Government Expenditure Determination on the Basis of Macroeconomics

durongkaveroj, wannaphong

chiang mai university

4 April 2014
Government Expenditure Determination on the Basis of Macroeconomics

Wannaphong Durongkaveroj

April 2014

Abstract

Government expenditure is realized to be the exogenous variable and its change impacts national income through Aggregate Demand expression. The purpose of this study is to derive new macroeconomic expression based on Keynesian basis with SAM multiplier through mathematical approach. The study reveals that there are factors determined government spending including exogenous shock (government subsidy), taxation, and money supply.

JEL classification: C02, E00, E10, E16
Keywords: government expenditure, macroeconomics
Introduction

According to Aggregate Demand (A.D.) expression implemented by Romer (1987), Sargent (1987), Blanchard & Fischer (1989), Turnovsky (2000), Ljungqvist & Sargent (2004), Wickens (2008), and Mankiw (2010), the existing strand of economics is fully completed by the basis of mathematics by many authors, for example, Henderson (1980), Simon & Blume (1994), Sydsaeter & Storm (1999), and Snyder and Nicholson (2010). For the intuition of Keynesian economics, demand side of country becomes the main economic stimulator. Aggregate Demand can be mathematically expressed as written:

\[ dY = \frac{dG + \frac{I_R}{P} \left( \frac{dM}{M} - \frac{dP}{P} \right)}{\left( 1 - C_{Y,T}(1 - T_r) + \frac{b}{I_R} \right)} \]

From expression above, an increase in price reduces real value of money balances \( \frac{M}{P} \). From money market equilibrium \( \frac{M}{P} = \lambda(Y, R) \) which states the demand for money is equal to supply of money. Relationship between income and money demand is positive while relationship between income and interest rate is negative. When there is an increase in income, demand for money goes up because people normally consume more of commodity and they need money to be the device in buying. When there is an increase in interest rate (said interest rate at bank), people use money for speculative prospect. A higher interest rate offers a good opportunity to everyone to enjoy the reward of saving. Then, people normally save more when interest rate increases. Money demand get reduced. From this relationship, real value of money balances is kept for the same concept of money demand. Thus, a decrease in real value of money balances can be incurred to an increase in interest rate. However, interest is not only the reward of saving but also the cost of borrowing for investing. When interest rate increase, cost of investment goes up and the amount demanded of investment is reduced. It is why A.D. curve is downward sloping due to the long and complex relationship between price and marketed commodity.

To make a shift in A.D. curve, it requires the changes in I.S. and L.M. curve which are fiscal and monetary policy including a policy on tax, government expenditure, and money supply. Besides Keynesian basis of macroeconomics, Social Accounting Matrices (SAM) became another popular framework aimed at considering economy-wide system. Pyatt & Round (1979) and Round (2003) proposed the standard of intuition in this area. The mainstream of SAM is its multiplier which stand for the total impact to economy as a change of exogenous macroeconomic shock.

From the basic duty of government argued by Smith (1776), we, economists, take much effort to try to understand more about sources and uses of government. According to Hyman (2012), main source of government revenue is tax, especially individual income tax. However, the distribution of its revenue is still questionable. Unfortunately, macroeconomic expression explaining government expenditure is limited to only G variable standing for an abstract government spending.
Objective
Present new macroeconomic expression aimed at explaining government determination or spending through the basic mathematics approach.

Methodology
Macroeconomic expression is derived through mathematic approaches and descriptive analysis for its implementation

Result
According to closed economy model
\[ Y = C + I + G \]  \hspace{1cm} (1)
where \( Y \) is national income, \( I \) is level of investment, and \( G \) is government expenditure. IS and LM equation will be derived.
From money market, there are demand for and supply of money. The standard expression is
\[ \frac{M}{P} = l(Y, R) \]  \hspace{1cm} (2)
For (2), demand for money is equal to supply of money. After taking differential,
\[ d \left( \frac{M}{P} \right) = \frac{\partial l}{\partial Y} dY + \frac{\partial l}{\partial R} dR \]  \hspace{1cm} (3)
Rearrange (3),
\[ d \left( \frac{M}{P} \right) - l_Y dY = l_R dR \]  \hspace{1cm} (4)
By letting \( l_Y \) stands for \( \frac{\partial l}{\partial Y} \) and \( l_R \) stands for \( \frac{\partial l}{\partial R} \). Then, rearrange (4) again to get LM expression
\[ dR = \frac{d \left( \frac{M}{P} \right) - l_Y dY}{l_R} \]  \hspace{1cm} (5)
Expression (5) is LM expression. For IS expression, it is presented by national income identity.
\[ Y = C(Y - T)_Y + I_R + G \]  \hspace{1cm} (6)
Denote \( Y - T_Y \) equals to \( A \)
\[ Y = C(A) + I_R + G \]  \hspace{1cm} (7)
Let's taking first differential on eq. (7),
\[ dY = \frac{\partial C}{\partial A} dA + \frac{\partial I}{\partial Y} dY + \frac{\partial I}{\partial R} dR + dG \]  \hspace{1cm} (8)
Rearrange eq. (8),
\[ dY - C_{Y \rightarrow T}(1 - T_Y) dY = I_R dR + dG \]  \hspace{1cm} (9)
Then,
\[ dY = \frac{I_R dR + dG}{1 - C_{Y \rightarrow T}(1 - T_Y)} \]  \hspace{1cm} (10)
Expression (10) is IS equation. LM expression represents the equilibrium in money market while LS expression represents the equilibrium in commodity market. Then, Aggregate Demand expression captures the equilibrium in money market and commodity market.
Substitute eq. (5) in eq. (10),
\[
\begin{align*}
\frac{dY}{dy} &= \frac{l_R \left( d \left( \frac{M}{P} \right) - l_Y dY \right)}{1 - C_{Y-T}(1 - T_Y)} + dG \\
(1 - C_{Y-T}(1 - T_Y))dY &= \frac{l_R}{l_R} d \left( \frac{M}{P} \right) - \frac{l_R}{l_R} l_Y dY + dG \\
(1 - C_{Y-T}(1 - T_Y))dY + \frac{l_R}{l_R} l_Y dY &= \frac{l_R}{l_R} d \left( \frac{M}{P} \right) + dG \\
\end{align*}
\]

from mathematic basic on differentials, \( d \left( \frac{\gamma}{g} \right) = \frac{\gamma df - \gamma dg}{g^2} \), this basis is used for
\[
\begin{align*}
\left[ 1 - C_{Y-T}(1 - T_Y) + \frac{l_R}{l_R} l_Y \right] dY &= \frac{l_R}{l_R} \left[ \frac{PdM - MdP}{P^2} \right] + dG \\
\left[ 1 - C_{Y-T}(1 - T_Y) + \frac{l_R}{l_R} l_Y \right] dY &= \frac{l_R}{l_R} \left[ \frac{PdM}{P^2} \cdot \frac{M}{M} - \frac{MdP}{P^2} \right] + dG \\
\end{align*}
\]

Multiplier \( \frac{M}{P} \) (exact to 1) to \( \frac{PdM}{P^2} \),
\[
\begin{align*}
\left[ 1 - C_{Y-T}(1 - T_Y) + \frac{l_R}{l_R} l_Y \right] dY &= \frac{l_R}{l_R} \left[ \frac{PdM}{P^2} \cdot \frac{M}{M} - \frac{MdP}{P^2} \right] + dG \\
\end{align*}
\]

Cancel out \( \frac{P}{P} \),
\[
\begin{align*}
\left[ 1 - C_{Y-T}(1 - T_Y) + \frac{l_R}{l_R} l_Y \right] dY &= \frac{l_R}{l_R} \left[ \frac{M}{P} \cdot \frac{dM}{M} - \frac{M}{P} \cdot \frac{dP}{P} \right] + dG \\
\end{align*}
\]

collect term \( \frac{M}{P} \),
\[
\begin{align*}
\left[ 1 - C_{Y-T}(1 - T_Y) + \frac{l_R}{l_R} l_Y \right] dY &= \frac{l_R}{l_R} \left[ \frac{M}{P} \left( \frac{dM}{M} - \frac{dP}{P} \right) \right] + dG \\
\end{align*}
\]

Finally, A.D. expression is derived as following,
\[
\begin{align*}
dY &= \frac{\frac{\frac{dM}{P}}{l_R} + \frac{dG}{l_R}}{1 - C_{Y-T}(1 - T_Y) + \frac{l_R}{l_R} l_Y} (11)
\end{align*}
\]

According to eq. (11), a change in money supply (M), government expenditure (G), and tax (T) can affect national income through a shift of A.D. curve.

After deriving A.D. expression, the next step is to link basic macroeconomic expression (eq. 11) with the intuition of Social Accounting Matrix,
From Pyatt & Round (1979),

$$dY = MdX$$  \hspace{1cm} (12)$$

where $dY$ is a change in income, $m$ is SAM multiplier, and $dX$ is exogenous change. Substitute eq. (12) in eq. (11),

$$mdX = \frac{IG[M/dM - dP]}{1-C_{Y-T}(1-T_Y) + \frac{IG}{i_R}I_Y} + dG$$  \hspace{1cm} (13)$$

Equation 13 tells us that a change in people’s income responses to the changes in government expenditure, taxation, and money supply (which can be injected to economy as a form of subsidy/grant).

This study aims at considering government determination, then, rearrange eq. 13,

$$dG = mdX \cdot \left[1 - C_{Y-T}(1 - T_Y) + \frac{IG}{i_R}I_Y \right] - \frac{IG}{i_R} \left[\frac{M}{M} \left(\frac{dM}{M} - \frac{dP}{P}\right)\right]$$  \hspace{1cm} (14)$$

For the role of tax, in equation 11, an increase in tax make the value of denominator higher and the value of $dY$ decreases. It is a general implication because a higher tax rate limits purchasing power when other variable constant. However, in equation 14, an increase in tax make the value of first term on the right hand side ($mdX \cdot \left[1 - C_{Y-T}(1 - T_Y) + \frac{IG}{i_R}I_Y \right]$) higher. Thus, $dG$ will increase. As you know that tax is a source of government revenue. When there is an increase in tax rate, government income rises. There are more funds available to use/finance national welfare program or other kind of spending.

For the role of constant SAM multiplier, it acts like coefficient in regression model. When there is an increase in exogenous macroeconomic shocks ($dX$), for example, subsidy of regional household income (grants) or subsidy to exporter, the first term on the right hand side increase which lead to an increase in government expenditure ($dG$).

For the role of money supply, money supply ($M$) is put to the second term on the right hans side. An increase in money supply causes a decrease in the first term which normally negatively affect government expenditure. For its intuition, when the monetary policy issued by central bank increases to economy as a whole, every sector in economy is encourage. This positive incidence is assumed to distribute to labor, owner of land, capital, and natural resources, owner of firm, producer of marketed commodity, and everyone. An increase in money in pocket normally make everyone better off. So, this scenario reduce "the need of incremental government spending". Simply put, the duty of government in taking care of its citizen is shared.

So, equation 14 make government account from exogenous variable like it was ever assumed by most of economist to endogenous account which means that government expenditure is dependent or responsible to three kinds of changes including exogenous macroeconomic shock (subsidy), taxation, and money supply.

**Conclusion and Critique**

This study mainly extend macroeconomic ideas through mathematical approach. Government is considered as endogenous variable and there are many factors in consideration including exogenous shock (government programs), taxation, and money supply. Government expenditure is positively related with a revenue from tax and government subsidy (called shock) while it is negatively relate with expanding monetary policy from central bank which reduce the need of government intervention. For three roles of factor, rational reason and explanation is crucial and requires the basic understanding of economy-wide progress.
To further study, I recommend the empirical study by exploring the evidence in many countries, especially developing countries which government expenditure become the main source of economic growth and share the main portion of political concern.

References