The seasonal KPSS Test: some extensions and further results

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Abstract

The literature distinguishes finite sample studies of seasonal stationarity quite less intensely than it shows for seasonal unit root tests. Therefore, the use of both types of tests for better exploring time series dynamics is seldom noticed in the relative studies on such a topic. Recently, Lyhagen (2006) introduced for quarterly data the seasonal KPSS test which null hypothesis is no seasonal unit roots. In the same manner, as most unit root limit theory, the asymptotic theory of the seasonal KPSS test depends on whether the data has been filtered by a preliminary regression. More specifically, one may proceed to the extraction of deterministic components – such as the mean and trend – from the data before testing. In this paper, I took account of de-trending on the seasonal KPSS test. A sketch of its limit theory was provided in this case. Also, I studied in finite sample the behaviour of the test for monthly time series. This could enrich our knowledge about it since it was – as I mentioned above – early introduced for quarterly data. Overall, the obtained results showed that the seasonal KPSS test preserved its good size and power properties. Furthermore, like the test of Kwiatkowski *et al.* [KPSS] (1992), the nonparametric corrections of residual variances may smooth the wide variations of the seasonal KPSS empirical sizes.

Keywords: KPSS test, deterministic seasonality, Brownian motion, LM test

JEL classification: C32 Time series models

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1. Introduction

Nowadays, the use of seasonally unadjusted data is on the increase. Behind that, the inference distortion and the detrimental information loss in dynamic models could be caused by seasonal adjustment. In this respect, Ghysels and Perron (1993) showed that seasonal adjustment filters can seriously affect seasonal unit roots. Furthermore, researchers, in their quest to reveal as much time series information, have shown great interest in studying their unobserved components.

Especially, several authors have shown that the seasonal and cyclical components are linked; see, *inter alia*, Canova and Ghysels (1994). That's why the systematic elimination of the seasonal component can generate non-rigorous deductions. However, having decided not to eliminate this component, the question that comes immediately after: what model should be given to seasonality?

The literature has considered several different seasonality models. The first approach is to model seasonality as a deterministic component; see Barsky and Miron (1989). The second approach is to consider seasonality as a deterministic variable within its stationary stochastic model; see Canova (1992). Finally, the third approach is to consider seasonality as stochastic. In this approach, the authors' concern is the development of seasonal unit root tests. The test of Hylleberg *et al.* [HEGY] (1990) is now the preeminent seasonal unit root test, with its asymptotic orthogonality as a key property allowing its generalization at any observational frequency. The subsequent rejection of their null hypothesis implies a strong result that the series exhibits a stationary seasonal pattern, but their test is found to suffer from the problem of low power in moderate sample sizes. In agreement with what was found in the conventional case, Hylleberg (1995) suggested the joint use of seasonal unit root and stationarity tests. Literature was relatively small in seasonal stationarity tests. One can refer to the tests of Canova and Hansen (1995) and Caner (1998). The difference between the two tests lies at the correction of the error term when the standard assumptions, which it should verify in regression analysis, do not apply. The first test used a non-parametric correction like the test of Kwiatkowski *et al.* [KPSS] (1992) and the second used a parametric correction. Likewise, Lyhagen (2006) proposed another version of the KPSS test in the seasonal context which resulted in a frequency-based test. More explicitly, Lyhagen (2006) tested the hypothesis of level stationarity against a single seasonal unit root. Thus, this test can be termed seasonal KPSS test.

It was shown by Khedhiri and El Montasser (2010) with a Monte Carlo method that the seasonal KPSS test is robust to the magnitude and the number of additive outliers. Furthermore, the statistical results obtained cast an overall good performance of the finite-sample properties of the test. Khédhiri and El Montasser (2012) provided a representation of the seasonal KPSS test in the time domain and established its asymptotic theory. This representation allows the generalization of the test's asymptotic theory when the basic equation incorporates other additional dynamics.

This paper differs from Khédhiri and El Montasser (2012) in that it takes into account the presence of a linear trend in the basic equation of the seasonal KPSS test and the monthly observational frequency. In doing so, I will conduct a Monte Carlo analysis to study the test's properties of size and power in such circumstances. In addition, a sketch of its asymptotic theory is provided in the presence of a linear trend.

The outline of this paper is as follows. In section 2, I introduce some preliminaries of the seasonal KPSS. In section 3, I conduct a Monte Carlo simulation study to assess the finite sample properties of the test in terms of its size and power performance when including a linear trend in its basic equations. Also, I consider in this study the effect of the observational frequency on the test properties. To this aim, I involve monthly data. The last section concludes.

2. Preliminaries on the Seasonal KPSS Test

Let y_t be a time series observed quarterly. Since the goal is to test for the presence of negative unit root, it would be suitable to use the appropriate filter in order to isolate the effects of other unit roots in the series. Therefore, the test will be applied to the transformed series: $y_t^{(1)} = (1 - L + L^2 - L^3)\, y_t$, where L is the lag operator. This transformation is obtained from the seasonal difference filter $1 - L^4 = (1-L)(1+L)(1+L^2) = (1-L^2)(1+L^2)$.

Next, one test the unit root of –1 in the series

$$y_t^{(1)} = x_t^{'}\beta + r_t + u_t, \quad t = 1, \ldots, T, \tag{1}$$

where $T = 4N$, $\beta^{'} x_t = \sum_{i=1}^{4} a_i D_{it}$ and the shorthand notation $D_{it} = \delta(i, t - 4[(t-1)/4])$ and also where [.] denotes the largest integer function and $\delta(i, j)$ is the Kronecker's δ function.

The term u_t is zero mean weakly dependent process with autocovariogram $\gamma_h = E(u_t u_{t+h})$ and a strictly positive long run variance ω_u^2.

The component r_t is drawn from the following process:

$$r_t = -r_{t-1} + v_t \,, \qquad (2)$$

where v_t is zero mean weakly process with variance σ_v^2 and long run variance $\omega_v^2 > 0$.

The transformation needed to carry out the seasonal KPSS test for complex unit roots $\pm i$ is given by the following variable,

$$y_t^{(2)} = (1 - L^2) y_t \,.$$

The test of such complex unit roots is based on the regression,

$$y_t^{(2)} = x_t^{'} \lambda + c_t + e_t \,, \qquad (3)$$

where e_t is zero mean weakly dependent process with long run variance $\omega_e^2 > 0$ and $\lambda^{'} x_t = \sum_{i=1}^{4} b_i D_{it}$. The component c_t is given by

$$c_t = -c_{t-2} + \varepsilon_t \,, \qquad (4)$$

where ε_t is another zero mean weakly dependent process with variance σ_ε^2 and strictly positive long run variance ω_ε^2.

Adding the deterministic terms in (1) and (3) is very important because it allows the seasonal KPSS test to include deterministic seasonality. The testing procedure follows in two steps: First, the unit root of -1 is tested, and then the complex roots are tested where their null hypothesis will be specified thereafter.

The seasonal KPSS test is a Lagrange Multiplier-based test. Hence, the null hypothesis of a root equals to -1 is $H_0 : \sigma_v^2 = 0$. Under this null hypothesis, $y_t^{(1)}$ is written as:

$$y_t^{(1)} = x_t^{'} \beta + u_t \,, \qquad (5)$$

where the series is trend stationary after seasonal mean correction. Under the alternative hypothesis $H_1 : \sigma_v^2 > 0,\ \ y_t^{(1)}$ has a unit root corresponding to Nyquist frequency.

Let $\tilde{u}_t$ be the residual series obtained from least squares regression applied to equation (5), $t = 1, 2, \ldots, T$. Following Breitung and Franses (1998, eq. (18), p. 209), and Busetti and Harvey (2003, eq. (8), p. 422) and Taylor (2003, eq. (38), p. 605), we replace the long-run variance ω_u^2 by an estimate of ($2\ \pi$ times) the spectrum at the observed frequency in order to deal with unconditional heteroscedasticity and serial correlation. This nonparametric estimation of the long-run variance is a useful solution to the nuisance parameter problem (Taylor, 2003). Thus, for the Nyquist frequency, this nonparametric estimation is written as follows:

$$\tilde{\omega}_{u,\pi}^2(l) = T^{-1}\sum_{t=1}^{T}\tilde{u}_t^2 + 2T^{-1}\sum_{k=1}^{l} w(k,l)(\sum_{t=k+1}^{T}\tilde{u}_t\tilde{u}_{t-k})\cos(\pi\ k), \quad (6)$$

where the weight function $w(k,l) = 1 - \dfrac{k}{l+1}$ and l is a lag truncation parameter such that $l \to \infty$ as $T \to \infty$ and $l = o(n^{1/2})$. Now from equation (6), I choose a Bartlett kernel following Newey and West (1987). It should be noted that Andrews (1991) showed that such a truncation lag can produce good results in practice, as also shown in KPSS (1992). Similarly, the null hypothesis of the test regarding complex unit roots is given by $H_0 : \sigma_\varepsilon^2 = 0$. Under this null hypothesis, $y_t^{(2)}$ is written as follows:

$$y_t^{(2)} = x_t^{'}\lambda + e_t \quad (7)$$

Using the residuals $\tilde{e}_t$ obtained from the least squares regression of equation (7), the Bartlett kernel estimator of ω_e^2 can be computed as follows:

$$\tilde{\omega}^2_{e,\frac{\pi}{2}}(l) = T^{-1}\sum_{t=1}^{T}\tilde{e}_t^2 + 2T^{-1}\sum_{k=1}^{l} w(k,l)(\sum_{t=k+1}^{T}\tilde{e}_t\tilde{e}_{t-k})\cos(\frac{\pi}{2}k) \quad (8)$$

Define the partial sums $\tilde{S}_t = \sum_{j=1}^{t} e^{i\pi\ j}\tilde{u}_j$ and $\tilde{P}_t = \sum_{j=1}^{t} e^{i\frac{\pi}{2}j}\tilde{e}_t$.

It follows that the test statistics for unit root of -1 is given by:

$$\eta^{(\pi)} = \frac{1}{T^2}\frac{\sum_{t=1}^{T}\tilde{S}_t\bar{\tilde{S}}_t}{\tilde{\omega}_{u,\pi}^2(l)} \quad (9)$$

This statistic may be written for the complex unit roots, as

$$\eta^{(\frac{\pi}{2})} = \frac{1}{T^2} \frac{\sum_{t=1}^{T} \tilde{P}_t \bar{\tilde{P}}_t}{\tilde{\omega}^2_{e,\frac{\pi}{2}}(l)}, \tag{10}$$

where $\bar{\tilde{S}}_t$ and $\bar{\tilde{P}}_t$ are the conjugate numbers of $\tilde{S}_t$ and $\tilde{P}_t$, respectively.

Khédhiri and El Montasser (2012) have shown under $H_0 : \sigma_v^2 = 0$, $\eta^{(\pi)} \to_d \int_0^1 V_\pi(r)^2 dr$ where $V_\pi(r)$ is a standard Brownian bridge, "$\to_d$" denotes weak convergence in probability and $r \in [0,1]$. However, for $H_0 : \sigma_\varepsilon^2 = 0$, the authors have shown that $\eta^{(\frac{\pi}{2})} \to_d \frac{1}{2}\int_0^1 [V^R_{\frac{\pi}{2}}(\tau)^2 + V^I_{\frac{\pi}{2}}(\tau)^2] d\tau$ where $V^R_{\frac{\pi}{2}}(\tau)$ and $V^I_{\frac{\pi}{2}}(\tau)$ are two independent standard Brownian bridges and $\tau \in [0,1]$.

<u>*Remark 1*</u>**:** Asymptotically $\eta^{(\pi)}$ has the first level Cramer-von Mises distribution (CvM_1) under the null hypothesis while the limit theory of $\eta^{(\pi/2)}$ was shown as a function of a generalized Cramer-von Mises with two degrees of freedom. Specifically, the asymptotic theory of this statistic is as follows: $\eta^{(\frac{\pi}{2})} \to_d \frac{1}{2} CvM_1(2)$.

The reader can refer to Anderson and Darling (1952) for this type of distributions. The critical values of the seasonal KPSS test with seasonal dummies can be computed from Nyblom (1989) or from Canova and Hansen (1995). These critical values are also shown in Table 1 of Khédhiri and El Montasser (2012).

<u>*Remark 2*:</u> It can be shown that the seasonal frequency has no effect on the asymptotic distribution of test statistics. In other words, $\eta^{(\pi)}$ may retain the same limit distribution as above and the statistic associated with the complex unit roots in question has the same limit distribution as $\eta^{(\frac{\pi}{2})}$. Only the set of seasonal unit roots will change and it may not include the unit root which corresponds to the Nyquist frequency, i.e. when the periodicity is odd.

<u>*Remark 3*: </u>Recall that if there is a time trend in the regression of the standard KPSS test, the partial sum of residuals from a first order polynomial regression weakly converges to a second level Brownian bridge denoted B_2 where, as in McNeill (1978),

$$B_2(r) = W(r) - rW(1) + 6r(1-r)\left[\frac{1}{2}W(1) - \int_0^1 W(s)ds\right], \qquad (11)$$

with $W(.)$ being a standard Wiener process or Brownian motion.

Then the test statistic follows the so-called second level Cramer von Mises distribution; see Harvey (2005). However, this result cannot be generalized to seasonal KPSS test. Indeed, the statistics $\eta^{(\pi)}$ follows the so called zero level Cramer von Mises noted CvM_0; see Harvey (2005). Specifically,

$$\eta^{(\pi)} \to_d \int_0^1 W(r)^2 dr. \qquad (12)$$

Meanwhile, when the deterministic component is represented by only a trend in Eq. (3), it can be shown that

$$\eta^{(\frac{\pi}{2})} \to_d \frac{1}{2} CvM_0(2). \qquad (13)$$

The critical values of the seasonal KPSS test in this case can be obtained from Nyblom (1989, Table 1) and they are shown in Table 1.

Table 1: Critical values of the seasonal KPSS test in the case of first order polynomial trend

	1%	5%	10%
Root -1	2.787	1.656	1.196
Roots $\pm i$	1.9645	1.3120	1.031

Even though only a constant is included in the Eqs. (1) and (3), these critical values are still appropriate. These findings show indeed that the generalization of the asymptotic results of the standard KPSS test should not be done in an automatic way, but rather it is advisable to take some serious reflection to establish equivalent results for the seasonal KPSS test.

3. The Monte Carlo Analysis

To evaluate the size performance of the seasonal KPSS statistic in presence of first order linear trend, I conduct Monte Carlo simulation experiments with seasonal roots of a quarterly process. The data generating process (DGP) for the negative unit root is

$$y_t = x_t^{'}\beta + r_t, \quad t = 1,\ldots,T, \tag{14a}$$

where $x_t^{'}\beta$ only represents a first order linear trend and the autoregressive process r_t is given by:

$$r_t = \alpha \; r_{t-1} + v_t, \tag{14b}$$

The error terms v_t are normally distributed with zero mean and unit variance.

The DGP for complex unit roots is given by:

$$y_t = x_t^{'}\lambda + c_t, \quad t = 1,\ldots,T, \tag{15a}$$

where $x_t^{'}\lambda$ only includes a first order linear trend and the process c_t is given by:

$$c_t = \alpha \; c_{t-2} + \varepsilon_t, \tag{15b}$$

ε_t are normally distributed with zero mean and unit variance.

I choose alternative values of $\alpha \in \{-1, -0.8, -0.2, 0, 0.2, 0.8\}$ and I only consider the 5% nominal size. The bandwidth values chosen in our experiments are given by:

$l0 = 0$, $l4 =$ integer $\left[4(T/100)^{1/4}\right]$ and $l12 =$ integer $\left[12(T/100)^{1/4}\right]$.

I use 20000 replications and all the simulation experiments were carried out with Matlab programs. The corresponding results are summarized in Table 2.

Table 2: Rejection frequencies for the seasonal KPSS test with a first order polynomial trend for seasonal quarterly unit roots, significance level: 5% (size and power)

			$\eta^{(-1)}$				$\eta^{(\pm i)}$
α	T	$l0$	$l4$	$l12$	$l0$	$l4$	$l12$
-1	80	0.9492	0.7218	0.2127	0.9774	0.9092	0.3863
	200	0.9936	0.8498	0.5981	0.9987	0.9734	0.8138
-0.9	80	0.7894	0.4153	0.1103	0.9123	0.7575	0.2929
	200	0.8398	0.5981	0.1481	0.9627	0.7694	0.3841
-0.2	80	0.1146	0.0534	0.0210	0.1368	0.0786	0.0243
	200	0.1158	0.0577	0.0398	0.1461	0.0761	0.0403
0	80	0.0514	0.0398	0.0176	0.0510	0.0398	0.0173
	200	0.0522	0.0473	0.0369	0.0479	0.0435	0.0331
0.2	80	0.0181	0.0296	0.0145	0.0126	0.0204	0.0116
	200	0.0164	0.0382	0.0336	0.0100	0.0255	0.0264
0.9	80	0.00	0.0053	0.0026	0.00	0.0019	0.0007
	200	0.00	0.0006	0.0074	0.00	0.00	0.0007

What has been observed in Table 2 of Khédhiri and El Montasser (2012) still be seen from Table 2. Indeed, the size of the test increases with decreasing values of α. Similarly, the sample size does not noticeably affect the test's size not markedly improved with the non-parametric corrections $(l4)$ and $(l12)$.

To see the effect of observational frequency on the seasonal KPSS test in finite samples, monthly periodicity is taken into account. I only consider a deterministic seasonality. More specifically, I assume that the deterministic component is represented by 12 seasonal dummy variables. Remember that seasonal unit roots exhibited by the filter $S(L)=(1+L+L^2+...+L^{11})$ corresponding to the seasonal frequencies $\lambda_i=\frac{2\pi\ i}{12}$, $i=1,2,....6$. For size experiments, I assume a particular value of the null hypothesis specifying an i.i.d. process as a data generating process and corresponding to $\alpha=0$ in (14b) and (15b) for the quarterly case. For power experiments, I suppose that the process r_t, for the seasonal frequencies other than the Nyquist one, is outlined by:

$$r_t = 2\ \cos\lambda_i\, r_{t-1} - r_{t-2} + \varepsilon_t,\quad i=1,2,...5. \tag{16}$$

However, when the process shows a unit root corresponding to the Nyquist frequency, it will be generated by:

$$r_t = -r_{t-1} + v_t \tag{17}$$

The considered sample sizes are T=240 et T=600 which display the same number of years as in table 2. As mentioned above, the critical values of the test are obtained from Table 1 of Khédhiri and El Montasser (2012) where the first line corresponds to the unit root -1 and the second one to complex unit roots.

Table 3: The size of the seasonal KPSS test for monthly data, significance level 5%

	T=240	T=600
$\eta^{(\frac{\pi}{6})}(l0)$	0.0583	0.0533
$\eta^{(\frac{\pi}{6})}(l4)$	0.0556	0.0491
$\eta^{(\frac{\pi}{6})}(l12)$	0.0472	0.0470
$\eta^{(\frac{\pi}{3})}(l0)$	0.0581	0.0524
$\eta^{(\frac{\pi}{3})}(l4)$	0.0531	0.0500
$\eta^{(\frac{\pi}{3})}(l12)$	0.0457	0.0471
$\eta^{(\frac{\pi}{2})}(l0)$	0.0590	0.0527
$\eta^{(\frac{\pi}{2})}(l4)$	0.0546	0.0499
$\eta^{(\frac{\pi}{2})}(l12)$	0.0473	0.0469
$\eta^{(\frac{2\pi}{3})}(l0)$	0.0592	0.0522
$\eta^{(\frac{2\pi}{3})}(l4)$	0.0568	0.0505
$\eta^{(\frac{2\pi}{3})}(l12)$	0.0471	0.0482
$\eta^{(\frac{5\pi}{6})}(l0)$	0.0568	0.0515
$\eta^{(\frac{5\pi}{6})}(l4)$	0.0529	0.0503
$\eta^{(\frac{5\pi}{6})}(l12)$	0.0454	0.0474
$\eta^{(\pi)}(l0)$	0.0583	0.0530
$\eta^{(\pi)}(l4)$	0.0538	0.0500
$\eta^{(\pi)}(l12)$	0.0444	0.0459

Table 4: The power of the seasonal KPSS test for monthly data, significance level 5%

	T=240	T=600
$\eta^{(\frac{\pi}{6})}(l0)$	1	1
$\eta^{(\frac{\pi}{6})}(l4)$	0.9988	0.9998
$\eta^{(\frac{\pi}{6})}(l12)$	0.9493	0.9944
$\eta^{(\frac{\pi}{3})}(l0)$	1	1
$\eta^{(\frac{\pi}{3})}(l4)$	0.9990	1
$\eta^{(\frac{\pi}{3})}(l12)$	0.95	0.9949
$\eta^{(\frac{\pi}{2})}(l0)$	1	1
$\eta^{(\frac{\pi}{2})}(l4)$	0.9987	1
$\eta^{(\frac{\pi}{2})}(l12)$	0.9517	0.9943
$\eta^{(\frac{2\pi}{3})}(l0)$	1	1
$\eta^{(\frac{2\pi}{3})}(l4)$	0.9986	1
$\eta^{(\frac{2\pi}{3})}(l12)$	0.9515	0.9942
$\eta^{(\frac{5\pi}{6})}(l0)$	1	1
$\eta^{(\frac{5\pi}{6})}(l4)$	0.9982	1
$\eta^{(\frac{5\pi}{6})}(l12)$	0.9450	0.9942
$\eta^{(\pi)}(l0)$	1	1
$\eta^{(\pi)}(l4)$	0.9648	0.9929
$\eta^{(\pi)}(l12)$	0.7744	0.9194

According to Table 3, all empirical rejection frequencies approach the theoretical significance level of 5%. This shows indeed an excellent empirical size not subject to any distortion. Also, the increase of sample size mostly results in a slight decrease in size. Table 4 shows again that the seasonal KPSS test for monthly data preserves its good power properties. In this regard, a reduction of power corresponding to the root of -1 and the function l12 is notable but not surprising. Indeed the value 0.7744 that appears in the last box of the first column of Table 4 is very close to the values provided by KPSS (1992, Table 4) for the conventional unit root. This similarity is due to the mirror effect situation that occurs between the unit roots at frequencies zero and π.

4. Conclusion

As pointed by Hylleberg (1995), the most important reserve against the seasonal unit root test was that the null hypothesis of a unit root at the seasonal frequencies is problematic because seasonal unit root allows more variation in the seasonal pattern that is actually observed. So if the data generating process (DGP) is a seasonal unit-root- process, 'winter may become summer'. Another limitation going along with the first one is manifested by the fact that the HEGY test, like the Dickey-Fuller test, has low power against reasonable alternatives and that the existence of moving average terms with roots close to the unit circle imply that the power is almost equal to the size. Even though there were some recommendations to handle such situations, interest has been granted for the construction of tests with better properties than the existing ones either against similar or different alternatives or for different established assumptions. In this research spectrum, one may refer to the tests of Canova and Hansen (1995) and Lyhagen (2006) adopting a very similar framework. In this paper, I studied the finite sample properties of the second one in presence of a linear trend and also by considering a monthly periodicity. The effect of changing observational frequencies should be studied since this test was early set for quarterly data. The bottom line of this Monte Carlo study is that the seasonal KPSS test preserves good size and power properties both in including a linear trend and considering monthly time series. Moreover, its empirical rejection frequencies often approximate nominal sizes when using the nonparametric corrections of the residual variances.

The extension of the seasonal KPSS to a vector of time series is a future avenue of research. In that framework, one can examine if a set of data exhibit a common deterministic seasonality. This extension would be analogous to that which Nyblom and Harvey (2000) made to the KPSS test.

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