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Abstract

Economists have developed theoretical models identifying self-fulfilling expectations as an important source of statistical discrimination practices in labor markets (Arrow, 1973). The static models dominating the literature of statistical discrimination, however, may leave the false impression that a bad equilibrium is as fragile as a "bubble" and can burst at any moment when expectations flip. Such models thus understate the adversity that disadvantaged groups face in seeking to escape bad equilibria. By developing a dynamic version of a statistical discrimination model based on Coate and Loury's (1993) original setup, we clarify the limits of expectations-related fragility. We show that when a group is strongly affected by negative reputational externalities, the group cannot escape a low skill investment trap, regardless of how expectations are formed. By examining the evolution of stereotypes in this way, we also provide new insights into egalitarian policies.

Keywords: Statistical Discrimination, Collective Reputation, Reputation Trap, Forward-Looking Behavior.
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1 Introduction

Statistical discrimination is a theory of inequality among demographic groups, where discrimination is based on stereotypes that do not arise from prejudice. When rational, information-seeking decision makers use aggregate group characteristics to evaluate relevant personal characteristics of individuals with whom they interact, individuals from different groups may be treated differently, even if they share identical observable characteristics (Moro, 2009). Economists have developed theoretical models identifying self-fulfilling expectations as an important source of statistical discrimination as practiced in labor markets.

Most notably, Arrow (1973) identified the ingredients for the existence of multiple equilibria, in which principals have different self-confirming beliefs about different social groups with identical fundamentals. Coate and Loury (1993) extend Arrow's approach and present a theory of discrimination in job assignment, where two ex ante identical groups may end up in different, Pareto-ranked, skill investment equilibria. As in Arrow's model, discrimination is generated by coordination failure: the disadvantaged group fails to coordinate on the good equilibrium. When employers expect (correctly) that fewer workers in some visibly distinct group invest in human capital, employers use more stringent hiring standards that, in equilibrium, provide lower incentives to invest in human capital, fulfilling the original expectation. Thus, when groups coordinate on different equilibria, the model displays inequality across ex-ante identical groups.

A rich literature followed these early contributions based on the standard statistical discrimination frameworks, including a general equilibrium model with endogenous wages (Moro and Norman, 2004), an analysis of an economy with public and private sectors (Fang and Norman, 2006), an examination of the possibility of belief flipping in the promotion stage (Fryer, 2007) and a study of social integration and negative stereotypes (Chaudhuri and Sethi, 2008).

The static models that dominate the literature, however, produce the unintended impression that group inequality would be fully eliminated if the disadvantaged group and firms could somehow revise their beliefs and expectations and coordinate on the good equilibrium. Static models suggests a simple change in expectations can trigger a sudden shift in behavior of employers and the disadvantaged group members, leading to equality. The models may even produce the impression that a bad equilibrium is as fragile as a "bubble" and can burst at any moment when expectations flip. Finally, and most importantly, this misperception may understate the adversity a disadvantaged group faces in escaping a bad equilibrium.

Additionally, static models, by their very nature, do not explicitly consider how the average investment level of a group might evolve over time and, consequently, cannot explain under what conditions stereotypes leading to permanent inequality might arise. So, the policy implications of static models are somewhat limited.

Seeking to overcome these shortcomings, we suggest a dynamic version of the statistical discrimination model based on Coate and Loury's original setup. Dynamics are added to this setup by assuming, in a continuous time set-up, that older workers retire and are replaced with new workers at the same rate. These new members—maximizing the total discounted value of future payoffs—are forward-looking. Their investment decisions are based on how they anticipate their group will be treated in the future. As new members are born and older members disappear, employers' beliefs about a group's average productivity are revised over time, leading to cross-generational linkages in agents' incentives to invest. While the steady states of our dynamic model essentially replicate the equilibria of static models, the process of revision just described provides insights regarding the stability of equilibria and the paths that lead to different steady states when a group's average investment level is out of equilibrium.

Using the characterization of steady states and their converging paths, we can identify an "overlap" range—that is, a set of initial conditions that give rise to multiple dynamic equilibrium paths leading to both high and low levels of human capital in the steady states. Outside the overlap, however, there exist unique converging paths that lead to either high- or low-level human capital steady states. This formalizes the intuition that discriminatory outcomes may be persistent and that discriminated groups may be trapped by the negative influence of reputational externalities: that is, under certain conditions which we are able to identify, no change in expectations about the future can affect the behavior of disadvantaged group members.

Thus, our analysis clarifies the limits of the above-noted expectations-related fragility, providing conditions for the persistence of statistical discrimination. Under this dynamic perspective, if the initial collective reputation of a disadvantaged group is too unfavorable—that is, below the overlap range—then the group cannot escape the bad equilibrium, regardless of how expectations are formed.

In addition, we provide a more intuitive analysis of the workings of some egalitarian policies by examining the forward-looking decision-making behaviors of principals and agents. Outside the overlap range, when a group is trapped by a negative reputational externality, any collective action to manage expectations would be ineffective, and the group cannot escape a low human capital investment rate in the absence of some other concerted external intervention. For instance, in this scenario, a policy of temporary affirmative action may advance the group's skill level into the overlap range, thereby initiating a recovery of reputation. It is worth noting that in our dynamic framework, if the goal is to ensure that group equality eventually obtains, then any egalitarian polices designed to benefit the group can be withdrawn once the group escapes the reputation trap.

If a disadvantaged group's initial condition is within the overlap range, however, an active state role may not be necessary to achieve the recovery of the group's reputation. Instead, an emphasis on coordinated optimism could be pursued to achieve the end. (Young members of the group will not increase their human capital investment if they continue to take a pessimistic view of the group's future.) The decisive role of collective optimism as a coordinating device among group members is often ignored in contemporary policy debates. Just as a persistent negative reputation for a group can be self-fulfilling prophecy in the bad steady state, so too can the optimism needed for the reputational recovery be self-justifying: if newly entering group members are sufficiently optimistic about the future to invest in human capital at a high enough rate, then the group's reputation can be improved to the point that future employers have the incentive to treat its members in a favorable manner, thereby justifying their optimism. Various social entities, such as civic organizations, religious groups and government offices, may all contribute to the encouragement of shared optimism.

The reminder of the paper is organized as follows. In the next section, we review the relevant literature and explain its relationship to our dynamic model. In Section 3, we incorporate dynamics into the simplified statistical discrimination model of Coate and Loury (1993) and describe a primary implication of the feasibility of the dynamic reputation recovery path. In Section 4, we propose a formal dynamic system with differential equations and identify both overlap and reputation trap ranges. Section 5 follows with a discussion of the properties of dynamic equilibrium paths and policy interventions from a dynamic perspective. Section 6 presents the study's conclusions.

2 Relevant Literature

In contrast to the rich literature on static models of statistical discrimination developed in the past few decades, the literature on the dynamic evolution of discrimination is sparse. As a result, our understanding of the evolution of stereotypes remains relatively poor, as summarized in Fang and Moro (2011). Nevertheless, there have been several important developments in recent years that are worthy of discussion in relation to our dynamic model.

Blume (2006) adds learning dynamics to a simplified version of Coate and Loury's model of statistical discrimination. Firms' beliefs regarding the productivity of a group are revised for each new cohort based on firms' collective experiences with the previous cohort, and workers' beliefs about labor market conditions are similarly updated. Presenting an asymptotic analysis of the long-run behavior of employment outcomes and agents' beliefs for each parameter level, Blume identifies which equilibria are robust or stochastically stable to small individual-level randomness. Although his approach differs significantly from ours—which relies on economic agents' forward-looking behaviors—the two approaches share common ground in that both seek to identify which equilibria are robust in a long-term horizon, given multiple static equilibria. For instance, both emphasize that, under some circumstances, a low skill investment rate can be a robust equilibrium that cannot be escaped by a disadvantaged group in the absence of external intervention.

Antonovics (2006) presents a dynamic model of statistical discrimination that accounts for intergenerational income mobility. When income is transmitted across generations through parental investment in the human capital of children, statistical discrimination can lead racial groups with low endowments of human capital to become trapped in an inferior stationary equilibrium because future discrimination lowers parents' incentives to invest in their children's human capital. As a result, racial discrimination and racial wage inequality can be self-reinforcing. Although her approach and ours both utilize an overlapping-generations framework to prove the existence of a low human capital investment trap, the underlying forces differ in that the trap in her model originates from the discouragement of parental investment in human capital, while in ours, it arises from the negative influence of the collective reputation of previous cohorts.

Finally, Tirole (1996) explains persistent patterns of behavior, such as the prevalence of hard work and corruption, in professional groups when agents care about their own individual reputations and the behavior and reputations of their peers. Although his paper is not about discriminatory practices but societal behavior, his approach and ours have many similarities: group members make binary decisions (whether to cheat or invest) and firms' hiring decisions are partly based on their beliefs about the average behavior of the group. Both studies show the persistence of a poor collective reputation by proving that the more efficient steady state may be unattainable if initial conditions are poor. The difference is that, in our model, for certain initial states (in the so-called overlap region), groups can converge to either the high-reputation state or the low-reputation state, depending on whether group members have an optimistic or a pessimistic view of the future. This point is important from a policy perspective because groups can become trapped, even when the more efficient steady state is attainable, if their members' collective expectations about the future remain pessimistic. The importance of coordinated expectations is neglected both in Tirole (1996) and in Levin's (2009) subsequent dynamic framework, which illustrates stochastic versions of Tirole's and Coate and Loury's models, allowing the cost of effort to evolve randomly, following driftless Brownian motion.

In the development of the dynamics, we are indebted to Krugman's (1991) insight regarding the interpretation of multiple self-confirming equilibrium paths. He denoted the range of multiple equilibrium paths by *overlap* in his influential argument for the relative importance of history and expectations in the determination of final economic outcomes. Within an overlap, the final state is determined by expectations about the future, while outside the overlap, it is determined by history. We incorporate his general arguments into the dynamic structure of statistical discrimination. Unlike his model, however, which assumes a fixed population, our dynamic model is developed in an overlapping-generations framework. We emphasize the importance of belief coordination over a long-term horizon: expectations coordinated across different time cohorts impact the dynamic self-confirming path to be taken. In this regard, we open the door to a discussion of how to address cross-generational belief coordination in terms of the human capital development of demographic groups.

3 A Dynamic Setup of Statistical Discrimination

In this section, we propose a dynamic set-up of the job assignment model introduced by Coate and Loury (1993). To illustrate how a group's collective reputation evolves over time, we present the simplest possible fixed-wage market equilibrium model. Imagine a large number of identical employers and a larger population of workers, with each employer randomly matched to many workers from this population. Employers assign each worker to one of two jobs, referred to as Task One and Task Zero. Task One is the more demanding and rewarding assignment: workers receive a gross benefit of ω per unit period if assigned to Task One. All workers prefer to be assigned to Task One, whether they are qualified for the task. Employers gain a net return of x_q per unit period if they assign a qualified worker to Task One and suffer a net loss of x_u per unit period if they assign an unqualified worker to Task One. Define $\rho \equiv x_q/x_u$ as the ratio of net gain to net loss. For the sake of simplicity, a worker's gross returns and an employer's net return from an assignment to Task Zero are normalized to zero: the less demanding task with an insignificant reward is performed equally well by qualified and unqualified agents.

Consider an identity group with a large population of workers. Each worker is subject to the "Poisson death process" with parameter λ: in a unit period, each individual faces a probability of dying of λ. We assume that the total population of each identity group is constant at N^i for group i, implying that a fraction λ of the group's population is replaced by newborn group members in each unit period. Each newborn group member makes a skill investment decision in the early stage of his life and then works for the rest of his life. Only those who make some ex ante skill investment are qualified for Task One. The cost of obtaining a skill varies among the newborns. We assume the simplest form of the cost distribution where each cohort is composed of three types of agents, as in other dynamic frameworks suggested by Tirole (1996) and Blume (2006): Π_l is the proportion of workers whose investment cost is very small and close to zero, $1 - \Pi_h$ is the proportion of workers whose investment cost is very high and beyond the highest possible benefit from skill investment, and $\Pi_h - \Pi_l$ is the proportion of workers whose investment cost is intermediate and fixed as c_m. The cost distribution $G(c)$ is then Π_l for $c \in (\epsilon, c_m)$ and Π_h for $c \in [c_m, K)$, for a very large number K.

3.1 Labor Market Composition

The labor market is composed of two parts: the so-called *temporary job market* and the *permanent job market*. After the skill investment period, each worker enters the temporary job market in which his qualification is not yet known to employers and thus employers cannot determine whether a job candidate is qualified for Task One. Employers, however, observe each candidate's group identity and a noisy signal θ, the distribution of which depends on whether he is qualified. The signal might be the result of a test, an interview, or some form of on-the-job monitoring: it is distributed for a qualified worker as $f_q(\theta)$ and for an unqualified worker as $f_u(\theta)$, where $F_q(\theta) \leq F_u(\theta)$ for all θ. Without knowing a worker's true qualification, employers' assignment decisions depend entirely on both the group identity i and the signal θ of a job candidate.

We assume that in the temporary job market, the job contract between a worker and a randomly-matched employer is short-term and that there is a very high turnover rate so that workers must recontract with different employers in each short-term period. We further assume that there is no "memory" of individual workers and that employers cannot perfectly distinguish the dates at which job candidates invested in skills. Therefore, newborn workers are "mixed" with the rest of the group population, and "age" becomes another source of imperfect information that makes the behavior of previous cohorts relevant to decisions made today, as in other collective reputation models, such as Tirole (1996) and Levin (2009).[1]

At some point, however, the true characteristics of each worker are revealed to employers. We assume that as a worker spends more time in the temporary job market, the worker's true capabilities are more likely to be revealed under the Poisson process with parameter η: a worker in the temporary job market reveals his true quality with probability η in each unit period. Once his true capabilities are revealed, he leaves the temporary job market and joins the permanent job market, where he secures a permanent job and is assigned to a task according to his qualifications: Task One, if qualified; Task Zero, otherwise. Thus, the permanent job market is composed of relatively older workers, while the temporary job market is composed of relatively younger workers.

Finally, we assume that the size of the temporary job market and that of the permanent job market are fixed: a fraction n of all workers are in the temporary job market, while a fraction $(1 - n)$ of all workers are in the permanent job market. Among group i members, inflows into the temporary job market within a short time period Δt are $N^i \lambda \Delta t$, and outflows from the market are $N^i n(\eta \Delta t + \lambda \Delta t - \eta \lambda \Delta t^2)$. Because we assume a fixed population in the temporary job market, the following holds: $\lambda = n\eta + n\lambda - n\eta\lambda\Delta t$. Taking the limit as

[1] If "age" is perfectly observable, a cohort's investment decision is not affected by the group's collective reputation. Therefore, the complementarity between employers' beliefs and the cohort's investment fully determine the economic outcome. No dynamic framework may be applicable in that case.

$\Delta t \to 0$, we obtain the following condition:

$$\lambda + \eta = \frac{\lambda}{n} \quad (\equiv \lambda'), \tag{1}$$

where λ' indicates the rate at which the population in the temporary job market is replaced by newborn group members: $\lambda' \equiv \lambda + \eta$.

3.2 Employers' Assignment Decision

In the following discussion, we focus on the temporary job market in which between-group unequal treatment occurs via job assignment decisions by employers. Consider an example from Coate and Loury(1993), where the job market screening process produces only three outcomes: "pass," which is achieved only by those who are qualified; "fail," which is achieved only by those who are not qualified; and "unclear," which can be achieved by either group but is less likely to be achieved if the worker is unqualified. That is, the screening process is more effective in identifying the unqualified than the qualified.

To describe the three possible outcomes, we adopt the simplest signal functions: $f_u(\theta)$ is uniformly distributed in $[0, \theta_u]$, and $f_q(\theta)$ is uniformly distributed in $[\theta_q, \bar{\theta}]$, where $\theta_q < \theta_u$. If the signal is below θ_q, the worker must be unqualified, and if the signal is above θ_u, the worker must be qualified. If the signal is between θ_q and θ_u, the signal cannot indicate the true characteristic of the worker. Let us denote the probability that if a worker does invest, his test outcome proves that he is qualified by $P_q(= \frac{\bar{\theta}-\theta_u}{\bar{\theta}-\theta_q})$. Similarly, let us denote the probability that if a worker does not invest, his test outcome proves that he is unqualified by $P_u(= \frac{\theta_q}{\theta_u})$.[2]

In this environment, we know that employers will assign all who "pass" to the more valued task (Task One) and all who "fail" to the less valued task (Task Zero). Additionally, we know that the employer accords the "benefit of the doubt" (BOD) to a worker with an "unclear" test result by assigning him or her to Task One. Let $\xi_\tau \in [0, 1]$ denote the probability that a worker will receive the benefit of the doubt in the event that this worker presents an "unclear" test result at time τ in the temporary job market. When BOD is given, the expected net return from investing in human capital is ωP_u, and ωP_q otherwise, given a wage rate of ω for Task One. Then the expected return from investing in human capital accrued at time τ in the temporary job market (β_τ) is summarized by a function of ξ_τ:

$$\beta_\tau(\xi_\tau) = \omega(P_q + \xi_\tau(P_u - P_q)). \tag{2}$$

As we assume that the unqualified are more effectively screened, the following holds: $0 < P_q < P_u < 1$. Therefore, the net return $\beta_\tau(\xi_\tau)$ is increasing in the probability that the worker receives the benefit of the doubt (ξ_τ). It is notable that if the opposite assumption holds ($0 < P_u < P_q < 1$), the expected net return to the skill investment $\beta_\tau(\xi_\tau)$ is decreasing in the probability that the worker receives the benefit of the doubt: the greater net return is expected to be accrued when the group is more likely to fail to achieve employers' favor.

[2] For improved disposition of the equations, we slightly alter the notation of Coate and Loury (1993). While they denote the probability that if a worker does (does not) invest, his test outcome proves that he is qualified (unqualified) by $1 - p_q$ $(1 - p_u)$, we use the notation P_q (P_u) to denote this probability.

This is not an ordinary economy that we examine.

What about the employer's decision to give the worker the benefit of the doubt? The stronger is the group's collective reputation in the job market, the more likely employers are to give workers the benefit of the doubt. In the given economy, they will give the benefit of the doubt if and only if the expected net return from doing so is non-negative: $x_q \cdot Prob[\text{qualified}|\theta] - x_u \cdot Prob[\text{unqualified}|\theta] \geq 0$, for $\theta_q \leq \theta \leq \theta_u$. Using Bayes' rule, the posterior probability that the worker with group identity i and an unclear signal ($\theta_q \leq \theta \leq \theta_u$) is qualified is $\frac{\Pi^i(1-P_q)}{\Pi^i(1-P_q)+(1-\Pi^i)(1-P_u)}(= Prob[\text{qualified}|\theta])$, given the employers' belief about the proportion of qualified workers (Π^i) among group i members in the temporary job market. The above inequality is equivalent to the requirement that $\Pi^i \geq \Pi^*$, where $\Pi^* \equiv \frac{1-P_u}{\rho(1-P_q)+1-P_u}$ with $\rho = \frac{x_q}{x_u}$. That is, the benefit of the doubt is given only when the "collective reputation" of the identity group to which a worker belongs is equivalent to or exceeds some threshold level Π^*, in that $\xi_\tau = 1$ for $\Pi_\tau \geq \Pi^*$, while $\xi_\tau = 0$ for $\Pi_\tau < \Pi^*$. Thus, a worker receives the benefit of the doubt if and only if the employer is sufficiently optimistic about the rate of human capital acquisition in the population from which the worker is drawn.

3.3 Workers' Skill Investment

Likewise, in this environment, a worker will acquire human capital if and only if that worker's cost of human capital acquisition is not greater than the anticipated net return. We assume that time is continuous and that workers discount future receipts at a rate $\delta > 0$. All workers also face the instantaneous probability $\lambda\Delta t$ of "dying" during time interval $(t, t + \Delta t)$. Workers in the temporary job market face the instantaneous probability $\eta\Delta t$ of revealing their true capabilities and, consequently, joining the permanent job market during time interval $(t, t+\Delta t)$. We further assume that newborn group members replacing "dying" workers must make their human capital investment decisions at the moment of birth and cannot alter such decisions subsequently. Thus, newborns either invest in human capital or not, based on whether the expected discounted net return from doing so is greater or less than that worker's investment cost, c.

Now, imagine that workers formulate expectations concerning employers' future behavior, which, for our purposes, can be fully described by the time-varying function ξ_τ: $\{\xi^e_\tau\}^\infty_t$. Then a worker newly born at time t anticipates an expected present discounted return to human capital investment as a function of the anticipated future behavior of employers, $R(\{\xi^e_\tau\}^\infty_t)$:

$$\begin{aligned} R(\{\xi^e_\tau\}^\infty_t) &= (\text{Return in Temporary Job Market}) + (\text{Return in Permanent Job Market}) \\ &= \int_t^\infty [\beta_\tau(\xi^e_\tau) \cdot e^{-\eta(\tau-t)}]e^{-(\delta+\lambda)(\tau-t)}\mathrm{d}\tau + \int_t^\infty [\omega \cdot (1 - e^{-\eta(\tau-t)})]e^{-(\delta+\lambda)(\tau-t)}\mathrm{d}\tau. \\ &= \int_t^\infty [\beta_\tau(\xi^e_\tau) \cdot e^{-(\delta+\lambda+\eta)(\tau-t)}]\mathrm{d}\tau + \frac{\eta\omega}{(\delta+\lambda)(\delta+\lambda+\eta)}, \end{aligned} \tag{3}$$

where $e^{-\eta(\tau-t)}$ indicates the probability that a worker's true capabilities are not yet revealed over the time interval $(t, \tau - t)$. For notational simplicity, we denote the level of the net return in the permanent job market by $\omega'(\equiv \frac{\eta\omega}{\delta+\lambda})$. Thus, using the compressed notation, λ' and ω', the above expected present discounted return is expressed in simpler form by

$$R(\{\xi^e_\tau\}^\infty_t) = \int_t^\infty [\beta_\tau(\xi^e_\tau) \cdot e^{-(\delta+\lambda')(\tau-t)}]\mathrm{d}\tau + \frac{\omega'}{\delta+\lambda'}. \tag{4}$$

Now, it is the complementarity between a group's reputation and its members' human capital incentives that creates the possibility of multiple self-confirming reputations. Noting that $R(\{0\}^\infty_t) = \frac{\omega P_q+\omega'}{\delta+\lambda'}$ and $R(\{1\}^\infty_t) = \frac{\omega P_u+\omega'}{\delta+\lambda'}$, we have the following lemma:

Lemma 1. *Assuming that* $\frac{\omega P_q+\omega'}{\delta+\lambda'} < c_m < \frac{\omega P_u+\omega'}{\delta+\lambda'}$, *if the following inequality holds, then both* Π_l *and* Π_h *are equilibrium group reputations in the given model:*

$$G(R(\{0\}^\infty_t)) = \Pi_l < \Pi^* < G(R(\{1\}^\infty_t)) = \Pi_h. \tag{5}$$

First, imagine that a group's current reputation, that is, the employers' belief about the rate of human capital acquisition among the group's population, is Π_l in the temporary job market. If newborns believe that employers will withhold the benefit of the doubt forever (that is, $\{\xi^e_\tau\}^\infty_t = \{0\}^\infty_t$), then current newborns invest in human capital acquisition at the rate Π_l because $G(R(\{0\}^\infty_t)) = \Pi_l$, thus maintaining a poor collective reputation. As the newborns that follow continue to hold the same expectations regarding the future behavior of employers, they will also invest at the rate Π_l, and the current reputation Π_l becomes a steady state. The benefit of the doubt is then denied forever, self-confirming the workers' earlier pessimistic expectations regarding employers' future behavior. Secondly, imagine that a group's current reputation is Π_h in the temporary job market. An analogous explanation can be given for the high reputation equilibrium Π_h, in which workers' expectations that employers will give them the benefit of the doubt forever is self-confirmed.

In the following discussion, we assume that the above lemma holds, so that both equilibrium group reputations, Π_l and Π_h, exist. Therefore, different groups may undertake differing equilibrium human capital investments, exhibit contrasting levels of employment in the more demanding job, and obtain different average returns to skill investment, despite having identical fundamentals with respect to investment cost and information technology.

3.4 Reputation Recovery

Our primary focus is the recovery path from the low equilibrium reputation Π_l to the high equilibrium reputation Π_h. Is it possible that a group with a low reputation can recover its reputation through increased skill investment? Yes, it is possible under some limited conditions. If newborns expect that while their group is being denied the benefit of the doubt today, they may still be extended this benefit in the future, and if the anticipated date at which employers' behavior will shift is not too distant in the future, then newborns may wish to acquire human capital at a higher rate than Π_l. This enhanced human capital acquisition may raise the groups' reputation sufficiently that employers would, indeed, change their treatment of the group at some point in the future.

That is, there may exist a date $T > 0$ such that $\xi_t = 0$ for $t < T$ and $\xi_t = 1$ for $t \geq T$. The poorer is the initial collective reputation Π_l or, equivalently, the larger is the distance between Π_l and Π^*, the more time T may be required for employers to change their treatment of the group. Anticipating that current and subsequent newborns enhance each cohort's skill investment rate up to Π_h and that the benefit of the doubt will be given beginning at date T, the return to human capital acquisition for a current newborn worker at time $t = 0$ is

$$\begin{aligned} R(\{\xi^e_\tau\}_0^\infty) &= \int_0^T [\beta_\tau(0) \cdot e^{-(\delta+\lambda')\tau}]\mathrm{d}\tau + \int_T^\infty [\beta_\tau(1) \cdot e^{-(\delta+\lambda')\tau}]\mathrm{d}\tau + \frac{\omega'}{\delta+\lambda'} \\ &= \frac{1}{\delta+\lambda'}\left\{\omega P_q + \omega' + (\omega P_u - \omega P_q)e^{-(\delta+\lambda')T}\right\}. \end{aligned} \tag{6}$$

Current newborns will invest at the rate Π_h if and only if the above return is not less than c_m. Subsequent newborn cohorts sharing the same optimistic expectations will then continue to invest at the high rate because the anticipated date at which employers' behavior shifts will be still less distant for them.

However, it is important to acknowledge that the recovery path is not always feasible. If the above return is smaller than c_m, current newborns have no incentive to enhance their human capital acquisition. That is, if the current reputation level Π_l is so low that even with increased skill investment, employers' change of treatment is seen as far in the future for current newborns, the medium cost newborns will stop investing in skills and the group's collective reputation will never improve. In this case, we may say that the group is stuck in a "reputation trap."

The above discussion can be summarized by the following lemma:

Lemma 2. *The conjectured dynamic recovery equilibrium exists if and only if the conjectured time T at which employers shift their treatment of the low reputation group satisfies the following condition:*

$$R(\{\xi^e_\tau\}_0^\infty) \geq c_m \quad \Longleftrightarrow \quad T \leq \frac{1}{\delta+\lambda'}\ln\left(\frac{\omega P_u - \omega P_q}{(\delta+\lambda')c_m - \omega P_q - \omega'}\right). \tag{7}$$

Therefore, the key problem is to characterize the circumstances under which reputational recovery can occur in the dynamic equilibrium of the model such that newborns' optimism about the future enables a group's reputation to recover from the initial low state within a finite period of time. For this purpose, we must find the determinants of the shift time T at which employers change their treatment of the group.

First, we construct the differential equation of the collective reputation variable Π_t. Let ϕ_t denote the rate of human capital acquisition among workers newly entering the market at time t. We assume that employers have correct beliefs at each date about human capital acquisition within each population group. This implies that the collective reputation variable Π^i_t indicates the true skill investment rate of the group i population in the temporary job market. Now, consider a very short time interval, from t to $t + \Delta t$. Suppose that at the beginning of the interval, a randomly chosen fraction $(\lambda\Delta t + \eta\Delta t - \lambda\eta\Delta t^2)$ of the group's population in the temporary job market $(N^i n)$ moves out of the market either by "dying" or through revelation of workers' true capabilities, as discussed in section 3.1. At the same time, a fraction $\lambda\Delta t$ of the group's total population (N^i) is newly born. At the end of the interval, the newborn agents incur the cost of skill achievement and enter the temporary job market. The total number of skilled workers in the temporary job market at time $t + \Delta t$ will be the sum of skilled workers in the surviving population and those in the newborn cohort. Thus, the total number of skilled workers belonging to group i in the temporary job market at time $t + \Delta t$, $N^i n \cdot \Pi^i_{t+\Delta t}$, is approximated as

$$\begin{aligned} N^i n \cdot \Pi^i_{t+\Delta t} &\approx (N^i n - \text{“outflow” in } \Delta t) \times \Pi^i_t + (\text{“inflow” in } \Delta t) \times \phi^i_t \\ &\approx (1 - \lambda\Delta t - \eta\Delta t + \lambda\eta\Delta t^2) N^i n \cdot \Pi^i_t + N^i \lambda \Delta t \cdot \phi_t. \end{aligned} \tag{8}$$

Rearranging, we have
$$\frac{\Delta\Pi^i_t}{\Delta t} \equiv \frac{\Pi^i_{t+\Delta t} - \Pi^i_t}{\Delta t} \approx -(\lambda + \eta)\Pi^i_t + \frac{\lambda}{n} \cdot \phi^i_t + \lambda\eta\Delta t \cdot \Pi^i_t.$$

Taking the limit as $\Delta t \to 0$ and using equation (1), we can express how Π^i_t evolves over time for each population group in the temporary labor market:

$$\dot{\Pi}^i_t = \lambda'[\phi^i_t - \Pi^i_t], \text{ in which } \lambda' \equiv \lambda + \eta. \tag{9}$$

The differential equation indicates that $\dot{\Pi}^i_t > 0$ when $\phi^i_t > \Pi^i_t$: if the fraction of newborn agents who invest in skill (ϕ^i_t) is greater than the current collective reputation level in the temporary job market (Π^i_t), the group's reputation improves at time t. It does not change when the two terms are equal; otherwise, it declines. The speed of group reputational change is also determined by the difference between the two terms, ($|\phi^i_t - \Pi^i_t|$) and the rate at which the population is replaced in the market (λ').

Because the group's collective reputation improves from Π_l to Π^* given $\phi^i_t = \Pi_h$ over the time span [0, T], we can calculate the time T, applying the above differential equation (9):

$$T = \frac{1}{\lambda'} \cdot \ln\left(\frac{\Pi_h - \Pi_l}{\Pi_h - \Pi^*}\right). \tag{10}$$

Applying time T thus calculated to Lemma 2 directly, we obtain the following conclusive result:

Proposition 1 (Recovery Path)**.** *The conjectured dynamic recovery from a poor initial collective reputation* Π_l *is not possible when the initial skill level of the group is sufficiently low that it satisfies*

$$\Pi_l < \Pi_h - (\Pi_h - \Pi^*)\left(\frac{\omega P_u - \omega P_q}{(\delta + \lambda')c_m - \omega P_q - \omega'}\right)^{\frac{\lambda'}{\delta+\lambda'}}. \tag{11}$$

The proposition implies that a group with a poor collective reputation may be unable to recover its reputation through the coordinated expectations of group members because current newborns may not find that conjectured future benefits can compensate for the necessary human capital acquisition costs.

Additionally, although a group's initial collective reputation Π_l may not be especially poor, the group may be trapped in a low reputation equilibrium if group members weight too heavily the near future or are strongly myopic. Thus, from the above proposition, we obtain the following equivalent result, given the two reputational equilibria, Π_h and Π_l:

Corollary 1. *If the group members' time discount rate δ is sufficiently high that it satisfies the following condition, the conjectured dynamic recovery from the poor initial collective reputation Π_l is not possible:*

$$\frac{\delta}{\lambda'} > \frac{\ln(\omega P_u - \omega P_q) - \ln((\delta + \lambda')c_m - \omega P_q - \omega')}{\ln(\Pi_h - \Pi_l) - \ln(\Pi_h - \Pi^*)} - 1. \tag{12}$$

4 Dynamic System with a Collective Reputation

Whether the recovery path is available for a group with an arbitrary initial reputation can be examined using a formal dynamic system with two differential equations: one for the collective reputation variable Π_t, which is constructed above in equation (9), and one for the expected net return variable $R(\{\xi_\tau^e\}_t^\infty)$. For convenience, multiplying the net return displayed in equation (4) by the sum of the discounting factors, $\delta + \lambda + \eta$ $(= \delta + \lambda')$, we denote as V_t^e the "normalized" expected net return from investing in human capital acquisition:

$$V_t^e = (\delta + \lambda') \int_t^\infty [\beta_\tau(\xi_\tau^e) \cdot e^{-(\delta+\lambda')(\tau - t)}]\mathrm{d}\tau + \omega'. \tag{13}$$

Taking the derivative of this expression with respect to t, the following differential equation, which describes how the anticipated discounted net return V_t^e evolves over time when group members share the same future expectations regarding employers' behavior $\{\xi_\tau^e\}_t^\infty$, must obtain:

$$\dot{V}_t^e = (\delta + \lambda')[V_t^e - \beta_t(\xi_t) - \omega']. \tag{14}$$

The differential equation indicates that $\dot{V}_t^e > 0$ when $\beta_t(\xi_t) < V_t^e - \omega'$: if the currently accrued net return to skill investment in the temporary job market ($\beta_t(\xi_t)$) is smaller than a normalized level of the discounted net return from the temporary job market (i.e., the normalized "total" discounted net return (V_t^e) minus the normalized discounted net return from the "permanent" job market (ω')), then the total discounted net return expected to accrue between time $t + \Delta t$ and death ($V_{t+\Delta t}^e$) would exceed its current level: $V_{t+\Delta t}^e > V_t^e$.

For each population group that shares a common collective reputation, the evolution rule of the skill investment rate in the temporary job market, $\dot{\Pi}_t^i$, and that of the expected net return to human capital acquisition, $\dot{V}_t^e$, are displayed in the following theorem, where Π_t is a flow variable and V_t^e is a jumping variable. The group skill level Π_t in the temporary job market is constantly adjusted by the skill investment rate of the newborn cohort (ϕ_t), implying that Π_t is a flow variable that cannot suddenly jump at some moment in time. However, the net return to human capital acquisition V_t^e depends on expectations regarding employers' assignment behaviors in the future. Through an alteration of the expectation of $\{\xi_\tau^e\}_t^\infty$, V_t^e can make a sudden jump at any point in time.

Theorem 1 (Dynamic System)**.** *The dynamic system with the flow variable Π_t and the jumping variable V_t^e is summarized by the following two-variable differential equations:*

$$\begin{aligned} \dot{\Pi}_t &= \lambda'[\phi_t - \Pi_t] \\ \dot{V}_t^e &= (\delta + \lambda')[V_t^e - \beta_t(\xi_t) - \omega'], \end{aligned}$$

where ϕ_t is a function of V_t^e, as $\phi_t = G(\frac{V_t^e}{\delta+\lambda'})$, and $\beta_t(\xi_t)$ is a function of Π_t, as $\xi_t = 1$ for $\Pi_t \geq \Pi^*$, while $\xi_t = 0$ for $\Pi_t < \Pi^*$, with $\lambda' \equiv \lambda + \eta$ and $\omega' \equiv \frac{\eta\omega}{\delta+\lambda}$. The two isoclines of the time dependent variables are represented by

$$\dot{\Pi}_t = 0 \textit{ Locus} \quad : \quad \Pi_t = \phi_t$$
$$\dot{V}_t^e = 0 \textit{ Locus} \quad : \quad V_t^e = \beta_t(\xi_t) + \omega'.$$

The two isolines and the direction arrows are displayed in Figure 1. The first isocline is determined by the newborns' skill investment rate ϕ_t, which equals $G(\frac{V_t^e}{\delta+\lambda'})$ so that the $\dot{\Pi}_t = 0$ locus is represented by $\Pi_t = \Pi_l$ for any $V_t^e \in [(\delta+\lambda')\epsilon, (\delta+\lambda')c_m)$ and by $\Pi_t = \Pi_h$ for any $V_t^e \in [(\delta+\lambda')c_m, (\delta+\lambda')K]$, where ϵ is close to zero and K is a very large number. Above (below) the $\dot{\Pi}_t = 0$ locus, the movement of Π_t is southward (northward). The second isocline is determined by the currently accrued net return to skill investment in the temporary job market ($\beta_t(\xi_t)$) plus the normalized discounted net return in the permanent job market (ω'). Using equation (2), the $\dot{V}_t^e = 0$ locus is represented by $V_t^e = \omega P_q + \omega'$ for any $\Pi_t \in [0, \Pi^*)$ and by $V_t^e = \omega P_u + \omega'$ for any $\Pi_t \in [\Pi^*, 1]$. To the left (right) of the $\dot{V}_t^e = 0$ locus, the movement of V_t^e is westward (eastward).

4.1 Equilibrium Paths Leading to Steady States

Under Lemma 1, the dynamic system has two steady states, ($\omega P_q + \omega'$, Π_l) and ($\omega P_u + \omega'$, Π_h), denoted by Q_l and Q_h, respectively. First, consider the low reputation steady state, Q_l. Given a low initial collective reputation Π_l of one's identity group, the normalized net return to human capital acquisition is $\omega P_q + \omega'$ in so far as current and subsequent newborns hold the pessimistic expectation that employers will withhold the benefit of the doubt forever ($\xi_\tau^e = 0, \forall\tau \geq 0$). As newborns' skill investment rate will continue to be as low as Π_l, with pessimism spread across the different time cohorts, employers never accord the benefit of the doubt and the newborns' earlier expectations regarding employers' unfavorable future behavior are self-confirmed.

However, there is another possibility in the low reputation state, Q_l. If newborns hold the optimistic expectation that employers will change their behavior within a finite period of time ($\xi_\tau^e = 1, \forall\tau \geq T,$ while $\xi_\tau^e = 0, \forall\tau < T$), then the normalized net return to human capital acquisition may exceed $(\delta + \lambda')c_m$. In this case, newborns increase their skill investment rate up to Π_h. In so far as subsequent newborns maintain the optimistic expectation, the group's collective reputation improves overtime, and eventually employers will start to accord group members the benefit of the doubt when $\Pi_t^i \geq \Pi^*$. Thus, newborns' earlier optimistic expectations regarding employers' favorable treatment in the future are self-confirmed.

Both possibilities are well depicted in Panel A of Figure 2, which shows two equilibrium paths: a pessimistic path leading to a low reputation steady state Q_l and an optimistic path leading to a high reputation steady state Q_h. Given a group's initial low reputation Π_l, whether the group's reputation can recover depends on group members' coordinated expectations regarding the future (i.e. whether the group in state Q_l "takes" the optimistic path in the phase diagram).

Next, consider a high reputation steady state Q_h. Given the high initial collective reputation Π_h of a group, the normalized net return to human capital acquisition is $\omega P_u + \omega'$ in so far as current and subsequent newborns optimistically expect that employers will continue to accord group members the benefit of the doubt forever ($\xi^e_\tau = 1, \forall \tau \geq 0$). Because the net return exceeds the medium cost level, $\omega P_u + \omega' > (\delta + \lambda')c_m$, the skill investment rates of newborn cohorts are Π_h. Then, the newborns' earlier optimistic expectations are self-confirmed by employers' continuing favorable treatment of the group in the future.

However, we cannot rule out another possibility, namely, that current and subsequent newborns pessimistically expect that employers will stop giving group members the benefit of the doubt at some point in the future ($\xi^e_\tau = 0, \forall \tau \geq T'$, while $\xi^e_\tau = 1, \forall \tau < T'$). Then, the normalized net return to human capital acquisition may fall below $(\delta + \lambda')c_m$, as depicted in Panel A of the figure. In this case, newborns decrease their skill investment rate to Π_l. As subsequent newborns maintain the same pessimism into the future, the group's collective reputation deteriorates overtime, and eventually, when $\Pi^i_t < \Pi^*$, employers cease according group members the benefit of the doubt. The reputation of the group will gradually fall to a low level Π_l. Therefore, given the initial high reputation Π_h of a group, whether the group's high reputation can be sustained also depends on group members' coordinated expectations regarding the future (i.e. whether the group in state Q_h "takes" the pessimistic path in the phase diagram).

4.2 Feasibility of Dynamic Recovery Path

Our primary interest is in characterizing the circumstances under which a reputational recovery to the state Q_h can occur in the given dynamic model of statistical discrimination. In fact, we can imagine many situations in which the recovery is not feasible. For example, with greater δ, the speed of the expected net return change $|\dot{V}^e_t|$ is also greater, and consequently, the dynamic equilibrium paths in the given phase diagram become "flatter." Then the dynamic reputational recovery path may not be feasible for an identity group, given a collective reputation of Π_l, as depicted in Panel B of Figure 2.

In the analysis below, we denote the lowest reputation level at which the optimistic equilibrium path leading to Q_h passes the vertical line $V^e_t = (\delta + \lambda')c_m$ by π^o. Along the optimistic path, a group's state $((\delta + \lambda')c_m, \, \pi^o)$ moves to state $(\omega P_u + \omega', \, \Pi^*)$, given the high skill investment rate of the newborn cohorts of ($\phi_t = \Pi_h, \forall t \in [0, T]$), while the benefit of the doubt is withheld by employers ($\beta_t(0) = \omega P_q, \forall t \in [0, T]$). In symmetric fashion, we denote the highest reputation level at which the pessimistic equilibrium path leading to Q_l passes the vertical line $V^e_t = (\delta + \lambda')c_m$ by π^p. Along the pessimistic path, a group's state $((\delta + \lambda')c_m, \, \pi^p)$ moves to state $(\omega P_q + \omega', \, \Pi^*)$, given the low skill investment rate of newborns ($\phi_t = \Pi_l, \forall t \in [0, T']$), while employers accord group members the benefit of the doubt ($\beta_t(1) = \omega P_u, \forall t \in [0, T']$). Using the given dynamic system, π^o and π^p are directly computed from the following equations:

$$\pi^o \quad : \quad \int_{\pi^o}^{\Pi^*} \frac{\mathrm{d}\Pi}{\lambda'(\Pi_h - \Pi)} = \int_{(\delta+\lambda')c_m}^{\omega P_u + \omega'} \frac{\mathrm{d}V^e}{(\delta + \lambda')(V^e - \omega P_q - \omega')} \tag{15}$$

$$\pi^p \quad : \quad \int_{\pi^p}^{\Pi^*} \frac{\mathrm{d}\Pi}{\lambda'(\Pi_l - \Pi)} = \int_{(\delta+\lambda')c_m}^{\omega P_q + \omega'} \frac{\mathrm{d}V^e}{(\delta + \lambda')(V^e - \omega P_u - \omega')} \tag{16}$$

Therefore, we obtain the following result:

$$\pi^o = \Pi_h + (\Pi^* - \Pi_h) \cdot v^{*-\frac{\lambda'}{\delta+\lambda'}}, \tag{17}$$

$$\pi^p = \Pi_l + (\Pi^* - \Pi_l) \cdot (1 - v^*)^{-\frac{\lambda'}{\delta+\lambda'}}, \tag{18}$$

where we define $v^* \equiv \frac{(\delta+\lambda')c_m - \omega P_q - \omega'}{\omega P_u - \omega P_q}$ (< 1). Note that π^o (π^p) increases (decreases) with the time discount factor δ: $\frac{\partial \pi^o}{\partial \delta} > 0$ and $\frac{\partial \pi^p}{\partial \delta} < 0$.

For the dynamic recovery path to be feasible for the poor reputation group in state Q_l, π^o should not be greater than Π_l: $\pi^o \leq \Pi_l$, as depicted in Panel A of Figure 2. If $\pi^o > \Pi_l$, as depicted in Panel B of the figure, dynamic recovery is unobtainable for the group because the discounted net return from the optimistic scenario that employers change their behaviors within a finite period of time cannot exceed the incurred medium cost level c_m. This implies that if the initial skill level of a group in the low reputation equilibrium (Π_l) is sufficiently low that it is even less than the critical level π^o, or if group members are myopic, placing too much weight on the near future (that is, their time discount factor δ is sufficiently large), then π^o becomes greater than Π_l, and consequently, the dynamic reputation recovery path is not feasible for the poor reputation group in state Q_l. It can easily be checked that these results are equivalent to those summarized in Proposition 1 and Corollary 1:

Proposition 2 (Recovery Path)**.** *Under the dynamic system stated in Theorem 1, the dynamic reputation recovery path is not feasible for group members whose collective reputation is the low equilibrium reputation* Π_l *when the following condition holds, which is equivalent to the conditions provided in Proposition 1 and Corollary 1:*

$$\Pi_l < \pi^o \left(= \Pi_h + (\Pi^* - \Pi_h) \cdot v^{*-\frac{\lambda'}{\delta+\lambda'}} \right).$$

4.3 Overlap and Reputation Trap

In this section, we extend our analysis and examine possible paths from an arbitrary initial reputation of group i, $\Pi_0^i \in [0, 1]$. First, consider the case where a group's initial reputation is below the BOD standard Π^*: $\Pi_0^i < \Pi^*$. One possibility is that group members expect employers to continue to withhold the benefit of the doubt forever: $\xi_t^e = 0$, $\forall t \geq 0$. Then the normalized expected net return V_t^e is fixed at $\omega P_q + \omega'$ for all $t \geq 0$, a level that is below $(\delta + \lambda')c_m$. Because newborn workers will invest at the rate Π_l, the group's collective reputation converges to the low steady state level Π_l as $t \to \infty$. This possibility is described by the pessimistic path in Figure 2.

Another possibility is that newborns expect that while the group is being denied the benefit of the doubt today, it may yet be extended to group members at some future time T: $\xi_t^e = 1$, $\forall t \geq T$. If the anticipated date at which employer's behavior will shift is not too distant, then newborns may wish to acquire human capital at a higher rate Π_h: $\phi_0 = \Pi_h$ if $V_0^e > (\delta + \lambda')c_m$. Provided that subsequent newborn workers do not alter these optimistic expectations, the higher skill investment rate will continue, and the group's reputation will improve overtime, eventually converging to Π^* at time T, when employers indeed begin to accord group members the benefit of the doubt. This possibility is described by the optimistic path in Figure 2. It is notable that given that $\pi^o > \Pi_l$, coordinated optimistic expectations may not be feasible when the group's initial reputation level is below the critical level π^o: $\Pi_0^i < \pi^o$, as depicted in Panel B of the figure.

Second, consider the case where the group's initial reputation is above the BOD standard Π^*: $\Pi_0^i \geq \Pi^*$. Again, one possibility is that group members believe that the employers will continue to accord them the benefit of the doubt forever: $\xi_\tau^e = 1$, $\forall \tau \geq 0$. Because the normalized expected net return V_t^e is fixed at $\omega P_u + \omega'$ for all $t \geq 0$, a level that is above $(\delta + \lambda')c_m$, newborn workers invest at the rate Π_h, and the group's reputation converges to Π_h as $t \to \infty$. This possibility is also described by the optimistic path in Figure 2. There exists another possibility, however, namely, that newborns expect that while the group is receiving the benefit of the doubt today, employers will stop giving it at some point in the future T': $\xi_\tau^e = 0$, $\forall \tau \geq T'$. If the anticipated date at which employers' behavior is to shift is not very distant, then newborns with a medium cost of skill investment may not wish to acquire human capital today: $\phi_0 = \Pi_l$ if $V_0^e < (\delta + \lambda')c_m$. Provided that newborn workers do not alter these pessimistic expectations, the group's reputation will deteriorate overtime and converge to Π^* at time T', when employers indeed change their treatment of the group. This possibility is also described by the pessimistic path in Figure 2. It is also notable that given that $\pi^p < \Pi_h$, coordinated pessimistic expectations may not be feasible when the group's initial reputation is sufficiently strong that $\Pi_0^i > \pi^p$, as depicted in Panel B of the figure.

Thus far, we have described two self-confirming equilibrium paths: an optimistic (pessimistic) path along which all group members are well coordinated in their optimism (pessimism) and where the optimistic (pessimistic) result is self-confirmed in the future. Nonetheless, we cannot rule out other possible paths that are not "self-confirming." For example, some fraction (α) of every cohort may be optimistic, while others $(1 - \alpha)$ are pessimistic when both scenarios seem plausible, where the final state, either high or low equilibrium steady state, depends on the relative size of the optimistic fraction of the group, determined by the parameter α. Although there may exist many different combinations worthy of consideration, the feasibility of the dynamic recovery path to high equilibrium steady state (or that of the dynamic "deteriorating" path to low equilibrium steady state) is exclusively confined to some range of a group's current reputation level, Π_0^i.

For example, as discussed above, given that $\pi^o > \Pi_l$, the dynamic recovery path is never feasible if the initial reputation level is so poor that it is less than π^o: that is, even if optimism is ready to be fully shared among all group members ($\alpha = 1$), newborns will not invest at the high rate Π_h because the anticipated date of employers' behavioral change toward favorable treatment is too distant, implying that the reputation recovery never happen. Similarly, given that $\pi^p < \Pi_h$, the group's reputation will never deteriorate to the low equilibrium reputation state if the current reputation level is sufficiently strong that $\Pi_0^i > \pi^p$: that is, even if pessimism is ready to be shared among all group members ($\alpha = 0$), newborns who know the anticipated date of employers' change of treatment is too distant will invest at the high rate Π_h and a deterioration in reputation is not possible. This feasibility discussion is summarized, using the term "overlap", borrowed from Krugman(1991):

Definition 1 (Overlap)**.** *A range of a group's reputation level is called "overlap". If the group's initial reputation Π_0^i is within the overlap, then the group's collective reputation can converge either to the high equilibrium reputation state, if an optimistic view is shared among group members, or to the low equilibrium reputation state, if a pessimistic view is shared among group members. On the other hand, if a group's current reputation* *level is below (above) the overlap, the dynamic recovery (deteriorating) path is not feasible.*

That is, outside the overlap, there exists a unique equilibrium path either to Q_h or to Q_l so that a group's final state cannot be altered by the choice of coordinated expectations among group members. The boundary of overlap is characterized by the following property, as contrasted in Panels A and B in Figure 2:

Lemma 3. *The lower boundary of overlap is π^o when $\pi^o > \Pi_l$ and is zero otherwise. The upper boundary of overlap is π^p when $\pi^p < \Pi_h$ and is one otherwise.*

Using the above lemma, we directly obtain the following result regarding the size of overlap.

Proposition 3. *In the simple dynamic model of statistical discrimination, the overlap is $[\pi^o, \pi^p]$ when both $\pi^o > \Pi_l$ and $\pi^p < \Pi_h$ are satisfied, while it is the entire range of group reputation level $[0, 1]$ when both $\pi^o \leq \Pi_l$ and $\pi^p \geq \Pi_h$ are satisfied.*

We now define another term, "reputation trap," whereby a group is trapped with its low initial collective reputation and cannot recover its reputation in the absence of external intervention:

Definition 2. *A range of a group's reputation level is called a "reputation trap". If the group's initial reputation Π_0^i falls within the reputation trap, the group's reputation cannot be recovered even through collective optimism among group members about the future.*

Using the definition of overlap, the reputation trap is simply the range below overlap. According to Lemma 3, the existence of a reputation trap is guaranteed by $\Pi_l < \pi^o$, as summarized in the following Theorem. It is notable that the condition is also equivalent to those in Propositions 1 and 2. Any group with the low equilibrium reputation Π_l is trapped by the negative influence of reputational externalities when a reputation trap $[0, \pi^o)$ exists.

Theorem 2. *Under Lemmas 1 and 3, if the following condition holds, there exists a reputation trap $[0, \pi^o)$ within which a group's collective reputation cannot be recovered through group members' coordinated expectations about the future. Also, the low equilibrium reputation Π_l falls within this trap:*

$$\Pi_l < \pi^o \quad \Longleftrightarrow \quad \left(\frac{\Pi_h - \Pi^*}{\Pi_h - \Pi_l}\right) < v^{*\frac{\lambda'}{\delta+\lambda'}}, \text{ where } v^* \equiv \frac{(\delta + \lambda')c_m - \omega P_q - \omega'}{\omega P_u - \omega P_q}. \tag{19}$$

This theorem suggests one possible source of persistent disparities between racial groups. For example, during the Jim Crow period and until the civil rights movement in the 1960s, African-Americans were overtly discriminated against in the US labor market. Although such discrimination has decreased significantly in recent decades, we continue to observe persistent black-white disparities in skill achievement. According to the theorem, when overt discrimination in American history results in a very low ratio of qualified workers among blacks, the black population may become stuck in a "reputation trap" and suffer from continuing "statistical" discrimination practices in the labor market. Any collective action through coordinated expectations regarding the future would be ineffective in this situation. Rational agents may not invest in skills when it is unlikely that their skill achievement will be properly compensated in the foreseeable future.

5 Further Discussion

In the dynamic model presented, we have adopted the simplest signal functions of uniformly distributed $f_u(\theta)$ and $f_q(\theta)$ and a simple cost distribution function $G(c)$ with three types of agents: high, low and medium cost individuals. This simplification aids an intuitive understanding of the essential structure of reputational dynamics. Avoiding massive complications, we can derive clean and transparent theoretical results. However, some readers may be uncomfortable with a simple set-up in which employers decide only whether to accord workers the benefit of the doubt when the signal is unclear so that the hiring standard is chosen among only two specific signals, θ_u or θ_q. Moreover, assuming three types of agents, the skill investment decision of a medium cost population exclusively affects the evolution of a group's collective reputation. To mitigate such concerns, the Appendix presents a generalized dynamic model and shows that the major findings can be replicated without the above simplifications.[3]

Furthermore, in the proposed model, we adopt continuous time functions to make the dynamic model easily tractable. Owing to the continuity of the time functions, workers are rematched (or newly contracted) with employers in each instant in the temporary job market. Readers will find that the major results of this paper are also derived using discrete-time functions, where worker are rematched every period instead of every instant.

In the following discussion, we further examine the properties of the dynamic equilibrium paths identified earlier and provide the policy implications that can be derived from the dynamics of statistical discrimination. Readers eager to learn about the policy implications may skip the next section without loss of continuity.

5.1 Properties of Dynamic Equilibrium Paths

The shape of the self-confirming dynamic equilibrium path to the high reputation steady state Q_h is manifest in the phase diagram when $0 < \pi^o \leq \Pi_l$, as displayed in Panel A of Figure 2. As we examine the slope $\frac{\dot{\Pi}_t}{\dot{V}_t^e}$, we directly observe that the equilibrium path is composed of two concave parts in the region below the horizontal line representing $\Pi_t = \Pi^*$. Symmetrically, the shape of the dynamic equilibrium path to the low reputation steady states Q_l is also manifest when $\Pi_h \leq \pi^p < 1$, as displayed in the same panel. The equilibrium path is composed of two convex parts in the region above the horizontal line $\Pi_t = \Pi^*$, as the slope $\frac{\dot{\Pi}_t}{\dot{V}_t^e}$ changes along the path.

The shapes of the dynamic equilibrium paths are more complex when either $\pi^o > \Pi_l$ or $\pi^p < \Pi_h$. For example, the specific case in which both conditions obtain is depicted in Panel B of Figure 2, where the overlap is $[\pi^o, \pi^p]$, in accordance with Proposition 3. In terms of the shapes and curvatures of the dynamic equilibrium paths leading to the two steady states, we obtain the following two major results:

Proposition 4 (Spiraling Paths). *Given $\pi^o > \Pi_l$, the optimistic equilibrium path leading to the high reputation steady state Q_h spirals out around the central state $((\delta + \lambda')c_m, \Pi^*)$. Given $\pi^p < \Pi_h$, the pessimistic equilibrium path leading to the low reputation steady state Q_l also spirals out around the same state.*

Proof. See the proof in the appendix. ■

[3] In addition, unlike under the simplified set-up, the net benefit of investing in skills in the generalized model can be either increasing or decreasing in a firm's prior beliefs, as in Coate and Loury's (1993) original work.

The proposition implies that when$\pi^o > \Pi_l$ ($\pi^p < \Pi_h$), the spiraling optimistic (pessimistic) equilibrium path originates from the central state $((\delta + \lambda')c_m, \Pi^*)$. In addition, the following corollary explains its curvature.

Corollary 2 (Curvature of Paths)**.** *Given $\pi^o > \Pi_l$, the spiraling optimistic equilibrium path leading to the high reputation steady state Q_h is concave on the right-hand side of the $V_t^e = (\delta + \lambda')c_m$ line and convex on the left-hand side of the $V_t^e = (\delta + \lambda')c_m$ line. The same is true of the spiraling pessimistic equilibrium path leading to the low reputation steady state Q_l, given $\pi^p < \Pi_h$.*

Proof. See the proof in the appendix. ■

5.2 Policy Interventions from a Dynamic Perspective

In the given dynamic structure, the steady state Q_h strictly Pareto dominates the steady state Q_l in that all economic agents, including employers and workers with different investment costs, are better off when the group state (v_t^e, Π_t) remains at the steady state Q_h rather than at the steady state Q_l, as summarized in the following corollary:

Corollary 3 (Pareto Dominance)**.** *In the proposed dynamic reputation model, the steady state Q_h strictly Pareto dominates the steady state Q_l.*

Proof. See the proof in the appendix. ■

Pareto dominance implies that groups in the high reputation state Q_h are socially more desirable than those in the low reputation state Q_l. As more group members invest in skill, additional accumulated benefits are distributed among all economic agents.[4] In this respect, government intervention to mobilize a disadvantaged group out of the low reputation steady state is needed to improve social outcomes.

Imagine an identity group that remains at the low reputation steady state Q_l. If the current reputation level Π_l is already in the overlap range, as in Panel A in Figure 2, then the economic performance of the group can be improved simply by increasing its expectations of a brighter future. Collective action to manage expectations may occasion significant behavioral change among young members of the disadvantaged group. The direct government intervention would not be needed to impact the group's skill level, in this case. Instead, an emphasis on coordinated optimism could be pursued. The decisive role of collective optimism is often ignored in contemporary policy debates. Not only the government but also civic organizations and religious groups may seek to encourage shared optimism among the group members.

However, if the current reputation level is far below the overlap, as in Panel B in Figure 2, the group may be trapped by the negative influence of reputational externalities, and an active state role will be needed to enable the group's collective reputation to enter the overlap range. Temporary affirmative action measures can be employed, however, which enhance the group's skill level to the point that it enters the overlap range, after

[4] Pareto dominance may not characterize thejob assignment framework introduced by Coate and Loury(1993), when production complementarities between high and low skilled workers and competitive rather than exogenous wages are considered (Moro and Norman, 2004). For example, the wages of agents with low investment costs may decline as more group members obtain skills. Skilled workers can be negatively affected by increased skill investment of other workers.

which private optimism-promoting efforts can achieve the group's transition to the high-reputation steady state. In the following discussion, we examine two policies that the government may implement to make the dynamic reputation recovery path feasible for a disadvantaged group whose current reputation level Π_l is below the lower boundary of the overlap π^o: 1) cost reduction through training subsidies; and 2) favorable hiring standards. The patronizing policy intervention, discussed in Coate and Loury(1993), is also examined in the given dynamic context.

5.2.1 Cost Reduction through Subsidy

The most direct form of government intervention is to support the training costs of disadvantaged group members. As discussed in section 3.4, if and only if the expected return to human capital acquisition $R(\{\xi_\tau^e\}_0^\infty)$ in equation (6) is no less than the cost, current newborns start to invest in skills, given the time T at which employers are conjectured to shift their treatment of the group so that $\xi_t^e = 0$ for $t < T$ and $\xi_t^e = 1$ for $t \geq T$. As the government implements the training subsidy program, the cost level of the medium ability newborns (c_m) can be reduced up to the conjectured return level $R(\{\xi_\tau^e\}_0^\infty)$ accrued in the reputation recovery scenario. Combining equations (6) and (10), we can compute the target cost level $\tilde{c_m}$ directly, such that medium ability newborns are induced to start investing in skills:[5]

$$\tilde{c_m} = \frac{1}{\delta + \lambda'} \left\{ \omega P_q + \omega' + (\omega P_u - \omega P_q) \left(\frac{\Pi_h - \Pi^*}{\Pi_h - \Pi_l} \right)^{\frac{\delta+\lambda'}{\lambda'}} \right\}. \tag{20}$$

The subsidy(S) must at least equal the difference between the target cost level $\tilde{c_m}$ and the original cost level c_m: $S \equiv c_m - \tilde{c_m}$. The subsidy program is described in Panel A of Figure 3. Given the government subsidy, the "normalized" cost distribution $G(c)$ shifts to the left by the amount of the subsidy S times the normalization factor $(\delta + \lambda')$, rendering the optimistic equilibrium path feasible for the disadvantaged group in the reputation trap.

The model implies that to escape the low investment trap, a subsidy intervention that lowers investment costs of the disadvantaged group must be sufficiently large. At the same time, an emphasis on coordinated optimism should be pursued, as young members of the group will not alter their behavior even with the lowered investment cost if they continue to take a pessimistic view of the group's future. It is also notable that in the dynamic perspective considered, the subsidy program need not be long-standing. Assuming that optimism about the future is sustained, the subsidy can be withdrawn as soon as the group's collective reputation exceeds the lower boundary of the overlap π^o, and thus enters the overlap range.

5.2.2 Favorable Hiring Standard

Given the uniformly distributed signal functions cited above, employers impose the stricter hiring standard θ_u on job candidates from the disadvantaged group. As the government intervenes with the objective of elevating the

[5] The target cost level $\tilde{c_m}$ can also be derived from the differential equations in Theorem 1. Once the target cost level is matched by the government, the group's reputation can recover, with its state moving from state $((\delta + \lambda')\tilde{c_m}, \Pi_l)$ to state $(\omega P_u + \omega', \Pi^*)$ in the phase diagram along the optimistic equilibrium path. Thus, the target cost level $\tilde{c_m}$ must satisfy the following condition: $\int_{(\delta+\lambda')\tilde{c_m}}^{\omega P_u + \omega'} \frac{\mathrm{d}V_t}{(\delta+\lambda')(V_t - \omega P_q - \omega')} = \int_{\Pi_l}^{\Pi^*} \frac{\mathrm{d}\Pi_t}{\lambda'(\Pi_h - \Pi_t)} (\equiv T)$.

group to the socially desirable state Q_h, employers may reduce the hiring standard imposed on group members. The degree of affirmative action regulation can be measured by the newly introduced hiring standard $\tilde{\theta}(< \theta_u)$ of employers. We assume that employers impose the favorable hiring standard $\tilde{\theta}$ as long as the group's collective reputation (belief about the average productivity of its members) remains below the threshold level Π^*.

Let us denote the expected return from investing in human capital accrued at time τ in the temporary job market by $\beta_\tau(\tilde{\theta})$, where $\tilde{\theta}$ indicates the policy-induced hiring standard. Suppose that workers from the disadvantaged group speculate that the favorable hiring standard $\tilde{\theta}$ will continue from the present until time T, at which the group's collective reputation is expected to have attained the threshold level Π^*, so that employers start to accord group members the benefit of the doubt. Replacing $\beta_\tau(0)(= \omega P_q)$ with $\beta_\tau(\tilde{\theta})$ in equation (6), we obtain the return to human capital acquisition for a newborn worker of the group:

$$R(\tilde{\theta}) = \frac{1}{\delta + \lambda'} \left\{ \beta_\tau(\tilde{\theta}) + \omega' + (\omega P_u - \beta_\tau(\tilde{\theta})) e^{-(\delta+\lambda')T} \right\}. \tag{21}$$

Only when $R(\tilde{\theta}) \geq c_m$ will newborn workers start to invest at the high rate Π_h. Using the time span T determined by equation (10), we obtain the following lemma:

Lemma 4. *The favorable hiring standard $\tilde{\theta}(< \theta_u)$ imposed by employers can lead to reputational recovery of the disadvantaged group, provided that the expected return from investing in human capital accrued at time τ in the temporary job market, $\beta_\tau(\tilde{\theta})$, is not less than $\hat{\beta}$, given by*[6]

$$\hat{\beta} \equiv \frac{(\delta + \lambda')c_m - \omega' - \omega P_u \cdot e^{-(\delta+\lambda')T}}{1 - e^{-(\delta+\lambda')T}}, \ \textit{with } T = \frac{1}{\lambda'} \cdot \ln\left(\frac{\Pi_h - \Pi_l}{\Pi_h - \Pi^*}\right). \tag{22}$$

The dynamic recovery path with the critical return level of $\hat{\beta}$ is depicted in Panel B of Figure 3. As the return from investing in human capital accrued at time τ (β_τ) increases, owing to the introduction of affirmative action, $\dot{V}_t^e (= (\delta + \lambda')[V_t^e - \beta_t - \omega'])$ becomes smaller, and consequently, the direction arrows become steeper. This structural change makes the dynamic optimistic path feasible for the group trapped at the low reputation level Π_l. From the dynamic structure, we can also infer that the critical return level $\hat{\beta}$ satisfies $\omega P_q < \hat{\beta} < (\delta + \lambda')c_m - \omega'$.

First, imagine that employers impose the favorable hiring standard $\tilde{\theta}$ within the range (θ_q, θ_u). Given the uniformly distributed signal functions, the expected net return from investment accrued at time τ, $\beta_\tau(\tilde{\theta})$, is the return from investment $\left(\frac{\bar{\theta}-\tilde{\theta}}{\bar{\theta}-\theta_q} \cdot \omega\right)$ minus the return from non-investment $\left(\frac{\theta_u - \tilde{\theta}}{\theta_u} \cdot \omega\right)$:

$$\beta_\tau(\tilde{\theta}) = \frac{\theta_q \theta_u - (\theta_q + \theta_u - \bar{\theta})\tilde{\theta}}{(\bar{\theta} - \theta_q)\theta_u} \cdot w, \text{ for } \tilde{\theta} \in (\theta_q, \theta_u). \tag{23}$$

Note that $\beta_\tau(\tilde{\theta})$ is a decreasing function of $\tilde{\theta} \in (\theta_q, \theta_u)$, as we assume that $P_q < P_u$ or, equivalently, that $\theta_q + \theta_u > \bar{\theta}$, in section 3.2. Therefore, the disadvantaged group can recover its reputation through collective

[6] The critical level $\hat{\beta}$ is also derived from the differential equations in Theorem 1. Once the time τ return from skill investment (β_τ) is enhanced, owing to the affirmative action program, the group's reputation can recover, with its state moving from state $((\delta + \lambda')c_m, \Pi_l)$ to state $(\omega P_u + \omega', \Pi^*)$ in the phase diagram along the optimistic equilibrium path. Thus, the critical return level $\hat{\beta}$ must satisfy the following condition: $\int_{(\delta+\lambda')c_m}^{\omega P_u + \omega'} \frac{\mathrm{d}V_t}{(\delta+\lambda')(V_t - \hat{\beta} - \omega')} = \int_{\Pi_l}^{\Pi^*} \frac{\mathrm{d}\Pi_t}{\lambda'(\Pi_h - \Pi_t)} (\equiv T)$.

optimism, provided the hiring standard $\tilde{\theta}$ is lowered sufficiently that it is below some threshold level $\tilde{\theta}'$:

$$\beta_\tau(\tilde{\theta}) \geq \hat{\beta}, \text{ given } \tilde{\theta} \in (\theta_q, \theta_u) \quad \iff \quad \tilde{\theta} \leq \frac{\theta_q \theta_u \omega - (\bar{\theta} - \theta_q)\theta_u \hat{\beta}}{(\theta_q + \theta_u - \bar{\theta})\omega} \, (\equiv \tilde{\theta}'), \tag{24}$$

where the existence of $\tilde{\theta}'$ is guaranteed by $\beta_\tau(\theta_u)(\equiv \omega P_q) < \hat{\beta} < \beta_\tau(\theta_q)(\equiv \omega P_u)$.

However, imagine that the degree of affirmative action is very strong and that the imposed hiring standard is below θ_q: $\tilde{\theta} \in (0, \theta_q)$. That is, the policy-induced hiring standard imposed on the disadvantaged group is even more favorable than that imposed on the advantaged group (θ_q) with a high reputation level Π_h. In this case, the expected net return from investment accrued at time τ (β_τ) is $\beta_\tau(\tilde{\theta}) = \omega - \frac{\theta_u - \tilde{\theta}}{\theta_u} \cdot w$, which is equivalent to $\frac{\tilde{\theta}}{\theta_u} \cdot w$, given$\tilde{\theta} \in (0, \theta_q)$. This implies that the lower is the hiring standard, the less is the expected net return accrued in the future. That is, the incentive for the skill achievement is lowered with the more favorable treatment in the labor market. Surprisingly, the dynamic recovery path would then not be feasible for the disadvantaged group members if the hiring standard $\tilde{\theta}$ is lowered so much that it is even below some threshold level $\tilde{\theta}''$:

$$\beta_\tau(\tilde{\theta}) < \hat{\beta}, \text{ given } \tilde{\theta} \in (0, \theta_q) \quad \iff \quad \tilde{\theta} < \frac{\theta_u \hat{\beta}}{\omega} \, (\equiv \tilde{\theta}''), \tag{25}$$

where the existence of $\tilde{\theta}''$ is guaranteed by $\beta_\tau(0)(\equiv 0) < \hat{\beta} < \beta_\tau(\theta_q)(\equiv \omega P_u)$. This can be called a "patronizing" government intervention: under an excessive affirmative action program, group members are discouraged from enhancing their human capital acquisition. The following proposition summarizes the above discussion:

Proposition 5. *A government intervention, by lowering the hiring standard of employers, can render the reputation recovery path feasible for a disadvantaged group trapped at the low reputation level* $\Pi_l (< \pi^o)$*, provided that the policy-induced hiring standard satisfies the following condition:* $\tilde{\theta}'' \leq \tilde{\theta} \leq \tilde{\theta}'$*. However, an excessive government intervention, where the induced hiring standard* $\tilde{\theta}$ *falls below* $\tilde{\theta}''$*, may generate the patronizing policy outcome, failing to pull the group out of the trap.*

Therefore, there are circumstances under which affirmative action will necessarily eliminate negative stereotypes, while there are equally plausible circumstances under which it will fail to do so, undercutting workers' incentives to acquire necessary skills. This possibly perverse effect of affirmative action was also highlighted in a static framework by Coate and Loury (1993), who analyzed how affirmative action in the form of an employment quota may affect incentives to invest in skills. In so-called "patronizing equilibria," incentives to invest in skills may be reduced in the static equilibrium with affirmative action relative to that without affirmative action.

We can now confirm this possibility of "patronizing" quota policy in the dynamic framework considered here. For example, imagine that the government imposes a strict quota on hiring, requiring that members of two identity groups, B and W, be hired at equal rates, where group B's current reputation is Π_l in the reputation trap range, while group W's reputation is Π_h. For simplicity, let us suppose that employers adjust the hiring standard imposed on group B members while maintaining the hiring standard on group W members at the lenient level θ_q. Then the target hiring standard of group B, denoted by $\tilde{\theta}^*$, must satisfy the following condition: $\Pi_l + (1 - \Pi_l)(1 - \tilde{\theta}^*/\theta_u) = \Pi_h + (1 - \Pi_h)(1 - \theta_q/\theta_u)$ so that $\tilde{\theta}^* = \frac{1-\Pi_h}{1-\Pi_l}\theta_q$, which is within the range $(0, \theta_q)$. Therefore, the strict quota policy might be patronizing, without altering the incentives of disadvantaged group members to invest in skills, if the quota-induced hiring standard ($\tilde{\theta}^*$) falls below the threshold $\tilde{\theta}''$: $\frac{1-\Pi_h}{1-\Pi_l}\theta_q < \frac{\theta_u \hat{\beta}}{\omega}$ $(\equiv \tilde{\theta}'')$.

6 Conclusion

This paper adds a new twist to the highly developed literature on self-fulfilling reputation through an application to racial discrimination (Arrow, 1973; Coate and Loury, 1993). Past studies have shown that multiple equilibria may exist when poor group reputation reduces the value to group members of investment in human capital so that a poor group reputation becomes self-fulfilling. The present paper adds a dynamic analysis of a somewhat specialized model that shows how reputation can evolve from different starting points. When initial conditions are not too extreme, if workers are sufficiently patient and if new workers replace old ones at a sufficiently rapid rate, then self-confirming equilibrium paths to low and high steady states overlap, so that either path can be attained, depending on the expectations of group members. If initial group reputations are too extreme (or workers too impatient, or worker replacement takes place at too slow a rate), however, then the paths do not overlap, and groups with a poor collective reputation will be stuck in a "reputation trap." Thus, the dynamic model shows that although between-group disparities may originate from a failure of expectation coordination, in some circumstances, a group cannot escape a once-developed low skill investment trap, regardless of how the expectations of its members are formed.

This approach affords new insights into the question of how policy can affect the equilibrium in which the economy finally settles. If a disadvantaged group's initial condition is within the overlap range, the direct government intervention would not be needed to impact the group's skill investment activities. Instead, collective optimism among disadvantaged group members can be crucial to altering the behavior of group members, although this fact is often ignored in policy debates. If the group's initial condition is within the trap, however, policies designed to mobilize the group out of the trap may include directly subsidizing human capital investment and relaxing the hiring standards that face the group, policies that will help make the reputation recovery path feasible for a group trapped in low skill investment activities. Nevertheless, an overly liberal relaxation of standards may generate unintended patronizing outcomes by reducing skill investment incentives, a conclusion that is consistent with the findings of Coate and Loury (1993).

Although we have discussed the importance of expectations in population-specific discriminatory dynamics, we have not yet discussed how expectations about the future are formed when both optimistic and pessimistic expectations are feasible. The mechanism of belief formation is an important topic but one that is beyond the scope of this paper and thus is left for future research. Additionally, in our model, given the assumption that wages are exogenously determined, inter-group interactions in the labor market are not fully examined. This also provides an avenue for future research.

7 Appendix A: Case with Generalized Functional Forms

In the earlier discussion, we have assumed the simplest signal functions where both $f_u(\theta)$ and $f_q(\theta)$ are uniformly distributed, generating only three possible outcomes of the job market screening process: "pass," "fail" and "unclear" test results. The cost distribution $G(c)$ is also assumed to be a step function that represents only three types of agents: "low," "intermediate" and "high" cost individuals. The simplified functional forms aid comprehension of the key intuition and the essential feature of the proposed dynamic model. However, some readers may wonder whether the major findings could be generated from other functional forms of signal functions and cost distributions. Some may also question whether the findings may be attributable to the artificial functional specifications. In this appendix, we address those concerns by showing that the major findings in this paper can be obtained from generalized signal functions and a generalized cost distribution.

Let us define $\psi(\theta) \equiv f_u(\theta)/f_q(\theta)$ as the likelihood ratio at θ. We assume that $\psi(\theta)$ is nonincreasing on $[0, \bar{\theta}]$, which implies that $F_q(\theta) \leq F_u(\theta)$ for all θ. Employers' assignment policies will be characterized by the choice of hiring standard s for each group, such that only those workers whose observed signals exceeds the standard are assigned to the more demanding task. Given the proportion of qualified workers Π^i among the group i population in the temporary job market, employers assign a group i worker who "emits" signal θ to Task One if the expected payoff, $x_q \cdot Prob[\text{qualified}|\theta] - x_u \cdot Prob[\text{unqualified}|\theta]$, is nonnegative. Using Bayes' rule, the posterior probability that he is qualified is $\frac{\Pi^i f_q(\theta)}{\Pi^i f_q(\theta)+(1-\Pi^i) f_u(\theta)}$. Therefore, the hiring standard s is a function of Π^i, as suggested in Coate and Loury (1993):

$$s(\Pi^i) \equiv min\left\{\theta \in [0, \bar{\theta}] \,\middle|\, \psi(\theta) \leq \frac{\rho \Pi^i}{1-\Pi^i}\right\}, \text{ with } \rho \equiv \frac{x_q}{x_u}, \tag{26}$$

where $s(\Pi^i)$ is a nonincreasing function of Π^i. Note that $s(0) \leq \bar{\theta}$ and $s(1) = 0$.

If the assignment standard is s, the probability of assignment to Task One is $1 - F_q(s)$, when qualified, and $1 - F_u(s)$, when unqualified. The expected net return to being qualified at time τ ($\beta_\tau(s^e_\tau)$) is $\omega[F_u(s^e_\tau) - F_q(s^e_\tau)]$, which is hump-shaped, where ω is the wage rate for Task One. Then $\beta_t(\xi_t)$ is replaced by $\beta_t(s_t(\Pi_t)) \equiv \beta_t(\Pi_t)$ in the dynamic system summarized in Theorem 1. Additionally, note that $\beta_t(\Pi_t) \geq 0$, given $\Pi_t = 0$, while $\beta_t(\Pi_t) = 0$, given $\Pi_t = 1$.

The cost of becoming qualified varies among workers and is distributed as CDF $G(c)$ in $(0, \infty)$. We assume that $G(0) > 0$, which implies that a fraction of workers will invest for very small expected returns on their investment. Suppose $G(c)$ is S-shaped, so that its PDF $g(a)$ has one peak (e.g., is bell-shaped). Given $G(c)$, the skill investment rate of the newborn cohort, ϕ_t, is $\phi_t = G(\frac{V^e_t}{\delta+\lambda'})$.

Therefore, the dynamic system is summarized by the following two-variable differential equations:

$$\begin{aligned} \dot{\Pi}_t &= \lambda'[\phi_t(V^e_t) - \Pi_t] \\ \dot{V}^e_t &= (\delta + \lambda')[V^e_t - \beta_t(\Pi_t) - \omega'], \end{aligned}$$

where the two isoclines of the time dependent variables are represented by:

$$\dot{\Pi}_t = 0 \text{ Locus} \quad : \quad \Pi_t = \phi_t(V_t^e)$$
$$\dot{V}_t^e = 0 \text{ Locus} \quad : \quad V_t^e = \beta_t(\Pi_t) + \omega'.$$

The $\dot{\Pi}_t = 0$ locus is S-shaped, while the $\dot{V}_t^e = 0$ locus is hump-shaped, as depicted in Appendix Figure 1. Under the boundary conditions that $\phi_t(0) > 0$ and $\beta_t(0) + \omega' \geq 0$, there must be at least one steady state. We suppose that there exist only three steady states, which we denote by $Q_h(V_h, \Pi_h)$, $Q_m(V_m, \Pi_m)$ and $Q_l(V_l, \Pi_l)$, where $\Pi_h > \Pi_m > \Pi_l$. Regarding the properties of these steady states, we obtain the following direct result.

Lemma 5 (Saddle Points)**.** *Among the three steady states (Q_h, Q_m and Q_l), Q_h and Q_l are saddle points, and Q_m is a source.*

Proof. See the proof in the appendix. ■

The lemma implies that we can find unique saddle paths converging to high and low reputation steady states, Q_h and Q_l. Although Q_m is another steady state, no phase path converges to it because it is a source. First, consider Panel A of the appendix figure, where the saddle path converging to the high reputation steady state Q_h is feasible for any initial reputation level of group i, Π_0^i. In this case, a group with a lower initial collective reputation level may recover its reputation through coordinated optimistic expectations among group members regarding employers' behavior in the future: although they are unfavorably treated now and a high hiring standard is imposed, employers will lower the standard in the future. Given the shared optimism, the expected net return to human capital acquisition V_0^e increases, and a greater fraction of the newborn cohort invests in skills. As more of the group's newborn workers acquire human capital, the group's collective reputation improves over time, and employers' hiring standard is eventually lowered, self-confirming earlier optimistic expectations regarding employers' behavior.

However, this optimistic expectation coordination may not be feasible in some cases. Consider Panel B of the same figure, where the saddle path converging to the high reputation steady state Q_h is feasible only when a group's initial collective reputation is sufficiently strong. Note that the slope of a phase path at an arbitrary state (V_t^e, Π_t) is determined by $\frac{\dot{\Pi}_t}{\dot{V}_t^e} = \frac{\lambda'[\phi_t(V_t^e) - \Pi_t]}{(\delta+\lambda')[V_t^e - \beta_t(\Pi_t) - \omega']}$, according to the above dynamic system. The larger is the time discount factor δ, the flatter is the phase path for a given state (V_t^e, Π_t). If δ is sufficiently large, then the converging path to the high reputation steady state may not reach the V_t^e axis, as the path originates from the source Q_m. That is, we may be in the range of the reputation trap, that is, the range below the overlap. If so, then the trap must cover the low equilibrium reputation Π_l. Thus, a group at the low equilibrium state Q_l may be unable to recover its reputation in the absence of external intervention.

Furthermore, we obtain the following lemma regarding the phase paths near the source Q_m:

Lemma 6 (Spiraling Out)**.** *There exists a critical level of $(\frac{\lambda'}{\delta})^*$, above which phase paths spiral out, and below which they do not spiral out, in the neighborhood of Q_m, where $(\frac{\lambda'}{\delta})^*$ satisfies $\left(1 + \frac{\lambda'}{\delta}\right) \frac{\lambda'}{\delta} = \frac{1}{4(\phi_t' \beta_t' - 1)}\Big|_{(V_m, \Pi_m)}$.*

Proof. See the proof in the appendix. ■

This lemma provides a sufficient condition for the generation of spiraling converging paths to the steady states, Q_h and Q_l, when the paths originate from the source Q_m. In addition, the lemma shows that if the time discount factor δ is very large, then the converging paths to Q_h and Q_l do not spiral out near their origin Q_m. This implies that the converging equilibrium paths may directly connect the origin Q_m and the equilibrium states, Q_h and Q_l, without a spiraling interval. In this extreme case, there is no overlap at all, and the size of the reputation trap is maximized: the dynamic reputation recovery path is not feasible for any group with its initial collective reputation below the intermediate level Π_m.

8 Appendix B: Proofs

8.1 Proof of Proposition 4

To examine the proposition more effectively, we focus on the limited range of $V_t^e \in [\omega P_q + \omega', \omega P_u + \omega']$ and linearly transform V_t^e so that $V_t^e = \omega P_q + \omega' + \omega(P_u - P_q)v_t$. Now, v_t ranges over $[0,1]$ as V_t^e ranges over $[\omega P_q + \omega', \omega P_u + \omega']$. Applying the function $\beta_\tau(\xi_\tau^e)$ in equation (2) to the definition of V_t^e in equation (13), the function V_t^e can be displayed as follows:

$$V_t^e = \omega P_q + \omega' + \omega(P_u - P_q)\left[(\delta+\lambda')\int_t^\infty \xi_\tau^e e^{-(\delta+\lambda')(\tau-t)}\mathrm{d}\tau\right]. \tag{27}$$

Therefore, the linear transformation of V_t^e implies that v_t indicates the normalized level of present discounted lifetime BOD:

$$v_t = (\delta+\lambda')\int_t^\infty \xi_\tau^e e^{-(\delta+\lambda')(\tau-t)}\mathrm{d}\tau. \tag{28}$$

Using this equation for v_t, both the differential equation $\dot{V}_t^e$ and its isocline(the $\dot{V}_t^e = 0$ locus) in Theorem 1 are replaced as follows: $\dot{v}_t = (\delta+\lambda')[v_t - \xi_t]$ and its isocline, the $\dot{v}_t = 0$ locus, is $v_t = \xi_t$, where $\xi_t = 1$ for $\Pi_t \geq \Pi^*$, while $\xi_t = 0$ for $\Pi_t < \Pi^*$. In this (v_t, Π_t) domain, the critical level of V_t^e, $(\delta+\lambda')c_m$, is also transformed to $v^* \equiv \frac{(\delta+\lambda')c_m - \omega P_q - \omega'}{\omega P_u - \omega P_q}$. The new phase diagram, with v^*, and the two corresponding steady states, $Q_h(1, \Pi_h)$ and $Q_l(0, \Pi_l)$, is displayed in Panel A of Appendix Figure 2. For convenience, let us divide the (v_t, Π_t) domain into four regions classified by two straight lines, $v_t = v^*$ and $\Pi_t = \Pi^*$, and name them regions I, II, III and IV, going counterclockwise, as marked in the panel.

First, we prove that an arbitrary phase path around the state (v^*, Π^*) is spiraling. Assume an arbitrary starting point a on the $\Pi_t = \Pi^*$ line near state (v^*, Π^*): $a(v_a, \Pi^*)$. The initial state at a moves counterclockwise, according to the directional arrows depicted in Panel A of Appendix Figure 2. Suppose the path starting from a passes across the $v = v^*$ line at $b(v^*, \Pi_b)$, the $\Pi_t = \Pi^*$ line at $c(v_c, \Pi^*)$, the $v_t = v^*$ line at $d(v^*, \Pi_d)$ and the $\Pi_t = \Pi^*$ line at $a'(v_{a'}, \Pi^*)$, as described in the same panel. In region I, the slope of the phase path is represented by $\frac{\dot{v}_t}{\dot{\Pi}_t} = \frac{(\delta+\lambda')(v_t-1)}{\lambda'(\Pi_h - \Pi_t)}$. Thus, we can obtain the relationship between v_a and Π_b:

$$\int_{v_a}^{v^*} \frac{\mathrm{d}v}{(\delta+\lambda')(v-1)} = \int_{\Pi^*}^{\Pi_b} \frac{\mathrm{d}\Pi}{\lambda'(\Pi_h - \Pi)} \Longrightarrow \left(\frac{1-v_a}{1-v^*}\right)^{\frac{\lambda'}{\delta+\lambda'}} = \frac{\Pi_h - \Pi_b}{\Pi_h - \Pi^*}. \tag{29}$$

Additionally, in region II, the slope of the phase path is represented by $\frac{\dot{v}_t}{\dot{\Pi}_t} = \frac{(\delta+\lambda')(v_t-1)}{\lambda'(\Pi_l - \Pi_t)}$. Thus, we can obtain the relationship between v_c and Π_b:

$$\int_{v^*}^{v_c} \frac{\mathrm{d}v}{(\delta+\lambda')(v-1)} = \int_{\Pi_b}^{\Pi^*} \frac{\mathrm{d}\Pi}{\lambda'(\Pi_l - \Pi)} \Longrightarrow \left(\frac{1-v_c}{1-v^*}\right)^{\frac{\lambda'}{\delta+\lambda'}} = \frac{\Pi_b - \Pi_l}{\Pi^* - \Pi_l}. \tag{30}$$

From (29) and (30), we can derive the relationship between v_a and v_c, eliminating Π_b. The following formula summarizes the result, denoting $\frac{\lambda'}{\delta+\lambda'}$ as ρ':

$$(1-v_a)^{\rho'}(\Pi_h-\Pi^*)+(1-v_c)^{\rho'}(\Pi^*-\Pi_l)=(1-v^*)^{\rho'}(\Pi_h-\Pi_l). \tag{31}$$

Therefore, v_a is a function of v_c in $v_c \in [0, v^*]$: $v_a(v_c)$. Note that $v_a(v^*) = v^*$.

Similarly, from regions III and IV, we can derive the relationship between v_a' and v_c, eliminating Π_d. In region III, the slope of the phase path is represented by $\frac{\dot{v}_t}{\dot{\Pi}_t} = \frac{(\delta+\lambda')v_t}{\lambda'(\Pi_l-\Pi_t)}$. Thus, we can find the relationship between v_c and Π_d,

$$\int_{v_c}^{v^*} \frac{\mathrm{d}v}{(\delta+\lambda')v} = \int_{\Pi^*}^{\Pi_d} \frac{\mathrm{d}\Pi}{\lambda'(\Pi_l-\Pi)} \Longrightarrow \left(\frac{v_c}{v^*}\right)^{\frac{\lambda'}{\delta+\lambda'}} = \frac{\Pi_d-\Pi_l}{\Pi^*-\Pi_l}. \tag{32}$$

Additionally, in region IV, the slope of the phase path is represented by $\frac{\dot{v}_t}{\dot{\Pi}_t} = \frac{(\delta+\lambda')v_t}{\lambda'(\Pi_h-\Pi_t)}$. Thus, we obtain the relationship between $v_{a'}$ and Π_d:

$$\int_{v^*}^{v_{a'}} \frac{\mathrm{d}v}{(\delta+\lambda')v} = \int_{\Pi_d}^{\Pi^*} \frac{\mathrm{d}\Pi}{\lambda'(\Pi_h-\Pi)} \Longrightarrow \left(\frac{v_{a'}}{v^*}\right)^{\frac{\lambda'}{\delta+\lambda'}} = \frac{\Pi_h-\Pi_d}{\Pi_h-\Pi^*}. \tag{33}$$

From (32) and (33), we obtain the following formula, which indicates the relationship between $v_{a'}$ and v_c, denoting $\frac{\lambda'}{\delta+\lambda'}$ as ρ':

$$v_{a'}^{\rho'}(\Pi_h-\Pi^*)+v_c^{\rho'}(\Pi^*-\Pi_l)=v^{*\rho'}(\Pi_h-\Pi_l). \tag{34}$$

Therefore, $v_{a'}$ is a function of v_c in $v_c \in [0, v^*]$: $v_{a'}(v_c)$. Note that $v_{a'}(v^*) = v^*$.

To prove the spiraling equilibrium path, the following two conditions, as depicted in Panel B of Appendix Figure 2, must be satisfied: **(Condition 1)** Two curves, $v_a(v_c)$ and $v_{a'}(v_c)$, are tangent at (v^*, v^*), as $\frac{\mathrm{d}v_a}{\mathrm{d}v_c}\big|_{(v^*,v^*)} = \frac{\mathrm{d}v_{a'}}{\mathrm{d}v_c}\big|_{(v^*,v^*)}$; and **(Condition 2)** $v_a(v_c)$ is concave and $v_{a'}(v_c)$ is convex in $[0, v^*]$, as $\frac{\mathrm{d}^2 v_a}{\mathrm{d}v_c^2} < 0$ and $\frac{\mathrm{d}^2 v_{a'}}{\mathrm{d}v_c^2} > 0, \forall v_c \in [0, v^*]$.

Proof of Condition 1. From equation (31), let us define the function F as $F = (1-v_a)^{\rho'}(\Pi_h-\Pi^*)+(1-v_c)^{\rho'}(\Pi^*-\Pi_l)-(1-v^*)^{\rho'}(\Pi_h-\Pi_l) = 0$. By the implicit function theorem, $F_{v_a}\mathrm{d}v_a + F_{v_c}\mathrm{d}v_c = 0$. Therefore, we have

$$\frac{\mathrm{d}v_a}{\mathrm{d}v_c} = -\frac{(1-v_c)^{\rho'-1}}{(1-v_a)^{\rho'-1}} \cdot \frac{\Pi^*-\Pi_l}{\Pi_h-\Pi^*}. \tag{35}$$

This indicates the slope of the curve $v_a(v_c)$ at (v^*, v^*): $\frac{dv_a}{dv_c}\Big|_{(v^*,v^*)} = -\frac{\Pi^*-\Pi_l}{\Pi_h-\Pi^*}$. From equation (34), let us define the function Z as $Z = v_{a'}^{\rho'}(\Pi_h-\Pi^*)+v_c^{\rho'}(\Pi^*-\Pi_l)-v^{*\rho'}(\Pi_h-\Pi_l) = 0$. By the implicit function theorem, $Z_{v_{a'}}\mathrm{d}v_{a'} + Z_{v_c}\mathrm{d}v_c = 0$. Therefore, we have

$$\frac{\mathrm{d}v_{a'}}{\mathrm{d}v_c} = -\frac{v_c^{\rho'-1}}{v_{a'}^{\rho'-1}} \cdot \frac{\Pi^*-\Pi_l}{\Pi_h-\Pi^*}. \tag{36}$$

This indicates the slope of the curve $v_{a'}(v_c)$ at (v^*, v^*): $\frac{\mathrm{d}v_{a'}}{\mathrm{d}v_c}\Big|_{(v^*,v^*)} = -\frac{\Pi^*-\Pi_l}{\Pi_h-\Pi^*}$. Therefore, condition 1 is satisfied.

Proof of Condition 2. From equations (31) and (35), $\frac{\mathrm{d}v_a}{\mathrm{d}v_c}$ can be expressed in terms of v_c:

$$\frac{\mathrm{d}v_a}{\mathrm{d}v_c} = -\frac{(1-v_c)^{\rho'-1}}{\left[-(1-v_c)^{\rho'}(\Pi^*-\Pi_l)+(1-v^*)^{\rho'}(\Pi_h-\Pi_l)\right]^{\frac{\rho'-1}{\rho'}}} \cdot \frac{\Pi^*-\Pi_l}{(\Pi_h-\Pi^*)^{\frac{1}{\rho'}}}. \tag{37}$$

Taking the second derivative of this expression with respect to v_c, we find that $\frac{\mathrm{d}^2 v_a}{\mathrm{d}v_c^2} < 0$ for $\delta > 0$. From equations (34) and (36), $\frac{\mathrm{d}v_{a'}}{\mathrm{d}v_c}$ can be expressed in terms of v_c,

$$\frac{\mathrm{d}v_{a'}}{\mathrm{d}v_c} = -\frac{v_c^{\rho'-1}}{\left[-v_c^{\rho'}(\Pi^*-\Pi_l)+v^{*\rho'}(\Pi_h-\Pi_l)\right]^{\frac{\rho'-1}{\rho'}}} \cdot \frac{\Pi^*-\Pi_l}{(\Pi_h-\Pi^*)^{\frac{1}{\rho'}}}. \tag{38}$$

Taking the second derivative of this expression with respect to v_c, we find that $\frac{\mathrm{d}^2 v_{a'}}{\mathrm{d}v_c^2} > 0$ for $\delta > 0$. Therefore, condition 2 is also satisfied.

Because the above two conditions are satisfied for $\delta > 0$, we prove that the phase paths around the state (v^*, Π^*) are spiraling. For example, given $\pi^o > \Pi_l$, when we assume that point d is (v^*, π^o) and that the point a' is $(1, \Pi^*)$, the above proof directly implies that the spiraling optimistic equilibrium path leads to $(1, \Pi^*)$ and then to $Q_h(1, \Pi_h)$ in the (v_t, Π_t) domain. Given $\pi^p < \Pi_h$, the proof also implies that the spiraling pessimistic equilibrium path leads to $(0, \Pi^*)$ and then to $Q_l(0, \Pi_l)$ in the (v_t, Π_t) domain. QED.

8.2 Proof of Corollary 2

We follow the modified dynamic system and the notation suggested in the proof of Proposition 4. Accordingly, the following proof is based on the phase space in the (v_t, Π_t) domain. The slope of the equilibrium path in region IV is $\frac{\dot{\Pi}_t}{\dot{v}_t}\Big|_{IV} = \frac{\lambda'(\Pi_h - \Pi_t)}{(\delta+\lambda')v_t}$. The first derivative of the slope is

$$\frac{\mathrm{d}}{\mathrm{d}v_t}\left[\frac{\dot{\Pi}_t}{\dot{v}_t}\Big|_{IV}\right] = -\frac{\lambda'(\Pi_h-\Pi_t)}{(\delta+\lambda')v_t^2} - \frac{\lambda}{(\delta+\lambda)v_t} \cdot \frac{\mathrm{d}\Pi_t}{\mathrm{d}v_t} < 0.$$

Therefore, in region IV, the equilibrium paths are concave. The slope of the equilibrium path in region I is $\frac{\dot{\Pi}_t}{\dot{v}_t}\Big|_{IV} = \frac{\lambda(\Pi_h-\Pi_t)}{(\delta+\lambda)(v_t-1)}$. The first derivative of the slope is

$$\frac{\mathrm{d}}{\mathrm{d}v_t}\left[\frac{\dot{\Pi}_t}{\dot{v}_t}\Big|_{I}\right] = -\frac{\lambda'(\Pi_h-\Pi_t)}{(\delta+\lambda')(v_t-1)^2} - \frac{\lambda'}{(\delta+\lambda')(v_t-1)} \cdot \frac{\mathrm{d}\Pi_t}{\mathrm{d}v_t} < 0.$$

Therefore, in region I, the equilibrium paths are concave. In sum, on the right-hand side of the $v_t = v^*$ line (regions I and IV), the equilibrium paths are concave. Similarly, we can prove that the equilibrium paths are convex on the left-hand side of the $v_t = v^*$ line (regions II and III). QED.

8.3 Proof of Corollary 3

It is clear that agents with low investment costs who always invest are better off in steady state Q_h than in steady state Q_l because they receive the benefit of the doubt in the temporary job market. Similarly, agents with high investment costs who do not always invest are also better off in state Q_h because they receive the benefit of the doubt, even though they did not invest in skills. The agents with the medium investment cost c_m invest in state Q_h and do not invest in state Q_l. Because the benefit of the doubt is given in state Q_h, the expected lifetime benefits are $\frac{\omega}{\delta+\lambda} - c_m$ in this steady state, while the expected lifetime benefits are zero in state Q_l. The assumption in Lemma 1 guarantees that $\frac{\omega}{\delta+\lambda} - c_m > 0$. Therefore, all agents with different investment costs are better off in steady state Q_h than in steady state Q_l. Let us now compare the profits of employers in steady state Q_h with those in steady state Q_l.

First, consider agents in the temporary job market: the size of the population is $N^i n$. Expected profits at a point of time in steady state Q_h are the sum of $N^i n \cdot \Pi_l \cdot x_q$ (profits from workers with a low investment cost), $N^i n \cdot (1 - \Pi_h) \cdot (1 - P_u)(-x_u)$ (profits from workers with a high investment cost) and $N^i n \cdot (\Pi_h - \Pi_l) \cdot x_q$ (profits from workers with a medium investment cost). Therefore, total profits at a point of time are $N^i n[\Pi_h \cdot x_q - (1 - \Pi_h) \cdot (1 - P_u)x_u]$ in steady state Q_h, while expected profits in steady state Q_l are $N^i n \cdot \Pi_l \cdot P_q \cdot x_q$ (profits from workers with a low investment cost) because the profits from workers with a high investment cost and those with a medium cost are zero. The condition $\Pi_l < \Pi^* < \Pi_h$ guarantees that profits from the population $N^i n$ are greater in state Q_h than in state Q_l.

Second, consider agents in the permanent job market: the size of the population is $N^i(1 - n)$. Expected profits at a point in time are $N^i(1 - n) \cdot \Pi_h \cdot x_q$ in steady state Q_h because both workers with a low investment cost and those with a medium investment cost invest in skills, while profits in steady state Q_l are $N^i(1-n) \cdot \Pi_l \cdot x_q$ because only workers with a low investment cost invest in skills. Thus, profits from the population $N^i(1 - n)$ are greater in state Q_h than in state Q_l. QED.

8.4 Proof of Lemma 5

Consider a steady state $(\bar{V}, \bar{\Pi})$ in the given the dynamic system. Linearization around the steady state $(\bar{V}, \bar{\Pi})$ yields

$$
\begin{aligned}
\dot{V}_t &= (\delta + \lambda')(\bar{V} - \beta_t(\bar{\Pi}) - \omega') - (\delta + \lambda')\beta_t'(\bar{\Pi})(\Pi_t - \bar{\Pi}) + (\delta + \lambda')(V_t - \bar{V}) \\
\dot{\Pi}_t &= \lambda'(\phi_t(\bar{V}) - \bar{\Pi}) - \lambda'(\Pi_t - \bar{\Pi}) + \lambda'\phi_t'(\bar{V})(V_t - \bar{V}).
\end{aligned}
$$

Because $(\bar{V}, \bar{\Pi})$ is a steady state, both $(\delta + \lambda')(\bar{V} - \beta_t(\bar{\Pi}) - \omega')$ and $\lambda'(\phi_t(\bar{V}) - \bar{\Pi})$ are zero by definition. Rearranging terms, we have

$$
\begin{aligned}
\dot{V}_t &= (\delta + \lambda') \cdot V_t - (\delta + \lambda')\beta_t'(\bar{\Pi}) \cdot \Pi_t + (\delta + \lambda')[-\bar{V} + \beta_t'(\bar{\Pi})\bar{\Pi}] \\
\dot{\Pi}_t &= \lambda'\phi_t'(\bar{V}) \cdot V_t - \lambda' \cdot \Pi_t + \lambda'[-\phi_t'(\bar{V})\bar{V} + \bar{\Pi}].
\end{aligned}
$$

Therefore, the Jacobian matrix J_E , evaluated at the steady state, is

$$
J_E \equiv \begin{bmatrix} \delta + \lambda' & -(\delta + \lambda')\beta_t' \\ \lambda'\phi_t' & -\lambda' \end{bmatrix}_{(\bar{V}, \bar{\Pi})}.
$$

Consequently, its transpose is $trJ_E = \delta$, and the determinant is $|J_E| = \lambda'(\delta + \lambda')[\beta_t'\phi_t' - 1]$. Because trJ_E is positive, every steady state is unstable.

Note that $|J_E|$ is negative if and only if $\left(\frac{\partial \beta_t}{\partial \Pi_t}\right)^{-1} > \frac{\partial \phi_t}{\partial V_t}$ at $(\bar{V}, \bar{\Pi})$. The first term, $\left(\frac{\partial \beta_t}{\partial \Pi_t}\right)^{-1}$, indicates the slope of the $\dot{V}_t^e = 0$ locus in steady state $(\bar{V}, \bar{\Pi})$ in the given (V_t^e, Π_t) domain, while the second term, $\frac{\partial \phi_t}{\partial V_t}$, indicates the slope of the $\dot{\Pi}_t = 0$ locus in the state $(\bar{V}, \bar{\Pi})$. Therefore, the inequality requires that the slope of the $\dot{V}_t^e = 0$ locus is greater than the $\dot{\Pi}_t = 0$ locus in the state $(\bar{V}, \bar{\Pi})$ in the given domain. This is always true for the two steady states, Q_h and Q_l, provided there are a total of three steady states in the given dynamic system, as is easily confirmed in Appendix Figure 1. In conclusion, each of the steady states, Q_h and Q_l, has one positive and one negative characteristic roots. Therefore, they are saddle points.

Similarly, we can confirm that $|J_E|$ is positive at Q_m because the following holds: $\left(\frac{\partial \beta_t}{\partial \Pi_t}\right)^{-1} < \frac{\partial \phi_t}{\partial V_t}$. This implies that Q_m is a source, either an unstable node or an unstable focus. QED.

8.5 Proof of Lemma 6

Note that based on the proof of Lemma 5, the characteristic roots are $r_1, r_2 = \frac{\delta \pm \sqrt{\delta^2 - 4\lambda'(\delta+\lambda')[\beta_t'\phi_t' - 1]}}{2}$. Because the proof of Lemma 5 shows that $|J_E| = \lambda'(\delta+\lambda')[\beta_t'\phi_t' - 1] > 0$ at Q_m, provided that $\delta^2 - 4\lambda'(\delta+\lambda')[\beta_t'\phi_t' - 1](\equiv D)$ is positive, both characteristic roots, r_1 and r_2, are distinct and positive. Therefore, when $D > 0$, Q_m is an unstable node, implying that the trajectories originating from the source Q_m do not spiral out in the neighborhood of Q_m. Additionally, provided that $D < 0$, both characteristic roots are imaginary numbers, implying that Q_m is an unstable focus: the trajectories originating from the source spiral out. Note that $D > (<) 0 \iff \left(1 + \frac{\lambda'}{\delta}\right) \frac{\lambda'}{\delta} < (>) \frac{1}{4(\phi_t'\beta_t' - 1)}\Big|_{(V_m, \Pi_m)}$. QED.

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Figure 1. Phase Diagram

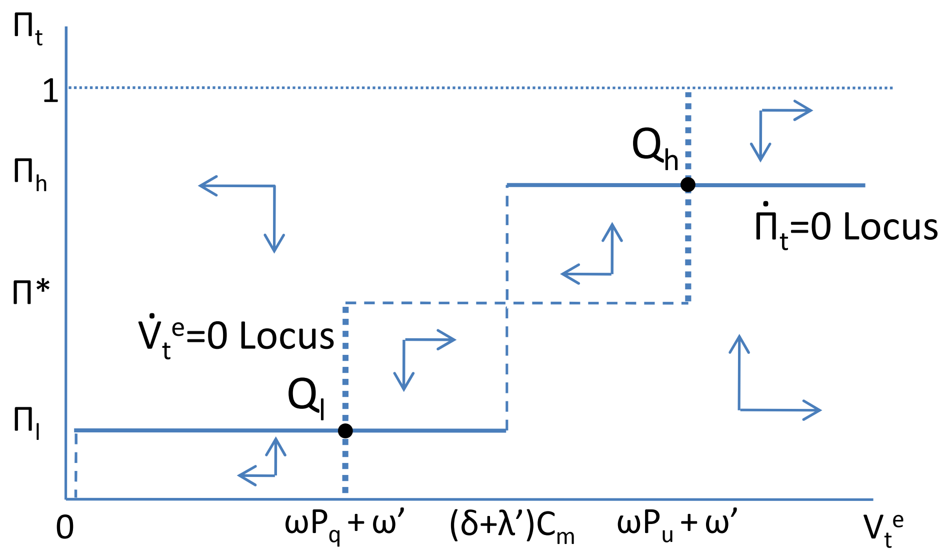
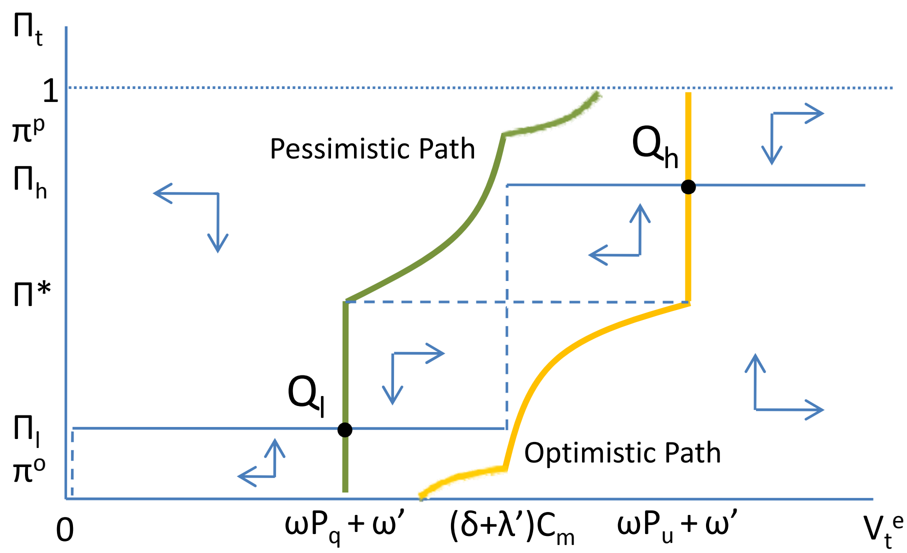


Figure 2. Dynamic Equilibrium Paths

Panel A. Equilibrium Paths Given $\pi^o \leq \Pi_l$ and $\pi^p \geq \Pi_h$



Panel B. Equilibrium Paths Given $\pi^o > \Pi_l$ and $\pi^p < \Pi_h$

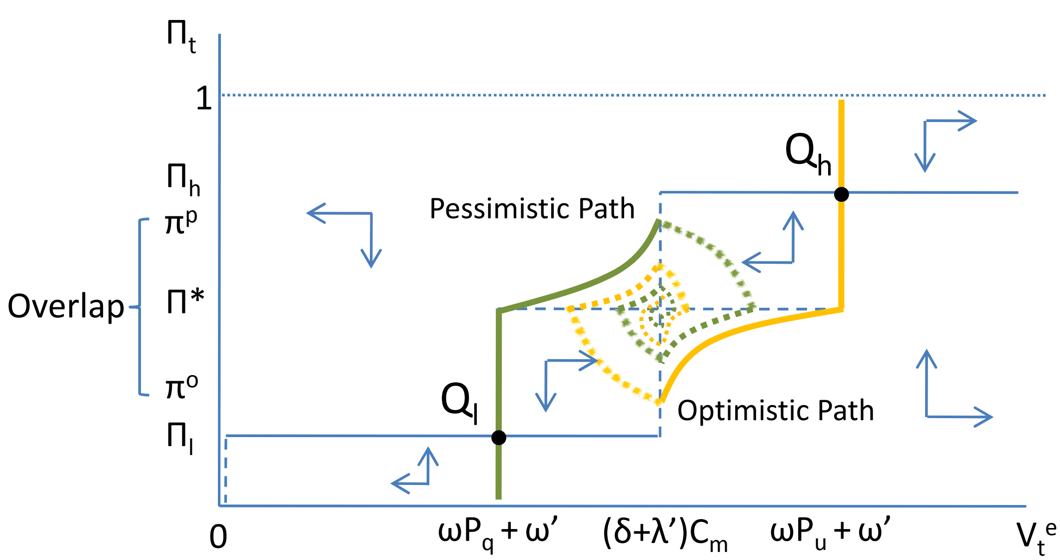
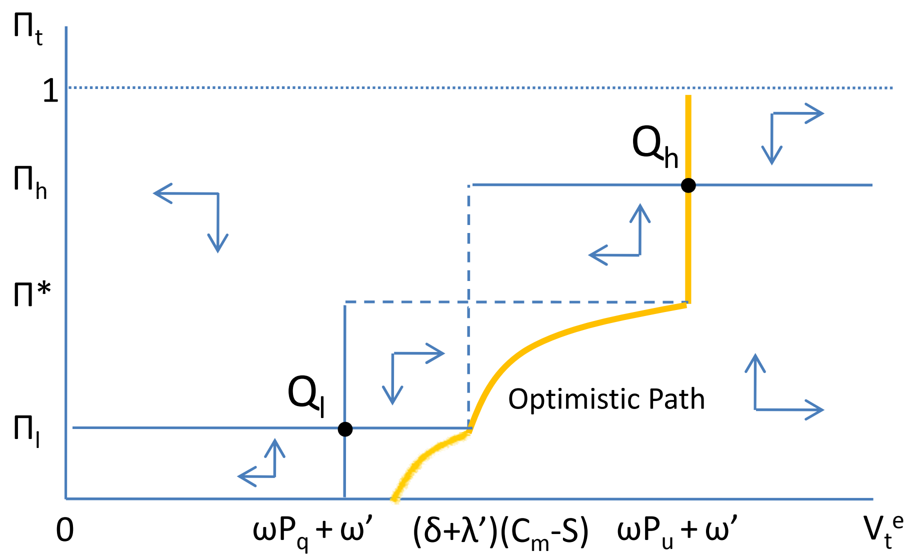
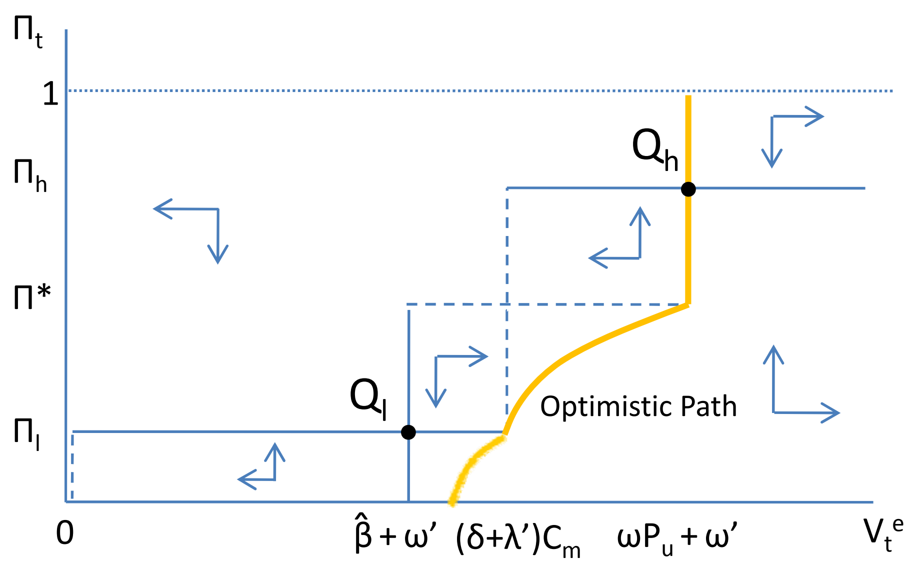


Figure 3. Policy Intervention

Panel A. Cost Down through Subsidy

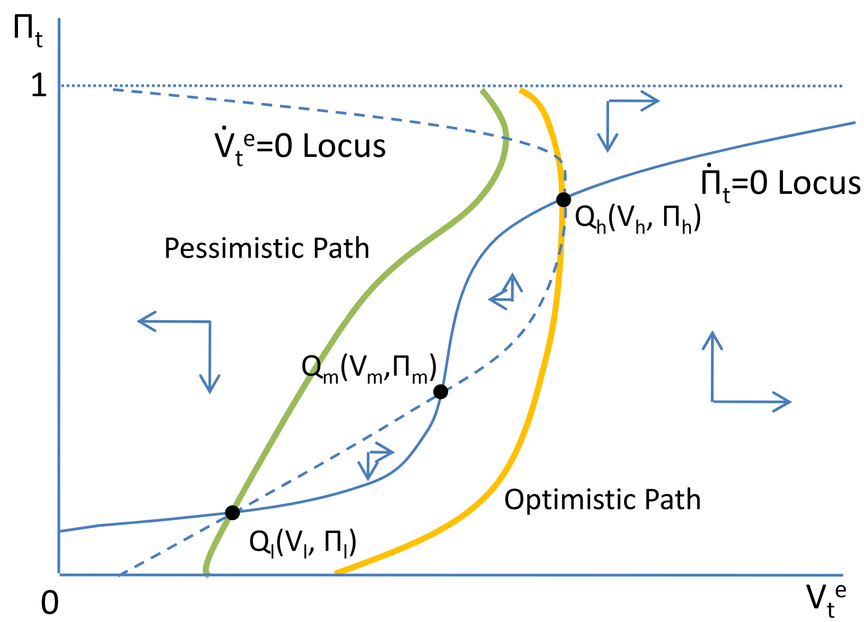


Panel B. Favorable Hiring Standard

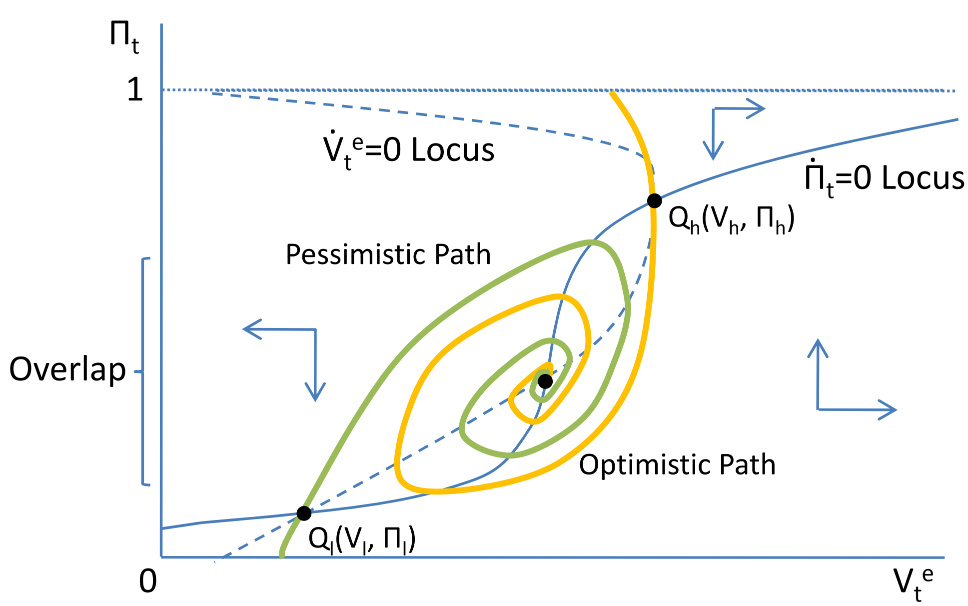


[Appendix Figure 1] Case with Generalized Functional Forms

Panel A. Equilibrium Paths with Overlap [0,1]

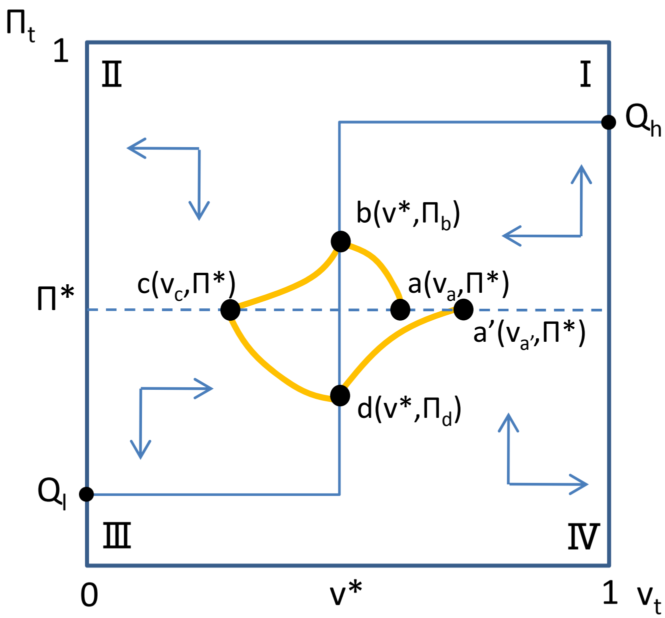


Panel B. Equilibrium Paths with Limited Overlap



[Appendix Figure 2] Proof of Proposition 4

Panel A. Phase Path Passing *a*



Panel B. Compare v_a and $v_{a'}$