\underline{p}, \overline{p} \in Y$ such that $\forall u \in U$, $u\left(\underline{p}\right) < u(\overline{p})$.

Moreover, the set U is unique up to positive linear transformations. [11]

2.1.4 Sketch of Proof

The formal proof of Theorem 1, like all other proofs in the paper, appears in Section 4. In what follows we briefly sketch the main parts of the proof, in order to explain the intuition of the result.

Eliaz and Ok (2006) show that WARNI and the standard axioms yield a multiple-utility representation in which an element is chosen if it can beat any other alternative in the

[9] Eliaz and Ok (2006)'s representation is somewhat different than (1) due to their different continuity requirements. Their representation is as follows: $q \in C(A) \Leftrightarrow \forall r \in A$, $(\exists u_r \in U, \text{s.t.}\, u_r(q) > u_r(r)$ or $\forall u \in U\, u(q) = u(r))$.

[10] CARNI could be stated as an if and only if property (see Lemma 3): q is chosen in A if and only if it is revealed not to be inferior to all the mixtures of the elements chosen from A.

[11] That is, if both U and V are convex compact sets that represent the same choice correspondence then $\forall u \in U$, $\exists v \in V$ such that $u = a \cdot v + b$ where $a > 0$ and $b \in R$.

choice set for at least one vN-M utility (as described in (1)). Replacing WARNI by CARNI yields a similar representation, only this time the chosen element has to beat any other alternative in the convex hull of the choice set:

$$q \in C(A) \Leftrightarrow \forall r \in conv\left(A\right), \, \exists u_r \in U, \text{ s.t. } u_r\left(q\right) \geq u_r\left(r\right).$$

This representation is equivalent to:

$$q \in C(A) \Leftrightarrow \min_{r \in conv(A)} \max_{u \in U} \left(u\left(q\right) - u\left(r\right)\right) \geq 0.$$

Since both the set of utilities and the convex hull of the menu are convex, and since all the utilities in U are linear, we can now use the minimax theorem to reverse the order of the minimization and maximization above and obtain that:

$$q \in C(A) \Leftrightarrow \max_{u \in U} \min_{r \in conv(A)} \left(u\left(q\right) - u\left(r\right)\right) \geq 0.$$

This is equivalent to:

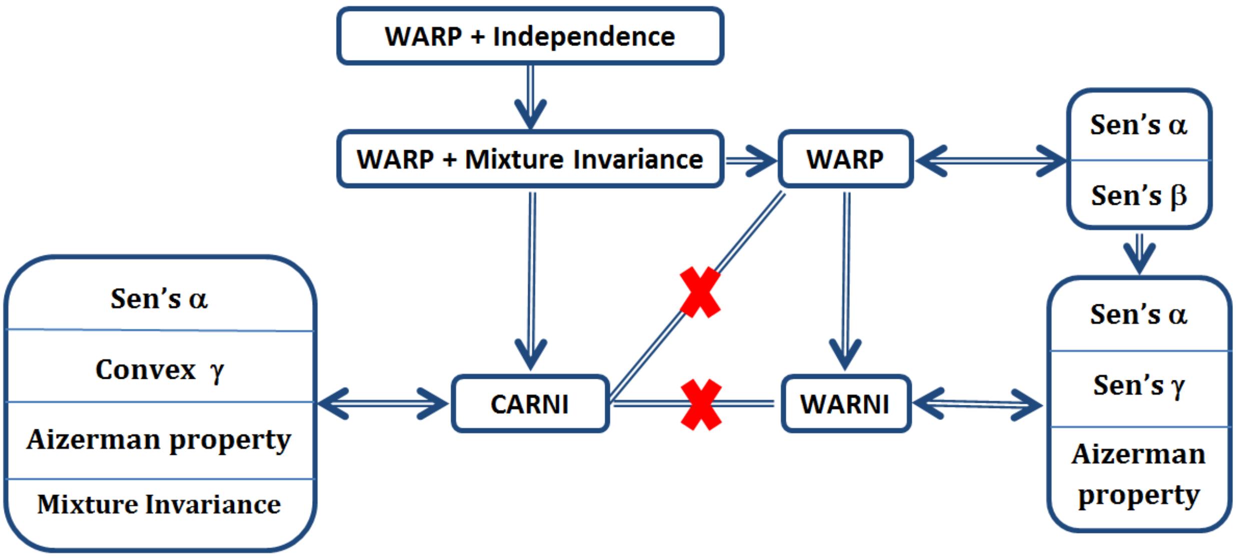
$$q \in C(A) \Leftrightarrow \exists u \in U \text{ s.t. } \forall r \in conv(A), \, u\left(q\right) \geq u\left(r\right).$$

Since all the utilities in U are linear, one can replace $conv\left(A\right)$ with A and obtain (2).

2.2 Relationships Between CARNI and Existing Axioms

For expositional purpose, we present in Figure 2 the relations between CARNI and existing axioms in the literature. We hope that this presentation will be beneficial to the reader and will simplify the understanding of CARNI. However, note that the results in this paper do not rely on these relations.

Figure 2. Logical Implications of CARNI and Related Axioms



The axioms that are presented in Figure 2 are defined as follows:

- **Mixture Invariance** (Seidenfeld et al., 2010, axiom 2b) - $q \in C(A) \Rightarrow q \in C(conv(A))$: If an element is chosen in a set, it is also chosen in the convex hull of the set. The intuition of Mixture Invariance is that if the decision maker can implicitly use internal randomization devices and she satisfies the *reduction of compound lotteries*, then a chosen element should still be chosen when a mixture of existing alternatives is added to the choice set.
- **Sen's α** (Sen, 1971; also called contraction property)- For any $A, B \in \mathcal{Y}$, if $q \in A \subseteq B$ and $q \in C(B)$, then $q \in C(A)$: If an element is chosen in a set, it should also be chosen in a subset.
- **Sen's β** (Sen, 1971) - For any $A, B \in \mathcal{Y}$ with $A \subseteq B$, if $q, r \in C(A)$ and $q \in C(B)$, then $r \in C(B)$: If two elements are chosen in a set, and one of them is also chosen in a superset, then both of them should be chosen in the superset.
- **Sen's γ** (Sen, 1971) - Let $M \subseteq \mathcal{Y}$ be a collection of sets and let A be the union of all these sets. If $A \in \mathcal{Y}$ and $q \in C(B)$ for each $B \in M$, then $q \in C(A)$: If an element is chosen in each set of some class, it should also be chosen in the union of all these sets.
- **Convex γ** - Let $M \subseteq \mathcal{Y}$ be a collection of sets and let A be the union of all these sets. If $A \in \mathcal{Y}$, A is convex, and $q \in C(B)$ for each $B \in M$, then $q \in C(A)$: This axiom is the same as Sen's γ except that it is restricted to cases where the union of the sets (A) is convex. It is immediate to see that Sen's γ implies Convex γ.
- **Aizerman property** (Aizerman and Malishevski, 1981, property O; Chernoff, 1954, postulate 5*) - For any $A, B \in \mathcal{Y}$, and for any $q \in C(A)$, if $C(B) \subseteq A \subseteq B$, then $q \in C(B)$: A choosable element is still chosen after adding unchosen alternatives to the menu.

The following observations are implied from Figure 2:

(1) There is no logical implication between CARNI and WARNI (or WARP). On the one hand, CARNI requires non-inferiority against a larger set of alternatives as a necessary condition for being chosen. On the other hand CARNI defines non-inferiority in a weaker way (there is a larger collection of sets in which an element may be revealed not to be inferior to another element). Example 1 demonstrates a choice correspondence that satisfies CARNI and violates WARNI. Modifying the choice in that example by having $\{bn, cn, q\} = C(\{bn, cn, q\})$, would give a choice correspondence that satisfies WARNI and violates CARNI (given that q is inferior to $0.5bn + 0.5cn$).
(2) WARP together with Mixture Invariance implies CARNI.
(3) It is well known that WARP can be decomposed into two independent axioms: Sen's α and Sen's β. CARNI (like WARNI) only satisfies Sen's α. As discussed in Eliaz and Ok (2006, Remark 1) Sen's α has a strong normative appeal, while the normative appeal of Sen's β is ambiguous.
(4) CARNI can be decomposed into four independent axioms: Sen's α, Convex γ (a weakening of Sen's γ to convex sets), Aizerman Property and Mixture Invariance.
(5) It may be of independent interest to note that WARNI can be decomposed into three well-known independent axioms: Sen's α, Sen's γ and Aizerman Property.
(6) It is well-known (Sen, 1971) that a choice correspondence is consistent with preference maximization (also called binariness or normality) if and only if it satisfies both Sen's α and Sen's γ. This implies that a choice correspondence which satisfies CARNI is

in general inconsistent with preference maximization (unlike WARNI or WARP).

2.3 Uncertainty (Anscombe-Aumann Framework)

2.3.1 Model

In this model we follow the framework of Anscombe-Aumann (1963, as reformulated in Fishburn, 1970). Similar to the first model, X is a finite set of outcomes and $Y = \triangle(X)$ is the set of lotteries. Let S be a finite set of states of nature, and, abusing notation, let $S = |S|$. Let $L = Y^S$ be the set of all functions from states of nature to lotteries. Such functions are referred to as acts. Endow this set with the product topology, where the topology on Y is the relative topology inherited from $[0,1]^X$. Let $\mathcal{L}$ be the set of all closed and non-empty sets in L. Abusing notation, for an act $f \in L$ and a state $s \in S$, we denote by $f(s)$ the constant act that assigns the lottery $f(s)$ to every state of nature. Similarly for set $A \in \mathcal{L}$ and state $s \in S$, let $A(s)$ denote the act-wise set of constant acts: $A(s) = \{f(s) \,|f \in A\}$.

Mixtures (convex combinations) of acts are performed point-wise. In particular if $f, g \in L$ and $\alpha \in [0,1]$, then $(\alpha f + (1-\alpha)g)(s) = \alpha f(s) + (1-\alpha)g(s)$ for every $s \in S$. Similarly, let $(\alpha f + (1-\alpha)A)$ denote the set where each $g \in A$ is replaced by $\alpha f + (1-\alpha)g$: $(\alpha f + (1-\alpha)A) = \{\alpha f + (1-\alpha)g | g \in A\}$. As in the former model, the primitive is a choice correspondence C over $\mathcal{L}$, which satisfies that for each $A \in \mathcal{L}$, $C(A)$ is a non-empty subset of A.

The following five axioms are imposed on the choice correspondence:

B0 *Monotonicity.* Let $f \in A \in \mathcal{L}$ and $g \in B \in \mathcal{L}$. If $\forall s \in S$, $f(s) \in C(f(s), g(s))$ then: (i) $g \in C(B) \Rightarrow f \in C(B \cup \{f\})$, and (ii) $C(A) \subseteq C(A \cup \{g\})$.
B1 *Non-triviality.* There is an act $f \in A \in \mathcal{L}$ such that $f \notin C(A)$.
B2 *Continuity.* For any act $f \in L$, the set $\{g \in L | g \in C(\{f, g\})\}$ is closed, and the set $\{g \in L | \{g\} = C(\{f, g\})\}$ is open.
B3 *Independence.* Let $f \in A \in \mathcal{L}$, $h \in L$ and $\alpha \in (0,1)$. $f \in C(A) \iff \alpha h + (1-\alpha) f \in C(\alpha h + (1-\alpha) A)$.
B4 *Convex Axiom of Revealed Non-Inferiority (CARNI).* Let $f \in A \in \mathcal{L}$. If $\forall g \in conv(C(A))$ there exists $B \in \mathcal{Y}$ such that $f \in C(B)$ and $g \in conv(B)$, then $f \in C(A)$.

We say that act f (weakly) dominates act g if for every state of nature $s \in S$ $f(s) \in C(\{f(s), g(s)\})$. That is, for every state of nature s, if the decision maker knows s, act f is chosen in the pair $\{f, g\}$. Thus, f is better than g in all states of nature. Axiom B0 (monotonicity) requires that if f dominates g, then: (i) f is chosen whenever it is added to a set where g was a choosable alternative, and (ii) any alternative that is chosen in a set that includes f is also chosen after adding g to this set. Axioms B1-B4 are analogous to axioms A1-A4, which were discussed in the first model.

Axioms B0-B3 and WARP [12] are equivalent to the subjective expected utility representation (Anscombe and Aumann, 1963; see also Savage, 1954): There exists a unique vN-M utility function u, and a unique probability distribution p over S (prior), such that for

[12] In the Anscombe-Aumann framework WARP is formulated as follows: Let $A, B \in \mathcal{L}$ and $f, g \in A \cap B$. $f \in C(A)$ and $g \in C(B)$ implies $f \in C(B)$.

every $A \in \mathcal{L}$ and every act $f \in A$: $f \in C(A) \Longleftrightarrow E_p(u(f)) \geq E_p(u(g)) \;\; \forall g \in A$.

Axioms B0-B3 and WARNI [13] are equivalent to the following representation: There exists a unique non-degenerate vN-M utility function u, and a unique closed and convex set $P \subseteq \triangle(S)$ of priors, such that for every $A \in \mathcal{L}$ and every act $f \in A$:

$$f \in C(A) \Longleftrightarrow \forall g \in A, \exists p_g \in P \text{ s.t. } E_{p_g}(u(f)) \geq E_{p_g}(u(g)). \tag{3}$$

This representation is equivalent to the binary choice correspondence that is induced from Knightian preferences (Bewley, 2002) or from justifiable preferences (Lehrer and Teper, 2011).

2.3.2 Representation Theorem

The standard axioms B0-B3 (monotonicity, non-triviality, continuity, independence) and CARNI (B4) yield a multiple-prior representation in which an act is chosen if and only if it is best with respect to one of the priors. Formally:

Theorem 2 *Let C be a choice correspondence over $\mathcal{L}$. The following are equivalent:*

(1) C satisfies axioms B0-B4.
(2) There exists a non-constant vN-M utility function u, and a closed and convex set $P \subseteq \triangle(S)$ of probability distributions over S (priors), such that for every set $A \in \mathcal{L}$ and every act $f \in A$:

$$f \in C(A) \Leftrightarrow \exists p \in P \text{ s.t. } \forall g \in A, \;\; E_p(u(f)) \geq E_p(u(g)). \tag{4}$$

Moreover, P is unique and u is unique up to positive linear transformations.

Remark 1 As in the previous model, the extra convexity of CARNI allows us to change the order of the quantifiers in the representation. In particular, in (3), each comparison of a chosen act f with some act $g \in A$ may be based on a different prior $p_g \in P$, while in (4), all comparisons are based on the same prior $p \in P$.

3 Psychological Preferences and Indecisiveness

Incomplete preferences allows a decision maker to exhibit indecisiveness. In this section we characterize the notions of indecisiveness and being more decisive in our models. This characterization may be of independent interest, as it can also be applied to other models of incomplete preferences. (e.g., Bewley, 2002; Dubra, Maccheroni and Ok, 2004; and Eliaz and Ok, 2006).

[13] In the Anscombe-Aumann framework WARNI is formulated as follows: Let $f \in A \in \mathcal{L}$. $f \in C(A)$ if and only if for every $g \in C(A)$ there exists $B \in \mathcal{L}$ such that $f \in C(B)$ and $g \in B$.

The decision maker's revealed psychological preference relation $\succeq^*$ is defined for each $q, r \in Y$ as follows:[14]

$$q \succeq^* r \Leftrightarrow \left(\forall A \in \mathcal{Y} \begin{array}{l} \text{(I)} \qquad\quad r \in C(A \cup \{r\}) \Rightarrow q \in C(A \cup \{q\}) \\ \text{(II) } \forall p \in A: \; p \in C(A \cup \{q\}) \Rightarrow p \in C(A \cup \{r\}) \end{array} \right). \tag{5}$$

Part (I) compares the set of menus in which each alternative is chosen. It requires that if r is chosen when it is added to menu A, then q must also be chosen when it is added to the same menu. Part (II) compares the choices of other alternatives when q or r are available. It requires that if an element is selected when q is added to menu A, then it should also be chosen when r is added to the same menu. Define $q \succeq^I r$ if part (I) holds (i.e, $q \succeq^I r \Leftrightarrow (\forall A \in \mathcal{Y},\ r \in C(A \cup \{r\}) \Rightarrow q \in C(A \cup \{q\}))$) and define $q \succeq^{II} r$ if part (II) holds (i.e., $q \succeq^{II} r \Leftrightarrow (\forall A \in \mathcal{Y}, \forall p \in A: \; p \in C(A \cup \{q\}) \Rightarrow p \in C(A \cup \{r\})))$. Observe that all these relations ($q \succeq^* r$, $q \succeq^I r$, $q \succeq^{II} r$) are transitive.

Define a decision maker to be *indifferent* between q and r, and denote it by $q \sim^* r$, if $q \succeq^* r$ and $r \succeq^* q$. Define a decision maker to be *indecisive* between q and r, and denote it by $q \bowtie^* r$, if $\neg q \succeq^* r$ and $\neg r \succeq^* q$.[15] Define a decision maker to have *incomplete* (*complete*) *preferences* if the relation $\succeq^*$ is incomplete (complete).

Bernheim and Rangel (2009) define revealed psychological preference relation (denoted by R' in their paper) as follows:

$$q \succeq^{BR} r \Leftrightarrow (\forall A \in \mathcal{Y} \text{ with } q, r \in A: \; r \in C(A) \Rightarrow q \in C(A)).$$

That is, q is revealed better than r à la Bernheim and Rangel (2009) if whenever both elements are available and r is selected, so is q. As noted by Bernheim and Rangel $\succeq^{BR}$ is not necessarily transitive.[16] The relation $\succeq^I$ is a natural transitive strengthening of $\succeq^{BR}$ ($q \succeq^I r \Rightarrow q \succeq^{BR} r$). The following example demonstrates why Part (II) is also required when evaluating the revealed psychological preferences.

Example 2 *Consider the following choice correspondence C over finite set $X = \{x, y, z\}$: $C(X) = \{x, y\}$, $C(\{x, y\}) = \{x, y\}$, $C(\{x, z\}) = \{x\}$, and $C(\{y, z\}) = \{y, z\}$. Both relations $\succeq^I$ and $\succeq^{BR}$ imply that the decision maker is indifferent between x and y. However, the fact that z is selected from $\{y, z\}$ but not from $\{x, z\}$, indicates that the decision maker is not indifferent between x and y: she selects z when y is available, but she does not choose z when x is available.*

We conclude by defining the notion of being more decisive. Let Alice and Bob be two

[14] For brevity, we state our definitions only in the von Neumann-Morgenstern framework, but they apply very similarly also in the Anscombe-Aumann framework.

[15] Indecisiveness is closely related to Eliaz and Ok (2006)'s notion of incomparability.

[16] Consider, for example, the following choice correspondence C over finite set $X = \{x, y, z\}$: $C(X) = \{x, y\}$, $C(\{x, y\}) = \{x, y\}$, $C(\{x, z\}) = \{x\}$, and $C(\{y, z\}) = \{z\}$. In this case, $y \succeq^{BR} x \succeq^{BR} z$ but $y \not\succeq^{BR} z$.

decision makers with respective indecisiveness relations $\bowtie_A^*$ and $\bowtie_B^*$. Alice is *more decisive* than Bob if $q \bowtie_A^* r \Rightarrow q \bowtie_B^* r$. That is, whenever Alice is indecisive between two alternatives, so is Bob.

Observe that when Bob prefers q over r ($q \succeq_B r$), Alice is required to have a preference between the two alternatives, but her preference may either be $q \succeq_A r$ or $r \succeq_A q$, and this may depend on q and r. Two special cases of being more decisive are the extreme cases of full consistency and full inconsistency. Alice is *fully consistent* (*fully inconsistent*) with Bob if for each $q, r \in \mathcal{Y}$: $q \succeq_B r$ implies that $q \succeq_A r$ ($r \succeq_A q$).

Remark 2 *If one assumes that there is a best element in X ($\exists x_b \in X$ such that $x_b \in A \Rightarrow \{x_b\} = C(A)$), then all of the results of the following subsections hold if one does either of the following two changes (or both): (a) one replaces CARNI with WARNI; and (b) one replaces $\succeq^*$ with $\succeq^I$ or $\succeq^{BR}$.*

3.2 Multiple-Utility Characterization

Intuitively, a decision maker with a multiple-utility representation prefers q over r if all his utilities assign q a better value. The following proposition shows the equivalence between this definition and the choice-derived definition given in the previous subsection.

Proposition 1 *Let C be a choice correspondence over $\mathcal{Y}$ that satisfies axioms A1-A4. Let U be the multiple-utility representation. Then for each $q, r \in Y$: $q \succeq^* r \Leftrightarrow q \succeq^{II} r \Leftrightarrow \forall u \in U,\ u(q) \geq u(r)$.*

An immediate corollary of Proposition 1 characterizes indecisiveness and indifference in terms of the representation.

Corollary 1 *Let C be a choice correspondence over $\mathcal{Y}$ that satisfies axioms A1-A4. Let U be the multiple-utility representation. Then for each $q, r \in Y$:*

(1) $r \sim^* q \Leftrightarrow \forall u \in U,\ u(r) = u(q)$.
(2) $r \bowtie^* q \Leftrightarrow \exists u_1, u_2 \in U,\ u_1(r) > u_1(q)$ and $u_2(r) < u_2(q)$.

Proposition 1 shows that $\succeq^*$ has a multiple-utility representation. It is well known that such a preference relation is complete if and only if its set of utilities is a singleton (up to positive linear transformations). Formally (proof is omitted):

Lemma 1 *Let C be a choice correspondence over $\mathcal{Y}$ that satisfies axioms A1-A4. Let U be the respective multiple-utility representation. Then the indecisiveness relation is empty if and only if U is a singleton (up to positive linear transformations: every $u_1, u_2 \in U$ satisfy $u_1 = a \cdot u_2 + b$ for some $a > 0$ and $b \in \mathbb{R}$).*

The following proposition shows that Alice is more decisive than Bob if either of the following conditions hold: 1) Alice has a single utility, or 2) Alice's set of utilities is included in Bob's set of utilities., or 3) Alice's set of utilities is included in Bob's set of opposite utilities.

Proposition 2 *Let Alice and Bob be two decision makers with respective choice correspondences (C_A, C_B) over $\mathcal{Y}$ that satisfy axioms A1-A4 with respective multiple-utility representations (U_A, U_B). Then Alice is more decisive than Bob if and only if at least one of the following holds:*

(1) U_A is a singleton (up to positive linear transformations).
(2) $U_A \subseteq U_B$ (up to positive linear transformations: for each $u_A \in U_A$ there exist $u_B \in U_B$, $a > 0$, and $b \in \mathbb{R}$ such that $u_B = a \cdot u_A + b$)
(3) $U_A \subseteq -U_B$ (up to positive linear transformations).

An immediate corollary of Proposition 2 is the following: if Alice has incomplete preferences and she is more decisive than Bob, then she is either fully-consistent or fully-inconsistent with him.

Corollary 2 *Let Alice and Bob be two decision makers with choice correspondences that satisfy axioms A1-A4. Assume that Alice has incomplete preferences and that she is more decisive than Bob. Then Alice is either fully-consistent or fully-inconsistent with Bob.*

3.3 Multiple-Prior Characterization

The following proposition shows that a decision maker with a multiple-prior representation prefers act f over g if all her priors assign f a better value.

Proposition 3 *Let C be a choice correspondence over $\mathcal{L}$ that satisfies axioms B0-B4. Let u be the utility and P the set of priors in the multiple-prior representation. Then for each $f, g \in L$: $f \succeq^* g \Leftrightarrow f \succeq^{II} g \Leftrightarrow \forall p \in P,\ E_p(u(f)) \geq E_p(u(g))$.*

An immediate corollary of Proposition 3 characterizes indecisiveness and indifference in terms of the representation.

Corollary 3 *Let C be a choice correspondence over $\mathcal{L}$ that satisfies axioms B0-B4. Let u be the respective utility and P the respective set of priors in the representation. Then for each $f, g \in L$:*

(1) $f \sim^* g \Leftrightarrow \forall p \in P,\ E_p(u(f)) = E_p(u(g))$.
(2) $f \bowtie^* g \Leftrightarrow \exists p_1, p_2 \in P,\ E_{p_1}(u(f)) > E_{p_1}(u(g))$ and $E_{p_2}(u(f)) < E_{p_2}(u(g))$.

The following lemma shows that a decision maker with a multiple-prior representation has complete preferences if and only if her set of priors is a singleton.

Lemma 2 *Let C be a choice correspondence over $\mathcal{L}$ that satisfies axioms B0-B4. Let P be the set of priors in the multiple-prior representation. Then the indecisiveness relation $\bowtie^*$ is empty if and only if P is a singleton.*

The following proposition shows that Alice is more decisive than Bob if either of the following conditions hold: 1) Alice has a single prior, or 2) Alice's set of priors is included in Bob's set of priors, and in addition Alice's utility is equal to Bob's utility or exactly the opposite of Bob's utility.

Proposition 4 *Let Alice and Bob be two decision makers with respective choice correspondences (C_A, C_B) over L that satisfy axioms B0-B4 with respective multiple-prior representations $((u_A, P_A), (u_B, P_B))$. Then Alice is more decisive than Bob if and only if at least one of the following holds:*

(1) P_A is a singleton.
(2) $P_A \subseteq P_B$ and $u_A = u_B$ (up to positive linear transformations).
(3) $P_A \subseteq P_B$ and $u_A = -u_B$ (up to positive linear transformations).

An immediate corollary of Proposition 4 is that if Alice has incomplete preferences and she is more decisive than Bob, then she is either fully-consistent or fully-inconsistent with him.

Corollary 4 *Let Alice and Bob be two decision makers with choice correspondences that satisfy axioms B0-B4. Assume that Alice has incomplete preferences and that she is more decisive than Bob. Then Alice is either fully-consistent or fully-inconsistent with Bob.*

4 Proofs

4.1 Equivalent Formulation of CARNI

The following lemma, which will be useful in later proofs, shows that CARNI can be stated also as an "if and only if" statement:

Lemma 3 *The following properties are equivalent:*

(1) $\forall r \in conv\,(C(A))\ \exists B_r \in \mathcal{Y}$ such that $q \in C\,(B_r)$ and $r \in conv\,(B_r) \Rightarrow q \in C(A)$.
(2) $\forall r \in conv\,(C(A))\ \exists B_r \in \mathcal{Y}$ such that $q \in C\,(B_r)$ and $r \in conv\,(B_r) \Leftrightarrow q \in C(A)$.

PROOF. The equivalence holds due to the observation that $q \in C(A)$ implies that $\forall r \in conv\,(C(A))$, $\exists B_r = A$ such that $q \in C\,(B_r) = C\,(A)$ and $r \in conv\,(C(A)) \subseteq conv\,(A) = conv\,(B_r)$. $\square$

4.2 Logical Implications of CARNI and Related Axioms

In this subsection we prove the logical implications of CARNI and related axioms which were presented in Subsection 2.2. The results are not used elsewhere in the paper (except the fact that CARNI implies Sen's α).

The first lemma shows that given WARP, Independence implies Mixture Invariance.

Lemma 4 *Let C be a choice correspondence that satisfies WARP and Independence. Then C satisfies Mixture Invariance: $q \in C\,(A) \Rightarrow q \in C\,(conv\,(A))$.*

PROOF. WARP implies that C is consistent with preference maximization of transitive and complete preference relation $\succeq$. Let $q \in C\,(A)$. Assume to the contrary that $q \notin C\,(conv\,(A))$. Let $r \in C\,(conv\,(A))$. Preference maximization implies that $r \in C\,(conv\,(A)) \setminus A$ and that $r \succ t$ for each $t \in A$ (because $r \succ q$ and $q \succeq t$ for each $t \in A$). Thus there exist $t \in A$ and $s \in conv\,(A)$ such that $r = \alpha \cdot s + (1-\alpha)\,t$ where $0 < \alpha < 1$. Independence implies: $r \succ t \Rightarrow s \succ t \Rightarrow s \succ r$ and this contradicts $r \in C\,(conv\,(A))$.

The next lemma shows that WARNI can be decomposed into the following three (independent) axioms: Sen's α, Sen's γ, and Aizerman Property.

Lemma 5 *Let C be a choice correspondence. Then C satisfies WARNI if and only if it satisfies the following 3 axioms: Sen's α, Sen's γ, and Aizerman Property .*

PROOF. WARNI $\Rightarrow$ Sen's α: Proved in Eliaz and Ok (2006).

WARNI $\Rightarrow$ Sen's γ: Let $M \subseteq \mathcal{Y}$ be a collection of sets and let A be the union of all these sets. Assume that $A \in \mathcal{Y}$, and $q \in C(B)$ for each $B \in M$. Observe that $\forall r \in C(A) \subseteq A$, $\exists B_r \in M \subseteq \mathcal{Y}$ such that $q \in C(B_r)$ and $r \in B_r$. By WARNI this implies that $q \in C(A)$.

WARNI $\Rightarrow$ Aizerman Property: Let $A, B \in \mathcal{Y}$ satisfying $C(B) \subseteq A \subseteq B$. Let $q \in C(A)$. Assume to the contrary that $q \notin C(B)$. By WARNI there exists $r \in C(B) \subseteq A$ such that q is never chosen when r is present in the choice set. This implies $q \notin C(A)$ and we get a contradiction.

Sen's α + Sen's γ + Aizerman Property $\Rightarrow$ WARNI: Let $A \in \mathcal{Y}$ and $q \in A$. Assume that $\forall r \in C(A)$, $\exists B_r \in \mathcal{Y}$ such that $q \in C(B_r)$ and $r \in B_r$. By Sen's α, $q \in C(\{q, r\})$ for each $r \in C(A)$. Sen's γ implies that $q \in C(C(A) \cup \{q\})$. Finally, by Aizerman Property, $q \in C(A)$.

The next lemma shows that CARNI can be decomposed into the following four (independent) axioms: Sen's α, Convex γ, Aizerman Property and Mixture Invariance.

Lemma 6 *Let C be a choice correspondence. Then C satisfies CARNI if and only if C satisfies the following 4 axioms: Sen's α, Convex γ, Aizerman Property, and Mixture Invariance.*

PROOF. CARNI $\Rightarrow$ Sen's α: Let $A, B \subseteq \mathcal{Y}$ with $A \subseteq B$ and $q \in C(B)$. Assume to the contrary that and $q \notin C(A)$. Then by CARNI there is $r \in conv(C(A))$ such that for every $B_r \in \mathcal{Y}$ with $r \in conv(B_r) \Rightarrow q \notin C(B_r)$. Observe that $r \in conv(C(A)) \subseteq conv(A) \subseteq conv(B)$ and this implies that $q \notin C(B)$ and this leads to a contradiction.

CARNI $\Rightarrow$ Convex γ: Let $M \subseteq \mathcal{Y}$ be a collection of sets and let A be the union of all these sets. Assume that $A \in \mathcal{Y}$, A is convex, and $q \in C(B)$ for each $B \in M$. Observe that $\forall r \in conv(C(A)) \subseteq conv(A) = A$, $\exists B_r \in M \subseteq \mathcal{Y}$ such that $q \in C(B_r)$ and $r \in B_r \subseteq conv(B_r)$. By CARNI this implies that $q \in C(A)$.

CARNI $\Rightarrow$ Aizerman Property: Let $A, B \in \mathcal{Y}$ satisfying $C(B) \subseteq A \subseteq B$. Let $q \in C(A)$. Assume to the contrary that $q \notin C(B)$. By CARNI there exists $r \in conv(C(B)) \subseteq conv(A)$ such that q is never chosen when r is present in the convex hull of the choice set. This implies $q \notin C(A)$ and we get a contradiction.

CARNI $\Rightarrow$ Mixture Invariance: Let $A \in \mathcal{Y}$ and $q \in C(A)$. By CARNI (and Lemma 3) $q \in C(A)$ implies that $\forall r \in conv(C(A))\ \exists B_r \in \mathcal{Y}$ such that $q \in C(B_r)$ and $r \in conv(B_r)$. This implies (again by CARNI) that $q \in C(conv(A))$.

Sen's α + Convex γ + Aizerman Property + Mixture Invariance $\Rightarrow$ CARNI: Let $A \in \mathcal{Y}$ and $q \in A$. Assume that $\forall r \in conv(C(A))\ \exists B_r \in \mathcal{Y}$ such that $q \in C(B_r)$ and $r \in conv(B_r)$. By Mixture Invariance $q \in C(conv(B_r))$. Sen's α implies that $q \in C(conv(\{q, r\}))$ for each $r \in conv(C(A))$. By Convex γ, $q \in C(conv(C(A) \cup \{q\}))$. Sen's α implies that $q \in C(C(A) \cup \{q\})$. Finally, by Aizerman Property, $q \in C(A)$.

It is well known that WARP implies Sen's α, Sen's $\gamma \Rightarrow$Convex γ, and Aizerman Property (see Aizerman and Malishevski, 1981). Thus Lemma 6 implies that WARP + Mixture Invariance $\Rightarrow$ CARNI.

4.3 Risk (von Neumann-Morgenstern framework)

In this subsection we prove Theorem 1. We begin by showing that the multiple-utility representation implies axioms A1-A4. Let U be a compact and convex set of linear (vN-M) utilities such that: 1) $\forall A \in \mathcal{Y}$ and $q \in A$: $q \in C(A) \Leftrightarrow \exists u \in U$, s.t. $\forall r \in A$, $u(q) \geq u(r)$, and 2) there are two lotteries $\underline{p}, \overline{p} \in Y$ such that $\forall u \in U$, $u\left(\underline{p}\right) < u(\overline{p})$. Axiom A1 (non-triviality) holds because $\{\overline{p}\} = C\left(\left\{\underline{p}, \overline{p}\right\}\right)$. Axioms A2 (continuity) and A3 (independence) are immediate from the compactness of U and the linearity of each $u \in U$. Let $q \in A \in \mathcal{Y}$. In order to prove axiom A4 we have to show that $q \in C(A)$ if $\forall r \in conv(C(A))$ $\exists B_r \in \mathcal{Y}$ such that $q \in C(B_r)$ and $r \in conv(B_r)$. This is done as follows:

$$
\begin{aligned}
& \forall r \in conv(C(A)) \ \exists B_r \in \mathcal{Y} \text{ s.t. } q \in C(B_r) \text{ and } r \in conv(B_r) \\
& \Rightarrow \forall r \in conv(C(A)) \ \exists u_r \in U \ \ u_r(q) \geq u_r(r) && (6) \\
& \Rightarrow \forall r \in conv(C(A)) \ \max_{u \in U}(u(q) - u(r)) \geq 0 \\
& \Rightarrow \min_{r \in conv(C(A))} \max_{u \in U}(u(q) - u(r)) \geq 0 \\
& \Rightarrow \max_{u \in U} \min_{r \in conv(C(A))}(u(q) - u(r)) \geq 0 && (7) \\
& \Rightarrow \exists u_0 \in U \text{ s.t. } \forall r \in conv(C(A)),\ u_0(q) \geq u_0(r) \\
& \Rightarrow \exists u_0 \in U \text{ s.t. } \forall r \in C(A),\ u_0(q) \geq u_0(r) && (8) \\
& \Rightarrow \exists u_0 \in U \text{ s.t. } \forall r \in A,\ u_0(q) \geq u_0(r) && (9) \\
& \Rightarrow q \in C(A) && (10)
\end{aligned}
$$

Where (6) is implied by the representation and the linearity of the utilities; (7) is due to the minimax theorem (von Neumann and Morgenstern, 1944) using the linearity of the utilities, and the convexity and compactness of U and $conv(C(A))$; and (10) is implied by the representation. We are left with showing that (9) holds. Assume to the contrary that (9) does not hold. Let $t \in A \backslash C(A)$ s.t. $u_0(t) > u_0(q)$. Let t' be an element in A that maximizes u_0. By (8) t' must be in $A \backslash C(A)$, while the representation implies that t' must be in $C(A)$ (contradiction).

We now show that axioms A1-A4 imply the multiple-utility representation. Let $\succ$ denote the *revealed (irreflexive) strict preference relation* that is induced from C: $q \succ r \Leftrightarrow \{q\} = C(\{q, r\})$ $(q \neq r)$.

The following lemma shows that $\succ$ satisfies transitivity, non-triviality, continuity and independence.

Lemma 7 *Let C be a choice correspondence that satisfies axioms A1-A4, and let $\succ$ be the revealed strict preference. Then $\succ$ satisfies the following properties:*

C1 *Non-triviality* - There are $q, r \in Y$ such that $q \succ r$.
C2 *Continuity* - For each $q \in Y$ the sets $\{q | q \succ r\}$ and $\{q | q \prec r\}$ are open.

C3 *Independence* - For any $p, q, r \in Y$ and any $\alpha \in (0,1)$, $q \succ r \Leftrightarrow \alpha p + (1-\alpha) q \succ \alpha p + (1-\alpha) r$

C4 *Transitivity* - For any $p, q, r \in Y$, $p \succ q$ and $q \succ r$ implies that $p \succ r$.

PROOF. Axiom C1 is implied by axiom A1 (non-triviality of C) together with CARNI: by Axiom A1 there exists $q \in A \backslash C(A)$; due to CARNI there exists $r \in conv(C(A))$ such that q is never chosen when r is present in the convex hull of the choice set; in particular, $q \notin C(\{q, r\})$. Axioms C2-C3 are immediately implied from the analogous properties of C (A2-A3). C4 (transitivity) is proved as follows. Let $p \succ q$ *and* $q \succ r$. CARNI implies that: $\{q\} = C(\{q, r\}) \Rightarrow r \notin C(\{p, q, r\})$, and $\{p\} = C(\{p, q\}) \Rightarrow q \notin C(\{p, q, r\})$.So it must be that $\{p\} = C(\{p, q, r\})$. Assume to the contrary that $r \in C(\{p, r\})$. CARNI implies that $r \in C(\{p, q, r\})$ and we get a contradiction. □ (Lemma 7)

The following proposition (Theorem 1 in Evren, 2010) shows that $\succ$ has a unique multiple-utility representation.

Proposition 5 *(Evren, 2010, Theorem 1) Let $\succ$ be a strict binary relation over Y. The following are equivalent:*

(1) $\succ$ satisfies axioms C1-C4 (transitivity, non-triviality, continuity and independence).
(2) There exists a nonempty convex compact set U of linear (vN-M) utility functions, such that:
  (a) for every two lotteries $q, r \in Y$, $q \succ r \Leftrightarrow \forall u \in U$, $u(q) > u(r)$.
  (b) There are two outcomes $\underline{q}, \overline{q} \in Y$ such that $\forall u \in U$, $u\left(\underline{q}\right) < u(\overline{q})$.

Moreover (Evren, 2010, Theorem 2),U is unique up to positive linear transformations. That is if both U and V are convex compact sets that represent the same choice correspondence then $\forall u \in U$, $\exists v \in V$ such that $u = a \cdot v + b$ where $a > 0$ and $b \in R$.

We use Proposition 5 to finish Theorem 1's proof, by showing that axioms A1-A4 imply the multiple-prior representation. Let C be a choice correspondence that satisfies these axioms, and let $\succ$ be the revealed strict preference. Let U be the unique (up to linear transformations) convex and compact set of utilities of Prop. 5. We have to show for each $q \in A \in \mathcal{Y}$, $q \in C(A) \Leftrightarrow \exists u \in U$, s.t. $u(q) \geq u(r)$ $\forall r \in A$. This is done as follows:

$$\begin{aligned}
q \in C(A) &\Longleftrightarrow \forall r \in conv(C(A)) \ \exists B_r \in \mathcal{Y} \text{ s.t. } q \in C(B_r) \text{ and } r \in conv(B_r) && (11)\\
&\Longleftrightarrow \neg \exists r \in conv(C(A)) \, s.t. \, r \succ q && (12)\\
&\Longleftrightarrow \forall r \in conv(C(A)) \ \exists u_r \in U \text{ such that } u_r(q) \geq u_r(r) && (13)\\
&\Longleftrightarrow \min_{r \in conv(C(A))} \max_{u \in U} (u(q) - u(r)) \geq 0 \\
&\Longleftrightarrow \max_{u \in U} \min_{r \in conv(C(A))} (u(q) - u(r)) \geq 0 && (14)\\
&\Longleftrightarrow \exists u_0 \in U \text{ s.t. } \forall r \in conv(C(A)), \ u_0(q) \geq u_0(r) && (15)\\
&\Longleftrightarrow \exists u_0 \in U \text{ s.t. } \forall r \in C(A), \ u_0(q) \geq u_0(r) && (16)\\
&\Longleftrightarrow \exists u_0 \in U \text{ s.t. } \forall r \in A, \ u_0(q) \geq u_0(r) && (17)
\end{aligned}$$

Where (11) is implied by CARNI and Lemma 3; (12) is due to the definition of $\succ$ and CARNI; (13) is implied by Proposition 5; (14) is due to the minimax theorem using the convexity and compactness of the sets U and $conv(C(A))$ and the linearity of each utility $u \in U$; and (16) is implied by the linearity of u. We are left with showing that (17) holds. Assume to the contrary that (17) does not hold. Let $t \in A \backslash C(A)$ s.t. $u_0(t) > u_0(q)$. Let t' be an element in A that maximizes u_0. Observe that t' must be in $A \backslash C(A)$ due to (15). By Proposition 5 $\neg \exists r \in A$ s.t. $r \succ t'$. By CARNI, t' must be chosen in A (contradiction).

Uniqueness (up to positive linear transformations) follows from the uniqueness of Proposition 5 as follows. Let C be a choice correspondence and let $\succ$ be its revealed strict preference. Due to Proposition 5 $\succ$ has a unique multiple-utility representation U. Let U' be a utility set that also represents C. Let $\succ'$ be the unique strict preference that is represented by U' (due to Proposition 5). Assume to the contrary that U and U' are not equivalent under positive linear transformations. Then there are $q, r \in Y$ such that either: ($q \succ r$ and $q \not\succ' r$) or ($q \succ' r$ and $q \not\succ r$). Both cases imply a contradiction with respect to the choice from $\{q, r\}$. $\square$ (Theorem 1)

4.4 Uncertainty (Anscombe-Aumann Framework)

In this subsection we prove Theorem 2. We begin by showing that the multiple-prior representation implies axioms B0-B4. Let u be a non-constant linear (vN-M) utility and let P be a set of priors such that for every $A \in \mathcal{L}$ and every act $f \in A$: $f \in C(A) \Leftrightarrow \exists p \in P$ s.t. $\forall g \in A$, $E_p(u(f)) \geq E_p(u(g))$. Axiom B0 (monotonicity) holds because $\forall s \in S$ $f(s) \in C(f(s), g(s))$ implies $E_p(f) \geq E_p(g)$ for every $p \in \triangle(S)$, which implies (i) and (ii) in B0. Axiom B1 (non-triviality) holds because of the non triviality of u. Axioms B2 (continuity) and B3 (independence) are immediate from the linearity of u and the closedness of P. Axiom B4 (CARNI) is implied by the representation due to the same argument that was given in the previous subsection for axiom A4.

We now show that axioms B0-B4 imply the multiple-prior representation. Let $\succeq$ denote the *revealed (weak) preference relation* that is induced from C: $q \succeq r \Leftrightarrow q \in C(\{q, r\})$, and let $\succ$ be its strict part (which is defined as in the previous subsection: $q \succ r \Leftrightarrow \{q\} = C(\{q, r\})$. The following proposition shows that $\succeq$ satisfies unambiguous transitivity, non-triviality, continuity, independence, completeness and favorable mixing.

Proposition 6 *Let C be a choice correspondence that satisfies axioms B0-B4, and let $\succeq$ be the revealed preference relation . Then $\succeq$ satisfies the following properties:*

D0 *Unambiguous Transitivity.* Let $f, g, h \in L$ such that $\forall s \in S$ $f(s) \succeq g(s)$. Then, (i) $h \succeq f \Rightarrow h \succeq g$, and (ii) $g \succeq h \Rightarrow f \succeq h$.
D1 *Non-triviality.* There are acts $f, g \in L$ s.t. $f \succ g$.
D2 *Continuity.* For any $f \in L$, the sets $\{g | g \succeq f\}$ and $\{g | g \preceq f\}$ are closed.
D3 *Independence.* Let $f, g \in L$. $f \succeq g$ if and only if $\alpha h + (1-\alpha) f \succeq \alpha h + (1-\alpha) g$ for every $h \in L$ and $\alpha \in (0,1)$.
D4 *Completeness and reflexivity.* For any $f, g \in L$, $f \succeq g$ or $g \succeq f$, and $f \sim f$.
D5 *Favorable mixing.* For every $f, g, h \in L$ *and* $\alpha \in (0,1)$, *if* $g \succ f$ *and* $\alpha f + (1-\alpha) h \succeq g$, *then* $\lambda f + (1-\lambda) h \succeq g$, *for every* $0 < \lambda \leq \alpha$.

PROOF. Axiom D0 is implied by axiom B0 (monotonicity) and by Sen's α (Lemma 6). Axiom D1 is implied by axiom B1 (non triviality of C) and CARNI. Axioms D2-D3 are implied by the analogous properties B2-B3. Axiom D4 follows from the definition of $\succeq$ as a revealed preference relation. Axiom D5 is proved as follows. Let $h' = \lambda f + (1-\lambda)\, h$ where $0 < \lambda \leq \alpha$. Assume to the contrary that $h' \prec g$. Observe that there exists $\beta \in (0,1)$ such that $\alpha f + (1-\alpha)\, h = \beta f + (1-\beta)\, h'$. Independence (D3) implies that $h' \prec g \Rightarrow \beta g + (1-\beta)\, h' \prec \beta g + (1-\beta)\, g = g$, and $f \prec g \Rightarrow \beta f + (1-\beta)\, h' \prec \beta g + (1-\beta)\, h'$. The transitivity of the strict preference $\succ$ (which is proved as in the previous subsection) implies that $\alpha f + (1-\alpha)\, h = \beta f + (1-\beta)\, h' \prec g$, which contradicts the fact that $\alpha f + (1-\alpha)\, h \succeq g$.

The following proposition (Lehrer and Teper, 2011, Theorem 1) shows that $\succeq$ has a unique multiple-prior representation.

Proposition 7 *(Lehrer and Teper, 2011, Theorem 1). Let $\succeq$ be a binary relation over L. The following are equivalent:*

(1) $\succeq$ satisfies axioms D0-D5.
(2) There exists a non-degenerate vN-M utility u, and a convex closed set P of priors over the states of nature, such that for every two acts $f, g \in L$: $f \succeq g \Leftrightarrow \exists p \in P$ with $E_p(u(f)) \geq E_p(u(g))$.

Moreover, P is unique and u is unique up to positive linear transformations.

Observe that Proposition 7 immediately implies that the strict relation $\succ$ has Knightian representation (Bewely, 2002): $f \succ g \Leftrightarrow \forall p \in P$, $E_p(u(f)) > E_p(u(g))$. We use Proposition 7 to finish the proof of Theorem 2, by showing that axioms B0-B4 imply the multiple-prior representation. Let C be a choice correspondence that satisfies these axioms, and let $\succ$ be the revealed strict preference. Let u be the unique utility (up to linear transformations), and let P be the unique convex and closed set of priors of Proposition 7. We have to show, for each $f \in A \in \mathcal{L}$, $f \in C(A) \Leftrightarrow \exists p \in P$, s.t. $E_p(u(f)) \geq E_p(u(g))\ \forall g \in A$. This is done as follows:

$$\begin{aligned}
f \in C(A) &\Longleftrightarrow \neg\exists g \in conv(C(A))\; s.t.\; g \succ f && (18)\\
&\Longleftrightarrow \forall g \in conv(C(A))\; \exists p \in P\, \text{s.t.}\; E_p(u(f)) \geq E_p(u(g)) && (19)\\
&\Longleftrightarrow \min_{g \in conv(C(A))} \max_{p \in P} E_p(u(f) - u(g)) \geq 0 &&\\
&\Longleftrightarrow \max_{p \in P} \min_{g \in conv(C(A))} E_p(u(f) - u(g)) \geq 0 && (20)\\
&\Longleftrightarrow \exists p_0 \in P\, \text{s.t.}\, \forall g \in conv(C(A)),\; E_{p_0}(u(f)) \geq E_{p_0}(u(g)) &&\\
&\Longleftrightarrow \exists p_0 \in P\, \text{s.t.}\, \forall g \in C(A),\; E_{p_0}(u(f)) \geq E_{p_0}(u(g)) && (21)\\
&\Longleftrightarrow \exists p_0 \in P\, \text{s.t.}\, \forall g \in A,\; E_{p_0}(u(f)) \geq E_{p_0}(u(g)) && (22)
\end{aligned}$$

Where (18) is implied by CARNI, Lemma 3 and the definition of $\succ$; (19) is due to Proposition 7, (20) is implied by the minimax theorem using the convexity and closedness of the sets P and $conv(A)$ and the linearity of each $u \in U$, (21) is due to the linearity of u; and (22) is proved in the same way that (17) is proved in the previous subsection. The uniqueness of P and u (up to linear transformations) is implied by the uniqueness in Proposition 7. $\square$ (Theorem 2).

4.5 Indecisiveness and indifference

In this section we prove the results of Section 3.

4.5.1 Multiple-Utility Characterization

We begin by proving Proposition 1, which characterizes $\succeq^*$ in terms of the representation:

Proposition 1 Let C be a choice correspondence over Y that satisfies axioms A1-A4. Let U be the multiple-utility representation. Then for each $q, r \in Y$: $q \succeq^* r \Leftrightarrow q \succeq^{II} r \Leftrightarrow \forall u \in U,\ u(q) \geq u(r)$.

PROOF. It is immediate that $q \succeq^* r \Rightarrow q \succeq^{II} r$ and that $\forall u \in U,\ u(q) \geq u(r) \Rightarrow q \succeq^* r$. We now show that $q \succeq^{II} r \Rightarrow \forall u \in U,\ u(q) \geq u(r)$. Assume to the contrary that there exists $u_0 \in U$ such that $u_0(r) > u_0(q)$. We have to show that there exist $A \in \mathcal{Y}$ and $p \in A$, such that $p \in C(A \cup \{q\})$ and $p \notin C(A \cup \{r\})$. Let $\underline{p}, \overline{p} \in Y$ be alternatives such that $u\left(\underline{p}\right) < u(\overline{p})$ for every utility $u \in U$. For each $\epsilon > 0$, let $p_\epsilon = \epsilon \underline{p} + (0.5 - \epsilon) q + 0.5r$, and let $A_\epsilon = \{p_\epsilon, (2\epsilon\overline{p} + (1 - 2\epsilon) q)\}$. For sufficiently small ϵ, $u_0(p_\epsilon) > u_0(q)$ and $u_0(p_\epsilon) > u_0(2\epsilon\overline{p} + (1 - 2\epsilon) q)$. This implies that $p_\epsilon \in C(A_\epsilon \cup \{q\})$. In addition, for every $\epsilon > 0$ and every $u \in U$, $u(p_\epsilon) < u(\epsilon\overline{p} + (0.5 - \epsilon) q + 0.5r) = 0.5u(2\epsilon\overline{p} + (1 - 2\epsilon) q) + 0.5u(r)$. This implies that $p_\epsilon \notin C(A_\epsilon \cup \{r\})$.

Finally, we prove Proposition 2, which characterizes when Alice is more decisive than Bob in terms of multiple-utility representation. It shows that Alice is more decisive if: 1) Alice has a single utility, or 2) Alice's set of utilities is included in Bob's set of utilities., or 3) Alice's set of utilities is included in Bob's set of opposite utilities.

Proposition 2 Let Alice and Bob be two decision makers with respective choice correspondences (C_A, C_B) over $\mathcal{Y}$ that satisfy axioms A1-A4 with respect to multiple-utility representations (U_A, U_B). Then Alice is more decisive than Bob if and only if at least one of the following holds:
(1) U_A is a singleton (up to positive linear transformations).
(2) $U_A \subseteq U_B$ (up to positive linear transformations).
(3) $U_A \subseteq -U_B$ (up to positive linear transformations).

PROOF. The 'if' part is straightforward. The 'only if' part is proved as follows. Assume that Alice is more decisive than Bob and that U_A is not a singleton. Let $\overline{p}_B, \underline{p}_B \in Y$ be elements such that $u_B(\overline{p}_B) > u_B\left(\underline{p}_B\right)$ for each $u_B \in U_B$, and let $\overline{p}_A, \underline{p}_A \in Y$ be elements such that $u_A(\overline{p}_A) > u_A\left(\underline{p}_A\right)$ for each $u_A \in U_A$. Let $q, r \in Y$ be elements such that Alice is indecisive between them ($q \bowtie^*_A r$, such elements exist due to Lemma 1).

We begin by showing that $u_A(\overline{p}_B) \neq u_A\left(\underline{p}_B\right)$ for every $u_A \in U_A$. If $u_A(\overline{p}_B) = u_A\left(\underline{p}_B\right)$ for every $u_A \in U_A$. Then by Corollary 1, for sufficiently small $\epsilon > 0$, Bob is decisive between $(1 - \epsilon)\overline{p}_B + \epsilon q$ and $(1 - \epsilon)\underline{p}_B + \epsilon r$, while Alice is indecisive between these alternatives. If there exist $u_1, u_2 \in U_A$ such that $u_1(\overline{p}_B) = u_1\left(\underline{p}_B\right)$ and $u_2(\overline{p}_B) > u_2\left(\underline{p}_B\right)$ ($u_2(\overline{p}_B) <$

$u_2\left(\underline{p}_B\right)$), then by using Corollary 1, for sufficiently small $\epsilon > 0$, Bob is decisive between $(1-\epsilon)\,\overline{p}_B + \epsilon\underline{p}_A$ and $(1-\epsilon)\,\underline{p}_B + \epsilon\overline{p}_A$ (between $(1-\epsilon)\,\overline{p}_B + \epsilon\overline{p}_A$ and $(1-\epsilon)\,\underline{p}_B + \epsilon\underline{p}_A$), while Alice is indecisive between these alternatives. The convexity of U_A then implies that either $u_A(\overline{p}_B) > u_A\left(\underline{p}_B\right)$ for every $u_A \in U_A$ or $u_A(\overline{p}_B) < u_A\left(\underline{p}_B\right)$ for every $u_A \in U_A$.

Assume first that $u_A(\overline{p}_B) > u_A\left(\underline{p}_B\right)$ for every $u_A \in U_A$. Normalize every utility u in $U_A \cup U_B$ to satisfy $u(\overline{p}_B) = 1$ and $u\left(\underline{p}_B\right) = 0$. Assume to the contrary that there exists $u_A \in U_A \backslash U_B$. By a standard separation theorem (using the convexity and the compactness of U_B) there exist $q, r \in Y$ such that $\alpha = u_A(r) - u_A(q) > u_B(r) - u_B(q)$ for each $u_B \in U_B$.[17] Let $\beta = max_{u_B \in U_B}(u_B(r) - u_B(q))$. Assume first that there is $u'_A \in U_A$ such that $\gamma = u'_A(r) - u'_A(q) \neq \alpha$. By the convexity of U_A one can assume that $\beta < \gamma$. This implies that Alice is indecisive between the following lotteries:

$$\frac{1}{1+\frac{\alpha+\gamma}{2}}r + \left(\frac{\frac{\alpha+\gamma}{2}}{1+\frac{\alpha+\gamma}{2}}\right)\underline{p}_B \text{ and } \frac{1}{1+\frac{\alpha+\gamma}{2}}q + \left(\frac{\frac{\alpha+\gamma}{2}}{1+\frac{\alpha+\gamma}{2}}\right)\overline{p}_B$$

(because if $\gamma > \alpha$ then the first lottery is better according to u'_A and the second lottery is better according to u_A, and if $\gamma < \alpha$ the opposite holds), while Bob is decisive (the second lottery is better according to all of Bob's utilities) - a contradiction. So we are left with the case that $u'_A(r) - u'_A(q) = \alpha$ for every $u'_A \in U_A$. As U_A is not a singleton, there is $p \in Y$ and $u^1_A, u^2_A \in U_A$ such that $u^1_A(p) > u^2_A(p)$. For sufficiently small $\delta > 0$, $r' = (1-\delta)\,r + \delta p$ and $q' = (1-\delta)\,q + \delta p$ satisfy: 1) $u^1_A(r') - u^1_A(q')$, $u^2_A(r') - u^2_A(q') > u_B(r') - u_B(q')$ for each $u_B \in U_B$, 2) $u^2_A(r') - u^2_A(q') \neq u^1_A(r') - u^1_A(q')$. By the previous argument, this leads to a contradiction. Thus, we have proved that in this case $U_A \subseteq U_B$.

We are left with the case that $u_A(\overline{p}_B) < u_A\left(\underline{p}_B\right)$ for every $u_A \in U_A$. Let Charlie be a decision maker with the exact opposite multiple-utility representation with respect to Bob ($U_C = -U_B$). Observe that Charlie is as decisive as Bob. This implies that Alice is more decisive than Charlie. Let $\overline{p}_C = \underline{p}_B$ and $\underline{p}_C = \overline{p}_B$. Observe that $u_A(\overline{p}_C) > u_A\left(\underline{p}_C\right)$ for every $u_A \in U_A$. By using the proof of the previous case, it follows that $U_A \subseteq U_C = -U_B$, which completes the proof. □ (2).

4.5.2 Multiple-Prior Representation

We begin by proving Proposition 3, which characterizes $\succeq^*$ in terms of the representation:

Proposition Let C be a choice correspondence over $\mathcal{L}$ that satisfies axioms B0-B4. Let u be the utility and P the set of priors in the multiple-prior representation. Then for each $f, g \in L$: $f \succeq^* g \Leftrightarrow f \succeq^{II} g \Leftrightarrow \forall p \in P$, $E_p(u(f)) \geq E_p(u(g))$.

PROOF. It is immediate that $f \succeq^* g \Rightarrow f \succeq^{II} g$, and that $\forall p \in P$, $E_p(u(f)) \geq E_p(u(g)) \Rightarrow f \succeq^* g$. We now show that $f \succeq^{II} g \Rightarrow \forall p \in P$, $E_p(u(f)) \geq E_p(u(g))$. Assume to the contrary that there exists $p_0 \in P$ such that $E_{p_0}(u(g)) > E_{p_0}(u(f))$.

[17] Extending each utility u from $\triangle(X)$ to $\mathbb{R}^{|X|}$, a standard separation theorem yields a signed unit vector v (possibly with negative values) such that $u_A(v) > u_B(v)$ for each $u_B \in U_B$. This vector v induces the two lotteries $q, r \in Y$ as follows: $q = \frac{v^+}{\|v^+\|}$ (and $q = \underline{p}$ if $v^+ = \vec{0}$) and $r = \frac{v^-}{\|v^-\|}$ (and $r = \underline{p}$ if $v^- = \vec{0}$), where $v^+_i = \max(v_i, 0)$ and $v^-_i =$ -min$(v_i, 0)$.

We have to show that there exist $A \in \mathcal{L}$ and $h \in A$, such that $h \in C(A \cup \{f\})$ and $h \notin C(A) \cup \{g\}$. Let $\underline{x}, \overline{x} \in X$ be alternatives such that $u(\underline{x}) < u(\overline{x})$. For each $\epsilon > 0$, let $h_\epsilon = \epsilon\underline{x} + (0.5-\epsilon) f + 0.5g$, and let $A_\epsilon = \{h_\epsilon, (2\epsilon\overline{x} + (1-2\epsilon) f)\}$. For sufficiently small ϵ, $E_{p_0}(u(h_\epsilon)) \geq E_{p_0}(u(f))$ and $E_{p_0}(u(h_\epsilon)) \geq E_{p_0}(u(2\epsilon\overline{x} + (1-2\epsilon) f))$. This implies that $h_\epsilon \in C(A_\epsilon \cup \{f\})$. In addition, for every $\epsilon > 0$ and every $p \in P$, $E_p(u(h_\epsilon)) < E_p(u(\epsilon\overline{x} + (0.5-\epsilon) f + 0.5g)) = 0.5E_p(u(2\epsilon\overline{x} + (1-2\epsilon) f)) + 0.5E_p(u(g))$. This implies that $h_\epsilon \notin C(A_\epsilon \cup \{g\})$. $\square$

Next we prove Lemma 2, which shows that a decision maker has complete preferences if and only if her set of priors is a singleton:

Lemma 2 Let C be a choice correspondence over $\mathcal{L}$ that satisfies axioms B0-B4. Let P be the set of priors in the multiple-prior representation. Then the decision maker has complete preferences if and only if P is a singleton.

PROOF. The 'if' part is straightforward. The 'only if' part is proved as follows. Assume that the decision maker has complete preferences (i.e., relation $\bowtie^*$ is empty). Assume to the contrary that there are $p_1 \neq p_2 \in P$. The fact that $p_1 \neq p_2$ implies that there are $s_1, s_2 \in S$ such that: $p_1(s_1) > p_2(s_1)$ and $p_2(s_2) > p_1(s_2)$. Let $\underline{x}, \overline{x} \in X$ be alternatives such that $u(\underline{x}) < u(\overline{x})$. Let

$$f_1 = \begin{cases} \overline{x} & s_1 \\ \underline{x} & \text{all other states} \end{cases}, \quad \text{and} \quad f_2 = \begin{cases} \overline{x} & s_2 \\ \underline{x} & \text{all other states} \end{cases}.$$

It follows that $E_{p_1}(u(f_1)) > E_{p_2}(u(f_1))$ and $E_{p_2}(u(f_2)) > E_{p_1}(u(f_2))$ and by Corollary 3 $f_1 \bowtie^* f_2$. $\square$ (Lemma 2)

Finally, we prove Proposition 4, which characterizes when Alice is more decisive than Bob in terms of a multiple-prior representation. It shows that Alice is more decisive if: 1) Alice has a single prior, or 2) Alice's set of priors is included in Bob's set of priors, and in addition Alice's utility is equal to Bob's utility or exactly the opposite of Bob's utility.

Proposition 4 Let Alice and Bob be two decision makers with respective choice correspondences (C_A, C_B) over L that satisfy axioms B0-B4 with respective multiple-prior representations $((u_A, P_A), (u_B, P_B))$. Then Alice is more decisive than Bob if and only if at least one of the following holds:

(1) P_A is a singleton (includes a single prior).
(2) $P_A \subseteq P_B$ and $u_A = u_B$ (up to positive linear transformations).
(3) $P_A \subseteq P_B$ and $u_A = -u_B$ (up to positive linear transformations).

PROOF. The 'if' part is straightforward. The 'only if' part is proved as follows. Assume that Alice is more decisive than Bob and that Alice has incomplete preferences. By Lemma **3** P_A is not a singleton. Let $\overline{x}_B, \underline{x}_B \in X$ be elements such that: 1) $u_B(\overline{x}_B) > u_B(\underline{x}_B)$, and 2) for each $x \in X$ $u_B(\overline{x}_B) \geq u_B(x) \geq u_B(\underline{x}_B)$. Let $f_0, g_0 \in L$ be elements that Alice is indecisive between them ($f_0 \bowtie_A^* g_0$, such elements exist because Alice has incomplete preferences).

We begin by showing that $u_A(\overline{x}_B) \neq u_A(\underline{x}_B)$. If $u_A(\overline{x}_B) = u_A(\underline{x}_B)$, then by Proposition 4, for sufficiently small $\epsilon > 0$, Bob is decisive between $(1-\epsilon)\overline{x}_B + \epsilon f_0$ and $(1-\epsilon)\underline{x}_B + \epsilon g_0$, while Alice is indecisive between these alternatives.

Case 1: Assume first that $u_A(\overline{x}_B) > u_A(\underline{x}_B)$. Normalize utilities u_A and u_B to satisfy $u_B(\overline{x}_B) = u_A(\overline{x}_B) = 1$ and $u_B(\underline{x}_B) = u_A(\underline{x}_B) = 0$. We show that for every $x \in X$, $0 \leq u_A(x) \leq 1$. Assume to the contrary that there exists $x \in X$ with $u_A(x) > 1$ ($u_A(x) < 0$). There there exists $0 < \alpha < 1$ such that $1 = u_A(\overline{x}_B) = u_A(\alpha x + (1-\alpha)\underline{x}_B)$ ($0 = u_A(\underline{x}_B) = u_A(\alpha x + (1-\alpha)\overline{x}_B)$). For sufficiently small $\epsilon > 0$, Alice is indecisive between $(1-\epsilon)\overline{x}_B + \epsilon f_0$ and $(1-\epsilon)(\alpha x + (1-\alpha)\underline{x}_B) + \epsilon g_0$ (between $(1-\epsilon)\underline{x}_B + \epsilon f_0$) and $(1-\epsilon)(\alpha x + (1-\alpha)\overline{x}_B) + \epsilon g_0$) while Bob is decisive (he prefers the first act).

Next, we show that for each $x \in X$ $u_A(x) = u_B(x)$. Assume to the contrary that $\alpha = u_A(x) \neq u_B(x)$ where $0 \leq \alpha \leq 1$. For sufficiently small $\epsilon > 0$, Alice is indecisive between $(1-\epsilon)x + \epsilon f_0$ and $(1-\epsilon)(\alpha\overline{x}_B + (1-\alpha)\underline{x}_B) + \epsilon g_0$, while Bob is decisive (contradiction).

This shows that both decision makers have the same utility. Let $u = u_A = u_B$. We now prove that $P_A \subseteq P_B$. Assume to the contrary that $P_A \nsubseteq P_B$. Let $p_A \in P_A \backslash P_B$. By a standard separation theorem (using the convexity and the compactness of P_B, see footnote 17) there are $f, g \in L$ such that $1 > \alpha = E_{p_A}(u(f) - u(g)) > E_{p_B}(u(f) - u(g))$ for each $p_B \in P_B$. Let $\beta = max_{p_B \in P_B} E_{p_B}(u(r) - u(q))$. Assume first that there is $p'_A \in P_A$ such that $\gamma = E_{p'_A}(u(f) - u(g)) \neq \alpha$. By the convexity of P_A we can assume that $\beta < \gamma$. This implies that Alice is indecisive between these acts

$$\frac{1}{1+\frac{\alpha+\gamma}{2}} f + \left(1 - \frac{1}{1+\frac{\alpha+\gamma}{2}}\right)\underline{x}_B \text{ and } \frac{1}{1+\frac{\alpha+\gamma}{2}} g + \left(1 - \frac{1}{1+\frac{\alpha+\gamma}{2}}\right)\overline{x}_B$$

(because if $\gamma > \alpha$ then the first act is better according to p'_A and the second act is better according to p_A and if $\gamma < \alpha$ the opposite holds), while Bob is decisive (the second act is better according to all of Bob's utilities) - a contradiction. We are left with the case that $E_{p'_A}(u(f) - u(g)) = \alpha$ for every $p'_A \in P_A$. As P_A is not a singleton, there are $h \in L$ and $p^1_A, p^2_A \in P_A$ such that $E_{p^1_A}(u(h)) > E_{p^2_A}(u(h))$. For sufficiently small $\delta > 0$, $f' = (1-\delta)f + \delta h$ and $g' = (1-\delta)g + \delta h$ satisfy: 1) $E_{p^1_A}(u(f') - u(g'))$, $E_{p^2_A}(u(f') - u(g')) > E_{p_B}(u(f') - u(g'))$ for each $p_B \in P_B$, 2) $E_{p^1_A}(u(f') - u(g')) \neq E_{p^2_A}(u(f') - u(g'))$. By the previous argument, this leads to a contradiction. Thus, we have proved that in this case $P_A \subseteq P_B$.

Case 2: Let Charlie be a decision maker with the opposite of Bob's utility ($u_C = -u_B$) and the same set of priors as Bob ($P_C = P_B$). This implies that Alice and Charlie fits case 1. By the proof of this case, $u_A = u_C = -u_B$ and $P_A \subseteq P_C = P_B$, which completes the proof.

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