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# The Illiquidity of Water Markets\*

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## Abstract

We explore a particular historical episode that switched from a market institution (auctions) to a non-market institution (fixed quotas with a ban on trading) to allocate water. This water is used by farmers for agricultural purposes; some of the farmers are liquidity constrained. We present a model in which farmers face liquidity constraints to explain why the change took place. From a positive perspective, we show that demand is underestimated if these liquidity constraints are not taken into account. We use a dynamic discrete choice model to estimate demand during the auction period; we also estimate the probability of being liquidity constrained by a farmer. From a normative perspective, auctions achieve the first-best allocation only in the absence of liquidity constraints; the quota achieves the first best allocation only if farmers are homogeneous in productivity. We compute the welfare under both institutions using the estimated parameters of the structural model.

**JEL Codes:** L23, D02, D53, G14, Q25

**Keywords:** Organization of Production, Institutions, Financial Markets, Market Efficiency, Water

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# 1 Introduction

Water regulations are central in policy discussions in many regions in the world today. Seventy percent of fresh water consumption worldwide is used for irrigation. Water is becoming increasingly scarce at an accelerated rate in regions such as India, Latin America and, more recently, the U.S. (Barnett et al., 2005). Water markets are emerging as a preferred institution in the developed world, in particular in dry regions of the U.S. and Australia (Grafton et al., 2011). In these cases, the implementation of markets increases efficiency because the users are heterogeneous in demand and gains from trade are realized. However, there is controversy about their efficiency in general (Johansson, 2000). In particular, when farmers are relatively homogeneous in demand (i.e., there are little gains from trading to be realized) and when farmers face liquidity constraints (LC) (i.e., they might not have the cash to pay for the water when they want to buy), there might be other mechanisms that perform better than markets.

Water markets, where they are used, are usually heavily regulated. These regulations and the overlap of public and private water rights imply that we rarely see undistorted water markets to estimate water demand. As noted by Libecap (2011), price differences signal gains from trade, but such comparisons are difficult due to barriers across districts and different regulatory frameworks.

In this paper we look at a centralized free market, within a stable regulatory framework, that was in place for over 700 years. Citizens in the Spanish town of Mula ran auctions to allocate (scarce) water from the river among farmers beginning in the Middle Ages, soon after the Christians recovered the city from the Muslims, in 1244 (see Espín-Sánchez, 2013, for the details on the institutional persistence and change). In 1966, the auction was replaced by a system of fixed quotas: each farmer owning a plot of land near the water channel was entitled to some water for irrigation, in proportion to the size of their plot.

This institutional change might be puzzling for economists, who regard auctions as an “ideal” allocation mechanism. This change is even more puzzling if we consider that most of the other towns in the region employed the quota system for centuries: the auction system was the oddity, not the rule. Moreover, the farmers of Mula were happy to end it (González Castaño and Llamas Ruiz, 1991).

Contemporaneous observers did not agree on whether the auctions were efficient or not. The historian Musso y Fontes (1847) argued that auctions were unequivocally good: “When the farmer irrigates for free, he demands a lot of water. When he is paying for the water, he demands as little as possible. With the auction, the allocation [of water] occurs at the proper level.” However, we should take this opinion

with a grain of salt, since Musso y Fontes owned water property rights. A more neutral commentator, Juan Subercase (1783-1856), Director of the National Engineering School, stated: “[The Waterlords sell the water] piece by piece, during the critical season when the crops are at risk, speculating [...] over the desperate and distressed farmer, who is willing to make the highest sacrifice in order to get a drop of water” (cited by Muñoz, 2001), suggesting that auctions are a bad way to allocate water during the “critical” season.<sup>1</sup>

Katherine Coman’s “Some Unsettled Problems of Irrigation” (1911), the lead article in the first issue of the *American Economic Review*, also hinted at the problem with the market during the critical season: “In southern Spain, where this system obtains and water is sold at auction, the water rates mount in a dry season to an all but prohibitive point.” Meaning that during the dry season only wealthy farmers could afford to buy water. Since poor farmers would also benefit from buying water during the dry season, one plausible theoretical explanation is that poor farmers faced LC. We indeed find that “poor” farmers buy less water during the critical season (when prices are higher) than “wealthy” farmers with the same crop type and number of trees.

We propose a theoretical model in which water for irrigation has diminishing returns and farmers are heterogeneous in both their productivity and their ability to pay for the water (cash holdings). We show that when farmers do not face LC, an auction system achieves the first-best (FB) allocation. However, when farmers are homogeneous in their productivity, a fixed quota system will achieve the FB allocation.<sup>2</sup> In general, farmers are heterogeneous in their productivity and some farmers might face LC. In this general case the relative efficiency of both institutions is ambiguous. It is then an empirical question to assess which of these institutions is more efficient.

The interaction between the LC and the strategic timing of purchases implies that liquidity constrained farmers are more likely to buy off-season than unconstrained farmers. In order to estimate demand, we account for inter-temporal substitution. Water increases the moisture of the land, thus reducing future demand. Hence, irrigation demand is similar to demand for durable or storable goods. Hendel and Nevo (2011) study inter-temporal price discrimination with unobserved inventory. They find that storability creates incentives for consumers to strategically delay their purchases in order to benefit from future price reductions. In our case, in addition to the strategic delay in purchases, there is also strategic anticipation

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<sup>1</sup>Both translations from the original in Spanish are ours.

<sup>2</sup>In particular, there are two limiting cases. If capital markets are perfect or all farmers are sufficiently wealthy, then the auction system achieves FB. If farmers are completely homogeneous, i.e., they have the exact same production function, then the quota system achieves the FB allocation. If all farmers are homogeneous in their productivity and are sufficiently wealthy, both mechanisms achieve the FB allocation (see sub-section 3.3 for details).

in purchases due to LC. Wealthy farmers strategically delay their purchases and buy (expensive) water if it does not rain. Poor farmers who expect an increase in prices during the harvest season will buy beforehand, since they could not afford water during the harvest season.

The data from water auctions in Mula is ideal because we observe detailed individual characteristics, both financial and demand-related. Moreover, we study the market for an intermediate good, thus we can disregard income effects on the demand for water, i.e., the demand for water is independent of the income (or wealth) of the farmer as long as the farmer has enough cash to pay for the water. We study the last years of the auction system in order to estimate demand in the presence of LC. We show that not accounting for the constraints biases estimated demand downward. We use the estimated demand to show that quotas outperformed the auctions due to LC.

The evolution over time of land moisture is a key determinant for demand, but it is not directly observable. However, we observe both rain and irrigation and we apply results from the agricultural engineering literature to construct a moistness variable for each farmer. After controlling for moistness, crop type and number of trees, productivity is assumed to be homogeneous up to an idiosyncratic shock across farmers. This assumption helps us to identify the other source of heterogeneity, liquidity constraints.

Our identification strategy is straightforward. Wealthy farmers face no LC while poor farmers might not have the cash needed nor have access to credit. Estimating demand with data on all farmers results in a demand underestimation. Wealthy farmers demand water without any constraints. Hence, we estimate the demand parameters of the model using data on wealthy farmers. We employ conditional choice probability (CCP) estimation (Hotz and Miller, 1993). We use the estimated demand parameters and the data on poor farmers to estimate the financial parameters of the model. There are two observed sources of financial heterogeneity that we exploit: real estate value and revenue from the harvest of the previous year. There might also be unobserved sources of financial heterogeneity. Our econometric estimation is flexible enough to take these sources of unobserved heterogeneity into account.

We use the estimated demand parameters to compute welfare under the quota system. We compute several counterfactual scenarios in order to decompose the changes in efficiency due to different factors. We conclude that the institutional change improved efficiency. In the intensive margin, the presence of LC and the smaller sized units used in the quota improved efficiency while the fact that farmers cannot choose when to irrigate with the quotas reduced efficiency. There was a net increase in efficiency, especially for the poor farmers. In the extensive margin, the quota improved efficiency because it allows farmers to undertake risky investments (trees) without risk. This is a direct consequence of the structure of the

quotas.

By exploring a particular historical episode, in which an institutional change from a market mechanism (auctions) to a non-market mechanism (quotas) took place, we compare the two allocation systems in the presence of LC. We combine a new model in which agents face LC with a novel data set (with detailed information regarding both financial and demand aspects). We propose a new structural estimation method to identify demand and LC. We estimate whether the output generated by the quota system is greater than the output generated by the auction system.

## 1.1 Literature Review

This review is not intended to be exhaustive but rather to present the reader with the most representative references from each field to which this paper is related.

Scholars studying the efficiency of irrigation communities in Spain have proposed two competing hypotheses to explain the duality in institutions. On the one hand, Glick (1967) and Anderson and Mass (1978) claim that the auctions are more efficient if we do not take into account operational costs. According to them, we observe both systems because the less efficient system (quotas) is simpler and easier to maintain. This hypothesis is based on the fact that we observe auctions in places where water is extremely scarce (Musso y Fontes, 1847; Pérez Picazo and Lemeunier, 1985). However, it has an important flaw: the size of the land used for irrigation is, at least in part, endogenous. Farmers could increase the land designated for irrigation (*regadío*) if needed. Hence, the causation could go the other way: in places with auctions, the owners of the water would allow for more lands to be irrigated, than in places with quotas. Ruiz Funés (1916) also shares this criticism.

On the other hand, both contemporaneous and current historians who study the traditional organizations of the *Huertas* (irrigated orchards) in Spain take a different approach.<sup>3</sup> They argue that the owners of the water rights had political power and were concerned only about their revenues, regardless of the overall efficiency of the system. This might be the reason why Mula and Lorca (both cities of Murcia, Spain) established different institutions than neighboring cities. Politics might have differed in these two towns during the 13 – 15<sup>th</sup> centuries due to their strategic position on the border between a Christian and a Muslim kingdom and due to military rule.

Along the same lines, Garrido (2011) has claimed that auctions were used in places where the local elite was powerful. Therefore, we would expect a quota system only when/if the local elite is not powerful

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<sup>3</sup>Contemporaneous historians include Aymard (1864), Passa (1844), Díaz Cassou (1889) and Brunhes (1902). Current historians include González Castaño and Llamas Ruiz (1991) and Gil Olcina (1994).

(Acemoglu and Robinson, 2008). As Rodriguez Llopis (1998) pointed out, the institutional configuration in place in each town by the end of the Middle Ages was the outcome of the tensions between the Crown, the Castilian aristocracy, the regional nobility and the local elites since the 13<sup>th</sup> century. Nonetheless, none of these scholars have considered that auctions might be less efficient than quotas. Hence, none of these hypotheses can explain the change from auctions to quotas, unless there was a shift in political power.

Elinor Ostrom (1990, 1992) extensively studied self-governed irrigation communities. However, both institutions are self-governed and self-regulated (see also Ellickson, 1991; Posner, 2000). Hence, self-governance is not of interest here, so, we focus on the relative efficiency of both institutions. Our paper is also related to the recent literature on institutional persistence (Guiso, Sapienza and Zingales, 2008; Jha, 2012) and competing institutions (Greif, 2006).

The theoretical literature on auctions with LC is recent (Maskin, 2000; Pai and Vohra, 2008). Che, Gale and Kim (2012) assume that agents can consume at most one unit of the good with linear utility in their type. They conclude that markets are always more efficient than quotas. We instead consider a model in which agents can consume as much as they want and the utility function is concave. In our setting there is no strict ranking between markets and quotas.

The empirical literature on the effect of LC in auction settings is non-existent. There are, however, some papers in the industrial organization literature that use supermarket data and use constraints similar to ours in their estimation. Gilbride and Allenby (2004) propose an estimator for a two stage decision process. In the first stage, agents set a maximum price they will pay. In the second stage, agents choose among all objects with a price lower than the threshold. Pires and Salvo (2013), using a similar method, find that low income households buy smaller sized storable products (detergent, toilet paper, etc.) than high income households, even though smaller sized products are more expensive per pound. They attribute this puzzle to low income households being liquidity constrained.

In addition to LC, we estimate a dynamic demand model with seasonal demand and storable goods. Storability and the implications that inter-temporal substitution has on demand estimation have been studied by Boizot et al. (2001), Pesendorfer (2002) and Hendel and Nevo (2006a). Gowrisankaran and Rysman (2012) estimate a model with new durable goods, and seasonal demand and found the same incentives for strategic delay as in Hendel and Nevo (2011). None of these papers addresses LC. As far as we know, ours is the first paper that proposes and estimates a model with durable/storable goods, seasonal demand and LC.

There are not many papers in the dynamic demand estimation literature that deal with water markets.

Timmins (2002) is the closest to our paper but he estimates demand for urban consumption and not for irrigation. He uses parameters from the engineering literature to estimate the supply of water, while we use parameters from the agricultural engineering literature to determine the demand structure as well as the evolution of the moisture in the farmer's plot (see Appendix A.2).

## 2 Historical background and Data

### 2.1 History and Origins<sup>4</sup>

During the reign of Ibn Hud (1228-1238), the Kingdom of Murcia enjoyed some prosperity and stability. When Ibn Hud was murdered in 1238, the kingdom was dismembered. This same year Jaime I (King of Aragon) conquered Valencia and prepared to march south. Castile was also advancing to the south, expanding its territory at the expense of the now fragile Kingdom of Murcia. By 1242, Castile had conquered most of the Kingdom. Ahmed, the son of Ibn Hud, traveled to Alcaraz (Toledo) to meet the (then) prince Alfonso. They agreed that what remained of the Kingdom of Murcia would become a protectorate of Castile.

The cities of Cartagena, Mula and Lorca rejected the agreement. In April 1244, Alfonso was in Murcia with his army ready to attack Mula (the closest of the three rebel cities). After Mula was conquered, the army moved to Lorca, which surrendered by the end of June. The government of Mula and Lorca was given to the Order of Santiago, while the government of the city of Murcia was given, in part, to the descendants of Ibn Hud according to the terms of the Alcaraz Treaty. However, the rulers of Castile had absolute authority on the cities of Mula and Lorca, since those were conquered by force.<sup>5</sup> As with much of the Spanish *Reconquista*, Christian populations were brought to the area with the goal of establishing a Christian base. Hence, the new Christian settlers in Mula started *tabula rasa* and created new institutions.<sup>6</sup>

Mula and Lorca were both frontier cities between a Christian kingdom and a Muslim kingdom, and, until the conquest of Granada in 1492, were in a constant state of war. This meant that the City Council was always in need of money, even though as a frontier city they were exempted from paying taxes to the Crown. The Council permitted the separation of ownership of water and land and eliminated the ban imposed on water rights' trading. There is no exact date for this process for the city of Mula, although this probably happened during the middle third of the 13<sup>th</sup> century, since the first document that explicitly

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<sup>4</sup>This section is based on Rodriguez Llopis (1998).

<sup>5</sup>This event had, as a consequence, stronger reprisals taken against the (mostly Muslims) citizens of Mula, which increased the local demand for new Christians settlers.

<sup>6</sup>Notice that this initial shock in institutions is similar to that in Chaney (2008).



shows evidence about auctions dates from this time. We know that this was indeed the case for Lorca (Musso y Fontes, 1847).<sup>7</sup>

After that, the owners of the water property rights (Waterlords) were clearly different persons than the land-owners (farmers). The Waterlords then established a well-functioning cartel. The situation did not change during the pre-modern era, despite the many political changes that occurred in Spain. It was not until the 19<sup>th</sup> century (with the creation of the 1843 ordinances) that the cartel was formalized under the name of *Heredamiento de Aguas*.<sup>8</sup> The land-owners were small proprietors, with family-size plots, who soon after created their own association, *Sindicato de Regantes*. The aim of this association was to regulate and settle disputes that arose between neighbors, as well as to keep the balance of power in the market for water.

## 2.2 Environment

Southeastern Spain is the most arid region of Europe. It is located on the east of a mountain chain (the Prebaetic System, which includes the Mulhacen, the second highest mountain in Europe).<sup>9</sup> The rainfall frequency distribution is skewed: most years are dryer than the average. The number of days of torrential rain is not very high but when they occur, they can reach high intensity (for example, 681 millimeters (*mm*) of water fell in Mula on one day, 10<sup>th</sup> October 1943, while the yearly average in Mula is 320 *mm*). Summers are dry and rain occurs mostly during fall and spring. Insolation is very high, with more than 3,000 hours of solar exposure per year, the highest in Europe. Despite the fact that this region is dry, rivers flowing down the Prebaetic System provide the region with the water needed for irrigation.

Weekly prices for water are very volatile and depend mostly on the season and rain. However, rain is hard to predict accurately, making the need for cash hard to predict months in advance. Additionally, water demand is seasonal, being especially high during the weeks before the harvest when the fruit is growing most rapidly. Farmers sell their output after the harvest, and only then have cash. Hence, the weeks when the farmers most need the cash to pay for the water are precisely the weeks farthest from the last harvest. As a consequence, they might be liquidity constrained.

Given that demand is seasonal, the farmers should take into account the joint dynamics of their demand and that of the prices, when making purchasing decisions. The farmers should notice that water today is

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<sup>7</sup>However, as noted by Rodriguez Llopis (1998), documents at the time of the conquest of Lorca (1244) suggest that the separation between land and water ownership happened right after the conquest.

<sup>8</sup>According to Rodriguez-Llopis (1998), the formalization was a response to the threat of broad disentailments and confiscations from the central government during the 19<sup>th</sup> century.

<sup>9</sup>The Prebaetic System is located on Southeastern Spain.

an imperfect substitute for water tomorrow and, adjusting for the current price and their expectation about future prices, act accordingly. This problem becomes even more complicated when the farmer might be facing liquidity constraints (LC) in the future. A farmer who expects to be liquidity constrained during the harvest season (when her demand is highest), might decide to buy water several weeks before the harvest, when the price is lower. This is true even if she would have bought during the harvest season had she known that she would not be constrained.

A farmer who expects to be constrained in the future would try to borrow money. However, even if a credit market is in place, she might not get the loan she needs. In the presence of limited liability (the farmer is poor) and non-enforceable contracts, endogenous borrowing constraints emerge (see sub-section 3.4). Hence, even if a credit market exists, non-enforceable contracts would prevent the farmer from having the cash when she needs it most. It is irrelevant in our case whether the credit market existed or not. What matters is whether the farmers have access to it and whether the LC affected their behavior. Personal interviews with surviving farmers confirm that farmers were usually constrained (they have less cash than needed to buy the water they demanded) yet they do not borrow money from others.

## 2.3 Institutions

In this sub-section, we describe two different institutions/mechanisms used in several cities of Southeastern Spain to allocate water from the river.

**Auctions** Although the process of allocating water in Mula has varied slightly over the years, it is remarkable that its basic structure has been unchanged since the 15<sup>th</sup> century. The mechanism to allocate water to those farmers is a sequential English-auction. The auctioneer sells each of the units sequentially and independently from each other, keeping track of the name of the buyer of every unit and the price. The units bought need to be paid in cash the day of the auction.<sup>10</sup>

The basic selling unit is a cuarta (quarter): the right to the water that flows through the main channel during three hours at a specific date and time. The property rights of water and land are independent: some people are the Waterlords (that is, they own the right to use the water flowing through the channel) and some people are the land-owners. The Waterlords will meet once a week and decide how many units of water are going to be sold.

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<sup>10</sup>Allowing the farmer to pay after the harvest would mitigate the problems created by the LC, and would increase the revenue obtained in the auction. The fact that the payment should be made in cash and reasons explained in the ordinances, suggest that the water owners were concerned with not getting their money back after the harvest, i.e., contracts were not enforceable.

Water storage is done at the main dam (*Embalse de La Cierva*).<sup>11</sup> Water will be delivered through a system of channels to the farmer's plot. Each unit corresponded to the right to use the water flowing from the river for three hours. Water flows from the dam through the channels at 40 liters per second (*l/s*). As a result, one unit carries 432,000 liters of water. During our sample period auctions were carried out once a week, every Friday.

In every session, forty units were auctioned: four units for irrigation during the day (from 7:00 AM to 7:00 PM) and four units for irrigation during the night (from 7:00 PM to 7:00 AM), every weekday (Monday to Friday). The auctioneer sells, first, twenty units corresponding to the night-time and, afterwards, twenty units corresponding to the day-time. Within each of these groups (day and night), units are sold starting from Monday (four units), and finishing with Friday's units.

**Quotas** Our sample consist of all water auctions in Mula from January 1955 until July 1966, when the last auction was run. On August 1, 1966 the allocation system was modified from being an auction allocation system to a two-sided bargaining system. In the bargaining system, the *Heredamiento the Aguas* (water-owners) and the *Sindicato de Regantes* (land-owners association) arranged a fixed price (renegotiated at the beginning of every six months) for the water. Gradually, the *Sindicato de Regantes* bought shares in the *Heredamiento the Aguas* association until they finally merged in 1974.<sup>12</sup> Since 1966, the *Sindicato de Regantes* allocates the water to each farmer following a fixed quota.

Under this system, water ownership is tied to land ownership. Every plot of land has assigned some amount of time of irrigation during each *tanda* (quota) and every *tanda* lasts three weeks. The amount of time allocated to every farmer is proportional to the size of her plot. Every year, in December, there will be a lottery to assign the order of irrigation of each farmer, within each *tanda*. The order will not change during the entire year. At the end of the year farmers pay a fee to the *Sindicato*, that is proportional to the size of their plot.<sup>13</sup> The fees paid by all farmers should cover all costs of operations (paying the guards, cleaning the channels, etc) incurred during the year.<sup>14</sup>

This system has the advantage that every farmer gets some "fair" amount of water once in a while, so it is especially desirable during a drought. Another important feature is that because of the insurance

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<sup>11</sup>This dam was built in 1929. Before this dam was built, water was stored at the smaller dam *Azud del Gallardo*.

<sup>12</sup>The purchase of shares was possible due to a line of credit obtained by the *Sindicato de Regantes* soon after the end of the auctions. According to surviving farmers from this time, the transition would not have been possible without the credit line.

<sup>13</sup>Notice that, since the farmer has to pay after the harvest, there are no LC. Moreover, the farmer is the owner of the water under the quota system, so the price the farmer pay is the average exploitation cost, which is much smaller that the price paid under the auction system.

<sup>14</sup>During the first year of the quota system, the fee also included the payments made to the *Heredamiento de Aguas* to buy the water rights.

property of this institution, farmers have less uncertainty when carrying out risky investments, such as trees. A tree will take several years to be fully productive, but it can die if it does not get enough water in a given year. On the other hand, vegetables grow faster, and can be harvested within a year of being planted. Hence, a farmer with a secure supply of water is more likely to plant trees and get a higher expected profit from them.

## 2.4 Data

The data set consist of a panel in which each period represents one week and each individual represents one farmer. The unit of observation then is a farmer-week. The data set is collected from four different sources. The first source contains information regarding the weekly auction: how many units each farmer bought and at what price, from January 1955 until July 1966 (when the last auction was run). This data is obtained from the historical archive of Mula.<sup>15</sup> We also have information on rainfall in Mula.<sup>16</sup> Since the frequency of the auctions is weekly (there is an auction every Friday), we compute the sum of rain during the seven days prior to each auction. We merge this data set with a cross sectional agricultural census (1955) that contains information regarding the farmer's plots, including crop types, number of trees, production and output price.<sup>17</sup> Finally, we also merge the data set with financial information regarding the real estate tax records in 1955. This last piece of information is crucial to identify liquidity constraints (LC) from demand shocks as we will show later.

Table 1 shows the summary statistics of some of the variables used in the empirical analysis. Detailed information about the data can be found in the Appendix A.1.

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<sup>15</sup>From the section *Heredamiento de Aguas*, boxes No.: HA 167, HA 168, HA 169 and HA 170.

<sup>16</sup>We obtain the rainfall information from the *Agencia Estatal de Meteorología*, AEMET (the Spanish National Meteorological Agency).

<sup>17</sup>Detailed census data is obtained from the section *Heredamiento de Aguas* in the historical archive of Mula, box No. 1,210.

Table 1: Summary Statistics of Selected Variables

Variable	Mean	SD	Min	Med	Max	Obs
Weekly Rain ( <i>mm</i> )	8.29	37.08	0	0	423.00	602
Water Price ( <i>pesetas</i> ) <sup>a</sup>	326.157	328.45	0.005	217.9	2,007	602
Real Estate Tax ( <i>pesetas</i> )	482.10	1,053.6	0	48	8,715	496
Area ( <i>ha</i> )	2.52	5.89	0.024	1.22	100.1	496
# Trees <sup>b</sup>	311.3	726.72	3	150	12,360	496
Units bought	0.0295	0.3020	0	0	4	145,684

*Source:* Own elaboration. We found 496 census cards in the archive. We were able to fully match 242 individuals to the auction data. The agricultural census include farmers that have only *secano* lands and thus, are not in our sample. The sample after matching consist on 602 weeks and 242 individuals, thus 145,684 observations. a) Water Price is the Weekly Average price. b) # Trees includes vines.

**Auction data** Based on bidding behavior and water availability, auction data (602 weeks) can be divided into three categories: (i) Normal periods (300 weeks), where for each transaction the name of the winner, price paid, date and time of the irrigation for each auction is registered, (ii) No-supply periods (295), where due to water shortage in the river or dam/channel damages (usually because of intense rain), no auction is carried out, and finally (iii) No-demand periods (7 weeks), where not all 40 units are sold due to lack of demand. In the main estimation we use data for the period 1955-66.

**Rainfall data** Mediterranean climate rainfall occurs mainly in spring and fall and peak water requirements for the products cultivated in the region are reached in spring and summer, between April and August. The rainfall is also very volatile, we can see in Table 1 that the standard deviation is several times greater than the mean.<sup>18</sup>

**Agricultural Census data** The 1955 census was conducted by the Spanish government to enumerate all cultivated soil, producing crops and agricultural assets available in the country. Individual characteristics for the farmers' land include the type of land and location, area, number of trees, production and the price at which this production was sold in the census year. We match the name of the farmer in each census card with the name of the winner in each auction from the auction data. In this paper we focus on farmers with only apricot trees.

<sup>18</sup>The reason that the mean rainfall presented in Table 1 (326 *mm*) is slightly greater than the 320 *mm* mentioned in the introduction is that the 326 *mm* correspond our estimation period (1955-66) while the 320 *mm* correspond to the complete series (1933-2010).

**Real Estate Tax data** In order to credibly identify the source of financial constraints we need a variable that is related to the farmers' wealth but unrelated to their demand for water (production function). We use the tax records paid by the farmers for urban real estate ownership. The idea is that farmers with big/expensive real estate are wealthier than farmers who own small/inexpensive (or no) real estate and, thus, are less likely to be financially constrained. On the other hand, owning more or less (urban) real estate should not affect the farmer's production function (farmer's willingness to pay), once we condition on type of crop and the size of the plot. Hence, after controlling for all the other variables, especially the type of crop, the number of trees and the area under cultivation, the value of the real estate should not be correlated with the farmer's demand for water.

## 2.5 Preliminary Analysis

In this sub-section, we show some patterns in the data. In Table 2, we restrict attention to farmers that only have apricot trees. We regress the number of units bought by each farmer in a given week on several covariates. The variable "Real Estate (dummy)" is a variable that equals 1 if the value of the real estate owned by the farmer is greater than the sample median, and 0 otherwise.<sup>19</sup> In columns 1 and 2 we see that farmers that are "wealthy" buy more water overall. We then include in the regressions the interaction between "Real Estate" and "Harvest Season". "Harvest Season" is a dummy variable that equals 1 if the observation belongs to a week during the harvest season and 0 otherwise.<sup>20</sup> This interaction captures precisely the effect we are interested in: farmers that face liquidity constraints (LC) are not able to buy water precisely during the weeks in which they need it the most: the harvest season. In the case of apricots the harvest season also coincide with the beginning of summer, when the prices are highest. This makes the LC more likely to be binding for apricot farmers. What we see in columns 3 and 4 is that the effect that LC have on the demand for water is concentrated mostly in the harvest season. The results are robust when we include relevant variables like the number of trees in the farmer's plot, the moisture in the farmer's plot, the price of the water in the given week and the rain during the week before the auction.

As seen in Table 4, wealthy farmers tend to have bigger plots. Since farmers can only buy whole units, the effect that we see in Table 2 could be explained by differences in the size of the plots: there are economies of scale when purchasing water, and only wealthy farmers (who own big plots) could take

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<sup>19</sup>Using other percentiles to define this dummy variable produce similar results. The effect that LC have on water demand is not linear. Moreover, it is not strictly increasing: two farmers that are wealthy enough to buy the water (unconstrained) should have exactly the same demand, regardless of how much cash they have left. Hence, using a dummy variable is consistent with the idea that demand is identical for all unconstrained farmers. Another advantage of using a dummy variable is that we have a direct interpretation of the coefficient in the regression.

<sup>20</sup>See sub-section 5 for a discussion on how the harvest season is defined.

Table 2: Demand for Water.

# Units Bought	(1)	(2)	(3)	(4)
Real Estate	<b>0.0255 ***</b> (0.0063)	<b>0.0235 ***</b> (0.0062)	<b>0.0133 **</b> (0.0066)	0.0126 * (0.0066)
Real Estate X Harvest Season			<b>0.0702 ***</b> (0.0117)	<b>0.0602 ***</b> (0.0122)
Covariates	N	Y	N	Y
Sample Size	14,448	14,448	14,448	14,448

\* Covariates include: Price, Rain, Moisture (individual) and # trees (individual). standard errors in parentheses (\*  $p < 0.10$ ; \*\*  $p < 0.05$ ; \*\*\*  $p < 0.01$ )

Table 3: Demand for Water. Variables normalized per tree.

# Units Bought	(1)	(2)	(3)	(4)
Real Estate	<b>0.0131***</b> (0.0042)	0.0073 (0.0044)	0.0066 (0.0044)	0.0017 (0.0047)
Real Estate X Harvest Season			<b>0.0374***</b> (0.0091)	<b>0.0315***</b> (0.0094)
Covariates	N	N	Y	Y
Sample Size	14,448	14,448	14,448	14,448

\* Covariates include: Price, Rain, Moisture (individual) and # trees (individual). standard errors in parentheses (\*  $p < 0.10$ ; \*\*  $p < 0.05$ ; \*\*\*  $p < 0.01$ )

advantage of them. In Table 3, we normalize for the number of trees in the plot of each farmer. We can see that the wealth of the farmer has a small effect on the amount of water bought year long. However, the effect during the harvest season is still present. The magnitude of the effect during the harvest season is similar to that in Table 2.

We now focus on the *extensive* margin in which LC can affect welfare. Since the composition and size of the plots are (partially) endogenous, this is just another effect of the LC of the farmers. Some farmers have smaller-than-optimal plots and hence, cannot buy water during the harvest season. Here the financial constraints are affecting the inefficiency of the system through the *extensive* margin: size and composition of the plots. The size and composition of the plot of each farmer are correlated with the wealth of each farmer. A poor farmer might not be able to buy a big plot of land, or maybe the reason that she is poor is that she only owns a small plot of land. Moreover, a poor farmer, in anticipation of her inability to buy water during the harvest season, would choose not to own a plot of land with trees. Trees are a risky investment and require more care than vegetables. A tree will usually take five years to become fully productive, so the farmer will have forgo earnings, and it could die during a drought if not irrigated.

In Table 4 (columns 1 and 2) we can see that wealthy farmers own bigger plots than poor farmers. In column 3 we see that the fraction of the land that is planted with trees is not correlated with wealth. This

Table 4: Relation between Size and Composition of the plots, and wealth.

	(1) Area Total (Ha)	(2) Area w trees (Ha)	(3) Fraction w trees	(4) Revenue (pesetas)	(5) Rev/area (pesetas/m <sup>2</sup> )
Real Estate	<b>34,023***</b> (9,747)	<b>22,069***</b> (7,031)	-0.0355 (0.0320)	<b>23,894***</b> (4,024)	-0.1797 (0.7543)
N	388	388	388	388	388

standard errors in parentheses (\* p<0.10; \*\* p<0.05; \*\*\* p<0.01)

Table 5: Revenue, Apricot trees.

Revenue per tree	(1)	(2)	(3)	(4)	(5)
# trees	<b>107.53***</b> (31.098)	<b>245.61**</b> (104.18)			-0.3857 (0.9668)
(# trees) <sup>2</sup>		-0.6059 (0.4372)			0.0000 (0.0040)
Real Estate			<b>0.0383**</b> (0.1418)	-0.0009 (0.0710)	-0.0017 (0.0740)
(RealEstate) <sup>2</sup>				0.0000 (0.0000)	0.0000 (0.0000)
Sample Size	24	24	24	24	24

standard errors in parentheses (\* p<0.10; \*\* p<0.05; \*\*\* p<0.01)

suggest that the allocation of crops with trees is mostly exogenous. This is not surprising since many of the trees were centenarian. Column 4 shows that wealthy farmers get more money from their plots. This is just a mechanical result, since wealthy farmers own bigger plots. Moreover, as column 5 shows, wealthy farmers are not getting more revenue per unit of area.

Finally, we can also use the census data to see the determinants of revenue. In Table 5 we restrict attention to farmers that only grow apricot trees. The data base is cross sectional and it is based on the agricultural census of 1955. In columns 1 and 2 we can see, as expected, that the revenue is increasing and concave in the number of trees owned. In the absence of LC, the revenue obtained in a given plot should be independent of the wealth of the farmers. Column 3 suggest that revenue is increasing on wealth. Notice that this is in contrast with the result of column 5 in Table 4. This is consistent with the fact that LC are important for apricots trees, because the harvest season takes place in the summer.

There is a concern for an omitted variable bias here. If farmer is more productive (i.e., better), she will earn more revenues and will also invest more in trees. She will then become wealthier. Although this story is appealing, it is unlikely to be driven the results. The differences in revenue per farmer are too small compared with the differences in wealth. A back of the envelope calculation shows that it will take



more than a century for a farmer to use the excess in revenues to buy the real estate needed for the story to be true.

Finally, one can express the lower-than optimal purchases of the poor farmers as a problem of input misallocation. In this case, one solution to the misallocation would be for the wealthy farmers to buy the land from the poor farmers and then irrigate the land properly. This would indeed be the solution if there were no dis-economies of scale. If there are dis-economies of scale or the optimal exploitation size is small, then the reduction in output due to the diminishing return might overcome the gains from the optimal irrigation (see Banerjee and Moll, 2010). The land distribution for orchards and the technology for exploitation suggest that dis-economies of scale are important.<sup>21</sup> The same argument is present in Hoffman (1996).

### 3 Model

In this section we present the theoretical model. In sub-section 3.1 we present the general model that we will estimate in Section 4. It is a model with infinite horizon and in which the farmers demand includes storability, seasonality and liquidity constraints (LC). Due to the complexity of the general model we will not solve for the equilibrium.<sup>22</sup>

Nonetheless, we show two particular cases of the general model in order to stress the main results that arise in the general case.

The first case is presented in sub-section 3.2, we provide a two-period model in which farmers value differently the water bought during the harvest season and the off season, but water could be stored. The second case is presented in sub-section 3.3 we present a static model, with farmers being heterogeneous in both their productivity and their wealth. We show that the relative efficiency of each institution depend on the parameters of the model: if differences in productivity are important and LC are not, then Markets are more efficient than Quotas, and *vice versa*. In each case, the particular model proposed is the simplest model we can construct that still has the properties that we want to highlight. When omitted the proofs are in Appendix B.

In sub-section 3.4 we show a model of endogenous LC. LC do not arise exogenously. Rather they are the consequence of limited liability (farmers are poor) and non-enforceable property rights (poor institutions).

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<sup>21</sup>Dis-economies of scale here take the form of diminishing return of effort and increasing marginal cost of monitoring employees.

<sup>22</sup>We do not need to solve for the equilibrium in order to estimate the parameters (see Section 4).

The combination of these two factors make the existence of a credit market unlikely.<sup>23</sup>

### 3.1 General Model

The economy consists of  $N$  farmers, denoted by  $i$ , and one auctioneer. There are two goods in the economy: water  $x$  (moisture) measured in *liters* and money  $\mu$  measured in *pesetas*. Time is denoted by  $t$ , the horizon is infinite and the discount between periods (weeks) is  $\beta$ . Demand is seasonal, hence some of the functions will depend on the season. We denote the season by  $w \in \{1, 2, \dots, 52\}$ , representing each of the 52 weeks of any given year. The supply of water in the economy is stochastic and equals  $X_t \equiv X(w_t)$  in period  $t$ .<sup>24</sup> In particular, supply follows a binomial conditional on the week, there will be an auction during week  $w_t = w$  with probability  $\rho_w$ . If there is an auction there will be  $X$  units to be sold.

Farmers will only get utility for the water consumed during the harvest season. Notice that water here is an intermediate good. Hence, utility here refers to profit or outcome and is measured in *pesetas*, not in *utils*. Water bought in any period could be carried forward into the next period, but it will depreciate according to some function  $\delta$ . Farmers' preferences over water and money are represented by  $u(j_t, M_t, w_t; \mu_t; \theta_t) = h(j_t, M_t, w_t; \theta_t) + (\mu_t - p_t j_t)$  where the production function  $h(\cdot)$  is twice continuously differentiable and strictly increasing in  $M_t$  and  $\theta_t$  and concave in the moisture on the farmer's plot  $M_t$ ;  $j_t$  is the number of units bought in period  $t$ ;  $p_t$  is a scalar that represents price in period  $t$ ; and  $\mu_t$  is the amount of cash that the farmer has in period  $t$ . Limited liability requires that  $(\mu_t - p_t j_t) \geq 0, \forall j_t > 0$ . Finally, the trees on the farmers plot will die if the water (moisture) in her plot decreases beyond the Permanent Wilting point  $PW$ .

Farmers in the economy differ from each other in two ways. First, a productivity shock  $\theta_{it}$  is drawn from a distribution  $F(\theta)$ , with  $f(\theta) > 0$ , defined on a compact interval of  $\mathbb{R}_+, [\underline{\theta}, \bar{\theta}]$ , independently from other farmers' draws.<sup>25</sup> Second, their initial wealth levels  $\mu_{it}$  are drawn from a distribution  $G(\mu)$ , with  $g(\mu) > 0$ , defined on a compact interval  $\mathbb{R}_+, [\underline{\mu}, \bar{\mu}]$ , where we assume that  $\underline{\mu} > 0$ . The realization of  $\theta_{it}$  is independent of the realization of  $\mu_{it}$ ; and both  $\theta_{it}$  and  $\mu_{it}$  are private information.<sup>26</sup>

The expected discounted utility of farmer  $i$  at  $t = 0$  is then:

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<sup>23</sup>Moreover, even if such a market exists, it would require a high interest rate, which was forbidden under the Spanish Usury Law.

<sup>24</sup>In our empirical application the supply can be considered exogenous. Although the seller had the authority to cancel an auction in any given week, they rarely do so and only when the price drops to zero.

<sup>25</sup>In the empirical application this productivity shock can be decompose into a permanent (or persistent) attribute of farmer  $i$  production function and a idiosyncratic time independent productivity shock.

<sup>26</sup>In section 4 we characterize the evolution of all state variables.

$$E \left[ \sum_{t=0}^{\infty} \beta^t u(j_{it}, M_{it}, w_{it}; \mu_{it}; \theta_{it}) \mid j_{it}, M_{it}, w_{it}; \mu_{it}; \theta_{it} \right]$$

s.t.  $m_{it} \geq PW$

s.t.  $j_{it} p_t \leq \mu_{it}, \forall j_{it} > 0$

### 3.2 Dynamic Model: *On-Season vs Off-Season*

In this sub-section, we propose a simple two-period model that captures the dynamics of an economy with storability, seasonal demand and liquidity constraints (LC). This is a simplified version of the model presented in sub-section 3.1. Here we only consider two periods, with a particular evolution for the moisture stored in a farmer's plot and a particular evolution for the cash that the farmer has. Due to these simplifications we are able to solve the model analytically while still preserving the main features of the dynamics of the general model.

The economy consists of a continuum of unit mass of farmers, denoted by  $i$ , and one auctioneer. There are two goods in the economy: water ( $x$ ) measured in *liters* and money ( $\mu$ ) measured in *pesetas*. There are two periods, denoted by  $t$ , and there is no discounting between periods. The supply of water in the economy is constant and equals  $X_t$  in period  $t$ . There will be an auction in the first period (*Off-Season*) and  $X_1$  units of water will be auctioned. There will also be another auction in the second period (*On-Season*) and  $X_2$  units of water will be auctioned.

Farmers will only get utility for the water consumed in the second period. Water bought in the first period, however, could be carried forward into the second period, but it will depreciate at a rate  $\delta$ . That means that for every unit of water bought in the first period, the farmer will only consume  $(1 - \delta)$  units in the second period. In the second period, there will be an amount of water equal to  $X \equiv (1 - \delta) X_1 + X_2$  to be consumed in this economy. For ease of exposition we consider the case in which farmers are homogeneous in productivity.<sup>27</sup> Farmers, however, will differ in their wealth  $\mu_i$ , which are drawn from a distribution  $G(\mu)$ , with  $g(\mu) > 0$ , defined on a compact interval  $\mathbb{R}_+, [\underline{\mu}, \bar{\mu}]$ . Farmers can buy only a discrete amount of water  $x_t \in \mathbb{N}$  in each period and will get a utility of  $u(x_1, x_2, p_1, p_2; \mu) = h((1 - \delta)x_1 + x_2) + (\mu - p_1 x_1 - p_2 x_2)$  where  $h(x)$  is the production function that transforms water into output (*pesetas*), and it is increasing and concave. Moreover, LC imply that  $(\mu - p_1 x_1 - p_2 x_2) \geq 0$ .

We first define and solve for an equilibrium in an economy without LC. It serves as a benchmark when analyzing the results in the general case. If there are no LC, farmers will be indifferent between buying 1

<sup>27</sup>The results are similar when farmers are also heterogeneous in productivity.

unit in the first period or  $(1 - \delta)$  units in the second period. We will restrict attention to the case in which water is scarce, i.e.,  $X_1 + X_2 < 1$ . This assumption implies that, in the unconstrained case, farmers are buying at most one unit of water.

Let the vector  $p$  be a pair of prices, i.e.,  $p \equiv (p_1, p_2)$ , where  $p_t$  is the equilibrium price in period  $t$ . The allocation of water in this economy is characterized by the allocation matrix:

$$Q \equiv \begin{bmatrix} q_{00} & q_{10} & q_{20} \\ q_{01} & q_{11} & q_{21} \\ q_{02} & q_{12} & q_{22} \end{bmatrix}$$

where  $q_{x_1, x_2}$  represents the mass of individuals that buy  $x_1$  units in period 1 and buy  $x_2$  units in period 2. Each allocation matrix satisfies  $\sum_{x_1} \sum_{x_2} (q_{x_1, x_2}) = 1$ . We call a pair  $(x_1, x_2)$  an optimal allocation if there is no other pair  $(x'_1, x'_2)$  such that  $u(x'_1, x'_2, p_1, p_2; \mu) > u(x_1, x_2, p_1, p_2; \mu)$ .

**Definition.** A *Equilibrium* is characterized by a price vector  $p^{FB}$  and an allocation matrix  $Q^{FB}$  that satisfy:<sup>28</sup>

- **Optimality (O).** At prices  $p = p^{FB}$  each farmer is maximizing her expected utility, i.e., when  $p = p^{FB}$  and each pair  $(x_1, x_2)$  such that  $q_{x_1, x_2} > 0$ , we have  $u(x_1, x_2, p_1, p_2; \mu) \geq u(x'_1, x'_2, p_1, p_2; \mu)$  for any other pair  $(x'_1, x'_2)$ .

- **Resource Constraint (RC).** The allocation  $Q$  satisfies the Resource Constraint in each period, i.e.,

$$\sum_{x_1} \sum_{x_2} (x_1 \cdot q_{x_1, x_2}) = X_1 \text{ and } \sum_{x_1} \sum_{x_2} (x_2 \cdot q_{x_1, x_2}) = X_2$$

The only prices that are consistent with equilibrium in this case are  $p_1^{FB} \equiv h(1 - \delta)$  and  $p_2^{FB} \equiv h(1)$  in the first and second period respectively.<sup>29</sup> The only allocation consistent with equilibrium is:

$$Q^{FB} \equiv \begin{bmatrix} (1 - X_1 - X_2) & X_2 & 0 \\ X_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; p^{FB} \equiv [h(1 - \delta), h(1)]$$

Hence, the mass of farmers buying one unit in the first period, one unit in the second period and buying no water are  $q_{10}^{FB} = X_1$ ,  $q_{01}^{FB} = X_2$  and  $q_{00}^{FB} = 1 - X_1 - X_2$  respectively. Since each unit of

<sup>28</sup>Since this is a dynamic game, we also have to check for Dynamic Consistency (DC): for every farmer, the surplus in the second period is no greater than the surplus in the first period.

<sup>29</sup>Given the discreteness in the allocation, if instead of an auction we had a price posting scheme, we will always have a continuum of prices consistent with equilibrium, but a unique allocation. The unique solution in the case of the auction coincide with the highest of those prices.

water is being allocated to a different farmer, the equilibrium achieves the first-best (FB) allocation. The equilibrium is also revenue maximizing because it extracts all the surplus from the farmers.

We now define and solve for the equilibrium when some farmers face LC. We restrict attention to the case in which the wealth can take only two values  $\{\mu_L, \mu_H\}$  with  $\mu_L < \mu_H$ ,  $Pr(\mu_i = \mu_L) = g_L$  and  $Pr(\mu_i = \mu_H) = g_H = 1 - g_L$ . For simplicity, we focus on the case when  $\mu_H \rightarrow \infty$ . We will focus on the cases where  $g_H < X_1 + X_2$ , otherwise the equilibrium is trivial, and wealthy farmers can buy all the water at the FB prices. For simplicity of exposition, we also restrict attention to the case when wealthy farmers buy at most two units and there are “enough” wealthy farmers to buy all the water in the second period, i.e.,  $2g_H > X_1 + X_2$  and  $g_H > X_2$ .<sup>30</sup>

The allocation of water in this economy is characterized by two allocation matrices:

$$Q_L \equiv \begin{bmatrix} q_{L00} & q_{L10} & q_{L20} \\ q_{L01} & q_{L11} & q_{L21} \\ q_{L02} & q_{L12} & q_{L22} \end{bmatrix} \text{ and } Q_H \equiv \begin{bmatrix} q_{H00} & q_{H10} & q_{H20} \\ q_{H01} & q_{H11} & q_{H21} \\ q_{H02} & q_{H12} & q_{H22} \end{bmatrix},$$

where  $q_{i,x_1,x_2}$  represents the mass of individuals with wealth  $\mu_i$  that buy  $x_{i1}$  units in period 1 and buy  $x_{i2}$  units in period 2. Each allocation matrix satisfies  $\sum_{x_{i1}} \sum_{x_{i2}} (q_{x_{i1},x_{i2}}) = g_i$ . We call a pair  $(x_{i1}, x_{i2})$  an optimal allocation for farmer  $i$  if there is no other pair  $(x'_{i1}, x'_{i2})$  such that  $u(x'_{i1}, x'_{i2}, p_1, p_2; \mu_i) > u(x_{i1}, x_{i2}, p_1, p_2; \mu_i)$ .

**Definition.** A *Constrained Equilibrium* is characterized by a price vector  $p^* \equiv (p_1^*, p_2^*)$  and an allocation matrix  $[Q_L^*; Q_H^*]$  that satisfy:<sup>31</sup>

- **Optimality (O).** At prices  $p = p^*$  each farmer is maximizing her expected utility, i.e., when  $p = p^*$  and each pair  $(x_{i1}, x_{i2})$  such that  $q_{x_{i1},x_{i2}} > 0$ , we have  $u(x_{i1}, x_{i2}, p_1, p_2; \mu_i) \geq u(x'_{i1}, x'_{i2}, p_1, p_2; \mu_i)$  for any other pair  $(x'_{i1}, x'_{i2})$ .
- **Resource Constraint (RC).** The allocation  $Q_i$  satisfies RC in each period,
   
i.e.,  $\sum_{i=L,H} \sum_{x_{i1}} \sum_{x_{i2}} (x_{i1} \cdot q_{x_{i1},x_{i2}}) = X_1$  and  $\sum_{i=L,H} \sum_{x_{i1}} \sum_{x_{i2}} (x_{i2} \cdot q_{x_{i1},x_{i2}}) = X_2$ .
- **Liquidity Constraint (LC).** For each  $i$ , the allocation  $Q_i$  satisfies LC, i.e.,  $(x_{i1} \cdot p_1 + x_{i2} \cdot p_2) \leq \mu_i, \forall i$ .

<sup>30</sup>When  $2g_H < X_1 + X_2$  the intuition of the results is the same, but we need to keep track of the amount of units bought by each farmer in each period. The equilibrium in this case has many cases and it is not worth presenting here. The results are similar when  $g_H < X_2$ .

<sup>31</sup>Since this is a dynamic game, we also have to check for Dynamic Consistency (DC): for every type, the surplus in the second period is no greater than the surplus in the first period.

We say that a *Constrained Equilibrium* is *Efficient* if all the units are consumed by some farmer and there are no farmers consuming more than one unit. We say that a *Constrained Equilibrium* is *Revenue Maximizing* if  $p = p^{FB}$ .

We now solve for the equilibrium assuming that  $\mu_L \geq [h(2 - 2\delta) - h(1 - \delta)]$ . For the complete solution and the proof see Appendix B.1.

Poor farmers would buy one unit in the first period, but not in the second period. Since there are “enough” wealthy farmers, i.e.,  $g_H > X_2$ , they will buy all the water in the second period and some of the water in the first period.

$(X_2 - g_H)$  wealthy farmers will buy one unit in the first period and  $X_2$  wealthy farmers will buy one unit in the second period, i.e.,  $q_{H10} = (X_2 - g_H)$  and  $q_{H01} = X_2$ .

$(X_1 - X_2 + g_H)$  poor farmers will buy water in the first period, and  $(g_L - X_1 + X_2 - g_H)$  poor farmers will not buy water, i.e.,  $q_{L10} = (X_1 - X_2 + g_H)$  and  $q_{L00} = g_L - X_1 + X_2 - g_H$ .

Therefore, in this simple model, due to the concavity of the production function, the homogeneity of productivity across farmers and the scarcity of water ( $X_1 + X_2 < 1$ ), any allocation in which a farmer consumes more than one unit of water is inefficient.

It should be noticed, however, that when farmers are heterogeneous in productivity, the allocation is also inefficient when LC are binding. If farmers are heterogeneous in productivity, efficiency requires not only that farmers consume at most one unit, but also that no low-productivity farmer is consuming any water unless all high productivity farmers are consuming one unit, and that more productive farmers consume *on-season* and less productive farmers consume *off-season*.<sup>32</sup>

When LC are binding, the model predicts that poor farmers will not buy water in the second period. Moreover, poor farmers never buy more water than wealthy farmers *On-Season*. However, it could be the case that the poor farmers are buying more water than the wealthy farmers *Off-Season*.

### 3.3 Static Model: Auction vs Quotas

In this sub-section, we propose a static version of the general model presented in sub-section 3.1. Due to the static nature of this model, there is no storability nor seasonality, but LC are still present. In this simplified version we are able to solve the model analytically and make normative claims about the efficiency of both the auctions and the quotas. The reader should notice that the claims made about the static model should apply *mutatis mutandi* to the dynamic case.

<sup>32</sup>We do not report in this paper the solution for the case in which farmers are also heterogeneous in productivity.

The model presented here is a generalization of Che, Gale and Kim (2012).<sup>33</sup> The economy consists of a continuum of farmers with unit mass and one auctioneer. Farmers will be denoted by  $i$ . There are two goods in the economy: water ( $x$ ) measured in liters and money ( $\mu$ ) measured in *pesetas*. The supply of water in the economy is constant and represented by  $X$ . Farmers' preferences over water and money are represented by  $u(x; \mu; \theta) = h(\theta, x) + (\mu - px)$  where  $h(\cdot)$  is twice continuously differentiable and strictly increasing in each argument and concave in  $x$ ;  $p$  is a scalar that represents the transfer per unit of water received and  $(\mu - px) \geq 0$ . We also required the cross derivative to be positive, i.e.,  $h_{\theta x}(\cdot) > 0$ . Farmers in the economy differ from each other in two ways. First, a productivity shock  $\theta_i$  is drawn from a distribution  $F(\theta)$ , with  $f(\theta) > 0$ , defined on a compact interval of  $\mathbb{R}_+, [\underline{\theta}, \bar{\theta}]$ , independently from other farmers' draws. Second, their wealth levels  $\mu_i$  are drawn from a distribution  $G(\mu)$ , with  $g(\mu) > 0$ , defined on a compact interval  $\mathbb{R}_+, [\underline{\mu}, \bar{\mu}]$ , where we assume that  $\underline{\mu} > 0$ . The realization of  $\theta_i$  is independent of the realization of  $\mu_i$ .<sup>34</sup> Both  $\theta_i$  and  $\mu_i$  are private information.

Given a number  $p$ , define the *optimal allocation*  $x^*(\theta_i, p)$ , to be the one that solves:

$$h_x[\theta_i, x^*(\theta_i, p)] = p \quad (1)$$

Under our assumptions on  $h(\cdot)$ , we have that  $x^*(\theta_i, p) = (h_x)^{-1}[\theta_i, x^*(\theta_i, p)]$ . We say that an agent is *liquidity constrained* at price  $p$  if  $p \cdot x^*(\theta_i, p) > \mu_i$ .

In order to characterize the first-best (FB) allocation in this economy, we need a welfare criterion. The utility function here represents a production function in which  $h(\theta_i, x_i)$  is the output measured in *pesetas*. Thus, we will use the utilitarian welfare criterion, which in a quasi-linear economy corresponds to maximizing the sum of utilities. FB allocations are defined as those allocations that maximize welfare in the absence of wealth constraints subject to the feasibility constraint:

$$\begin{aligned} & \max_{\{x(\theta_i)\}} \int_{\underline{\theta}}^{\bar{\theta}} h(\theta_i, x) dF \\ & \text{s.t. } \int_{\underline{\theta}}^{\bar{\theta}} x dF = X \end{aligned} \quad (2)$$

Given a number  $p^{FB}$ , define the *FB allocation*  $x^{FB}(\theta_i, p^{FB})$ , to be the one that solves:

<sup>33</sup>Che, Gale and Kim (2012) restrict attention to the special case in which the utility function is linear until  $x = 1$ . Strictly speaking, they consider the case in which there is a continuum of indivisible objects and consumers have unit demand. Mathematically, their model and results is identical to our model when we consider a utility function that is linear, with slope equal to 1, until  $x = 1$  and is flat afterwards.

<sup>34</sup>The independence of  $\theta_i$  and  $\mu_i$  is irrelevant for the results presented here. The correlation between  $\theta_i$  and  $\mu_i$  will matter when trying to solve for the optimal mechanism.

$$\begin{aligned}
h_x [\theta_i, x^{FB} (\theta_i, p^{FB})] &= p^{FB} \\
\int_{\underline{\theta}}^{\bar{\theta}} x^{FB} (\theta_i, p^{FB}) dF &= X
\end{aligned} \tag{3}$$

Notice that, without liquidity constrained agents the optimal allocation coincide with the FB allocation. We denote the welfare level achieved in the FB allocation by:

$$\Omega^{FB} \equiv \int_{\underline{\theta}}^{\bar{\theta}} h [\theta_i, x^{FB} (\theta_i, p^{FB})] dF \tag{4}$$

### 3.3.1 Auctions

We study the allocation that arises under the auction system assuming that all the units are allocated simultaneously.<sup>35</sup> At each price  $p$  each farmer demands the quantity of water she would be willing to purchase at that price, and the price adjusts so that the market clears. In the static case, a simultaneous auction is equivalent to a centralized market, since the supply of water is fixed.

Given a price  $p$ , an agent of type  $(\theta_i, \mu_i)$  will demand:

$$y (\theta_i, \mu_i, p) = \begin{cases} x^* (\theta_i, p) & \text{if } \mu_i > p \cdot x^* (\theta_i, p) \\ \frac{\mu_i}{p} & \text{otherwise} \end{cases} \tag{5}$$

Farmers with sufficient wealth will buy the optimal amount of water, given the price, while farmers that do not have enough wealth will spend all their wealth. Denote by  $\hat{\theta}_i \equiv \hat{\theta}_i (\mu_i, p)$  a farmer of type  $(\theta_i, \mu_i)$  such that  $\mu_i = p \cdot x [\hat{\theta}_i (\mu_i, p), p]$ , i.e., the marginal farmer. This is a farmer that is using all her wealth but would not buy more water even if she has more wealth. Aggregate demand at price  $p$  is given by:

$$Y (p) \equiv \int_{\underline{\mu}}^{\bar{\mu}} \int_{\underline{\theta}}^{\bar{\theta}} y (\theta_i, \mu_i, p) dF dG \tag{6}$$

Aggregate demand can be decomposed into constrained and unconstrained farmers:

$$Y (p) = \int_{\underline{\mu}}^{\bar{\mu}} \int_{\underline{\theta}}^{\hat{\theta}_i} x^* (\theta_i, p) dF dG + \int_{\underline{\mu}}^{\bar{\mu}} \frac{\mu_i}{p} \left( 1 - F [\hat{\theta}_i (\mu_i, p)] \right) dG \tag{7}$$

The first term corresponds to farmers who are unconstrained, thus they buy the optimal amount at

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<sup>35</sup>In the next subsection we consider the case in which units are allocated sequentially.



price  $p$ . The second term corresponds to farmers who are constrained, thus they expend all their wealth. In equilibrium we need demand to equal supply:  $Y(p^A) = X$ . Using this fact, we can decompose the aggregate demand and compare it with the FB case. Rearranging we get:

$$\int_{\underline{\mu}}^{\bar{\mu}} \int_{\underline{\theta}}^{\bar{\theta}} \left[ \frac{\mu_i}{p^A} - x^*(\theta_i, p^A) \right] dF dG = \int_{\underline{\mu}}^{\bar{\mu}} \int_{\underline{\theta}}^{\bar{\theta}} [x^*(\theta_i, p^{FB}) - x^*(\theta_i, p^A)] dF dG$$

Since the left hand side (LHS) is non-positive we need the right hand side (RHS) to be non-positive as well. If  $p^{FB} < p^A$  then the RHS is positive, because demand is decreasing in  $p$ . Hence, we have  $p^A < p^{FB}$ . Welfare in a centralized auction would be equal to:

$$\Omega^A \equiv \int_{\underline{\mu}}^{\bar{\mu}} \int_{\underline{\theta}}^{\hat{\theta}_i} h[\theta_i, x^*(\theta_i, p^A)] dF dG + \int_{\underline{\mu}}^{\bar{\mu}} h\left(\theta_i, \frac{\mu_i}{p^A}\right) \left(1 - F[\hat{\theta}_i(\mu_i, p^A)]\right) dG \quad (8)$$

We can establish the following results:

**Proposition 1.** *Under the Auction system:*

i) *Welfare is lower than it would have been without LC. In particular, the Auction system does not achieve the FB allocation when LC are binding for some farmers.*

ii) *The equilibrium price is lower than it would have been without LC. Moreover, there are farmers whose marginal utility in equilibrium is greater than the equilibrium price.*

### 3.3.2 Quotas

A quota system means that each farmer will get the same amount of water, regardless of their type  $(\theta_i, \mu_i)$ .<sup>36</sup> Hence,  $x(\theta_i, \mu_i) = X$ . Welfare in this case is:

$$\Omega^Q \equiv \int_{\underline{\theta}}^{\bar{\theta}} h(\theta_i, X) dF \quad (9)$$

We can establish an efficiency result here too:

**Proposition 2.** *Under the Quota system:*

i) *Welfare is lower than it would have been with heterogeneous farmers, i.e.,  $\theta_i = \theta \forall i$ . In particular, the Quota system does not achieve the FB allocation when farmers are heterogeneous.*

<sup>36</sup>In a dynamic setting the amount of water allocated at every given week could be different for each farmer based on observables like the type of crop and past rain.

ii) When farmers are homogeneous, i.e.,  $\theta_i = \theta \forall i$ , the Quota system does achieve the FB allocation.

The explanation for these results is straightforward. When farmers are homogeneous, since  $h(\theta, x)$  is concave in  $x$ , the FB allocation requires that all farmers are allocated the same amount of water. Hence, the Quota system achieves FB. When farmers are heterogeneous, FB allocation requires more productive farmers to be allocated a greater amount of water.

### 3.3.3 Auction vs Quotas

The previous results imply that there is not a complete ranking in efficiency between the Auction and the Quota system. Since each of them achieves full efficiency under particular circumstances, it is easy to find a pair of distributions  $F(\theta)$  and  $G(\mu)$  in which either system outperforms the other. In particular, when  $F(\theta)$  is degenerate (homogenous farmers) and  $G(\mu)$  is binding (constrained farmers), the Quota system outperforms the Auction system and achieves FB. On the other hand, when  $G(\mu)$  is not binding (unconstrained farmers) and  $F(\theta)$  is not degenerated (heterogeneous farmers), the Auction system outperforms the Quota system and achieves FB.

The intuition behind the previous argument is that when farmers have similar productivity and LC are important, we will expect the Quota system outperforms the Auction system. Moreover, we can identify, given the parameters, when the Quota system outperforms the Auction system:

**Proposition 3.** *The Quota system outperforms the Auction system if and only if:*

$$\int_{\underline{\mu}}^{\bar{\mu}} \int_{\hat{\theta}_i}^{\bar{\theta}} \left[ h(\theta_i, X) - h\left(\theta_i, \frac{\mu_i}{p^A}\right) \right] dF dG \geq \int_{\underline{\mu}}^{\bar{\mu}} \int_{\hat{\theta}}^{\hat{\theta}_i} \left[ h(\theta_i, x^*(\theta_i, p^A)) - h(\theta_i, X) \right] dF dG \quad (10)$$

*Proof.* Use equations 8 and 9, and rearrange terms. □

The objects inside the brackets in each side of the equation represent the gains and losses respectively of the Quota system with respect to the Auction system. Broadly speaking, constrained farmers will get more water under the Quota system, hence the expression in brackets in the LHS is positive. Along the same lines, unconstrained farmers will get more water under the Auction system than under the Quota system, hence the expression in brackets in the RHS is positive. Given the concavity of the production function, which system is more efficient will depend then on the differences of utility of each group. These relative gains in efficiency would have to be weighted by the number of farmers in each group. Hence, the efficiency ranking will also depend on the relative size of each group.

Given the structure of the model, we would expect a market for water to exist between regions. Between regions, each individual in the model represents a whole region. It is unlikely that a whole region or a big association of farmers is liquidity constrained. Also, it is very likely that different regions, specially if they are far apart, are affected by idiosyncratic shocks and, thus, can benefit from trading. We would also expect a non-market mechanism for water to exist within a region. Within a region, each individual in the model represents a farmer. It is likely that a given farmer is liquidity constrained. Also, it is unlikely that farmers within the same region are affected by important idiosyncratic shocks. They are more likely to be affected by aggregate shocks.

### 3.4 Endogenous Borrowing Constraints

This sub-section is a direct application of Albuquerque and Hopenhayn (2004). Interviews with farmers reveal that credit markets were not used by farmers. There was no centralized credit market and farmers will not ask for a loan from a relative unless their situation was desperate. Moreover, there was no easy way to enforce a loan but reputation (poor macro institutions).<sup>37</sup>

The intention of this sub-section is twofold. On the one hand, to show that it is unlikely that a short term credit market would emerge, hence the farmers would be financially constrained. On the other hand, even if such short term credits did exist, the amount of the loan would be sub-optimal, meaning that the farmers would be partially financially constrained. There are two characteristics of the situation depicted here that make this situation extremely likely: lack of perfect enforceable property rights and the relatively low long-term profits compared with the price of water. Lack of enforceable property rights means that the farmer can take the money from the loan (or the money from the harvest) and walk away, without repaying the loan. This imposes a constraint in the amount of cash that the potential lender would give to the farmer. The expenses in water during a dry year suppose a big part of the cost of that year. Long term profits are not very sensitive to past rain. Hence, the maximum amount of cash that the potential lender would lend to the farmer will be particularly low in those years in which the farmers need a particularly high amount of cash.

Time is discrete and infinite. At time zero the farmer is pursuing a project (planting some trees) and needs an initial investment of  $I_0 \geq 0$ .<sup>38</sup> The plot of the farmer will produce a flow of revenues each period

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<sup>37</sup>In contrast to German credits cooperatives (Guinnane, 2001), the farmers in southeastern Spain were not able to create an efficient credit market. Spanish farmers were poorer than German farmers and, more importantly, the weather shocks were aggregate (not idiosyncratic) and greater in magnitude. Hence, in order to reduce the risk, Spanish farmers should resort to external financing. However, external financing have problems such as monitoring costs and information acquisition that credit cooperatives do not have.

<sup>38</sup>Notice that  $I_0 = 0$  is a possibility here, meaning that the results will follow even if the farmer only needs short term

that depend on the amount of water purchased and the rain  $\tilde{R}(w, r)$ . Without loss of generality, we can define  $R(k, s) \equiv \tilde{R}(w, r)$ , where  $k$  is the cash that the lender lends to the farmer (and the farmer uses to buy water) and  $s \in S \subset \Re$  is a revenue shock. Notice that  $s$  includes both the variability in rain and in the price of water. The revenue shock  $s$  follows a Markov process with conditional cumulative distribution function  $L(s', s)$ .  $L(\cdot)$  is jointly continuous. In every period, the shock  $s$  (publicly known) is realized, and then revenues  $R(k, s)$  are collected.

The farmer has limited liability; she starts with zero wealth and the lender is required to finance both the initial investment and the advancement of cash every period.<sup>39</sup> Both the farmer and the lender have the same discount rate  $\beta$ . The lender can commit to a long-term contract with the farmer, but contracts have limited enforceability as the farmer can choose to default. This means that the lender will continue with the policy agreed in the contract until the farmer defaults, while the farmer can walk away any time. If the match is ended, the residual value for the farmer is  $O(k, s)$ .

A long-term contract specifies a contingent liquidation policy  $e_t \in \{0, 1\}$ , cash advancements  $k_t$  and a cash flow distribution consistent on a dividend  $d_t \geq 0$  for the farmer (and its complement  $R(k_t, s_t) - d_t$  for the lender). We assume that there is competition in the credit market. Hence, lenders will break even in equilibrium while farmers will make profits. Notice that this is the most conservative approach since it maximizes the set of parameters under which there will be a loan.

Under perfect enforceability is easy to see that the lender will give the farmer the money she needs for the initial investment, provided that the project is profitable. The lender will also give the farmer in each period the optimal amount of cash  $k^* = \underset{k}{\operatorname{argmax}} \{R(k, s) - (1 + \beta)k\}$ . However, under imperfect enforceability there would be inefficiencies both in the extensive margin (the lender would not offer any contract to the farmer) and the intensive margin (the lender would only advance an amount of cash lower than the optimal). If the contract is implemented it will have two phases, depending on the history. At the beginning, the farmer will pay all the revenues generated to the lender in order to repay the loan. During this phase, the cash advances that the lender provides the farmer will be suboptimal. If the match is not broken before that, there will be a point at which the farmer has a sufficiently high value of the match (since the farmer has paid a big amount of the loan) so that the lender will lend him the efficient amount of cash advances, and the farmer will get part of the revenue generated.<sup>40</sup>

Following Albuquerque and Hopenhayn (2004) the optimal contract will have the following properties:

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financing.

<sup>39</sup>If the farmer starts with some wealth  $\tilde{I}_0$ , then the project only needs financing of  $I'_0 = I_0 - \tilde{I}_0$ .

<sup>40</sup>Notice that, due to the stochastic process of  $s$  the relation could go back and forth between the two phases. Moreover, since the revenue could be negative the principal of the loan could increase over time.

- **Inefficiency at the extensive margin:** The set of parameters under which a credit contract is feasible is strictly smaller than the set of parameters under which the investment is profitable.
- **Inefficiency at the intensive margin:** Even when a loan is awarded, the amount of cash advancements will be suboptimal. This case is indistinguishable from the case in which the farmer has a fixed amount of cash to expend or has exogenous financial constraints. Since the amount of cash is lower than optimal, the amount of water bought at the auction will also be lower than optimal.
- **Inefficient liquidations:** Although the farmer cannot commit to a contract, the lender can. The lender will commit to early (inefficient) liquidation in order to prevent the farmer from walking away with the cash. Remember that, given the perfect observability of this model, in the perfect enforceability case, a relation was never liquidated.

As a summary, the credit is not awarded to the farmer even though it would have been profitable. Even when a loan is awarded, the amount of water bought at the auction will also be lower than optimal. Whether because of lack of credit or lack of sufficient credit, the farmer will be liquidity constrained. Finally, due to imperfect enforceability, the lender will commit to early liquidation. This could be the reason why the *Heredamiento* committed to a policy of “only cash”, even though it seemed to reduce its revenue.

## 4 Econometrics

In Section 3 we showed evidence of LC. However, a reduced-form analysis, although useful to identify the patterns on the data, is not so useful if we are trying to construct a counterfactual. In particular, it is unclear how to incorporate some features of the empirical setting that are essential when estimating demand, such as the seasonality of demand, the inter-temporal substitution due to the “storability” of water and the inter-temporal dependence of cash holdings and the interaction among them. Hence, we need to incorporate these singularities into an econometric model. In this section, we propose an econometric model that takes into account all those features and an estimation procedure.

The database is a panel. Each week the farmers can buy up to  $J$  units of water. Each individual  $i$  represents a farmer while each period  $t$  represents a week. We restrict the data to analyze this simple case to farmers that have only apricot trees. We are down to 24 farmers. We are going to use the following variables:

- $j_{it} \in \{0, 1, \dots, J\}$ , is the number of units that farmer  $i$  buys at period  $t$ .

- $p_t$  is the price (in *pesetas*) per unit of water during period  $t$ .
- $r_t$  is the amount of rain (measured in *mm*) that fall in the town of Mula during period  $t$ . We actually compute the amount of water during the seven days prior to the auction.
- $area_i$  (*m*) is the farmer's plot area.
- $re_i$  is the value of the real state that the farmer owns (measured in *pesetas*).

We are going to estimate the following parameters:

- $\gamma$  is a vector of parameters that determine the payoff function.
- $\phi$  is a vector of parameters that determine the cash flow function.

#### 4.1 Model without Liquidity Constraints

The value function has five arguments:

- $M_{it}$  (deterministic, measured in  $l/m^2$ ): is the moisture of the plot. It represents the amount of water "stored" in the farmer's plot.
- $w_t$  (deterministic): is the weekly seasonal effect. Its support is  $\{1, 2, \dots, 51, 52\}$ .
- $p_t$  (random, measured in *pesetas*): is the price for each unit of water during week  $t$ . Prices are a big determinant of demand. Here, prices play a twofold role. Higher prices means that farmer would demand less water or that farmer will not demand any water at all, if the price is above their cash holdings.
- $r_t$  (random, measured in  $l/m^2$ ): is the amount of rain that fell on the town during period  $t$ .
- $\epsilon_{it} \equiv (\epsilon_{i0t}, \dots, \epsilon_{iDt})$  (random): is a choice specific component of the utility function.

The law of motion for the moisture  $M_{it}$  is:

$$M_{it} = \min \left\{ M_{i,t-1} + r_t + \frac{j_{it} \cdot 432,000}{area_i} - ET(M_{it}, w_t), FC \right\} \quad (11)$$

where  $j_t$  is the option chosen by the farmer at period  $t$  (here the option chosen is equal to the number of units bought),  $ET(M_{it}, w_t)$  is the adjusted Evapotranspiration at period  $t$  and  $FC$  is the Full Capacity of the farmer's plot. For details about this formula see Appendix A.2. Moisture is the main determinant of

demand (together with the seasonality). Although, we do not directly observe the moisture in each plot, we can compute it. The moisture of a given plot will increase only with rain or irrigation, both of which are observable, and will decrease due to Evapotranspiration (Evaporation and Transpiration ET). We follow the literature in agricultural engineering to compute the ET, which will depend on the season of the year and on the level of moisture on the plot (see equation 11 below).

The evolution of the weekly season is mechanical:

$$w_t = \begin{cases} w_{t-1} + 1 & \text{if } w_{t-1} < 52 \\ 1 & \text{if } w_{t-1} = 52 \end{cases} \quad (12)$$

Farming is a seasonal activity and each crop has different water requirements depending on the season. Since the market for water has a weekly frequency, we have a state variable with a different value for every week of the year.

We assume that  $(p_t, r_t)$  is jointly i.i.d. conditional on  $w_t$ . We can compute (non-parametrically) the joint probability distribution of prices and rain. Hence, this assumption is testable. While assuming that  $(p_t, r_t)$  is jointly i.i.d. unconditionally is unrealistic (and we can reject it empirically), assuming that  $(p_t, r_t)$  is jointly i.i.d. conditional on  $w_t$  is both realistic and testable. Price is fully determined by the rain during last week, the season and some measure of the moisture in all farmers plot. In our data, it is the case that rain and season  $(r_t, w_t)$  are a sufficient statistic to predict price. In other words, after controlling for rain and season, the remaining “error” on prices is a white noise (uncorrelated to past rain, past prices or past “errors”).

The error term  $\epsilon_{ijt}$  is choice-specific. Hence, we are more interested on the differences in  $\epsilon_{ijt}$  across choices than in  $\epsilon_{ijt}$  *per se*. For example, in the case in which  $J = 1$ , the farmer has to choose whether to buy 1 unit or buy nothing. In this case the farmer will measure the difference in utility between buying or not for the observable components and the unobservable components. Taking the observable components (and the parameters) fixed, the probability of a farmer buying the good is increasing on the expectation of the difference in  $\epsilon_{ijt}$ , i.e.  $E[\epsilon_{i1t} - \epsilon_{i0t}]$ . If we assume that  $\epsilon_{ijt}$  follows a extreme type I distribution then  $(\epsilon_{i1t} - \epsilon_{i0t})$  follows a logistic distribution.

We allow the flow payoff function  $h(\cdot)$  to depend on the moisture of the plot every week, the season and an unobserved vector of parameters  $\gamma$ . We will also allow for an unobserved choice-specific component of the purchase/irrigation  $\zeta_j$ . The intuition for including  $\zeta_j$  is that the farmer might have to incur in additional cost (disutility) when irrigating, and this cost depend on the amount of units bought, e.g., if

the farmer has to hire a laborer to help him during the irrigation, the wage of the laborer will be increasing on the number of unit bought. The value function is then:

$$\begin{aligned}
V(M_{it}, w_t, p_t, r_t, \epsilon_{it}) &\equiv \max_{j_{it} \in \{0, 1, \dots, J\}} \{h(M_{i,t}, w_t; \gamma) - (j_{it}p_t + \zeta_j + \epsilon_{ijt}) + \\
&+ \beta E[V(M_{i,t+1}, w_{t+1}, p_{t+1}, r_{t+1}, \epsilon_{i,t+1}) | M_{it}, w_t, p_t, r_t, j_{it}]\} \\
&\text{s.t. } M_{it} \geq PW
\end{aligned} \tag{13}$$

where  $h(M_{it}, w_t; \gamma)$  is the payoff function and  $PW$  is the Permanent Wilting point, i.e., the level of moisture below which the tree will die (see Appendix A.2).

## 4.2 Model with Liquidity Constraints

The value function has six arguments. In addition to the five arguments used in Subsection 4.1, we have now an additional state variable:

- $\mu_{it}$ : represents the amount of cash that the individual has at period  $t$ .

The law of motion for the "cash" variable:

$$\mu_{it} = \mu_{i,t-1} - p_{t-1}j_{i,t-1} + \phi_{i0} + \eta_{it} + \nu_{it} \tag{14}$$

where  $\phi_{i0}$  is the weekly net cash flow of the farmer,  $\eta_{it}$  is the revenue that the farmer gets when she sells the harvest and  $\nu_{it}$  is an idiosyncratic financial shock with variance equal to  $\sigma_{\nu}^2$ .<sup>41</sup> We should interpret  $\phi_{i0}$  not only as the cash generated (or not expended) by the real estate owned by each farmer but also as the cash generated by other activities which in turn are correlated with real estate. Other activities include another job the farmer may have or other investments.

The value function is then:

$$\begin{aligned}
V(M_{it}, w_t, p_t, r_t, \mu_{it}, \epsilon_{it}) &\equiv \max_{j_{it} \in \{0, 1, \dots, J\}} \{h(M_{it}, w_t; \gamma) - (j_{it}p_t + \zeta_j + \epsilon_{ijt}) + \\
&+ \beta E[V(M_{i,t+1}, \mu_{i,t+1}, w_{t+1}, p_{t+1}, r_{t+1}, \epsilon_{i,t+1}) | M_{it}, w_t, p_t, r_t, \mu_{it}, j_{it}]\} \\
&\text{s.t. } M_{it} \geq PW \\
&\text{s.t. } j_{it}p_t \leq \mu_{it}, \forall j_{it} > 0
\end{aligned} \tag{15}$$

<sup>41</sup>Hence,  $\eta_{it}$  is equal to 0 all weeks except the week after the harvest when it is positive. We estimate this revenue non-parametrically using out-of-sample data and the rain and purchases of each harvest year.



## 5 Estimation

We estimate the parameters of the model using a three-step estimator. This estimator is an expansion of the two-step estimator proposed by Hotz and Miller (1993), in which we include a third step in order to estimate the parameters of the liquidity constraint. In the first step we estimate (non-parametrically) the transition probability matrices as well as the conditional choice probabilities (CCP).<sup>42</sup> In the second step, we use only the data of those farmers that we know are not liquidity constrained, using a CCP estimator (see Hotz and Miller, 1993) and the econometric model in sub-section 4.1.<sup>43</sup> With this estimator, we will get a consistent estimate of  $\Theta \equiv (\gamma, \zeta)$ , because these farmers are not constrained. We call this estimator  $\hat{\Theta}^0$ . We will then treat this estimator as the “true” value of  $\Theta$ . In the third stage, we will estimate the vector of financial parameters  $\phi$  using the econometric model in sub-section 4.2, taking the parameters estimated in the first and second stage as given.<sup>44</sup> See Appendix C for details about the estimation.

### 5.1 First Step

The first step of the estimator includes the estimation procedures outside the dynamic routine. We estimate the transition probability matrices for the relevant states using a (non-parametric) bin estimator. We also compute smooth conditional choice probabilities using the methods described in Srisuma and Linton (2012).

#### 5.1.1 Transition Probabilities

We estimate the transition matrices for each of the state variables of the model (moisture, week, price, rain) except the cash holdings. The transition probability of the cash holdings will be estimated in the third stage. As explained in the previous section the transition of the moisture and rain are deterministic. Rain and price are assumed to be jointly i.i.d. conditional on the weekly seasonal effect.

#### 5.1.2 Conditional Choice Probabilities

We also compute smooth conditional choice probabilities (CCP) in the first step (see Appendix C.1). One of the shortcomings of dynamic discrete choice estimation methods is that the state space needs to be finite and discrete. If one or more of the state variables are continuous, the econometrician usually

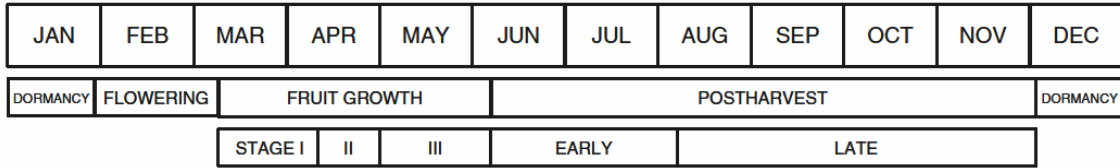
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<sup>42</sup>In our data set there are many states, this means that the probability of purchasing in a given (discretized) state is very low. Instead of defining a coarser state space we compute non-parametric smooth CCP. See Appendix C.1 for details.

<sup>43</sup>See Appendix C.

<sup>44</sup>Aguirregabiria and Mira (2007) proposed an algorithm to incorporate permanent unobserved heterogeneity into dynamic games. However, their algorithm only allows for unobserved heterogeneity in the payoff function. In addition to that, CCP is much faster than other estimation methods, and speed is also a binding constraint when estimating a model with a big parameter space dimensionality.

Figure 1: Seasonal Stages for “Búlida” Apricot trees



Source: Pérez-Pastor *et al* (2009).

resort to “discretize” the continuous variables. Sometimes, even if the state variable is discrete in nature the econometrician group several values together in order to reduce the dimensionality of the problem. Without this discretization, the high dimensionality of the state space would make some problem intractable. Moreover, if some state variable has a big support, there might be no observations in some of the bins, making it impossible to compute the CCP. This is an issue in our specification in which we allow for 52 seasonal effects and want the moisture variable to be as fine as possible.

An alternative method to deal with continuous or very fine discrete variables is to compute smooth CCP. With smooth CCP we can have a richer state space. Smooth CCP are probabilities created from the raw probabilities and a smoothing non-parametric kernel. The kernel assigns positive probability not only to the bin that corresponds to the observation, but also to bins that are “close” to it. Moreover, grouping several values of a discrete variable can be seen as a particular case of smooth CCP, in which the probability assigned to each data point is positive and uniform within the new bin and zero outside the bin.

## 5.2 Second Step

In this sub-section, we estimate the parameters affecting the production function  $(\gamma; \zeta)$  using the CCP estimated in the first step and the model presented in sub-section 4.1. Since we are estimating a model without liquidity constraints (LC), in the second step we will only use data on unconstrained farmers.

Following Torrecillas *et al* (2000) we can specify the weeks of the year in which irrigation is “critical” for apricot trees, as shown in Figure 1. The critical weeks include the second rapid fruit growth period (Stage III) and two months after the harvest, i.e., Early Post-Harvest (EPH). Both periods are located consecutively: before and after the harvest.

Stage III corresponds to the period of high growth before the harvest. This stage is critical because it is the stage at which the trees “transform” water into fruit at the highest rate. The EPH period is also important because of the stress that the trees suffer during the summer after the harvest. Before and during the harvest the trees use the water at a high rate. Hence, the levels of moisture in the trees are

very low after the harvest. In order for the trees to survive the summer, they need to be irrigated. Failure to do so will result in a lower output during the next season (see Pérez-Pastor *et al*, 2009).

For the main estimation we will consider a simple payoff function with  $\gamma \equiv (\gamma_1, \gamma_2)$ :

$$h(M_{t-1}, w_t; \gamma) = [\gamma_1 \cdot (M_{t-1} - PW) \cdot KS(M_t) \cdot Z_1(w_t) + \gamma_2 \cdot (M_{t-1} - PW) \cdot KS(M_t) \cdot Z_2(w_t)] \cdot area_i \quad (16)$$

where  $h(M_{t-1}, w_t; \gamma)$  is the harvest at period  $t$ ,  $area_i$  is the size of the land ( $m^2$ ) that farmer  $i$  owns,  $KS(M_t)$  is the hydric stress coefficient (see Appendix A.2),  $Z_1(w_t)$  is a dummy variable that equals 1 during weeks 18-23 and 0 otherwise:

$$Z_1(w_t) = \begin{cases} 1 & \text{if } 18 \leq week \leq 23 \\ 0 & \text{otherwise} \end{cases} \quad (17)$$

and  $Z_2(w_t)$  is a dummy variable that equals 1 during weeks 24-32 and 0 otherwise:

$$Z_2(w_t) = \begin{cases} 1 & \text{if } 24 \leq week \leq 32 \\ 0 & \text{otherwise} \end{cases} \quad (18)$$

The characterization of  $\gamma$  is just a direct application of the results in the agricultural engineer literature (Torrecillas *et al*, 2000; Pérez-Pastor *et al*, 2009).  $\gamma_1$  measures the transformation rate of the fruit during the fruit growth (stage III) season.  $\gamma_2$  measures the recovery of the tree during the early post-harvest stress season. Both parameters are measured in *pesetas per millimeters* and *square meter*, or in *pesetas per liter*.

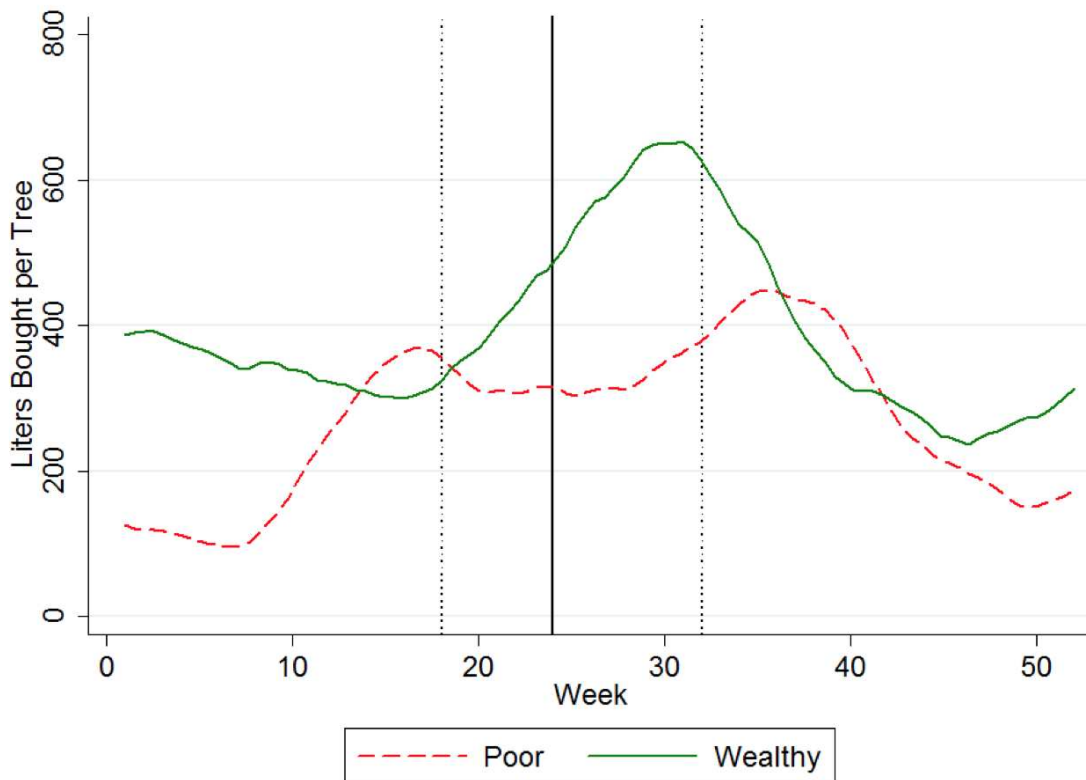
With this payoff function we can compute the revenue that the farmer obtains in a given year:

$$Revenue_t = \sum_{w_t=1}^{52} h(M_{t-1}, w_t) = \sum_{w_t=18}^{23} \gamma_1 \cdot (M_{t-1} - PW) \cdot KS(M_t) \cdot area_i + \sum_{w_t=24}^{32} \gamma_2 \cdot (M_{t-1} - PW) \cdot KS(M_t) \cdot area_i \quad (19)$$

### 5.3 Third Step

Figure 2 shows the liters of water bought, on average, for both the wealthy and the poor farmers, as a function of the season (week). We can see from the data that, for the wealthy farmers, which are unconstrained, the periods in which irrigation is more likely are the weeks before the harvest (weeks 18-23) that correspond to Stage III and the weeks after the harvest (weeks 24-32) that correspond to EPH.

Figure 2: Liters bought by week.



\* The vertical lines mark the critical irrigation period. The vertical solid line indicate the harvest. The first vertical dotted line indicates the beginning of the pre-harvest season (Stage III) and the second vertical dotted line indicates the end of the post-harvest season (EPH).

Table 6: Structural Estimation. Demand Parameters.

Structural Parameters		Baseline (1)
Transformation rate <i>pre-season</i>	$\gamma_1$	0.4404
		-
Transformation rate <i>post-season</i>	$\gamma_2$	0.4404
		-
Irrigation cost	$\zeta_{-0}$	10.0149
		-

In specification 1 we restrict the value of the transformation rate to be the same pre- and post-season, i.e.  $\gamma_1 = \gamma_2$ .

## 5.4 Estimation Results

In this sub-section, we present the estimation results of the structural model under different specifications. We present the structural estimates obtained using a tolerance level of  $1.0e - 25$ . In table 6 we present the results of the second stage of the estimator, i.e., the demand parameters  $(\gamma; \zeta)$ , of equation 13. We use the functional form presented in equation 19. In the baseline case we restrict the values of the vector  $\zeta$  so that it is constant if the farmer irrigates a positive amount and zero if the farmer does not irrigate, i.e.,  $\zeta_0 = 0$  and  $\zeta_j = \zeta_k = \zeta_{-0} \forall j, k > 0$ . Hence, we are estimating three parameters in the second stage  $(\gamma_1, \gamma_2; \zeta_{-0})$ .

The results in Table 6 refer to the estimation of the model expressed in equation 13 using the specification shown in sub-section 5.2 (see Appendix C.2 for details in the estimation). The value of  $\gamma_i$  correspond to the transformation rate of the median farmer, with 76 apricot trees. Hence, a value of 0.44 corresponds to a transformation rate of 0.006 *pesetas* per tree, per millimeter of moisture above the Permanent Wilting point. The irrigation cost represents the cost in *pesetas* that a farmer must incur every time he wants to irrigate.

In Table 7 we show the values of the estimated parameters of equation 23 in the third stage. We can see that there is a lot of variation both in the estimated consumption rate  $\phi_{i0}$  and in the estimated variance of the idiosyncratic shock  $\nu_{it}$ . The consumption rate is an estimation of the net consumption of each farmer. The differences in mean annual revenue are driven by both differences in purchase patterns and differences on the number of trees.

## 6 Discussion

In this section we compute the welfare under both the auctions and the quotas. The structural model allow us to see what differences in the allocation from the two institutions are more important, including Liquidity Constraints (LC). In sub-section 6.1 we describe our welfare estimations for both markets and

Table 7: Structural Estimation. Liquidity Parameters.

Farmer ID	Consumption Rate	Variance	Mean Yearly Revenue
$i$	$\phi_{i0}$	$\sigma_{iv}^2$	$\eta_i$
1	12.1	18.9	924.4
2	7.7	8.6	1,980.9
3	51.8	0.2	5,089.4
4	10.3	144.6	1,646.1
5	17.9	30.1	2,664.8
6	11.2	13.9	1,696.0
7	8.5	51.2	1,216.5
8	3.0	2.5	1,102.1
9	11.3	20.2	1,820.2
10	0.2	0.7	1,955.3
11	51.9	162.9	3,508.4
12	22.5	17.0	2,475.8
Mean	17.2	39.2	2,173.3
SD	17.2	55.5	1,166.1

All terms expressed in pesetas. Results for the third stage using the values in specification 1 in the previous table for the second stage.

quotas, under different assumptions. In sub-section 6.2 we show a summary of the results. In sub-section 6.3 we show the results disaggregated by year. In sub-section 6.4 we discuss the limitations of our analysis.

## 6.1 Welfare Measures

In this section we use the results from the previous section and perform counterfactual analysis. The goal of this section is to use the demand parameters estimated in the previous section  $\hat{\gamma}$  and compute the welfare under different scenarios. For the auction system we compute both the Revenue and the Welfare per tree and year. In the quotas system, Revenue and Welfare coincide in expectation. In particular, following equation 19 we have:

$$Revenue_i = \frac{1}{\# trees_i} \frac{1}{T} \sum_{t=1}^T [Revenue_{it}] = \frac{1}{\# trees_i} \frac{1}{T} \sum_{t=1}^T \left[ \sum_{w_t=1}^{52} h(M_{i,t-1}, w_t) - (\zeta_j) \right] \quad (20)$$

$$Welfare_i = \frac{1}{\# trees_i} \frac{1}{T} \sum_{t=1}^T [Welfare_{it}] = \frac{1}{\# trees_i} \frac{1}{T} \sum_{t=1}^T \left[ \sum_{w_t=1}^{52} h(M_{i,t-1}, w_t) - (\zeta_j + \epsilon_{ijt}) \right] \quad (21)$$

Notice that we do not take into account the expenses in water. Since this is a welfare comparison, transfers should not be taken into account. Notice also that the only difference between Revenue and Welfare corresponds to the choice specific unobservable component. Since the error term  $\epsilon_{ijt}$  is choice-

specific the relevant elements are differences in  $\epsilon_{ijt}$  across choices, and not  $\epsilon_{ijt}$ . For example, in the case in which  $J = 1$ , the farmer has to choose whether to buy 1 unit or buy nothing. In this case the farmer will balance the difference in utility between buying or not, considering both the observable components and the unobservable components. The probability of a farmer buying water increases with the expectation of the difference in  $\epsilon_{ijt}$ , i.e.,  $E[\epsilon_{i1t} - \epsilon_{i0t}]$ .

By construction, the unconditional mean of the differences in the error term is zero. Hence, in the quota system, since the farmers cannot choose when to irrigate, the expectation of the differences in the error term is zero, i.e.,  $E[\epsilon_{i1t} - \epsilon_{i0t}] = 0$ . However, in the auction system, farmers can choose when to irrigate. Hence, the expectation is not zero. Moreover, farmers are more likely to irrigate when their (unobserved) utility of irrigation is high, i.e.,  $\hat{\epsilon}_{i1t} > \hat{\epsilon}_{i0t}$ . This implies that under the auction system we have  $E[\epsilon_{i1t} - \epsilon_{i0t}|j = 1] > 0$  and  $E[\epsilon_{i0t} - \epsilon_{i1t}|j = 0] > 0$ . In other words, with the auction, gains from trade can be realized. In the model presented here gains from trade are translated into the timing of the irrigation. Farmers “trade” with each other in order to irrigate at their preferred time.<sup>45</sup>

We compute the revenue generated by the auction system, for poor and wealthy farmers, using the actual allocation of water during the sample period. We also compute the revenue generated by the counterfactual allocation under the quota system under different scenarios:<sup>46</sup>

## Auctions

- **Poor farmers:** We compute the revenue produced during the period of study using  $\hat{\gamma}$  and the actual purchases made by the poor farmers. We use equation 19 and the actual moisture in the farmers’ plots and compute the revenue for each farmer and for the whole economy.
- **Wealthy farmers:** We compute the revenue produced during the period of study using  $\hat{\gamma}$  and the actual purchases made by the wealthy farmers. We use equation 19 and the actual moisture in the farmers’ plots and compute the revenue for each farmer and for the whole economy. Notice that this welfare could be greater than the first best average welfare. Since poor farmers are sometimes constrained, wealthy farmers are buying more water than what the first best establishes and thus, getting a greater revenue.

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<sup>45</sup>Farmers do not actually trade with each other, they just “select” when they want to trade. The auction system is more flexible than the quota system in this aspect.

<sup>46</sup>Alternatively, we could simulate the optimal decision of each farmer in the case in which they are not liquidity constrained and compute the revenue. We could use the actual prices and rain patterns, and then compute the counterfactual moisture. Notice that this case is an overestimation of what the welfare under the auction without liquidity constraints would be, because we are using the actual prices. If all farmers were not liquidity constrained, as we have seen in sub-section 3.2, the prices would be greater. With greater prices the farmers would buy less water overall and thus, will produce a lower revenue.

## Quotas

- **Based on # of trees:** There were 53,020 trees in the *Huerta* of Mula in 1955 according to the agricultural census and there are 24 individuals with 2,069 apricot trees in total (86.2 trees on average) in our sample. We consider a counterfactual case in which all the irrigated land was planted with apricot trees. Hence, we consider an economy with 616 farmers, each of them with 86 apricot trees. In this case each farmer consumes 0.2 units of water every three weeks, if there was any water available during that week.
- **Based on # units bought:** The 24 apricot farmers bought 750 units over our sample period. We consider as a counterfactual an economy with 24 farmers with 86.2 apricot trees each. Hence, this economy has also 2,069 apricot trees. In this case, every time one unit was bought in the real data, all farmers will receive an equal amount of water.
- **Based on # units bought, adjusted:** We also adjust the previous case by making farmers receive water only once every three weeks, if there is any water available during that three-week period. Hence, farmers will receive a greater amount of water each time, compared to the previous case. However, farmers will irrigate at most once every three weeks.

## 6.2 Welfare Results

In Table 8 we show the weighted (by # of trees) average of revenue and welfare, across farmers. The first thing to notice in Table 8 is that our estimation for 1955-66 has a greater average revenue and a lower dispersion than the data corresponding to 1954. One explanation is that 1954 was a dry year, hence the revenue was lower than average and the gap in revenue was greater than average.<sup>47</sup> In addition to that, we cannot rule out that the wealthy farmers are more productive due to unobservables that the poor farmers. The differences in revenue estimated in Table 8 are based on differences in moisture only, since our specification assumed that all farmers are equally productive, up to an idiosyncratic shock. One explanation for this unobserved differences in productivity could be that wealthy farmers also used other productive input (such as manure or hired labor) in greater quantities than poor farmers. This explanation would be consistent with the idea that poor farmers get a lower revenue due to their LC, but also through a different input.

We can also see in Table 8 the results from the counterfactual welfare measures. As expected, under

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<sup>47</sup>In Figure 3 we can see that this was indeed the case for 1958 and 1963.



the auction system, poor farmers have a lower revenue than wealthy farmers (17% less). Also expected is the fact that, under the auction system welfare is greater than revenue, while welfare is identical to revenue under the quota system. Finally, we can see that the Quota system increases the revenue of the poor farmers (17% increase in revenue), but not for the wealthy farmers (0% increase in revenue). Overall, the change to a quotas improved the welfare of the average farmer (8% increase in revenue).

The welfare computed using the # trees in the sample seems too high. The reason is that not all the area under cultivation is planted with trees. If we do not take into account that fact we are left with a smaller area. This translates into a greater amount of water available for irrigation per tree. The high value is a direct consequence of farmers getting a lot of water for irrigation using this counterfactual.<sup>48</sup> In order to avoid this issue, we compute another counterfactual allowing the farmers to use only the amount of water actually bought in the sample. This computation is more realistic as we can see in column 4. The results in columns 4 and 5 are very similar since the only thing that changes is the frequency of irrigation.

The counterfactual results in columns 4 and 5 are just a redistribution of water from wealthy apricot farmers to poor apricot farmers. In reality, if apricot farmers are poorer than the rest of the farmers, there would also be a redistribution from non-apricot farmers to apricot farmers, and *vice versa*.

Table 8: Welfare (*pesetas* per tree and year) - Counterfactual.

	Auctions			Quotas		
	Revenue 1954 (0)	Revenue (1)	Welfare (2)	# trees (3)	# units (4)	# units* (5)
ALL	118.8	122.1	-	169.2	132.2	135.8
Poor	82.8	112.0	-	169.2	132.2	135.8
Wealthy	145.3	132.2	-	169.2	132.2	135.8

In column (0) we report the revenue reported by the farmers in the Agricultural census. In column 1 we report just the value of the production  $h(\cdot)$  minus the cost of irrigation  $\zeta$ , under the auction system. In column 2 we report the total welfare, that is including the  $\epsilon$  when farmers irrigate. Notice that the quantities in column 2 are always greater than the quantities in column 1: in the auction system, since farmers can choose when to irrigate, they irrigate when the value of  $(\epsilon_1 - \epsilon_0)$  is big. In columns 3, 4 and 5 we report the revenue, since in the case of quotas the expectation of  $(\epsilon_1 - \epsilon_0) = 0$ . In column 3 we report the revenue of the counterfactual quota system when we use a uniform amount of water in each week, assuming there are 53,020 apricot trees in the economy. In column 4 we report the revenue of the counterfactual quota system when we redistribute the water that farmers actually used and they irrigate once every week. In column 5 we report the revenue of the counterfactual quota system when we redistribute the water that farmers actually used and they irrigate once every three weeks.

Table 9 shows the estimated revenue for both institutions disaggregated by season. We can see there that quotas outperform auctions in terms of revenue. However, this comparison is made without considering the irrigation costs. Since the quota system requires a more frequent irrigation, adding these costs means

<sup>48</sup>Alternatively one could compute the welfare if we assume that all the area not planted with trees (but planted with vegetables) were to be planted with trees. In this case the area is too big, and the welfare computations too small.

Table 9: Revenue comparison.

	<i>pesetas</i> per tree	Pre-season	Post-season	Total
Quotas	All	50.9	81.3	132.2
	All	47.5	74.6	122.1
Auctions	Poor	44.1	67.9	112.0
	Wealthy	50.8	81.4	132.2

\* All terms are expressed in *pesetas* per tree. Quotas based on # units bought.

that the reduction in revenues in the quota system will be greater than the reduction in revenues in the auction. Moreover, in order to fully compare both mechanisms, we need to add to the auction system the gains in welfare due to the gains from exchange, as explained above.

### 6.3 Yearly Results

In Figure 3 we can see that there is substantial variation on revenue across time. This variation is due to the variation in rainfall. Revenue is lowest for both poor and wealthy farmers during 1961-63, which were also the driest years in our sample (see Figure 9 in Appendix A). It is also interesting to see how the profits, i.e., the difference between expenses and revenue per tree, decreases during the drought of 1961-63 and then increases again after it.

In Figure 4 we can see the profits of the poor and wealthy farmers. We can see in the figure that the profit of both types of farmers is low when the price is high (1957 and 1962). What is more interesting is that the gap between profits goes up the year after the price is high (1958 and 1963). We interpret this increase in the profit gap as the financial effect that a drought has on the savings of the poor farmers. The poor farmers can “survive” during a dry year, by using some of the accumulated savings from previous years. However, if the drought persists, then the farmers cannot keep up and the profit gap increases.

In Figure 5 we decompose both the expenses and the revenue into pre- and post-season. We can then see how the revenue in the pre-season in 1961 was high, due to the rains of the spring of 1961 and the generous winter of 1960-61 (see Figure 9 in Appendix A). After that we can see the drop in pre-season revenue in 1962-63. It is worth noticing that the differences in revenue between poor and wealthy are mostly driven by the pre-season revenue after 1961. This is consistent with the idea that poor farmers will buy water and get a level of moisture similar to the wealthy farmers only during the post-season when they have the money from the harvest. This effect is more clear after the drought of 1961-63, since the savings of the poor farmers are more exhausted than during the rest of the period.

In Figure 6 we can compare the revenue obtained with the auction system for both poor and wealthy

Figure 3: Average Revenue (estimated) and Expenses (data)

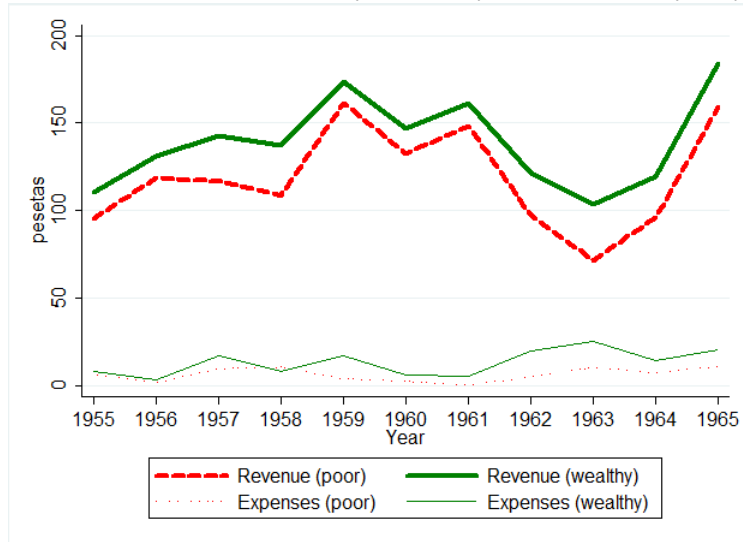


Figure 4: Profits (Revenue - Expenses)

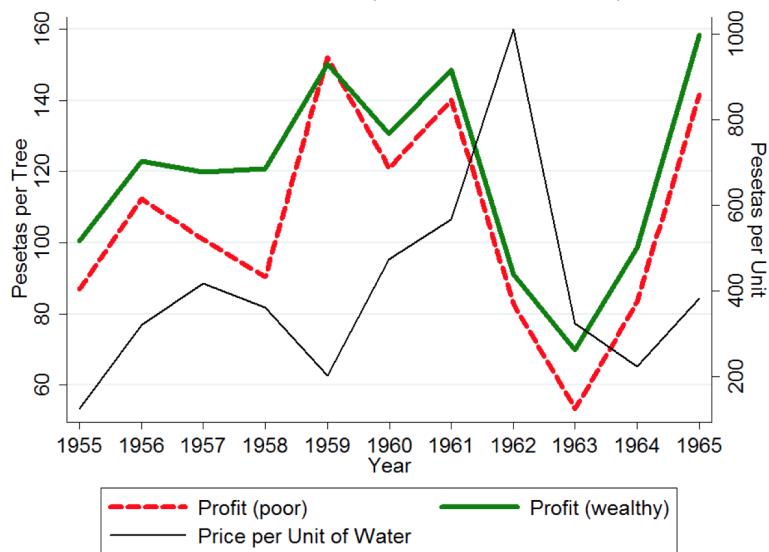


Figure 5: Average Revenue (estimated) and Expenses (data).

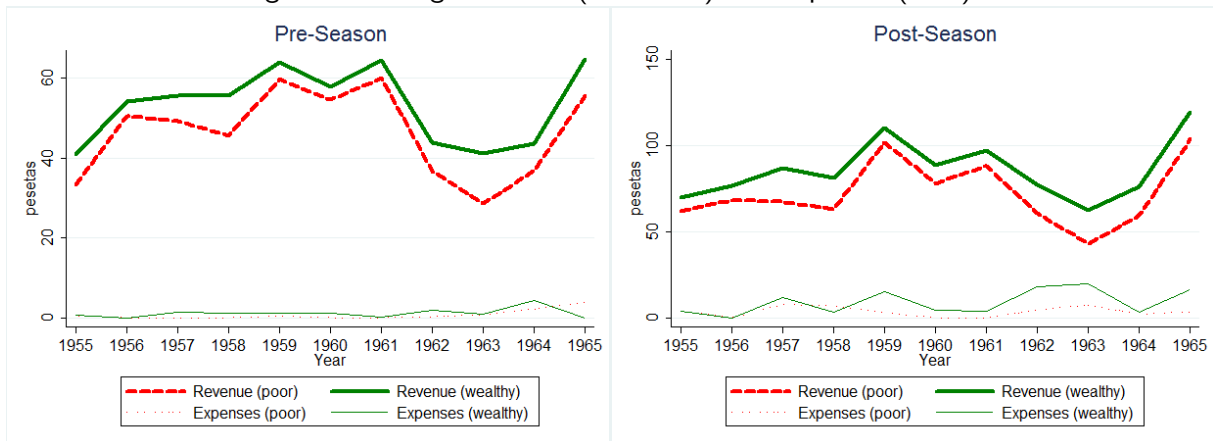
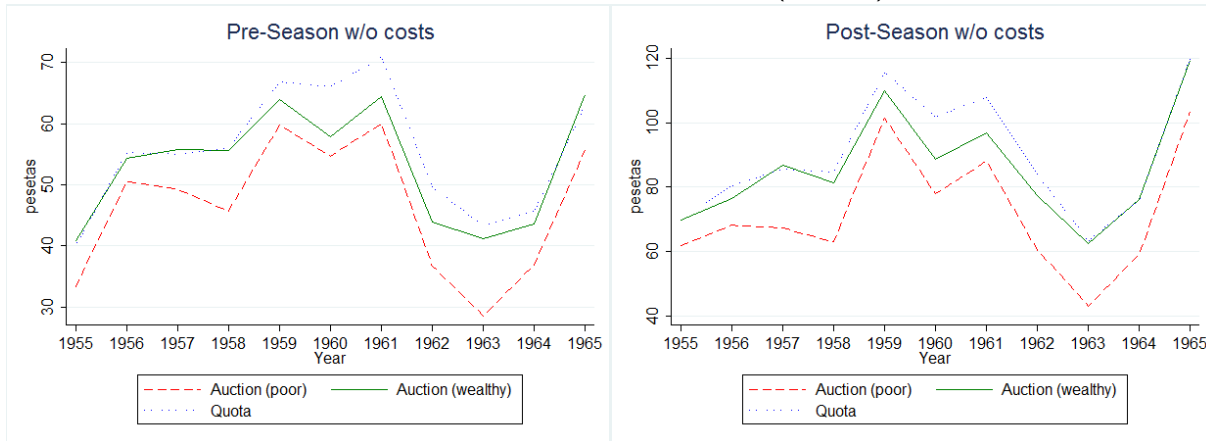


Figure 6: Seasonal Revenue per tree (*pesetas*).



farmers as well as the counterfactual with quotas (based on # units bought). We can see that, in terms of revenue, the quotas perform best overall. Farmers under quotas would obtain a similar revenue to the wealthy farmers with the auction during the beginning and the end of our sample period and a greater revenue during the middle of the sample period. Notice as well that the greatest difference between the revenue of the poor and the quotas, especially during the dry years of 1962-64, happen during the pre-season.

#### 6.4 Limitations

In addition to the redistribution of water, there are other margins in which a system of quotas could improve the efficiency with respect to the market. We have taken into account some of them when making the comparison, but others are harder to quantify. Here are those that are harder to quantify:

**Strategic Supply** Whether to run an auction or not was a decision made by the president of the *Heredamiento the Aguas*. There is no evidence of a strategic decision on whether to run the auction: if there was enough water in the dam, the auction was run. However, we know that the president could stop the auction at any time, and indeed used to do so if the price fell considerably (usually to less than 1 *peseta*). This situation was uncommon and happened only after an extraordinary rainy season. However, we should take into account that the decision on when and whether to sell water, made by the seller, need not be welfare-maximizing but would be revenue-maximizing.

**Strategic Size and Sunk Cost** The results obtained when comparing revenue from quotas and auctions suggest that the choice of size of the units allocated is not innocuous. In particular, the fact that in some

years the farmers under the quota system produce a greater revenue than the wealthy farmers under the auction system suggest that the size of the units sold at the auction (3 hours) might be too big. The size of the units sold at the auction has not changed since the Middle Ages. This could be due to institutional persistence or due to technical reasons, i.e., 3 hours could be the size that maximizes revenue. Based on the results, it could be the case that 3 hours maximizes revenue but not welfare. The optimal size would be determined by a trade-off between the sunk cost incurred every time a farmer irrigates (due to the loss of water flowing through a dry channel) and the diminishing returns of water.

As shown in Donna and Espín-Sánchez (2013), the first unit of water allocated to a plot has to flow through a dry channel, thus, some of the water will be lost. Subsequent units associated with the same channel will then flow through a wet channel, thus not losing any water. In the auction system, subsequent units are allocated to different farmers, depending on who has won each unit. However, in the quota system, units are allocated to each farmer in geographical order, i.e., every unit will be allocated to a neighbor farmer down the channel with respect to the previous farmer.<sup>49</sup>

The sunk cost implied by the dry channel would only be incurred by the farmer that irrigates first. In our estimation,  $\zeta$  is capturing the effect of both the sunk cost and the irrigation cost. Ideally we would like to include the irrigation cost but not the sunk cost when computing the welfare with Quotas. Hence, if we include the  $\zeta$  in the welfare analysis for the quotas, we might be underestimating the welfare, and if we do not include it, we might be overestimating it.

**Optimal crop mix** In our computations and counterfactuals, we are only considering cases in which the farmers are growing one crop only. Since different crops have different needs for irrigation in different seasons, the optimal crop mix will involve several crops. For example, oranges are harvested in winter, and their need for water peaks in December. Apricots are harvested in summer, and their need for water peaks in June. Hence, a crop mix with apricot and orange trees would outperform a single crop. This is indeed what we see in the data; many farmers have orange trees and either apricot, peach or lemon trees (all three are harvested during summer).

## 7 Conclusions

In this paper, we investigated a unique historical episode. A market that was active for more than 700 years came to an end and was replaced by a system of fixed quotas. The puzzle here is not so much

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<sup>49</sup>In the neighboring city of Lorca, auctions are carried out independently for farmers with lands in each sub-channel. This way, the water has to travel shorter distances and the amount of water lost is smaller.

that the institution changed, as it is that the old institution was a market institution while the new institution prohibited trading. Under general conditions, markets are considered the most efficient allocation mechanism. Theoretically, we showed that when agents face liquidity constraints (LC), markets are no longer the most efficient allocation. Moreover, a mechanism as simple as a fixed quota could outperform markets if LC are sufficiently severe. However, if LC are not sufficiently severe, the market would still be more efficient than the quota. Hence, whether the institutional change improved efficiency is an empirical question.

As suggested by some historians, we showed empirically that LC were present. Poor farmers bought less water than rich farmers during the critical seasons and they had lower revenue per tree as a consequence. However, estimating demand when LC are present is not simple: a reduction in the amount purchased after an increase in price could be due to either downward sloping demand or LC. The inter-temporal substitution of water demand further complicates the analysis. We used a detailed data set and structural dynamic demand estimation methods to identify LC from demand. We showed that neglecting LC would result in an underestimation of the demand parameters.

With the recovered demand parameters we computed the welfare under the market and under the quotas. We also proposed an econometric test, based on the model proposed, to test which system was more efficient. Based on the results we concluded that the institutional change improved efficiency, i.e., the quotas generated greater revenue than the market. Hence, the end of the water market in Mula was a “settled problem of irrigation”.<sup>50</sup>

The contributions of this paper are manifold. From a historical perspective we have provided empirical evidence of a source of inefficiency in water markets, as well as empirical support for the institutional change proposed by Espín-Sánchez (2013). From a theoretical perspective we have proposed a dynamic model that includes storability, seasonality and LC, and shown the dynamics of this economy under a market institution. Moreover, we have discussed the relation between storability and LC and shown how ignoring LC would result in biased estimates. We also expanded the conditional choice probability estimator proposed by Hotz and Miller (1993) and computed unobserved heterogeneity outside the payoff function. We believe the three-step estimator proposed here could also be used in other applications.

Finally, the empirical results in this paper apply only to this specific setting, and one should not conclude

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<sup>50</sup>Nowadays the institution in place is a two-tiered pricing. Farmers pay a low price for the water used up to a certain threshold (quota) and they pay a high price for the water if they want to use more water than their quota. The two prices are set every year and are meant to cover all operational costs. This system has the best features of Markets (people get to choose when to irrigate, and if they want to irrigate “more”, they can do so at a premium) and Quotas (payments are made after the harvest and each farmer is entitled to some water at a low price).

that all water markets are inefficient. We have presented an empirical framework with the main ingredients found in water markets: seasonal demand, storability and LC. The empirical framework can be used by other researchers in other cases to assess the efficiency of water markets. We have also shown how ignoring the financial situation of the farmers will lead to biased demand estimations. This result applies to a more general case and implies that researchers should place more emphasis on the financial characteristics of the farmers and not only on their demand characteristics.

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## A Data Appendix

In this section we add detailed information regarding the data gathering and the moisture computation.

### A.1 Detailed Data Information

In this sub-section we describe in greater detail the data set and show some graphs to better understand the context of the imperial setting. We also present sample of the pictures used to create the data set.

#### Auction Data

In this paper we will not take into account the price differences within each week. Although we have all the prices (40 prices per week), we only use the average price paid during that week and assume that all farmers pay the same price. This simplification is not without cost but it greatly simplifies our analysis and helps us focus on the main points of the paper: Liquidity Constraints and its implications for efficiency and dynamic demand estimation. Nonetheless, in Appendix C.5 we provide an econometric model to estimate the demand when we average over 4-unit auctions rather than the 40-unit auction. For details about the dynamics and strategic behavior within 4-unit auctions see Donna and Espín-Sánchez (2013).

Figure 7 shows a sample picture of auction data. We can see the names of the 40 farmers that bought water during that week and the prices they paid, which corresponds to May 17 1963.

Figure 8 shows the weekly average price paid by the farmers during our sample period. There is a lot of variation with prices ranging from (virtually) zero to 2000 *pesetas*. The fall of 1955 saw a big flood that damaged the dam for several months, thus auctions could not be carried out until the next fall. We can also see some especially dry years like 1961-63 when there were no auctions in winter, causing the prices to soar in spring and summer. Finally, after 1964, prices are less volatile than in the rest of the sample.

Figure 7: Sample of Auction Sheet

*Heredamiento de Aguas de Mula*  
**Venta Pública de Agua**  
 HOY DIA 17 DE Mayo DE 1963

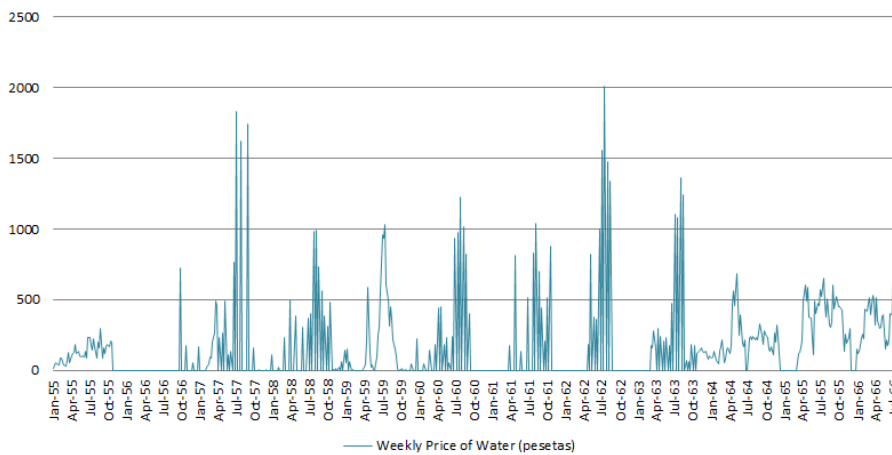
NOCHE		Puntos	Cts
1	D. Juan Vázquez	410	-
2	Francisco Jacinto	400	-
3	Juan Vázquez	400	-
4	Francisco Jacinto	161	-
5	José Belmonte	203	-
6	Carlos Belmonte	100	-
7	Cristóbal Belmonte	210	-
8	Salvador Pastor	153	-
9	Francisco Belmonte	162	-
10	Leandro Belmonte	161	-
11	Juan Belmonte	120	-
12	Juan López	121	-
13	Juan Belmonte	91	-
14	Juan Belmonte	101	-
15	Cristóbal Belmonte	120	-
16	Francisco Jacinto	101	-
17	Juan Belmonte	101	-
18	Juan Belmonte	20	-
19	Valentín Pastor Aguirre	108	-
20	Juan Belmonte	20	-
Suma		3201	-

DIA		Puntos	Cts
1	D. Pedro Jacinto	571	-
2	Juan Belmonte	200	-
3	Juan Belmonte	251	-
4	Juan Belmonte	151	-
5	Juan Belmonte	410	-
6	Leandro Belmonte	300	-
7	Juan Belmonte	250	-
8	Juan Belmonte	250	-
9	Juan Belmonte	250	-
10	Juan Belmonte	251	-
11	Juan Belmonte	235	-
12	Leandro Belmonte	200	-
13	Pedro Belmonte	301	-
14	Juan Belmonte	201	-
15	Juan Belmonte	201	-
16	Juan Belmonte	228	-
17	Juan Belmonte	150	-
18	Juan Belmonte	150	-
19	Juan Belmonte	150	-
20	Pedro Belmonte	101	-
Dia		4886	-
Noche		3115	-
Suma		8001	-
Vta pública		0	-
Total líquido		8001	-

Mula 17 de Mayo de 1963.  
 El Presidente: [Signature]  
 El Secretario: [Signature]

Figure 8: Prices of water 1955-66 (pesetas)

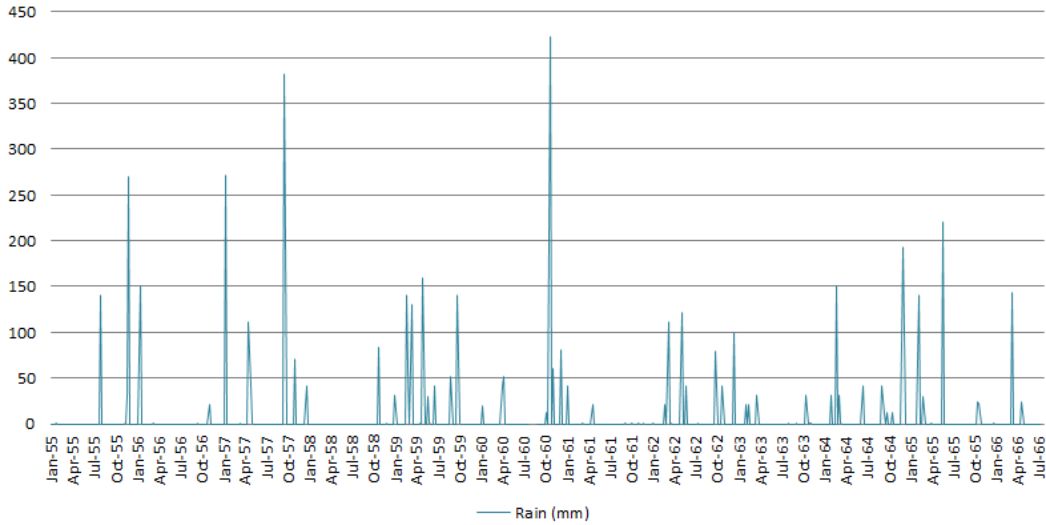


Source: Own elaboration from the data from the Municipal Archive in Mula. Weekly average.

**Rainfall Data**

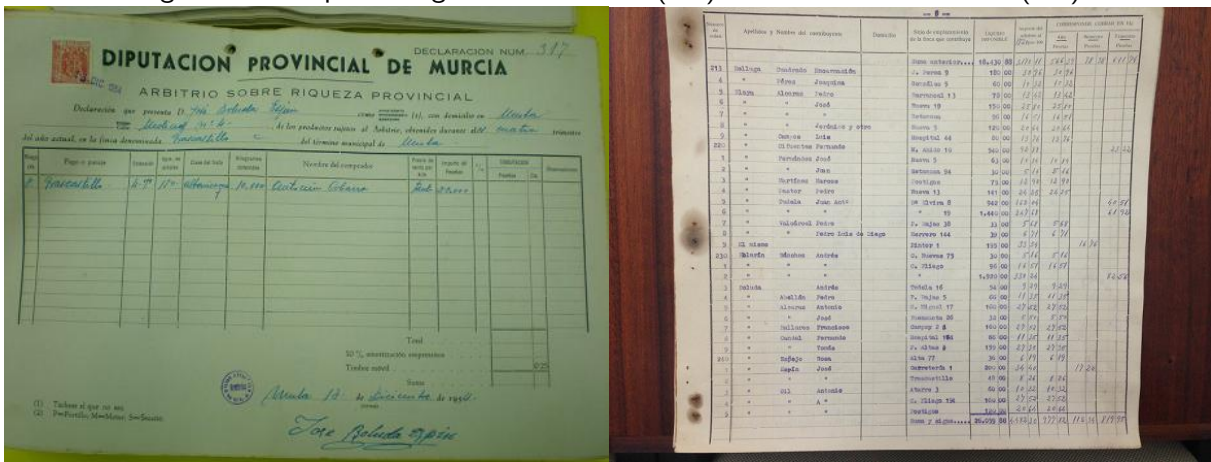
Mediterranean climate rainfall occurs mainly in spring and fall and peak water requirements for the products cultivated in the region are reached in spring and summer, between April and August. Between these months, more frequent irrigation is recommended because during this period trees quality of production is

Figure 9: Weekly Rainfall in Mula (mm).



Source: Own elaboration from the data from AEMET.

Figure 10: Samples of Agricultural Census (left) and Real Estate Census (left)



more sensitive to water deficits. In Figure 9, we can see that there are very few weeks in which it rains. Moreover, its median is zero. However, in some of the rainy weeks the amount of rain is substantial. In our short sample, on two occasions - September 1957 and October 1960 - the weekly rain exceeded the yearly average.

### Agricultural Census Data

Figure 10 shows a sample card of a farmer taken from the census data. It can be seen in Table 1, that Area and # Trees varies considerably across farmers. If we focus on apricot trees, on average, each farmer has 86 trees and buys 31.5 units for the period 1955-66.

Figure 11: Composition of the plots in Mula.

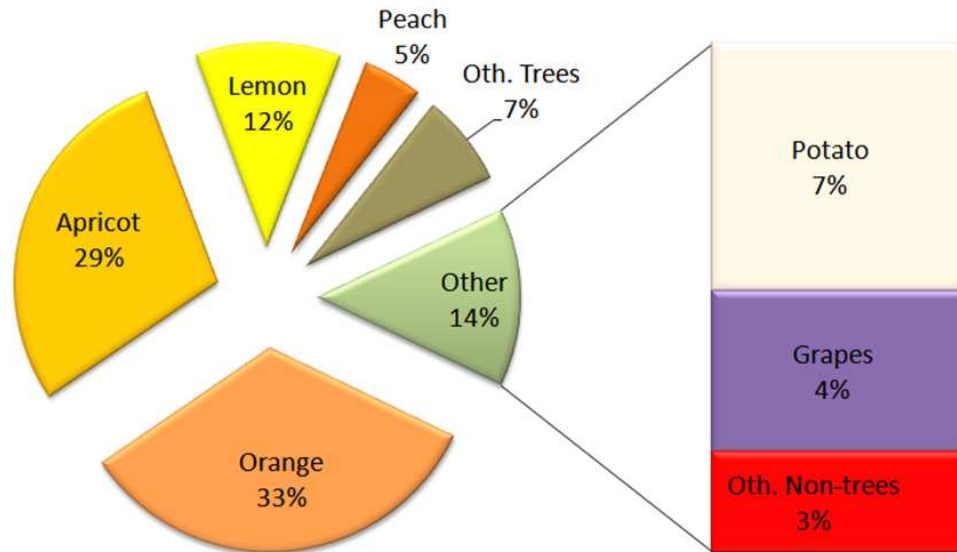


Figure 11 shows the composition of the land in Mula, based on the area of the plots. We can see that the most common crops are apricot and orange trees, followed by lemon and peach. There are also other trees such as pear and apple trees present in the area. Finally, there is a variety of vegetables (tomatoes, red peppers, cucumbers) and a considerable area planted with potatoes. The role of the vegetables and potatoes is complementary to the trees. Fruit trees produce greater returns than vegetables, but they require irrigation at specific times of the year and up to five years to reach maturity. By contrast, vegetables, although they have lower returns, can be harvested a few months after the sowing. Hence, they can produce a high output during a rainy year and the cost of drying up during a drought is not very high; they can be sown again and be ready to produce the year after.

## Real Estate Tax Data

The value in the real estate data records corresponds to the taxable income for urban real estate only. Farmers have to pay an annual tax equal to 17% of the taxable income. The rural real estate holdings are subject to different taxes and are kept in a different directory. This is important because the real estate tax data is capturing precisely the effect that we are interested in: non-agricultural wealth.

## A.2 Moisture Computation<sup>51</sup>

Trees are traditionally positioned in a square grid, each trunk 9 meters ( $m$ ) from each other. Hence, there is a tree for every  $81 m^2$ . This corresponds to our data in which for apricot trees the average ratio of trees

<sup>51</sup>This section follows closely Allen *et al.* (2006)



per  $m^2$  is  $79.96 m^2/tree$  and the ratio between total number of trees and total area is  $78.25 m^2/tree$ . These numbers are slightly smaller than  $81 m^2/tree$  because some farmers place some trees very close to the edge of their plot.

Evapotranspiration (ET) is the loss of water suffered by the trees due to both Evaporation (E) of the water stored underground and Transpiration (T) of the water stored within the plant through the surface of the leaves. We use the method recommended by the Food and Agriculture Organization (FAO) to compute the evolution of the moisture due to ET:

$$ET_c = K_c \cdot ET_0$$

where  $ET_c$  is the weekly ET of crop  $c$ ,  $ET_0$  is the weekly reference ET and  $K_c$  is the crop coefficient. Both  $ET_c$  and  $ET_0$  are measured as millimeters per week ( $mm/week$ ). ET is affected by climatic factors: radiation, air temperature, atmospheric humidity and wind speed. The effect of those parameters is summarized in  $ET_0$ . We will use the estimations of  $ET_0$  in Franco et al (2000).

ET would also change depending on the phase of the growing cycle:

$$ET_{cb,t} = K_{cb,t} \cdot ET_0$$

We can then distinguish four phases (initial, development, median and final) in the growing season. Following (Allen *et al.* p 107) we have that  $L_{ini} = 20$ ,  $L_{dev} = 70$ ,  $L_{med} = 120$  and  $L_{fin} = 60$ ; 270 days in total, finishing at the harvest season. The coefficient  $K_{cb,t}$  will be flat during the initial period (with  $K_{cb,ini} = 0.35$ ). It will be linearly increasing during the development period until it reaches the median period. It will be flat during the median period (with  $K_{cb,med} = 0.85$ ). It will be linearly decreasing during the final period until it reaches the harvest (with  $K_{cb,fin} = 0.60$  during the estimated harvest day). It will then be linear during the no-growth period until it reaches the initial period of the next year at  $K_{cb,ini}$ .<sup>52</sup>

### Evapotranspiration Under Hydric Stress

$ET_c$  refers to the ET of crop  $c$  under standard conditions. We should nonetheless adjust the value of  $ET_c$  ( $ET_{c,adj}$ ) when those conditions are not met. When the soil is wet, the water has a high potential energy, meaning that it can be easily absorbed by the roots of the tree. When the soil is dry, water is not so easily absorbed by the roots. When the moisture of the plot falls below a certain threshold, we say that the crop

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<sup>52</sup>Allen *et al.* (2006) formula (66).

is under Hydric Stress (HS). The effects of HS are incorporated by multiplying  $K_{cb}$  by the Hydric Stress coefficient  $KS$ :

$$ET_{c,adj} = KS \cdot K_{cb} \cdot ET_0$$

Water availability refers to the ability of a soil to keep water available for plants. After a heavy rain or irrigation, the soil will drain water until the full capacity is reached. The Full Capacity (FC) of a soil represents the moisture that a well drained soil keeps against gravitational forces, i.e., the moisture of a soil when the downward vertical drainage has decreased substantially. In our case:

$$FC = 1000 \cdot \theta_{FC} \cdot Z_r$$

where  $\theta_{FC}$  is the moisture content of the soil at Full Capacity ( $m^3 m^{-3}$ ) and  $Z_r$  is the depth of the tree's roots ( $m$ ).

In absence of a source of water, the moisture in the soil will decrease due to the water consumption of the tree. As this consumption increases, the moisture level will go down, making it harder for the tree to absorb the remaining water. Eventually, a point will be reached beyond which the tree could no longer absorb any water: the Permanent Wilting (PW) point. The PW point is the moisture level of the soil at which the tree will permanently die. In our case:

$$PW = 1000 \cdot \theta_{WP} \cdot Z_r$$

where  $\theta_{WP}$  is the moisture content of the soil at the Permanent Wilting point ( $m^3 m^{-3}$ ) and  $Z_r$  is the depth of the tree's roots ( $m$ ).

Moisture levels above FC cannot be sustained, given the effect of gravity. Moisture levels below PW cannot be extracted by the roots of the trees. Hence, the Total Available Water (TAW) will be the difference between both:

$$TAW = FC - PW$$

$Z_r = 4m$  in the case of apricot trees irrigated with traditional flooding methods. The soil in Murcia is limestone, hence  $(\theta_{FC} - \theta_{WP}) \in [0.13, 0.19]$  and  $\theta_{WP} \in [0.09, 0.21]$ . For our estimation we take the middle point, i.e.,  $\theta_{FC} = 1240$ ,  $\theta_{WP} = 600$  and  $TAW = 640$ .

In practice, the tree will absorb water from the soil at a slower rate, even before reaching the PW point. When the tree is under HS, the tree is not absorbing water at the proper rate. The fraction of water that the tree can absorb without suffering HS is the Easily Absorbed Water (EAW):

$$EAW = p_c TAW$$

where  $p_c \in [0, 1]$ . For the case of the apricot tree  $p_c = 0.5$ , thus  $EAW = 320$ .<sup>53</sup> The Hydric Stress coefficient  $KS \equiv KS(M_t)$  is a function of the moistness of the plot  $M_t$ :

$$KS(M_t) = \begin{cases} 1 & \text{if } M_t > EAW \\ \frac{M_t - PW}{EAW} & \text{if } EAW \geq M_t > PW \\ 0 & \text{if } M_t \leq PW \end{cases}$$

Adding the subscripts for the periods we can write:

$$ET_{c,adj,t}(M_t) = KS(M_t) \cdot K_{cb,t} \cdot ET_0$$

Finally, we have to take into account that, regardless of the amount of rain or irrigation, the moistness of the soil can never get beyond the  $TAW$ . The evolution of the moisture  $M_t$  over time is then:

$$M_t = \min \{M_{t-1} + rain_{t-1} + irrigation_{t-1} - ET_{c,adj,t-1}(M_t), TAW\}$$

We get an average value for  $ET_c$  of 8.77, which is smaller than Franco et al (2000) who find values of 23.1-30.8 *mm* per week (3.3-4.3 *mm* per day) for almond trees in the same region. Pérez-Pastor et al (2009) report an Evapotranspiration of 1,476 *mm* per year (28.4 *mm* per week). This difference is due to the fact that recent studies are done using intensive dripping irrigation. Since the level of moisture of the land is greater, so is the level of Evapotranspiration.

We can also look at empirical methods used in the literature to estimate  $ET$  based on the annual rain  $R$  (*mm*) and is the annual temperature  $T$  ( $^{\circ}C$ ). For Mula we have  $R = 347$  and  $T = 16.7$ .

- Turc formula:

$$ET = \frac{1}{52} \left[ \frac{R}{\sqrt{0.9 + \left( \frac{R}{300 + 25T + 0.05T^2} \right)^2}} \right] = \frac{327.15}{7} = 6.29$$

<sup>53</sup>Sometimes  $p_c$  is adjusted using the formula  $p_c = 0.5 + 0.04(5 - ET_c)$ .

- Coutagne formula:

$$ET = \frac{1}{52} \left[ R - \frac{R^2 10^{-3}}{0.8 + 0.14T} \right] = \frac{308.63}{7} = 5.94$$

Our value of  $ET_c$ , computed following Allen et al (2006) is greater than those computed using these empirical methods. They do not take into account irrigation, but are used as an average value. As expected, our estimation of ET for traditional flooding irrigation is greater than the estimation without irrigation but smaller than the estimation for intensive dripping irrigation.

## B Theory Appendix

In this section we show the proof and the details of the models presented in sub-sections 3.2 and 3.3.

### B.1 Dynamic Model

In this sub-section we show the proofs corresponding to the static model presented in sub-section 3.2. Farmers can buy only a discrete amount of water  $x \in \mathbb{N}$  in each period and will get a utility of  $u(x_1, x_2, p_1, p_2) = h((1 - \delta)x_1 + x_2) - p_1 x_1 - p_2 x_2$ . The only prices that are consistent with equilibrium in the unconstrained case are  $p_1^{FB} \equiv h(1 - \delta)$  and  $p_2^{FB} \equiv h(1)$ , in the first and second period respectively. The only allocation consistent with equilibrium is:

$$Q^{FB} \equiv \begin{bmatrix} (1 - X_1 - X_2) & X_2 & 0 \\ X_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; p^{FB} \equiv [h(1 - \delta), h(1)]$$

Hence, the mass of farmers buying one unit of water in the first period, in the second period or buying no water is  $q_{10}^{FB} = X_1$ ,  $q_{01}^{FB} = X_2$  and  $q_{00}^{FB} = 1 - X_1 - X_2$ , respectively. Since each unit of water is being used by a different farmer, the equilibrium is efficient. The equilibrium is also revenue maximizing because it extracts all the surplus from the farmers.

**O:** We need to check that this is an equilibrium for each allocation such that  $q_{x_1, x_2} > 0$ . We need to check that  $u(x_1, x_2, p_1^{FB}, p_2^{FB}) \geq \max_{x'_1, x'_2} \left\{ u(x'_1, x'_2, p_1^{FB}, p_2^{FB}) \right\}$ . This case is trivial since  $u(x_1, x_2, p_1^{FB}, p_2^{FB}) = 0$  and  $\max_{x'_1, x'_2} \left\{ u(x'_1, x'_2, p_1^{FB}, p_2^{FB}) \right\}$ . It is trivial to see why this is the unique equilibrium. If  $p_t > p_t^{FB}$ , then the utility of the farmers buying only at period  $t$  would be negative, and they would prefer not to buy. If  $p_t < p_t^{FB}$ , then the utility of the farmers buying only

at period  $t$  would be positive, while the utility of farmers not buying water at all would be zero, thus those farmers would be willing to pay a price  $p'_t$  such that  $p_t^{FB} > p'_t > p_t$  and get a positive utility. They can do that as long as  $p_t < p_t^{FB}$ .

$$\mathbf{Q:} \sum_{x_1} \sum_{x_2} (q_{x_1, x_2}) = q_{01} + q_{10} + q_{00} = X_1 - X_2 + (1 - X_1 - X_2) = 1$$

$$\mathbf{RC:} \sum_{x_1} \sum_{x_2} (\tau_1 \cdot q_{x_1, x_2}) = q_{10}^{FB} = X_1 \text{ and } \sum_{x_1} \sum_{x_2} (x_2 \cdot q_{x_1, x_2}) = q_{01}^{FB} = X_2$$

**DC:** Since the surplus is zero in both periods for all farmers, DC is trivially satisfied.

We will say that a *Constrained Competitive Equilibrium* is *Efficient* if all the units are consumed by some farmer and there are no farmers consuming more than one unit. We will say that a *Constrained Competitive Equilibrium* is *Revenue Maximizing* if  $p = p^{FB}$ .

**Case a) No Binding Liquidity Constraints,  $p_1^{FB} < p_2^{FB} < \mu_L$**

If there are no binding liquidity constraints the equilibrium will be the same as in the FB case.

**Case b) Mild Liquidity Constraints,  $p_1^{FB} < \mu_L < p_2^{FB}$**

If  $p_1^{FB} < \mu_L < p_2^{FB}$ , then poor ( $\mu_i = \mu_L$ ) farmers would buy one unit in the first period, but not in the second period. Since there are "enough" wealthy ( $\mu_i = \mu_H$ ) farmers, i.e.,  $g_H > X_2$ , they will buy all the water in the second period and some of the water in the first period.

$q_{H01} = X_2$  wealthy farmers will buy one unit in the second period and  $q_{H10} = (X_2 - g_H)$  wealthy farmers will buy one unit in the first period.  $q_{L10} = (X_1 - X_2 + g_H)$  poor farmers will buy water in the first period and  $q_{L00} = (g_L - X_1 + X_2 - g_H)$  poor farmers will not buy water.

Prices will not be affected by the liquidity constraints (LC), i.e.,  $p^* = p^{FB}$ . In this case, the auction will still be efficient, but the model predicts that poor farmers will not buy water in the second period.

**O:** The utility of all farmers is zero. Hence, this condition is trivially satisfied as in the case with no LC.

$$\mathbf{Q:} \sum_{x_{H1} x_{H2}} (q_{x_{H1}, x_{H2}}) = q_{H01} + q_{H10} = X_2 + X_2 - g_H = g_H \text{ and } \sum_{x_{L1} x_{L2}} (q_{x_{L1}, x_{L2}}) = q_{L10} + q_{L00} = X_1 - X_2 + g_H + g_L - X_1 + X_2 - g_H = g_L$$

$$\mathbf{RC:} \sum_{i=L, H} \sum_{x_{i1} x_{i2}} (x_{i1} \cdot q_{x_{i1}, x_{i2}}) = 1 \cdot q_{H10} + 1 \cdot q_{L10} = (X_2 - g_H) + (X_1 - X_2 + g_H) = X_1 \text{ and}$$

$$\sum_{i=L, H} \sum_{x_{i1} x_{i2}} (x_{i2} \cdot q_{x_{i1}, x_{i2}}) = 1 \cdot q_{H01} = X_2 = X_2$$

**LC:** Since  $\mu_L > p_1^{FB} = p_1^*$ , LC is trivially satisfied.

**DC:** Since  $p^* = p^{FB}$ , DC is trivially satisfied.

**Case c) Severe Liquidity Constraints,  $\mu_L < p_1^{FB} < p_2^{FB}$**

If  $\mu_L < p_1^{FB} < p_2^{FB}$ , then poor farmers will not be able to buy one unit of water in the first period at  $p_1^A = p_1^{FB}$ . Water in the second period will be bought by wealthy farmers only. However, since  $g_H > X_2$ , there will be  $(g_H - X_2) < X_1$  wealthy farmers that will not buy any water in the second period. Those farmers will be willing to pay  $p_1^{FB}$  for a unit of water in the first period. We will have to check whether wealthy farmers will buy a second unit of water or poor farmers will buy one unit of water. Since  $h'(1 - \delta) > h'(1)$ , all the wealthy farmers that are already buying one unit in the first period will be the ones competing with the poor farmers for the remaining units in the first period. The price in the first period will have to go down until supply meets demand:

1.  $p_1^* = \mu_L > [h(2 - 2\delta) - h(1 - \delta)]$ . In this case  $q_{L10} = (X_1 - X_2 + g_H)$  poor farmers will buy one unit of water in the first period and  $q_{L00} = (g_L - X_1 + X_2 - g_H)$  poor farmers will buy no water.  $q_{H01} = X_2$  wealthy farmers will buy one unit in the second period and  $q_{H10} = (X_2 - g_H)$  wealthy farmers will buy one unit in the first period. The equilibrium is efficient but not revenue maximizing.

**O:** We need to check that this is an equilibrium for each allocation such that  $q_{i,x_1,x_2} > 0$  for each type  $i$ .

**Poor Farmers.** Since  $\mu_L < p_2^*$  they cannot afford to buy any water in the second period:

- $q_{L10}$  farmers are buying 1 unit in the first period and expending all their cash, hence they cannot buy any more water. Also, since  $\mu_L < p_1^{FB}$  they are getting a positive utility so they are better off than if they were not buying any water.
- $q_{L00}$  farmers are not buying any water. They would like to buy a unit of water in the first period so they participated in the rationing but were unlucky and got no water.

**Wealthy Farmers.** Since  $p_1^* - p_1^{FB} = p_2^* - p_2^{FB}$ , they are indifferent between buying one unit in the first period or one unit in the second period. Moreover, they are all getting a positive utility, so they are better off than if they were not buying any water. Notice that  $p_2^* = p_1^* - p_1^{FB} + p_2^{FB}$  is an equilibrium because, in the second period there are exactly  $X_2$  wealthy farmers, and in the sub-game of the second period where there are the same number of farmers and units, any price such that  $p_2^* \in [0, p_2^{FB}]$  is an equilibrium.

$$\mathbf{Q:} \sum_{x_{H1} x_{H2}} \sum (q_{x_{H1}, x_{H2}}) = q_{H01} + q_{H10} = X_2 + X_2 - g_H = g_H \quad \text{and} \quad \sum_{x_{L1} x_{L2}} \sum (q_{x_{L1}, x_{L2}}) = q_{L10} + q_{L00} = X_1 - X_2 + g_H + g_L - X_1 + X_2 - g_H = g_L$$

$$\mathbf{RC:} \sum_{i=L, H} \sum_{x_{i1} x_{i2}} \sum (x_{i1} \cdot q_{x_{i1}, x_{i2}}) = 1 \cdot q_{H10} + 1 \cdot q_{L10} = (X_2 - g_H) + (X_1 - X_2 + g_H) = X_1 \quad \text{and}$$

$$\sum_{i=L, H} \sum_{x_{i1} x_{i2}} \sum (x_{i2} \cdot q_{x_{i1}, x_{i2}}) = 1 \cdot q_{H01} = X_2$$

**LC:** Since  $p_1^* = \mu_L$ , LC is trivially satisfied.

**DC:** Farmers buying water in the first period are obtaining a surplus, i.e.,  $p_1^* = \mu_L < p_1^{FB} \equiv v(1 - \delta)$ , while the wealthy farmers buying water in the second period are getting zero surplus, i.e.,  $p_2^* = p_2^{FB} \equiv v(1)$ . Hence, we do not have to worry about strategic delay of purchases. Since  $p_2^* = p_2^{FB}$ , DC is satisfied.

2. If the price goes down enough before reaching  $\mu_L$ , i.e.,  $p_1^* = [h(2 - 2\delta) - h(1 - \delta)] > \mu_L$ , then wealthy farmers who are not buying any unit in the second period will be willing to buy two units in the first period. Poor farmers will buy no water, i.e.,  $q_{L00} = g_L$ .  $q_{H10} = [2(g_H - X_2) - X_1]$  wealthy farmers will buy one unit in the first period,  $q_{H20} = (X_1 + X_2 - g_H)$  wealthy farmers will buy two units in the first period and  $q_{H01} = X_2$  wealthy farmers will buy one unit in the second period. In this situation, all farmers buying water in the first period are obtaining a surplus, i.e.,  $p_1^* < p_1^{FB} \equiv h(1 - \delta)$ , while the wealthy farmers buying water in the second period are getting a smaller surplus, i.e.,  $p_2^* \in [p_1^* - p_1^{FB} + p_2^{FB}, p_2^{FB}]$ . Hence, we do not have to worry about strategic delay of purchases. The equilibrium is inefficient and not revenue maximizing.

**O:** We need to check that this is an equilibrium for each allocation such that  $q_{i, x_1, x_2} > 0$  for each type  $i$ .

**Poor Farmers.** Since  $\mu_L < p_t^*$  they cannot afford to buy any water in any period.

**Wealthy Farmers.** If  $p_1^* - p_1^{FB} = p_2^* - p_2^{FB}$  they are indifferent between buying one unit in the first period or one unit in the second period. Moreover, they are all getting a positive utility, so they are better off than if they were not buying any water. Since there are exactly  $X_2$  wealthy farmers that have not bought any water when the auction in the second period begins, any price between that price and the FB is consistent with equilibrium, i.e.,  $p_2^* \in [p_1^* - p_1^{FB} + p_2^{FB}, p_2^{FB}]$ .

$$\mathbf{Q:} \sum_{x_{H1} x_{H2}} \sum (q_{x_{H1}, x_{H2}}) = q_{H10} + q_{H20} + q_{H01} = [2(g_H - X_2) - X_1] + (X_1 + X_2 - g_H) + X_2 = g_H$$

$$\text{and} \quad \sum_{x_{L1} x_{L2}} \sum (q_{x_{L1}, x_{L2}}) = q_{L00} = g_L$$

$$\text{RC: } \sum_{i=L,H} \sum_{x_{i1}, x_{i2}} (x_{i1} \cdot q_{\tau_{i1}, \tau_{i2}}) = 1 \cdot q_{H10} + 1 \cdot q_{H20} = [2(g_H - X_2) - X_1] + 2(X_1 + X_2 - g_H) = X_1$$

$$\text{and } \sum_{i=L,H} \sum_{x_{i1}, x_{i2}} (x_{i2} \cdot q_{x_{i1}, x_{i2}}) = 1 \cdot q_{H01} = X_2$$

**LC:** Since  $p_1^* = \mu_L$ , LC is trivially satisfied.

**DC:** Farmers buying water in the first period are obtaining a surplus, i.e.,  $p_1^* < p_1^{FB} \equiv h(1 - \delta)$ .

Since  $p_2^* > p_1^* - p_1^{FB} + p_2^{FB}$ , DC is trivially satisfied.

## B.2 Static Model

In this sub-section we show the proofs corresponding to the static model presented in sub-section 3.3.

### B.2.1 Auctions

**Proposition.** *Under the Auction system we have:*

*i) Welfare is lower than it would have been without LC. In particular, the Auction system achieves the FB allocation if and only if LC are not binding for any farmers.*

*ii) The equilibrium price is lower than it would have been without LC. Moreover, there are farmers whose marginal utility in equilibrium is greater than the equilibrium price.*

*Proof.* i) This result is a direct consequence of equation 8. The definition of welfare in the FB case and the conditions that it satisfies are identical of those under the auction mechanism, except for the liquidity constraint. Hence, the welfare under the auction system is no greater than the FB. It is also immediate to see that if the constraint is binding for some farmers, these farmers will get less water than in the FB case, and other farmers will get more water than in the FB case (since the total amount allocated is the same).

ii) Price is determined by demand and supply. Supply in this case is constant. Demand, as shown by equation 5 is decreasing in wealth  $\mu_i$ . In particular, when the liquidity constraint is not binding, demand is independent of wealth, but when the liquidity constraint is binding for some farmers, demand is strictly decreasing in the wealth of each farmer.

A simple look at equation 5 shows that farmers that are constrained are buying less water than they would, if they were not constrained. The amount of water that they would buy in the unconstrained case is, by definition, the amount of water that make their utility equal to the price. Since the utility is concave, consuming a lower amount of water means that the marginal utility is greater. Hence, the marginal utility of constrained farmers is greater than the price.  $\square$



## B.2.2 Quotas

**Proposition.** *Under the Quotas system we have:*

*i) Welfare is lower than it would have been with heterogeneous farmers, i.e.,  $\theta_i = \theta \forall i$ . In particular, the Quotas system does not achieve the FB allocation when farmers are heterogeneous.*

*ii) When farmers are homogeneous, i.e.,  $\theta_i = \theta \forall i$ , the Quotas system does achieve the FB allocation.*

*Proof.* i) When farmers are heterogeneous the FB allocation implies that more productive farmers will receive more water. Since the quota system assigns all farmers the same amount of water, it cannot achieve the FB allocation.

ii) When farmers are homogeneous, the FB allocation implies that all farmers will receive the same amount of water. Since the quota system assigns all farmers the same amount of water, it does achieve the FB allocation. □

## C Estimation Appendix

### C.1 First Stage

We estimate the conditional choice probabilities (CCP) non-parametrically. There are four observable state variables in the structural model without liquidity constraints: moistness, week of the year, price of water, and rain. Moistness is a deterministic continuous variable that represents the amount of water accumulated in the farmers' plot; it goes from 300 to 1200. Week of the year is a deterministic discrete variable; it goes from 1 to 52. Price of water and rain are random variables. We model the joint probability distribution of prices and rain as an independent and identically distributed (i.i.d.) process conditional on the week of the year. Note that seasonality is the main determinant of prices. Thus, the *week-conditional i.i.d.* assumption seems reasonable in our setting. Each week, prices may take three discrete values: low, high, or no-auction. Each week, rain may take two discrete values: zero (no rain) or 31 *mm* (positive rain).<sup>54</sup> For each week, *low price* is the mean price below the median of the same week across years; *high price* is the mean price above the median of the same week across years. We estimate the joint distribution of prices and rain non-parametrically using a frequency estimator.

Rather than using a traditional frequency-based approach in the presence of discrete variables, to compute the CCP we smooth both discrete and continuous variables. There are two reasons for this. First, it allows us to extend the reach of the nonparametric methods to our empirical model. It is well known that

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<sup>54</sup>31 *mm* is the median of the rain distribution, conditional on rain being positive.

nonparametric frequency methods are useful only when the sample size is large and the discrete variables take a limited number of values: this allows the number of discrete cells to be smaller than the sample size.<sup>55</sup>

Second, since moistness is a continuous variable and its evolution over time depends on both the decisions to buy water of the farmers and the realizations of rain. Therefore, certain values of moistness are never reached in the sample even when their probability of occurrence is not zero.<sup>56</sup> To estimate the demand, however, we need to integrate the value function for each possible combination of the state variables in the state space.<sup>57</sup> Thus, we estimate the CCP non-parametrically using kernel methods to smooth both discrete and continuous variables.

We define now the nonparametric CCP estimator. Following Li and Racine (2003) we use generalized product kernels for a mix of continuous and discrete variables. Let  $S_t = (M_t, S_t^d) \in \mathbb{R} \times \mathbb{R}^3$  be the vector of state variables, where  $M_t \in \mathbb{R}$  is moistness and  $S_t^d = (w_t, p_t, r_t) \in \mathbb{R}^3$  is the vector of discrete state variables: week, price, and rain. Let  $s_k^d$  be the  $k$ th component of  $s^d$  and  $S_t^d = (t = 1, \dots, T)$ . For  $S_{tk}^d, s_k^d \in \{0, 1, \dots, c_k - 1\}$  (the support of each discrete variable) define the univariate kernel (Aitchison and Aitken, 1976):

$$l^u \left( S_{tk}^d, s_k^d, \lambda_k \right) = \begin{cases} 1 - \lambda_k & \text{if } S_{tk}^d = s_k^d \\ \frac{\lambda_k}{c_k - 1} & \text{if } S_{tk}^d \neq s_k^d \end{cases}$$

We use the above kernel for prices and rain. For the ordered discrete variable week we use the following kernel function (Wang and van Ryzin, 1981):

$$l^o(w_t, v, \lambda_1) = \lambda_1^{|w-v|}$$

<sup>55</sup>The frequency approach would not be feasible in our setting, even if we discretize the (continuous variable) moistness in a reasonable number of values. With four discrete variables and assuming we discretize moistness into just 22 values the number of discrete cells that arise is  $22 \times 52 \times 3 \times 2 = 6864$ . Thus, the average number of observations (the effective sample size) in each cell would be  $T/6864 = 6864/6864 = 1$ , where  $T = 6864$  is our sample size. Note that:  $T = 6864 = 12$  unconstrained farmers  $\times$  52 weeks per year  $\times$  11 years. Discretizing moistness into 22 values would be too low and will not capture the variability of the data.

<sup>56</sup>For example, for week 23 the joint probability of *no rain* and *low price* conditional on this week is 9.1%. This is because only in 1 out of 11 years was registered *low rain* and *low price* in the week 23 ( $1/11 = .0909$ ). The observed different values of inventories for the 12 unconstrained farmers are (at most)  $12 \times 1 = 12$ . In the simulation, however, a value of moistness different from (although close to) these 12 observed values may be reached. But the frequency estimator would not be defined for any value of moistness different from those 12 values.

<sup>57</sup>That is, we also need to integrate over values of moistness discussed in previous footnote where the frequency estimator is not defined. These values of moistness are never reached in our finite sample.

where  $\lambda_1 \in [0, 1]$ .

Therefore, for the multivariate vector of discrete state variables we use the product kernel:

$$L(S_t^d, s^d, \lambda) = l^o(w_t, v, \lambda_1) \prod_{k=2}^3 l^u(S_{tk}^d, s_k^d, \lambda_k) = \lambda_1^{|w-v|} \prod_{k=2}^3 \left( \frac{\lambda_k}{c_k - 1} \right)^{N_{tk}(s)} (1 - \lambda_k)^{1 - N_{tk}(s)} \quad (22)$$

where  $\lambda = (\lambda_2, \lambda_3)$  and  $N_{tk}(s) = \mathbf{1}(S_{tk}^d \neq s_k^d)$  is an indicator function that equals 1 if  $S_{tk}^d \neq s_k^d$  and 0 otherwise.

Let  $f(s) = f(m, s^d)$  be the joint probability density function (PDF) of  $S_t = (M_t, S_t^d)$ . We use the following kernel estimator of  $f(s)$ :

$$\hat{f}(s) = \frac{1}{T} \sum_{t=1}^T L_{ts^d} W_{h,tM}$$

where  $W_{h,tM} = h^{-1} w\left(\frac{M_t - m}{h}\right)$ ,  $w(\cdot)$  is a standard univariate second order Gaussian kernel, and  $L_{ts^d} = L(S_t^d, s^d, \lambda)$  is given by equation 22. We select bandwidth using likelihood cross-validation.

We estimate the  $\hat{f}(s)$  using the observed values of the variables in the state space (in the sample). We then use the estimated density and evaluate it at the unobserved values of the state space needed to integrate the value function (out of the sample).

## C.2 Second Stage

We restrict the sample to the twelve farmers that are unconstrained. We estimate the vector of structural parameters,  $(\gamma, \zeta)$ , of the dynamic model in Section 4 using the conditional choice simulation estimator proposed by Hotz et al (1994) which is based on the inversion theorem by Hotz and Miller (1993). We integrate the value function using the smoothed CCP as computed in the previous sub-section. We set the discount factor  $\beta$  equal to 0.99.<sup>58</sup> Prices and rain are simulated using the joint distribution of prices and rain estimated with the procedure described in the previous sub-section. When rain is positive we assign an amount of rainfall equals to 31 mm, the median rain in the sample conditional on rain being positive. We normalize the number of trees of the farmers using the median number of apricots trees for the unconstrained farmers, 76 trees. We let moistness follow the evolution described by equations in Appendix A.2 with the following values calibrated for our setting:  $TAW = 1200$ ,  $PW = 300$ ,  $EAW = 0.5 \cdot TAW$ ,  $E = 4$  and ET is 5% higher than the values from Appendix A.2 .

<sup>58</sup>Since each period represents one week, using a smaller value of  $\beta$  would be unrealistic.

The observed number of units that farmers buy varies from 0 to 4 units per week. To compute the smooth CCP as described in previous sub-section we model farmers' decision as binary: to buy ( $j_{it} = 1$ ) or not to buy ( $j_{it} = 0$ ). To compute the evolution of moistness in the estimation, when a farmer buys water we assign the median number of units in the sample conditional on buying: two units. We extrapolate CCP on unobserved states using the estimated density from the sample (see previous sub-section).

For the estimation, we minimize the distance between the smooth CCP and the predicted choice probabilities from our model. We use 200 simulations with  $T = 11 \text{ years} \times 52 \text{ weeks} \times 12 \text{ individuals} = 6864$  observations in each simulation. We use the contraction by Berry, Levinsohn, and Pakes (1995). We perform the estimation using KNITRO, a solver for non-linear optimization, with tolerance level of  $1.0e-25$ .

With the estimated demand we recover the annual revenue for all farmers (constrained and unconstrained). We compute farmer-specific revenue by adjusting the revenue predicted by the model for the representative farmer (that uses the median number of trees) multiplying it by the number of trees of each farmer relative to the median number of trees.

### C.3 Third Stage

Cash of farmer  $i$  in period  $t$ ,  $\mu_{it}$ , evolves according to:

$$\mu_{it} = \mu_{i,t-1} - p_{t-1}j_{i,t-1} + \Phi_t(re_i; \phi) + \eta_{it} + \nu_{it} \quad (23)$$

where  $\Phi_t(re_i; \phi) = \phi_{i0} + \phi_1 re_i$  captures the (weekly) cash flow function derived from the real estate  $\phi re_i$  minus the weekly consumption of individual  $i$  that is constant over time,  $\phi_{i0}$ ;  $\eta_{it}$  is the revenue that the farmer obtains when he sells the harvest (more about this below), and  $\nu_{it}$  is an idiosyncratic financial shock.

The farmer obtains its revenue after the harvest, in week 24. Thus, the revenue is:

$$\eta_{it} = \begin{cases} 0 & \text{if } w_t \neq 24 \\ R_{it} & \text{if } w_t = 24 \end{cases}$$

where  $R_{it} = \sum_{w_t=1}^{52} h(M_{t-1}, w_t) = \sum_{w_t=18}^{23} \gamma_1 \cdot (M_{t-1} - PW) \cdot KS(M_t) \cdot area_i + \sum_{w_t=24}^{32} \gamma_2 \cdot (M_{t-1} - PW) \cdot KS(M_t) \cdot area_i$ .

For the estimation in the text we set  $\phi_1 = 0$  (that is, we did not include the cash flow derived from the farmers' real state) and we let  $\nu_{it} \sim \mathbf{N}(0, \sigma_\nu^2)$ .

For the initial condition of the cash flow we use the revenue after the first harvest ( $w_t = 24$ ) in 1955, the first year in our sample (which varies by farmer), assuming that all farmers had no cash before. That is, we use the first 24 weeks of the first year to generate the accumulated cash flow of each farmer assuming that before January 1955 (unobserved in the data) the amount of cash is zero (for all farmers).

In the data we only observe if the farmer buys water or not (and the number of units he buys in case he buys). When a farmer does not buy water, we do not know whether it is because he does not need the water (no demand) or because she is liquidity constrained. That is, for the liquidity constrained farmers, the dependent variable is censored. An additional complication is that we know which farmers are not liquidity constrained (the wealthy ones), but we do not observe which farmers are liquidity constrained and in which week. This is why we need the structural model to compute the probability that farmer  $i$  is liquidity constrained in week  $t$  given his demand in that week:  $\mathbb{P}(p_t j_{it} > \mu_{it})$ .

Using equation 23:

$$\mathbb{P}(p_t j_{it} < \mu_{it}) = \mathbb{P}(p_t j_{it} < \mu_{i,t-1} - p_{t-1} j_{i,t-1} - \phi_{i0} + \eta_{it} + \nu_{it}) = \mathbb{P}(\nu_{it} > C_t)$$

where  $C_t \equiv \mu_{i,t-1} - p_{t-1} j_{i,t-1} - \phi_{i0} + \eta_{it} - p_t j_{i,t}$ . Using the distribution of  $\nu_{it}$  we have:

$$\mathbb{P}(p_t j_{it} < \mu_{it}) = \mathbb{P}(\nu_{it} > -C_t) = 1 - \mathbb{F}_\nu(-C_t)$$

where we have used the symmetry of the distribution of  $\nu_{it}$ , and  $\mathbb{F}_\nu(\cdot)$  is its cumulative distribution function.

Similarly:

$$\mathbb{P}(p_t j_{it} \geq \mu_{it}) = \mathbb{P}(\nu_{it} \leq C_t) = \mathbb{F}_\nu(-C_t)$$

To simplify notation, in what follows we omit conditioning on the state variables. Everything is, however, conditional on the state. Let the estimated smooth CCP (from the first stage) of not buying water, i.e.,  $j_{it} = 0$ , for an unconstrained farmer be  $\hat{\mathbb{P}}_{CCP}(j_{it} = 0)$ . Similarly, let the estimated smooth CCP of buying water, i.e.,  $j_{it} = 1$ , for an unconstrained farmer be  $\hat{\mathbb{P}}_{CCP}(j_{it} = 1)$ . For (potentially) liquidity constrained farmers, define the latent variable:

$$\tilde{j}_{it} = \begin{cases} j_{it} & \text{if } p_t j_{it} < \mu_{it} \\ 0 & \text{if } p_t j_{it} \geq \mu_{it} \end{cases}$$

Then:

$$\begin{aligned} \mathbb{P}(\tilde{j}_{it} = 0) &= \mathbb{P}[(j_{it} = 0 \wedge p_t j_{it} < \mu_{it}) \vee (p_t j_{it} \geq \mu_{it})] \\ &= \mathbb{P}(j_{it} = 0)\mathbb{P}(p_t j_{it} < \mu_{it}) + \mathbb{P}(p_t j_{it} \geq \mu_{it}) \end{aligned}$$

Thus:

$$\hat{\mathbb{P}}(\tilde{j}_{it} = 0; \chi) = \hat{\mathbb{P}}_{CCP}(j_{it} = 0) [1 - \mathbb{F}_\nu(-C_t; \chi)] + \mathbb{F}_\nu(-C_t; \chi) \quad (24)$$

where  $\chi \equiv (\phi_{i0}, \sigma_\nu^2)$  is a parameter vector.

Similarly:

$$\begin{aligned} \mathbb{P}(\tilde{j}_{it} = 1) &= \mathbb{P}(j_{it} = 1 \wedge p_t j_{it} < \mu_{it}) \\ &= \mathbb{P}(j_{it} = 1)\mathbb{P}(p_t j_{it} < \mu_{it}) \end{aligned}$$

Thus:

$$\hat{\mathbb{P}}(\tilde{j}_{it} = 1; \chi) = \hat{\mathbb{P}}_{CCP}(j_{it} = 1) [1 - \mathbb{F}_\nu(-C_t; \chi)] \quad (25)$$

Note that  $\mathbb{P}(\tilde{j}_{it} = 0) + \mathbb{P}(\tilde{j}_{it} = 1) = 1$ .

We estimate the parameter vector in the third stage,  $\chi \equiv (\phi_{i0}, \sigma_\nu^2)$ , by maximizing the log-likelihood function:

$$\chi = \arg \max_{\chi} \mathcal{L} = \arg \max_{\chi} \sum_{i=1}^I \sum_{t=1}^T \mathbf{1}(\tilde{j}_{it} = 0) \log \hat{\mathbb{P}}(\tilde{j}_{it} = 0; \chi) + \mathbf{1}(\tilde{j}_{it} = 1) \log \hat{\mathbb{P}}(\tilde{j}_{it} = 1; \chi)$$

where  $\hat{\mathbb{P}}(\tilde{j}_{it} = 0; \chi)$  and  $\hat{\mathbb{P}}(\tilde{j}_{it} = 1; \chi)$  are given by equations 24 and 25, respectively.

We perform the estimation using KNITRO, a solver for non-linear optimization, with tolerance level of 1.0e-25.

## C.4 Aggregate Demand, No liquidity Constraints

Following Aguirregaviria and Mira (2007) we now establish some properties of our estimator. Time is discrete and indexed by  $t$ . Each period represents a week. We index agents by  $i$ . Agents have preferences defined over a sequence of states of the world from period  $t = 0$  until period  $t = \infty$ . The state of the world at period  $t$  for individual  $i$  has two components: a vector of state variables  $s_{it} = (x_{it}, w_t, M_{it}; \epsilon_{it}) = (M_{it}, w_t, p_t, r_t; \epsilon_{it})$  that is known at period  $t$ ; and a decision vector  $j_{it}$  chosen at period  $t$  that belongs to the discrete set  $J \in \{0, \dots, J\}$ . The vector of state variables  $s_{it}$  also includes the error vector  $\epsilon_{it} \equiv (\epsilon_{i1t}, \dots, \epsilon_{iJt})$ . The time index  $t$  can be a component of the state vector  $s_{it}$ , which may also contain time-invariant individual characteristics. Farmer's preferences over possible sequences of states of the world can be represented by a utility function  $\sum_{\tau=0}^{\infty} \beta^\tau U(j_{i,t+\tau}, s_{i,t+\tau})$ , where  $\beta \in (0, 1)$  is the discount factor and  $U(j_{it}, s_{it}) = h(M_{it}, w_t) - (j_{it}p_t + \zeta_j + \epsilon_{ijt})$  is the current utility function. The decision at period  $t$  affects the evolution of future values of the state variables, but the agent faces uncertainty about these future values. The farmer's beliefs about future states can be represented by a Markov transition distribution function  $F(s_{i,t+1}|j_{it}, s_{it})$ . These beliefs are *rational* in the sense that they are the true transition probabilities of the state variables. Every period  $t$  the farmer observes the vector of state variables  $s_{it}$  and chooses her action  $j_{it} \in J$  to maximize the expected utility

$$E \left( \sum_{\tau=0}^{\infty} \beta^\tau U(j_{i,t+\tau}, s_{i,t+\tau}) | j_{it}, s_{it} \right)$$

This is the farmer's Dynamic Programming (DP) problem. Let  $\alpha(s_{i,t})$  and  $V(s_{i,t})$  be the optimal decision rule and the value function of the DP problem, respectively. By Bellman's principle of optimality the value function can be obtained using the recursive expression:

$$V(s_{it}) = \max_{j \in J} \left\{ h(M_{it}, w_t; \gamma) - (j_{it}p_t + \zeta_j + \epsilon_{jt}) + \beta \int V(s_{i,t+1}) dF(s_{i,t+1}|j, s_{it}) \right\}$$

and the optimal decision rule is then  $\alpha(s_{i,t}) = \underset{j \in J}{\operatorname{argmax}} \{v(j, s_{i,t})\}$  where, for every  $j \in J$ ,

$$v(j, s_{i,t}) \equiv h(M_{it}, w_t; \gamma) - (j_{it}p_t + \zeta_j + \epsilon_{jt}) + \beta \int V(s_{i,t+1}) dF(s_{i,t+1}|j, s_{it})$$

is a choice-specific value function.

We are interested in the estimation of the structural parameters in preferences, transition probabilities, and the discount factor  $\beta$ . Suppose that a researcher has panel data for  $N$  individuals who behave

according to this decision model. For every observation  $(i, t)$  in this panel data-set, the researcher observes the individual's action  $j_{it}$  and a sub-vector  $x_{it}$  of the state vector  $s_{it}$ . Therefore, from an econometric point of view, we can distinguish two subsets of state variables:  $s_{it} = (x_{it}, w_t, M_{it}; \epsilon_{it})$ , where the sub-vector  $(M_{it}, \epsilon_{it})$  is observed by the agent but not by the researcher. Note that  $\epsilon_{it}$  is a source of variation in the decisions of agents conditional on the variables observed by the researcher. It is the models "econometric error", which is given a structural interpretation as an unobserved state variable.

In summary, the researcher's data set is:

$$Data = \{j_{it}, x_{it} : i = 1, 2, \dots, N; t = 1, 2, \dots, \infty\}$$

Let  $\Theta$  be the vector of structural parameters and let  $g_N(\Theta)$  be an estimation criterion. For instance, if the data are a random sample over individuals and the criterion is a log-likelihood, then  $g_N(\Theta) = \sum_{i=1}^N l_i(\Theta)$ , where  $l_i(\Theta)$  is the contribution to the log-likelihood function of individual  $i$  history:

$$\begin{aligned} l_i(\Theta) &= \log \Pr\{j_{it}, x_{it} : t = 1, 2, \dots, \infty | \Theta\} \\ &= \log \Pr\{\alpha(x_{it}, \epsilon_{it}, \Theta) = d_{it}, x_{it} : t = 1, 2, \dots, \infty | \Theta\} \end{aligned}$$

Whatever estimation criterion, in order to evaluate it for a particular value of  $\Theta$  it is necessary to know the optimal decision rules  $\Delta(x_{it}, \epsilon_{it}, \Theta)$ . Therefore, for each trial value of  $\Theta$  it is necessary to know the optimal decision rules  $\Delta(x_{it}, \epsilon_{it}, \Theta)$ . Therefore, for each trial value of  $\Theta$  the DP problem needs to be solved exactly, or its solution approximated in some way.

So far we have not made any assumption on the relationship between observable and unobservable variables. These are key modeling decisions in the econometrics of dynamic discrete structural models. The form of  $l_i(\Theta)$  and the choice of the appropriate solution and estimation methods crucially depend on these assumptions. The first 6 assumptions define the Rust's model.

**ASSUMPTION AS (Additive Separability).** The one-period utility function is additively separable in the observable and unobservable components:  $U(j_{it}, s_{it}) \equiv U(j_{it}, x_{it}, w_t, M_{it}; \epsilon_{it}) = h(M_{it}, w_t; \gamma) - (j_{it}p_t + \zeta_j + \epsilon_{jt})$ , where  $\epsilon_{ijt}$  is a zero mean real random variable with unbounded support. That is, there is one unobservable state variable for each choice alternative, so the dimension of  $\epsilon_{it}$  is  $(J + 1) \times 1$ .

**ASSUMPTION IID (i.i.d. Unobservables).** The unobserved state variables in  $\epsilon_{it}$  are independently and identically distributed over agents and over time with cumulative density function (CDF)  $G_\epsilon(\epsilon_{it})$  which has finite first moments and is continuous and twice differentiable in  $\epsilon_{it}$ .

**ASSUMPTION CI-X (Conditional Independence of Future  $x$ ).** Conditional on the current values of



the decision and the observable state variables, next period observable state variables do not depend on current  $\epsilon_{it}$ :

$$CDF(x_{i,t+1}|j_{it}, x_{it}, w_t, M_{it}, \epsilon_{it}) = F_x(x_{i,t+1}|j_{it}, x_{it}, w_t, M_{it})$$

$$CDF(w_{t+1}|j_{it}, x_{it}, w_t, M_{it}, \epsilon_{it}) = F_w(w_{t+1}|j_{it}, x_{it}, w_t, M_{it})$$

$$CDF(M_{i,t+1}|j_{it}, x_{it}, w_t, M_{it}, \epsilon_{it}) = F_M(M_{i,t+1}|j_{it}, x_{it}, w_t, M_{it})$$

This assumption holds trivially for  $w_t$ . It also holds trivially for  $x_{it}$ , since the covariates are constant for a given individual, calendar effects, rain (which is absolutely exogenous) and prices. Notice that this assumption holds for prices because the auction format is a English (Second Price) auction. Thus, the price that individual  $i$  is paying is independent of her bid or her type. Finally, the law of motion of the inventory is independent of  $\epsilon_{it}$ . We use  $\Theta_f$  to represent the vector of parameters that describe the transition probability function  $F_x$ .

**ASSUMPTION CLOGIT.** The unobserved state variables  $\{\epsilon_{ijt} : j = 0, 1, \dots, J\}$  are independent across alternatives and have an extreme value type 1 distribution.

**ASSUMPTION DIS** (*Discrete Support of  $x$* ). The support of  $(x_{it}, w_t, M_{it})$  is discrete and finite:  $(x_{it}, w_t, M_{it}) \in X = \{x^{(1)}, \dots, x^{(|X|)}\}$  with  $|X| < \infty$ .

Since our model fits with all those assumptions, we can use a simulation-based CCP estimator, Hotz, Miller Sanders and Smith (1994).

## C.5 Disaggregated Demand, No Liquidity Constraints

In this section, we consider the case in which every week the farmer can buy several units of water. The purchase will be sequential. The farmer is offered a price for the first unit and has to decide whether to purchase the unit or not. After this decision is made, the farmer is offered a price for the second unit, and so forth. The prices the farmer is offered follow a stochastic Markov process. The farmer knows the parameters governing this process.

There will be forty units auctioned every week. Before the first price is offered, the farmer observes the rain in the previous week and a  $10 \times 1$  vector containing the shocks to her utility for the next week (Monday to Friday=5 days; day or night=2;  $\epsilon_{it} = (\epsilon_{it1}, \dots, \epsilon_{it10})$ ). Each value of the shock represents a shock to the utility for all four units in a 4-unit auction. We abstract here from the equilibrium played within each 4-unit auction, see Donna and Espín-Sánchez (2013) for details.

We index each of the ten units by  $k$ . We denote a purchase of  $j$  units of water by farmer  $i$ , during period  $t$  and within the  $k^{th}$  4-unit auction by  $j_{itk}$ , with  $0 \leq j_{itk} \leq 4$ , and  $\sum_{k=1}^{k=10} j_{itk} = j_{it} \leq 40$ . We

denote by  $p_{tk}$  the price associated with buying any unit within the  $k^{th}$  4-unit auction in period  $t$ . We denote by  $V_{it}(s_{it}, M_{it}, x_{it})$  the value of a farmer to participate in a 40-unit auction at week  $t$ , where  $s_t$  is an unobserved state and  $x_t$  is an observed state.  $s_t$  is now a vector of epsilons  $\epsilon_{it} = (\epsilon_{it1}, \dots, \epsilon_{it10})$ , thus  $s_{it} = (\epsilon_{it}) = (\epsilon_{it1}, \dots, \epsilon_{it10})$ . Let  $\sigma_{itk}$  be a state variable in the within-period game.  $\sigma_{itk}$  includes the units of water already bought by the farmer in period  $t$  up to auction  $k-1$ , thus  $\sigma_{it1} = 0$  and  $\sigma_{itk} = \sum_{l=1}^{k-1} j_{itl}$ .

Hence:

$$V_{it}(s_{it}, M_{it}, x_{it}) = h(M_{it}, w_t; \gamma) - \sum_{k=1}^{k=10} (j_{itk}^* p_{tk} + \zeta_{jk} + \epsilon_{ijtk}) + \beta V_{i,t+1}(s_{i,t+1}, \sigma_{i,t+1,1}, x_{i,t+1})$$

where  $j_{itk}^*$  are the elements of the solution to the game below. We define the value of the farmer of entering the within-week game as:

$$V_{it}(s_{it}, M_{it}, x_{it}) \equiv W_{it1}(s_{it}, 0, M_{it}, x_{it})$$

The (finite) within-week game then is:

$$\begin{aligned} W_{it1}(s_{it}, 0, M_{it}, x_{it}) &= \underset{j_{it1} \in \{0,1,2,3,4\}}{\text{Max}} \{h(M_{it}, w_t; \gamma) - (j_{it1} p_{t1} + \zeta_{j1} + \epsilon_{ijt1}) + W_{it2}(s_{it}, \sigma_{it2}, M_{it}, x_{it})\} \\ &\quad \vdots \\ W_{itk}(s_{it}, \sigma_{itk}, M_{it}, x_{it}) &= \underset{j_{itk} \in \{0,1,2,3,4\}}{\text{Max}} \{- (j_{itk} p_{tk} + \zeta_{jk} + \epsilon_{ijtk}) + W_{it,k+1}(s_{it}, \sigma_{it,k+1}, M_{it}, x_{it})\} \\ &\quad \vdots \\ W_{it10}(s_{it}, \sigma_{it10}, M_{it}, x_{it}) &= \underset{j_{it,10} \in \{0,1,2,3,4\}}{\text{Max}} \{- (j_{it,10} p_{t,10} + \zeta_{j,10} + \epsilon_{ij,10}) + \beta V_{i,t+1}(s_{i,t+1}, M_{it} + \sigma_{it,11}, x_{i,t+1})\} \end{aligned}$$

Or, if we do not assume that all prices are learn at the beginning of the week, but rather, prices are learnt at the beginning of each 4-unit auction, then we have:

$$\begin{aligned}
W_{it1}(s_{it}, 0, M_{it}, x_{it}) &= \underset{j_{it1} \in \{0,1,2,3,4\}}{\text{Max}} \{h(M_{it}, w_t; \gamma) - (j_{it1}p_{t1} + \zeta_{j1} + \epsilon_{ijt1}) + E[W_{it2}(s_{it}, \sigma_{it2}, M_{it}, x_{it})]\} \\
&\quad \vdots \\
W_{itk}(s_{it}, \sigma_{itk}, M_{it}, x_{it}) &= \underset{j_{itk} \in \{0,1,2,3,4\}}{\text{Max}} \{- (j_{itk}p_{tk} + \zeta_{jk} + \epsilon_{ijt}) + E[W_{it,k+1}(s_{it}, \sigma_{it,k+1}, M_{it}, x_{it})]\} \\
&\quad \vdots \\
W_{it10}(s_{it}, \sigma_{it10}, M_{it}, x_{it}) &= \underset{d_{i,t,10} \in \{0,1,2,3,4\}}{\text{Max}} \{- (j_{it,10}p_{t,10} + \zeta_{j,10} + \epsilon_{ijt,10}) + \beta V_{i,t+1}(s_{i,t+1}, M_{i,t} + \sigma_{it,11}, x_{i,t+1})\}
\end{aligned}$$

where the expectation is taken with respect to the remaining prices to be disclose in the current week, and the price sequence follows a Markov chain.

We do not include a discount factor because the time from one auction to the next is just a few minutes and the discount factor in this case is virtually 1. Notice that, since this is finite game, we do not need a discount factor to have solution. Instead, we can solve this game by backward induction if we know the value of  $V_{i,t+1}(\cdot)$ . The solution concept will be Sub-game perfection (or Bayesian Perfect if price are learned at every step).