Money Demand and Inflation: A Cointegration Analysis for Canada

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Abstract

The aim of this paper is to investigate the presence of long-run equilibrium relationships among variables that explain money demand in Canada during the period 1983–2011. To this end, I set up a vector-error correction model with an appropriate lag order and test for cointegration by means of the Bartlett corrected trace test. I estimate the long-run money demand parameters by means of the maximum likelihood method of Johansen, comparing an unconstrained benchmark model against a constrained model. I find the latter to not be better than the benchmark one. Finally, I perform sensitivity analysis and check the stability of the resulting cointegration relationships.

JEL classification: C3; E31; E41

Keywords: Canada, cointegration, money demand, vector autoregression, VECM

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1 Theoretical background

This empirical work aims at investigating the presence of cointegrating relationships among the variables that explain money demand in Canada. Cointegration arises when two non–stationary series have in common the same stochastic trend. In particular, let us consider two series, Y_t and X_t, integrated of order one and suppose that a linear relation between them exists. If there exists some value of β such that $Y_t - \beta X_t$ is integrated of order zero, then it is said that Y_t and X_t are cointegrated and that they share a common trend. In this work, I will explore whether and to which extend such a relation exists between the determinants of Canadian money demand. The set of variables considered are:

m_t: log of real M1 money balances
$infl_t$: quarterly inflation rate (in % per year)
cpr_t: commercial paper rate
y_t: log real GDP (in billions of 2002 dollars)
trb_t: treasury bill rate.

The commercial paper rate and the treasury bill rate are considered as risky and risk-free returns, respectively. The series for M1 and GDP are seasonally adjusted.

In principle, we could dispute the presence of a unit root in some of these series. As customary in the literature, I assume that these variables are well described by an I(1) process. One can think of three possible cointegrating relationships governing the long-run behavior of these variables. First, we can specify money demand as

$$m_t = \alpha_1 + \beta_{14} y_t + \beta_{15} tbr_t + \varepsilon_{1t} \tag{1}$$

where β_{14} denotes the income elasticity and β_{15} the interest rate elasticity. It can be expected that it is close to unity, corresponding to a unitary income elasticity, and that $\beta_{15} < 0$. Second, if real interest rate is stationary we can express an equation for inflation

$$infl_t = \alpha_2 + \beta_{25} tbr_t + \varepsilon_{2t} \tag{2}$$

which corresponds to a cointegrating relationship with $\beta_{25} = 1$. This is referred to as the Fisher equation where we are using actual inflation as a proxy for expected inflation. Third, we can expect the risk-premium to be stationary so that a third cointegrating relationship can be expressed as

$$cpr_t = \alpha_3 + \beta_{35} tbr_t + \varepsilon_{3t} \tag{3}$$

with $\beta_{35} = 1$.

2 Empirical analysis

The data have been collected on a quarterly horizon for the period ranging between June 1983:2 to March 2011:2. The total number of observation equals 112. [1] Relaying on the literature we can expect our variables to be stationary, but following Hoffman and Rasche (1996) let us assume for them to be I(1) and therefore that they turn to be stationary after taking their first differences. The more standard way to test the presence of unit root is the Dicky-Fuller test and its augmented version. For comparison, in Table (1) are reported these tests performed on the original series and in Table (2) the same analysis performed on the their first differences.

Before proceeding to the vector error-correction process of these five variables, let me consider the OLS estimates of the above three regressions, which

[1] The data have been collected from the National Bureau Statistics of Canada: http://www.statcan.gc.ca/

Table 1: Dicky-Fuller test I(1)

	m_t	$infl_t$	cpr_t	y_t	tbr_t
DF	-1.241	-3.163 **	-1.163	-2.018	-0.904
ADF (6)	-0.117	-1.993	-1.993	-0.547	-1.638

*** 1%, ** 5%, * 10%

Table 2: Dicky-Fuller Test First Difference

	m_t	$infl_t$	cpr_t	y_t	tbr_t
DF	-6.671 ***	-10.689 ***	-9.971 ***	-5.545 ***	-8.422 ***
ADF (6)	-3.137 **	-3.917 ***	-3.955 ***	-3.273 **	-4.443 ***

are presented in Table (3) and show that with the exception for the Fisher equation, both money demand and risk-premium have an R^2 close to unity which is an informal requirement for a cointegration relationship. In addition, we can test for cointegration by means of the usual Durbin–Watson statistic. Under the null hypothesis of a unit root, the appropriate test is wether DW is significantly larger than zero (Verbeek, 2004). Given the standards critical values, we should reject the null hypothesis for risk-premium and Fisher equations but not for money demand. However, these results have to be considered only partially reliable since the test is based on the random walk assumption for the data generating process of all series, that is not the case for the GDP and money supply which are clearly trended-series. Nevertheless, the value of the Durbin–Watson statistics is often useful to obtain a general idea about wether or not a cointegrating relationship might exist.

We can also test for a unit root in the residuals of the regressions by means of the augmented Dickey-Fuller tests. The results, for 6 lags, are also reported in Table (3).

Table 3: Univariate cointegrating regressions by OLS (standard errors in parentheses), intercept estimates not reported.

	Money Demand	Fisher Equation	Risk-premium
m_t	-1	0	0
$infl_t$	0	-1	0
cpr_t	0	0	-1
y_t	2.727 (0.062)	0	0
tbr_t	-2.822 (0.399)	0.323 (0.029)	0.974 (0.011)
R^2	0.9868	0.527	0.986
dw	0.108	0.577	1.770
ADF (6)	-1.988	-2.376	-4.100

Given the 5% asymptotic critical values of -3.37 and -3.77 for the regression involving three and two variables respectively, we can reject the null hypothesis of no cointegration only for the risk-premium equation. Given the results obtained up to now, it is not that straightforward to state something about the presence of cointegration relationships between our variables. Recalling that cointegration requires R^2 values close to unity, high values of the DW test and no serial correlation in regression residuals, our results show that these requirements are only partially fulfilled, and even when they are it is not the case always for the same variables at the same time, providing results that are therefore mixed. In fact, summing up we obtain R^2 values close to unity for money demand and risk-premium equations and sufficiently high DW and ADF values only for the risk-premium equation. In order to get additional information it may be useful to plot the residuals of the three regressions. We can interpret cointegration as long-run stable equilibrium, so that regression residuals should therefore look like a stationary mean reverting process fluctuating around zero. The residuals of the three regressions are displayed in Fig.(1), Fig.(2) and Fig.(3), respectively. However, in our case we can observe by visual inspection only some evidence of mean reversion for the Fisher equation and the risk-premium equation, but there is less evidence for money demand.

Figure 1: Residuals of money demand regression

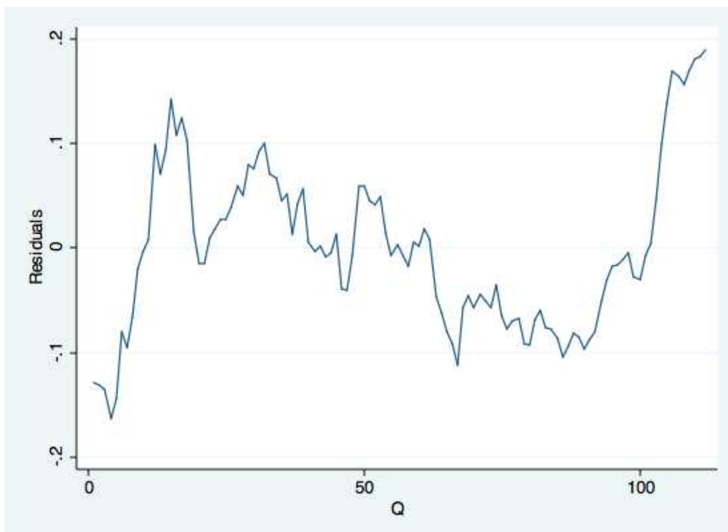


Figure 2: Residuals of Fisher regression

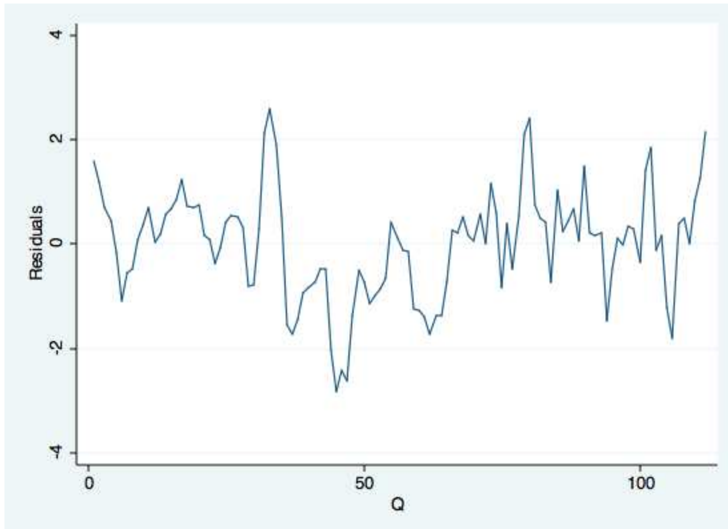
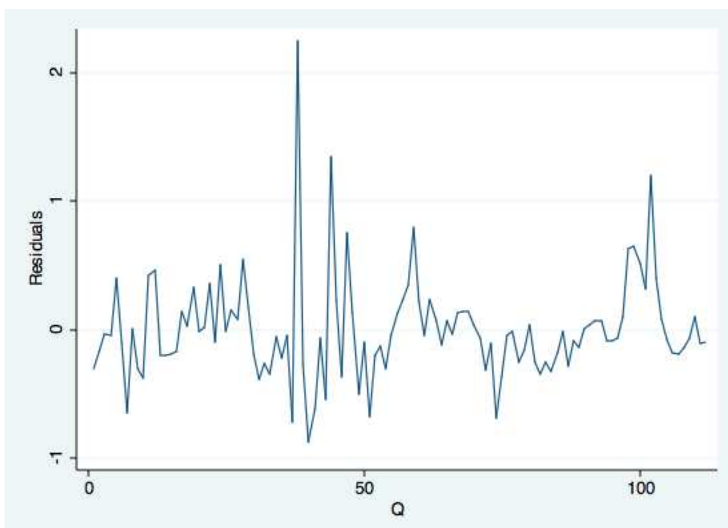


Figure 3: Residuals of risk-premium regression



The first step in the Johansen approach is to test for the cointegration rank r. It means that we want to estimate the number of cointegrating relationships which may exist in our five-dimensional vector process. In order to compute this tests, we first need to choose how many lags (lags length p) to be used in the vector error-correction model. This can be obtained by means of a specific test, which provide the lag-order selection statistics for our VECM, at a pre-estimation level. As it can be seen in Table (4), the Akaike information criterion, the Hannan-Quinn information criterion and the final prediction error jointly suggest to select as optimal lag-order $p = 2$. However, the existing literature suggest that choosing p too small will invalidate results and choosing p too high may results in loss of power (Veerbeek, 2004). For this reasons, I found it more optimal to choose a value of $p = 4$ [2].

Table 4: Lag-Order Selection Statistics

Lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	1138.73				1.5e-16	-22.2301	-22.178	-22.1014
1	1812.65	1347.8	25	0.000	4.5e-22	-34.9539	-34.6413	-34.1818*
2	1859.32	93.337	25	0.000	3.0e-22*	-35.3788*	-34.8056*	-33.9633
3	1869.51	20.382	25	0.727	4.0e-22	-35.0884	-34.2547	-33.0296
4	1890.93	42.853	25	0.015	4.4e-22	-35.0183	-33.9241	-32.3162
5	1924.37	66.864	25	0.000	3.8e-22	-35.1837	-33.8289	-31.8381
6	1946.17	43.608	25	0.012	4.2e-22	-35.121	-33.5057	-31.1321
7	1974.44	56.545	25	0.000	4.2e-22	-35.1852	-33.3094	-30.5529
8	1995.26	41.626	25	0.020	5.0e-22	-35.1031	-32.9668	-29.8274
9	2022.64	54.772	25	0.01	5.3e-22	-35.1499	-32.753	-29.2308
10	2048.43	51.574*	25	0.01	6.0e-22	-35.1653	-32.5079	-28.6028

Endogenous: cpr infl tbr logdp logm1
Exogenous: _cons
Maxlag(10)

After having determined the optimal lags length $p = 4$, I proceed to the determination of the number of cointegrating relationships to be used for the error-

[2] Analysis performed by using $p = 2$ displayed highly correlated residuals. I then have selected $p = 4$, which is closer to the value chosen in literature for the same typology of data and analysis.

correction model estimation. The tests for cointegration are based on Johansen's method. If the log likelihood of the unconstrained model that includes the cointegrating equations is significantly different from the log likelihood of the constrained model that does not include the cointegrating equations, we reject the null hypothesis of no cointegration. Results are provided in Table (5) indicating the presence of two cointegrating relationships.

Table 5: Johansen's Cointegration Test

Max Rank	Parms	LL	Eigenvalue	Trace Statistic	5% Critic. Value
0	105	-1456.567	.	105.1762	68.52
1	114	-1432.6779	0.36015	57.3981	47.21
2	121	-1418.598	0.23139	29.2383 *	29.68
3	126	-1411.9966	0.11608	16.0353	15.41
4	129	-1406.8365	0.09194	5.7153	3.76
5	130	-1403.9789	0.05201	-	-

To identify individual cointegrating relationships I need to normalize the cointegrating vectors. Since $r = 2$, I need to impose two normalization constraints on each cointegrating vector. In this case, I impose m_t and cpr_t to have coefficients of -1, 0 and 0, -1, respectively in each constraint. I shall estimate the cointegrating vectors by maximum likelihood jointly with the coefficients in the vector error-correction model, which takes the following general form:

$$ecm_i = \alpha_0 + \beta_{i1} m_t + \beta_{i2} infl_t + \beta_{i3} cpr_t + \beta_{i4} y_t + \beta_{i5} tbr_t \quad (4)$$

With the restrictions imposed above we therefore have: $\beta_{11} = -1$, $\beta_{13} = 0$ and $\beta_{21} = 0$, $\beta_{23} = -1$. After estimation, I get the results reported in Table (6).

Table 6: ML estimates of cointegrating vectors (after normalization) based on VAR with $p = 4$ (standard errors in parentheses), intercept estimates not reported.

	Money demand	Risk-premium
m_t	-1	0
$infl_t$	22.283 (5.638)	0.238 (0.108)
cpr_t	0	-1
y_t	0.627 (0.477)	-0.292 (0.009)
tbr_t	-22.576 (3.939)	0.710 (0.076)

Log likelihood value: 1975.709

For a fair interpretation of these results, and in order to determine which variables enter actually the equations and which ones do not, I need to check the significance of their coefficients. It is possible to do this by means of the t-statistics. They are reported in Table (7). With regards to the cointegrating vector corresponding to the risk premium equation, I reject the null hypothesis for for all coefficients to be zero, and they are therefore significantly different from zero. This means that all the variables involved in the risk premium relation are contributing significantly in determining that relation. In addition, I also reject the null hypothesis for the treasury bills rate coefficient to be equal to one, so that our a priori expectations (see Table 3) are not confirmed. Regarding the cointegrating vector corresponding to the money demand equation, I cannot reject the null hypothesis only for the output coefficient. Therefore, both inflation and treasury bills rate enter significantly into the money demand equation. This contradicts our a priori expectations (again Table 3) where inflation is not entering the equation.

Table 7: T-Statistics based on Table (6)

	Money demand	Risk premium
m_t	-	-
$infl_t$	3.952	2.204
cpr_t	-	-
y_t	1.314	-32.444
tbr_t	-5.731	9.342

I can also test our a priori cointegrating vectors by using likelihood ratio tests and by imposing additional constraints on the cointegration vectors, which are:

$H_0^a : \beta_{12} = 0, \beta_{14} = 1;$
$H_0^{\mathrm{b}} : \beta_{22} = \beta_{24} = 0, \beta_{25} = 1;$
$H_0^{\mathrm{c}} : \beta_{12} = \beta_{22} = \beta_{24} = 0, \beta_{14} = \beta_{25} = 1,$

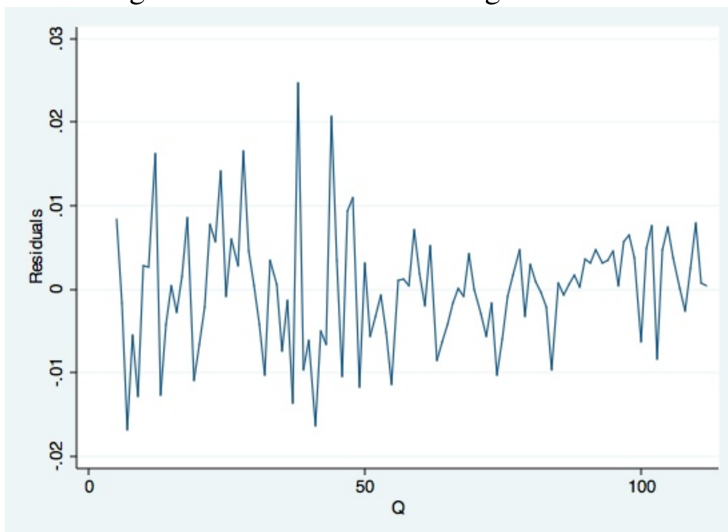
The model is estimated testing all three hypotheses. In Table (8) the likelihood values and the likelihood ratio tests for each case are reported.

Table 8: ML estimates for the complete a priori model

	Log-likelihood values	Likelihood Ratio Tests
H_0^a	1970.345	10.728
H_0^b	1969.139	13.14
H_0^c	1971.329	8.76

The likelihood ratios are defined as twice the difference between the Likelihood of the estimated model (constrained) and the benchmark (unconstrained) one. The asymptotic distribution under the null hypotheses are the Chi-squared

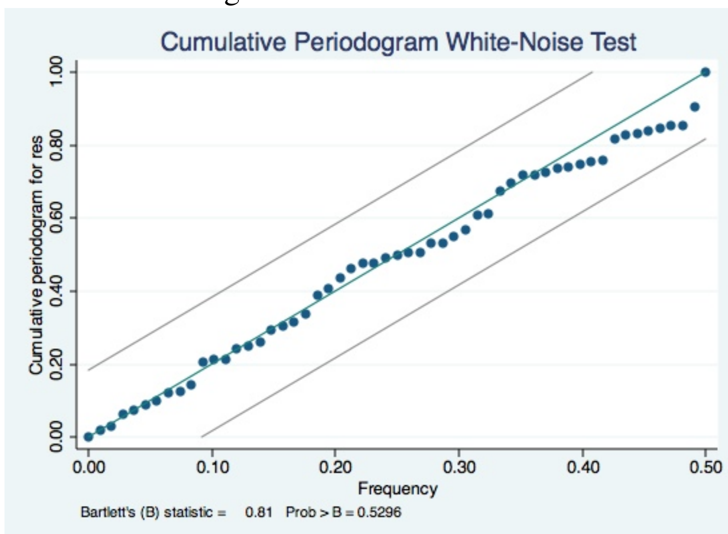
Figure 4: Residuals of the Original Model



distributions with degrees of freedom given by the number of restrictions that I have imposed. In the present case, I imposed two restrictions on coefficients in H_0^a , three in H_0^b and five in H_0^c. The Chi-squared critical values for 2, 3 and 5 degrees of freedom are 5.991, 7.815 and 11.070, respectively and therefore I can only reject H_0^c meaning that the unconstrained model is as good as the unconstrained one and not better. In order to evaluate the original model, I can perform additional residual analysis and plotted in Fig. (4) .

In Fig. (5) it is shown the periodgramm as a result of the Bartlett's test, which tests the null hypothesis that the data come from a white-noise process of uncorrelated random variables having a constant mean and a constant variance. I can see in the graph below that the values never appear outside the confidence bands. The test statistic has a p-value of 0.81, so I conclude that the process is not different from a white noise.

Figure 5: White Noise Test



In addition, I perform a test for autocorrelation in the residuals of vector error-correction model. The test implements Lagrange-multiplier (LM) test for autocorrelation in the residuals of vector error-correction model where for each lag j the null hypothesis of the test is that there is no autocorrelation at a specific lag j. From results, reported in Table (9), it can be realized that we cannot reject the null hypothesis of no autocorrelation from the second lag toward, but in the first lag the serial correlation is statistically significant. At this point, in order to determine whether this autocorrelation coefficient is statistically close to 1, I can regress residuals on lags. Results show a coefficient equal to 0.0057 and a standard error of 0.0969, so that calculating the appropriate t-ratio it turns out to be different then 1.

Table 9: Lagrange Multipliers Test

Lag	Chi2	df	Prob>Chi2
1	55.0209	25	0.00049
2	21.2437	25	0.67895
3	31.3888	25	0.17646
4	52.3511	25	0.00108
5	26.4747	25	0.38263
6	40.6415	25	0.02503
7	24.9622	25	0.46450
8	22.0622	25	0.63215
9	19.7791	25	0.75831
10	23.5782	25	0.54384
11	31.8810	25	0.16148
12	30.2011	25	0.21685

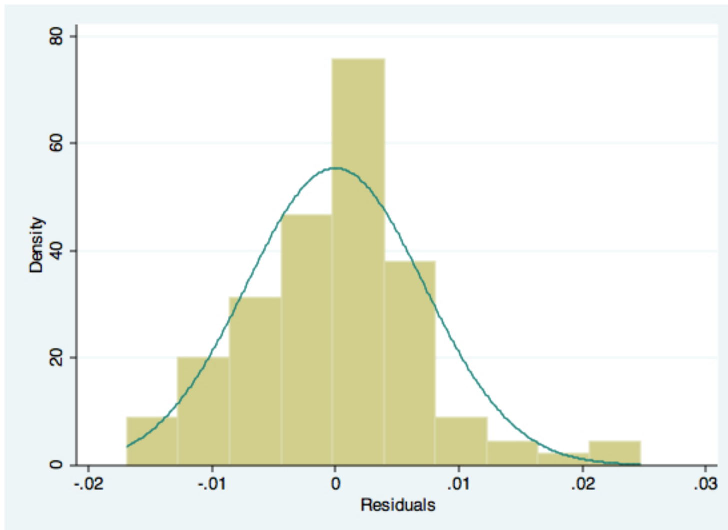
Some consideration may be done about the stability conditions of VECM estimates. I perform a test that provides indicators of whether the number of cointegrating equations is misspecified or whether the cointegrating equations, which are assumed to be stationary, are not stationary. From the test I expect to obtain that the number of eigenvalues having unit moduli is equal to $K - r$, that is the difference between the number of variable K and the number of cointegrating relationships r. The results of this analysis reported in Table (10) confirm these expectations.

Table 10: Eigenvalue Stability Condition

Eigenvalue	Modulus
1	1
1	1
1	1
0.7529544 + 0.2866931i	0.805688
0.7529544 - 0.2866931i	0.805688
0.756527	0.756527
0.6470258 + 0.3871149i	0.75399
0.6470258 - 0.3871149i	0.75399
-0.4473219 + 0.5838522i	0.735514
-0.4473219 - 0.5838522i	0.735514
-0.1979945 + 0.5781694i	0.611131
-0.1979945 - 0.5781694i	0.611131
0.6047231	0.604723
0.02288368 + .5999055i	0.600342
0.02288368 - 0.5999055i	0.600342
-0.357975 + 0.3968059i	0.534416
-.357975 - .3968059i	0.534416
-.1759097 + .2917176i	0.340651
-.1759097 - .2917176i	0.340651
-.1168251	0.116825

Moreover, I perform a further test to check normality of residuals which involves Jarque-Bera, Skewness and Kurtosis tests. For a first visual inspection about these statistical properties, it would be useful to have a look to Fig. (6) where the empirical probability distribution function (PDF) of residuals is plotted against the normal probability density function.

Figure 6: Empirical PDF against (histograms) Normal Distribution (solid line)



The tests results are instead reported in Tables (11), (12) and (13). They show how visual inspection-based tests may be very misleading. Despite a good result of the white noise test, these tests are now showing that only for some variables the null hypothesis of normality of residuals is not rejected. Looking at the Jarque–Bera, skewness and kurtosis tests in fact, the null hypothesis is not rejected only for inflation and the (log) GDP and the therefore the hypothesis of normality holds only for the commercial paper rate, the treasury bills rate and the (log) money balances. In addition, considering all variables jointly, the overall p-value is too low so that we reject the hypothesis of normality in residuals, which indicates a low power of the model that I have been considering up to now.

Table 11: Jarque-Bera Test

Equation	Chi2	df	Prob>Chi2
D_cpr	7.624	2	0.02210
D_inf	0.474	2	0.78897
D_tbr	51.747	2	0.0000
D_logdp	2.513	2	0.28466
D_logm1	145.042	2	0.0000
ALL	207.400	10	0.0000

Table 12: Skewness Test

Equation	Skewness	Chi2	df	Prob>Chi2
D_cpr	0.34818	2.182	1	0.13962
D_inf	0.09653	0.168	1	0.68214
D_tbr	-0.36305	19.137	1	0.0001
D_logdp	0.56294	2.373	1	0.12349
D_logm1		5.704	1	0.01692
ALL		29.564	5	0.0002

Table 13: Kurtosis Test

Equation	Kurtosis	Chi2	df	Prob>Chi2
D_cpr	4.0997	5.442	1	0.001965
D_inf	3.2609	0.306	1	0.57993
D_tbr	5.692	32.610	1	0.0000
D_logdp	3.1766	0.140	1	0.70900
D_logm1	8.5645	139.337	1	0.0000
ALL		177.836	5	0.0000

Let us consider now the vector error-correction model for this system. From Table (6) we can write the error-correction terms as follows:

$$ecm_1 = -m_t + 22.283\, infl_t + 0.627\, y_t - 22.576\, tbr_t + 4.121;$$

$$ecm_2 = -cpr_t + 0.238\, infl_t - 0.292\, y_t + 0.710\, tbr_t + 0.414;$$

and in Table (14) the estimated adjustment coefficients matrix of the VECM and their associated standards errors are reported.

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Table 14: Estimated matrix of adjustment coefficients (standard errors in parentheses), * indicates 5% significance level.

Equation	$ecm1_{t-1}$	$ecm2_{t-2}$
Δm_t	0.0111 (0.0144)	0.7768 (0.8599)
$\Delta infl_t$	-0.0229 (0.0062) *	0.6465 (0.3692)
Δcpr_t	0.0018 (0.0064)	0.1390 (0.3801)
Δy_t	-0.0092 (0.0044) *	0.9331 (0.2624) *
Δtbr_t	0.0078 (0.0045)	0.0045 (0.2689)

The meaning of these coefficients is that that they represent at which rate errors change to bring the system back to the long-run equilibrium. The long-run money demand appears to be significantly affected by inflation and income. The latter is also the only one variable that significantly affects the long-run risk premium relationship.

Conclusions

The objective of this empirical work has been to investigate the presence of cointegrating relationships among the variables that explain Canadian money demand. The reason of doing this is that cointegration in multivariate time series models may bring significant improvements in forecasting, as their analysis allow the researcher to discover relations with stationary properties as a result of non-stationary time series interactions.

The analysis have been carried out on quarterly data covering a time range going from June 1983:2 to March 2011:2. The first step of these analysis was to identify the optimal lag p in order to determine the number r of cointegrating relationships involved in the variables, determining $p = 4$ and $r = 2$. With these values, I have estimated the model setting some theoretical assumptions useful to express a theoretical (and a priori) relation for money demand, inflation and risk premium. This (unconstrained) model has been considered as a benchmark against which other models –constrained on purpose by means of different restrictions imposed on coefficients in some variables. The results obtained by means of maximum likelihood estimation have shown that no constrained model is better then the unconstrained one. Therefore, I considered the unconstrained model and evaluated its goodness by means of normality tests, a white noise test, a Lagrange Multipliers and a serial correlation test performed on residuals. Results have shown that residuals are not significantly different from a white noise process. However, the null hypothesis of normally distributed residuals is rejected. These results invalidate the power of the model in describing the system.

Finally, the sensitivity tests have shown that the long-run money demand is significantly affected both by inflation and income, while the risk-premium relationship is affected only by income.

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