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Abstract

Previous studies find that some social groups are stuck in poverty traps because of network effects. However, these studies do not carefully analyze how these groups overcome low human capital investment activities. Unlike previous studies, the model in this paper includes network externalities in both the human capital investment stage and the subsequent career stages. This implies that not only the current network quality, but also the expectations about future network quality affect the current investment decision. Consequently, the coordinated expectation among the group members can play a crucial role in the determination of the final state. We define "overlap" for some initial skill ranges, whereby the economic performance of a group can be improved simply by increasing expectations of a brighter future. We also define "poverty trap" for some ranges, wherein a disadvantaged group is constrained by its history, and we explore the egalitarian policies to mobilize the group out of the trap.

KEYWORDS: Group Inequality, Network Externality, Overlap, Poverty Trap.

JEL CODE: I30, J15, Z13

1 Introduction

A human being is socially situated such that familial and communal resources explicitly influence his acquisition of human capital through various routes, including through training resources, nutritional and medical provision, after-school parenting, peer effects, role models, and even the psychological processes that shape one's outlook on life. Even after the skill acquisition period, one's social network influences his economic success through various routes, such as mentoring, job searches, business connections, and information channeling.

A number of theoretical works emphasize the network effects and the subsequent development bias that generates between-group disparities. Some of these studies focus on network externalities in the human capital investment stage. For example, Becker and Tomes (1979) and Loury (1981) explain the persistence in relative economic status across generations via the effects of parental income on offspring's education. Lundberg and Startz (1998) argue that group disparities in earnings can persist indefinitely when the average level of human capital in a community affects the accumulation of human capital of the following generations. In a related work, Bowles, Loury and Sethi (2007) prove the instability of an equal society in a highly segregated economy under the interpersonal spillovers in human capital accumulation and the production complementarity between high and low skill labor. Other works focus on network externalities in the subsequent career stages. For example, Montgomery (1991, 1994) suggests that the widespread use of employee referrals combined with a tendency to refer others within individuals' social networks might generate persistent inequalities between groups of workers. More recently, Calvó-Armengol and Jackson (2004, 2007) argue that differences in collective employment histories and the consequent asymmetry of job information produce sustained inequality in wages and drop-out rates across social groups.

Although theoretical works on the topic find that some groups are stuck in poverty traps because of network effects, they do not provide a rich analysis of how these groups can overcome the low human capital investment activities. In their models, multiple equilibria of skill investment rates are self-confirmed under the presence of strong network externalities. What can we say about the skill levels between those stationary points? How can a group with a lower skill investment rate change the skill investment behaviors of its group members? In the theoretical models, players are assumed to be myopic. However, what if they are farsighted and can cooperate with each other? This paper tries to answer these questions by suggesting that coordinated expectations among group members play a critical role in the determination of a group's skill investment activities.

As mentioned earlier, previous studies focus on network externalities in either the human capital investment stage or the subsequent career stages. Externalities in the former stage alter the cost to achieve skills and those in the latter stages alter the benefits from skill investments. Unlike previous studies, the theoretical model in this paper allows both types of externalities. Therefore, not only the current network quality, but also the expectations regarding future network quality affect current invest decisions. This implies that multiple equilibrium paths can exist for a certain range of initial skill levels. Within that range, a group's economic performance can be improved simply by increasing expectations for a brighter future. This provides rational support for the argument that some social groups are not constrained by their history but can raise themselves up "by their bootstraps." However, some groups may suffer from very low network quality if their skill levels are far below a certain range. This provides rational support for the argument that some groups may not escape their low skill investment activities without external interventions.

This paper illustrates the mechanism through concrete examples. For network externalities in the human capital investment stage, the change in a group's status tends to be caused by factors in the past; by altering skill investment cost, the current stock of network human capital directly affects the investment rate in a newborn cohort. In contrast, for network externalities in the career stage, the change in a group's status tends to be affected by factors in the future; by altering the future benefits to skill acquisition, the expected success of one's network influences skill investment in an entering cohort.

The latter effect implies a unique feature of collective action: the possibility of group members acting together to improve (or deteriorate) the quality of a group's social network. For instance, suppose that a group's network quality is relatively poor but that a newborn cohort happens to believe that the quality of the group's network will be better in the future. If this belief leads more newborn group members to acquire skills, then the next cohort will find that the overall network quality has improved due to the enhanced skill investment of the previous cohort. If they and the following cohorts continue to hold the optimistic view toward the future, they will maintain the enhanced skill investment rate and the quality of group's social network will improve over time – thereby justifying the optimistic beliefs of earlier cohorts. However, suppose that the newborn cohort holds a pessimistic view that the network quality will be even worse in the future. Fewer members invest in skill achievement because the expected benefits are fewer. If the following cohorts continue to hold the pessimistic view, the network quality will deteriorate over time. Thus, this pessimistic belief could also be self-fulfilling. This nature of possible collective actions stresses the importance of coordinated expectations across different time cohorts. Whether optimism or pessimism persists across cohorts determines the final economic state of a social group.

However, collective action through coordinated beliefs may not be feasible for all social groups with unequal network quality. For example, the skill improvement that occurs due to coordinated optimism may not be feasible if the negative influence of the current network quality is too strong. This is the situation of "the past" that traps disadvantaged groups.

Therefore, the analysis of the dynamic structure of network externalities focuses on the identification of the network quality range in which both the optimistic and the pessimistic expectations are feasible for the group members. Many theoretical papers including Murphy et al.(1989), Matsuyama(1991) and Kremer(1993) highlights the importance of coordinated expectations in equilibrium determination. Among them, Krugman (1991) denoted the range with multiple equilibrium paths by *overlap* in his influential argument for the relative importance of history and expectations. In his argument, within an overlap, the final economic state is determined by expectations toward the future, while it is determined by history outside an overlap.

Unlike their models with a fixed population, our model is developed based on the overlapping generation framework. The model emphasizes the importance of belief coordination over the long-term horizon: the expectations coordinated across the different time cohorts impact the dynamic path to be taken. In addition, the model developed in the social externalities setting proves that the size of overlap is determined by the relative strength of working-period network externalities over the skill investment period network externality.

Finally, the model provides some new perspectives on egalitarian policies such as affirmative action by considering economic agents' forward-looking behaviors. This point distinguishes the paper from other papers concerning the egalitarian policies whose main focus is on the equilibrium analysis (e.g., Coate and Loury 1993, Fryer and Loury 2007). If the initial network quality of a social group is far below the "overlap", the group may be trapped by the negative influence of network effects, and an active state role is required to enhance the group's skill level to enter the "overlap". However, if the network quality of a social group is already in the overlap range, the active state role would not have a significant impact on the group's skill level. Instead, an emphasis on coordinated optimism among the disadvantaged group members should be pursued, although this fact is often ignored in policy debates. Civic leaders, civic organizations, religious groups, and governments may all contribute to the encouragement of collective optimism. Therefore, an effective policy to mobilize a disadvantaged group out of the poverty trap first requires active governmental intervention and then requires societal belief coordination. A policy that fails in either respect cannot be successful in helping the group to advance as much as an advantaged group.

The paper is organized into the following sections. Section 2 describes the basic structure of the model with social network externalities. Section 3 develops the dynamic model with the newborn cohort's forward-looking decision making and the dynamic evolution of group skill levels. Section 4 identifies multiple stationary states in the dynamic model. Section 5 identifies the equilibrium paths to those stationary states and the consequent overlap. In Section 6, we discuss the egalitarian policies and some theoretical issues. Section 7 provides study conclusions.

2 Social Externalities and Skill Investment Decision

Consider a social group with a large population of workers. A worker is subject to the "Poisson death process" with parameter α: in a unit period, each individual faces α chances to die. We assume that the total population of the group is constant at N, implying that the α fraction of the group's population is replaced by newborn group members in a unit period. Each worker is either skilled or unskilled. Let $s_t \in [0, 1]$ denote the fraction of skilled workers in the group at time t, which is called *the group skill level at time t*. An agent's neighbors are n random draws from the group's population (N), and n is large enough that the quality of an agent's network is approximately equal to the group skill level s_t.[2]

A newborn's innate ability a is a random draw from a distribution $G(a)$. Each newborn agent born with an innate ability level a decides whether to be skilled or not during his early days of life. Each newborn individual at time t makes a skill investment decision by comparing the cost of skill acquisition with the expected benefits of investment. The cost to achieve a skill at time t depends on innate ability $a \in (-\infty, \infty)$ and the quality of social network at time t as suggested by Bowles, Loury and Sethi (2007): $C_t \equiv C(a, s_t)$. The $C(a, s_t)$ is a decreasing function in both arguments a and s_t. The cost includes both the mental and physical costs that are incurred for the skill achievement. The lower one's innate ability or the worse the quality of one's social network, the more mentally stressful the skill acquisition process is or the more materials he must spend on the achievement.

The expected benefits of investment to a newborn individual born at time t, $\Pi_t \in (0, \infty)$, is the net benefits

[2]In the extreme case that n is equal to the population size minus one ($N - 1$), agents are all connected to one another. This extreme network is called *complete network* in the network literature (Jackson, 2008).

of his skill investment to be realized over his whole lifetime from time t until he dies. Let us assume the base level salary for a skilled worker is w_1 and w_0 for an unskilled worker, in which $w_1 > w_0$. A worker's neighbors at time t are composed of the ns_t number of skilled workers and the $n(1-s_t)$ number of unskilled workers. Let $\phi_{ij}(x)$ denote the extra benefits of having x number of j type workers in a i type worker's social network, in which $\phi_{ij}(0) = 0$ for any i, $j \in \{s, u\}$.

First, $\phi_{ss}(ns_t)$ denote the extra benefits of having ns_t skilled workers in a skilled worker's social network, which is an increasing function of ns_t. The benefits are both psychological and material. For instance, job information flows along the synapses of the social network (Granovetter 1975). The more skilled workers he has in his network, the more likely he is to find an appropriate job position for his specific skills (Holzer 1988). A skilled worker can be more efficient in contacting customers and handling specific work troubles when he has more skilled workers in his network (Ozgen and Baron 2007). He may gain comfort and mentoring from the informal network, and the cost of maintaining jobs may decline with more skilled workers around him (Castilla 2005, Rockoff 2008).

In the same way, unskilled workers such as car mechanics, construction workers and shop attendants would obtain extra benefits from having unskilled workers in his network. $\phi_{uu}(n(1-s_t))$ denote the extra benefits of having $n(1-s_t)$ unskilled workers in an unskilled worker's social network, which is an increasing function of $n(1-s_t)$. For example, a car mechanic searching for a place to work for will find a car center that fits his specialty better when he has more car mechanics in his network. He can be more efficient in handling specific mechanical problems in his work when he can confer with more mechanics. A construction worker (or a shop attendant) will have more chances to find new job openings with more construction workers (or shop attendants) in his network.

Even a skilled worker may obtain extra benefits from having unskilled workers in his network, denoted by $\phi_{su}(n(1-s_t))$, but to a lesser degree than an unskilled worker would obtain: $\phi'_{su}(n(1-s_t)) < \phi'_{uu}(n(1-s_t))$. In a symmetric way for having skilled workers, we have $\phi'_{us}(ns_t) < \phi'_{ss}(ns_t)$. Thus, the net benefits of being a skilled worker realized at time τ is $w_s + \phi_{ss}(ns^e_\tau) + \phi_{su}(n(1-s^e_\tau)) - w_u - \phi_{uu}(n(1-s^e_\tau)) - \phi_{us}(ns^e_\tau)$, in which s^e_τ indicates the expected network quality at the future point of time τ. Replacing the baseline salary differential $w_s - w_u$ with $\bar{\delta}$, and $\phi_{ss}(ns^e_\tau) + \phi_{su}(n(1-s^e_\tau)) - \phi_{uu}(n(1-s^e_\tau)) - \phi_{us}(ns^e_\tau)$ with $f(s^e_\tau)$, the lifetime net benefits of skill investment to an agent born at time t are summarized as

$$\Pi_t^e = \int_t^\infty [\bar{\delta} + f(s_\tau^e)] e^{-(\rho+\alpha)(\tau-t)} \mathrm{d}\tau, \tag{1}$$

where ρ is a time-discounting factor and α is a Poisson death rate. Also note that $f'(s_\tau^e) > 0$, which implies the social increasing returns emphasized by Acemoglu(1996). We assume that $\bar{\delta}$ is big enough that $\bar{\delta} + f(0) > 0$ so that the net benefits of being skilled is always positive. Thus, the net benefits of skill investment Π_t^e are an increasing function of both the baseline salary differential $\bar{\delta}$ and the sequences of expected network quality $\{s_\tau^e\}_{\tau=t}^\infty$: $\Pi_t^e \equiv \Pi(\bar{\delta}, \{s_\tau^e\}_{\tau=t}^\infty)$. The higher rate of return on skill investment with the more skilled workers in the social group seems natural with the existence of social network externalities as long as the group's skill level does not affect significantly the aggregate skill composition of the economy.[3]

An agent born at time t with an innate ability a commits a skill investment if and only if

$$C(a, s_t) \leq \Pi(\bar{\delta}, \{s_\tau^e\}_{\tau=t}^\infty). \tag{2}$$

The notable feature of this argument is that one's skill investment is affected by both the current network externalities in the skill acquisition period and the future network externalities over the labor market phase of his career. The interplay between the two kinds of network externalities has not yet been explored by other theoretical works including recent developments by Bowles et al. (2007) and Calvó-Armengol and Jackson (2004). The above formula generates a unique threshold ability level such that newborn individuals of the social group whose innate ability is at least the threshold invest in the skill acquisition. Let us define a function A that represents the unique threshold ability $\tilde{a}$: $\tilde{a}_t \equiv A(s_t, \Pi_t^e)$. Using the distribution of the innate ability level $G(a)$, the fraction of individuals born at time t who invest in the skill, denoted by x_t, is expressed by

$$x_t = 1 - G(A(s_t, \Pi_t^e)). \tag{3}$$

[3]However, some scholars even suggest that the skill premium may depend positively on the the aggregate skill composition given the market frictions. For example, Acemoglu (1996) develops a mechanism for social increasing returns that the rate of return on human capital of a worker is increasing in the average human capital of the workforce when the labor market is characterized by costly search.

3 Dynamic System with Social Network Externalities

The skill investment of the newborns can be approximated by the following procedure. Consider a very short time interval between t and $t + \Delta t$. Suppose that, at the beginning of the interval, the randomly chosen $\alpha\Delta t$ fraction of the group's population, which is the $N \cdot \alpha\Delta t$ number of workers, die, and the same number of agents are newly born. The $N \cdot (1 - \alpha\Delta t)$ workers of the group survive until $t + \Delta t$. At the end of the interval, the newborn agents incur the cost of skill achievement. Then, the threshold level of ability $\tilde{a}$ for the skill investment is determined by the following equation: $C(\tilde{a}, s_t) = \Pi(\bar{\delta}, \{s^e_\tau\}^\infty_{\tau=t+\Delta t})$. The fraction of the individuals who are born at time t who invest in skill (x_t) is $1 - G(A(s_t, \Pi^e_{t+\Delta t}))$. The total number of skilled workers at time $t + \Delta t$ will be the sum of skilled workers in the surviving population and those in the newborn cohort: $N \cdot (1 - \alpha\Delta t) \cdot s_t + N \cdot \alpha\Delta t \cdot [1 - G(A(s_t, \Pi^e_{t+\Delta t}))]$. Thus, the group skill level at time $t + \Delta t$ is approximated as

$$s_{t+\Delta t} \approx (1 - \alpha\Delta t) \cdot s_t + \alpha\Delta t \cdot [1 - G(A(s_t, \Pi^e_{t+\Delta t}))]. \tag{4}$$

Rearranging the equation gives us

$$\frac{\Delta s_t}{\Delta t} \equiv \frac{s_{t+\Delta t} - s_t}{\Delta t} \approx \alpha \left[1 - G(A(s_t, \Pi^e_{t+\Delta t})) - s_t\right]. \tag{5}$$

Taking $\Delta t \to 0$, we achieve the evolution rule of a group's skill level s_t,

$$\dot{s}_t = \alpha \left[1 - G(A(s_t, \Pi^e_t)) - s_t\right]. \tag{6}$$

This can be expressed as $\dot{s}_t = \alpha [x_t - s_t]$ because of equation (3). If the fraction of newborn agents who invest in skill (x_t) is greater than the current skill level of the group (s_t), the network quality improves at time t. Otherwise, it declines.

Also, taking the derivative with respect to time t in equation (1), we have the evolution rule of the net benefits of skill investment Π^e_t,

$$\dot{\Pi}^e_t = (\rho + \alpha) \left[\Pi^e_t - \frac{\bar{\delta} + f(s_t)}{\rho + \alpha}\right]. \tag{7}$$

If a normalized level of the currently accrued benefits of being skilled $\left(\frac{\bar{\delta}+f(s_t)}{\rho+\alpha}\right)$ is greater than the lifetime benefits of being skilled that are expected to accrue from now until death (Π^e_t), the lifetime benefits of being skilled expected to accrue from the next time point $t + \Delta t$ to the death ($\Pi^e_{t+\Delta t}$) would be smaller than its current level: $\Pi^e_{t+\Delta t} < \Pi^e_t$. If they are equal, the lifetime net benefits of skill investment would not change within the short time interval Δt: $\Pi^e_{t+\Delta t} = \Pi^e_t$.

The group skill level s_t is constantly adjusted by the level of skill investments among the newborn cohort, which means that it is a flow variable, which cannot make a sudden jump at a point of time. However, the lifetime benefits of skill investment Π^e_t depend on the expectations about future network quality. By altering the expectation of $\{s^e_\tau\}^\infty_{\tau=t}$, the lifetime benefits can make a sudden jump at any point of time. Thus, it is a jumping variable. The dynamic system with network externalities that includes a flow variable s_t and a jumping variable Π^e_t is therefore summarized by the following equations:

$$\dot{s}_t = \alpha(x_t - s_t), \text{ in which } x_t = 1 - G(A(s_t, \Pi^e_t)), \tag{8}$$

$$\dot{\Pi}^e_t = (\rho + \alpha)\left[\Pi^e_t - \frac{\bar{\delta} + f(s_t)}{\rho + \alpha}\right], \tag{9}$$

and the two isoclines of the time dependent variables are represented by

$$\dot{s}_t = 0 \text{ Locus} \quad : \quad s_t = 1 - G(A(s_t, \Pi^e_t)), \tag{10}$$

$$\dot{\Pi}^e_t = 0 \text{ Locus} \quad : \quad \Pi^e_t = \frac{\bar{\delta} + f(s_t)}{\rho + \alpha}. \tag{11}$$

For further analysis of this dynamic system without damaging its essential structure, we introduce the following linear functional forms of the cost function $C(a, s_t)$ and the benefits function $f(s_t)$: $C(a, s_t) = c_0 - \psi a - p s_t$ and $f(s_t) = f_0 + q s_t$, where p represents the influence of the education period network externality, and q represents the influence of the working period network externality. ψ represents the cost sensitivity to the innate ability level a. The threshold ability level ($\tilde{a}$) for the skill achievement is obtained from the equation $C(\tilde{a}, s_t) = \Pi^e_t$:

$$A(s_t, \Pi^e_t) = \frac{c_0 - p s_t - \Pi^e_t}{\psi}. \tag{12}$$

In order to avoid the massive complications, the model in this paper uses the above linear functional forms. However, readers will be able to find that the major results of this paper are derived with the general forms of the cost function $C(a, s_t)$ and the benefits function $f(s_t)$ without the artificial linearization.

4 Multiple Steady States

In this section, we check the possible multiple steady states in the given dynamic system. We start with the simplest case wherein the innate ability is equal across the population: $a \equiv \bar{a}$. Then, we examine the case with the general form of the ability distribution.

4.1 Steady States with Unique Ability Level

In the unique system with the equal innate ability $\bar{a}$, the investment rate among the newborns is either $x_t = 1$ or $x_t = 0$. All of the newborn agents at time t invest in skill achievements when the expected benefits are no less than the cost: $x_t = 1$ with $\Pi_t^e \geq C(\bar{a}, s_t)(= c_0 - \psi\bar{a} - ps_t)$. This implies that the equation $\dot{s}_t = 0$ holds when $\Pi_t^e \geq c_0 - \psi\bar{a} - p$ and $s_t = 1$ as $x_t = s_t = 1$. Also, for the given $s_t = 1$, the equation $\dot{\Pi}_t^e = 0$ holds when $\Pi_t^e = \frac{\bar{\delta}+f_0+q}{\rho+\alpha}$, according to equation (11). Therefore, there exists a steady state with $s_t = 1$ when $\frac{\bar{\delta}+f_0+q}{\rho+\alpha} \geq c_0 - \psi\bar{a} - p$. Let us normalize $\frac{\bar{\delta}}{\rho+\alpha}$ as $\bar{\delta}'$, which represents the lifetime level of the wage differential $\int_t^\infty \bar{\delta} \cdot e^{-(\rho+\alpha)(\tau-t)}\mathrm{d}\tau$, and normalize $\frac{\rho}{\rho+\alpha}$, $\frac{f_0}{\rho+\alpha}$ and $\frac{q}{\rho+\alpha}$ as ρ', f_0' and q'. The high investment rate with $x_t = 1$ and the high quality of the social network with $s_t = 1$ is a steady state when the composite network externalities $(p + q')$ are large enough that $p + q' \geq c_0 - \psi\bar{a} - \bar{\delta}' - f_0'$.

None of the newborns invest in the skill when the expected benefits is smaller than the cost, $x_t = 0$ with $\Pi_t^e < C(\bar{a}, s_t)(= c_0 - \psi\bar{a} - ps_t)$. This implies that the equation $\dot{s}_t = 0$ holds when $\Pi_t^e < c_0 - \psi\bar{a}$ and $s_t = 0$ as $x_t = s_t = 0$. For the given $s_t = 0$, the equation $\dot{\Pi}_t^e = 0$ holds when $\Pi_t^e = \frac{\bar{\delta}+f_0}{\rho+\alpha}$ (equation (11)). Therefore, another steady state that includes the low investment rate $x_t = 0$ and the low quality social network with $s_t = 0$ exists when the following holds: $c_0 - \psi\bar{a} - \bar{\delta}' - f_0' > 0$. The two steady states are displayed in Figure 1 with the two isocline loci.

Proposition 1. *Given a unique innate ability level $\bar{a}$ for all the newborns and assuming $c_0 - \psi\bar{a} - \bar{\delta}' - f_0' > 0$, multiple steady states exist, $(s_l, \Pi_l) = (0, \bar{\delta}' + f_0')$ and $(s_h, \Pi_h) = (1, \bar{\delta}' + f_0' + q')$, if and only if the composite network externalities $(p + q')$ are big enough that $p + q' \geq c_0 - \psi\bar{a} - \bar{\delta}' - f_0'$.*

This proposition proves that sufficient network externalities must be present for multiple steady states to exist. In addition, under the absence of network externalities, a unique steady state always exists. If the base salary differential $\bar{\delta}'$ is big enough, then the high network quality $(s_t = 1)$ is self-confirmed at the unique steady state. Otherwise, the low network quality $(s_t = 0)$ is self-confirmed.

Corollary 1. *Given a unique innate ability level $\bar{a}$ for all the newborns, there exists a unique steady state under the absence of network externalities ($p = q = 0$). The unique steady state is $(1, \bar{\delta}' + f_0')$ if $\bar{\delta}' \geq c_0 - \psi\bar{a} - f_0'$ and $(0, \bar{\delta}' + f_0')$ otherwise.*

4.2 Steady States with Ability Distribution $G(a)$

In the simplest case with a unique ability label $\bar{a}$, we have shown that the multiplicity of steady states is generated by the influence of network externalities, and, without the network externalities, the multiplicity is not achieved. In this section, we confirm this conclusion with the more general form of the innate ability function $G(a)$. Suppose $G(a)$ is S-shaped. Then, there exists $\hat{a}$ such that $G(a)'' > 0$ for any $a \in (-\infty, \hat{a})$ and $G(a)'' < 0$ for any $a \in (\hat{a}, \infty)$, which implies that its PDF $g(a)$ has one peak at $\hat{a}$ (e.g., a bell-shaped $g(a)$).

The $\dot{s}_t = 0$ locus in equation (10) is represented by (s_t, Π_t^e)s that satisfy the following two equations, which are associated with the threshold ability level ($\tilde{a}$) for the skill achievement.

$$\begin{cases} s_t = 1 - G(\tilde{a}) \\ \tilde{a} = A(s_t, \Pi_t^e) \end{cases} \tag{13}$$

The first is denoted by the solid curve in Panel A of Figure 2 in the $(s_t, \tilde{a})$ domain, and the second is denoted by the dotted lines for each level of Π_t^e (iso-Π lines) in the same panel. The slope of the $\dot{s}_t = 0$ locus is obtained through the implicit function theorem: defining a function F as $F = 1 - G(A(s_t, \Pi_t^e)) - s_t$, we have $\frac{\mathrm{d}\Pi_t^e}{\mathrm{d}s_t}\big|_{\dot{s}_t=0} \left(\equiv -\frac{F_s}{F_\Pi}\right) = -\frac{G'(\tilde{a}) \cdot A_s + 1}{G'(\tilde{a}) \cdot A_\Pi}$. Because $A_s = -p\psi^{-1}$ and $A_\Pi = -\psi^{-1}$ (equation (12)), we have the following lemma.

Lemma 1. *The slope at an arbitrary point (s', Π') on the $\dot{s}_t = 0$ locus is $\frac{\psi}{g(\tilde{a}')} - p$, in which $\tilde{a}' = A(s', \Pi')$ and $\tilde{a}' = G^{-1}(1 - s')$.*

For the specific ability level $\hat{a}$ under which $g(a)$ is maximized, the corresponding $\hat{s}$ on the $\dot{s}_t = 0$ locus is $\hat{s} \equiv 1 - G(\hat{a})$, and the corresponding $\hat{\Pi}$ is the Π_t^e that satisfies $\hat{a} = A(\hat{s}, \Pi_t^e)$: $\hat{\Pi} \equiv c_0 - \psi\hat{a} - p\hat{s}$.

Suppose the $\dot{\Pi}_t^e = 0$ locus passes through the specific point $(\hat{s}, \hat{\Pi})$, as displayed in Panel B of Figure 2. Note that the slope of the $\dot{s}_t = 0$ locus, $\frac{\psi}{g(\tilde{a})} - p$, is minimized at the point because $g(\tilde{a})$ is maximized with $\tilde{a} = \hat{a}$. In this case, it is obvious that multiple steady states exist if and only if the slope of the $\dot{\Pi}_t^e = 0$ locus is greater than that of the $\dot{s}_t = 0$ locus at the point $(\hat{s}, \hat{\Pi})$: $q' > \frac{\psi}{g(\hat{a})} - p$. Thus, we conclude that the multiplicity of the steady states is achieved when the composite influence of the network externalities measured by $p + q'$ is big enough that $p + q' > \frac{\psi}{g(\hat{a})}$.

Proposition 2. *Suppose that the $\dot{\Pi}_t^e = 0$ locus passes through $(\hat{s}, \hat{\Pi})$ on the $\dot{s}_t = 0$ locus, in which $A(\hat{s}, \hat{\Pi}) = \hat{a}$. Three steady states exist if and only if the composite network externalities $(p + q')$ are big enough that $p + q' > \frac{\psi}{g(\hat{a})}$.*

Let us denote the three steady states by $E_l(s_l, \Pi_l)$, $E_m(s_m, \Pi_m)$ and $E_h(s_h, \Pi_h)$, in which $s_l < s_m (= \hat{s}) < s_h$, as displayed in Figure 2. With the low quality social network s_l now and in the future, the expected benefits of skill investment are low (Π_l). With the low level of the network quality s_l together with the low level of benefits to investment Π_l, the ability threshold for skill achievement among the newborns is high and a relatively small fraction of the newborns invest in skills. In this manner, the low quality social network is self-confirmed, which is represented by the steady state $E_l(s_l, \Pi_l)$. With the high quality social network s_h now and in the future, the expected benefits of skill investment are high (Π_h). With the high levels of network quality (s_h) and the net benefits to investment (Π_h), the ability threshold for skill achievement is lower and a relatively large fraction of the newborns invest in skills. Thus, the high quality social network is also self-confirmed, which is represented by the steady state $E_h(s_h, \Pi_h)$.

The existence of multiple steady states is possible only when the influence of network externalities is sufficiently strong. Consider the economy under the absence of network externalities $(p = q = 0)$. The $\dot{\Pi}_t^e = 0$ locus is flat because of $q = 0$: $\Pi_t^e = \bar{\delta}' + f_0'$. The $\dot{s}_t = 0$ locus is $\Pi_t^e = -\psi G^{-1}(1 - s_t) + c_0$ because of $p = 0$, according to equations (10) and (12), which is a monotonically increasing function. Therefore, a unique steady state always exists at $(1 - G((c_0 - \bar{\delta}' - f_0')\psi^{-1}), \bar{\delta}' + f_0')$ without the network externalities.

Corollary 2. *A unique steady state $(1 - G((c_0 - \bar{\delta}' - f_0')\psi^{-1}), \bar{\delta}' + f_0')$ exists under the absence of the network externalities $(p = q = 0)$.*

Proposition 2 can be generalized further as follows.

Theorem 1. *If and only if the composite network externalities $(p + q')$ are big enough that $p + q' > \frac{\psi}{g(\hat{a})}$, does a range of the base salary differential $[\bar{\delta}_2, \bar{\delta}_1]$ exist such that the multiple steady states exist with any $\bar{\delta}$ within the range: $\bar{\delta}_j = (\rho + \alpha)(c_0 - \psi G^{-1}(1 - s_j) - (p + q')s_j - f_0'), \forall j \in \{1, 2\}$, in which both s_1 and s_2 satisfy $g(G^{-1}(1 - s_j)) = \psi(p + q')^{-1}$ and $s_1 < \hat{s} < s_2$. Otherwise, a unique steady state exists regardless of the base salary differential level $\bar{\delta}$.*

Proof. For the proof, see the appendix. ■

The theorem implies there are three steady states with $\bar{\delta}$ between $\bar{\delta}_2$ and $\bar{\delta}_1$ given $p + q' > \frac{\psi}{g(\hat{a})}$ and two with either $\bar{\delta} = \bar{\delta}_2$ or $\bar{\delta} = \bar{\delta}_1$ because the $\dot{\Pi}^e_t = 0$ locus is tangent to the $\dot{s}_t = 0$ locus. The above theorem confirms the importance of network externalities in generating multiple steady states. If the influence of network externalities $(p + q')$ is weak, the different steady states never include multiple social groups. When the influence is sufficiently strong, however, we can have multiple social groups at the steady states with the different network qualities.

5 Dynamic Equilibrium Paths

In this section, we identify the converging dynamic paths to the steady states and provide the economic interpretation of the paths.

5.1 Dynamic Paths with Unique Ability Level

In the simplest case of the identical ability level $(\bar{a})$ across the population, we have identified two possible steady states, assuming the conditions in Proposition 1 are satisfied. Denoting them by E_l and E_h, they are $E_l(0, \bar{\delta}' + f_0')$ and $E_h(1, \bar{\delta}' + f_0' + q')$. To examine the converging dynamic paths to the steady states, we need a phase diagram with direction arrows, which are displayed in Figure 3, in which the four dynamic regimes are classified by the two straight lines, $\Pi^e_t = c_0 - \psi\bar{a} - ps_t$ and $\Pi^e_t = \bar{\delta}' + f_0' + q's_t$. Let us denote the intersection of the two straight lines by E_m: $E_m(\frac{c_0 - \psi\bar{a} - \bar{\delta}' - f_0'}{p+q'}, \frac{p(\bar{\delta}' + f_0') + q'(c_0 - \psi\bar{a})}{p+q'})$. The two converging paths to E_h and E_l spiral out of the intersection E_m. The dynamic path converging to E_h above the two straight lines is determined by the following dynamic system, $\dot{s}_t = -\alpha s_t + \alpha$ and $\dot{\Pi}^e_t = (\rho+\alpha)\Pi^e_t - qs_t - \bar{\delta} - f_0$ because of $x_t = 1$ in the regime (equations (8) and (9)). The optimistic path is summarized by $\Pi^e_t = \frac{q}{\rho+2\alpha}s_t + \bar{\delta}' + f_0' + \frac{\alpha q'}{\rho+2\alpha}$. Also, the dynamic path converging to E_l below the two straight lines is determined by the dynamic system, $\dot{s}_t = -\alpha s_t$ and $\dot{\Pi}^e_t = (\rho + \alpha)\Pi^e_t - qs_t - \bar{\delta} - f_0$ because $x_t = 0$. This pessimistic path is summarized by $\Pi^e_t = \frac{q}{\rho+2\alpha}s_t + \bar{\delta}' + f_0'$. Therefore, we can calculate both the lower bound of s_t for the converging path to E_h

and the upper bound of s_t for the converging path to E_l, denoted by e_o and e_p for each:

$$\begin{cases} e_o = \max\left\{\left[p + \frac{q}{\rho+2\alpha}\right]^{-1}\left(c_0 - \psi\bar{a} - \bar{\delta}' - f_0' - \frac{\alpha q'}{\rho+2\alpha}\right), 0\right\} \\ e_p = \min\left\{\left[p + \frac{q}{\rho+2\alpha}\right]^{-1}\left(c_0 - \psi\bar{a} - \bar{\delta}' - f_0'\right), 1\right\}. \end{cases} \tag{14}$$

With an initial social network quality s_0 between the two bounds, $s_0 \in [e_o, e_p]$, two coordinated equilibrium paths are available to the social group: the optimistic path to E_h and the pessimistic path to E_l. If the coordinated expectation about the future is optimistic across the generations, the expected benefits of the skill achievement at time zero (Π_0^{op}) are $\frac{q}{\rho+2\alpha}s_0 + \bar{\delta}' + f_0' + \frac{\alpha q'}{\rho+2\alpha}$, and the expected benefits of the skill achievement among the following newborn cohorts are greater than the level: $\Pi_t^{op} > \Pi_0^{op}, \forall t \in (0, \infty)$. The newborn cohort and all following cohorts invest in skills: $x_t = 1, \forall t \in [0, \infty)$, which means that the skill level of the group s_t improves over time until it reaches one. However, if the coordinated expectation is pessimistic across the generations, the expected benefits at the initial point (Π_0^{pe}) is $\frac{q}{\rho+2\alpha}s_0 + \bar{\delta}' + f_0'$, which is smaller than Π_0^{op} by as much as $\frac{\alpha q'}{\rho+2\alpha}$, and the expected benefits of investment among the following newborn cohorts are smaller than the level for the current newborns: $\Pi_t^{pe} < \Pi_0^{pe}, \forall t \in (0, \infty)$. The newborn cohort and all following cohorts do not invest in skills: $x_t = 0, \forall t \in [0, \infty)$, which means that the skill level of the group deteriorates over time until it reaches zero.

Let us denote the range $[e_o, e_p]$ by *overlap*, in which multiple coordinated equilibrium paths are available, as suggested by Krugman (1991). Outside the overlap, a unique equilibrium path exists that is either optimistic or pessimistic. If the initial network quality is good enough that $s_0 > e_p$, the only reasonable expectation about the newborns' investments is $x_t = 1, \forall t \in [0, \infty)$. If it is poor enough that $s_0 < e_o$, the only reasonable expectation is $x_t = 0, \forall t \in [0, \infty)$.

The size of the overlap, denoted by $L(\equiv e_p - e_o)$, is essential to understand the characteristics of the economy: $L = \frac{\alpha}{(\rho+2\alpha)(\alpha+\rho)\frac{p}{q}+(\alpha+\rho)}$, as far as $e_o, e_p \in (0, 1)$. The bigger the L, the more likely it is that the coordinated expectation about the future is critical in the determination of the final skill level of the group. Because L is a decreasing function with respect to p/q, we have the following result.

Proposition 3. *Given a unique innate ability level $\bar{a}$ for all newborns, the size of overlap (L) is positively related to the relative strength of the working period network externality in comparison with the education period network externality (q/p). As the influence of the working period network externalities (q) increases* *and the influence of the education period network externalities* (p) *decreases, the more likely it is that the coordinated expectation will determine the final state of a social group's skill level instead of the history.*

Moreover, the overlap does not exist under the absence of working period network externalities: $L = 0$ when $q = 0$. This means that the skill investment activities of the newborns are subject to the "past" if network externalities are active only over the education period. That is, the initial quality of the social network determines the future, and the belief coordination across the generations does not have an effect. However, the size of the overlap is maximized under the absence of the education period network externality: $\arg\max_p L = 0, \forall q \in (0, \infty)$. The belief coordination across generations and the consequent collective action are most crucial when network externalities are not active over the education period.

Suppose a group's current skill level is in the overlap range. Then, the economic performance of the group can be improved simply by increasing expectations of a brighter future. This rationalizes the arguments of those who suggest that social groups are not constrained by their history but can raise themselves up "by their bootstraps." However, suppose that a group is poor enough that its current skill level (s_0) is below the lower bound of the optimistic path (e_o). The group cannot escape its miserable condition through belief coordination or collective actions among the group members. Under this situation, a disadvantaged group is trapped by its own history.

Definition 1 (Poverty Trap)**.** *A social group is in the poverty trap if its network quality* (s_0) *is below the lower bound of the optimistic path:* $s_0 < e_o$.

Consider two social groups, A and B, at the different steady states. Group A's skill level is one, and group B's is zero. Assuming that the overlap range is between zero and one, group B is in the poverty trap and group A is out of it. The disparity between the two groups cannot be overcome without governmental intervention. Suppose that the government helps group B improve its skill level and enter the overlap range. At this stage, the crucial point is the belief coordination among group B members. If optimism prevails, the newborn cohorts invest in skills and the group's skill level improves consistently up to the level of group A. If pessimism prevails, the newborn cohorts do not invest in skills and the skill level can even deteriorate over time until reaching group B's original level of zero, making the earlier governmental intervention useless.

5.2 Dynamic Paths with Ability Distribution $G(a)$

In this section, we confirm the findings presented in the earlier section using the more general form of the ability distribution, S-shaped $G(a)$. Suppose that the condition for multiplicity is satisfied in Theorem 1. Suppose that the following three distinct steady states exist: $E_l(s_l, \Pi_l)$, $E_m(s_m, \Pi_m)$ and $E_h(s_h, \Pi_h)$, in which $s_l < s_m < s_h$. This is achieved with $\bar{\delta} \in (\bar{\delta}_2, \bar{\delta}_1)$ in Theorem 1. Using equations (8) and (9), $\dot{s}_t$ is positive (negative) above (below) the $\dot{s}_t = 0$ locus, $\dot{\Pi}_t^e$ is positive (negative) above (below) the $\dot{\Pi}_t^e = 0$ locus. The phase diagram with direction arrows is displayed in Figure 4. The characteristics of the steady states are summarized by the following lemma.

Lemma 2. *Among the three steady states, $E_l(s_l, \Pi_l)$, $E_m(s_m, \Pi_m)$ and $E_h(s_h, \Pi_h)$, in which $s_l < s_m < s_h$, E_l and E_h are saddle points and E_m is a source.*

Proof. For the proof, see the appendix. ■

We can identify the equilibrium path (saddle path) to each saddle point, E_l and E_h, as described in Figure 4. The equilibrium paths spiral out of a source E_m. The lower bound of the optimistic path to E_h (e_o) is smaller than the upper bound of the pessimistic path (e_p): $e_o < e_p$. Within the overlap range $[e_o, e_p]$, multiple coordinated equilibrium paths exist. If the coordinated expectation is optimistic, the upper path is taken and the skill level approaches s_h. If it is pessimistic, the lower path is taken and the skill level approaches s_l. Outside the overlap, a unique reasonable equilibrium path exists, which is either an optimistic path to s_h or a pessimistic path to s_l. Thus, within the overlap, the coordinated expectation determines the final state, and the history determines the final state outside the overlap.

The existence of the overlap range is related to the existence of working period network externalities. First, consider the case where working period network externalities are absent ($q = 0$). Because the benefits of skill acquisition are fixed as $\bar{\delta}' + f_0'$ in this case, the expectation about the future does not play any role. This is displayed in the phase diagram of Panel A in Figure 5 as the flat $\dot{\Pi}_t^e = 0$ locus ($\Pi_t^e \equiv \bar{\delta}' + f_0'$) and the non-existence of overlap, in which the final economic state depends entirely on the initial network quality of the group. Also, the existence of the overlap range is guaranteed by the existence of working period network externalities ($q > 0$). When the future benefits of skill acquisition are affected by the future network quality, one's skill investment should be influenced by other group members' skill investments now and in the future. Collective action to mange expectation can therefore play an important role in the determination of the overall skill investment rate among the newborn cohorts.

Proposition 4. *Suppose that the condition in Theorem 1 is satisfied with* $\bar{\delta} \in (\bar{\delta}_2, \bar{\delta}_1)$ *such that there exist three distinct steady states* (E_l, E_m *and* E_h). *The overlap range does not exist under the absence of the working period network externalities* ($q = 0$), *and it always exists with the existence of the working period network externalities* ($q > 0$).

Proof. For the proof, see the appendix. ■

Furthermore, the size of the overlap is determined by the relative influence of the working period network externalities over the education period network externalities. With the greater q, the size of the overlap tends to be larger. This point is illustrated by the following theorem, which is consistent with the earlier result with a unique ability level $\bar{a}$ in Proposition 3.

Theorem 2. *Suppose that the condition in Theorem 1 is satisfied with* $\bar{\delta} \in (\bar{\delta}_2, \bar{\delta}_1)$ *such that there exist three distinct steady states* (E_l, E_m *and* E_h). *With the increased influence of the working period network externalities (greater* q*) while holding a steady state* E_m *at* (s_m, Π_m), *the optimistic path to* E_h *with greater* q *is placed above the original optimistic path to* E_h *for any* $s_t \geq s_m$, *and the pessimistic path to* E_l *with greater* q *is placed below the original pessimistic path to* E_l *for any* $s_t \leq s_m$ *(Refer to Panel B of Figure 5).*

Proof. For the proof, see the appendix. ■

The theorem also suggests that, given any initial s_0 within the overlap, the difference between the expected benefits of investments with an optimistic view (Π_0^{op}) and that with a pessimistic view (Π_0^{pe}) tends to be greater with the increased influence of working period network externalities (greater q) because the distance between the optimistic path and the pessimistic path gets wider: the greater $\Pi_0^{op} - \Pi_0^{pe}$ with the greater q given p. This means that, with the greater q relative to p, we are more likely to observe a greater difference in the newborns' skill investment activities between the case with the coordinated optimism and that with the coordinated pessimism because $x_0^{op} - x_0^{pe} = G(A(s_0, \Pi_0^{pe})) - G(A(s_0, \Pi_0^{op}))$ in equation (3). This further illustrates that the coordinated expectation can generate a greater difference in newborns' skill investment activities with the greater influence of working period network externalities relative to the education period network externalities.

6 Policies Implications and Further Discussion

The developed dynamic model implies that if a social group is trapped by poor network quality, an effective egalitarian policy of either social integration or affirmative action requires both active governmental intervention and societal belief coordination to mobilize the group out of the poverty trap and to help it advance as far as the advantaged group. A policy that fails in either respect would not be successful in eliminating a persistent skill disparity between social groups. However, if the network quality of a disadvantaged group is already in the overlap range, the active governmental intervention would not have a significant impact on the advancement of the group. Instead, an emphasis on coordinated optimism should be pursued, although this point is often ignored in policy debates. Collective action to manage expectation can generate a significant impact on behavioral changes among young members of the disadvantaged group.

During the Jim Crow period of US history and until the civil rights movement in the 1960s, African Americans were segregated from whites and discriminated against in an overt manner in the US labor market. Although overt discrimination decreased in recent decades, we still observe persistent skill disparities between the blacks and the whites. In particular, Black youths have significantly lower academic achievement than white youths. The dynamic model in this paper suggests that Blacks with more low-skilled people in their social network tend to be trapped by the adverse effects of poor network quality in the segregated American society. Further, the model implies that the optimistic coordinated expectation is important for behavioral changes among Black youth. The persistent group disparity that occurs in many other segregated societies including South Africa, Australia and many countries in Latin America may also be attributable to the poverty trap through the social externalities channel and the failure of belief coordination.

So far we have not discussed fully the spiraling out equilibrium paths. There often exist multiple points of lifetime benefits of skill investment (Π_0) that are available given an initial skill level (s_0) even on the same equilibrium path. For instance, as denoted in Figure 3, the group with a certain skill level may choose either point a and point b (or others if available) on the optimistic path. What would make the difference between choosing point a or choosing point b? The answer is related to the expectation about the length of time to arrive at high equilibrium state E_h. Choosing point a means that the group believes that the high network quality will be realized as soon as it can be. With the high lifetime benefits of investment, the newborn cohorts invest in skills right away: $x_0 = 1$. This is a case of *strong optimism.* On the other hand, choosing point b

means that the group believes that the high network quality may take longer to come. If they believe in that way, the benefits of investment would be lowered and the newborn cohorts will not invest in skills: $x_0 = 0$. This causes the group skill level to drop for a while, even when they share the optimistic view that the group will arrive at E_h in the long run. Therefore, this is a case of *weaker optimism*.

Finally, Adsera and Ray (1998) argue that *overlap* is generated only when agents have an incentive to choose the option that offers less appealing benefits at the moment of decision. In our model, the incentive originates in the overlapping generation structure. Since agents are given only one chance to choose their occupational type at the early stage of their lives, they choose a type with less appealing benefits at the moment of the skill investment decision while expecting greater benefits to accrue in his lifetime. The dynamic structure with an overlapping generational framework provided here can be useful in the examination of other topics in which external economies exist and populations are replaced by new cohorts. Hauk and Saez-Marti(2002)'s work on the cultural transmission of corruption and Bisin and Verdier(2001)'s work on intergenerational transmission of values adopt the similar structure though they do not explicitly analyze the existence of multiple equilibrium paths and the size of overlap. We expect future theoretical works that include external economies and overlapping generations to adopt the given dynamic structure.

7 Conclusion

The importance of social externalities in the skill acquisition stage has been extensively studied in the literature, with the conclusion that history might be important in the determination of the skill acquisition equilibrium in social groups. The importance of externalities in subsequent career stage has been less extensively studied in this context. This paper examines the effect of coordinated expectation on the improvement of a group' skill level under the presence of social externalities in both stages. The consideration of economic agents' forward-looking behaviors contributed to the development of this dynamic model.

The model proves that the skill investment period network externality operates as *a historical force* that restricts a group to be subject to the current network quality and that the working period network externalities operate as *a mobilization force* that leads a group to enhance (shrink) the skill investment activities by holding an optimistic (pessimistic) view about the future network quality. The relative strength of working-period network externalities over the skill investment period network externalities determines the size of overlap(Krugman 1991). In terms of the policy perspectives, the model emphasizes that an effective policy to mobilize a disadvantaged social group trapped by the negative influence of network effects first requires active governmental intervention to enhance the group's skill level to enter the "overlap", and then requires societal belief coordination for the collective optimism. A policy that fails in either respect may not be successful.

8 Appendix: Proofs

8.1 Proof of Theorem 1

Lemma 1 indicates the slope of the $\dot{s}_t = 0$ locus can be represented by $\frac{d\Pi_t^e}{ds_t}\Big|_{\dot{s}_t=0} = \frac{\psi}{g(G^{-1}(1-s))} - p$, in which $\frac{d\Pi_t^e}{ds_t}\Big|_{\dot{s}_t=0} = \infty$ as either $s \to 0$ or $s \to 1$. This implies that there must be at least one steady state. The slope is minimized at $\hat{s}(= 1 - G(\hat{a}))$ with its minimum $\frac{\psi}{g(\hat{a})} - p$, and the slope is decreasing in $(0, \hat{s})$, and increasing in $(\hat{s}, 1)$. Therefore, if the slope of the $\dot{\Pi}_t^e = 0$ locus (q') is greater than the minimum $\frac{\psi}{g(\hat{a})} - p$, we can find two network quality levels, s_1 and s_2, with which the slopes of the two loci are equalized: $\frac{\psi}{g(G^{-1}(1-s))} - p = q'$ for $s \in \{s_1, s_2\}$, in which $s_1 < \hat{s} < s_2$. When the $\dot{\Pi}_t^e = 0$ locus is tangent to the $\dot{s}_t = 0$ locus at either s_1 or s_2, the corresponding Π_1 and Π_2 on the $\dot{s}_t = 0$ locus are $\Pi_j = -\psi G^{-1}(1 - s_j) - ps_j + c_0, \forall j \in \{1, 2\}$, because of equations (10) and (12). Therefore, noting that the $\dot{\Pi}_t^e = 0$ locus shifts up with the greater $\bar{\delta}$, we can find the corresponding $\bar{\delta}_1$ and $\bar{\delta}_2$ at s_1 and s_2: $\bar{\delta}_j(= \Pi_j(\rho+\alpha) - qs_j - f_0) = (\rho+\alpha)(c_0 - \psi G^{-1}(1-s_j) - (p+q')s_j - f_0'), \forall j \in \{1, 2\}$. Given the slope q' of the $\dot{\Pi}_t^e = 0$ locus greater than $\frac{\psi}{g(\hat{a})} - p$, there exist multiple steady states with $\bar{\delta} \in [\bar{\delta}_2, \bar{\delta}_1]$ and the number of steady states is three with $\bar{\delta} \in (\bar{\delta}_2, \bar{\delta}_1)$.

If the slope of the $\dot{\Pi}_t^e = 0$ locus (q') is smaller than the minimum $\frac{\psi}{g(\hat{a})} - p$, we have only one intercept between the two loci for any level of $\bar{\delta}$, because the slope of the $\dot{s}_t = 0$ locus is greater than that of the $\dot{\Pi}_t^e = 0$ locus (q') for any $s_t \in (0, 1)$. In the same reason, if the slope of the $\dot{\Pi}_t^e = 0$ locus (q') equals the minimum $\frac{\psi}{g(\hat{a})} - p$, the two loci cross each other only one time, because the slope of the $\dot{s}_t = 0$ locus is greater than that of the $\dot{\Pi}_t^e = 0$ locus for any $s_t \in (0, 1)$, except for $s_t = \hat{s}$. ■

8.2 Proof of Lemma 2

Given the dynamic system in equations (8) and (9), its linearization around a steady state $(\bar{s}, \bar{\Pi})$ is

$$\begin{aligned} \dot{s}_t &= \alpha[-G'A_s' - 1](s_t - \bar{s}) + \alpha[-G'A_\Pi'](\Pi_t^e - \bar{\Pi}) \\ \dot{\Pi}_t^e &= -f'(s_t - \bar{s}) + (\rho + \alpha)(\Pi_t^e - \bar{\Pi}). \end{aligned}$$

The Jacobian matrix J_E evaluated at a steady state is

$$J_E \equiv \begin{bmatrix} -\alpha G'A_s' - \alpha & -\alpha G'A_\Pi' \\ -f' & \rho + \alpha \end{bmatrix}_{(\bar{s},\bar{\Pi})} = \begin{bmatrix} \alpha \cdot \frac{g(\tilde{a}')p}{\psi} - \alpha & \alpha \cdot \frac{g(\tilde{a}')}{\psi} \\ -q & \rho + \alpha \end{bmatrix}_{(\bar{s},\bar{\Pi})},$$

, in which $\tilde{a}' = A(\bar{s}, \bar{\Pi})$. Consequently, its transpose is $trJ_E = \alpha \cdot \frac{g(\tilde{a}')p}{\psi} + \rho$ and the determinant is $|J_E| = \frac{\alpha(\alpha+\rho)g(\tilde{a}')}{\psi}\left[p + q' - \frac{\psi}{g(\tilde{a}')}\right]$. Since trJ_E is positive, every steady state is unstable. The slope of the $\dot{s}_t = 0$ locus is $\frac{\psi}{g(\tilde{a}')} - p$ (Lemma 1) and the slope of the $\dot{\Pi}_t^e = 0$ locus is q'. At the steady states E_l and E_h, the slope of the $\dot{s}_t = 0$ locus is greater than the slope of the $\dot{\Pi}_t^e = 0$ locus: $p + q' < \frac{\psi}{g(\tilde{a}')}$. At the steady state E_m, the slope of the $\dot{s}_t = 0$ locus is smaller than the slope of the $\dot{\Pi}_t^e = 0$ locus: $p + q' > \frac{\psi}{g(\tilde{a}')}$. Therefore, $|J_E|$ is negative at E_l and E_h, and positive at E_m, which implies E_l and E_h are saddle points and E_m is a source. ■

8.3 Proof of Proposition 4

First, consider the case with $q = 0$. Since the net benefits of being skilled is $\bar{\delta} + f_0$ at each point of time, the lifetime benefits for skill achievements is fixed as $\bar{\delta}' + f_0'$ because $\Pi_0^e = \int_0^\infty (\bar{\delta} + f_0) \cdot e^{-(\rho+\alpha)\tau} \mathrm{d}\tau$, for any s_0. Therefore, there cannot be multiple choices of the level of Π_0^e for any given s_0. Second, consider the case with $q > 0$. Consider an initial network quality $s_0 = s_m + \epsilon$ and its converging sequences $\{s_\tau\}_0^\infty$ to s_h. Then, we have $\Pi_0^{op} = \int_0^\infty [\bar{\delta} + f_0 + q s_\tau] e^{-(\rho+\alpha)\tau} \mathrm{d}\tau > \frac{\bar{\delta}+f_0+q(s_m+\epsilon)}{\rho+\alpha}$ because $\Pi_0^{op} = \frac{\bar{\delta}+f_0+q(s_m+\epsilon)}{\rho+\alpha} + \int_0^\infty q(s_\tau - (s_m+\epsilon)) e^{-(\rho+\alpha)\tau} \mathrm{d}\tau$ and $s_\tau > s_m + \epsilon, \forall \tau \in (0, \infty)$. Π_0^{op} is above the $\dot{\Pi}_t^e = 0$ locus for $s_0 = s_m + \epsilon$ for any small ϵ. Therefore, the converging path to E_h is placed above the $\dot{\Pi}_t^e = 0$ locus around s_m, implying $e_o < s_m$. Also, consider an initial network quality $s_0 = s_m - \epsilon$ and its converging sequences $\{s_\tau\}_0^\infty$ to s_l. Then, in the same way, we can show that Π_0^{pe} is below the $\dot{\Pi}_t^e = 0$ locus for $s_0 = s_m - \epsilon$ for any small ϵ. Therefore, the converging path to E_l is placed below the $\dot{\Pi}_t^e = 0$ locus around s_m, implying $e_p > s_m$. These imply the existence of an overlap in the neighborhood of s_m. ■

8.4 Proof of Theorem 2

Holding a steady state E_m at (s_m, Π_m) requires the $\dot{\Pi}_t^e = 0$ locus to pass through (s_m, Π_m) for different levels of q: the combination $(\bar{\delta}, q)$ should satisfy $\Pi_m = \frac{\bar{\delta}+f_0+q s_m}{\rho+\alpha}$. The q increase by Δq must come with $\bar{\delta}$ decrease by $s_m \Delta q$: $\Delta\bar{\delta} = -s_m \Delta q$. Note that the $\dot{\Pi}_t^e = 0$ locus is rotated in a counter-clockwise direction as displayed in Panel B of Figure 5. We compare the converging dynamic paths in a dynamic system with $q_0 > 0$ with those in a dynamic system with $q_0 + \Delta q$. Let us denote two stable steady states in the dynamic system with q_0 by $E_l(s_l, \Pi_l)$ and $E_h(s_h, \Pi_h)$, and those in the system with $q_0 + \Delta q$ by $E_l'(s_l', \Pi_l')$ and $E_h'(s_h', \Pi_h')$. The $\dot{\Pi}_t^e$ given $q_0 + \Delta q$, denoted by $\dot{\Pi}_t^e(q_0 + \Delta q)$, is smaller (greater) than the $\dot{\Pi}_t^e(q_0)$ for any $s_t > s_m$ ($s_t < s_m$), because $\dot{\Pi}_t^e(q_0 + \Delta q)$ is, using equation (9):

$$\begin{aligned}
\dot{\Pi}_t^e(q_0 + \Delta q) &= (\rho+\alpha)\left[\Pi_t^e - \frac{(\bar{\delta} + \Delta\bar{\delta}) + f_0 + (q_0 + \Delta q)s_t}{\rho+\alpha}\right] \\
&= (\rho+\alpha)\left[\Pi_t^e - \frac{(\bar{\delta} - s_m\Delta q) + f_0 + (q_0 + \Delta q)s_t}{\rho+\alpha}\right] \\
&= \dot{\Pi}_t^e(q_0) - \Delta q(s_t - s_m) \qquad (15)
\end{aligned}$$

There always exist $s_c \in (s_m, s_h)$ such that the optimistic path to E_h in the dynamic system with q_0 intercepts the $\dot{\Pi}_t^e = 0$ locus in the dynamic system with $q_0 + \Delta q$ at $s_t = s_c$. Let us denote Π_t^{op} on the optimistic path in the dynamic system with q_0 ($q_0 + \Delta q$) by C (C') at $s_t = s_c$, A (A') at $s_t = 1$, and B (B') at $s_t = s_m$. First, it is obvious that, over the range $[s_c, s_h)$, the optimistic path to E_h' with $q_0 + \Delta q$ is above the $\dot{\Pi}_t^e = 0$ locus with $q_0 + \Delta q$, and the optimistic path to E_h with q_0 is below the $\dot{\Pi}_t^e = 0$ locus with $q_0 + \Delta q$. Secondly, we show that the optimistic path to E_h' is above the optimistic path to E_h over the range $[s_h, 1]$. Consider a start point at $s_t = 1$: $(1, x)$. In order that the path from the point in the dynamic system with $q_0 + \Delta q$ passes through E_h, x should be greater than A because the state moves down faster with $\dot{\Pi}_t^e(q_0 + \Delta q) < \dot{\Pi}_t^e(q_0)$. A' should be greater than x because the path from the point $(1, A')$ approaches E_h', in which $s_h' > s_h$. Therefore, we have $A' > x > A$. The same logic can be applied for any s_t over the range

$[s_h, 1]$. Thus, the optimistic path to E_h' is above the optimistic path to E_h at any $s_t \in [s_h, 1]$. Lastly, we show that the optimistic path to E_h' is above the optimistic path to E_h over the range $[s_m, s_c)$. Consider a start point at $s_t = s_m$: (s_m, y). In order that the path from the point in the dynamic system with $q_0 + \Delta q$ passes through C, y should be greater than B because the state moves up more slowly with $\dot{\Pi}_t^e(q_0 + \Delta q) < \dot{\Pi}_t^e(q_0)$. B' should be greater than y because the path from the point (s_m, B') approaches (s_c, C') and $C' > C$. Therefore, we have $B' > y > B$. The same logic can be applied for any s_t in the range $[s_m, s_c)$. In sum, we conclude that the optimistic path to E_h' with $q_0 + \Delta q$ is above the path to E_h with q_0 for any $s_t \geq s_m$. In a symmetric way, we can show that the pessimistic path to E_l' with $q_0 + \Delta q$ is below the path to E_l with q_0 for any $s_t \leq s_m$. ■

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Figure 1. Steady States with Unique Ability Level

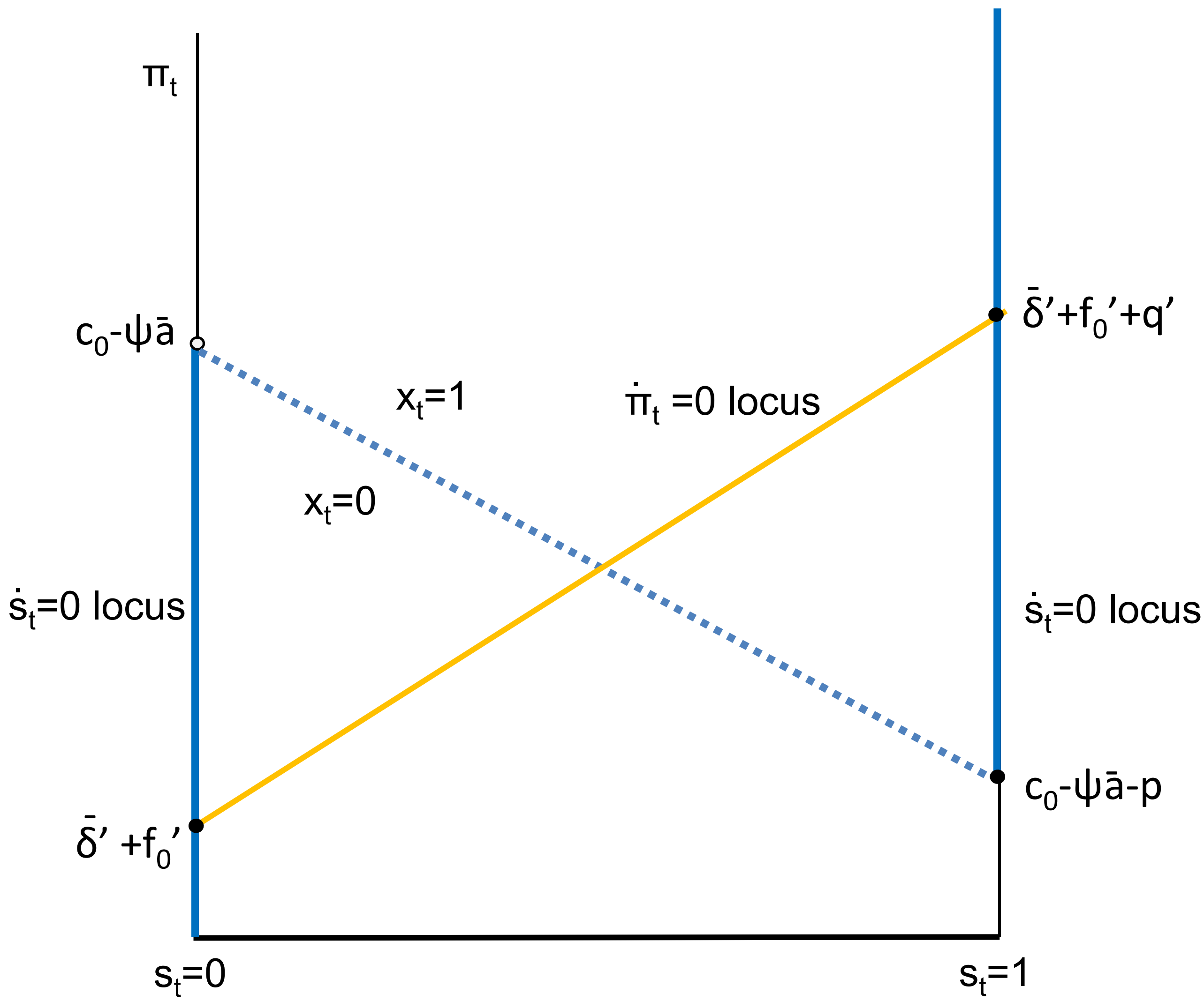
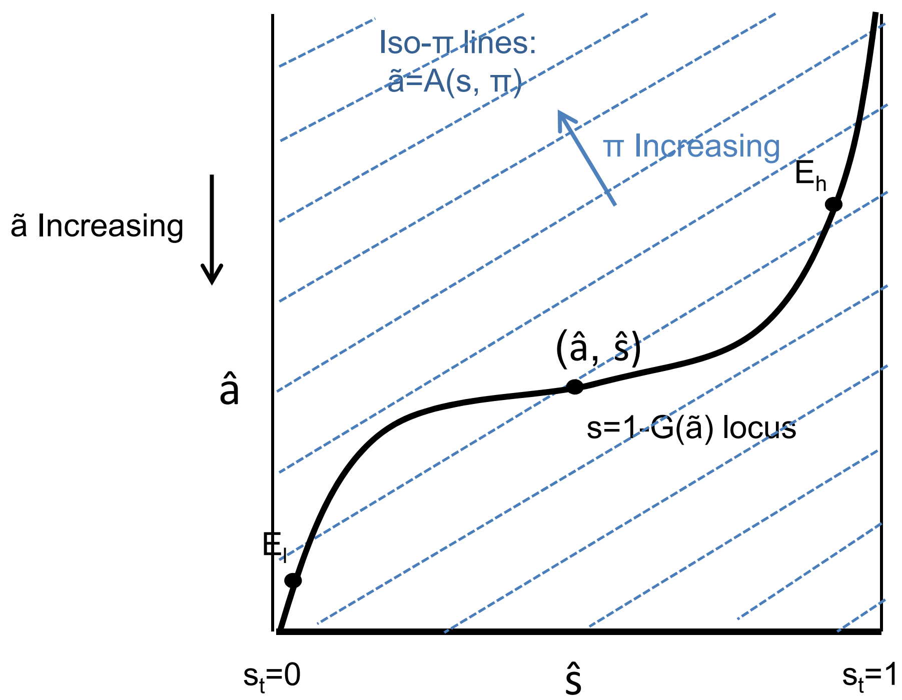


Figure 2. Steady States with Ability Distribution G(a)

Panel A Finding the $\dot{s}_t$=0 Locus



Panel B Steady States with the $\dot{s}_t$=0 and $\dot{\pi}_t$=0 Loci

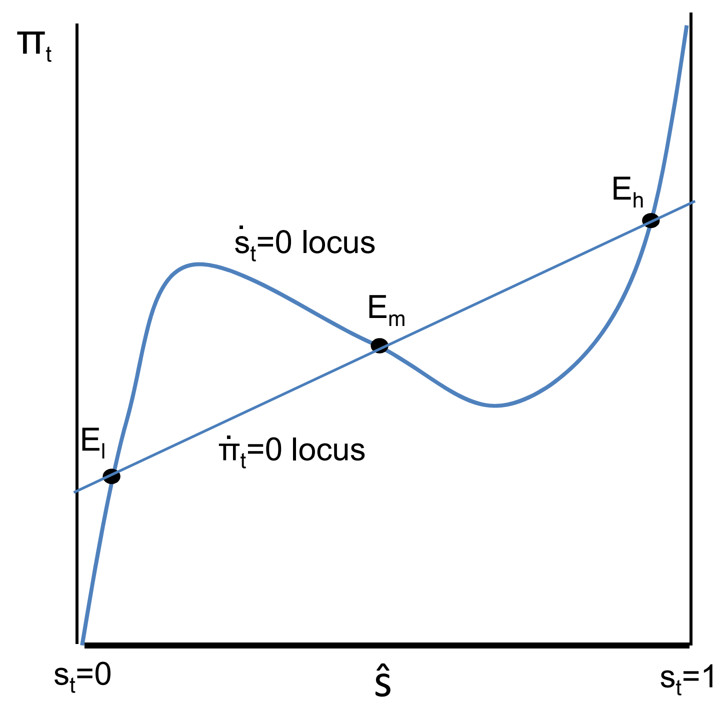


Figure 3. Equilibrium Paths with Unique Ability Level

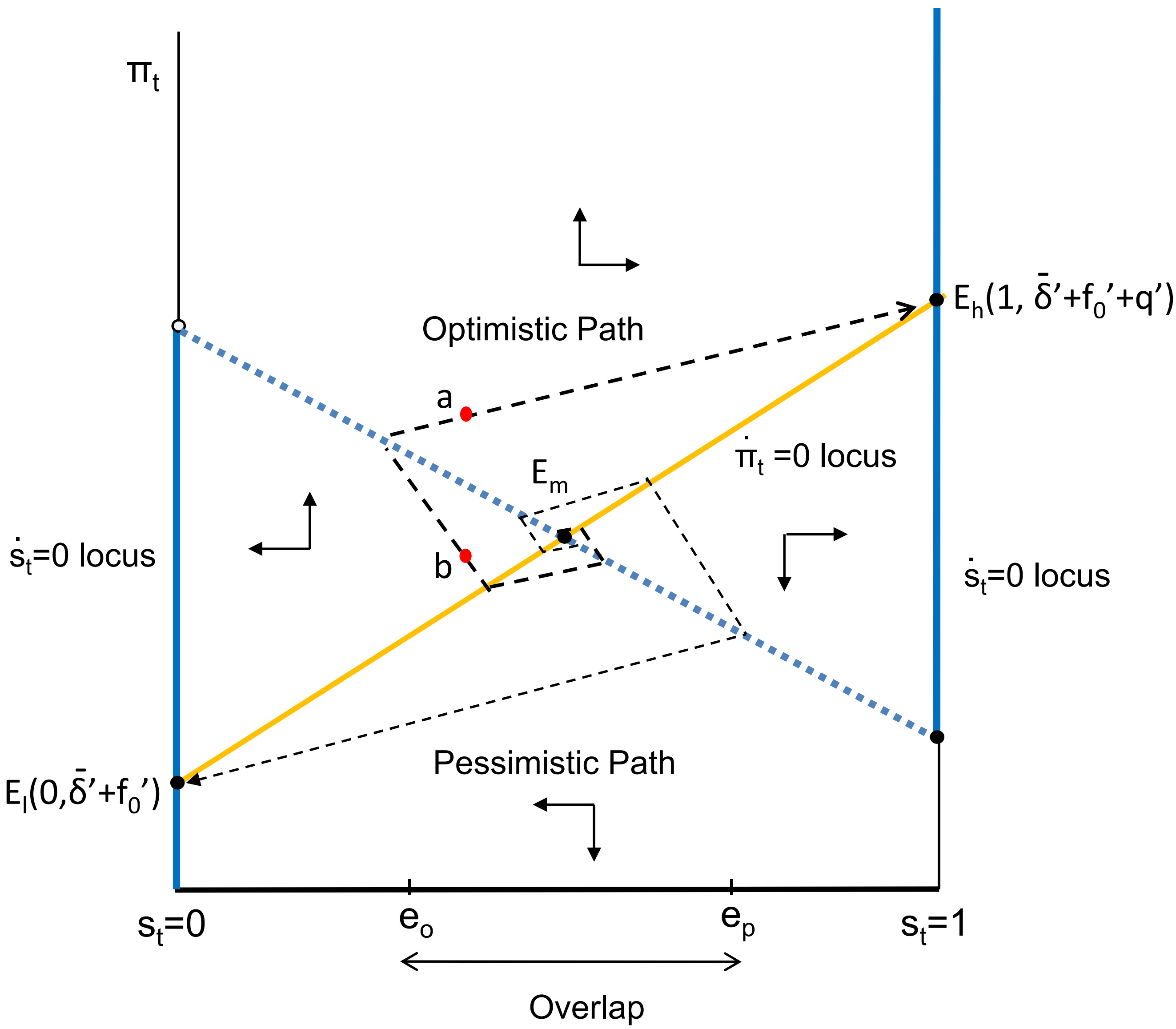


Figure 4. Equilibrium Paths with Ability Distribution G(a)

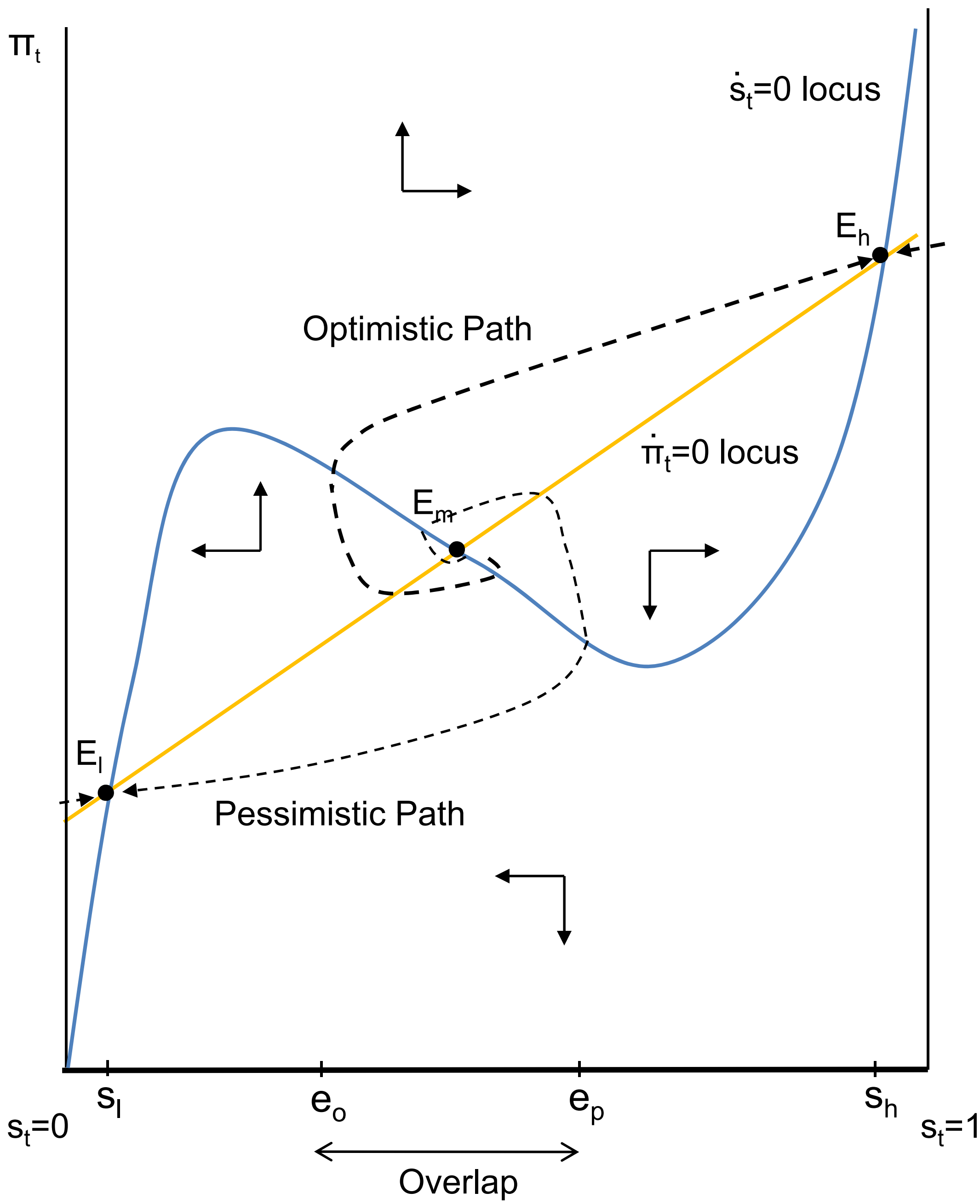


Figure 5. Different Levels of Working-period Network Externalities