How a small open economy's asset are priced by heterogeneous international investors

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Abstract

We study how a small open economy's assets are prices by heterogeneous international investors. We initially decompose the asset pricing issue into separate studies of its two ingredients: the asset's ex post return and the investors' stochastic discount factor.

The ex post asset return is examined in a small open economy RBC model featuring adjustment cost in investment. We derive an approximate closed-form solution for the ex post asset return using the Campbell (1994) log-linear technique. The international investors' stochastic discount factor is taken as given by this small open economy.

To examine the international investors' stochastic discount factor, general equilibrium analysis is called in. We do this by setting up a world economy model. In the world economy model, the production side features a world representative firm which produce the world aggregate output consumed as world aggregate consumption; the consumer side features heterogeneous international investors from N countries in a sense that there are exogenous consumption distribution shocks and the price variation across countries. The shock affects the cross-sectional distribution of consumption goods among international investors but won't affect the world aggregate level. The market stochastic discount factor hence is derived as a function of the world aggregate consumption growth, the world aggregate price growth and the cross-sectional variances and covariance terms of individual consumption growth and price growth.

We then derive the closed-form solutions for asset prices by substituting the two ingredients, the asset's ex post return from small open economy model and the investors' stochastic discount factor from a general equilibrium world economy model, into the basic asset pricing formulas. Our model generates a risk premium for a small economy's asset that tends to be low when the global economy is robust and to soar when global economy experiences a downturn. The main reason behind this is our assumption of heterogeneity across international investors. We also study the capital accumulation and capital loss/gain channels and explore their asset pricing implications. Our major finding is: For a small country that conducts fierce capital accumulation, our model predicts that its risk premium will fluctuate less broadly than one that conducts little capital accumulation.

1 Introduction

This paper is an application of general asset pricing theory to an analysis of a specific topic, that is, how a small open economy's assets are priced by heterogeneous international investors. Asset pricing theory has developed over several decades, from the partial equilibrium capital asset pricing model (CAPM),[1] to the general equilibrium consumption-based asset-pricing model (CCAPM).[2] The core question in asset pricing theory is what an asset's price is determined by investors. The latter so-called "general equilibrium" model is actually in an endowment economy. In this environment, the asset pricing issue becomes a study of what price a consumer (investor) will demand for an asset in order to hold it given its exogenous payoff (dividend). In an endowment economy, the asset's exogenous payoff is equal to the consumer's consumption. CCAPM answers the core question of the asset pricing: that is only the undiversified risk which is the covariance between an asset's ex post return and investors' stochastic discount factor, gets compensated and enters the asset price formulas.

First, we review the basic asset pricing formulas derived from the consumer's Euler equation

$$\begin{aligned} P_t^j U'\left(C_t^i\right) &= \beta E_t\left[U'\left(C_{t+1}^i\right)\left(D_t^j + P_{t+1}^j\right)\right] \\ U'\left(C_t^i\right) &= \beta E_t\left[U'\left(C_{t+1}^i\right)R_{t+1}^j\right] \end{aligned} \tag{1}$$

[1] See Sharpe (1964); Lintner (1965).

[2] See Lucas (1978).

where U is the consumer's utility from consumption and U' is his marginal utility,[3] C_t^i is the consumption level of a consumer indexed by i , at time t, β is his time-preference factor, P_t^j is asset j's price at time t, D_t^j is its payoff or dividend during period t, $R_{t+1}^j = \frac{D_t^j + P_{t+1}^j}{P_t^j}$ is hence asset j's gross rate of return from time t to time $t+1$.

Dividing Equation (1) by $U'\left(C_t^i\right)$ yields:

$$E_t\left(\beta\frac{\Lambda_{t+1}}{\Lambda_t}R_{t+1}^j\right)=1 \tag{2}$$

where marginal utility is denoted by Λ and $\beta\frac{\Lambda_{t+1}}{\Lambda_t}$ is known as the stochastic discount factor (SDF).[4] Since the existence of a common SDF across investors is guaranteed by the absence of arbitrage in the market (Campbell, 2003), we drop the subscript i in Equation (2).

To write the expectation of the product in Equation (2) as the product of expectations plus the covariance, we get

$$E_t\left(R_{t+1}^j\right)=\frac{1-Cov_t\left(\beta\frac{\Lambda_{t+1}}{\Lambda_t},R_{t+1}^j\right)}{E_t\left(\beta\frac{\Lambda_{t+1}}{\Lambda_t}\right)} \tag{3}$$

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[3] To write the consumer's Euler equation in the form of equation (1), we implicitly assume that utility is time-separable.

[4] $\beta\frac{\Lambda_{t+1}}{\Lambda_t}$ is also known as intertemporal marginal rate of substitution (MRS), price kernel, or marginal utility growth.

Equation (2) or Equation (3) expresses the most fundamental idea in asset pricing. They must hold true for any asset. Applying them to the riskless asset whose gross return $\Re_{t+1}^{f}$ is not a random variable and known at the beginning of period t, we get

$$\Re_{t+1}^{f} = \frac{1}{E_t\left(\beta \frac{\Lambda_{t+1}}{\Lambda_t}\right)} \tag{4}$$

Equation (4) shows that the riskless interest rate is just the reciprocal of the expectation of the market stochastic discount factor.

If we define the risk premium as $R_{j,t+1}^{rp} = R_{t+1}^{j} - \Re_{t+1}^{f}$, Equation (2) becomes

$$E_t\left(\beta \frac{\Lambda_{t+1}}{\Lambda_t} R_{j,t+1}^{rp}\right) = 0 \tag{5}$$

and Equation (3) becomes

$$E_t\left(R_{j,t+1}^{rp}\right) = -\Re_{t+1}^{f} Cov_t\left[\beta \frac{\Lambda_{t+1}}{\Lambda_t}, R_{j,t+1}^{rp}\right] \tag{6}$$

Equations (1) to (6) constitute the basic asset pricing formulas. They are the main results of the CCAPM. Notice that there are two ingredients appeared in each basic asset-pricing formula. The first ingredient is an asset's ex post return R_{t+1}^{j}. The second one is the stochastic discount factor, which is investors' intertemporal marginal rate of substitution. It is these two ingredients that determine an asset's price and its ex ante return.

In this paper, we make extensions and modifications; add details, to those two ingredients, to fit our goal: to determine how a small open economy's assets are priced in the global capital market by heterogeneous international investors. The rest of the paper is organized as follows. Section two reviews the related literature and shows the relationship between our model and the literature. Section three is about the first ingredient in basic asset pricing formulas: an asset's ex post return R_{t+1}^{j}. We present a small open economy model from which its asset return is derived. Section four is a study of the second ingredient in basic asset pricing formulas. In this section we present a model with heterogeneous international investors and examine the market stochastic discount factor. We assume for this purpose that the world is composed of N countries and each one has a representative agent. The SDF we derive in this section is a market SDF valid for every heterogeneous investor. In section five we derive the approximate closed-form solution for the asset price. This is done by putting the two ingredients, (which we have modified to fit our goal, in section 3 and 4 respectively), back into the basic asset pricing formulas. The results of our asset price analysis thus answer the central question we raise in this paper: how a small open economy's assets are priced by heterogeneous international investors. Section six contains our summary and conclusions.

2 Review of the Related Literature

In the introduction, we described how an asset's price is determined in the CCAPM. To focus on its main object, the model is simplified to an exchange economy without a nontrivial production sector. This simplification has its trade-offs. For example, in an exchange economy, a positive technology shock leads to a higher asset return. This is not necessarily true in a production economy. A positive technology shock causes capital accumulation which lowers the asset return due to the diminishing marginal returns. The effect of capital accumulation on the asset return can be strong enough to offset the positive direct effect of technology shock and causes a lower asset return. (Lettau, 2003) This capital accumulation channel is absent in an exchange economy.

Since the 1990s, the growing literature on this subject reflects the efforts of economists to fill this gap and extend the CCAPM into a general equilibrium framework with a nontrivial production sector. Examples include: Cochrane (1991), Rouwenhorst (1995), Jermann (1998), and Boldrin, Christiano, Fisher (2001). This strand of work is sometimes called the production-based capital asset pricing model (PCAPM) to differentiate it from CCAPM. PCAPM is an intersection between macroeconomics and finance. Since PCAPM studies asset pricing in a general equilibrium real business cycle (RBC) model, it is convenient to enrich models with tools developed in RBC models. Now we see PCAPM which has the habit formation utility (time inseparability utility) and incorporates costly adjustment in investment; which derives approximate closed-form solutions using log-linear method or conducts numerical simulation in general cases.

Another motivation to extend CCAPM model comes from its unsuccessful empirical performance. Using U.S. postwar quarterly data, the average real return on stock over the period 1947.2 to 1998.4 is 8.1% at an annual rate. The riskless real interest rate is low. The average real return on 3-month Treasury Bill is 0.9% at an annual rate. Therefore, the equity premium is about 7% per year. On the other hand, real consumption is very smooth. The annualized standard deviation of the growth rate of seasonally adjusted real consumption of nondurables and services is 1.1% (Campbell, 2003). For a constant relative risk aversion (CRRA) utility function, the high equity premium can only be explained by a very high coefficient of risk aversion. But a high level of risk aversion is against micro data. Moreover, a low elasticity of intertemporal substitution implied by high risk aversion from CRRA class of utility leads to a counterfactual high riskless interest rate. This has been referred to as the "equity premium puzzle" (Mehra and Prescott, 1985) and the related "low riskless interest rate puzzle" (Weil, 1989).

2.1 Previous Work on the Stochastic Discount Factor with Homogenous Agents

To generate a historical high equity premium, the standard Lucas (1978)-type CCAPM has been modified in various ways on the model's consumer side. New features with respect to the consumer's utility function have been incorporated. This line of work is on the first ingredient in the basic asset pricing formulas, that is, on the investor's stochastic discount of factor. Examples include: habit-formation (Abel, 1990, 1999; Constantinides, 1990; Campbell and Cochrane, 1999); recursive utility which can separate the risk aversion and the elasticity of intertemporal substitution (Epstein and Zin, 1989, 1991; Weil, 1989); and incomplete market model with heterogeneous agents which have either different risk aversion, different income stream or different market access, different borrowing constraints (Mankiw, 1986; Dumas, 1989; Mankiw and Zeldes, 1991; Constantinides and Duffie, 1996; Heaton and Lucas, 1996; Chan and Kogan, 2002).

Habit formation makes the utility function non-separable over time. With habit formation, the CRRA class of utility becomes a power function of either the ratio or the difference between consumption and habit.[5] Campbell (2003) claims that the choice between ratio models and difference models of habit is important because ratio models have constant risk aversion whereas difference models have time-varying risk aversion. Campbell and Cochrane (1999) have developed a model in which the consumer derives utility from the difference between his own

[5] Habit is defined as a slow-moving average of past consumption, either the consumption's own past consumption or the aggregate past consumption.

consumption and a habit level, which is the average of past aggregate consumption. This utility function makes the consumer more risk-averse in bad times when consumption is low relative to its past history, than in good times when consumption is relatively high. Therefore their model generates a time-varying countercyclical risk aversion, which has significant asset pricing implications.

Time-variation in the price of risk can also arise in other frameworks.[6] Models built on prospect theory argue that agents become less risk averse as their wealth has risen[7]. It can also arise in the models with heterogeneous agents. Constantinides and Duffie (1996) build a model with heterogeneous agents. They examine the market stochastic discount factor, an SDF valid for every heterogeneous investor. They claim that such an SDF does exist and depends on aggregate consumption growth rate, which solely determines SDF in the models with homogenous agents. Furthermore, their market SDF also depends on cross-sectional variance of individual consumer's consumption growths. This is a new feature for SDF and it only arises in a model with heterogeneous agents.

If the cross-sectional variance is assumed to be heteroskedasticity, and furthermore, negatively correlated with the level of aggregate consumption, so that idiosyncratic risk increases in economic downturn, then the market stochastic discount factor will be strongly countercyclical, very much in the spirit of Campbell and Cochrane's (1999) habit-formation model. Therefore, both habit-formation models and heterogeneous agent models can generate countercyclical stochastic discount factors. Since the model with heterogeneous investors in an incomplete international capital market also has significant implications for the international business cycles,

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[6] The price of risk is the coefficient of relative risk aversion of the investor (Campbell, 2003).

[7] See, for example, Kahneman and Tversky (1979); Benartzi and Thaler (1995); Barberis, Huang and Santos (2001).

in this paper, we adopt a model with heterogeneous investors rather than a habit-formation model with homogenous investors.

2.2 Previous Work on the Asset Return

To improve the model's empirical performance, another strand of literature works on the asset return, that is, on the second ingredient in basic asset pricing formulas. For the model to generate a high equity premium, the asset return needs to vary a lot. This can be done by imposing rigidity upon the model's investment process, such as adding adjustment cost[8] or constructing a separate capital goods production sector[9].

For the asset return to be derived endogenously, one need a model beyond the exchange economy environment, specifically, one need a general equilibrium model with a nontrivial production sector. Rouwenhorst (1995) introduces the nontrivial production sector into the standard CCAPM. Unlike in an exchange economy, consumption and dividend in PCAPM are determined endogenously. But this effort is less successful in the explanation of the equity premium. Rouwenhorst (1995) finds that his model's asset pricing implication is even worse than that from models of exchange economy. This is not a surprising finding since in a model with one sector and frictionless investment, an agent can easily and instantaneously alter the production plan to reduce fluctuations in his consumption. As a result, consumption becomes even smoother than in an exchange economy. A smooth consumption causes SDF to fluctuate less. This is the source of puzzling asset pricing implication arising in these models.

Jermann (1998) develops a production-based asset-pricing model in a general equilibrium closed economy environment. To enhance the model's asset pricing implication, on the

[8] See, for example, Jermann (1998).

[9] See, for example, Boldrin, Christiano and Fisher (2001)

consumer side, he incorporates habit formation into the utility function; on the production side, he imposes adjustment cost on the investment. For a model to generate a high equity premium, Jermann (1998) concludes that both of the above features are necessary. “[w]e find that a real business cycle model can generate the historical equity premium with both capital adjustment cost and habit formation, but not with either taken separately” (Jermman, 1998).

Boldrin, Christiano and Fisher (2001) build a general equilibrium closed economy model also featuring habit formation in consumer’s preference. Rather than adding adjustment cost, they imposed investment rigidity by constructing a separate sector for capital goods production. Since capital goods and consumption goods are now produced in two distinct sectors and they cannot be converted to each other frictionless, their model generated a volatile investment return to help reconcile the high variance of stock return we observe in reality.[10] In addition, they claim that their model’s business-cycle implications are improved over the standard growth model.

Hansen, Sargent and Tallarini (1998) deal with a general equilibrium model with a recursive utility function.[11] This class of utility function allows the separation of the risk aversion coefficient and the elasticity of intertemporal substitution, which always intertwine together in a power utility. Their finding is that what really matters for the model’s business cycle implications is the elasticity of intertemporal substitution, rather than the risk aversion coefficient. But the latter is important in calculating the welfare cost of risk sharing. This is positive news to RBC models, considering its bad asset pricing implication. The existing RBC models can always have modifications made for better asset pricing implication as long as its elasticity of intertemporal substitution does not get changed; the model’s business cycle implication will hence remain intact.

[10] The standard deviation of the stock return in U.S. is 17%.

[11] Recursive utility function form is explained in detail in Appendix A.1.

2.3 Previous Work on the SDF in an Incomplete Market with Heterogeneous Agents

A decade of research into the incomplete market and the idiosyncratic risk had stumbled against one difficulty after another until Constantinides and Duffie (1996) made a brilliant contribution (Cochrane, 2006). Their breakthrough work shows how an asset is priced by the heterogeneous agents facing uninsurable persistent idiosyncratic income risk. Their work makes both possible and easy for us to explore the asset pricing implication in a PCAPM open economy model featuring heterogeneous international investors.

If investors from different countries are subject to uninsurable persistent country-specific risk in their income, the consumption path of each country is more volatile than the world aggregate consumption. For each investor, his consumption growth is still the sole factor in determining his individual SDF. However, the world aggregate consumption is not the only factor in determining the market SDF in the international capital market. For example, considering a CRRA class of utility with risk aversion coefficient ρ, each investor's individual SDF is his consumption growth rate raised to the power $-\rho$; however, the world aggregate consumption growth raised to the power $-\rho$ may not be a valid SDF (Campbell, 2003). This follows from Jensen's inequality due to the non-linearity of the marginal utility.

Even though each investor's marginal rate of substitution is still valid as his SDF, it does not imply that we will then see a series of distinct asset prices applied to each investor in the market. The investors, even though with heterogeneity among each other, still face one market asset price, which in turn implies the existence of one market SDF, a stochastic discount factor valid for every investor in the market. The question is begged: Does this market SDF exist and if so what does it look like?

Constantinides and Duffie (1996) solve this problem in a brilliant way. In their closed exchange economy model, there are heterogeneous investors facing persistent, uninsurable, idiosyncratic income risk. They argue that a market SDF does exist and that it depends on the aggregate consumption growth and the cross-sectional variance of individual consumers' consumption growth.

In short, in an incomplete market with heterogeneous investors, the aggregate consumption growth is not a valid SDF. Since each investor's own intertemporal marginal rate of substitution is still a valid SDF for himself, it follows that the cross-sectional average of investors' intertemporal marginal rate of substitution is a valid stochastic discount factor in the market. This market SDF, which is valid for every investor, depends on the aggregate consumption growth rate and the cross-sectional variance of the individual consumers' consumption growth (Campbell 2003).

Applying this logic into an open economy model is straightforward. In Constantinides and Duffie's (1996) closed-economy model, it has one risk, namely the uninsurable, persistent, idiosyncratic consumption shock across agents within a country. In contrast, in this paper we assume that the agents within a country are homogenous. The uninsurable, persistent, idiosyncratic consumption shock occurs across countries, at the international level. This assumption is justified by the fact that the asset market is more integrated and complete within a country than across countries.

Moreover, in our model, there are differential of consumption goods prices across countries. The uninsurable, persistent, country-specific consumption shocks cause the uninsurable, persistent differential of consumption goods prices across countries. Even though there is only one good acting as "consumption good" in our model, one may think of its price differential across countries in this way. Imagine that there is a commodity with constant supply across countries. This commodity does not provide utility but rather acts as a unit of measurement. For example, the commodity could be gold. The country-specific shock on the endowment of the consumption goods causes its relative price to gold to vary. Note that, in our one-good model, a variation in the relative price of the consumption goods is equivalent to a fluctuation in the country's real exchange rate.[12]

In our model, which accounts for both consumption endowment shock and the accompanying goods price risk, it turns out that the market SDF depends on five factors. The first two are similar to Constantinides and Duffie (1996)-type market SDF: the world aggregate consumption growth and the cross-sectional variance of the individual countries' consumption growth. Beyond these, the additional factors include: the world aggregate goods price growth rate, the cross-sectional variance of individual countries' price growth, and the cross-sectional covariance between an individual country's price growth and its consumption growth.

A model featuring heterogeneous international investors might be a better environment in which to study the issues of international assets prices and international business cycle than would a model with homogenous agents. OECD countries' aggregate consumption volatility is small,[13] but in the real world we do not witness a low equity premium for emerging countries' risky assets, as CCAPM would predict. Moreover, we often observe international investors (mostly from developed countries) demanding positive risk premia over the assets issued by developing countries. This is a puzzle given the fact that the emerging countries' outputs usually have negative covariance with that of developed countries. It seems, hence, that equity premium is even more a puzzle at the international than at the domestic level. The model with

[12] The real exchange rate between two countries is the ratio of national price levels (CPI is a candidate index to measure a country's aggregate price level).

[13] By saying so, we imply that investors in the world capital market are mainly from OECD countries.

heterogeneous international investors can generate a more volatile SDF than a model featuring only homogenous agents. Moreover, the correlation between an asset's ex post return and the cross-sectional variance of the individual investors' consumption growth arises in a model with heterogeneous agents. It is this correlation that enables our model to generate the countercyclical[14] risk premia for emerging countries' assets, a phenomenon we observe in reality. This will be discussed in more detail in Section 5.

2.4 Log-linearization and the Approximate Closed-Form Solutions

With the development of real business cycle models, calibration and simulation have become a popular methodology.[15] Researchers impose complex structure on their models without worrying about the lack of closed-form solutions. The numerical and simulation approach has its trade-offs, however. As Campbell (1994) states "[m]ost of these methods are heavily numerical rather than analytical…[t]he methods are often mysterious to the noninitiate…[a] typical paper in the real business cycle literature states the model, then moves directly to the discussion of the properties of the solution without giving the reader the opportunity to understand the mechanism giving rise to these properties."

Campbell (1994) provides an analytical approach to solving the RBC model.[16] First, one must approximate all relevant equations in log-linear form around non-stochastic steady states. The model then becomes a system of log-linear difference equations, which can be solved by the method of undetermined coefficients. Following Campbell (1994), Lettau (2003) derives and

[14] "Countercyclical" is relative to the developed countries' economic condition. To put it another way, the risk premia for developing countries' assets will rise when developed countries experience economic downturns. In contrast, these risk premia will drop when the economies in developed countries are robust.

[15] Classic papers on this topic include Kydland and Prescott (1982), and King, Plosser and Rebelo (1988).

[16] For a step-by-step demonstration of this approach, see Uhlig (1999)

analyzes approximate closed-form solutions for asset prices in a closed-economy RBC model. Lettau (2003) argues that solving the model analytically rather than numerically makes the relationship between asset prices and the model's state variables particularly transparent. The approximate closed-form solution for risk premium is written as a function of elasticity of real variables given by the solution of the RBC model. Using Campbell's words, this analytical solution method can let us "[i]nspect the mechanism".

In order to develop a clear understanding of how a small open economy's assets are priced by international investors, we follow Campbell (1994) and Lettau (2003) by solving the model analytically rather than numerically.

To summarize, in this paper we will explore how a small open economy's assets are priced in the global capital market by heterogeneous international investors. The market stochastic discount factor, the first ingredient in the asset pricing formula, is derived in a world economy model featuring heterogeneous international investors. The small open economy's asset return, the second ingredient in asset pricing formula, is derived in a small open economy RBC model featuring adjustment cost in the investment process. As a result, the small open economy's asset price depends on both global factors and the small open economy's country-specific factors.

There are several strands in the literature related to our model. The first is PCAPM; that is, a general equilibrium asset-pricing model with a nontrivial production sector[17]. To our knowledge, major papers in this area deal with closed economies. In the strand of an asset pricing model with heterogeneous agents, Constantinides and Duffie (1996) is a breakthrough work and a major contribution. In their model, the environment is an exchange closed economy without a non-trivial production sector. In the strand of international asset pricing literature, to our knowledge, one approach extends the partial equilibrium CAPM model at an international level; the other

[17] See Jermann (1998); Hansen, Sargent and Tallarini (1998); Boldrin, Christiano, Fisher (2001)

approach modifies the Lucas (1978)-type exchange economy general equilibrium model in an open-economy environment[18]. In these models, the efforts of modifications focus on the consumer's preferences. They adopt the habit formation in the utility form or (and) incorporate heterogeneity among the agents. However, there is no production sector in these models, as exemplified by Li and Zhong (2004), and Li (2005).

To summarize, our model is an extension of the PCAMP from the closed-economy to an open economy environment. Other major features of our model include the adjustment cost on the production side, and consideration of the heterogeneous agents on the consumer side.

3 The Small Open Economy Model and the ex post Asset Return

The object in this section is to derive the small open economy's ex post asset return as a function of the model's state variables. Firstly we derive the firm's investment return from a small open economy RBC model featuring adjustment cost. To get the firm's ex post asset return, we then apply Restoy and Rockinger's (1994) result that, under Hayashi (1982) condition, a firm's investment return is equal to, state by state, the firm's asset return.[19]

In this section, the model is a small open economy with households and firms. Since we assume homogeneity among domestic consumers in this small open economy (heterogeneity arises in international level, among international investors) and constant return to scale in its production, the model in this section has a representative consumer and a representative firm.

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[18] See Dumas (1994); Stulz (1994); Karolyi and Stulz (2003) for surveys

[19] The Hayashi (1982) condition requires: 1) The firm is a price taker in its output market; 2) The capital installation function is linearly homogenous in I and K; 3) The production function is linearly homogenous in K and L.

Our model satisfies the Hayashi (1982) condition.

3.1 The Small Open Economy's Preference (the consumer side)

There is a single consumption good in the small open economy. The economy is populated with the infinitely lived identical households, represented in our model by a representative consumer, who derives utility from the consumption of the single good. The representative consumer maximizes the objective function

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} U\left(C_s^d\right) = E_t \sum_{s=t}^{\infty} \beta^{s-t} \frac{\left(C_s^d\right)^{1-\rho}}{1-\rho} \tag{7}$$

where C_t^d is the domestic consumer's consumption at time t, and β is the subjective discount factor, also known as time-preference factor. This equation is of the time-separable constant relative risk aversion (CRRA) class of utility functions. Furthermore, ρ is the Arrow-Pratt coefficient of relative risk aversion, and the intertemporal substitution elasticity is $1/\rho$.[20] King, Plosser and Rebelo (1988) claim that CRRA utility functions with fixed labour supply can generate a balanced growth.[21]

The domestic consumer gets income from 1) the labor income by offering a fixed amount of his labor endowment to the firm; 2) the financial income by investing in the shares of risky assets and the bonds of the riskless asset in the global financial market. Given the constant return of scale of the production function, we can treat the firms in each country as a representative firm. Therefore in the world asset market, there are N securities which are issued by the firm from

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[20] When ρ is larger than zero but not equal to one, the utility is a power function. When ρ equals one the utility is a log function.

21 When labor supply is varying and period utility is additively separable over consumption and leisure, log utility for consumption is required while the utility function for leisure is not restricted, in order to obtain a balanced growth.

each N country. Moreover, there are uninsurable, persistent, idiosyncratic consumption distribution shocks across countries. And these idiosyncratic shocks cannot be hedged away in the world asset market. Further, due to the persistent character of these shocks, they can neither be eliminated by investing in the riskless asset. With these assumptions the domestic household budget constraint becomes

$$\begin{aligned} C_t^d &= LI_t^d + FI_t^d + \aleph_t^d \\ &= W_t L_t^d + \sum_{j=1}^{N}\left[\theta_{jt}^d\left(\Omega_{jt} + D_{j,t-1}\right)\right] - \sum_{j=1}^{N}\left(\theta_{j,t+1}^d \Omega_{jt}\right) + B_t^d \Re_t^f - B_{t+1}^d + \aleph_t^d \end{aligned} \tag{8}$$

where j is the asset index; C_t^d is the domestic consumption during the period t; $\aleph_t^d$ denotes the idiosyncratic consumption distribution shock on the domestic country at time t. We will describe this shock in detail in section 4 when we examine the market SDF among heterogeneous international investors; LI_t^d denotes the domestic consumer's labor income, a product of the wage at the time t, W_t, and his labor supply during the period t, which is a fixed amount and equals to his labor endowment, L_t^d; FI_t^d denotes his financial income which is composed of the asset returns from his holding of N types of the world risky assets and a world riskless asset; θ_{jt}^d is the domestic consumer's holding of shares of the risky asset j at the beginning of the period t; analogously, $\theta_{j,t+1}^d$ is his shares at the end of the period t, or at the beginning of the period $t+1$; $D_{j,t-1}$ is the dividend from the risky asset j during the period $t-1$, which is available for consumption from the beginning of the period t; Ω_{jt} is the price of the risky asset j at the beginning of the period t; B_t^d is the domestic consumer's holding of the world riskless asset at the beginning of the period t; analogously, B_{t+1}^d is his holding of the riskless asset at the end of the period t, or at the beginning of the period $t+1$, $\Re_t^f$ is the riskless interest rate between time t and time $t+1$ at the global asset market.

The consumer's optimization problem is to maximize the utility of Equation (7) subject to his budget constraint of Equation (8). Substituting Equation (8) into Equation (7), we get the maximand:

$$\max_{B_{t+1}^d,\theta_{j,t+1}^d\left(j=1,2\ldots N\right)} E_t\sum_{s=t}^{\infty}\beta^{s-t}\frac{\left(W_tL_t^d+\sum_{j=1}^{N}\left[\theta_{jt}^d\left(\Omega_{jt}+D_{j,t-1}\right)\right]-\sum_{j=1}^{N}\left(\theta_{j,t+1}^d\Omega_{jt}\right)+B_t^d\Re_t^f-B_{t+1}^d+\aleph_t^d\right)^{1-\rho}}{1-\rho} \tag{9}$$

The first order conditions for a maximum are the following $(N+1)$ equations, with the first one for the world riskless asset and the rest of N equations for the world risky assets:

$$\frac{1}{\Re_{t+1}^f}=E_t\left[\beta\left(\frac{C_t^d}{C_{t+1}^d}\right)^{\rho}\right] \tag{10}$$

$$E_t\left[\beta\left(\frac{C_t^d}{C_{t+1}^d}\right)^{\rho}\left(\frac{\Omega_{j,t+1}+D_{jt}}{\Omega_{jt}}\right)\right]=E_t\left[\beta\left(\frac{C_t^d}{C_{t+1}^d}\right)^{\rho}R_{t+1}^j\right]=1 \qquad j=1,2\ldots N \tag{11}$$

Equation (10) and Equation (11) are asset pricing formulas. They come from the domestic consumer's Euler equations.

3.2 The Small Open Economy's Firm (the production side)

Due to the constant return of scale of the production function, the domestic firms can be treated as a representative firm which operates in a competitive environment. The firm maximizes its present value to owners,[22] subject to the capital stock law of motion and the technological shock evolution. The firm pays the worker the competitive wage rate, which is equal to the marginal product of labor. The firm then pays its shareholders dividends. We assume a Cobb-Douglas production function

$$Y_t^d = A_t^d \left(K_t^d\right)^{\alpha} \left(L_t^{fd}\right)^{1-\alpha} = A_t^d \left(K_t^d\right)^{\alpha} \tag{12}$$

where Y_t^d denotes the domestic firm's output at the time t , L_t^{fd} denotes the domestic firm's labor demand at the period t , which we normalized to be one; K_t^d denotes the domestic firm's capital stock at the beginning of the period t , α is the capital's share and $1-\alpha$ is the labor's share. Capital stock is chosen one period before it becomes productive and labor can be adjusted instantaneously. A_t^d is the domestic total factor productivity, which is assumed to be a random variable in this dynamic stochastic general equilibrium model. The stochastic process of A_t^d is assumed to follow an AR (1) in log form with i.i.d. normally distributed homoscedastic shock

[22] The firm's present value to owners is the sum total of all-its current and future expected dividends discounted by a market SDF deemed valid for every heterogeneous owner. We will explain this SDF in detail in Section 3.

$$\log A_t^d = (1-\phi)\log \overline{A}^d + \phi \log A_{t-1}^d + \mu_t$$
$$\mu_t \sim i.i.d.N\left(0;\sigma_\mu^2\right) \tag{13}$$

here ϕ measures the persistence of the domestic technological shock. Moreover we assume $0 < \phi < 1$. $\overline{A}^d$ is the steady state domestic productivity level.

Rigidity in investment is necessary for any PCAPM to generate a reasonable asset price. If investment is frictionless, a consumer can smooth his consumption even better than he could in an exchange economy. A smooth consumption path causes a low volatility in SDF, which leads to a counterfactual low equity premium. By incorporating rigidity into the investment process, such as imposing adjustment cost in the investment or constructing a separate sector to produce the capital goods, investment responds less to a positive technology shock and the consumer consumes more, than would be in a model with frictionless investment. A less smooth consumption path increases SDF volatility and helps to generate a high risk premium.

The second problem with the frictionless investment comes from the asset-pricing effect of the capital accumulation. Without any friction in the investment process, investment responds instantaneously and dramatically to a positive productivity shock. However this capital accumulation effect, which tends to reduce the investment return due to the diminishing marginal returns, can be strong enough to offset the original positive effect of the productivity shock on the investment return. As a result, without any adjustment cost, the asset return might even turn out to be countercyclical, so that the equity becomes a hedge against the technology shock.[23] This leads to a low or even negative risk premium. For example, Rouwenhorst (1995) reports

[23] A countercyclical equity return is counterfactual. Using U.S. data, Campbell (2003) displays the stylized fact that real stock return is procyclical, with a quarterly positive correlation with real consumption growth of 0.23. The correlation increases to 0.34 at a 1-year horizon.

that, for some certain parameter values, the equity premium from his model can be smaller than the long-term bond premium. In some cases it is even negative.

The third problem with the frictionless investment is the lack of variation in the marginal q, the relative price of the capital goods to the consumption goods.[24] Without any friction in the investment, the marginal q always equals to one. Since the capital is quite smooth compared with the output and the investment, if the investment return comes only from the capital's marginal product, the return tends to vary little. After imposing rigidity in the investment, the investment return then comes not only from the marginal product of capital but also from the capital gain/loss due to the variation in the capital good's relative price to the consumption goods. With a varying relative price of the capital goods, the model can generate a volatile investment return.

There are various ways to add friction into a model's investment process. Examples include the adjustment cost on the investment, or a separate capital goods production sector. In this paper, we adopt the adjustment cost approach.

The domestic firm's capital stock evolves according to the following law of motion

$$K_{t+1}^{d} = G\left(I_t^d, K_t^d\right) = \Psi\left(\frac{I_t^d}{K_t^d}\right) K_t^d + \left(1-\delta\right) K_t^d \tag{14}$$

where K_{t+1}^{d} is the domestic firm's capital stock at the beginning of the period $t+1$, K_t^d is its capital stock at the beginning of the period t, I_t^d is the investment made by the domestic firm during the period t, and δ is the depreciation rate. Ψ reflects the adjustment cost when

[24] Marginal q is the shadow price of installed capital, that is, the value generated by a unit of installed capital good in the next production period. At optimum, it equals to the relative price of installed capital good (capital good) with respect to the uninstalled capital good (consumption good).

We call Tobin's q the average q, which is the stock-market value of a unit of the firm's capital, given by V/K.

making the investment, which is positive near the steady state point. In the steady state, $\Psi(\delta)=\delta$ and $\Psi'(\delta)=1$. Thus the steady state level of the marginal q is one. We set these parameters so that the model with the adjustment cost has the same steady state as the model without it. Adjustment cost Ψ is also increasing and concave in I^d $(\Psi > 0, \Psi_I > 0, \Psi_{II} < 0)$. This specification reflects the idea that changing the capital stock rapidly is more costly than changing it slowly. In addition, $\left(\frac{1}{\Psi'}\right)$ is the marginal q, the relative price of the installed capital goods with respect to the consumption goods.

Following Jermann (1998), we assume that the domestic firm does not issue new shares, and that it finances its capital stock solely through its retained earning. The dividends to shareholders are equal to the output net of the investment and the wage payment to the workers. The second equality in Equation (15) is derived based on the fact that the labor market is competitive, hence the wage rate is equal to the marginal product of labor.

$$D_t^d = A_t^d \left(K_t^d\right)^\alpha - W_t - I_t^d = \alpha A_t^d \left(K_t^d\right)^\alpha - I_t^d \tag{15}$$

The domestic firm maximizes its value to shareholders subject to the production function, the law of motion of the capital stock and the stochastic process of domestic technology. That is, the domestic firm's optimization problem is:

$$\max E_t \sum_{s=t}^{\infty} \left\{ \frac{\beta^{s-t} \Lambda_s}{\Lambda_t} \left[Y_s^d - W_s L_s^{fd} - I_s^d \right] \right\}$$

$$s.t. \begin{cases} Y_s^d = A_s^d \left(K_s^d \right)^{\alpha} \left(L_s^{fd} \right)^{1-\alpha} \\ K_{s+1}^d = \Psi\left(\dfrac{I_s^d}{K_s^d} \right) K_s^d + (1-\delta) K_s^d \end{cases} \tag{16}$$

here β is the international investors' subjective discount factor, or time-preference factor, which we assume is the same as that of domestic consumers in this small country, and Λ_s is the international investor's marginal utility at the time s; hence $\frac{\beta^{s-t}\Lambda_s}{\Lambda_t}$ is the investor's intertemporal marginal rate of substitution, also known as the stochastic discount factor (SDF). In a complete market, SDF is unique because investors can trade with each other to eliminate any idiosyncratic variation in their marginal utilities. However, we assume heterogeneity across the international investors, that is, we assume that there are uninsurable, persistent, country-specific consumption distribution risks across nations. In this sense, the international asset market is not a complete market. As a result, SDF is not unique. Even though each investor's intertemporal marginal rate of substitution is still a valid SDF for himself, there exists a market SDF applied to every heterogeneous investor. In Equation (16) $\frac{\beta^{s-t}\Lambda_s}{\Lambda_t}$ refers to this market SDF. The existence of such a market SDF is guaranteed by the absence of arbitrage opportunity in the markets.

Substituting the production function into the firm's objective function and setting up the Lagrangian, we get:

$$\mathfrak{A}_t = E_t \sum_{s=t}^{\infty} \left\{ \frac{\beta^{s-t}\Lambda_s}{\Lambda_t} \left\{ \begin{array}{l} \left[A_s^d \left(K_s^d\right)^{\alpha} \left(L_s^{fd}\right)^{1-\alpha} - W_s L_s^{fd} - I_s^d \right] \\ -q_s \left[K_{s+1}^d - \Psi\left(\frac{I_s^d}{K_s^d}\right) K_s^d - \left(1-\delta\right) K_s^d \right] \end{array} \right\} \right\} \tag{17}$$

$$I_s^d, L_s^{fd}, K_{s+1}^d, q_s$$

The first order conditions for a maximum are:

$$\frac{\partial \mathfrak{A}_t}{\partial I_s^d} = 0 \rightarrow q_s = \frac{1}{\Psi'\left(\frac{I_s^d}{K_s^d}\right)} \tag{18}$$

$$\frac{\partial \mathfrak{A}_t}{\partial L_s^{fd}} = 0 \rightarrow W_s = \left(1-\alpha\right) A_s^d \left(K_s^d\right)^{\alpha} \tag{19}$$

$$\frac{\partial \mathfrak{A}_t}{\partial q_s} = 0 \rightarrow K_{s+1}^d = \Psi\left(\frac{I_s^d}{K_s^d}\right) K_s^d + \left(1-\delta\right) K_s^d \tag{20}$$

$$\frac{\partial \mathfrak{A}_t}{\partial K_{s+1}^d} = 0 \rightarrow$$

$$E_t \left\{ \left(\frac{\beta^{s+1-t}\Lambda_{s+1}}{\Lambda_t} \right) \left[\alpha A_{s+1}^d \left(K_{s+1}^d\right)^{\alpha-1} + q_{t+1} \left[\left(1-\delta\right) + \Psi\left(\frac{I_{s+1}^d}{K_{s+1}^d}\right) - \left(\frac{I_{s+1}^d}{K_{s+1}^d}\right) \Psi'\left(\frac{I_{s+1}^d}{K_{s+1}^d}\right) \right] \right] \frac{1}{q_t} \right\} = 1$$

(21)

Equation (21) is the basic asset pricing formula, which states that an asset's expected future gross return discounted by the investor's stochastic discount factor is equal to one. Equation (21) is also a condition guaranteed by the absence of arbitrage in the markets. Since the SDF is a discount factor to value the future uncertain payoff in terms of the present certain value, Equation (21) says nothing but that if you investment one unit today, it turns out that your expected return tomorrow is equivalent to a present certain value of one unit today, which of course holds if we rule out arbitrage.

From Equation (18) and Equation (21), we get:

$$R_{t+1}^{d} = \left[\alpha A_{t+1}^{d} \left(K_{t+1}^{d} \right)^{\alpha-1} + \frac{(1-\delta) + \Psi\left(\frac{I_{t+1}^{d}}{K_{t+1}^{d}} \right)}{\Psi'\left(\frac{I_{t+1}^{d}}{K_{t+1}^{d}} \right)} - \frac{I_{t+1}^{d}}{K_{t+1}^{d}} \right] \Psi'\left(\frac{I_{t}^{d}}{K_{t}^{d}} \right) \qquad (22)$$

$$E_{t}\left[\left(\beta \frac{\Lambda_{t+1}}{\Lambda_{t}} \right) R_{t+1}^{d} \right] = 1$$

Equation (22) is the gross rate of return of the risky asset in this small open economy. Recall Equation (18) stating that $\frac{1}{\Psi'(t)}$ is equal to the marginal q, q_t, the relative price of the installed capital goods with respect to the uninstalled capital. A marginal unit of the installed capital will cost $\frac{1}{\Psi'(t)}$ units of the uninstalled capital goods; therefore a marginal unit of the uninstalled capital will cost $\Psi'(t)$ units of the installed capital good. During the next production period, a marginal unit of the installed capital produces $F_K\left(t+1\right)$ or specifically $\alpha A_{t+1}^{d}\left(K_{t+1}^{d}\right)^{\alpha-1}$ units of the final goods (referring to the uninstalled capital goods) at the time $t+1$; but this marginal unit of the installed capital also depreciates into $\left(1-\delta\right)+\Psi\left(\frac{I_{t+1}^{d}}{K_{t+1}^{d}}\right)-\frac{I_{t+1}^{d}}{K_{t+1}^{d}}\Psi'\left(\frac{I_{t+1}^{d}}{K_{t+1}^{d}}\right)$ units of the installed capital, which are worth $\frac{\left(1-\delta\right)+\Psi\left(\frac{I_{t+1}^{d}}{K_{t+1}^{d}}\right)}{\Psi'\left(\frac{I_{t+1}^{d}}{K_{t+1}^{d}}\right)}-\frac{I_{t+1}^{d}}{K_{t+1}^{d}}$ units of the uninstalled capital goods at the time $t+1$. As a result, the investment return, in terms of the final goods (the uninstalled capital goods), is described by Equation (22). Again, Restoy and Rockinger (1994) prove that, under Hayashi (1982) condition, a firm's investment return equals to, state-by-state, the firm's asset return. Therefore the return of this small open economy's risky asset is also described by Equation (22).

3.3 The Market Clearing Conditions

The domestic goods market clearing condition is:

$$Y_t^d+\aleph_t^d=C_t^d+I_t^d+NX_t^d \tag{23}$$

where Y_t^d is the domestic output at the period t; $\aleph_t^d$ is the idiosyncratic consumption distribution shock on the domestic country at the period t; C_t^d is the domestic consumer's consumption at the period t; I_t^d is the investment the domestic firm made during the period t; NX_t^d is the domestic country's net export.

In addition, there is equilibrium in the domestic labor market, which means labor supply equals to labor demand and both are normalized to be one. Also there is equilibrium in the financial market, which requires that the international investors hold all the outstanding equity shares issued by the domestic firm. We normalize the equity share to be one. The risk-free bond in the global capital market is in zero net supply.

3.4 The First Order Conditions from the Consumer's and the Firm's Optimization Problems and the Market Clearing Conditions

The domestic consumer maximizes his lifetime utility subject to the budget constraint. The firm maximizes its discounted present value of all dividends subject to the production function, the law of motion of the capital stock, the capital installation cost and the stochastic process of the domestic technology. We rewrite here these first order conditions from the preceding consumer's and firm's optimization problems. Also we rewrite the market clearing conditions for domestic goods.

$$C_t^d + I_t^d + NX_t^d = A_t^d \left(K_t^d\right)^\alpha + \aleph_t^d \tag{24}$$

$$K_{t+1}^d = \Psi\left(\frac{I_t^d}{K_t^d}\right)K_t^d + \left(1-\delta\right)K_t^d \tag{25}$$

$$\frac{1}{\Re_{t+1}^f} = E_t\left[\beta\left(\frac{C_t^d}{C_{t+1}^d}\right)^\rho\right] \tag{26}$$

$$R_{t+1}^{d}=\left[\alpha A_{t+1}^{d}\left(K_{t+1}^{d}\right)^{\alpha-1}+\frac{\left(1-\delta\right)+\Psi\left(\dfrac{I_{t+1}^{d}}{K_{t+1}^{d}}\right)}{\Psi'\left(\dfrac{I_{t+1}^{d}}{K_{t+1}^{d}}\right)}-\frac{I_{t+1}^{d}}{K_{t+1}^{d}}\right]\Psi'\left(\frac{I_{t}^{d}}{K_{t}^{d}}\right) \qquad (27)$$

$$E_{t}\left[\left(\beta\frac{\Lambda_{t+1}}{\Lambda_{t}}\right)R_{t+1}^{d}\right]=1 \qquad (28)$$

3.5 The Nonstochastic Steady State

The above first order conditions constitute a system of nonlinear stochastic difference equations. There is no closed-form solution to this system. Kydland and Prescott (1982) put forward an approximate solution method by taking a linear-quadratic approximation to the true model around a nonstochastic steady state growth path. King, Plosser and Rebelo (1988) develop this method further by using a log-linear-quadratic approximation. In this paper, we follow Campbell (1994) approach to solving the RBC model. After approximating all relevant equations in log-linear form, Campbell (1994) presents analytical solutions for the elasticities of the endogenous variables with respect to the state variables.

First, we write down a system of the first order conditions in a nonstochastic steady state where all exogenous variables are constant. Variables in the steady state are denoted with a bar over them.

$$\overline{C}^{d}+\overline{I}^{d}+\overline{NX}^{d}=\overline{A}^{d}\left(\overline{K}^{d}\right)^{\alpha} \qquad (29)$$

$$\overline{I}^{d}=\delta\overline{K}^{d} \qquad (30)$$

$$\beta = \frac{1}{\bar{\Re}^f} \tag{31}$$

$$\bar{R}^d = \alpha \bar{A}^d \left(\bar{K}^d\right)^{\alpha-1} + 1 - \delta \tag{32}$$

$$\beta = \frac{1}{\bar{R}^d} = \frac{1}{\bar{\Re}^f} \tag{33}$$

Equation (29) specifies the resource constraint in the steady state given that $\bar{\aleph}^d = 0$. Equation (30) is the law of motion of the capital stock in the steady state. It shows that the steady state level of the investment is a level to cover the depreciation of the capital stock in order to keep the capital stock constant. Equation (31) ties down this small open economy's time-preference factor with the steady state world riskless interest rate. Equation (32) describes the domestic firm's asset return in the steady state. Given the specification of the capital installation cost in the steady state, that is, $\Psi(\delta) = \delta$ and $\Psi'(\delta) = 1$, it turns out that, in the steady state, the domestic asset return is the same whether there is the installation cost or not. Finally, Equation (33) states that, in the steady state, with the domestic asset return and the foreign investor's SDF both not random variables any more, the domestic firm earns exactly the world riskless rate.

3.6 Log-linear Approximation of the First Order Conditions around the Steady State

We now take the log-linear approximation of the first order conditions and Equation (13), which describes the domestic productivity evolution process, around their nonstochastic steady states. Following Campbell (1994), we derive analytical solutions for the elasticities of the control variables with respect to the state variables. In this small open economy model, the control variables are the domestic dividend D_t^d, the domestic investment I_t^d, the end of period domestic capital stock K_{t+1}^d and the domestic asset return R_t^d. The model's state variables are the domestic productivity A_t^d, the beginning of period domestic capital stock K_t^d and the world riskless interest rate $\Re_t^f$, the idiosyncratic consumption distribution shock $\aleph_t^d$.

Applying such a method to the basic asset pricing formula of Equation (28) is known for imposing equality on ex ante returns across different assets, which would disqualify it as a method for studying risk premium. Following Jermann (1998) and Lettau (2003), we will combine a linearization approach with nonlinear asset pricing formula. The closed-form solution for the risk premium is written as a function of the elasticities of the model's real variables. The latter is obtained by solving RBC model using Campbell's (1994) approach.

Loglinearly approximating the first order conditions of Equation (24), Equation (25), Equation (27) and productivity evolution process of Equation (13) yield respectively:

$$\bar{C}^d c_t^d + \delta \bar{K}^d i_t^d + \bar{N}\bar{X}^d nx_t^d = \bar{A}^d \left(\bar{K}^d\right)^{\alpha} a_t^d + \alpha \bar{A}^d \left(\bar{K}^d\right)^{\alpha} k_t^d \tag{34}$$

$$k_{t+1}^d = \delta i_t^d + \left(1-\delta\right) k_t^d \tag{35}$$

$$r_{t+1}^d = \left(\frac{\bar{\Re}^f - 1 + \delta}{\bar{\Re}^f}\right) a_{t+1}^d - \left(\frac{\bar{\Re}^f - 1 + \delta}{\bar{\Re}^f}\right)\left(1-\alpha\right) k_{t+1}^d - \zeta\left(i_t^d - k_t^d\right) + \frac{\zeta}{\bar{\Re}^f}\left(i_{t+1}^d - k_{t+1}^d\right) \tag{36}$$

$$a_t^d = \phi a_{t-1}^d + \mu_t \tag{37}$$

Each lowercase letter x_t is the logarithmic deviation of the corresponding uppercase letter X_t from its steady state value $\bar{X}$. Formally:

$$x_t = \log\left(X_t\right) - \log\left(\bar{X}\right) \qquad (38)$$

Therefore, d_t^d is the log deviation of the period t domestic dividend D_t^d from its steady state value $\bar{D}^d$. Analogously, $i_t^d, a_t^d, k_t^d, nx_t^d$ are, respectively, the log deviation of the period t domestic investment I_t^d, domestic productivity A_t^d, domestic capital stock K_t^d and domestic net export NX_t^d, from their steady state value $\bar{I}^d, \bar{A}^d, \bar{K}^d, \overline{NX}^d$; $i_{t+1}^d, a_{t+1}^d, k_{t+1}^d$ are, respectively, the log deviation of the period $t+1$ domestic investment I_{t+1}^d, domestic productivity A_{t+1}^d and domestic capital stock K_{t+1}^d, from their steady state value $\bar{I}^d, \bar{A}^d, \bar{K}^d$. r_{t+1}^d is the log deviation of R_{t+1}^d, the domestic firm's risky asset return between the period t and the period $t+1$, from its steady state value $\bar{R}^d$. At steady state, $\bar{R}^d$ is equal to $\bar{\Re}^f$, the steady state level of world riskless gross interest rate. ζ is defined by Equation (39) so that $\frac{1}{\zeta}$ is the elasticity of the investment capital ratio $\left(\frac{\bar{I}^d}{\bar{K}^d}\right)$ with respect to the marginal q at the steady state. The marginal q is equal to $\Psi'\left(\frac{I}{K}\right)$. Recall that δ is the capital depreciation rate; α is the capital's share in the Cobb-Douglas production function; ϕ measures the persistence of the domestic technology shocks and μ_t is the i.i.d. normally distributed shock in the domestic productivity's AR(1) process.

$$\zeta = \frac{-\Psi''\left(\frac{\bar{I}^d}{\bar{K}^d}\right)\frac{\bar{I}^d}{\bar{K}^d}}{\Psi'\left(\frac{\bar{I}^d}{\bar{K}^d}\right)} \tag{39}$$

Equation (34) is the log-linear approximation of Equation (24), the domestic goods market clearing condition, around its steady state, Equation (29). Equation (35) is the log-linear approximation of Equation (25), the domestic capital stock's law of motion, around its steady state, Equation (30). Notice that, the log-linear approximations of the capital's law of motion are identical whether there is adjustment cost or not. Equation (37) is derived from the domestic productivity stochastic process, Equation (13), which is linear in log and needs no approximation. Therefore Equation (37) holds exactly.

Equation (36) is the log-linear approximation of Equation (27), the domestic risky asset return, around its steady state, Equation (32). Without adjustment cost, the relative price of the capital goods, known as the marginal q, is always one. As a result the asset return comes only from the capital's marginal product. With adjustment cost, the relative price of the capital goods varies. The asset return is therefore composed of the capital's marginal product and the capital gain/loss from the relative price variation of the capital goods.

Equation (36) merits some discussion. The first two terms in Equation (36) are identical to the usual case without adjustment cost. Recall that, without adjustment cost

$$\Psi\left(\frac{I}{K}\right) = \frac{I}{K} \qquad \text{and} \qquad \Psi'\left(\frac{I}{K}\right) = 1$$

and Equation (27) becomes

$$R_{t+1}^{d}=\alpha A_{t+1}^{d}\left(K_{t+1}^{d}\right)^{\alpha-1}+\left(1-\delta\right) \quad (40)$$

Loglinearing Equation (40) around its steady state of Equation (32) yields

$$r_{t+1}^{d}=\left(\frac{\bar{\Re}^{f}-1+\delta}{\bar{\Re}^{f}}\right)a_{t+1}^{d}-\left(\frac{\bar{\Re}^{f}-1+\delta}{\bar{\Re}^{f}}\right)\left(1-\alpha\right)k_{t+1}^{d} \quad (41)$$

Equation (41) is the log-linear approximation of the asset return without adjustment cost, which is exactly the first two terms in Equation (36). Recall our argument that the asset return is composed of two parts; one is the marginal product of capital; another is the capital gain/loss from the marginal q variation. The first part exists in both cases with or without adjustment cost. Therefore Equation (41) reflects the effect of the marginal product of capital on the asset return.

The last two terms in Equation (36) reflect the asset return effect of the capital gain/loss from the marginal q variation. This channel is absent in the usual case without adjustment cost. Recall that $\frac{1}{\zeta}$ is the elasticity of the investment capital ratio with respect to the marginal q. Therefore $\zeta\left(i_{t}^{d}-k_{t}^{d}\right)$ is the logarithmic deviation of the marginal q at the time t from its steady state value, which is one. We denote the log deviation of the marginal q at time t as ς_{t}. Analogically $\zeta\left(i_{t+1}^{d}-k_{t+1}^{d}\right)$ is ς_{t+1}. A higher ς_{t}, *ceteris paribus*, results a capital loss and consequently a lower asset return. Therefore we see a negative sign before $\zeta\left(i_{t}^{d}-k_{t}^{d}\right)$ in Equation (36). Analogically, a higher ς_{t+1}, *ceteris paribus*, results a capital gain and a higher asset return. Therefore the sign before $\zeta\left(i_{t+1}^{d}-k_{t+1}^{d}\right)$ in Equation (36) is positive. Also note that item

$\zeta\left(i_{t+1}^{d}-k_{t+1}^{d}\right)$ is discounted by $\bar{\Re}^{f}$ while $\zeta\left(i_{t}^{d}-k_{t}^{d}\right)$ is not since the former is a variable measured at time $t+1$ and the latter is measured at time t. Given one time period lag, the comparison can be done only after the conversion, either the time $t+1$ variable being discounted by $\bar{\Re}^{f}$ or the time t variable being multiplied by $\bar{\Re}^{f}$.

Lettau (2003) decomposes the effect of the technology shocks on the asset prices into $i)$ the direct effect due to the shock itself and $ii)$ the indirect effect stemming from the capital accumulation. Recalling Equation (36), the first term on its right hand side, $\left(\frac{\bar{\Re}^{f}-1+\delta}{\bar{\Re}^{f}}\right)a_{t+1}^{d}$, reflects the direct effect from the technological shock itself. The second term in Equation (36), $-\left(\frac{\bar{\Re}^{f}-1+\delta}{\bar{\Re}^{f}}\right)(1-\alpha)k_{t+1}^{d}$, reflects the indirect effect from the capital accumulation. A positive technology shock has a positive direct effect and a negative indirect effect on the asset return. The latter is due to the law of diminishing marginal returns. The third and forth terms in Equation (36) also shows a third effect. It is absent in Lettau (2003). We call it the capital gain/loss effect. This effect only arises in the model where the relative price of the capital goods can vary, not always keep at one.

Without adjustment cost, the indirect effects of the capital accumulation could be strong enough to offset the positive direct effects. If this is the case, the model could generate a countercyclical asset return. As a result the equity becomes a hedge against the technology shock and therefore the equity premium is low or even turns to be negative. Lettau (2003) points out that the effect of the capital accumulation is the source of most of the puzzling asset pricing implications of the RBC models without the investment rigidity.

With adjustment cost, investment responds less dramatically to a technology shock. As a result, the asset return effect of the capital accumulation abates. If the positive direct effect dominates, the model could generate a procyclical asset return and a high equity premium.

3.7 The Method of Undetermined Coefficients

Equations (34), (35), (36) and (37) constitute a system of stochastic difference equations. Following Campbell (1994) the system can be solved by the method of undetermined coefficients. First we conjecture that the log of the control variables $C_t^d, NX_t^d, I_t^d, K_{t+1}^d, R_t^d$ is a form of the log of the state variables $A_t^d, K_t^d, \Re_t^f, \aleph_t^d$.

$$\begin{aligned}
c_t^d &= \omega_{ca} a_t^d + \omega_{ck} k_t^d + \omega_{cr} r_t^f + \omega_{cn} n_t^d \\
nx_t^d &= \omega_{xa} a_t^d + \omega_{xk} k_t^d + \omega_{xr} r_t^f + \omega_{xn} n_t^d \\
i_t^d &= \omega_{ia} a_t^d + \omega_{ik} k_t^d + \omega_{ir} r_t^f + \omega_{in} n_t^d \\
k_{t+1}^d &= \omega_{ka} a_t^d + \omega_{kk} k_t^d + \omega_{kr} r_t^f + \omega_{kn} n_t^d \\
r_t^d &= \omega_{ra} a_t^d + \omega_{rk} k_t^d + \omega_{rr} r_t^f + \omega_{rn} n_t^d
\end{aligned} \tag{42}$$

where ω_{xy} is the partial elasticity of x with respect to y. In Equation (42), x represents, respectively, the control variables $C_t^d, NX_t^d, I_t^d, K_{t+1}^d, R_t^d$ and y represents the state variables $A_t^d, K_t^d, \Re_t^f, \aleph_t^d$. ω_{xy} is an unknown parameter that is assumed to be constant. Then we verify the above conjecture by finding the value of ω_{xy} that satisfies the restrictions of the approximate log-linear model. Since $\bar{\aleph} = 0$, we define $n_t^d = \dfrac{\aleph_t^d - \bar{\aleph}}{\bar{C}} = \dfrac{\aleph_t^d}{\bar{C}}$. Combined with the domestic goods market clearing condition (23), we get:

$$\begin{aligned}&\omega_{cn}=1,\omega_{xn}=-\frac{\bar{C}}{\overline{NX}}\\&\omega_{in}=0,\omega_{kn}=0,\omega_{rn}=0\end{aligned}\tag{43}$$

3.8 The Small Economy Firm's ex post Asset Return

In this subsection, we derive the domestic firm's ex post asset return as a function of the elasticities of the real variables given by the solution of the above RBC model.

Plugging the last equation in Equation (42) into Equation (36) yields

$$\begin{aligned}r_{t+1}^{d}&=\omega_{ra}a_{t+1}^{d}+\omega_{rk}k_{t+1}^{d}+\omega_{rr}r_{t+1}^{f}\\&=\omega_{ra}a_{t+1}^{d}+\omega_{rr}r_{t+1}^{f}+\omega_{rk}\omega_{ka}a_{t}^{d}+\omega_{rk}\omega_{kk}k_{t}^{d}+\omega_{rk}\omega_{kr}r_{t}^{f}\\&=Xa_{t+1}^{d}+Tr_{t+1}^{f}+Sa_{t}^{d}+Vk_{t}^{d}+Hr_{t}^{f}\end{aligned}\tag{44}$$

where

$$\begin{aligned}X&=\frac{\bar{\Re}^{f}-1+\delta}{\bar{\Re}^{f}}+\frac{\zeta\omega_{ia}}{\bar{\Re}^{f}}\\T&=\frac{\zeta\omega_{ir}}{\bar{\Re}^{f}}\\S&=\frac{-(1-\alpha)\left(\bar{\Re}^{f}-1+\delta\right)\omega_{ka}-\zeta\omega_{ka}+\zeta\omega_{ik}\omega_{ka}}{\bar{\Re}^{f}}-\zeta\omega_{ia}\\V&=\frac{-(1-\alpha)\left(\bar{\Re}^{f}-1+\delta\right)\omega_{kk}+\zeta\omega_{ik}\omega_{kk}-\zeta\omega_{kk}}{\bar{\Re}^{f}}+\zeta-\zeta\omega_{ik}\\H&=\frac{-(1-\alpha)\left(\bar{\Re}^{f}-1+\delta\right)\omega_{kr}+\zeta\omega_{ik}\omega_{kr}-\zeta\omega_{kr}}{\bar{\Re}^{f}}-\zeta\omega_{ir}\end{aligned}\tag{45}$$

Considering a case in which the system is originally at a steady state and then an unexpected technology shock μ_{t+1} occurring at period $t+1$, according to Equation (44), r_{t+1}^{d} thus becomes $X\mu_{t+1} + Tr_{t+1}^{f}$, that is

$$\begin{aligned} r_{t+1}^{d} &= X\mu_{t+1} + Tr_{t+1}^{f} \\ &= \left(\frac{\bar{\Re}^{f} - 1 + \delta}{\bar{\Re}^{f}} + \frac{\zeta\omega_{ia}}{\bar{\Re}^{f}} \right)\mu_{t+1} + \frac{\zeta\omega_{ir}}{\bar{\Re}^{f}} r_{t+1}^{f} \end{aligned} \tag{46}$$

The first term in the parentheses measures the direct effect of the technology shock on the asset return. A positive technology shock leads to a higher interest rate by increasing the capital's marginal product. The second term in the parentheses measures the effect of the capital gain/loss due to the variation in the relative price of the capital goods. A positive technology shock causes a higher investment $\left(\omega_{ia}\mu_{t+1}\right)$ at period $t+1$. Recall $\frac{1}{\zeta}$ is the elasticity of the investment capital ratio with respect to the relative price of capital goods. A higher investment, by leading to a higher investment/capital ratio, causes capital goods price ς_{t+1} to go up by $\left(\zeta\omega_{ia}\mu_{t+1}\right)$ at the period $t+1$. As a result there is a capital gain that results in a high asset return r_{t+1}^{d}.

If the technology shock μ_{t} occurs at the period t and before that the system is at a steady state, its asset return effect becomes complicated due to the presence of a capital accumulation channel. r_{t+1}^{d} hence becomes

$$
\begin{aligned}
r_{t+1}^{d} &= X\left(\phi\mu_t + \mu_{t+1}\right) + S\mu_t + Tr_{t+1}^{f} + Hr_t^{f} \\
&= \left[\frac{\bar{\Re}^{f} - 1 + \delta}{\bar{\Re}^{f}} + \frac{\zeta\omega_{ia}}{\bar{\Re}^{f}}\right]\left(\phi\mu_t + \mu_{t+1}\right) \\
&+ \left[\frac{-(1-\alpha)\left(\bar{\Re}^{f} - 1 + \delta\right)\omega_{ka}}{\bar{\Re}^{f}} + \frac{-\zeta\omega_{ka}}{\bar{\Re}^{f}} + \frac{\zeta\omega_{ik}\omega_{ka}}{\bar{\Re}^{f}} - \zeta\omega_{ia}\right]\mu_t \\
&+ \frac{\zeta\omega_{ir}}{\bar{\Re}^{f}} r_{t+1}^{f} + \left[\frac{-(1-\alpha)\left(\bar{\Re}^{f} - 1 + \delta\right)\omega_{kr} + \zeta\omega_{ik}\omega_{kr} - \zeta\omega_{kr}}{\bar{\Re}^{f}} - \zeta\omega_{ir}\right] r_t^{f}
\end{aligned}
\tag{47}
$$

Recall ϕ measures the persistence of the domestic technology shock. $E_t\left(a_{t+1}^{d}\right) = \phi\mu_t$. A positive shock today will cause a positive shock tomorrow with a decayed magnitude. The first bracketed term in Equation (47) is the effect of a_{t+1}^{d} on r_{t+1}^{d}. a_{t+1}^{d} is composed of two parts: one is from the persistence of period t shock μ_t and another is an i.i.d. shock μ_{t+1} at the period $t+1$. The mechanism behind how a_{t+1}^{d} affects r_{t+1}^{d} was just analyzed in the last paragraph. Therefore, we see the first bracketed term in Equation (47) is exactly the same as that in Equation (46).

The term in the second bracket of Equation (47) merits some discussion. The first item, $\frac{-(1-\alpha)\left(\bar{\Re}^{f} - 1 + \delta\right)\omega_{ka}}{\bar{\Re}^{f}}\mu_t$, measures the asset return effect of the capital accumulation. A positive shock μ_t causes the capital accumulation, that is, an increase in k_{t+1}^{d}. But a larger capital stock drives down the capital's marginal product due to the diminishing marginal returns. That is the reason why we see a negative sign for the first item. This effect of capital accumulation exists in every PCAPM, with or without adjustment cost.

Notice that all elasticities in Equation (47) are partial elasticities. Therefore, their effects on the asset return are *ceteris paribus* effects. The second item in the second bracket, $\frac{-\zeta\omega_{ka}}{\bar{\Re}^f}\mu_t$, measures the asset return effect of the capital gain/loss. A positive shock μ_t leads to a larger capital stock k_{t+1}^d, which in turn causes a lower investment/capital ratio. A lower ratio, given a positive $\frac{1}{\zeta}$, results in a lower relative price of the capital goods ς_{t+1}, which is equivalent to a capital loss. A capital loss is a negative contribution to the asset return. Therefore, the sign of the second item is negative.

The third item, $\frac{\zeta\omega_{ik}\omega_{ka}}{\bar{\Re}^f}\mu_t$, is also from the effect of the capital gain/loss. Recall that a positive shock μ_t causes a larger capital stock k_{t+1}^d, which then affects i_{t+1}^d by ω_{ik}. If ω_{ik} is positive, investment rises in response to a higher capital stock. An increase in investment causes a higher investment/capital ratio, which results in a higher capital goods relative price ς_{t+1}. Again, a higher ς_{t+1} is a capital gain and it contributes positively to the asset return. Therefore, the sign of the third item is positive.

The last item, $-\zeta\omega_{ia}\mu_t$, is again from the effect of the capital gain/loss. This time the asset return effect comes from ς_t, instead of ς_{t+1}. A positive shock μ_t causes the investment i_t^d to rise, which in turn leads to a higher investment/capital ratio $\frac{i_t^d}{k_t^d}$. A higher ratio results in a higher relative price ς_t of the capital goods. A higher ς_t is a capital loss to r_{t+1}^d. Therefore, it causes r_{t+1}^d to fall. This is the reason behind a negative sign of the last item in the second bracket of Equation (47).

All the above effects of the capital gain/loss arise only in the PCAPM with investment friction, such as a model with adjustment cost so that the relative price of the capital goods could vary from one.

Our task in this section has been accomplished. A glance back at Equation (46) and Equation (47) shows that the approximate closed-form solution for the log of the ex post asset return is a function of the exogenous technology process and the elasticities of the control variables with respect to the model's state variables.

4 The General Equilibrium World Economy Model and the SDF of the Heterogeneous International Investors

A glance back at the basic asset pricing formula of Equation (2) shows that we are halfway home. We rewrite that formula here

$$E_t\left(\beta\frac{\Lambda_{t+1}}{\Lambda_t}R^j_{t+1}\right)=1$$

In section 3, we model a small open economy and derive the approximate closed-form solution for its ex-post asset return R^d_{t+1}, which is the first common ingredient in the basic asset pricing formulas. To obtain the model's asset pricing implication, we need to do the similar work on the investors' stochastic discount factor, $\beta\frac{\Lambda_{t+1}}{\Lambda_t}$, the second common ingredient in the basic asset pricing formulas. Since the domestic asset is also owned by foreign investors, their stochastic discount factors are taken as given by the small open economy we examined in section 3. For the investors' SDF to be endogenously examined, we need go to a general equilibrium world economy environment. Our goal in this section is to model the heterogeneous international investors in a world economy model. Moreover we write the market SDF as a function of the general equilibrium world economy model's state variables.

Why bother to model the international investors as heterogeneous rather than homogenous agents and the international asset market as an incomplete market rather than a complete one? The open economy model with a complete market, that is, a market with the existence of Arrow-Debreu security in each state of nature, generates a series of counterfactuals against the stylized facts in the international business cycle. The complete market model predicts a perfect correlation of the consumption across countries. In reality, the international correlation of consumption is low, even lower than the output correlation across countries.[25] Moreover, the complete market model predicts that agents across countries will share risk perfectly and hold exactly the same global portfolio. In reality, we see the home bias puzzle: residents hold a very large share of their equity wealth at home.[26] Therefore, in our model, we abandon the assumption of the complete asset market and the homogenous investors. Instead we assume that the global asset market is an incomplete market in a sense that there are uninsurable persistent country-specific consumption distribution risks and the accompanying real exchange rate risks across countries. Since not all risks can be diversified away in the global asset market, the incomplete international asset market leaves us with the heterogeneous international investors.

Our assumption of the heterogeneous international investors and the incomplete international asset market can be justified by the following reasons. There exists uninsurable labor income

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[25] Backus, Kehoe and Kydland (1993) show that the consumption correlation between the US and the European aggregate is 0.51 while the output correlation of 0.66.

[26] See, for example, French and Poterba (1991); Tesar and Werner (1995).

even within a country.[27] Furthermore, the international capital market is more problematic when treated as a complete market than a domestic one is. Markets seem to be better integrated within than among countries.[28] Fiscal federalism is one major reason to expect the higher consumption correlations within than across countries.[29] There are other reasons to justify an incomplete global capital market, from difficulty in the international contract enforcement to legal, information and regulation barriers across countries (Obstfeld and Rogoff, 1996). In addition, the existence of the nontradable goods in each country leads to the lower international consumption correlations.[30] Exchange rate risk cannot be completely hedged off by both parties due to the Siegel's paradox (Siegel, 1972).[31]

To keep our model simple but flexible, we assume that the world is composed of N countries. Each country has a representative investor involved in the international capital market. N countries are identical except that there are exogenous persistent consumption distribution shocks across countries. The international capital market is incomplete in a sense that these shocks are uninsurable. Moreover, since the shocks are persistent, they cannot be smoothed away by investing in riskless asset. These shocks cause the investors from the different countries to lead the different consumption paths that in turn result in the different intertemporal marginal rates of substitution (also known as SDF) across the international investors. It is in this sense that we called them the heterogeneous international investors. Constantinides and Duffie (1996) prove that, in an incomplete market, even though SDF is not unique, there does exist a market common SDF for every heterogeneous investor.

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[27] Constantinides and Duffie (1996) point out this in the justification of their heterogeneous assumption on investors within a country.

[28] See the evidence from Crucini (1992); Atkeson and Bayoumi (1993); Bayoumi and Klein (1995).

[29] See the evidence from Sachs and Sala-I-Martin (1992).

[30] See Stockman and Dellas (1989); Stockman and Tesar (1995) for more details.

[31] Siegel's paradox is another application of Jensen's inequality. See more detail in Obstfeld and Rogoff (1996).

In our model, the only good is a single consumption-investment good. Given the assumption that shocks are persistent and uninsurable, the law of one price therefore does not hold in our model. One can understand the price (the real exchange rate) effect of the consumption distribution shocks in the following way: Since the shock in our model cannot be hedged away, it is quite similar to a shock on the nontradable goods in a multi-goods setting. A country that experiences a positive consumption distribution shock will see its goods price plunge and go through the real exchange rate depreciation. In contrast, the country having a negative shock will see its goods price hike and go through the real exchange rate appreciation. Differential in the goods price and therefore in the real exchange rate cause the real return from investing in the same foreign asset not necessarily to be identical across countries.

In our model, heterogeneity across the international investors is not explicitly derived from the business cycle of each country, but from our ad-hoc assumption. The consumption distribution shocks are exogenously assumed, rather than derived from the model. We do not set up a model with each country having a tradable and a nontradable sector even though we realize that shock on the nontradable sector might be a good candidate to be persistent and uninsurable. Should we adopt the multi-goods setting, we would have to deal with the price index, the consumption index, and the consumption-based real interest rate. And the model tends to be quite complex. With such a thoroughly theoretical setting, the benefit is that the consumption shocks and the accompanying price (the real exchange rate) risks are both endogenously determined, rather than from the exogenously imposed arbitrary assumptions.

To keep the model simple and within our capability to handle, we simply assume that there are uninsurable persistence country-specific consumption distribution risks across countries, making the international investors be the heterogeneous agents and the international asset market be an incomplete market. We feel the difference between our model's setting and the thorough approach is similar to a difference between the endowment economy setting and the production economy environment. In an exchange economy model, one does not explicitly model the production process but simply makes ad-hoc assumption that the model economy exogenously has its output (endowment) in that way.

4.1 The Preference in a General Equilibrium World Economy Model and the SDF of the Heterogeneous International Investors

In the world economy, there are N countries. Each country has a representative agent. Initially these N countries are identical. There is a single consumption good in the world economy. Consumers from different countries have a homogeneous preference represented by the following utility function:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} U\left(C_s^i\right) = E_t \sum_{s=t}^{\infty} \beta^{s-t} \frac{\left(C_s^i\right)^{1-\rho}}{1-\rho} \tag{48}$$

where C_s^i is the consumption by the consumer from country i at the time s, ρ is the Arrow-Pratt coefficient of relative risk aversion.

The consumer gets his income from 1) the labor income by offering a fixed amount of his labor endowment to the firm; 2) the financial income by investing in shares of the risky assets and in bonds of the riskless asset in the global financial market. Given the constant return of scale of the production function, we can treat the firms in each country as a representative firm. In the world asset market, there are N securities which are issued by the firm from each N country. We normalize the number of each firm's share to be one. Also there is a world riskless asset with a zero net supply. We assume investors from N countries are heterogeneous since there are uninsurable, persistent, idiosyncratic consumption distribution shocks across countries. It is the fact that the world asset market cannot hedge these risks away makes it an incomplete market. With the above assumptions, the consumers' budget constraints become:

$$\begin{aligned} C_t^i &= LI_t^i + FI_t^i + \aleph_t^i \\ &= W_t L_t^i + \sum_{j=1}^{N}\left[\theta_{jt}^i\left(\Omega_{jt} + D_{j,t-1}\right)\right] - \sum_{j=1}^{N}\left(\theta_{j,t+1}^i \Omega_{jt}\right) + B_t^i \Re_t^f - B_{t+1}^i + \aleph_t^i \end{aligned} \tag{49}$$

where i is the consumer index and j is the asset index; C_t^i is the consumption by the consumer i during the period t; $\aleph_t^i$ denotes the idiosyncratic consumption distribution shock on the consumer i at the time t. We will describe this shock in detail below; LI_t^i denotes his labor income which is the product of the wage rate at time t, W_t, and his labor supply during the period t, L_t^i; FI_t^i denotes his financial income which is composed of the asset returns from his holding of N risky assets and a riskless asset; θ_{jt}^i is the consumer i's holding of shares of the risky asset j at the beginning of the period t; analogously, $\theta_{j,t+1}^i$ is his shares at the end of the period t, or at the beginning of the period $t+1$; $D_{j,t-1}$ is the dividend from the risky asset j during the period $t-1$, which is available for consumption from the beginning of the period t; Ω_{jt} is the price of the risky asset j at the beginning of the period t; B_t^i is the consumer i's holding of the riskless asset at the beginning of the period t; analogously, B_{t+1}^i is his holding of the riskless asset at the end of the period t, or at the beginning of the period $t+1$, $\Re_t^f$ is the riskless interest rate between the time t and the time $t+1$ at the global asset market.

The consumer's optimization problem is to maximize his utility of Equation (48) subject to the budget constraint of Equation (49). Substituting Equation (49) into Equation (48), we get the maximand:

$$\max_{B^i_{t+1},\theta^i_{j,t+1}(j=1,2\ldots N)} E_t\sum_{s=t}^{\infty}\beta^{s-t}\frac{\left(W_tL^i_t+\sum_{j=1}^{N}\left[\theta^i_{jt}\left(\Omega_{jt}+D_{j,t-1}\right)\right]-\sum_{j=1}^{N}\left(\theta^i_{j,t+1}\Omega_{jt}\right)+B^i_t\Re^f_t-B^i_{t+1}+\aleph^i_t\right)^{1-\rho}}{1-\rho} \tag{50}$$

The first order conditions for a maximum are the following $(N+1)$ equations, with the first one for riskless asset and the rest N equations for risky assets:

$$\frac{1}{\Re^f_{t+1}}=E_t\left[\beta\left(\frac{C^i_t}{C^i_{t+1}}\right)^{\rho}\right] \tag{51}$$

$$E_t\left[\beta\left(\frac{C^i_t}{C^i_{t+1}}\right)^{\rho}\left(\frac{\Omega_{j,t+1}+D_{jt}}{\Omega_{jt}}\right)\right]=E_t\left[\beta\left(\frac{C^i_t}{C^i_{t+1}}\right)^{\rho}R^j_{t+1}\right]=1 \qquad j=1,2\ldots N \tag{52}$$

Equation (51) and Equation (52) are asset pricing formulas. They are from each consumer's Euler equation. It shows that each consumer's marginal rate substitution is still valid to be his own stochastic discount factor.

Similar to Constantinides and Duffie (1996), the consumption distribution shock takes the form

$$\aleph_t^i = \tau_t^i C_t - \frac{1}{N}\left(\sum_{j=1}^{N} D_{jt}\right) - \frac{1}{N}\left(\sum_{j=1}^{N} W_t L_t^i\right) \tag{53}$$

$$\text{where}\begin{cases} \tau_t^i = \tau_{t-1}^i \exp\left[\upsilon_{it}^c \Upsilon_t^c - \frac{1}{2}\left(\Upsilon_t^c\right)^2\right] \\ \upsilon_{it}^c \sim i.i.d.N\left(0;1\right) \end{cases} \tag{54}$$

where C_t is the world aggregate consumption at the time t, υ_{it}^c is a random variable across countries. It is identical, independent and follows a standard normal distribution. Following Constantinides and Duffie (1996), $\left(\Upsilon_t^c\right)^2$ is interpreted as the variance of the cross-sectional distribution of $\log\left[\frac{\left(C_{t+1}^i / C_{t+1}\right)}{\left(C_t^i / C_t\right)}\right]$. See Appendix A.3 for a proof. $\aleph_t^i$ and τ_t^i are set up in their ways to make sure that these idiosyncratic shocks leave the world aggregate consumption intact:

$$E^{\oplus}\left(\tau_t^i\right) = 1 \tag{55}$$

$$E^{\oplus}\left(C_t^i\right) = E^{\oplus}\left(\tau_t^i C_t\right) = C_t \tag{56}$$

$$E^{\oplus}\left(\aleph_t^i\right) = 0 \tag{57}$$

where $E^{\oplus}$ is an expectation taking over the cross-sectional distribution. See Appendix A.4 for a proof.

The uninsurable, persistent and idiosyncratic consumption shocks across countries prevent the law of one price from holding. The consumption shock in each country leads to a fluctuation of its goods price level. The price fluctuation is a by-product of the exogenous consumption shock. Therefore, we assume the price fluctuation follows the same distribution as the consumption shock does.

$$P_t^i = \pi_t^i P_t \tag{58}$$

$$\text{where } \begin{cases} \pi_t^i = \pi_{t-1}^i \exp\left[\upsilon_{it}^p \Upsilon_t^p - \frac{1}{2}\left(\Upsilon_t^p\right)^2\right] \\ \upsilon_{it}^p \sim i.i.d.N\left(0;1\right) \end{cases} \tag{59}$$

where P_t is the world average price level at the time t, υ_{it}^p is a random variable across countries. It is identical, independent and follows a standard normal distribution. π_t^i is set up in its way to make sure that these idiosyncratic shocks leave the world average price level intact, that is, $E^{\oplus}\left(P_t^i\right) = P_t$.

Our task in this section is to find a market SDF which is valid for every heterogeneous international investor, while at the same time each investor's own intertemporal marginal rate of substitution is still valid to be his own SDF.

From each consumer's Euler equation, Equation (52), we get:

$$E_t\left[\beta\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\rho}\frac{P_t^i}{P_{t+1}^i}R_t^j\right] = 1 \tag{60}$$

The first term in Equation (60) is consumer i's intertemporal marginal rate of substitution. The remaining part is the consumer i's real return from holding asset j after applying Fisher parity.[32]

We need to find a market SDF, $\frac{\beta\Lambda_{t+1}}{\Lambda_t}$, which is valid to be a SDF for every heterogeneous investor. Since Equation (60) holds for each investor, its cross-sectional average holds true as well. We take an expectation of Equation (60) over the cross-sectional distribution:

$$E_t\left\{E^{\oplus}\left[\beta\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\rho}\frac{P_t^i}{P_{t+1}^i}\right]R_{t+1}^j\right\}=1 \tag{61}$$

Substituting Equation (56), Equation (58) and Equation (59) into Equation (61) and applying again the formula of the mean of the lognormal distribution,[33] we get:

$$\begin{aligned}&E_t\left\{\beta\exp\left[-\rho\ln\left(\frac{C_{t+1}}{C_t}\right)-\ln\left(\frac{P_{t+1}}{P_t}\right)+\frac{\rho(\rho+1)}{2}\left(\Upsilon_t^c\right)^2+\left(\Upsilon_t^p\right)^2+\rho\sigma_{cp}\right]R_{t+1}^j\right\}\\&=E_t\left\{\beta\frac{\Lambda_{t+1}}{\Lambda_t}R_{t+1}^j\right\}=1\end{aligned} \tag{62}$$

From Eequation (62) we get the market SDF among the heterogeneous international investors:

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[32] Fisher parity equation is $1+i_{t+1}=\left(1+r_{t+1}\right)\frac{P_{t+1}}{P_t}$, where i_{t+1} is the nominal interest rate between period t and period $t+1$, r_{t+1} is the real interest rate between period t and period $t+1$, P_t, P_{t+1} is, respectively, the price level at period t and at period $t+1$ (Obstfeld and Rogoff, 1996).

[33] See Appendix A.3 for details about the mean of lognormal distribution.

$$\beta\frac{\Lambda_{t+1}}{\Lambda_t}=\beta\exp\left[-\rho\ln\left(\frac{C_{t+1}}{C_t}\right)-\ln\left(\frac{P_{t+1}}{P_t}\right)+\frac{\rho(\rho+1)}{2}\left(\Upsilon_t^c\right)^2+\left(\Upsilon_t^p\right)^2+\rho\sigma_{cp}\right]$$

(63)

Applying Equation (63) to Equation (4), we get the world riskless interest rate:

$$\Re_{t+1}^f=\frac{1}{E_t\left(\beta\frac{\Lambda_{t+1}}{\Lambda_t}\right)}$$
$$=\frac{1}{E_t\left\{\beta\exp\left[-\rho\ln\left(\frac{C_{t+1}}{C_t}\right)-\ln\left(\frac{P_{t+1}}{P_t}\right)+\frac{\rho(\rho+1)}{2}\left(\Upsilon_t^c\right)^2+\left(\Upsilon_t^p\right)^2+\rho\sigma_{cp}\right]\right\}}$$

(64)

Applying Equation (63) to Equation (5), we get the risky premium for any risky asset:

$$E_t\left(\beta\frac{\Lambda_{t+1}}{\Lambda_t}R_{j,t+1}^{rp}\right)$$
$$=E_t\left\{\beta\exp\left[-\rho\ln\left(\frac{C_{t+1}}{C_t}\right)-\ln\left(\frac{P_{t+1}}{P_t}\right)+\frac{\rho(\rho+1)}{2}\left(\Upsilon_t^c\right)^2+\left(\Upsilon_t^p\right)^2+\rho\sigma_{cp}\right]R_{j,t+1}^{rp}\right\}$$
$$=0$$

(65)

Log of the market SDF is:

$$\ln\left(\beta\frac{\Lambda_{t+1}}{\Lambda_t}\right)=\ln\beta-\rho\ln\left(\frac{C_{t+1}}{C_t}\right)-\ln\left(\frac{P_{t+1}}{P_t}\right)+\frac{\rho(\rho+1)}{2}\left(\Upsilon_t^c\right)^2+\left(\Upsilon_t^p\right)^2+\rho\sigma_{cp}$$

(66)

Recall the log of SDF in a complete market with the homogenous agents is:

$$\ln\left(\beta\frac{\Lambda_{t+1}}{\Lambda_t}\right)=\ln\beta-\rho\ln\left(\frac{C_{t+1}}{C_t}\right) \tag{67}$$

Log of the market SDF in the Constantinides and Duffie (1996) model is:

$$\ln\left(\beta\frac{\Lambda_{t+1}}{\Lambda_t}\right)=\ln\beta-\rho\ln\left(\frac{C_{t+1}}{C_t}\right)+\frac{\rho\left(\rho+1\right)}{2}\left(\Upsilon_t^c\right)^2 \tag{68}$$

Equation (67) shows that the log SDF in a representative-agent model depends on the investors' time preference and the aggregate consumption growth. Equation (68) describes the log SDF in the Constantinides and Duffie's (1996) model, which depends on the investors' time preference, the aggregate consumption growth and the cross-sectional variance of the individual consumption growths. Equation (66) delineates the log SDF in our model. The terms belonging to the Constantinides and Duffie (1996)-type enter into Equation (66) as well. Moreover, Equation (66) also depends on the world average price level, the cross-sectional variance of the individual countries' price growths and the cross-sectional covariance between the individual countries' consumption growths and the price growths.

If the cross-sectional distribution is heteroskedasticity and further we assume that the cross-sectional variances and covariance terms are negatively correlated with the level of the world aggregate consumption, the market SDF in our model shown in Equation (66) will be more strongly countercyclical than the SDF in the homogenous agent case. A countercyclical SDF turns out to have significant asset pricing implications.

To summarize, Constantinides and Duffie (1996) study the heterogeneous agents in a closed economy. We study the heterogeneous international investors from N countries. Price and real exchange rate fluctuations are new features in our model due to our international setting. Country-specific consumption shock causes the country's real exchange rate to deviate from one and to fluctuate. Therefore, price terms enter into the market SDF equation in our model.

4.2 The Production Side of the General Equilibrium World Economy Model

Recall Equation (56) that the idiosyncratic consumption distribution shocks leave the world aggregate consumption intact.[34] What is that aggregate level? To answer this question, we need to study the production side of the general equilibrium world economy.

The production side is examined in this sub-section in a standard stochastic neoclassical growth model. Since we assume the production function is constant return of scale, the world aggregate output can be treated as produced by a representative global firm operating in a competitive environment. We assume a Cobb-Douglas production function

$$Y_t = Z_t \left(K_t\right)^{\alpha} \left(L_t^f\right)^{1-\alpha} \tag{69}$$

where Y_t denotes the world aggregate output at the time t ; L_t^f denotes global firm's labor demand; K_t denotes the global firm's capital stock at the beginning of the period t , α is the capital's share and $1-\alpha$ is the labor's share. The capital stock is chosen one period before it becomes productive and labor can be adjusted instantaneously. Z_t merits an explanation. Here

[34] See Appendix A.4 for a proof.

Z_t represents a level of the global total factor productivity such that the world aggregate output is equal to the sum of the individual country's output over N countries. See Appendix A.5 for more detail.

$$Y_t = Z_t \left(K_t\right)^{\alpha} \left(L_t^f\right)^{1-\alpha} = \sum_j^N A_t^j \left(K_t^j\right)^{\alpha} \left(L_t^{fj}\right)^{1-\alpha} \tag{70}$$

where $K_t = \sum_j^N K_t^j, L_t^f = \sum_j^N L_t^{fj}$

Moreover we assume the global technology Z_t is a random variable evolving following an $AR(1)$ process in log with i.i.d. normally distributed homoscedastic shock:

$$\begin{aligned} \log Z_t &= (1-\varphi)\log \overline{Z} + \varphi \log Z_{t-1} + \varepsilon_t \\ \varepsilon_t &\sim i.i.d.N\left(0;\sigma_\varepsilon^2\right) \end{aligned} \tag{71}$$

here φ measures the persistence of the global technological shock and we assume $0 < \varphi < 1$. $\overline{Z}$ is the steady state level of the global productivity.

The law of motion of the global capital stock is

$$K_{t+1} = (1-\delta)K_t + I_t \tag{72}$$

where I_t is the global firm's investment made during the period t. In this section, the central issue is the investor's SDF, not the firm's ex post asset return. Therefore, to keep the model simple, we do not impose adjustment cost on the firm's investment.

The dividends to shareholders are equal to the global output net of investment and wage payment to the workers:

$$\begin{aligned} D_t &= Z_t \left(K_t\right)^{\alpha} \left(L_t\right)^{1-\alpha} - W_t L_t - I_t \\ &= \alpha Z_t \left(K_t\right)^{\alpha} \left(L_t\right)^{1-\alpha} - I_t \end{aligned} \tag{73}$$

The global firm maximizes its value to shareholders subject to the production function, the law of motion of the capital stock and the stochastic process of the global technology. That is, the global firm's optimization problem is:

$$\begin{aligned} &\max E_t \sum_{s=t}^{\infty} \left\{ \frac{\beta^{s-t} \Lambda_s}{\Lambda_t} \left[Y_s - W_s L_s - I_s \right] \right\} \\ &s.t. \begin{cases} Y_s = Z_s \left(K_s\right)^{\alpha} \left(L_s\right)^{1-\alpha} \\ K_{s+1} = I_s + \left(1-\delta\right) K_s \end{cases} \end{aligned} \tag{74}$$

Substituting the production function into the global firm's objective function and setting up the Lagrangian, we get:

$$\mathfrak{A}_t = E_t \sum_{s=t}^{\infty} \left\{ \frac{\beta^{s-t} \Lambda_s}{\Lambda_t} \left\{ \begin{aligned} &\left[Z_s \left(K_s\right)^{\alpha} \left(L_s^f\right)^{1-\alpha} - W_s L_s^f - I_s \right] \\ &- \mathbb{Q}_s \left[K_{s+1} - I_s - \left(1-\delta\right) K_s \right] \end{aligned} \right\} \right\} \tag{75}$$

$$I_s, L_s^f, K_{s+1}, \mathbb{Q}_s$$

The first order conditions for a maximum are:

$$\frac{\partial \mathfrak{A}_t}{\partial I_s} = 0 \rightarrow \mathbb{Q}_s = 1 \tag{76}$$

$$\frac{\partial \mathfrak{A}_t}{\partial L_s^f} = 0 \rightarrow W_s = \left(1-\alpha\right) Z_s \left(K_s\right)^{\alpha} \left(L_s^f\right)^{-\alpha} \tag{77}$$

$$\frac{\partial \mathfrak{A}_t}{\partial \mathbb{Q}_s} = 0 \rightarrow K_{s+1} = I_s + \left(1-\delta\right) K_s \tag{78}$$

$$\frac{\partial \mathfrak{A}_t}{\partial K_{s+1}} = 0 \rightarrow$$

$$E_t \left\{ \left(\frac{\beta^{s+1-t} \Lambda_{s+1}}{\Lambda_t} \right) \left[\alpha Z_{s+1} \left(K_{s+1}\right)^{\alpha-1} \left(L_{s+1}^f\right)^{1-\alpha} + \mathbb{Q}_{t+1} \left(1-\delta\right) \right] \right\} = \mathbb{Q}_t \tag{79}$$

Comparing Equation (76) with Equation (18), it shows that, without adjustment cost, which is the case in this section, the marginal q is always equal to one. This result is reasonable. Without any friction in the investment process, the consumption goods and the capital goods are perfect substitutes and their relative price keeps to be one always.

From Equation (76) and Equation (79) we get:

$$R_{t+1} = \left[\alpha Z_{t+1} \left(K_{t+1}\right)^{\alpha-1} \left(L_{t+1}^f\right)^{1-\alpha} + 1 - \delta \right] \tag{80}$$

$$E_t \left[\left(\beta \frac{\Lambda_{t+1}}{\Lambda_t} \right) R_{t+1} \right] = 1$$

Comparing Equation (80) with Equation (22), we see that, without the adjustment cost, the investment return comes only from the marginal product of capital. The capital gain/loss channel arising from the variation of the marginal q is shut down.

4.3 The Market Clearing Conditions

Recall Equation (57): $E^{\oplus}\left(\aleph_t^i\right)=0$. The idiosyncratic country-specific consumption distribution shocks leave the following relationship hold true as before: the world aggregate consumption equals to the sum of each country's consumption level over N countries, therefore the global goods market clearing condition is:

$$Y_t = C_t + I_t \tag{81}$$

where Y_t is the world aggregate output at the period t, C_t is the world aggregate consumption at the period t; I_t is the investment the global firm made during the period t.

In addition, there is equilibrium in the world labor market, which means labor supply equals to labor demand and both equal to a fixed global labor endowment.

$$L_t^f = \sum_j^N \left(L_t^j\right) = L \tag{82}$$

And there is equilibrium in the financial market, which requires that the international investors hold all outstanding equity shares issued by the global firm. We normalize the equity share to be one. The risk-free bond in the global capital market is in zero net supply. We can drop one of these market clearing conditions by Walras' law.

$$\sum_{i=1}^{N} \theta_{t+1}^{i} = 1$$
$$\sum_{i=1}^{N} B_{t+1}^{i} = 0 \tag{83}$$

4.4 Log-linear Approximation of the First Order Conditions around the Steady State

Following Campbell (1994), we derive analytical solutions for the elasticities of the control variables with respect to the state variables. In the world economy model, the control variables are the world aggregate consumption C_t, the world aggregate investment I_t, the world riskless interest rate $\Re_t^f$, the global firm's risky return R_t and the end of period world capital stock K_{t+1}. The model's state variables are the global technological shock Z_t, the beginning of period world aggregate capital stock K_t.

Loglinearly approximating the first order conditions of Equation (81), Equation (72), Equation (80) and the productivity evolution process of Equation (71) yield respectively:

$$\bar{C}c_t + \delta\bar{K}i_t = \bar{Z}\left(\bar{K}\right)^{\alpha}\left(L\right)^{1-\alpha} z_t + \alpha\bar{Z}\left(\bar{K}\right)^{\alpha}\left(L\right)^{1-\alpha} k_t \tag{84}$$

$$k_{t+1} = \delta i_t + \left(1-\delta\right)k_t \tag{85}$$

$$r_{t+1} = \left(\frac{\bar{\Re}^f - 1 + \delta}{\bar{\Re}^f}\right) z_{t+1} - \left(\frac{\bar{\Re}^f - 1 + \delta}{\bar{\Re}^f}\right)\left(1-\alpha\right)k_{t+1} \tag{86}$$

$$z_t = \varphi z_{t-1} + \varepsilon_t \tag{87}$$

4.5 The Method of Undetermined Coefficients

Equations (84), (85), (86) and (87) constitute a system of stochastic difference equations. Following Campbell (1994) it can be solved by the method of undetermined coefficients. First we conjecture that the log of the control variable C_t is a form of the log of the state variables Z_t, K_t.

$$c_t = \eta_{ck} k_t + \eta_{cz} z_t \tag{88}$$

where η_{ck} is the elasticity of the world aggregate consumption C_t with respect to the beginning of period global capital stock K_t, η_{cz} is the elasticity of C_t with respect to the global technology shock Z_t. These elasticities can be expressed in the model's deep parameters.

Analogously

$$\begin{aligned} c_{t+1} &= \eta_{ck} k_{t+1} + \eta_{cz} z_{t+1} \\ &= \eta_{ck} \eta_{kk} k_t + \eta_{ck} \eta_{kz} z_t + \eta_{cz} z_{t+1} \end{aligned} \tag{89}$$

Suppose a global technology shock occurs at the time $t+1$ and before that the economy is in a steady state, we have

$$c_{t+1} - c_t = \eta_{cz} \varepsilon_{t+1} \tag{90}$$

If the technology shock occurs at the time t, we get

$$c_{t+1} - c_t = \eta_{ck} \eta_{kz} \varepsilon_t - \eta_{cz} \left(1 - \varphi\right) \varepsilon_t + \eta_{cz} \varepsilon_{t+1} \tag{91}$$

4.6 SDF as a Function of the Elasticities of the World Economy Model's State Variables

Recall Equation (66), the market SDF among the heterogeneous international investors:

$$\ln\left(\beta\frac{\Lambda_{t+1}}{\Lambda_t}\right)=\ln\beta-\rho\ln\left(\frac{C_{t+1}}{C_t}\right)-\ln\left(\frac{P_{t+1}}{P_t}\right)+\frac{\rho(\rho+1)}{2}\left(\Upsilon_t^c\right)^2+\left(\Upsilon_t^p\right)^2+\rho\sigma_{cp}$$

Subtracting $\ln\beta$ from both sides gives

$$\ln\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)=-\rho\ln\left(\frac{C_{t+1}}{C_t}\right)-\ln\left(\frac{P_{t+1}}{P_t}\right)+\frac{\rho(\rho+1)}{2}\left(\Upsilon_t^c\right)^2+\left(\Upsilon_t^p\right)^2+\rho\sigma_{cp}$$

(92)

Suppose cross-sectional distribution is heteroskedasticity and furthermore we assume that all cross-sectional variance and covariance terms are functions of $c_{t+1}-c_t$. The function forms are specified to guarantee that these cross-sectional variance and covariance terms are negatively correlated with the level of the world aggregate consumption growth. Specifically, the cross-sectional variance of individual countries' consumption growths, of individual country's price growths and the cross-sectional covariance between individual countries' consumption growths and price growths all increase when the world economy is in downturn. The formulation reflects the idea that the idiosyncratic risks increase in economic downturns (Campbell, 2003). In addition, we assume the log of the world average price growth $p_{t+1}-p_t$ is also a function of $c_{t+1}-c_t$:

$$
\begin{aligned}
p_{t+1} - p_t &= F_p\left(c_{t+1} - c_t\right) \\
&= \frac{F_p\left(c_{t+1} - c_t\right)}{c_{t+1} - c_t}\left(c_{t+1} - c_t\right) = f_p\left(c_{t+1} - c_t\right)
\end{aligned}
\tag{93}
$$

Analogously

$$
\left(\Upsilon_t^c\right)^2 = f_{\Upsilon_c}\left(c_{t+1} - c_t\right) \tag{94}
$$

$$
\left(\Upsilon_t^p\right)^2 = f_{\Upsilon^p}\left(c_{t+1} - c_t\right) \tag{95}
$$

$$
\sigma_{cp} = f_{\sigma_{cp}}\left(c_{t+1} - c_t\right) \tag{96}
$$

where f_p denotes $\dfrac{F_p\left(c_{t+1} - c_t\right)}{c_{t+1} - c_t}$ in Equation (93).

Inserting equations (93), (94), (95) and (96) into Equation (92) yields

$$
\begin{aligned}
\ln\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) &= \left(-\rho - f_p + \frac{\rho\left(\rho+1\right)}{2} f_{\Upsilon^c} + f_{\Upsilon^p} + \rho f_{\sigma_{cp}}\right)\left(c_{t+1} - c_t\right) \\
&= m\left(c_{t+1} - c_t\right)
\end{aligned}
\tag{97}
$$

where $m = -\rho - f_p + \dfrac{\rho(\rho+1)}{2} f_{\Upsilon^c} + f_{\Upsilon^p} + \rho f_{\sigma_{cp}}$. In the homogenous agent case, we have $m = -\rho$. In the Constantinides and Duffie's (1996) model, we have $m = -\rho + \dfrac{\rho(\rho+1)}{2} f_{\Upsilon^c}$. m turns out to be an important parameter when we explore the model's asset pricing implication.

Substituting Equation (90) and Equation (91) into Equation (97) gives respectively:

$$\begin{aligned}\ln\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) &= \lambda_{t+1} - \lambda_t \\ &= m\eta_{cz}\varepsilon_{t+1} = \hbar\varepsilon_{t+1}\end{aligned} \tag{98}$$

$$\begin{aligned}\ln\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) &= \lambda_{t+1} - \lambda_t \\ &= m\left[\eta_{ck}\eta_{kz} - \eta_{cz}\left(1-\varphi\right)\right]\varepsilon_t + m\eta_{cz}\varepsilon_{t+1} \\ &= m\lambda\!\!\!^{-}\varepsilon_t + \hbar\varepsilon_{t+1}\end{aligned} \tag{99}$$

where $\lambda_t = \ln\Lambda_t$, $\hbar$ denotes $m\eta_{cz}$; $\lambda\!\!\!^{-}$ denotes $\eta_{ck}\eta_{kz} - \eta_{cz}\left(1-\varphi\right)$.

Substituting Equation (98) and Equation (99) into Equation (64), we get respectively:

$$\begin{aligned}r_{t+1}^{f} &= \log R_{t+1}^{f} - \log \bar{R} \\ &= -\frac{1}{2}m^2\eta_{cz}^2\sigma_\varepsilon^2 = -J\sigma_\varepsilon^2\end{aligned} \tag{100}$$

$$\begin{aligned}r_{t+1}^{f} &= \log R_{t+1}^{f} - \log \bar{R} \\ &= m\left[\eta_{cz}\left(1-\varphi\right) - \eta_{ck}\eta_{kz}\right]\varepsilon_t - \frac{1}{2}m^2\eta_{cz}^2\sigma_\varepsilon^2 \\ &= -m\lambda\!\!\!^{-}\varepsilon_t - J\sigma_\varepsilon^2\end{aligned} \tag{101}$$

where J denotes $\frac{1}{2}m^2\eta_{cz}^2$.

The right hand side of Equation (100) is a Jensen's inequality adjustment arising from the fact that we are dealing with the expectation of log terms. So does the variance term on the right hand side of Equation (101). According to Lettau (2003), the technology shock affects the riskless rate through two channels: the direct consumption channel and the indirect consumption channel through the capital accumulation effect. They are expressed respectively in the first and the second bracketed terms.

Considering the homogenous agent case, Equation (101) becomes:

$$\begin{aligned} r_{t+1}^{f} &= \log R_{t+1}^{f} - \log \bar{R} \\ &= -\rho\left[\eta_{cz}\left(1-\varphi\right)-\eta_{ck}\eta_{kz}\right]\varepsilon_t - \frac{1}{2}m^2\eta_{cz}^2\sigma_\varepsilon^2 \\ &= \rho\tilde{\lambda}\varepsilon_t - J\sigma_\varepsilon^2 \end{aligned} \tag{102}$$

The direct consumption channel is described as follows. A positive technology shock at the period t, ε_t, given a positive η_{cz}, will cause an increase in the consumption at the period t, c_t. In the homogenous case when $m = -\rho$, a rise in c_t leads to a fall in the riskless rate between the time t and the time $t+1$, r_{t+1}^{f}. Due to the persistence of the technology shock, a positive shock at time t, ε_t, also leads to a positive shock at time $t+1$, $\varphi\varepsilon_t$, which cause an increase in the consumption at time $t+1$, c_{t+1}. In the homogenous case, a rise in c_{t+1} results in a rise in r_{t+1}^{f}. Therefore we see an opposite sign to ε_t in the first bracketed term on the right hand side of Equation (102).

There is also exists an indirect consumption channel through the capital accumulation effect. A positive shock ε_t, given a positive η_{kz}, causes an accumulation of the capital stock during the period t, which leads to a rise in k_{t+1}. The greater k_{t+1} is, given a positive η_{ck}, the larger is c_{t+1}. In the homogenous case, a rise in c_{t+1} results in a rise in r_{t+1}^{f}. Therefore we see the same sign to ε_t in the second bracketed term on the right hand side of Equation (102).

Our task in this section has been accomplished. We get an approximate closed-form solution for the market common SDF in Equation (66). It shows that the market SDF is a function of these factors: the world aggregate consumption growth, the world aggregate price growth and their cross-sectional variances and covariance. Moreover, we assume all the preceding factors are functions of the world aggregate consumption growth. And the latter is expressed, in Equation (88) and Equation (89), as functions of the exogenous global technology shock and the elasticities of the model's control variables to its state variables. Equation (99) shows that ultimately the market SDF is expressed as a function of the global technology shock and the elasticities of the model's control variables to its state variables. In turn, the world riskless interest rate is described in Equation (100) and Equation (101).

5 The Approximate Closed-Form Solutions for the Asset Prices

In section 3 and section 4, we derive the approximate closed-form solutions for the small open economy's ex post asset return and the international investors' SDF respectively. Now before completing our model and exploring the model's asset pricing implication, we have one last step to go. We need to put both the ex post asset return and the investors' SDF back into the basic asset pricing formulas. Our task in this section is to get this done and derive the approximate closed-form solutions for the small open economy's asset price and the ex ante asset return. We then discuss our results.

5.1 The Closed-form Solutions for the Asset Prices

We rewrite here the basic asset pricing formula, Equation (6):

$$E_t\left(R^{rp}_{j,t+1}\right) = -\Re^f_{t+1} Cov\left[\beta \frac{\Lambda_{t+1}}{\Lambda_t}, R^{rp}_{j,t+1}\right]$$

If we assume that all the logarithm terms are normal distributed random variables, that is, all the primitive terms follow lognormal distribution, Equation (6) can be written in logarithm form:

$$r^{rp}_{j,t+1} = \log E_t\left(R^j_{t+1}\right) - r^f_{t+1} = -cov_t\left(\Delta\lambda_{t+1}, r^j_{t+1}\right) \tag{103}$$

where $r^{rp}_{j,t+1}$ is the logarithm of the expected excess return of the asset j between the time t and the time $t+1$; $\log E_t\left(R^j_{t+1}\right)$ is the logarithm of the expected asset return; r^f_{t+1} is the logarithm of the riskless interest rate, that is, $r^f_{t+1} = \log\left(R^f_{t+1}\right)$; r^j_{t+1} is the logarithm of the ex post asset return, that is, $r^j_{t+1} = \log\left(R^j_{t+1}\right)$; $\Delta\lambda_{t+1} = \lambda_{t+1} - \lambda_t$ is the logarithm of the market SDF. See Appendix A.6 for a proof of Equation (103).

We complete the model by substituting r^d_{t+1}, derived in section 3, and $\Delta\lambda_{t+1}$, derived in section 4, into Equation (103). Then we explore the model's asset pricing implication.

Suppose the technology shock unexpectedly occurring from the time $t+1$. Substituting Equation (46), Equation (98) and Equation (100) into Equation (103), we get:

$$\begin{aligned} r^{rp}_{d,t+1} &= -Cov_t\left(\Delta\lambda_{t+1}, r^d_{t+1}\right) \\ &= -Cov_t\left(m\eta_{cz}\varepsilon_{t+1}, X\mu_{t+1} + Tr^f_{t+1}\right) \\ &= -Cov_t\left(m\eta_{cz}\varepsilon_{t+1}, X\mu_{t+1}\right) + Cov_t\left(m\eta_{cz}\varepsilon_{t+1}, \frac{1}{2}Tm^2\eta^2_{cz}\sigma^2_\varepsilon\right) \\ &= -Xm\eta_{cz}Cov_t\left(\varepsilon_{t+1}, \mu_{t+1}\right) \end{aligned} \tag{104}$$

If the technology shock unexpectedly occurs from the time t, to get the expected risk premium of the small open economy's asset, $r_{d,t+1}^{rp}$, we need to plug Equation (47), Equation (99), Equation (100) and Equation (101) into Equation (103). Firstly recall Equation (99):

$$\begin{aligned}\Delta\lambda_{t+1} &= m\left[\eta_{ck}\eta_{kz} - \eta_{cz}\left(1-\varphi\right)\right]\varepsilon_t + m\eta_{cz}\varepsilon_{t+1} \\ &= m\tilde{\lambda}\varepsilon_t + \hbar\varepsilon_{t+1}\end{aligned}$$

Recall Equation (47):

$$r_{t+1}^{d} = X\left(\phi\mu_t + \mu_{t+1}\right) + S\mu_t + Tr_{t+1}^{f} + Hr_t^{f}$$

If the technology shock unexpectedly occurs from the time t, analogous to Equation (100), we get:

$$r_t^{f} = -\frac{1}{2}m^2\eta_{cz}^2\sigma_\varepsilon^2 = -J\sigma_\varepsilon^2 \tag{105}$$

Recall Equation (101):

$$\begin{aligned}r_{t+1}^{f} &= m\left[\eta_{cz}\left(1-\varphi\right) - \eta_{ck}\eta_{kz}\right]\varepsilon_t - \frac{1}{2}m^2\eta_{cz}^2\sigma_\varepsilon^2 \\ &= -m\tilde{\lambda}\varepsilon_t - J\sigma_\varepsilon^2\end{aligned}$$

Substituting Equation (101) and Equation (105) into Equation (47), we get:

$$\begin{aligned}r_{t+1}^{d} &= X\left(\phi\mu_t + \mu_{t+1}\right) + S\mu_t + Tr_{t+1}^{f} + Hr_t^{f} \\ &= \left(\phi X + S\right)\mu_t + X\mu_{t+1} - Tm\tilde{\lambda}\varepsilon_t - TJ\sigma_\varepsilon^2 - HJ\sigma_\varepsilon^2\end{aligned} \tag{106}$$

We assume that $\operatorname{cov}\left(\varepsilon_{t+i}, \mu_{t+j}\right)=0 \quad \forall i \neq j$. Substituting Equation (99) and Equation (106) into Equation (103), we get:

$$\begin{aligned}
r_{d,t+1}^{rp} &= -Cov_t\left(\Delta\lambda_{t+1}, r_{t+1}^{d}\right) \\
&= -Cov_t\left[\left(m\tilde{\lambda}\varepsilon_t + \hbar\varepsilon_{t+1}\right), \left(\left(\phi X + S\right)\mu_t + X\mu_{t+1} - Tm\tilde{\lambda}\varepsilon_t - TJ\sigma_\varepsilon^2 - HJ\sigma_\varepsilon^2\right)\right] \\
&= -X\hbar Cov_t\left(\varepsilon_{t+1}, \mu_{t+1}\right) - m\tilde{\lambda}\left(\phi X + S\right)Cov_t\left(\varepsilon_t, \mu_t\right) + Tm^2\tilde{\lambda}^2\sigma_\varepsilon^2 \\
&= -Xm\eta_{cz}Cov_t\left(\varepsilon_{t+1}, \mu_{t+1}\right) - m\tilde{\lambda}\left(\phi X + S\right)Cov_t\left(\varepsilon_t, \mu_t\right) + Tm^2\tilde{\lambda}^2\sigma_\varepsilon^2
\end{aligned} \tag{107}$$

5.2 Discussion of Our Results

5.2.1 The Technology Shocks Occurring from Time $t+1$

We first discuss Equation (104), the expected risk premium of the small open economy's asset between the period t and the period $t+1$ when the technology shocks occur from time $t+1$. Notice that both X and η_{cz} in Equation (104) are positive.

For the homogenous agent case, $m=-\rho$, Equation (104) hence becomes

$$\begin{aligned}
r_{d,t+1}^{rp} &= -Cov_t\left(\Delta\lambda_{t+1}, r_{t+1}^{d}\right) \\
&= -Cov_t\left(m\eta_{cz}\varepsilon_{t+1}, X\mu_{t+1} + Tr_{t+1}^{f}\right) \\
&= -Cov_t\left(m\eta_{cz}\varepsilon_{t+1}, X\mu_{t+1}\right) + Cov_t\left(m\eta_{cz}\varepsilon_{t+1}, \frac{1}{2}Tm^2\eta_{cz}^2\sigma_\varepsilon^2\right) \\
&= X\rho\eta_{cz}Cov_t\left(\varepsilon_{t+1}, \mu_{t+1}\right)
\end{aligned} \tag{108}$$

If the covariance between a small open economy's country-specific technological shock and the global technological shock is positive, Equation (108) predicts a positive risk premium for the small open economy's asset. In contrast, for a negative covariance between ε and μ, Equation (108) predicts a negative risk premium for the small open economy's asset.

This result is standard and consistent with the economic intuition. A positive covariance implies that the global and the country-specific shock move in the same direction. The small open economy experiences a positive technology shock just at a time when the global economy encounters a positive shock as well. In the homogenous agent case, a positive global shock causes the market marginal rate of substitution to fall. A positive country-specific shock causes its ex post asset return to raise. A positive covariance between the two shocks implies that the small open economy's asset tends to pay off unexpectedly well when the international investors' marginal rate of substitution is unexpectedly low. The asset has no value as a consumption hedge to the investors and therefore will command a positive (high) risk premium.

In contrast, for a negative covariance between the two shocks, the small open economy's asset tends to pay off unexpectedly well when the international investors' marginal rate of substitution is unexpectedly high. It has value as a consumption hedge to the investors and therefore will command a negative (low) risk premium.

Developing countries usually have their country-specific shocks negatively correlated with those of developed countries.[35] As a result, the homogenous agent model predicts a counterfactual negative (low) risk premium for the developing countries' assets.

For the heterogeneous international investors case in our model, $m = -\rho - f_p + \frac{\rho(\rho+1)}{2} f_{\Upsilon^c} + f_{\Upsilon^p} + \rho f_{\sigma_{cp}}$, Equation (104) thus becomes

[35] We imply that the global shock is mainly determined by developed countries' country-specific shocks.

$$
\begin{aligned}
r_{d,t+1}^{rp} &= -Cov_t\left(\Delta\lambda_{t+1}, r_{t+1}^{d}\right) \\
&= -Cov_t\left[m\eta_{cz}\varepsilon_{t+1}, X\mu_{t+1} + Tr_{t+1}^{f}\right] \\
&= -Cov_t\left[m\eta_{cz}\varepsilon_{t+1}, X\mu_{t+1}\right] + Cov_t\left[m\eta_{cz}\varepsilon_{t+1}, \frac{1}{2}Tm^2\eta_{cz}^2\sigma_\varepsilon^2\right] \\
&= -Xm\eta_{cz}Cov_t\left(\varepsilon_{t+1}, \mu_{t+1}\right) \\
&= -X\eta_{cz}\left(-\rho - f_p + \frac{\rho(\rho+1)}{2} f_{\Upsilon^c} + f_{\Upsilon^p} + \rho f_{\sigma_{cp}}\right)Cov_t\left(\varepsilon_{t+1}, \mu_{t+1}\right)
\end{aligned} \tag{109}
$$

We call $m = -\rho - f_p + \frac{\rho(\rho+1)}{2} f_{\Upsilon^c} + f_{\Upsilon^p} + \rho f_{\sigma_{cp}}$, the core of the market SDF in a heterogeneous-agent case because the market SDF, in this case, is the product of m and $c_{t+1} - c_t$. m turns out to have very important asset pricing implication. This will be discussed in more detail below.

Recall the assumption we made on the cross-sectional variance and covariance terms. We assume that they are negatively correlated with $c_{t+1} - c_t$. When the world aggregate consumption growth is high, it is also a time when the cross-sectional variances and covariance terms are low. Therefore, it is also a time when negative terms in m, $\left(-\rho, -f_p\right)$, tend to dominate the positive terms (those variance and covariance terms). As a result, when the world economy is robust, m will remain to be a negative number, just like the homogenous case where m equals to a negative number $-\rho$.

Now consider an opposite scenario. When the world aggregate consumption growth is low, the cross-sectional variance and covariance terms tend to be high. Their effect on m can be strong enough and turn m from a negative to a positive number. Put it in another way, when consumption growth is low, the positive terms in m tend to be high according to our assumptions. When these positive terms dominate the negative terms, m will change sign from negative to positive.

A negative m has the same asset pricing implication as that of the homogenous agent case. A positive m merits some discussion. Given a positive m, a positive covariance between shocks μ and ε implies that the small open economy's asset tends to pay off unexpectedly well when the investors' marginal rate of substitution (measured by the market common SDF) is unexpectedly high. The asset has value as a consumption hedge to the investors and therefore will command a negative (low) risk premium. In contrast, for a negative covariance between shocks μ and ε, the asset tends to pay off unexpectedly well when the investors' SDF is unexpectedly low. It thus has no value as a consumption hedge to the investors and therefore will command a positive (high) risk premium.

Recall that the developing country' country-specific shock μ is usually negatively correlated with the global shock ε. First we study a case where the global shock ε is positive, which implies a negative shock μ occurring in the developing country. A positive ε leads to a high consumption growth. A global economic boom causes the cross-sectional variance and covariance terms to be low by lowering the idiosyncratic risks across countries. As a result, m remains negative when a positive ε happens. Given a negative correlation between μ and ε, a developing country's asset, at this time, commands a negative (low) risk premium.

When the global economy experiences a negative shock ε, the global economic downturns cause the cross-sectional variance and covariance terms to be high. As a result, m could change sign from a negative number to a positive number. If this indeed happens, given a positive m

and a negative correlation between μ and ε, a developing country's asset, at this time, commands a positive (high) risk premium.

Combining the results from the above two paragraphs together, we get the following important finding. When the developed countries' economies are robust, the developing country's asset commands a low risk premium. When the developed countries' economies are dismal, the developing country's asset commands a high risk premium.

Does this finding look familiar? Absolutely! It is exactly what we observe in reality, especially in those times when financial crises run rampantly. Eichengreen and Rose (2001) demonstrate the related empirical evidence: "[e]xternal factors are adverse during periods of Southern banking crisis and significantly so. The North tends to be in recession when banking crises break out in developing countries. There is much less evidence that macroeconomic conditions in the South vary systematically between periods of tranquility and banking crises….[t]here is a clear presumption that global conditions play a role in developing country financial crises." We believe our model offers a deep explanation of why risk premia of the developing countries' assets soar at a time when the developed countries experience economic downturns.

Some researchers try to study this issue from the different perspectives, such as the "financial accelerator" hypothesis and the "sudden stop" hypothesis.[36] In this paper, we put forward an explanation to the above economic phenomenon from a different angle. We believe our model offers a deeper and more fundamental answer to the question of why the developing countries' asset prices change procyclically with respect to the developed countries' economic conditions. The main feature in our model is the heterogeneity assumption on the international investors. It turns out that this assumption has very important asset pricing implications and it can be used to

[36] Examples include: Calvo and Reinhart (2000); Arellano and Mendoza (2002); Urib and Yue (2003); Kaminsky, Reinhart and Vegh (2004); Neumeyer and Perri (2004).

explain the above "puzzle" we observe in reality. The "puzzle" is hard to reconcile with the standard economic theory, which is usually built in a homogenous agent environment. That is why it is called "puzzle".[37] It seems that it is not a puzzle at all in our model featuring the heterogeneous agents.

5.2.2 The Technology Shocks Occurring from Time t

In this subsection, we discuss Equation (107), the expected risk premium of the small open economy's asset between the period t and the period $t+1$ when the technology shocks occur from time t. We rewrite it here:

$$\begin{aligned}
r_{d,t+1}^{rp} &= -Cov_t\left(\Delta\lambda_{t+1}, r_{t+1}^{d}\right) \\
&= -Cov_t\left[\left(m\bar{\lambda}\varepsilon_t + \hbar\varepsilon_{t+1}\right), \left(\left(\phi X + S\right)\mu_t + X\mu_{t+1} - Tm\bar{\lambda}\varepsilon_t - TJ\sigma_\varepsilon^2 - HJ\sigma_\varepsilon^2\right)\right] \\
&= -X\hbar Cov_t\left(\varepsilon_{t+1}, \mu_{t+1}\right) - m\bar{\lambda}\left(\phi X + S\right)Cov_t\left(\varepsilon_t, \mu_t\right) + Tm^2\bar{\lambda}^2\sigma_\varepsilon^2 \\
&= -Xm\eta_{cz}Cov_t\left(\varepsilon_{t+1}, \mu_{t+1}\right) - m\bar{\lambda}\left(\phi X + S\right)Cov_t\left(\varepsilon_t, \mu_t\right) + Tm^2\bar{\lambda}^2\sigma_\varepsilon^2
\end{aligned}$$

Below we focus on a case that the small open economy's country-specific technology shock μ is negatively correlated with the global shock ε since this is the case we often observe in reality for developing countries. When we study the risk premium between period t and the period $t+1$ in a case that the shocks occur from time t, the capital accumulation effect of shock arises, which is expressed in the second terms on the right hand side of Equation (107). Without the capital accumulation channel, Equation (107) will degrade to Equation (104) and a positive technology shock always leads to a higher asset return.

---

[37] "[e]conomists often use the term puzzle to refer to awkward empirical facts that refuse to comply with their established theoretical frameworks". (Coakley, Kulasi and Smith, 1998)

With the capital accumulation channel, capital accumulation will drag down the asset return due to the diminishing marginal returns. As we emphasized in section 3, there are also the capital gain/loss channels accompanying with the capital accumulation in a model with adjustment cost. Capital loss tends to drive down the asset return. The total negative effects, reflected in S, could be strong enough to offset the positive effect from shock per se, which is reflected in ϕX. If this is the case, we see a negative $\phi X + S$ in Equation (107). Conversely, a positive $\phi X + S$ is achieved when capital accumulation effect is not strong enough to offset the positive effect arising from the positive shock per se.

Next we consider $\tilde{\lambda} = \eta_{ck}\eta_{kz} - \eta_{cz}(1-\varphi)$ in Equation (107). The first term $\eta_{ck}\eta_{kz}$ measures the indirect effect of the global shock ε_t on the world aggregate consumption c_{t+1} through the capital accumulation channel, k_{t+1}; the second term $\eta_{cz}(1-\varphi)$ is the direct effect of the shock on the consumption. If the capital accumulation channel dominates, we will see a positive $\tilde{\lambda}$.

In short, when the capital accumulation effect is sufficiently strong and dominates, we will have a negative $\phi X + S$ and a positive $\tilde{\lambda}$. If this is the case, the second term acts as an offsetting term to the first term in Equation (107). Hence we get the following result: with capital accumulation, or put it in another way, when the shocks occur from the time t, a small open economy when facing a positive global shock will command a negative risk premium for its asset. In contrast, a negative global shock causes a positive risk premium. The result in Equation (107) is similar to the one we obtained for the risk premium when the shock occurs from the time $t+1$ in Equation (104). The difference between the risk premia when the shock is from the time $t+1$ and from the time t is quantitative, not qualitative. To put it in another way, the difference between the risk premia in these two cases is the magnitude, not the sign of the risk premia. With an offsetting term, the risk premium in the capital accumulation case becomes less volatile than that without the capital accumulation. With capital accumulation, the developing countries' assets still command low risk premia when developed countries' economies are robust,, but not as low as that in the case where the capital accumulation is absent. In contrast, when developed countries' economies are dismal, the developing countries' assets still command high risk premia, but not as high as that in the case where there is no capital accumulation.

The reason behind this is as follows: A negative global shock leads to a low market marginal rate of substitution (MRS) among heterogeneous investors (recall m changes sign from negative to positive then). At the same time, the small open economy experiences a positive country-specific shock. Without a capital accumulation channel, a positive country-specific shock causes the country's asset return to rise. As a result, the asset pays off well when international MRS is low and badly when it is high. The asset thus has no value as a consumption hedge and therefore will command a high risk premium. The capital accumulation channel tends to drive down the asset return when an economy encounters a positive shock. Capital accumulation channel per se makes the small economy's asset pay off badly when international MRS is low and well when MRS is high. Therefore capital accumulation per se makes the asset a consumption hedge to international investors. Capital accumulation offsets the original high risk premium. In short, the capital accumulation a country made moderates the fluctuation of its asset risk premium.

For the developing country that conducts fierce capital accumulation, our model predicts that its risk premium will fluctuate less broadly. When developed countries experience the economic downturns, the risk premium of a developing country's asset will soar, but with less magnitude. For the developing country that conducts little capital accumulation, our model predicts a large fluctuation in its risk premium. Its risk premium will incur a sharp and significant rise when developed countries experience the economic downturns. The role of capital accumulation has been extensively examined in the economic growth literature. We demonstrate here its role in a country's asset prices and in turn its effect on a country's welfare.

Notice the third term on the right hand side of Equation (107), $Tm^2\lambda^2\sigma_\varepsilon^2$. It shows that the larger the variance of the global economy is, the larger the risk premium a risky asset will command over the riskless rate. This result is intuitive. A large variance of the economy implies that there is big risk to hold the risky assets than the riskless one, therefore the risk premium of the risky assets will increase.

6 Conclusion

In this paper, we have studied how a small open economy's assets are being priced by heterogeneous international investors. We initially decomposed the asset pricing issue into a study of its two ingredients: the asset's ex post return and the investors' stochastic discount factor. Firstly we derived the ex post asset return from a small open economy RBC model featuring adjustment cost in investment process. Secondly we derived the market common stochastic discount factor among heterogeneous international investors. By substituting the asset return and market SDF into the basic asset pricing formula, we obtained the closed-form solutions for asset prices.

Our model generates a risk premium for a small economy's asset that tends to be low when the global economy is robust and to soar when global economy experiences a downturn. The main reason behind this is our assumption of heterogeneity across international investors.

Also we studied the capital accumulation and capital loss/gain channels and explored their asset pricing implications. The major finding is as follows: For a small country that conducts fierce capital accumulation, our model predicts that its risk premium will fluctuate less broadly. Its risk premium will soar, but with less intensity, when developed countries experience the economic downturns. For one that conducts little capital accumulation, our model predicts a large fluctuation in its risk premium. Its risk premium will incur a sharp and significant rise when developed countries experience the economic downturns.

Our model's finding and prediction are consistent with the stylized fact we observe in reality. And this economic phenomenon becomes even more apparent in times when financial crises run rampantly. Researchers have struggled and worked hard to get an answer for the question of why in reality the risk premia for developing countries' assets experience a hover when developed countries experience economic downturns. A lot of explanations have been offered from different perspectives. We hope our work, if we dare say so, contributes a little bit to people's gaining of a deeper and better understanding of this economic phenomenon.

Appendix

A.1 Recursive Utility Function

The time-separable power utility function we adopt in our model does not permit us to vary the consumer's aversion to risk and intertemporal substitution (two very different things) independently of each other. The reason is that utility is additive across states as well as time, with probabilities weighting the period utility function as applied to different states (measured by risk aversion coefficient) in the same multiplicative fashion that the temporal discount factor (measured by intertemporal substitution elasticity) weights the value of period utility on different dates (Obstfeld and Rogoff, 1996).

Yet it is not clear that these two concepts should be linked so tightly. Risk aversion describes the consumer's reluctance to substitute consumption across states and is meaningful even in an atemporal setting, whereas the elasticity of intertemporal substitution describes the consumer's willingness to substitute consumption over time and is meaningful even in a deterministic setting (Campbell, 2003). A high risk aversion helps to solve the equity premium puzzle while a high elasticity of intertemporal substitution is needed to reconcile the low riskless interest rate.

Epstein and Zin (1989, 1991) and Weil (1989) develop a recursive version of the basic power utility model

$$U=\frac{C_1^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}+\beta\frac{\left\{\left[\sum_{s=1}^{\Im}\pi(s)C_2(s)^{1-\rho}\right]^{\frac{1}{1-\rho}}\right\}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \qquad (110)$$

where ρ is the coefficient of relative risk aversion and σ is a distinct parameter for elasticity of intertemporal substitution. Only when $\sigma=\frac{1}{\rho}$ does the recursive utility reduce to the expected lifetime utility (Obstfeld and Rogoff, 1996).

The so-called Epstein-Zin-Weil utility breaks the link between risk aversion and elasticity of intertemporal substitution by making the utility state non-separable. State separable means utility is additive across states. One adds utility across states, so the marginal utility of consumption in one state is unaffected by what happens in another state.

To explain the equity premium puzzle, we need a high risk aversion. But to solve the related low riskless interest rate puzzle, we need a high elasticity of intertemporal substitution. With the regular power utility, $\sigma=\frac{1}{\rho}$, we cannot simultaneously get a high ρ and a high σ. Epstein-Zin-Weil model, by breaking the link between these two concepts, might explain the equity premium puzzle while still reconcile the low riskless interest rate. However, as Cochrane (1997) point out, this research is only starting to pay off in terms of plausible models that explain the facts, in this paper we do not adopt it in our modeling of the consumer's preference.

A.2 The Form of Adjustment Cost

There are two ways to add adjustment cost in a general equilibrium model. One is introduced by Lucas (1967), Gould (1968) and Treadway (1969).

$$GY_t = A_t K_t^{\alpha} L_t^{1-\alpha} = A_t K_t^{\alpha}$$

where GY_t denotes firm's gross output at time t , correspondingly Y_t is firm's output net of adjustment cost

$$Y_t = F\left(A_t, K_t, L\right) - Q\left(I_t, K_t\right) = A_t K_t^{\alpha} - \frac{\chi}{2}\frac{I_t^2}{K_t}$$

where $Q\left(.\right)$ denotes the function form of investment installation cost, I_t is the investment made by firm during period t , χ is a parameter. The specific installation cost function form shows that cost is an increasing and convex function of investment $\left(G_I > 0, G_{II} > 0\right)$. And the cost depends negatively on the amount of capital already in place. This specification captures the observation that a faster speed of change in capital stock requires a greater than proportional rise in installation cost and a firm with larger capital stock can absorb a given influx of new capital at lower cost (Obstfeld and Rogoff, 1996).

The firm's capital stock evolves according to the following law of motion

$$K_{t+1} = \left(1 - \delta\right) K_t + I_t$$

An alternative way to introduce adjustment cost was introduced by Uzawa (1969), Lucas and Prescott (1971). They leave firm's output intact but modify the law of motion of capital stock.

$$Y_t = F\left(A_t, K_t, L\right) = A_t K_t^{\alpha}$$

$$K_{t+1} = G\left(I_t, K_t\right) = \Psi\left(\frac{I_t}{K_t}\right)K_t + \left(1-\delta\right)K_t$$

where Ψ reflected the installation cost, which is positive near the steady state point. In steady state, $\Psi(\delta)=\delta$ and $\Psi'(\delta)=1$. Also the installation cost Ψ is increasing and concave in I $\left(\Psi > 0, \Psi_I > 0, \Psi_{II} < 0\right)$. This specification catches the same idea as function Q does, that is, changing the capital stock rapidly is more costly than changing it slowly.

Since the two formulations of adjustment costs give similar results concerning the optimal investment rule (Hayashi, 1982) and papers deriving the closed-form solution for asset prices often adopt Uzawa (1969) approach, we adopt Uzawa's formulation in the present paper.

A.3 Proof: $\left(\Upsilon_t^c\right)^2$ is the variance of the cross-sectional distribution of

$$\log\left[\frac{\left(C_{t+1}^i / C_{t+1}\right)}{\left(C_t^i / C_t\right)}\right]$$

We proof this by applying the mean of lognormal distribution: If X is a normally distributed random variable with mean μ_x and variance σ_x^2, then $\exp X$ is lognormal with mean $E^{\oplus}\left(\exp X\right) = \exp\left(\mu_x + \frac{1}{2}\sigma_x^2\right)$ (Obstfeld and Rogoff, 1996).

$$\log\left[\frac{C_{t+1}^{i}/C_{t+1}}{C_{t}^{i}/C_{t}}\right]=\log\left(\frac{\tau_{t+1}^{i}}{\tau_{t}^{i}}\right)$$

$$=\upsilon_{i,t+1}^{c}\Upsilon_{t+1}^{c}-\frac{1}{2}\left(\Upsilon_{t+1}^{c}\right)^{2}$$

$$\sim N\left(-\frac{1}{2}\left(\Upsilon_{t+1}^{c}\right)^{2};\left(\Upsilon_{t+1}^{c}\right)^{2}\right)$$

It shows that $\log\left[\frac{\left(C_{t+1}^{i}/C_{t+1}\right)}{\left(C_{t}^{i}/C_{t}\right)}\right]$ follow a normal distribution with $\left(\Upsilon_{t}^{c}\right)^{2}$ as its variance. Q.E.D.

A.4 Proof: $E^{\oplus}\left(\tau_{t}^{i}\right)=1,E^{\oplus}\left(\aleph_{t}^{i}\right)=0,E^{\oplus}\left(C_{t}^{i}\right)=E^{\oplus}\left(\tau_{t}^{i}C_{t}\right)=C_{t}$

We proof this by applying again the mean of lognormal distribution.

Proof:

From Equation (54), we get:

$$\begin{aligned} E^{\oplus}\left(C_t^i\right) &= E^{\oplus}\left(\tau_t^i C_t\right) \\ &= E^{\oplus}\left[\tau_{t-1}^i \exp\left(\upsilon_{it}^c \Upsilon_t^c - \frac{1}{2}\left(\Upsilon_t^c\right)^2\right)\right] \\ &= E^{\oplus}\left\{\exp\left[\sum_{s=1}^{t}\left(\upsilon_{is}^c \Upsilon_s^c - \frac{1}{2}\left(\Upsilon_s^c\right)^2\right)\right] C_t\right\} \\ &= \left\{\exp\left[\sum_{s=1}^{t}\left(\frac{1}{2}\left(\Upsilon_s^c\right)^2 - \frac{1}{2}\left(\Upsilon_s^c\right)^2\right)\right] C_t\right\} \\ &= C_t \end{aligned} \tag{111}$$

Recall Equation (53):

$$\aleph_t^i = \tau_t^i C_t - \frac{1}{N}\left(\sum_{j=1}^{N} D_{jt}\right) - \frac{1}{N}\left(\sum_{j=1}^{N} W_t^i L_t^i\right)$$

Taking the cross-sectional expectation, we get:

$$\begin{aligned} E^{\oplus}\left(\aleph_t^i\right) &= E^{\oplus}\left[\tau_t^i C_t - \frac{1}{N}\left(\sum_{j=1}^{N} D_{jt}\right) - \frac{1}{N}\left(\sum_{j=1}^{N} W_t^i L_t^i\right)\right] \\ &= C_t - \frac{1}{N}\left(\sum_{j=1}^{N} D_{jt}\right) - \frac{1}{N}\left(\sum_{j=1}^{N} W_t^i L_t^i\right) \end{aligned} \tag{112}$$

Recall the asset market clearing condition, that is, the supply of each risky asset is normalized to be one and the riskless bond is in zero net supply:

$$\sum_{i=1}^{N} \theta_{jt}^{i} = 1$$

$j = 1, 2 \ldots N$

$$\sum_{i=1}^{N} B_{t}^{i} = 0$$

Recall the consumer's budget constraint, Equation (49):

$$C_t^i = W_t^i L_t^i + \sum_{j=1}^{N} \left[\theta_{jt}^i \left(\Omega_{jt} + D_{j,t-1} \right) \right] - \sum_{j=1}^{N} \left(\theta_{j,t+1}^i \Omega_{jt} \right) + B_t^i \Re_t^f - B_{t+1}^i + \aleph_t^i$$

Taking the cross-sectional expectation, we get:

$$E^{\oplus}\left(C_t^i\right) = E^{\oplus}\left\{ W_t^i L_t^i + \sum_{j=1}^{N} \left[\theta_{jt}^i \left(\Omega_{jt} + D_{j,t-1} \right) \right] - \sum_{j=1}^{N} \left(\theta_{j,t+1}^i \Omega_{jt} \right) + B_t^i \Re_t^f - B_{t+1}^i + \aleph_t^i \right\}$$

$$= C_t = \frac{1}{N}\left(\sum_{j=1}^{N} D_{jt} \right) + \frac{1}{N}\left(\sum_{j=1}^{N} W_t^i L_t^i \right) + E^{\oplus}\left(\aleph_t^i\right)$$

The above equation generates the same $E^{\oplus}\left(\aleph_t^i\right)$ as Equation (112) does.

World goods market clearing condition implies that world aggregate consumption comes from either the financial income or the labor income:

$$C_t = \frac{1}{N}\left(\sum_{j=1}^{N} D_{jt} \right) + \frac{1}{N}\left(\sum_{j=1}^{N} W_t^i L_t^i \right)$$

Comparing the above equation with Equation (112), we get:

$$E^{\oplus}\left(\aleph_t^i\right)=0 \tag{113}$$

Equation (111) and Equation (113) show that $E^{\oplus}\left(\tau_t^i\right)=1, E^{\oplus}\left(\aleph_t^i\right)=0, E^{\oplus}\left(C_t^i\right)=E^{\oplus}\left(\tau_t^i C_t\right)=C_t$ Q.E.D.

A.5 More Detail on: $Y_t = Z_t\left(K_t\right)^{\alpha}\left(L_t^f\right)^{1-\alpha} = \sum_j^N A_t^j\left(K_t^j\right)^{\alpha}\left(L_t^{fj}\right)^{1-\alpha}$

We suppose there exists a global technology level such that:

$$Y_t = Z_t\sum_j^N\left[\left(K_t^j\right)^{\alpha}\left(L_t^{fj}\right)^{1-\alpha}\right] = \sum_j^N\left[A_t^j\left(K_t^j\right)^{\alpha}\left(L_t^{fj}\right)^{1-\alpha}\right] \tag{114}$$

According to our assumption of the constant return to scale of the production function, we can further assume that:

$$\begin{aligned} K_t^j &= \nu_j K_t^{\min} \\ L_t^{fj} &= \nu_j L_t^{f,\min} \end{aligned} \tag{115}$$

for $j=1,2,\ldots N$

where $\min$ represents the country with the smallest economy in the world.

From Equation (115), we get:

$$K_t = \sum_j^N \left(K_t^j\right) = K_t^{\min} \sum_j^N \left(\nu_j\right)$$

$$L_t^f = \sum_j^N \left(L_t^{fj}\right) = L_t^{f,\min} \sum_j^N \left(\nu_j\right)$$

Substituting Equation (115) into Equation (114), we get:

$$\begin{aligned}
Y_t &= \sum_j^N \left[A_t^j \left(K_t^j\right)^\alpha \left(L_t^{fj}\right)^{1-\alpha} \right] = Z_t \sum_j^N \left[\left(K_t^j\right)^\alpha \left(L_t^{fj}\right)^{1-\alpha} \right] \\
&= Z_t \sum_j^N \left[\left(\nu_j K_t^{\min}\right)^\alpha \left(\nu_j L_t^{f,\min}\right)^{1-\alpha} \right] \\
&= Z_t \left(K_t^{\min}\right)^\alpha \left(L_t^{f,\min}\right)^{1-\alpha} \sum_j^N \left(\nu_j\right) \qquad (116) \\
&= Z_t \left[K_t^{\min} \left(\sum_j^N \left(\nu_j\right) \right) \right]^\alpha \left[L_t^{f,\min} \left(\sum_j^N \left(\nu_j\right) \right) \right]^{1-\alpha} \\
&= Z_t \left(K_t\right)^\alpha \left(L_t^f\right)^{1-\alpha}
\end{aligned}$$

Equation (116) shows that $Y_t = Z_t \left(K_t\right)^\alpha \left(L_t^f\right)^{1-\alpha} = \sum_j^N A_t^j \left(K_t^j\right)^\alpha \left(L_t^{fj}\right)^{1-\alpha}$ can be warranted by our assumption that the production function has the character of constant return to scale.

A.6 Proof: $r_{j,t+1}^{rp} = \log\left(E_t\left(R_{t+1}^j\right)\right) - \log\left(R_{t+1}^f\right) = -cov_t\left(\Delta\lambda_{t+1}, r_{t+1}^j\right)$

We assume that all the logarithm terms are normal distributed random variables, which implies that all the primitive terms follow lognormal distribution. Recall Equation (2):

$$E_t\left(\beta\frac{\Lambda_{t+1}}{\Lambda_t}R^j_{t+1}\right)=1$$

Taking log on both sides, we get:

$$\log\left[E_t\left(\beta\frac{\Lambda_{t+1}}{\Lambda_t}R^j_{t+1}\right)\right]=0$$

Writing the random variables in logarithms, we get:

$$\log\left\{E_t\left[\exp\left(\log\beta+\log\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)+\log\left(R^j_{t+1}\right)\right)\right]\right\}=0 \tag{117}$$

Applying the mean of lognormal distribution to Equation (117), we get:

$$\log\left\{\exp\left[\begin{array}{l} E_t\left(\log\beta+\log\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)+\log\left(R^j_{t+1}\right)\right) \\ +\frac{1}{2}\mathrm{var}\left[\log\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\right]+\frac{1}{2}\mathrm{var}\left[\log\left(R^j_{t+1}\right)\right] \\ +\mathrm{cov}\left[\log\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right),\log\left(R^j_{t+1}\right)\right] \end{array}\right]\right\}=0$$

After simplifying we get:

$$
\begin{aligned}
&E_t\left[\log\left(R_{t+1}^j\right)\right] \\
&= -\log\beta - E_t\left[\log\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\right] \\
&-\frac{1}{2}\operatorname{var}\left[\log\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\right] - \frac{1}{2}\operatorname{var}\left[\log\left(R_{t+1}^j\right)\right] \\
&-\operatorname{cov}\left[\log\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right), \log\left(R_{t+1}^j\right)\right]
\end{aligned}
\tag{118}
$$

Recall Equation (4):

$$
\Re_{t+1}^f = \frac{1}{E_t\left(\beta\frac{\Lambda_{t+1}}{\Lambda_t}\right)}
$$

Taking log on both sides, we get:

$$
\log\left(\Re_{t+1}^f\right) = -\log\left[E_t\left(\beta\frac{\Lambda_{t+1}}{\Lambda_t}\right)\right]
$$

Writing the random variables in logarithms, we get:

$$
\log\left(\Re_{t+1}^f\right) = -\log\left\{E_t\left[\exp\left(\log\beta + \log\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\right)\right]\right\} \tag{119}
$$

Applying the mean of lognormal distribution to Equation (119), we get:

$$\log\left(\Re_{t+1}^{f}\right) = -\log\left\{\exp\left[\begin{array}{l}\log\beta + E_t\left(\log\left(\dfrac{\Lambda_{t+1}}{\Lambda_t}\right)\right) \\ +\dfrac{1}{2}\operatorname{var}\left(\log\left(\dfrac{\Lambda_{t+1}}{\Lambda_t}\right)\right)\end{array}\right]\right\}$$

$$= -\log\beta - E_t\left(\log\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\right) - \frac{1}{2}\operatorname{var}\left(\log\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\right) \tag{120}$$

Applying again the mean of lognormal distribution, we get:

$$\log E_t\left(R_{t+1}^{j}\right) = \log\left\{E_t\left[\exp\left(\log\left(R_{t+1}^{j}\right)\right)\right]\right\}$$

$$= \log\left\{\exp\left[E_t\left[\log\left(R_{t+1}^{j}\right)\right] + \frac{1}{2}\operatorname{var}\left[\log\left(R_{t+1}^{j}\right)\right]\right]\right\} \tag{121}$$

$$= E_t\left[\log\left(R_{t+1}^{j}\right)\right] + \frac{1}{2}\operatorname{var}\left[\log\left(R_{t+1}^{j}\right)\right]$$

By definition of logarithms of the excess return, the following equation holds:

$$r_{j,t+1}^{rp} = \log E_t\left(R_{t+1}^{j}\right) - \log\left(R_{t+1}^{f}\right) \tag{122}$$

Substituting Equation (118), Equation (120) and Equation (121) into Equation (122), we get:

$$
\begin{aligned}
r_{j,t+1}^{rp} &= \log E_t\left(R_{t+1}^{j}\right) - \log\left(R_{t+1}^{f}\right) \\
&= E_t\left[\log\left(R_{t+1}^{j}\right)\right] + \frac{1}{2}\mathrm{var}\left[\log\left(R_{t+1}^{j}\right)\right] - \log\left(R_{t+1}^{f}\right) \\
&= -\log\beta - E_t\left[\log\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\right] \\
&- \frac{1}{2}\mathrm{var}\left[\log\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\right] - \frac{1}{2}\mathrm{var}\left[\log\left(R_{t+1}^{j}\right)\right] \\
&- \mathrm{cov}\left[\log\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right), \log\left(R_{t+1}^{j}\right)\right] + \frac{1}{2}\mathrm{var}\left[\log\left(R_{t+1}^{j}\right)\right] \\
&+ \log\beta + E_t\left[\log\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\right] + \frac{1}{2}\mathrm{var}\left[\log\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\right] \\
&= -\mathrm{cov}\left[\log\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right), \log\left(R_{t+1}^{j}\right)\right] \\
&= -\mathrm{cov}\left[\Delta\lambda_{t+1}, r_{t+1}^{j}\right]
\end{aligned}
\tag{123}
$$

Equation (123) shows that $r_{j,t+1}^{rp} = \log\left[E_t\left(R_{t+1}^{j}\right)\right] - \log\left(R_{t+1}^{f}\right) = -cov_t\left(\Delta\lambda_{t+1}, r_{t+1}^{j}\right)$.

Q.E.D.

Reference

Abel, A. B., 1990, Asset prices under habit formation and catching up with the Joneses, *American Economic Review Paper and Proceedings*, 80, 38-42.

Abel, A. B., 1999, Risk premia and term premia in general equilibrium, *Journal of Monetary Economics*, 43, 3-33.

Arellano, C., and E. Mendoza, 2002, Credit frictions and "sudden stops" in small open economies: an equilibrium business cycle framework for emerging market crises, *NBER working paper*, No. w8880.

Atkeson, A., and T. Bayoumi, 1993, Do private capital markets insure regional risk? Evidence from Europe and the United States, *Open Economics Review*, 4 (3), 303-324.

Backus, D. K., P. J. Kehoe, and F. E. Kydland, 1992, International real business cycles, *Journal of Political Economy*, 100, 745-775.

Barberis, N., M. Huang and T. Santos, 2001, Prospect theory and asset prices, *Quarterly Journal of Economics*, 116, 1-53.

Bayoumi, T., and M. W. Klein, 1995, A provincial view of capital mobility, *International Monetary Fund Staff Papers*, 44 (4), 534-556.

Benartzi, S., and R. Thaler, 1995, Myopic loss aversion and the equity premium puzzle, *Quarterly Journal of Economics*, 110, 73-92.

Boldrin, M., L. J. Christiano and J. Fisher, 2001, Habit persistence, asset returns, and the business cycle, *American Economic Review*, 91, 149-166.

Calvo, G.A., C. Reinhart, 2000: When capital inflows come to a sudden stop: consequences and policy options, in P. Kenen, A. Swoboda, eds, *Key issues in reform of the international monetary and financial system,* Washington D.C., International Monetary Fund, 175-201.

Campbell, J. Y., 1994, Inspecting the mechanism: an analytical approach to the stochastic growth model, *Journal of Monetary Economics*, 33, 463-506.

Campbell, J. Y., 2003, Consumption-based asset pricing, In G. Constantinides, M. Harris and R. Stulz, eds, *Handbook of the economics of finance*, Vol. IB, North-Holland, Amsterdam, 803-887.

Campbell, J. Y., and J. H. Cochrane, 1999, By force of habit: A consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy*, 107, 205-252.

Chan, Y. L., and L. Kogan, 2002, Catching up with the Joneses: Heterogeneous preferences and the dynamic of asset prices, *Journal of Political Economy*, 110, 1255-1285.

Coakley, J., F. Kulasi and R. Smith, 1998, The Feldstein-Horioka puzzle and capital mobility: A review, *International Journal of Finance and Economics*, 3, 169-188.

Cochrane, J. H., 1991, Production-based asset pricing and the link between stock returns and economic fluctuations, *Journal of Finance*, 46, 207-234.

Cochrane, J. H., 1997, Where is the market going? Uncertain facts and novel theories, *Economic Perspectives* Federal Reserve Bank of Chicago, 21(6).

Cochrane, J. H., 2006, Financial markets and the real economy, In J. H. Cochrane, eds, *Financial markets and the real economy*, Volume 18 of the International library of critical writings in financial economics, London: Edward Elgar.

Constantinides, G. M., 1990, Habit formation: A resolution of the equity premium puzzle, *Journal of Political Economy*, 98, 519-543.

Constantinides, G. M., and D. Duffie, 1996, Asset pricing with heterogeneous consumers, *Journal of Political Economy*, 104, 219-240.

Crucini, M., 1992, International risk sharing: A simple comparative test, Mimeo, Ohio State Univeristy.

Dumas, B., 1989, Two-person dynamic equilibrium in the capital market, *Review of Financial Studies*, 2, 157-188.

Dumas, B., 1994, Partial equilibrium versus general equilibrium models of the international capital market, In F. Van Der Ploeg, eds., *The handbook of international macroeconomics*, Oxford, UK, basil Blackwell.

Eichengreen, B., and A. K. Rose, 2001, Staying afloat when the wind shifts: External factors and emerging-market banking crises, In Calvo, G. A., M. Obstfeld and R. Dornbusch, eds., *Money, factor mobility and trade: Essays in honor of Robert A. Mundell*, The MIT Press.

Epstein, L. G., and S. E. Zin, 1989, Substitution, risk aversion, and the temporal behavior of asset returns: A theoretical framework, *Econometrica*, 57, 937-968.

Epstein, L. G., and S. E. Zin, 1991, Substitution, risk aversion, and the temporal behavior of asset returns: An empirical investigation, *Journal of Political Economy*, 99, 263-286.

French, K. R., and J. M. Poterba, 1991, Investor diversification and international equity markets, *American Economic Review*, 81 (May), 222-226.

Gould, J. P., 1968, Adjustment costs in the theory of investment of the firm, *Review of Economic Studies*, 35, 47-56.

Hansen, L. P., T. J. Sargent, and T. Tallarini, 1998, Robust permanent income and pricing, *Review of Economic Studies*, 66, 873-907.

Hayashi, F., 1982, Tobin's marginal q and average q: A neoclassical interpretation, *Econometrica*, 50 (January), 213-234.

Heaton, J., and D. Lucas, 1996, Evaluating the effects of incomplete markets on risk sharing and asset pricing, *Journal of Political Economy*, 104, 443-487.

Jermann, U., 1998, Asset pricing in production economies, *Journal of Monetary Economics*, 41, 257-275.

Kahneman, D., and A. Tversky, 1979, Prospect theory: An analysis of decision under risk, *Econometrica*, 47, 263-291.

Kaminsky, G. L., and C. M. Reinhart, C.A. Vegh, 2004, When it rains, it pours: Procyclical capital flows and macroeconomic policies, *NBER working paper*, No. 10780.

Karolyi, G. A., and R. M. Stulz, 2003, Are financial assets priced locally or globally? In G. Constantinides, M. Harris and R. Stulz, eds, *Handbook of the economics of finance*, Vol. IB, North-Holland, Amsterdam, 976-1020.

King, R., C. Plosser and S. Rebelo, 1988, Production growth and business cycle I: The basic neoclassical model, *Journal of Monetary Economics*, 21, 195-232.

Kydland, F. E., and E. C. Prescott, 1982, Time to build and aggregate fluctuations, *Econometrica*, 50 (6), 1345-1370.

Lettau, M., 2003, Inspecting the mechanism: Closed-form solutions for asset prices in real business cycle models, *Economic Journal*, 113, 550-575.

Li, Y., 2005, Global asset returns and risk sharing under habit formation, Mimeo, California State University.

Li, Y., and M. Zhong, 2004, The predictability international stock returns under habit formation and idiosyncratic risks, Mimeo, California State University.

Lintner, J., 1965, The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets, *Review of Economics and Statistics*, 47, 13-37.

Lucas, R. E., 1967, Adjustment costs and the theory of supply, *Journal of Political Economy*, 75, 321-334.

Lucas, R. E., 1978, Asset prices in an exchange economy, *Econometrica*, 46, 1429-1446.

Lucas, R. E., and E. C. Prescott, 1971, Investment under uncertainty, *Econometrica*, 39, 659-682.

Mankiw, N. G., 1986, The equity premium and the concentration of aggregate shocks, *Journal of Financial Economics*, 17, 211-219.

Mankiw, N. G., and S. Zeldes, 1991, The consumption of stockholders and non-stockholders, *Journal of Financial Economics*, 29, 97-112.

Mehra, R., and E. C. Prescott, 1985, The equity premium: A puzzle, *Journal of Monetary Economics*, 15, 145-161.

Neumeyer, P. A., and F. Perri, 2004, The business cycle in emerging economies, the role of the interest rates, *NBER working paper*, No. 10387.

Obstfeld, M., and K. Rogoff, 1996, *Foundations of international Macroeconomics*, The MIT Press.

Restoy, F., and G. M. Rockinger, 1994, On stock market returns and returns on investment, *Journal of Finance*, 49 (2), 543-556.

Rouwenhorst, G., 1995, Asset pricing implications of equilibrium business cycle models, In T.F. Cooley, eds, Frontiers of business cycle research, Princeton, Princeton University Press.

Sachs, J. D., and X. Sala-I-Martin, 1992, Fiscal federalism and optimum currency areas, In M. B. Canzoneri, V. U. Grilli, and P. R. Masson, eds., *Establishing a central bank: Issues in Europe and lessons from the U.S.* Cambridge, UK, Cambridge University Press.

Sharpe, W. F., 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance*, 19, 425-442.

Siegel, J. J., 1972, Risk, interest rates and the forward exchange, *Quarterly Journal of Economics*, 86 (May), 303-309.

Stockman, A. C., and H. Dellas, 1989, International portfolio nondiversification and exchange rate variability, *Journal of International Economics*, 26 (May), 271-289.

Stockman, A. C., and L. L. Tesar, 1995, Tastes and technology in a two-country model of the business cycle: Explaining international comovements, *American Economic Review*, 85 (March), 168-185.

Stulz, R. M., 1994, International portfolio choice and asset pricing: An integrative survey, *NBER working paper*, No. 4645.

Tesar, L. L., and I. Werner, 1995, Home bias and high turnover, *Journal of International Money and Finance*, 14 (August), 467-492.

Treadway, A., 1969, On rational entrepreneurial behavior and the demand for investment, *Review of Economic Studies*, 36, 227-240.

Uhlig, H., 1999, A toolkit for analyzing nonlinear dynamic stochastic models easily, In R. Marimon and A Scott, eds, *Computational methods for the study of dynamic economics*, Oxford, Oxford Univeristy Press.

Uribe, M., and V.Z. Yue, 2006, Country spreads and emerging countries: Who drives whom, *Journal of International Economics*, 69 (June), 6-36.

Uzawa, H., 1969, Time preference and the penrose effect in a two-class model of economic growth, *Journal of Political Economy*, 77, 628-652.

Weil, P., 1989, The equity premium puzzle and the risk-free rate puzzle, *Journal of Monetary Economics*, 24, 401-421.