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Chen, Kaihua

February 2014

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Kaihua Chen[1]

Institution of Policy and Management, Chinese Academy of Sciences

No. 15 ZhongGuanCunBeiYitiao, Alley, Beijing 100190, People's Republic of China

Tel/Fax:+86-10-59358707; E-mail: chenkaihua@casipm.ac.cn

Mingting Kou

School of management, University of Chinese Academy of Sciences

No. 80 ZhongGuanCunDong, Road, Beijing 100190, People's Republic of China

Tel/Fax:+86-10-59358707; E-mail: koumingting@ucas.ac.cn

Abstract: The combination way of component efficiencies into the overall efficiency is a central topic in the efficiency modeling of network systems based on data envelopment analysis (DEA). In terms of the feature and advantage of DEA modeling as the multiplier generation on inputs/outputs, it is desirable that the combination weights are derived from the data and self-generated in calculation process. The prior weights choice makes DEA modeling lose the objectivity and generalization in efficiency measures. This study proposes a new formulating approach of dynamic network DEA (DN-DEA) models to measure and decompose the overall efficiency of multi-period and -division systems without the pre-specified weights to combine component efficiencies into the overall efficiency. In our formulating approach, the double identities of carry-overs connecting consecutive periods and linkers connecting consecutive divisions are fully accounted for. This approach is applicable for the formulations of both radial measures (DN-CCR and DN-BCC) and non-radial measures (DN-SBM). This study extends Kao's (in press) relational approach of dynamic DEA to dynamic network systems for empirical comparison. In contrast to Kao's (in press) approach, our approach can present a weighted average decomposition of the overall (in)efficiency score into components ones by a set of endogenous weight sets which are the most favorable for the tested multi-period and -division system. This makes sense of the comparison between overall and component (in)efficiency scores. In this context, the overall efficiency score is less or more than all component ones. We applied our models to evaluate the innovation efficiency of OECD (Organization for Economic Co-operation and Development) countries.

Keywords: Dynamic network DEA; Multi-period and -division systems; Efficiency measurement and decomposition; Innovation efficiency; OECD countries

[1] **Corresponding Author.**
Teh Tel/Fax:+86-10-59358707.
E-mail: chenkaihua@casipm.ac.cn (Kaihua C.)

1. Introduction

Traditional data envelopment analysis (DEA) models don't account for the multi-division (-stage) transformation process of decision-making units (DMUs), and present the "black-box" measurement of their efficiency scores. However, the operational information embedded into the internal structure is neglected, which may make efficiency scores overestimated or underestimated. This means that the "black-box" measurement of efficiency usually is biased. In this situation, the network DEA model was developed using link variables (usually referred as division-intermediate products) (see, Kao and Hwang, 2008; Kao, 2009; Chen et al., 2009; Cook et al., 2010; Guan and Chen, 2012). In addition, the operation of DMU in one period is not independent on that in the next one in some situations. There is an inter-relationship between consecutive periods by carry-overs (usually referred as time-intermediate products). Such stock variables usually serve as carry-overs, which form in one period becomes the source for growth in the next one. Moreover, in the actual world, a long time planning and investment is a subject of great concern. For this case, single period optimization model is not suitable. In such multi-period situation, the dynamic DEA model was proposed (see, Bogetoft et al., 2009; Tone and Tsutsui, 2010; Kao, in press). If multi-division situation and multi-period one coexist, network DEA model and dynamic DEA one independently cannot work. The dynamic network DEA (DN-DEA) is needed.

This study proposes a new formulating approach of dynamic network DEA model to measure and decompose the overall efficiency of multi-division and -period systems. In the extant literature, Tone and Tsutsui (2014) proposed the dynamic network slacks-based measures (SBM) (Tone, 2001), and Avkiran and McCrystal (2013, in press) proposed dynamic network range-adjusted measures (RAM) (Cooper, et al., 2001). In contrast to their approaches, our approach need not to depend on a set of pre-specified weighted to combine component efficiency scores when formulating the overall efficiency score. In our approach, a set of weights are generated endogenously based on the statistical data from the most favorable perspective for the tested multi-division and -period system like the multipliers on inputs and outputs. The sum of weights is "1", which builds a weighted average combination relationship between the overall efficiency score and component ones, and makes sense of the numerical size comparison between them. This indicates that our modeling approach is not straightforward extension of relevant studies about network DEA modeling (e.g., Chen et al., 2009 and Cook et al., 2010) although a same weighted average combination relationship between the overall efficiency score and component ones also holds. Our modeling approach is essentially different with theirs, which presents a post decomposition of the overall efficiency based on the endogenous relationship between the sum of the surplus variables associated with the component constraints and the surplus variable associated with system constraint. The combination weights of component efficiency scores into the overall one need not subjectively pre-specifying like the multipliers on inputs and outputs in our modeling approach, however which is needed in theirs.

The rest of this paper is as follows. In section 2, we extend the Kao's (in press) relational approach to a dynamic network system. We formulate our dynamic network DEA model associated with CCR model (Charnes et al., 1978) in Section 3. An application to a dataset of scientific and technological (S&T) innovation activities about OECD countries is presented in section 4, along with the comparison with the results by our extending model based on Kao's (in press) approach. Section 5 presents two extensions of our formulating approach respectively to radial BCC model (Banker et al., 1984) as well as non-radial and non-oriented slack-based measures (SBM) (Tone, 2001).

2. A modeling extension of Kao's (in press) relational approach to a dynamic network system

There are *n* DMUs ($j = 1, 2, \cdots, n$) consisting of *D* divisions ($d = 1, 2, \cdots, D$) over *T* time periods ($t = 1, 2, \cdots, T$). The conceptual graph for the internal structure of DMUs is depicted in Fig.1. Let $X_{i^d j}^{(t)}$ $(i^d = 1, 2, \cdots, m^d)$ and $Y_{r^d j}^{(t)}$ $(r^d = 1, 2, \cdots, s^d)$ be own independent inputs and outputs in DMU$_j$ for division *d* in period *t*. Let $Z_{p^d j}^{(t-1,t)}$ $(t = 1, 2, \cdots, T+1; p^d = 1, 2, \cdots, q^d; d = 1, 2, \cdots, D)$ denote carry-overs over period *t*-1 and *t,* and $Z_{p^{(d-1,d)} j}^{(t)}$ $(t = 1, 2, \cdots, T; p^{d-1,d} = 1, 2, \cdots, q^{d-1,d}; d = 1, 2, \cdots, D+1)$ denote linkers over division *d*-1 and *d*. Here, $Z_{p^{(0,1)} j}^{(t)}, Z_{p^{(D,D+1)} j}^{(t)} = 0 \, (\forall t, j)$. To strengthen the correlations between periods or divisions, the same factor has the same multiplier associated with it, regardless of whether it is an input or output in any period or division. We denote v_{i^d} , u_{r^d} , w_{p^d} and $w_{p^{(d-1,d)}}$ are virtual multipliers respectively associated with $X_{i^d j}^{(t)}$, $Y_{r^d j}^{(t)}$, $Z_{p^d j}^{(t-1,t)}$ and $Z_{p^{(d-1,d)} j}^{(t)}$. Here, $w_{p^{(0,1)}}$, $w_{p^{(D,D+1)}}$ =0.

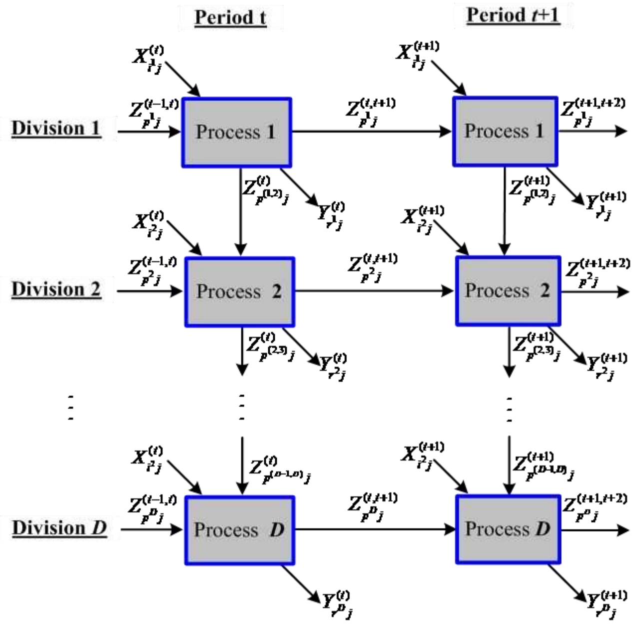

Fig. 1. A conceptual graph of one general multi-period and -division system

Kao (in press) proposed a relational approach for the efficiency measure of a dynamic system composed of the operation of one specific division over T periods, i.e., a dynamic DEA model. This section extends Kao's (in press) relational approach to one general dynamic network system depicted as Fig.1, i.e., a dynamic network DEA model. Based on the Kao's (in press) approach, the two additively aggregated terms, $\left(\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{r^d=1}^{s^d} u_{r^d} Y_{r^d j}^{(t)} + \sum_{d=1}^{D}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d j}^{(T,T+1)}\right)$ and $\left(\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d} X_{i^d j}^{(t)} + \sum_{d=1}^{D}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d j}^{(0,1)}\right)$ respectively present the aggregate inputs and outputs of the whole production system in DMU$_j$ during the T periods. Clearly, only initial carry-overs $Z_{p^d j}^{(0,1)}$ and final ones $Z_{p^d j}^{(T,T+1)}$ $(p^d = 1,2,\cdots,q^d; d = 1,2,\cdots,D)$ outside of systems are considered. As soon as the optimal multiplier set ($u_{r^d}^*, v_{i^d}^*, w_{p^d}^*$) are obtained, the overall (system) efficiency E_k^s of the tested DMU_k with the production structure as depicted in Fig. 1 is represented as

$$E_k^s = \frac{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{r^d=1}^{s^d} u_{r^d}^* Y_{r^d k}^{(t)} + \sum_{d=1}^{D}\sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(T,T+1)}}{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d}^* X_{i^d k}^{(t)} + \sum_{d=1}^{D}\sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(0,1)}} \tag{1}$$

Constraining the overall efficiency score and component ones not over "1", the program (2) is formulated in order to estimate the optimal multiplier set ($u_{r^d}^*, v_{i^d}^*, w_{p^d}^*, w_{p^{(d-1,d)}}^*$) .

$$E_k^{s} = \max \frac{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{r^d=1}^{s^d} u_{r^d} Y_{r^d k}^{(t)} + \sum_{d=1}^{D}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d k}^{(T,T+1)}}{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d} X_{i^d k}^{(t)} + \sum_{d=1}^{D}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d k}^{(0,1)}}$$

$$\begin{aligned}
& s.t. \\
& \frac{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{r^d=1}^{s^d} u_{r^d} Y_{r^d j}^{(t)} + \sum_{d=1}^{D}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d j}^{(T,T+1)}}{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d} X_{i^d j}^{(t)} + \sum_{d=1}^{D}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d j}^{(0,1)}} \le 1, j = 1,2,\cdots,n \\
& \frac{\sum_{r^d=1}^{s^d} u_{r^d} Y_{r^d j}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d j}^{(t,t+1)} + \sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}} Z_{p^{(d,d+1)} j}^{(t)}}{\sum_{i^d=1}^{m^d} v_{i^d} X_{i^d j}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d j}^{(t-1,t)} + \sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}} Z_{p^{(d-1,d)} j}^{(t)}} \le 1, \\
& j = 1,2,\cdots,n; d = 1,2,\cdots,D; t = 1,2,\cdots,T \\
& u_{r^d}, v_{i^d}, w_{p^d}, w_{p^{(d-1,d)}} \ge \varepsilon
\end{aligned} \tag{2}$$

Here, $\varepsilon > 0$ is a small "non-Archimedean" quantity. Model (2) can be reduced to the equivalent linear form in virtue of the Charnes and Cooper's (1962) transformation for the optimal multipliers ($u_{r^d}^*, v_{i^d}^*, w_{p^d}^*, w_{p^{(d-1,d)}}^*$). After the optimal multipliers are obtained, we can use

$$E_k^{(d)} = \frac{\sum_{t=1}^{T}\sum_{r^d=1}^{s^d} u_{r^d}^* Y_{r^d k}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(T,T+1)} + \sum_{t=1}^{T}\sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}}^* Z_{p^{(d,d+1)} k}^{(t)}}{\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d}^* X_{i^d k}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(0,1)} + \sum_{t=1}^{T}\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}}^* Z_{p^{(d-1,d)} k}^{(t)}}, \quad d = 1,2,\cdots,D \tag{3}$$

$$E_k^{(t)} = \frac{\sum_{d=1}^{D}\sum_{r^d=1}^{s^d} u_{r^d}^* Y_{r^d k}^{(t)} + \sum_{d=1}^{D}\sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(t,t+1)}}{\sum_{d=1}^{D}\sum_{i^d=1}^{m^d} v_{i^d}^* X_{i^d k}^{(t)} + \sum_{d=1}^{D}\sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(t-1,t)}}, t = 1,2,\cdots,T \tag{4}$$

$$E_k^{(td)} = \frac{\sum_{r^d=1}^{s^d} u_{r^d}^* Y_{r^d k}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(t,t+1)} + \sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}}^* Z_{p^{(d,d+1)} k}^{(t)}}{\sum_{i^d=1}^{m^d} v_{i^d}^* X_{i^d k}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(t-1,t)} + \sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}}^* Z_{p^{(d-1,d)} k}^{(t)}}, \quad t = 1,2,\cdots,T; d = 1,2,\cdots,D \tag{5}$$

to calculate division efficiency $E_k^{(d)}$, period efficiency $E_k^{(t)}$ and period-division efficiency $E_k^{(td)}$ for the tested DMU_k.

Since the sum of the constraints associated with all component processes is equal to the constraint associated with the system for DMU_k, that is, the sum of the surplus variables associated with the period-division constraints is equal to the surplus variable associated with system constraint, we can have:

$$
\begin{aligned}
&\left(\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d}^{*} X_{i^d k}^{(t)} + \sum_{d=1}^{D}\sum_{p^d=1}^{q^d} w_{p^d}^{*} Z_{p^d k}^{(0,1)}\right) \\
&\text{-}\left(\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{r^d=1}^{s^d} u_{r^d}^{*} Y_{r^d k}^{(t)} + \sum_{d=1}^{D}\sum_{p^d=1}^{q^d} w_{p^d}^{*} Z_{p^d k}^{(T,T+1)}\right) \\
&= \sum_{d=1}^{D}\sum_{t=1}^{T}\left[\begin{array}{l}\left(\sum_{i^d=1}^{m^d} v_{i^d}^{*} X_{i^d k}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d}^{*} Z_{p^d k}^{(t-1,t)} + \sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}}^{*} Z_{p^{(d-1,d)}k}^{(t)}\right) \\ -\left(\sum_{r^d=1}^{s^d} u_{r^d}^{*} Y_{r^d k}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d}^{*} Z_{p^d k}^{(t,t+1)} + \sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}}^{*} Z_{p^{(d,d+1)}k}^{(t)}\right)\end{array}\right]
\end{aligned} \tag{6}
$$

Dividing both sides by $\left(\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d}^{*} X_{i^d k}^{(t)} + \sum_{d=1}^{D}\sum_{p^d=1}^{q^d} w_{p^d}^{*} Z_{p^d k}^{(0,1)}\right)$ results in:

$$
1 - E_k^s = \sum_{d=1}^{D}\sum_{t=1}^{T} \omega^{(td)}\left(1 - E_k^{(td)}\right) \tag{7}
$$

where

$$
\omega^{(td)} = \frac{\sum_{i^d=1}^{m^d} v_{i^d}^{*} X_{i^d k}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d}^{*} Z_{p^d k}^{(t-1,t)} + \sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}}^{*} Z_{p^{(d-1,d)}k}^{(t)}}{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d}^{*} X_{i^d k}^{(t)} + \sum_{d=1}^{D}\sum_{p^d=1}^{q^d} w_{p^d}^{*} Z_{p^d k}^{(0,1)}}, \quad t = 1,2,\cdots,T; d = 1,2,\cdots,D. \tag{8}
$$

The derived weights $\omega^{(td)}$ represent the importance of component process for period *t* and division *d*. We can deduce:

$$
\begin{aligned}
&\sum_{d=1}^{D}\sum_{t=1}^{T}\omega^{(td)} \\
&= \sum_{d=1}^{D}\sum_{t=1}^{T} \frac{\sum_{i^d=1}^{m^d} v_{i^d}^{*} X_{i^d k}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d}^{*} Z_{p^d k}^{(t-1,t)} + \sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}}^{*} Z_{p^{(d-1,d)}k}^{(t)}}{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d}^{*} X_{i^d k}^{(t)} + \sum_{d=1}^{D}\sum_{p^d=1}^{q^d} w_{p^d}^{*} Z_{p^d k}^{(0,1)}} \\
&\geq \sum_{d=1}^{D}\sum_{t=1}^{T} \frac{\sum_{i^d=1}^{m^d} v_{i^d}^{*} X_{i^d k}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d}^{*} Z_{p^d k}^{(t-1,t)} + \sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}}^{*} Z_{p^{(d-1,d)}k}^{(t)}}{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d}^{*} X_{i^d k}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d}^{*} Z_{p^d k}^{(t-1,t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}}^{*} Z_{p^{(d-1,d)}k}^{(t)}} \\
&= 1
\end{aligned} \tag{9}
$$

Only $\sum_{d=1}^{D}\sum_{t=2}^{T}\sum_{p^d=1}^{q^d} w_{p^d}^{*} Z_{p^d k}^{(t-1,t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}}^{*} Z_{p^{(d-1,d)}k}^{(t,t)} = 0$, this means that there are no linkers and carry-overs, the equal sign in (9) exists. In fact, since $w_{p^d}^{*} > 0$ and $w_{p^{(d-1,d)}}^{*} > 0$ in practice, the equal situation usually does not exist in our extended model based on Kao's (in press) approach in the empirical study. That is, the equal formula $\sum_{d=1}^{D}\sum_{t=1}^{T}\omega^{(td)} = 1$ usually does not hold.

The inequality $\sum_{d=1}^{D}\sum_{t=1}^{T}\omega^{(td)} > 1$ in our extending model brings some confusion and difficulty in the performance comparison and management in practice, which does not assure that the overall efficiency is not less than the minimum period-division efficiency and not more than the maximum one. In this situation, it is difficult to understand the overall performance of dynamic network system is resulted from the component performance of all period-division processes in the average sense.

3. A new formulating approach of dynamic network DEA

3.1 Measurement of the overall efficiency

The essential reason for the existence of the puzzling relationship between overall and component efficiency scores in the relational framework is that the objective function and overall constraints in the program model do not incorporate the internal production information embedded on carry-overs connecting consecutive periods and linkers connecting consecutive divisions inside the observed time-system as depicted in Fig.1, but only include the initial and final carry-overs outside of the dynamic network system as depicted in Fig.1. However, the carry-overs and linkers inside systems may bring shortfalls as outputs and surpluses as inputs, and therefore influence the operational performance of the dynamic network system. This section will propose a flexible approach of dynamic network DEA associated with CCR model, i.e., dynamic network CCR, which can fully account for the information on carry-overs and linkers.

To facilitate our formulating approach, we present the conceptual framework of decomposed dynamic network systems to (see Fig.2) help in understanding the double identity of carry-overs and linkers. Here, $Z^{(t)}_{p^{(D,D+1)}j} = 0$ and $Z^{(t)}_{p^{(0,1)}j} = 0, \forall t, j$. We can extend the Cook et al.'s (2010) modeling approach of network DEA under the multi-period context to incorporate the production information embedded on the double identity of carry-overs and linkers. However, their approach needs pre-specified weights of component efficiency scores when formulating the overall one. This paper will present an essentially different formulating procedure based on the idea of CCR model (Charnes et al., 1978). This formulating procedure does not pre-specify weights, however which can present a weighted average decomposition of the overall (in)efficiency score into components ones by a set of endogenous weight sets which are the most favorable for the tested multi-period and -division system, and reduced based on the fact that the sum of the surplus variables associated with the period-division constraints is equal to the surplus variable associated with system constraint. Our model is formulated as following.

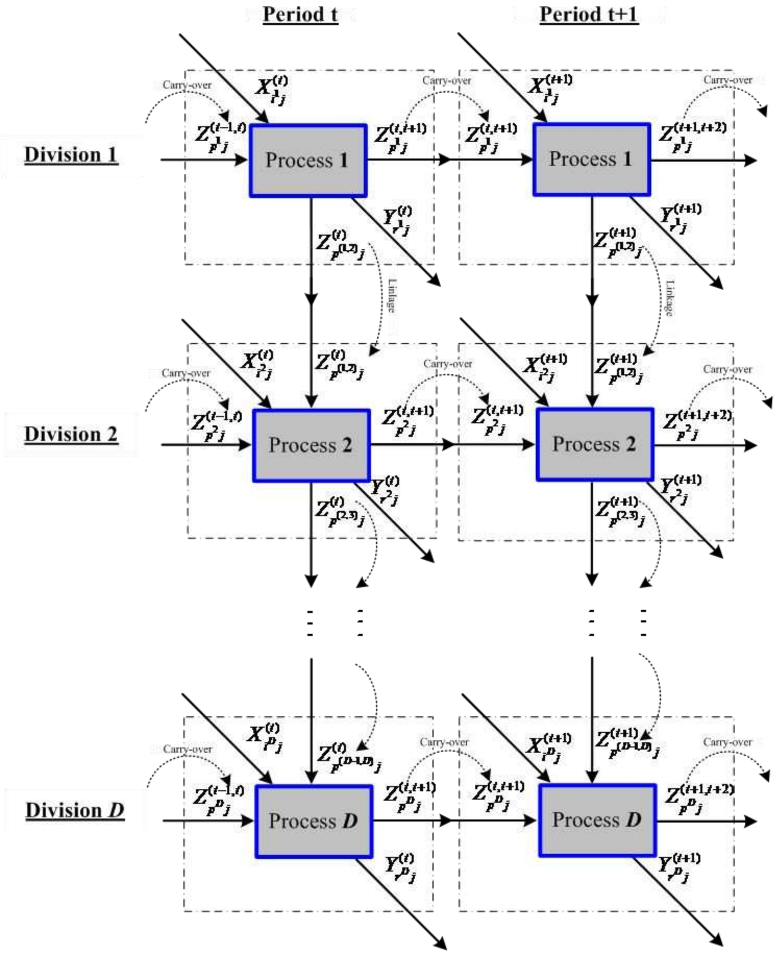

Fig. 2. Decomposition of a general multi-period and -division system

As indicated by Fig.2, the additively aggregated terms of inputs and outputs of all period-division processes, $\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{r^d=1}^{s^d} u_{r^d} Y_{r^d j}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d j}^{(t,t+1)}$ $+\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}} Z_{p^{(d,d+1)} j}^{(t)}$ and $\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d} X_{i^d j}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d j}^{(t-1,t)}$ $+\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}} Z_{p^{(d-1,d)} j}^{(t)}$, respectively present the nominal system-wide aggregate inputs and outputs of the whole dynamic network system as depicted in Fig. 2, which fully accounts for the double identity of carry-overs and linkers inside the dynamic network system. As soon as the optimal multiplier set $(u_{r^d}^*, v_{i^d}^*, w_{p^d}^*, w_{p^{(d-1,d)}}^*)$ is obtained, the overall efficiency E_k^s of the tested DMU_k with the production structure as depicted in Fig.1 is represented as

$$E_k^s = \frac{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{r^d=1}^{s^d} u_{r^d}^* Y_{r^d k}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(t,t+1)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}}^* Z_{p^{(d,d+1)}k}^{(t)}}{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d}^* X_{i^d k}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(t-1,t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}}^* Z_{p^{(d-1,d)}k}^{(t)}} \quad (10)$$

Constraining the overall efficiency score and component ones not over "1", the following fraction program (11) is formulated in order to derive the optimal multipliers.

$$\begin{aligned}
&E_k^s = \max \frac{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{r^d=1}^{s^d} u_{r^d} Y_{r^d k}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d k}^{(t,t+1)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}} Z_{p^{(d,d+1)}k}^{(t)}}{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d}^d X_{i^d k}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d}^d Z_{p^d k}^{(t-1,t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}}^{(d-1,d)} Z_{p^{(d-1,d)}k}^{(t)}}\\
&s.t.\\
&\frac{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{r^d=1}^{s^d} u_{r^d} Y_{r^d j}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d j}^{(t,t+1)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}} Z_{p^{(d,d+1)}j}^{(t)}}{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d} X_{i^d j}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d j}^{(t-1,t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}} Z_{p^{(d-1,d)}j}^{(t)}} \le 1,\\
&j = 1,2,\cdots,n\\
&\frac{\sum_{r^d=1}^{s^d} u_{r^d} Y_{r^d j}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d j}^{(t,t+!)} + \sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}} Z_{p^{(d,d+1)}j}^{(t)}}{\sum_{i^d=1}^{m^d} v_{i^d} X_{i^d j}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d j}^{(t-1,t)} + \sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}} Z_{p^{(d-1,d)}j}^{(t)}} \le 1, d = 1,2,\cdots,D; t = 1,2,\cdots,T, j = 1,2,\cdots,n\\
&u_{r^d}, v_{i^d}, w_{p^d}, w_{p^{(d-1,d)}} \ge \varepsilon.
\end{aligned} \quad (11)$$

The constraint on the system as well as the object function in the program model (11) fully incorporates the double identities of carry-overs and linkers during the operation of dynamic network systems. After the optimal multipliers ($u_{r^d}^*, v_{i^d}^*, w_{p^d}^*, w_{p^{(d-1,d)}}^*$) are obtained associated with the Charnes and Cooper's (1962) transformation, we can use

$$E_k^{(d)} = \frac{\sum_{t=1}^{T}\sum_{r^d=1}^{s^d} u_{r^d}^* Y_{r^d k}^{(t)} + \sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(t,t+1)} + \sum_{t=1}^{T}\sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}}^* Z_{p^{(d,d+1)}k}^{(t)}}{\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d}^* X_{i^d k}^{(t)} + \sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(t-1,t)} + \sum_{t=1}^{T}\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}}^* Z_{p^{(d-1,d)}k}^{(t)}} \quad (12)$$
$$d = 1,2,\cdots,D$$

$$E_k^{(t)} = \frac{\sum_{d=1}^{D}\sum_{r^d=1}^{s^d} u_{r^d}^* Y_{r^d k}^{(t)} + \sum_{d=1}^{D}\sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(t,t+1)} + \sum_{d=1}^{D}\sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}}^* Z_{p^{(d,d+1)}k}^{(t)}}{\sum_{d=1}^{D}\sum_{i^d=1}^{m^d} v_{i^d}^* X_{i^d k}^{(t)} + \sum_{d=1}^{D}\sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(t-1,t)} + \sum_{d=1}^{D}\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}}^* Z_{p^{(d-1,d)}k}^{(t)}} \quad (13)$$
$$t = 1,2,\cdots,T$$

$$E_k^{(td)} = \frac{\sum_{r^d=1}^{s^d} u_{r^d}^* Y_{r^d k}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(t,t+1)} + \sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}}^* Z_{p^{(d,d+1)}k}^{(t)}}{\sum_{i^d=1}^{m^d} v_{i^d}^* X_{i^d k}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(t-1,t)} + \sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}}^* Z_{p^{(d-1,d)}k}^{(t)}} \quad (14)$$
$$t = 1,2,\cdots,T; d = 1,2,\cdots,D$$

to calculate the period, division, and period-division efficiencies for DMU_k.

3.2 Ex-Post Decomposition of the overall efficiency

Since it is difficult to understand the combination relationship among component processes for one

DMU, and the importance of component processes varies across DMUs, it is not appropriate to pre-specify the combination weights. Inspired by Kao (In press), we implement an ex-post decomposition of the overall efficiency and obtain combination weights. Based on the fact that the inefficient slacks associated with all period-division processes is equal to the inefficient slack associated with the system for DMU_k , we can obtain:

$$\begin{aligned}
&\left(\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d}^* X_{i^d k}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(t-1,t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}}^* Z_{p^{(d-1,d)}k}^{(t)}\right) \\
&-\left(\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{r^d=1}^{s^d} u_{r^d}^* Y_{r^d k}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(t,t+1)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}}^* Z_{p^{(d,d+1)}k}^{(t)}\right) \\
&= \sum_{d=1}^{D}\sum_{t=1}^{T}\left[\begin{array}{l}\left(\sum_{i^d=1}^{m^d} v_{i^d}^* X_{i^d k}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(t-1,t)} + \sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}}^* Z_{p^{(d-1,d)}k}^{(t)}\right) \\ -\left(\sum_{r^d=1}^{s^d} u_{r^d}^* Y_{r^d k}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(t,t+1)} + \sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}}^* Z_{p^{(d,d+1)}k}^{(t)}\right)\end{array}\right]
\end{aligned} \tag{15}$$

Dividing the both sides by

$A = \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d}^* X_{i^d k}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(t-1,t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}}^* Z_{p^{(d-1,d)}k}^{(t)}$ results in:

$$1 - E_k^s = \sum_{d=1}^{D}\sum_{t=1}^{T} \omega^{(td)}\left(1 - E_k^{(td)}\right) \tag{16}$$

where

$$\begin{aligned}
&\omega^{(td)} = \frac{\sum_{i^d=1}^{m^d} v_{i^d}^* X_{i^d k}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(t-1)} + \sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}}^* Z_{p^{(d-1,d)}k}^{(t)}}{A}, \\
&t = 1,2,\cdots,T; d = 1,2,\cdots,D.
\end{aligned} \tag{17}$$

Since $\sum_{d=1}^{D}\sum_{t=1}^{T}\omega^{(td)} = 1$, we can get a generalized additive decomposition of the overall efficiency score into period-division ones as depicted by formula (18). This means that the overall efficiency score is a weighted average of period-division ones in the context of our approach.

$$E_k^s = \sum_{d=1}^{D}\sum_{t=1}^{T}\omega^{(td)} E_k^{(td)} \tag{18}$$

Similarly, we can obtain an additive decomposition of the overall efficiency score into period ones (see (19)) with weights (see (20),

$$E_k^s = \sum_{d=1}^{D}\omega^{(d)} E_k^{(d)} \tag{19}$$

$$\omega^{(d)}=\sum_{t=1}^{T}\omega^{(td)}$$
$$=\frac{\sum_{t=1}^{T}\sum_{i^d=1}^{m^d}v_{i^d}^{*}X_{i^dk}^{(t)}+\sum_{t=1}^{T}\sum_{p^d=1}^{q^d}w_{p^d}^{*}Z_{p^dk}^{(t-1,t)}+\sum_{t=1}^{T}\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}}w_{p^{(d-1,d)}}^{*}Z_{p^{(d-1,d)}k}^{(t)}}{A},\quad (20)$$
$$d=1,2,\cdots,D$$

and division ones (see (21)) with weights (see (22)).

$$E_k^s=\sum_{t=1}^{T}\omega^{(t)}E_k^{(t)} \quad (21)$$

$$\omega^{(t)}=\sum_{d=1}^{D}\omega^{(td)}$$
$$=\frac{\sum_{d=1}^{D}\sum_{i^d=1}^{m^d}v_{i^d}^{*}X_{i^dk}^{(t)}+\sum_{d=1}^{D}\sum_{p^d=1}^{q^d}w_{p^d}^{*}Z_{p^dk}^{(t-1)}+\sum_{d=1}^{D}\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}}w_{p^{(d-1,d)}}^{*}Z_{p^{(d-1,d)}k}^{(t)}}{A},\quad (22)$$
$$t=1,2,\cdots,T$$

So, our modeling approach presents a desirable aggregation way of component efficiencies into the overall efficiency. Besides, our efficiency measures make sense of the comparison between the overall efficiency score and component ones. Use the comparison between E_k^s and $E_k^{(td)}$ ($t=1,2,\cdots,T;d=1,2,\cdots,D$) as an example. If $E_k^{(td)\min}$ and $E_k^{(td)\max}$ is respectively the minimum and maximum period-division efficiency score of $E_k^{(td)}$ ($t=1,2,\cdots,T;d=1,2,\cdots,D$), then $E_k^s=\sum_{d=1}^{D}\sum_{t=1}^{T}\omega^{(td)}E_k^{(td)}\le E_k^{(td)\max}\sum_{d=1}^{D}\sum_{t=1}^{T}\omega^{(td)}=E_k^{(td)\max}$ and $E_k^s=\sum_{d=1}^{D}\sum_{t=1}^{T}\omega^{(td)}E_k^{(td)\min}\ge E_k^{(td)\min}\sum_{d=1}^{D}\sum_{t=1}^{T}\omega^{(td)}=E_k^{(td)\min}$ by $\sum_{d=1}^{D}\sum_{t=1}^{T}\omega^{(td)}=1$. The desirable property is not always founded in our extended approach of Kao's (in press).

Our approach can build a weighted average between overall inefficiency score and component scores. We still use the relationship between overall inefficiency score and period-division ones as an example. If let INE_k^s and $INE_k^{(td)}$ ($t=1,2,\cdots,T;d=1,2,\cdots,D$) respectively denote the overall inefficiency score of DMU_k and the period-division ones, then $INE_k^s=1-E_k^s$ and $INE_k^{(td)}=1-E_k^{(td)}$ ($t=1,2,\cdots,T;d=1,2,\cdots,D$). Based on the Eq. (16), we have

$$INE_k^s=\sum_{d=1}^{D}\sum_{t=1}^{T}\omega^{(td)}\cdot INE_k^{(td)}\ . \quad (23)$$

This means that the overall inefficiency is also a convex combination of period-division inefficiencies in our modeling approach.

3.3 Uniqueness of component efficiencies

Although the overall efficiency E_k^s is uniquely determined as the optimum value of the above program (11), the optimal multiplier set ($u^*_{r^d}, v^*_{i^d}, w^*_{p^d}, w^*_{p^{(d-1,d)}}$) is not necessarily unique. Hence, the component efficiencies in (12) - (14) may suffer from plurality. So, the uniqueness check of the optimal multiplier set is needed. We here present one post-program approach for checking the uniqueness of period efficiencies. The uniqueness of division and period-division efficiencies can follow this approach.

As argued in Tone and Tsutsui (2014), it would be reasonable that the last period *T* has the top priority and those of *T*-1, *T*-2, ⋯ , 1 decrease in this order. Under this priority principle, the following post-programming scheme can be employed to overcome this plurality problem.

We firstly maximize the period efficiency in *T* while keeping the overall efficiency at E_k^{s*} by the post-program (24).

$$
\begin{aligned}
&E_k^{(T)*} = \max \frac{\sum_{d=1}^{D}\sum_{r^d=1}^{s^d} u_{r^d} Y_{r^d k}^{(T)} + \sum_{d=1}^{D}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d k}^{(T,T+1)} + \sum_{d=1}^{D}\sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}} Z_{p^{(d,d+1)}k}^{(T)}}{\sum_{d=1}^{D}\sum_{i^d=1}^{m^d} v_{i^d} X_{i^d k}^{(T)} + \sum_{d=1}^{D}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d k}^{(T-1,T)} + \sum_{d=1}^{D}\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}} Z_{p^{(d-1,d)}k}^{(T)}} \\
&s.t. \\
&\frac{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{r^d=1}^{s^d} u_{r^d} Y_{r^d j}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d j}^{(t,t+1)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}} Z_{p^{(d,d+1)}j}^{(t)}}{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d} X_{i^d j}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d j}^{(t-1,t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}} Z_{p^{(d-1,d)}j}^{(t)}} \le 1, \\
&j = 1,2,\cdots,n \\
&\frac{\sum_{r^d=1}^{s^d} u_{r^d} Y_{r^d j}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d}^{d} Z_{p^d j}^{(t)} + \sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}} Z_{p^{(d,d+1)}j}^{(t)}}{\sum_{i^d=1}^{m^d} v_{i^d} X_{i^d j}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d}^{d} Z_{p^d j}^{(t-1)} + \sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}} Z_{p^{(d-1,d)}j}^{(t)}} \le 1,\ d = 1,2,\cdots,D; t = 1,2,\cdots,T \\
&and \\
&\frac{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{r^d=1}^{s^d} u_{r^d} Y_{r^d k}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d k}^{(t,t+1)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}} Z_{p^{(d,d+1)}k}^{(t)}}{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d} X_{i^d k}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d k}^{(t-1,t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}} Z_{p^{(d-1,d)}k}^{(t)}} = E_k^{s*} \\
&u_{r^d}, v_{i^d}, w_{p^d}, w_{p^{(d-1,d)}} \ge \varepsilon.
\end{aligned}
\qquad (24)
$$

And then we repeat this process to maximize $E_k^{(T-1)}$ while keeping the overall efficiency at E_k^{s*} and the period efficiency in *T* at $E_k^{(T)*}$ by the post-program (25).

$$E_k^{(T-1)*} = \max \frac{\sum_{d=1}^{D}\sum_{r^d=1}^{s^d} u_{r^d} Y_{r^d k}^{(T-1)} + \sum_{d=1}^{D}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d k}^{(T-1,T)} + \sum_{d=1}^{D}\sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}} Z_{p^{(d,d+1)}k}^{(T-1)}}{\sum_{d=1}^{D}\sum_{i^d=1}^{m^d} v_{i^d} X_{i^d k}^{(T-1)} + \sum_{d=1}^{D}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d k}^{(T-2,T-1)} + \sum_{d=1}^{D}\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}} Z_{p^{(d-1,d)}k}^{(T-1)}}$$

s.t.

$$\frac{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{r^d=1}^{s^d} u_{r^d} Y_{r^d j}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d j}^{(t,t+1)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}} Z_{p^{(d,d+1)}j}^{(t)}}{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d} X_{i^d j}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d j}^{(t-1,t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}} Z_{p^{(d-1,d)}j}^{(t)}} \le 1,$$

$j = 1, 2, \cdots, n$

$$\frac{\sum_{r^d=1}^{s^d} u_{r^d} Y_{r^d j}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d j}^{(t,t+1)} + \sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}} Z_{p^{(d,d+1)}j}^{(t)}}{\sum_{i^d=1}^{m^d} v_{i^d} X_{i^d j}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d j}^{(t-1,t)} + \sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}} Z_{p^{(d-1,d)}j}^{(t)}} \le 1,\ d = 1, 2, \cdots, D; t = 1, 2, \cdots, T$$

and

$$\frac{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{r^d=1}^{s^d} u_{r^d} Y_{r^d k}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d k}^{(t,t+1)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}} Z_{p^{(d,d+1)}k}^{(t)}}{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d} X_{i^d k}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d k}^{(t-1,t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}} Z_{p^{(d-1,d)}k}^{(t)}} = E_k^{s*}$$

$$\frac{\sum_{d=1}^{D}\sum_{r^d=1}^{s^d} u_{r^d} Y_{r^d k}^{(T)} + \sum_{d=1}^{D}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d k}^{(T,T+1)} + \sum_{d=1}^{D}\sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}} Z_{p^{(d,d+1)}k}^{(T)}}{\sum_{d=1}^{D}\sum_{i^d=1}^{m^d} v_{i^d} X_{i^d k}^{(T)} + \sum_{d=1}^{D}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d k}^{(T-1,T)} + \sum_{d=1}^{D}\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}} Z_{p^{(d-1,d)}k}^{(T)}} = E_k^{(T)*} \tag{25}$$

$u_{r^d}, v_{i^d}, w_{p^d}, w_{p^{(d-1,d)}} \ge \varepsilon.$

We repeat this process until t=2. The efficiency of the first period is calculated by

$$E_k^{(1)*} = \frac{E_k^{s*} - \sum_{t=2}^{T} E_k^{(t)*} \cdot \omega_k^{(t)}}{\omega_k^{(1)}} \tag{26}$$

5. Empirical study

To improve the management of increasing innovation investment in knowledge economy age, the measurement of innovation efficiency receives more and more attention from all aspects, which has been the hot topic of academic research the recent extant literature (see, Hollanders and Celikel-Esser, 2007; Guan and Chen, 2010; 2012; Chen and Guan, 2012; Chen et al., 2013; Chen, 2013). Assessing innovation efficiency helps both to identify the best innovation practitioners for benchmarking and to shed light on ways to improve efficiency by highlighting areas of weakness. The extant literature above has built a static two-stage (-division) measurement framework of innovation efficiency in the specific year associated with network DEA. In this section, we will add time element on the two-division analytical framework, and build a dynamic network system composed of two-division processes over multiple periods. We will use our dynamic network CCR model to measure the system based on the dataset about OECD countries' innovation inputs and outputs, with the comparison to the results with our extending model based on Kao' (in press) relational approach.

If not considering the internal structure of innovation input-output processes (see Fig.3 for the conceptual graph), a “black-box” measurement of innovation processes can be estimated by one traditional DEA model (see Wang and Huang, 2007; Guan and Chen, 2010; Chen, 2013). In this framework, the production information embedded on intermediate products is neglected, which may produce biased efficiency estimation.

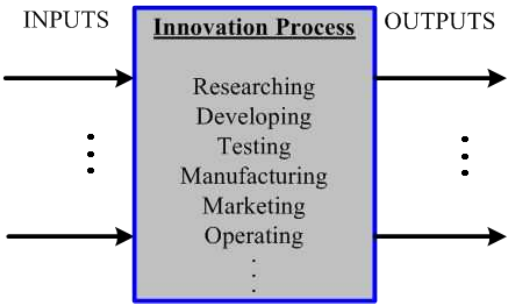

Fig. 3. A conceptual black-box framework of an innovation process

In order to unfold innovation “black-box” and account for the internal operation of innovation processes, some researchers (see, Hollanders and Celikel-Esser, 2007; Guan and Chen, 2010, 2012; Chen, 2013) constructed a two-division analytical framework by decomposing one innovation process into an upstream R&D process and a downstream application process as shows Fig. 4. Chen et al. (2013), Guan and Chen (2012), Chen and Guan (2010, 2012) and Chen (2013) introduced network DEA models to measure it at the industrial, provincial and national levels, which account for the interaction between R&D process and application one. In contrast to the independent measures by traditional DEA models, network DEA can present more desirable estimation of efficiencies, which makes sense of the comparison between overall efficiency and component ones.

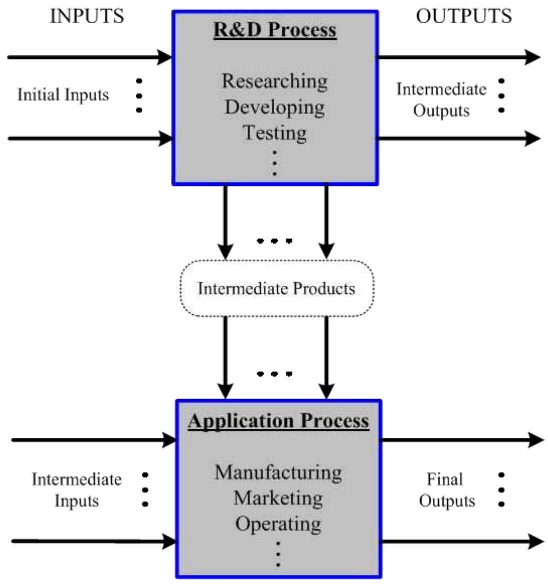

Fig. 4. An innovation process composed of an R&D process and an application process

If the multi-period performance of innovation processes in Fig. 4 is concerned, we can use the average of efficiencies over all periods to measure its overall efficiency. However, the operation of an innovation process in one period is not independent on that in the next one. There is an inter-relationship between consecutive periods by time-intermediate products for innovation processes. In the upstream R&D process, the RD_CS (R&D capital stock) serves as such a time-intermediate product, while in the downstream application process, the RD_CS (R&D capital stock) plays such a role. In this situation, the dynamic network production framework of one innovation input-output process forms. The average way of independent measures neglects the inter-relationship, and may produce a biased estimation of the overall efficiency. The dynamic network DEA will be an appropriate estimation technique. We use it to model the 30 OECD countries' innovation inputs and outputs over 2008-2010 year period. Fig. 5 displays the three-year and two-division dynamic network framework. Following Guan and Chen (2012) and Chen and Guan (2012), we here select RD_P (R&D personnels) and RD_E (R&D expenditure) as initial inputs, Tech_IM (Technology import) as intermediate inputs, S&T_PAP (S&T_papers) and EPO_PAT (EPO-patents) as intermediate outputs, TRI_PAT (Triadic patents) as intermediate products, EM_GDPP (GDPP of employment) and HI_Tech_EX (Export of high-tech products) as final outputs. Table 1 reports the descriptive statistics of them.

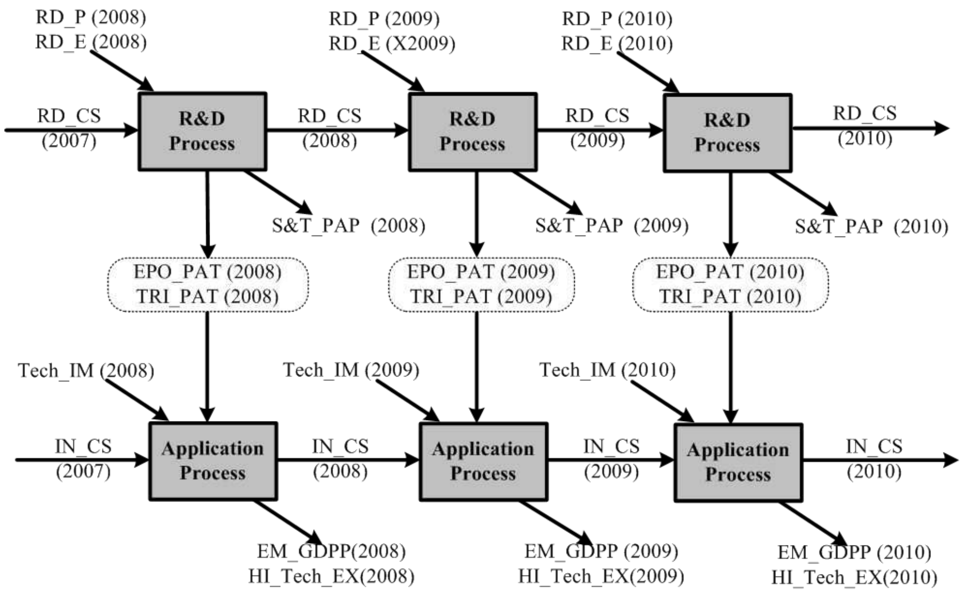

Fig. 5. A three-period and two-division network model of an innovation process

Table 1. Innovation inputs and outputs of 30 OECD countries over 2008-2010 year period

Variables	Year	Average	SD	Maximum	Minimum
R&D capital stock (RD_CS)	2007	5.388E+04	1.324E+05	6.946E+05	5.763E+02
Industry capital stock (IN_CS)	2007	1.519E+12	2.652E+12	1.383E+13	2.819E+10
R&D expenditure (RD_E)	2008	2.891E+04	7.113E+04	3.742E+05	3.076E+02
R&D capital stock (RD_CS)	2008	5.421E+04	1.332E+05	7.008E+05	5.845E+02
R&D personnels (RD_P)	2008	2.176E+05	4.615E+05	2.443E+06	3.117E+03
S&T_papers (S&T_PAP)	2008	4.960E+04	8.942E+04	4.773E+05	2.450E+02
EPO-patents (EPO_PAT)	2008	3.737E+03	7.226E+03	2.963E+04	2.237E+01
Triadic patents (TRI_PAT)	2008	1.486E+03	3.502E+03	1.407E+04	3.242E+00
Tech-payments (Tech_IM)	2007	1.036E+04	1.444E+04	5.671E+04	1.028E+02
Export of high-tech products (HI_Tech_EX)	2008	5.070E+04	7.208E+04	3.177E+05	3.894E+02
GDPP of employment (EM_GDPP)	2008	4.298E+04	1.080E+04	6.481E+04	2.006E+04
Overall-capital stock (IN_CS)	2008	1.415E+12	2.477E+12	1.295E+13	2.743E+10
R&D expenditure (RD_E)	2009	2.848E+04	6.981E+04	3.699E+05	3.068E+02
R&D capital stock (RD_CS)	2009	5.454E+04	1.339E+05	7.071E+05	5.837E+02
R&D personnels (RD_P)	2009	2.197E+05	4.667E+05	2.473E+06	3.753E+03
S&T_papers (S&T_PAP)	2009	5.287E+04	9.472E+04	5.059E+05	2.320E+02
EPO-patents (EPO_PAT)	2009	3.755E+03	7.206E+03	2.909E+04	2.366E+01
Triadic patents (TRI_PAT)	2009	1.502E+03	3.523E+03	1.401E+04	2.934E+00
Tech-payments (Tech_IM)	2009	1.049E+04	1.522E+04	6.028E+04	1.406E+02
Export of high-tech products (HI_Tech_EX)	2009	5.427E+04	7.567E+04	3.347E+05	8.685E+02
GDPP of employment (EM_GDPP)	2009	4.205E+04	1.065E+04	6.497E+04	1.908E+04
Overall-capital stock (IN_CS)	2009	1.623E+12	2.827E+12	1.471E+13	2.894E+10
R&D expenditure (RD_E)	2010	2.881E+04	6.970E+04	3.682E+05	3.060E+02

R&D capital stock (RD_CS)	2010	5.441E+04	1.325E+05	7.012E+05	5.821E+02
R&D personnels (RD_P)	2010	2.237E+05	4.746E+05	2.521E+06	4.389E+03
S&T_papers (S&T_PAP)	2010	5.516E+04	9.844E+04	5.268E+05	2.830E+02
EPO-patents (EPO_PAT)	2010	3.641E+03	6.912E+03	2.735E+04	2.215E+01
Triadic patents (TRI_PAT)	2010	1.577E+03	3.799E+03	1.571E+04	3.414E+00
Tech-payments (Tech_IM)	2010	1.095E+04	1.608E+04	6.728E+04	1.784E+02
Export of high-tech products (HI_Tech_EX)	2010	5.755E+04	7.854E+04	3.448E+05	5.328E+02
GDPP of employment (EM_GDPP)	2010	4.312E+04	1.085E+04	6.732E+04	1.989E+04
Overall-capital stock (IN_CS)	2010	1.849E+12	3.208E+12	1.664E+13	3.036E+10

Tables 2 and 3 report the calculated results respectively by our dynamic network CCR (DN-CCR) model and our extending model based on Kao’s (in press) approach. We checked the uniqueness of efficiency scores, and found no multiple solutions in this case.

To clarify the advantage of our model, we here compare our calculated results with those by our extending model based on Kao’s (in press) approach. We cannot compare both scores directly, because measure schemes are different in our dynamic network CCR and our extending model based on Kao’s (in press) approach. In term of the ranking order in the overall innovation efficiency, component R&D efficiency and component application efficiency, our model shows some variations. For example, there is a difference of above 5 between two approaches for several countries, such as Canada, France, Japan, and Sweden, in the ranking of overall innovation efficiency scores. Only 7 (less than 1/4) countries, Germany, Greece, Luxembourg, New Zealand, Portugal, Slovak Republic, and United States, in our DN-CCR are identical to that in our extending model based on Kao’s (in press) approach.

Table 2. Efficiency results estimated by dynamic network CCR

Countries	Innovation efficiency					R&D efficiency					Application efficiency				
	2008-2010	Rank	2008	2009	2010	2008-2010	Rank	2008	2009	2010	2008-2010	Rank	2008	2009	2010
Australia	0.8379	13	0.7917	0.8542	0.8662	0.8470	19	0.8056	0.8595	0.8743	0.7022	7	0.5875	0.7664	0.7530
Austria	0.7663	24	0.7459	0.7836	0.7697	0.8465	20	0.8245	0.8680	0.8473	0.1459	27	0.1288	0.1448	0.1636
Belgium	0.8269	15	0.7953	0.8346	0.8507	0.8853	14	0.8564	0.8918	0.9074	0.2131	23	0.1777	0.2183	0.2450
Canada	0.8640	9	0.8107	0.8777	0.9038	0.8349	22	0.8327	0.8319	0.8401	0.9109	4	0.7755	0.9562	1.0000
Czech Republic	0.8101	17	0.8029	0.8018	0.8251	0.8149	25	0.8094	0.8059	0.8291	0.5534	15	0.4729	0.5776	0.6116
Denmark	0.7947	23	0.7632	0.8171	0.8041	0.8960	12	0.8681	0.9188	0.9011	0.1111	28	0.0952	0.1150	0.1240
Estonia	0.8999	5	0.8562	0.9229	0.9193	0.9341	9	0.8707	0.9637	0.9669	0.6705	9	0.7455	0.6587	0.6233
Finland	0.7621	26	0.7463	0.7769	0.7634	0.8494	18	0.8309	0.8682	0.8495	0.0964	30	0.0878	0.0952	0.1060
France	0.8270	14	0.7823	0.8488	0.8492	0.8881	13	0.8767	0.8947	0.8929	0.7415	6	0.6496	0.7816	0.7910
Germany	0.6939	29	0.6721	0.7037	0.7056	0.9358	7	0.9173	0.9572	0.9332	0.2937	21	0.2592	0.2920	0.3285
Greece	0.9803	1	0.9523	0.9976	0.9903	0.9834	1	0.9567	1.0000	0.9929	0.8023	5	0.7069	0.8541	0.8475
Hungary	0.8412	12	0.8237	0.8503	0.8493	0.8672	17	0.8503	0.8770	0.8739	0.1862	25	0.1662	0.1852	0.2072
Iceland	0.8735	8	0.8866	0.8819	0.8535	0.8169	24	0.7999	0.8189	0.8321	0.9411	2	1.0000	0.9583	0.8767
Ireland	0.8974	6	0.8266	0.9198	0.9431	0.9358	8	0.8613	0.9592	0.9841	0.2005	24	0.1903	0.2045	0.2062
Italy	0.7991	21	0.7799	0.8092	0.8086	0.9160	10	0.9165	0.9205	0.9110	0.3745	20	0.3081	0.3895	0.4306
Japan	0.7969	22	0.7653	0.8073	0.8180	0.9764	3	0.9701	0.9597	1.0000	0.5619	14	0.4856	0.5942	0.6026
Korea	0.7502	28	0.7155	0.7655	0.7678	0.7742	28	0.7345	0.7910	0.7956	0.5279	16	0.5211	0.5314	0.5302
Luxembourg	0.7569	27	0.7456	0.7680	0.7575	0.7904	27	0.7956	0.7994	0.7759	0.4446	18	0.3590	0.4584	0.5554
Mexico	0.8738	7	0.8766	0.9009	0.8486	0.6674	29	0.6443	0.6901	0.6682	0.9680	1	1.0000	1.0000	0.9193
Netherlands	0.8459	11	0.8203	0.8717	0.8461	0.9668	4	0.9370	1.0000	0.9643	0.1813	26	0.1609	0.1828	0.1990
New Zealand	0.9545	3	0.9360	0.9612	0.9650	0.9571	5	0.9394	0.9633	0.9673	0.6577	10	0.5686	0.7017	0.7051
Norway	0.8181	16	0.7664	0.8363	0.8509	0.8405	21	0.7851	0.8599	0.8761	0.3856	19	0.3713	0.3823	0.4005
Poland	0.9279	4	0.8744	0.9625	0.9443	0.9364	6	0.8832	0.9708	0.9527	0.6397	11	0.5767	0.6762	0.6636
Portugal	0.8097	18	0.7329	0.8723	0.8248	0.8137	26	0.7377	0.8767	0.8277	0.5821	13	0.4803	0.6209	0.6540
Slovak Republic	0.9763	2	0.9569	0.9881	0.9833	0.9766	2	0.9581	0.9880	0.9832	0.9373	3	0.8175	1.0000	1.0000
Slovenia	0.8567	10	0.8094	0.8975	0.8638	0.9051	11	0.8602	0.9472	0.9086	0.2376	22	0.2143	0.2495	0.2513
Spain	0.8013	20	0.7690	0.8218	0.8134	0.8174	23	0.7905	0.8379	0.8241	0.6736	8	0.5963	0.6929	0.7298
Sweden	0.7638	25	0.7420	0.7803	0.7698	0.8801	15	0.8598	0.8971	0.8839	0.1060	29	0.0892	0.1080	0.1215
United Kingdom	0.8084	19	0.7813	0.8168	0.8270	0.8782	16	0.8590	0.8814	0.8938	0.4770	17	0.4202	0.4957	0.5160
United States	0.4947	30	0.4511	0.5079	0.5233	0.1924	30	0.2074	0.1923	0.1783	0.6054	12	0.5399	0.6283	0.6457
Average	0.8236		0.7926	0.8413	0.8369	0.8541		0.8280	0.8697	0.8645	0.4976		0.4517	0.5173	0.5269
SD	0.1085		0.1117	0.1107	0.1066	0.1806		0.1721	0.1846	0.1875	0.2843		0.2784	0.2994	0.2884
Maximum	0.9803		0.9569	0.9976	0.9903	0.9834		0.9701	1.0000	1.0000	0.9680		1.0000	1.0000	1.0000
Minimum	0.4947		0.4511	0.5079	0.5233	0.1924		0.2074	0.1923	0.1783	0.0964		0.0878	0.0952	0.1060

Table 3. Efficiency results estimated by our extending model based on Kao's (in press) approach

Countries	Innovation efficiency					R&D efficiency					Application efficiency				
	2008-2010	Rank	2008	2009	2010	2008-2010	Rank	2008	2009	2010	2008-2010	Rank	2008	2009	2010
Australia	0.7825	8	0.7943	0.8547	0.8679	0.7945	12	0.8050	0.8591	0.8742	0.4809	9	0.5875	0.7664	0.7530
Austria	0.6165	23	0.6873	0.7370	0.7303	0.6273	25	0.6962	0.7443	0.7364	0.2561	21	0.3870	0.4807	0.5264
Belgium	0.7727	10	0.7949	0.8364	0.8552	0.8017	11	0.8196	0.8582	0.8745	0.1607	24	0.2737	0.3388	0.3992
Canada	0.7215	15	0.7761	0.8447	0.8795	0.7215	17	0.7848	0.8158	0.8408	0.7998	4	0.7767	0.9561	1.0000
Czech Republic	0.6587	20	0.8014	0.8003	0.8244	0.6639	21	0.8051	0.8027	0.8268	0.3660	17	0.5008	0.6024	0.6363
Denmark	0.6090	24	0.6425	0.7227	0.7256	0.6304	24	0.6598	0.7382	0.7410	0.1506	25	0.2596	0.3286	0.3424
Estonia	0.8075	7	0.8463	0.9158	0.9113	0.8846	8	0.8707	0.9637	0.9669	0.5829	6	0.7455	0.6587	0.6233
Finland	0.5536	25	0.6433	0.6787	0.6806	0.5925	28	0.6730	0.7092	0.7088	0.0688	30	0.1428	0.1628	0.1896
France	0.6304	22	0.7628	0.8110	0.8116	0.8023	10	0.8767	0.8947	0.8929	0.5321	7	0.6333	0.7189	0.7302
Germany	0.4884	29	0.6130	0.6428	0.6470	0.6395	23	0.7267	0.7548	0.7462	0.1639	23	0.3019	0.3390	0.3797
Greece	0.9801	1	0.9523	0.9976	0.9903	0.9834	1	0.9567	1.0000	0.9929	0.6081	5	0.7069	0.8541	0.8475
Hungary	0.7082	16	0.8189	0.8458	0.8448	0.7313	15	0.8322	0.8588	0.8569	0.1036	28	0.1832	0.2029	0.2246
Iceland	0.7677	11	0.8831	0.8783	0.8495	0.6741	19	0.7999	0.8189	0.8321	0.8996	3	1.0000	0.9583	0.8767
Ireland	0.8887	4	0.8408	0.9411	0.9732	0.9274	5	0.8693	0.9696	1.0000	0.0874	29	0.1951	0.2125	0.2278
Italy	0.6964	17	0.7463	0.7752	0.7860	0.7289	16	0.7770	0.7904	0.8005	0.3671	16	0.5031	0.6451	0.6716
Japan	0.5156	28	0.7239	0.7573	0.7632	0.9291	4	0.9530	0.9425	0.9868	0.2850	20	0.4423	0.5387	0.5387
Korea	0.5269	26	0.7137	0.7655	0.7697	0.5546	29	0.7264	0.7815	0.7868	0.2988	19	0.5211	0.5314	0.5302
Luxembourg	0.5218	27	0.7176	0.7490	0.7403	0.6192	27	0.7923	0.7978	0.7745	0.3919	15	0.3590	0.4584	0.5554
Mexico	0.7663	12	0.8761	0.9006	0.8482	0.4142	30	0.6443	0.6901	0.6682	0.9398	1	1.0000	1.0000	0.9193
Netherlands	0.8153	6	0.7675	0.8712	0.8186	0.8933	6	0.8350	0.9548	0.8930	0.1081	27	0.2227	0.2467	0.2553
New Zealand	0.9497	3	0.9394	0.9516	0.9575	0.9524	3	0.9429	0.9536	0.9597	0.4420	12	0.5686	0.7017	0.7051
Norway	0.7592	13	0.7624	0.8363	0.8540	0.7659	14	0.7677	0.8400	0.8578	0.4045	13	0.5370	0.6692	0.6937
Poland	0.8791	5	0.8778	0.9658	0.9468	0.8881	7	0.8834	0.9703	0.9518	0.4032	14	0.5674	0.6912	0.6619
Portugal	0.6625	18	0.7305	0.8699	0.8235	0.6642	20	0.7317	0.8712	0.8241	0.4604	11	0.5852	0.7158	0.7503
Slovak Republic	0.9586	2	0.9571	0.9881	0.9832	0.9590	2	0.9579	0.9881	0.9832	0.9079	2	0.8175	1.0000	1.0000
Slovenia	0.7774	9	0.7967	0.8404	0.8188	0.7783	13	0.7974	0.8410	0.8193	0.4638	10	0.6155	0.6749	0.6984
Spain	0.6443	21	0.6855	0.7351	0.7514	0.6528	22	0.6891	0.7335	0.7474	0.5139	8	0.6512	0.7615	0.8044
Sweden	0.6596	19	0.7168	0.7707	0.7684	0.6900	18	0.7422	0.7931	0.7891	0.1252	26	0.2214	0.2853	0.3232
United Kingdom	0.7584	14	0.7945	0.8275	0.8387	0.8322	9	0.8539	0.8767	0.8893	0.2519	22	0.4202	0.4957	0.5160
United States	0.4402	30	0.6576	0.7063	0.7015	0.6244	26	0.7797	0.7989	0.7845	0.3526	18	0.5281	0.6140	0.6310
Average	0.7106		0.7773	0.8272	0.8254	0.7474		0.8017	0.8471	0.8469	0.3992		0.5085	0.5870	0.6004
SD	0.1085		0.1117	0.1107	0.1066	0.1806		0.1721	0.1846	0.1875	0.2843		0.2784	0.2994	0.2884
Maximum	0.9801		0.9571	0.9976	0.9903	0.9834		0.9579	1.0000	1.0000	0.9398		1.0000	1.0000	1.0000
Minimum	0.4402		0.6130	0.6428	0.6470	0.4142		0.6443	0.6901	0.6682	0.0688		0.1428	0.1628	0.1896

In contrast to our extending model based on Kao's (in press) approach, our DN-CCR model makes sense of the numerical size comparison between the overall efficiency and components ones. In the common sense, some of the component efficiency scores should be above or equal to the overall efficiency, and the other should be below or equal to the overall efficiency (see, Tone and Tsutsui, 2010). It seems unexpected that the overall efficiency score is above or below all component period efficiency scores. The results displayed in Tables 1 and 2 show that such relationship exists for our results. However, for our extending model based on Kao's (in press) approach, only 6 countries (Greece, Ireland, Netherlands, New Zealand, Poland and Slovak Republic) in the context of Kao's (in press) model have such efficiency results satisfying the attractive relationship in the overall innovation efficiency. More, 7 countries (Estonia, Greece, Ireland, Netherlands, New Zealand, Poland, Slovak Republic) in the component R&D efficiency, and 5 countries (Canada, Iceland, Luxembourg, Mexico, Mexico and Slovak Republic) satisfy the relationship in the component application efficiency.

Fig. 6 describes the trends of the country-average innovation efficiency, R&D efficiency and application efficiency. Fig. 7 presents the comparison among 30 OECD countries in the three-year average of innovation efficiency, R&D efficiency and application efficiency.

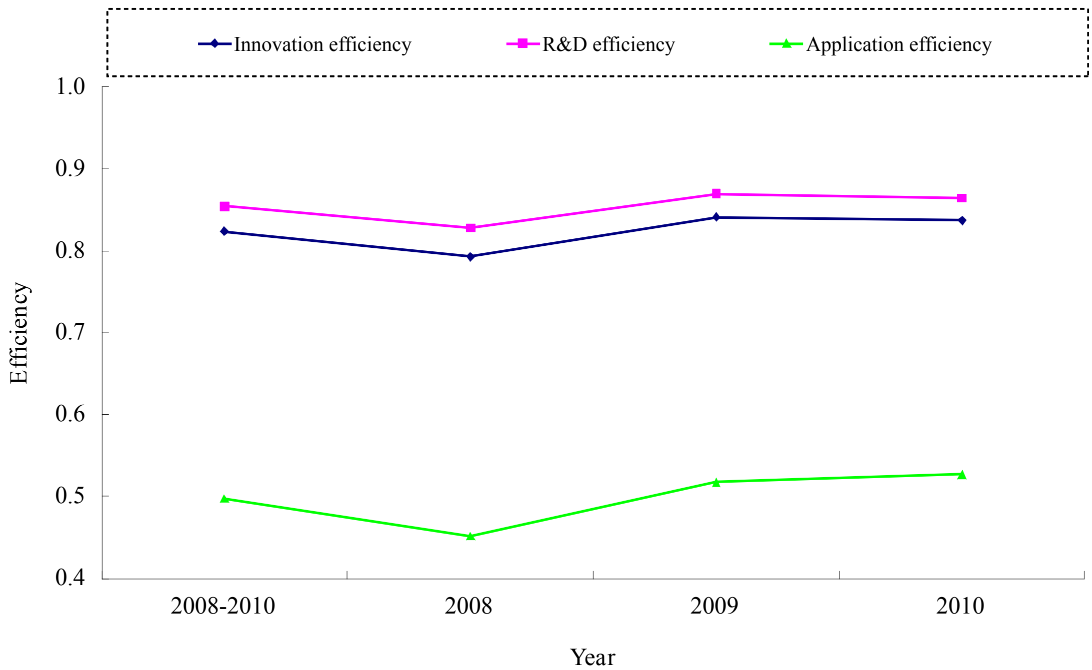

Fig. 6. Trends of the country-average innovation efficiency, R&D efficiency and application efficiency

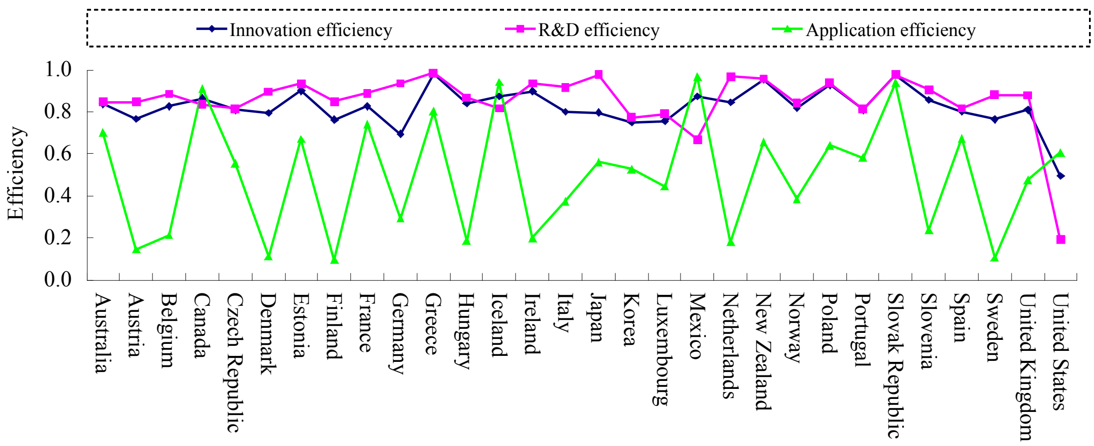

Fig. 7. Comparisons among 30 OECD countries in the three-year average of innovation efficiency, R&D efficiency and application efficiency

Clearly, OECD countries' R&D efficiency presents good performance at the average level (>0.8), however their application efficiency is relatively lower (<0.6). Fig. 7 shows that, in application efficiency, only four countries (Canada, Iceland, Mexico and Slovak Republic) display good efficiency performance (>0.8), and five ones (Austria, Denmark, Finland, Hungary, Netherlands and Sweden) display bad efficiency performance (<0.2). This means that there is a mismatching relationship between R&D efficiency and application one at the average level of OECD countries. Comparing the relative opposition between the curve of overall efficiency and the curves of two component efficiencies by Fig.6 and Fig.7 show that the innovation efficiency is mainly contributed by R&D efficiency, and the innovation inefficiency mainly originates from the inefficiency in the application process.

6. Discussions

This section will show that our modeling approach can be extended to variable returns to scale, i.e., dynamic network BCC, as well as non-radial and non-oriented SBM measures, i.e., dynamic network SBM.

6.1 Extension to dynamic network BCC

In this part, we will show that our modeling approach is also suitable to formulate dynamic network BCC (DN-BCC) model in the multi-period and –division situation. If let $\mu_k^{(td)}(t=1,2,\cdots,T;d=1,2,\cdots,D)$ be scale variables of division *d* at period *t*, the programming model of dynamic network BCC is formulated as:

$$E_k = \max \frac{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{r^d=1}^{s^d} u_{r^d} Y_{r^d k}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d k}^{(t,t+1)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}} Z_{p^{(d,d+1)}k}^{(t)} - \sum_{d=1}^{D}\sum_{t=1}^{T}\mu_k^{(td)}}{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d} X_{i^d k}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d k}^{(t-1,t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}} Z_{p^{(d-1,d)}k}^{(t)}}$$

$$\begin{aligned}
& s.t. \\
& \frac{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{r^d=1}^{s^d} u_{r^d} Y_{r^d j}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d j}^{(t,t+1)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}} Z_{p^{(d,d+1)}j}^{(t)} - \sum_{d=1}^{D}\sum_{t=1}^{T}\mu_k^{(td)}}{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d} X_{i^d j}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d j}^{(t-1,t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}} Z_{p^{(d-1,d)}j}^{(t)}} \le 1, \\
& j = 1,2,\cdots,n \\
& \frac{\sum_{r^d=1}^{s^d} u_{r^d} Y_{r^d j}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d j}^{(t,t+1)} + \sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}} Z_{p^{(d,d+1)}j}^{(t)} - \mu_k^{(td)}}{\sum_{i^d=1}^{m^d} v_{i^d} X_{i^d j}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d} Z_{p^d j}^{(t-1,t)} + \sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}} Z_{p^{(d-1,d)}j}^{(t)}} \le 1, t = 1,2,\cdots,T; d = 1,2,\cdots,D; j = 1,2,\cdots,n \\
& u_{r^d}, v_{i^d}, w_{p^d}, w_{p^{(d-1,d)}} \ge \varepsilon.
\end{aligned} \tag{27}$$

Based on the fact that the inefficient slacks associated with all period-division processes is equal to the inefficient slack associated with the system for DMU_k, we can obtain:

$$\begin{aligned}
& \left(\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d}^* X_{i^d k}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(t-1,t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}}^* Z_{p^{(d-1,d)}k}^{(t)}\right) \\
& -\left(\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{r^d=1}^{s^d} u_{r^d}^* Y_{r^d k}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(t,t+1)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}}^* Z_{p^{(d,d+1)}k}^{(t)} - \sum_{d=1}^{D}\sum_{t=1}^{T}\mu_k^{(td)}\right) \\
& = \sum_{d=1}^{D}\sum_{t=1}^{T}\left[\begin{array}{l}\left(\sum_{i^d=1}^{m^d} v_{i^d}^* X_{i^d k}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(t-1,t)} + \sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}}^* Z_{p^{(d-1,d)}k}^{(t)}\right) \\ -\left(\sum_{r^d=1}^{s^d} u_{r^d}^* Y_{r^d k}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(t,t+1)} + \sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} w_{p^{(d,d+1)}}^* Z_{p^{(d,d+1)}k}^{(t)} - \mu_k^{(td)}\right)\end{array}\right]
\end{aligned} \tag{28}$$

Dividing both sides by $\left(\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d}^* X_{i^d k}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(t-1,t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}}^* Z_{p^{(d-1,d)}k}^{(t)}\right)$, we can obtain an additive decomposition of the system inefficiency score into period-division ones (see (29)) with weights (see (30),

$$1 - E_k^s = \sum_{d=1}^{D}\sum_{t=1}^{T}\omega^{(td)}\left(1 - E_k^{(td)}\right) \tag{29}$$

$$\omega^{(td)} = \frac{\sum_{i^d=1}^{m^d} v_{i^d}^* X_{i^d k}^{(t)} + \sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(t-1,t)} + \sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}}^* Z_{p^{(d-1,d)}k}^{(t)}}{\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{i^d=1}^{m^d} v_{i^d}^* X_{i^d k}^{(t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^d=1}^{q^d} w_{p^d}^* Z_{p^d k}^{(t-1,t)} + \sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} w_{p^{(d-1,d)}}^* Z_{p^{(d-1,d)}k}^{(t)}}, \quad t = 1,2,\cdots,T; d = 1,2,\cdots,D \tag{30}$$

Since $\sum_{d=1}^{D}\sum_{t=1}^{T}\omega^{(td)} = 1$, we obtain an additive decomposition of the system inefficiency score into period-division ones:

$$E_k^s = \sum_{d=1}^{D}\sum_{t=1}^{T}\omega^{(td)} E_k^{(td)} \tag{31}$$

We can further obtain an additive decomposition of the overall efficiency score into period ones (see (32)) with weights (see (33),

$$E_k^s = \sum_{t=1}^{T} \omega^{(t)} E_k^{(t)} \tag{32}$$

$$\omega^t = \sum_{d=1}^{D} \omega^{(td)}, t = 1, 2, \cdots, T \tag{33}$$

and division ones (see (34)) with weights (see (35)).

$$E_k^s = \sum_{d=1}^{D} \omega^{(d)} E_k^{(d)} \tag{34}$$

$$\omega^d = \sum_{t=1}^{T} \omega^{(td)}, d = 1, 2, \cdots, D \tag{35}$$

6.2 Extension to dynamic network SBM

Our approach is not specific to radial measures of efficiency scores, and can be extended to non-radial measures. Inspired by Kao (2014) which presents efficiency decomposition in network DEA with slacks-based measures (SBM) (Tone, 2001), our modeling approach can be used to formulate non-radial and non-oriented dynamic network SBM (DN-SBM). The pre-specified weights here are no longer needed, which, however, is needed in Tone and Tsutsui (2014).

For the tested production unit *k*, let $s_{i^d k}^{(t)-}$ and $s_{r^d k}^{(t)+}$ be the slack variables on independent inputs $X_{i^d k}^{(t)}$ and outputs $Y_{r^d k}^{(t)}$, $s_{p^d k}^{(t-1,t)-}$ and $s_{p^d k}^{(t,t+1)+}$ be the slack variables associated with carry-overs respectively as inputs $Z_{p^d k}^{(t-1,t)}$ and outputs $Z_{p^d k}^{(t,t+1)}$, $s_{p^{(d-1,d)} k}^{(t)-}$ and $s_{p^{(d,d+1)} k}^{(t)+}$ be the slack variables associated with linkers respectively as inputs $Z_{p^{(d-1,d)} k}^{(t)}$ and outputs $Z_{p^{(d,d+1)} k}^{(t)}$, and $\lambda_{dj}^{(t)}$ be intensity vector variable. One new form of dynamic network SBM for the overall efficiency, E_k^s, of systems containing *D* divisions over *T* periods under CRS assumption is formulated as:

$$E_k^s = \min \frac{\sum_{d=1}^{D}\left\{\sum_{t=1}^{T}\left[1-\frac{1}{m^d+q^d+q^{(d-1,d)}}\cdot\left(\sum_{i^d=1}^{m^d}\frac{s_{i^d k}^{(t)-}}{X_{i^d k}^{(t)}}+\sum_{p^d=1}^{q^d}\frac{s_{p^d k}^{(t-1,t)-}}{Z_{p^d k}^{(t-1,t)}}+\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}}\frac{s_{p^{(d-1,d)}k}^{(t)-}}{Z_{p^{(d-1,d)}k}^{(t)}}\right)\right]\right\}}{\sum_{d=1}^{D}\left\{\sum_{t=1}^{T}\left[1+\frac{1}{s^d+q^d+q^{(d,d+1)}}\cdot\left(\sum_{r^d=1}^{s^d}\frac{s_{r^d k}^{(t)+}}{Y_{r^d k}^{(t)}}+\sum_{p^d=1}^{q^d}\frac{s_{p^d k}^{(t,t+1)+}}{Z_{p^d k}^{(t,t+1)}}+\sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}}\frac{s_{p^{(d,d+1)}k}^{(t)+}}{Z_{p^{(d,d+1)}k}^{(t)}}\right)\right]\right\}}$$

$s.t.$

$$X_{i^d k}^{(t)} = \sum_{j=1}^{n}\lambda_{dj}^{(t)}X_{i^d j}^{(t)} + s_{i^d k}^{(t)-}, \qquad d=1,2,\cdots,D; i^d=1,2,\cdots,m^d; t=1,2,\cdots,T$$

$$Z_{p^d k}^{(t-1,t)} = \sum_{j=1}^{n}\lambda_{dj}^{(t)}Z_{p^d j}^{(t-1,t)} + s_{p^d k}^{(t-1,t)-}, \qquad d=1,2,\cdots,D; p^d=1,2,\cdots,q^d; t=1,2,\cdots,T$$

$$Z_{p^{(d-1,d)}k}^{(t)} = \sum_{j=1}^{n}\lambda_{dj}^{(t)}Z_{p^{(d-1,d)}j}^{(t)} + s_{p^{(d-1,d)}k}^{(t)-}, \quad d=1,2,\cdots,D; p^{(d-1,d)}=1,2,\cdots,q^{(d-1,d)}; t=1,2,\cdots,T$$

$$Y_{r^d k}^{(t)} = \sum_{j=1}^{n}\lambda_{dj}^{(t)}Y_{r^d j}^{(t)} - s_{r^d k}^{(t)+}, \qquad d=1,2,\cdots,D; r^d=1,2,\cdots,s^d; t=1,2,\cdots,T$$

$$Z_{p^d k}^{(t,t+1)} = \sum_{j=1}^{n}\lambda_{dj}^{(t)}Z_{p^d j}^{(t,t+1)} - s_{p^d k}^{(t,t+1)+}, \qquad d=1,2,\cdots,D; p^d=1,2,\cdots,q^d; t=1,2,\cdots,T$$

$$Z_{p^{(d,d+1)}k}^{(t)} = \sum_{j=1}^{n}\lambda_{dj}^{(t)}Z_{p^{(d,d+1)}j}^{(t)} - s_{p^{(d,d+1)}k}^{(t)+}, \quad d=1,2,\cdots,D; p^{(d,d+1)}=1,2,\cdots,q^{(d,d+1)}; t=1,2,\cdots,T$$

$$0 \le \lambda_{dj}^{(t)}, s_{i^d k}^{(t)-}, s_{p^d k}^{(t-1,t)-}, s_{p^{(d-1,d)}k}^{(t)-}, s_{r^d k}^{(t)+}, s_{p^d k}^{(t,t+1)+}, s_{p^{(d,d+1)}k}^{(t)+}, \quad t=1,2,\cdots,T; j=1,2,\cdots,n$$

(36)

In this framework, the formulation of dynamic network SBM under VRS assumption is relative simple, where only $\sum_{j=1}^{n}\lambda_{dj}^{(t)}=1$ ($t=1,2,\cdots,T; d=1,2,\cdots,D$) are added. After the optimal solution $(s_{i^d k}^{(t)*-}, s_{p^d k}^{(t-1,t)*-}, s_{p^{(d-1,d)}k}^{(t)*-}, s_{r^d k}^{(t)*+}, s_{p^d k}^{(t,t+1)*+}, s_{p^{(d,d+1)}k}^{(t)*+}, \lambda_{dj}^{(t)*})$ is obtained, the overall efficiency, E_k^s , period t efficiency, $E_k^{(t)}$, and division d efficiency, $E_k^{(d)}$ are respectively calculated as

$$E_k^{(t)} = \frac{\sum_{d=1}^{D}\left[1-\frac{1}{m^d+q^d+q^{(d-1,d)}}\cdot\left(\sum_{i^d=1}^{m^d}\frac{s_{i^d k}^{(t)*-}}{X_{i^d k}^{(t)}}+\sum_{p^d=1}^{q^d}\frac{s_{p^d k}^{(t-1,t)*-}}{Z_{p^d k}^{(t-1,t)}}+\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}}\frac{s_{p^{(d-1,d)}k}^{(t)*-}}{Z_{p^{(d-1,d)}k}^{(t)}}\right)\right]}{\sum_{d=1}^{D}\left[1+\frac{1}{s^d+q^d+q^{(d,d+1)}}\cdot\left(\sum_{r^d=1}^{s^d}\frac{s_{r^d k}^{(t)*+}}{Y_{r^d k}^{(t)}}+\sum_{p^d=1}^{q^d}\frac{s_{p^d k}^{(t,t+1)*+}}{Z_{p^d k}^{(t,t+1)}}+\sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}}\frac{s_{p^{(d,d+1)}k}^{(t)*+}}{Z_{p^{(d,d+1)}k}^{(t)}}\right)\right]} \quad (37)$$

$$E_k^{(d)} = \frac{\sum_{t=1}^{T}\left[1 - \frac{1}{m^d + q^d + q^{(d-1,d)}} \cdot \left(\sum_{i^d=1}^{m^d} \frac{s_{i^d k}^{(t)*-}}{X_{i^d k}^{(t)}} + \sum_{p^d=1}^{q^d} \frac{s_{p^d k}^{(t-1,t)*-}}{Z_{p^d k}^{(t-1,t)}} + \sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} \frac{s_{p^{(d-1,d)} k}^{(t)*-}}{Z_{p^{(d-1,d)} k}^{(t)}} \right)\right]}{\sum_{t=1}^{T}\left[1 + \frac{1}{s^d + q^d + q^{(d,d+1)}} \cdot \left(\sum_{r^d=1}^{s^d} \frac{s_{r^d k}^{(t)*+}}{Y_{r^d k}^{(t)}} + \sum_{p^d=1}^{q^d} \frac{s_{p^d k}^{(t,t+1)*+}}{Z_{p^d k}^{(t,t+1)}} + \sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} \frac{s_{p^{(d,d+1)} k}^{(t)*+}}{Z_{p^{(d,d+1)} k}^{(t)}} \right)\right]} \quad (38)$$

$$E_k^{(td)} = \frac{1 - \frac{1}{m^d + q^d + q^{(d-1,d)}} \cdot \left(\sum_{i^d=1}^{m^d} \frac{s_{i^d k}^{(t)*-}}{X_{i^d k}^{(t)}} + \sum_{p^d=1}^{q^d} \frac{s_{p^d k}^{(t-1,t)*-}}{Z_{p^d k}^{(t-1,t)}} + \sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} \frac{s_{p^{(d-1,d)} k}^{(t)*-}}{Z_{p^{(d-1,d)} k}^{(t)}} \right)}{1 + \frac{1}{s^d + q^d + q^{(d,d+1)}} \cdot \left(\sum_{r^d=1}^{s^d} \frac{s_{r^d k}^{(t)*+}}{Y_{r^d k}^{(t)}} + \sum_{p^d=1}^{q^d} \frac{s_{p^d k}^{(t,t+1)*+}}{Z_{p^d k}^{(t,t+1)}} + \sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} \frac{s_{p^{(d,d+1)} k}^{(t)*+}}{Z_{p^{(d,d+1)} k}^{(t)}} \right)} \quad (39)$$

Since the inefficiencies on both inputs and outputs of the whole dynamic system are equal to the sum of the inefficiencies on inputs and outputs overall *TP* period-division processes, there is

$$\begin{aligned}
&\sum_{d=1}^{D}\left\{\sum_{t=1}^{T}\left[\frac{1}{s^d + q^d + q^{(d,d+1)}} \cdot \left(\sum_{r^d=1}^{s^d} \frac{s_{r^d k}^{(t)*+}}{Y_{r^d k}^{(t)}} + \sum_{p^d=1}^{q^d} \frac{s_{p^d k}^{(t,t+1)*+}}{Z_{p^d k}^{(t,t+1)}} + \sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} \frac{s_{p^{(d,d+1)} k}^{(t)*+}}{Z_{p^{(d,d+1)} k}^{(t)}} \right)\right]\right\} \\
&+\sum_{d=1}^{D}\left\{\sum_{t=1}^{T}\left[\frac{1}{m^d + q^d + q^{(d-1,d)}} \cdot \left(\sum_{i^d=1}^{m^d} \frac{s_{i^d k}^{(t)*-}}{X_{i^d k}^{(t)}} + \sum_{p^d=1}^{q^d} \frac{s_{p^d k}^{(t-1,t)*-}}{Z_{p^d k}^{(t-1,t)}} + \sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} \frac{s_{p^{(d-1,d)} k}^{(t)*-}}{Z_{p^{(d-1,d)} k}^{(t)}} \right)\right]\right\} \\
&= \sum_{d=1}^{D}\left\{\sum_{t=1}^{T}\left[\begin{aligned}&\frac{1}{s^d + q^d + q^{(d,d+1)}} \cdot \left(\sum_{r^d=1}^{s^d} \frac{s_{r^d k}^{(t)*+}}{Y_{r^d k}^{(t)}} + \sum_{p^d=1}^{q^d} \frac{s_{p^d k}^{(t,t+1)*+}}{Z_{p^d k}^{(t,t+!)}} + \sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}} \frac{s_{p^{(d,d+1)} k}^{(t)*+}}{Z_{p^{(d,d+1)} k}^{(t)}} \right) \\ &+\frac{1}{m^d + q^d + q^{(d-1,d)}} \cdot \left(\sum_{i^d=1}^{m^d} \frac{s_{i^d k}^{(t)*-}}{X_{i^d k}^{(t)}} + \sum_{p^d=1}^{q^d} \frac{s_{p^d k}^{(t-1,t)*-}}{Z_{p^d k}^{(t-1,t)}} + \sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}} \frac{s_{p^{(d-1,d)} k}^{(t)*-}}{Z_{p^{(d-1,d)} k}^{(t)}} \right)\end{aligned}\right]\right\}
\end{aligned} \quad (40)$$

It is converted into:

$$
\begin{aligned}
&\sum_{d=1}^{D}\left\{\sum_{t=1}^{T}\left[1+\frac{1}{s^d+q^d+q^{(d,d+1)}}\cdot\left(\sum_{r^d=1}^{s^d}\frac{s_{r^dk}^{(t)*+}}{Y_{r^dk}^{(t)}}+\sum_{p^d=1}^{q^d}\frac{s_{p^dk}^{(t,t+1)*+}}{Z_{p^dk}^{(t,t+1)}}+\sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}}\frac{s_{p^{(d,d+1)}k}^{(t)*+}}{Z_{p^{(d,d+1)}k}^{(t)}}\right)\right]\right\}\\
&-\sum_{d=1}^{D}\left\{\sum_{t=1}^{T}\left[1-\frac{1}{m^d+q^d+q^{(d-1,d)}}\cdot\left(\sum_{i^d=1}^{m^d}\frac{s_{i^dk}^{(t)*-}}{X_{i^dk}^{(t)}}+\sum_{p^d=1}^{q^d}\frac{s_{p^dk}^{(t-1,t)*-}}{Z_{p^dk}^{(t-1,t)}}+\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}}\frac{s_{p^{(d-1,d)}k}^{(t)*-}}{Z_{p^{(d-1,d)}k}^{(t)}}\right)\right]\right\}\\
&=\sum_{d=1}^{D}\left\{\sum_{t=1}^{T}\left[\begin{array}{l}\left[1+\frac{1}{s^d+q^d+q^{(d,d+1)}}\cdot\left(\sum_{r^d=1}^{s^d}\frac{s_{r^dk}^{(t)*+}}{Y_{r^dk}^{(t)}}+\sum_{p^d=1}^{q^d}\frac{s_{p^dk}^{(t,t+1)*+}}{Z_{p^dk}^{(t,t+1)}}+\sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}}\frac{s_{p^{(d,d+1)}k}^{(t)*+}}{Z_{p^{(d,d+1)}k}^{(t)}}\right)\right]\\-\left[1-\frac{1}{m^d+q^d+q^{(d-1,d)}}\cdot\left(\sum_{i^d=1}^{m^d}\frac{s_{i^dk}^{(t)*-}}{X_{i^dk}^{(t)}}+\sum_{p^d=1}^{q^d}\frac{s_{p^dk}^{(t-1,t)*-}}{Z_{p^dk}^{(t-1,t)}}+\sum_{p^{(d-1,d)}=1}^{q^{(d-1,d)}}\frac{s_{p^{(d-1,d)}k}^{(t)*-}}{Z_{p^{(d-1,d)}k}^{(t)}}\right)\right]\end{array}\right]\right\}
\end{aligned}
\tag{41}
$$

Dividing both sides by

$$\sum_{d=1}^{D}\left\{\sum_{t=1}^{T}\left[1+\frac{1}{s^d+q^d+q^{(d,d+1)}}\cdot\left(\sum_{r^d=1}^{s^d}\frac{s_{r^dk}^{(t)*+}}{Y_{r^dk}^{(t)}}+\sum_{p^d=1}^{q^d}\frac{s_{p^dk}^{(t,t+1)*+}}{Z_{p^dk}^{(t,t+1)}}+\sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}}\frac{s_{p^{(d,d+1)}k}^{(t)*+}}{Z_{p^{(d,d+1)}k}^{(t)}}\right)\right]\right\}$$ results

in:

$$1-E_k^s=\sum_{d=1}^{D}\sum_{t=1}^{T}\omega^{(td)}\left(1-E_k^{(td)}\right) \tag{42}$$

where the weight

$$\omega_k^{(td)}=\frac{\left[1+\frac{1}{s^d+q^d+q^{(d,d+1)}}\cdot\left(\sum_{r^d=1}^{s^d}\frac{s_{r^dk}^{(t)*+}}{Y_{r^dk}^{(t)}}+\sum_{p^d=1}^{q^d}\frac{s_{p^dk}^{(t,t+1)*+}}{Z_{p^dk}^{(t,t+1)}}+\sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}}\frac{s_{p^{(d,d+1)}k}^{(t)*+}}{Z_{p^{(d,d+1)}k}^{(t)}}\right)\right]}{\sum_{d=1}^{D}\left\{\sum_{t=1}^{T}\left[1+\frac{1}{s^d+q^d+q^{(d,d+1)}}\cdot\left(\sum_{r^d=1}^{s^d}\frac{s_{r^dk}^{(t)*+}}{Y_{r^dk}^{(t)}}+\sum_{p^d=1}^{q^d}\frac{s_{p^dk}^{(t,t+1)*+}}{Z_{p^dk}^{(t,t+1)}}+\sum_{p^{(d,d+1)}=1}^{q^{(d,d+1)}}\frac{s_{p^{(d,d+1)}k}^{(t)*+}}{Z_{p^{(d,d+1)}k}^{(t)}}\right)\right]\right\}},$$
$$t=1,2,\cdots,T;d=1,2,\cdots,D \tag{43}$$

Clearly, there is the equal relationship, $\sum_{t=1}^{T}\omega_k^{(t)}=1$, so it is obtained:

$$E_k^s=\sum_{t=1}^{T}\omega^{(td)}E_k^{(td)} \tag{44}$$

We can further obtain an additive decomposition of the overall efficiency score into period ones (see (45)) with weights (see (46)),

$$E_k^s = \sum_{t=1}^{T} \omega^{(t)} E_k^{(t)} \tag{45}$$

$$\omega^t = \sum_{d=1}^{D} \omega^{(td)}, t = 1, 2, \cdots, T \tag{46}$$

and division ones (see (47)) with weights (see (48)).

$$E_k^s = \sum_{d=1}^{D} \omega^{(d)} E_k^{(d)} \tag{47}$$

$$\omega^d = \sum_{t=1}^{T} \omega^{(td)}, d = 1, 2, \cdots D \tag{48}$$

This shows that dynamic SBM model by our formulating approach can produce an expected weighted average relationship between the overall efficiency score and period-division ones without depending on pre-specified weights, which is different with the dynamic network SBM proposed by Tone and Tsutsui (2014).

Note that models (27) and (37) need uniqueness check of component efficiency scores in the practical applications. The post-programming approach discussed in section 3 is still applicable.

7. Conclusions

In this paper, we proposed a formulating approach of dynamic network DEA models without a pre-specified weights set combining component efficiencies into the overall efficiency. This approach can be used to formulate the radial dynamic network models associated with CCR and BCC measures, and also formulate the non-radial and non-oriented dynamic network model associated with SBM. For comparison, we extend the relational dynamic DEA model in Kao (in press) to dynamic network systems.

We find that it is attractive to fully incorporate the production information embedded on carry-overs connecting consecutive periods and linkers connecting consecutive divisions into the objective function determining the efficiency score measure of the overall efficiency. As expected, our formulating approach produces a weighted average decomposition of the overall efficiency score into component ones in contrast to Kao's (in press) relational approach by a set of endogenous weights which are generated automatically based on statistical data from the most favorable perspective for the tested multi-period and -division system like the multipliers on inputs and outputs. This is different with Tone and Tsutsui (2014), where a set of pre-specified weights is exogenously supplied. If the efficiency results are sensitive to the change of weights, it is difficult to specify the exogenous weights agreeable to all DMUs. We have to argue that our modeling approach is not straightforward extension of relevant studies about network DEA modeling approaches (e.g., Chen et al., 2009 and Cook et al., 2010), and there is essential difference between our modeling approach and theirs although two modeling approaches produce a same weighted average combination relationship between the overall efficiency score and component ones. In contrast to their approach, our approach need not to depend on a set of pre-specified weights to combine component efficiency scores when formulating the overall efficiency score. However, their weights are pre-specified.

Another novelty of this paper is that the dynamic network DEA modeling technique is introduced to evaluate the dynamic network process of innovation based on the dataset of OECD countries. Our empirical results indicate that there is a mismatching relationship between R&D efficiency and application one at the average level of OECD countries. Another interesting finding is that the innovation inefficiency mainly originates from the inefficiency in the application process. The empirical example shows that our dynamic DEA models also has a good discrimination capability of efficiency scores, and furthermore presents logical estimation of efficiency scores in terms of the comparison between overall and period efficiency scores.

In view of future work, some important research subjects include uniqueness check of period efficiency scores and identification of returns to scale statuses. To be exciting, our modeling approach is not subject to the structure of dynamic network systems. This means that it can be extended to various situations such as the existence of shared inputs or outputs.

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