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**Abstract**

Emerging countries that have defaulted on their debt repayment obligations in the past are more likely to default again in the future than are non-defaulters even with the same debt-to-GDP ratio. This paper explains this stylized fact within a dynamic stochastic general equilibrium framework by explicitly modeling renegotiations between a defaulting country and its creditors. The quantitative analysis of the model reveals that the equilibrium probability of default for a given debt-to-GDP level is weakly increasing with the number of past defaults, consistent with empirical observations. The equilibrium of the model also accords with an additional observed fact: a country for which default terms require less than a 100 percent recovery rate tends to pay a higher rate of return (relative to a risk-free rate) on subsequently issued debt than do defaulting countries that agree to a full recovery rate.

**JEL Classification Codes**: E43; F32; F34; G12

**Key words**: Serial default; Debt renegotiation; Past credit history; Recovery rates; Interest spreads

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# 1 Introduction

Emerging countries that have defaulted on their debt repayment obligations in the past are more likely to default again in the future than are non-defaulters with the same debt-to-GDP ratio. This paper explains this stylized fact within a dynamic stochastic general equilibrium framework that explicitly models renegotiations between a defaulting country and its creditors. Specifically, the model extends the existing literature by allowing the defaulter and creditors to bargain not just over recovery rates, but also over the rate of return offered on newly-issued debt. Quantitative analysis of the model reveals that the equilibrium probability of default for a given debt-to-GDP level is weakly increasing with the number of past defaults, consistent with empirical observations. The equilibrium of the model also accords with an additional observed trend: a country for which default terms require less than a 100 percent recovery rate tend to pay a higher rate of return (relative to a risk-free rate) on debt that is issued subsequently than do defaulting countries that agree to a full recovery rate. These findings are robust to extensions that allow the renegotiation outcome to be modeled more flexibly.

This paper deals with endogenous debt renegotiation after default in a standard dynamic model of defaultable debt. The renegotiation process involves Nash bargaining between the defaulting debtor and creditors over both the recovery rate and increases in rates of return on new debt. Evidence suggests that the spread between the rate of return on new debt and the risk-free rate increases after default more for defaulters that pay less than a full recovery rate than for defaulters that agree to repay all of the defaulted debt (i.e. a 100 percent recovery rate). Thus, it appears that, at least implicitly, a country that defaults negotiates with its creditors both over recovery rates and over future rates of return. This reflects a trade-off for defaulting country: the defaulted debt can be repaid in the present at a high short-run cost in return for only a small or even negligible deterioration in long-term credit condition; or the short-run benefit of repaying the debt only partially will be offset by having to pay lenders a higher rate of return on future issuances. The trade-off for creditors is symmetric: if they are not appeased by a full recovery of funds in the short term, they can attempt to recoup their losses by demanding higher rates of return for holding the country's bonds in the future.

The present paper seeks to incorporate these trade-offs facing the debtor and creditors during renegotiations following defaults. In the model, the endogenously-determined terms of renegotiations following default present the observed pattern, i.e. lower recovery rates are associated with larger increases in yield spreads. An emerging country that defaults once therefore pays a penalty either through a large recovery rate in the short term or through higher borrowing costs in the long term. If it chooses to repay less than full recovery rates, it will face high borrowing costs, which lead to increase the risks that the country will default again in the future. This mechanism drives the equilibrium serial default behavior in the model, and it is a plausible explanation of the pattern of repeat defaults observed in the data. Hence, the model is able to jointly explain both stylized facts of debt renegotiations and repeat defaults.

We embed the debt renegotiation in a dynamic sovereign debt model with endogenous defaults where an emerging country is subject to exogenous income shocks. This part of the model builds on recent quantitative analysis of sovereign debt such as Aguiar and Gopinath (2006), Arellano (2008) and Tomz and Wright (2007) which are based on classical setup of Eaton and Gersovitz (1981). At the renegotiation, creditors and defaulting country bargain over increases in rate of return on new debt together with recovery rates. Outcomes of the renegotiation represent trade-offs of both defaulting country and creditors, as indicated above. Total spread between the rate of return on new debt and the risk-free rate, incorporates not only the probability of future default but also impacts on increases in rate of return on new debt agreed at the past renegotiations.

Our paper is most closely related with Yue (2010), in which a dynamic model of defaultable debt is argumented with an endogenous treatment of debt renegotiation after default. Our model differs from her model in that we incorporate the effects of increases in rate of return on new debt. At the renegotiation, both parties bargain not only over recovery rates, but also over increases in rate of return on new debt. Therefore, its credit condition, i.e. borrowing cost of the country after re-entry to the market, depends on how much the country pays at the debt renegotiation. Increase in borrowing costs accompanied by repaying the debt only partially will lead to increase future default probability. In special case where the country always repays in full the level of defaulted debt, increases in rate of return on new debt will be close to zero. As impacts of additional default premia are totally negligible, results will be quite similar to ones in Yue (2010).

The rest of the paper is organized as follows. Section 2 reviews two strands of literature. Section 3 overviews stylized facts of debt negotiations and serial defaults. We provide our stochastic dynamic general equilibrium model in Section 4. We define recursive equilibrium of the model in Section 5. Quantitative analysis of the model is shown in Section 6. Model implications are indicated in

Section 7. A short conclusion summarizes the discussion. The computation algorithm is provided in Appendix A.

## 2 Literature Review

This paper is related to the literature of serial default. Reinhart, Rogoff and Savastano (2003) and Reinhart and Rogoff (2005) both advocate the role of past credit history in debt intolerance. On contrary, Eichengreen, Hausmann, and Panizza (2003) show that countries with "original sin", inability to issue bonds in their domestic currencies, must pay an additional risk premium when they borrow, increasing their solvency risks since the financial market knows this inability is a source of financial fragility. However, none provides economic models describing how weak credit history or "original sin" features are associated with serial defaults. With stochastic dynamic model, Kovrijnykh and Szentes (2007) explain the equilibrium default cycles, but they do not derive any relation between default occurrences and outcomes of negotiations. This paper improves these papers by explaining how outcomes of current debt renegotiation, such as additional spread premia, lead to higher probability of next default in future.

The other strand of literature models the sovereign default and renegotiation as a game between a sovereign debtor and its creditors.[1] Yue (2010) treats debt renegotiation process using a one-round Nash bargaining game. Moreover, Bai and Zhang (2010), Benjamin and Wright (2009) and Bi (2008) presume a multi-round bargaining to analyze delay in renegotiation. Benjamin and Wright (2009) assume that debtor and representative creditor randomly alternate in their ability to propose a bargaining outcome with changes in the probability of making future proposals serving to capture changes in bargaining power, while Bi (2008) supposes that lenders have an option to "pass" proposing to the debtor. Bai and Zhang (2010) focus on the role of information friction which generates the delay. Furthermore, Pitchford and Wright (2007) regard multi-creditor renegotiation process as a series of bilateral bargaining games to explain delays in renegotiation. Similarly, Kovrijnykh and Szentes (2007) also study multi-creditor renegotiation and makes the time of exclusion from the financial market endogenously and potentially long. Our paper differs from this literature in that

[1] Our borrowing environment, besides the debt renegotiation, is a version of the Eaton and Gersovitz (1981) model of defaultable debt, which has been used recently by a number of authors including Arellano (2008), Aguiar and Gopinath (2006), and Tomz and Wright (2007).

we concentrate on the observed pattern that lower recovery rates at the renegotiation are highly associated with larger increases in yield spreads.[2]

Lastly, our empirical finding is linked to studies analyzing the impacts of past defaults on future spreads. Ozler (1993) finds that past defaulters had to pay a premium on the interest rate for the sovereign debt issued in the 1970s and defaults previous to 1930 did not affect the premium paid but defaults after that did affect it. In a similar context, Ozler (1992) empirically shows that borrower's repeated experience in the market contributes significantly to the variation of spreads. Cantor and Packer also confirm that sovereign yields tend to rise as sovereign has a bad default history.[3] On the contrary, Lindert and Morton (1989) focusing on borrowing experience in late 1970s, find no evidence that defaulters were punished by creditors through higher interest rates on new loans. What is distinctive in our paper relative to previous work is that we analyze the deterioration of long-term borrowing in the short window after the renegotiations on bonds during 1986-2007 and how it differs in terms of agreed recovery rates.

# 3 Stylized facts

Evidence of serial defaults reflects that past defaulters are more likely to default in the future than are non-defaulters given the debt-to-GDP ratio. Moreover, from recent debt renegotiation episodes, we observe that lower recovery rates at the renegotiation are highly associated with larger increases in yield spreads between the rate of return on new debt and the risk-free rate.

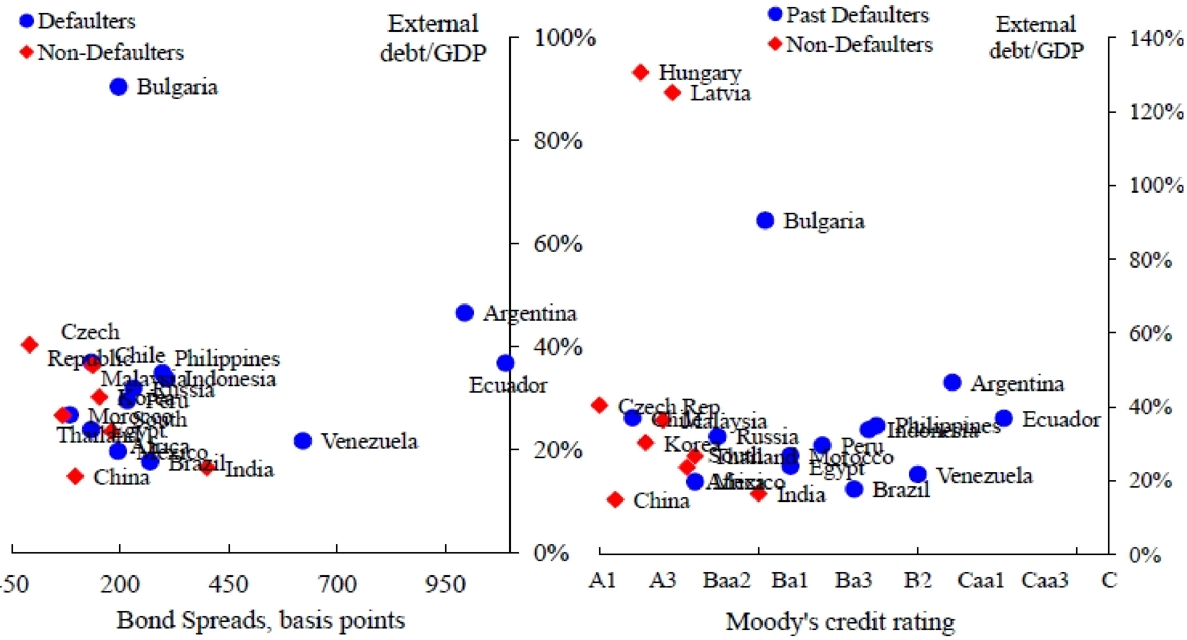
## 3.1 Evidence on serial defaults

In this subsection, we cover stylized facts of serial defaults, especially some features differing by countries' history of defaults.

[2] We assume that debt renegotiation takes place only once for each default.

[3] Trebesch (2009) indicates that unilateral, aggressive sovereign debt policies lead to a stronger decline in corporate access to external finance (loans and bond issuance).

Figure 1: External debt/GDP, bond spreads, and credit ratings, average 2005-2010




Source: Bloomberg, Datastream, De Paoli, Hoggarth and Saporta (2006), Haver, IMF WB Quarterly External Debt Statistics, IMF World Economic Outlook and Moody's.

Figure 1 reports external debt-to-GDP ratio, bond spreads and credit ratings. Bond spreads of past defaulters are higher than those of non-defaulters given external debt-to-GDP ratio. Past defaulters tend to suffer higher spreads on the newly issued bonds in the future after default, even if they have the same level of foreign debt relative to GDP as before. Similarly, past defaulters have lower credit ratings than non-defaulters, reflecting higher default probability.

Moreover, Reinhart, Rogoff and Savastano (2003) show that countries with a weak credit history may become more vulnerable even at much lower levels of external debt, relative to countries with a sound credit history. Table 1 illustrates predicted Institutional Investor ratings and debt intolerance regions for Argentina and Malaysia.[4]

[Insert Table 1 here]

[4] In order to address this point, Reinhart, Rogoff and Savastano (2003) use the estimated coefficients from the regression which analyzes the role of history and "club" in Institutional Investor Ratings (IIR), together with actual values of external debt/GNP, to predict values of the IIR for varying ratios of external debt/GNP for two countries, Argentina and Malaysia, which were member of "club B" based on their classifications.

It is apparent that precarious debt intolerance situation of Argentina is more severe than one of Malaysia.[5] Since Argentina is representative of many countries with a weak credit history and Malaysia is representative of countries with a sound credit history, this result reflects that the debt thresholds of countries with a weak credit history are lower than that of countries with a sound credit history. In other words, the default probability of countries with a weak credit history is higher than one of countries with a sound credit history, given the same level of debt-to-GNP.

In addition, Reinhart, Rogoff and Savastano (2003) report that defaulters repeat defaults or restructurings in short periods: emerging countries with at least one external default or restructuring since 1824, have experienced 5.2 defaults or restructurings in average as shown in Table 2.

[Insert Table 2 here]

### 3.2 Recent sovereign debt renegotiations

We start with an overview of recent debt renegotiation episodes. Table 3 summarizes 15 cases of expost-default and preemptive restructurings in the ten years from 1998 to 2007.[6] We present default year, defaulted debts, recovery rates, and increases in interest spreads for each episode. One feature which stands out is that recovery rates vary depending on the cases.

[Insert Table 3 here]

Furthermore, Figure 2 displays recovery rates and increases in spreads for 35 sovereign debt renegotiation episodes during 1986-2007.[7][8]

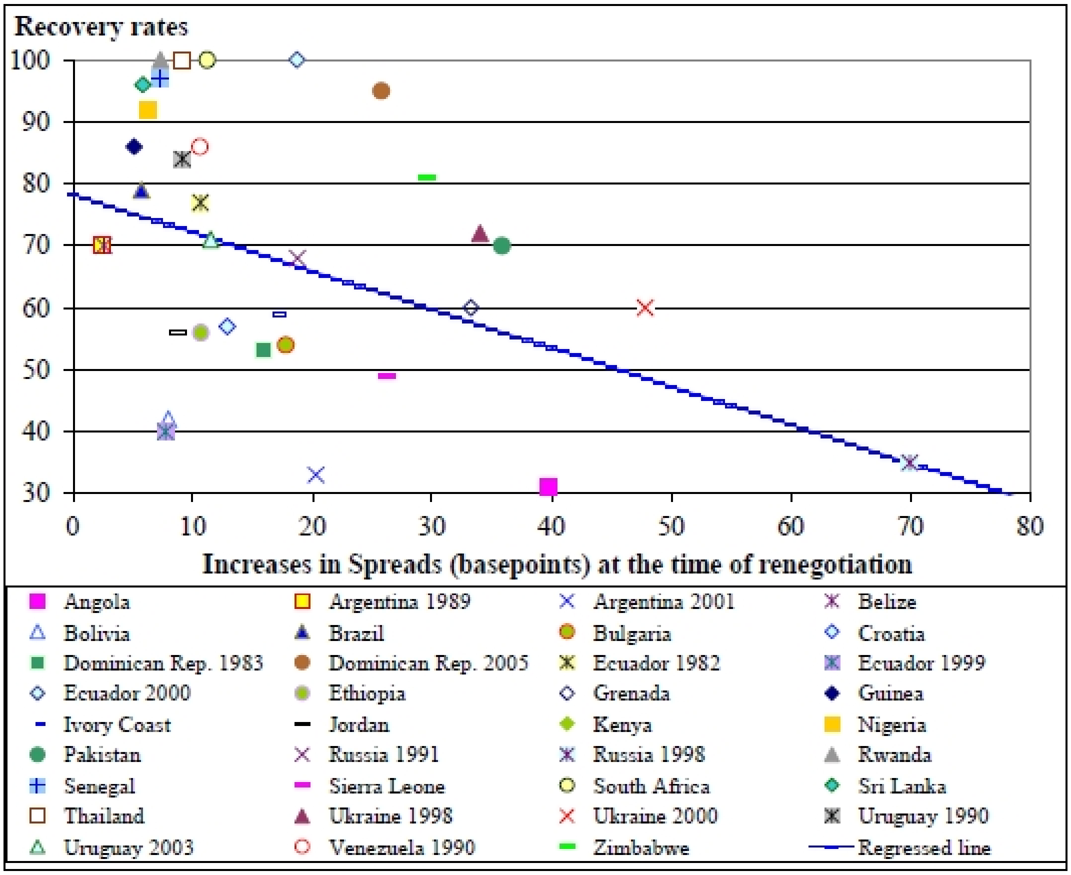
[5] Argentina only remains in the relatively safe "region **1**" as long as its external debt is below 15 percent of GNP, whereas Malaysia stays in "region **1**" up to a debt-to-GNP ratio of 30 percent, and it is still in the relatively safe "region **2**" with a debt of 35 percent of GNP.

[6] We exclude the cases of swap agreements and delay in payment such as Venezuela in 1995, 1998 and 2005, Peru in 2000 and Paraguay in 2003.

[7] For 6 cases such as Argentina 2001, Ecuador 1999, Pakistan, Russia 1998, Ukraine 1998, Uruguay 2003, we use recovery rates in Sturzenegger and Zettlemeyer (2008). Recovery rates for Grenada, Dominican Rep.2005 and Belize are from Bedford, Penalver and Salmon (2005). The remaining cases are based on Benjamin and Wright (2009).

[8] Sturzenegger and Zettelmeyer (2006, 2008) define recovery rates as the market value of the new instruments, plus any cash payment received, relative to the net present value (NPV) of the remaining contractual payments on the old instruments (inclusive of any principal or interest arrears). They attempt to compare the value of the new instruments to the value of the old debt in a situation in which the sovereign would not have defaulted. Contrary to that, Bedford, Penalver and Salmon (2005) and Benjamin and Wright (2009) define recovery rates as the market value of the new debt and cash received to the sum of *outstanding face value* of the old debt and past due interest. The justification for using the face value - apart from the fact that it makes recovery rates much easier to compute, since it is based only on the total volume of outstanding debt, not the payments terms of the old bonds - is that in a default situation, payments due under the old bonds are usually accelerated, so that the contractual right of the creditor shifts from being entitled to a future payment stream to the right to immediate payment of the principal.

Figure 2: Recovery rates and increases in spreads for recent debt renegotiations




Source: Bedford, Penalver and Salmon (2005), Benjamin and Wright (2009), Datastream, and Sturzenegger and Zettelmeyer (2006 and 2008)

We focus only on expost-default and preemptive renegotiation episodes in the sample periods, and we exclude examples of delays in payment such as Paraguay in 2003, and Venezuela in 1995, 1998, 2005, and swap agreement for Peru in 2000. We define "increase in spreads" as the difference in spreads between the time of renegotiation and one year before the renegotiation.[9][10] The fitted line is obtained by regressing recovery rates on increases in spreads controlling for actual detrended GDP and political indicators as indicated in the third column of Table 4. This negative relationship is robust even controlling for debt/GDP ratio and omitting an outlier case of Russia 1998 shown

[9] The bond spreads data are from the J.P. Morgan's Emerging Markets Bond Index Global (EMBIG) data for respective countries. Included in the EMBI Global are U.S.-dollar-denominated Brady bonds, Eurobonds, traded loans, and local market debt instruments issued by sovereign and quasi-sovereign entities. The spreads are computed as an arithmetic, market capitalization-weighte average of bond spreads over U.S. treasury bonds of comparable duration.

[10] According to J.P. Morgan (1999), a new issue that meets the EMBI Global's admission requirements is added to the index on the first monthend business date after its issuance, provided its issue date falls before the 15th of the month. A new issue whose issue date falls on or after the 15th of the month is added to the index on the last business day of the next month. Thus, the EMBI Global spreads reflect spreads on newly issued bonds.

in the fourth and fifth columns respectively.[11] These results reflect that lower recovery rates at the renegotiation are associated with larger increases in yield spreads between the rates of return on new debt and the risk-free rate. This presents a trade-off for defaulting countries; if the countries recover a larger fraction of debt at the renegotiations, long-term borrowing costs will be smaller. At the same time, we can interpret it as a trade-off of creditors. If the creditors receive payments for only a small fraction of defaulted debt, they can recoup their losses by demanding higher rates of return for the newly issued bonds.

Table 4: Regression results

| Variable[*1] | (1) Baseline | (2) Political factors | (3) w/o Russia 1998 | (4) with Debt/GDP |
|---|---|---|---|---|
| *Constant* | 80.72*** | 79.48*** | 79.26*** | 98.03*** |
| *Increases in spreads* | -0.71*** | -0.63** | -0.61** | -0.82** |
| *GDP Deviation*[*2] | 18.64 | 20.46 | 20.67 | -2.08 |
| *Debt/GDP ratio* | | | - | -0.36* |
| *Political System*[*3] | | -0.02 | -0.02 | 15.39* |
| *Years-in-term*[*4] | | -0.34 | -0.33 | 1.47 |
| *Percent-of-votes*[*5] | | -4.2E-3 | -4.2E-3 | -2.6E-3 |
| Sample | 33 | 33 | 32 | 22 |

Source: Bedford, Penalver and Salmon (2005), Benjamin and Wright (2009), Datastream, Sturzenegger and Zettelmeyer (2006 and 2008), Reinhart and Rogoff (2010), and World Bank The Database for Political Institutions (PDI). Note: Standard errors are in parentheses. ***, **, * show significance at 1, 5, and 10 percent levels respectively. *1: All regression results are based on least square estimations. *2: GDP deviation from the trend is a percentage deviation from the trend obtained by applying the Hodrick-Prescott (H-P) filter. *3: "Political system" indicator is differenciated by parliamentary, assembly-elected president, and presidential ststems. *4: "Years-in-term" indicator defines years left in current term. *5: "Percent-of-votes" indicator specifies percentages of votes the current president got in the 1st round of election.

[11] When we define "increase in spreads" for 2-year window, such as the difference between one year before and after the renegotiation, we still obtain the negative relationship with a flatter slope shown in Appendix B.

# 4 Model environment

The basic structure of the model follows previous work that extends the model of sovereign default by Eaton and Gersovitz (1981) and applies its quantitative analysis. Among these studies, the closest reference to our paper is Yue (2010). The distinctive feature in our model with respect to her model is that we introduce effects of increases in rate of return on new debt after the re-entry to the market. Since both recovery rates and increases in rate of return on new debt are determined endogenously, how much the country pays at the renegotiation will affect its credit condition in the future, i.e. borrowing costs of the country after re-entry to the market, which will have impacts on default probability.[12]

## 4.1 General points

The model analyzes sovereign default and negotiation in a stochastic dynamic equilibrium model. We consider a risk-averse country that can't affect world risk-free interest rate. The country's preference is given by following utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where $0 < \beta < 1$ is a discount factor, $c_t$ denotes consumption in period $t$, and $u(.)$ is its one-period utility function, which is continuous, strictly increasing and strictly concave and satisfies the Inada conditions. A discount factor reflects both pure time preference and probability that the current sovereignty will survive into next period.

In each period, the country starts with its credit history $h_t$, which satisfies $h_t \in H$ where $H = [0, 1, 2, ..., h_{\max}]$. The credit history expresses number of debt renegotiations the country has experienced in the past.[13] The reason why we assume multi-state credit history rather than two-state credit history as in Yue (2010) is to analyze how the outcomes of past debt renegotiations associated with defaults affect the probability of next default. Moreover, we assume that the credit

[12] On contrary, Yue (2010) has not taken into account impacts of increases in rate of return on new debt. In her model, both parties negotiate over only recovery rate after default. The reason why effects of increases in rate of return on new debt are missing in her model is that the country's credit condition will always return to the same level irrelevant to recovery rate which is determined at the renegotiation.

[13] The model simply distinguishes $h_t = 0$ and $h_t > 0$ as the non-default history and defaulting history, not as the non-exclusion and exclusion periods. After default and renegotiation, the country will be excluded from the market for a short period.

history reverts with exogenous probability $\chi$ conditional on that the country chooses to pay the spread returns after defaults.[14]

The country receives an exogenous income shock $y_t$. Income shock $(y_t)$ is stochastic, drawn from a compact set $Y = [y_{\min},\ y_{\max}] \subset \mathbb{R}_+$. $\mu(y_{t+1}|y_t)$ is the probability distribution of a shock $y_{t+1}$ conditional on previous realization $y_t$.

There is an infinite number of investors who are risk-neutral and behave competitively in the international capital market. They have perfect information on the country's assets, credit history, income shocks and additional spread premia agreed to at previous debt renegotiation. We also assume that they can borrow or lend as much as needed at a constant risk-free interest rate $(r)$ in the market. Since they are symmetric and similarly ranked, we can interpret them as "a representative investor" lending money to the country. The country borrows the money from the same representative investor though bond exchanges even after it defaults.[15] As investors are able to collude at the debt renegotaition, "a representative investor" has a bargaining power at the renegotiation in order to impose higher spreads on future bonds, though its bargaining power is low compared to that of country.[16] Moreover, we assume that all the investors behave in the same manner: they all lend the money to the country every time the country issues bonds, and there is no sub-group of investors who behave differently from the majority of investors such as they still lend money to the country even if the country defaults and refuse to negotiate with the majority of investors.[17]

The international capital market is incomplete. The country and foreign investors can borrow and lend only via one-period zero-coupon bonds where $b_{t+1}$ denotes amount of bonds to be repaid next period. When the country purchases bonds, $b_{t+1} > 0$, and when it issues new bonds, $b_{t+1} < 0$.

[14] Following the consumer defaults as in Chatterjee et al (2007), we assume that the record of the recent default remains on the country's credit history for only a finite number of years.

[15] The country negotiates wtih the creditors who hold its debts and the creditors receive the recovered debts as in current model. Thus, it is true that the country borrows again from the same creditors. While in the reality, there exists the secondary markets where the creditors can sell and purchase the exchanged bonds, the current model abstracts this feature.

[16] In usual debt restructurings, the bond holders organize a committee, which conduct research on the soverign and consolidate creditors' view to faciliate the discussion. Given the restructuring plan proposed by the soverign, all creditors vote on it. If a critical mass of the creditor approve, the proposal is passed and finalized. Otherwise, the government has to revise the proposal until it passes. In order to smooth the renegotiation, the committee plays an important role to reflect the creditors' view on the sovereign's proposal. Thus, it is identical to say that a committee has a barganing power, but it is relatively low as the committee has a difficulity to consolidate views across investors. Rieffel (2003) provides description of sovereign debt renegotiation.

[17] It is true that the current model abstracts elements of entry of new creditors and existance of secondary markets. The rationale of this is to keep the model trackable to deliver the main implications. Thus, if there has not been a default in the past, creditors behave competitively, making zero profits. When the sovereign defaults, they are to collude and ask the sovereign for higher spreads in future.

The set of amount of bonds is $B = [b_{\min},\ b_{\max}] \subset \mathbb{R}$ where $b_{\min} \leq 0 \leq b_{\max}$. The upper bound is the highest level of assets that the country can accumulate and the lower bound is the highest level of debts that the country can hold. We assume $q(b_{t+1}, h_t, y_t)$ is the price of a bond with asset position $(b_{t+1})$, credit history $(h_t)$, and income level $(y_t)$. The bond price will be determined in equilibrium.

We assume that foreign investors always commit to repay their debt. However, the country is free to decide whether to repay its debt or to default. If the country chooses to repay its debt, it will preserve access to the international capital market next period.

If the country chooses not to pay its debt, it is subject to both exclusion from the international capital markets and direct output cost.[18][19] When a default occurs, the country and foreign investors negotiate reduction of unpaid debt via Nash bargaining. At the renegotiation, both recovery rates and additional spread premia on the newly issued bonds are agreed to by both parties.[20] The country regains access to the market after excluded from the market a short period, but the country's credit history records the current debt renegotiation.[21] In order to avoid permanent exclusion from the international capital market, the country has an incentive to negotiate over haircut rates (recovery rates) and additional default premia. From foreign investors' point of view, Foreign investors want to maximize the payment from recovered debt and spread returns on newly issued bonds after default, so they are also willing to negotiate over reduction of unpaid debt.

All the information on the country's asset, credit history, and income realization is perfect.[22]

## 4.2 Timing of the model

Timing of decisions within each period is summarized in Figure 3.

[18] There are several estimates for output loss at the time of default. Sturzeneger (2002) esimates output loss as around 2% of GDP. On contrary, De Paoli, Hoggarth, and Saporta (2006) suggest that the output loss in the wake of sovereign default apprears to be very large - around 7% a year on the median measure - as well as long lasting.

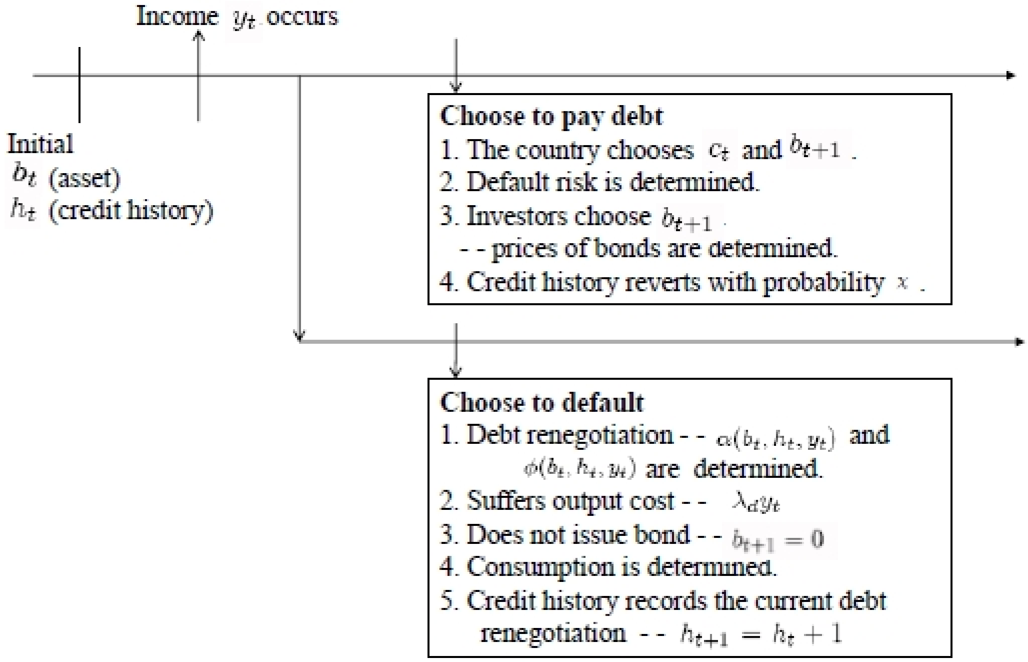
[19] Mendoza and Yue (2011) explain that output cost associated sovereign default is efficiency loss of production through two channels: inefficient production using domestic inputs which are imperfect substitutable with imported inputs, and labor reallocation away from final good production.

[20] After the bond exchanges are annouced, the creditors at the market price the yields and spreads of exchanged bonds depending on recovered level of bonds. At each round of debt renegotaitions, both parties take into account the possible impacts of spreads depending on proposed recovery rates. Thus, it is identical to say that both recovery rates and increases in spreads are determined by both parties at the renegotaition.

[21] In our model, the period of exclusion from the market is fixed as in Yue (2010). Bi (2008) and Benjamin and Wright (2009) replicate the endogenous periods of exclusion from the market by assuming multi-round renegotaitions.

[22] Alfaro and Kanczuk (2005) and D'Erasmo (2011) develop a model of sovereign debt with heterogenous governments where a players' type changes over time and there is private information. Focusing on consumer credits, Chatterjee, Corbae and Rio Rull (2010) consider an environment with heterogenous borrowers and private information. These papers have advantages of incorporating reputation effects, but have some weakness of not taking into account persistent impacts of past events.

Figure 3: Timing of the model




The country starts the current period with initial asset position ($b_t$) and credit history ($h_t$). After observing the current income shock ($y_t$), the country chooses either to pay the debt or to default. If the country decides to pay the debt, given the bond price schedule, the country chooses next period assets ($b_{t+1}$) and consumption ($c_t$). Then the default probability and price of bond are determined by the market equilibrium. Given the price of bonds, foreign investors choose $b_{t+1}$ consistent with belief of default probability. Its credit history will be upgraded with exogenous probability $\chi$.

If the country chooses to default, the country and foreign investors negotiate a debt reduction. Both recovery rates $\alpha(b_t, h_t, y_t)$, and additional spread premia $\phi(b_t, h_t, y_t)$ are agreed to by both sides. After negotiation, the country pays the recovered debt $\alpha(b_t, h_t, y_t)b_t$ and suffers direct output cost due to default, $\lambda_d y_t$. The country can not raise funds in the international capital market this period ($b_{t+1} = 0$), but will regain access to the market next period. The consumption level is $c_t = (1 - \lambda_d)\, y_t + \alpha(b_t, h_t, y_t)b_t$. The country's credit history records the current debt renegotiation $h_{t+1} = h_t + 1$.

# 5 Recursive Equilibrium

In this section, we define stationary recursive equilibrium of the model.

## 5.1 Sovereign country's problem

The country's problem is to maximize its expected lifetime utility. The country makes its default decision and determines its assets for next period ($b_{t+1}$), given its current asset position ($b_t$), credit history ($h_t$), and income shock ($y_t$). Let $V(b_t, h_t, y_t)$ be one value function of the country that starts the current period with initial asset ($b_t$), credit history ($h_t$), and income ($y_t$).

Given with the bond market price $q(b_{t+1}, h_t, y_t)$, debt recovery rates $\alpha(b_t, h_t, y_t)$, and additional spread premia $\phi(b_t, h_t, y_t)$, the country solves its optimization problem. We assume both the debt recovery rates and additional spread premia determined at current debt negotiation depend on these state variables.

For simplicity, we consider the problem with $h_t = 0$, indicating that the country has never experienced the debt renegotiation in the past. Later, we consider the problem with general cases $h_t \geq 1$.

For $b_t \geq 0$ ($h_t = 0$), the country has savings. The country receives payments from foreign investors and determines its next-period asset position $b_{t+1}$ and its consumption $c_t$ to maximize utility, given the price of bond $q(b_{t+1}, 0, y_t)$. Thus the value function is

$$V(b_t, 0, y_t) = \max_{c_t, b_{t+1}} u(c_t) + \beta \int_Y V(b_{t+1}, 0, y_{t+1}) d\mu(y_{t+1}, y_t) \tag{1}$$

$$s.t. \quad c_t + q(b_{t+1}, 0, y_t) b_{t+1} = y_t + b_t$$

For $b_t < 0$ ($h_t = 0$), the country has the debt. If the country decides to pay its debt, it chooses its next-period asset position $b_{t+1}$ and consumption $c_t$. On contrary, if the country chooses to default, it become financial autarky for this period and its credit history deteriorates to $h_{t+1} = 1$ next period. Due to agreement in debt renegotiation, the country must pay $-\alpha(b_t, 0, y_t) b_t$ in current period, and it regains access to the international capital market next period with history $h_{t+1} = 1$. With credit history $h_{t+1} = 1$, when the country issues new bonds, it must pay interests on newly issued bonds equal to the sum of the risk-free rate ($r$) and the spread premia agreed at the last renegotiation ($\phi(b_{t+1}, 1, y_{t+1})$). Thus, the price of bonds after default $q(b_{t+2}, 1, y_{t+1})$ incorporates $\phi(b_{t+1}, 1, y_{t+1})$.

Given the option to default, $V(b_t, 0, y_t)$ satisfies

$$V(b_t, 0, y_t) = \max\left[V^R(b_t, 0, y_t), V^D(b_t, 0, y_t; \alpha(b_t, 0, y_t), \phi(b_t, 0, y_t))\right] \tag{2}$$

where $V^R(b_t, 0, y_t)$ is the value associated with paying debt:

$$V^R(b_t, 0, y_t) = \max_{c_t, b_{t+1}} u(c_t) + \beta \int_Y V(b_{t+1}, 0, y_{t+1}) d\mu(y_{t+1,} y_t) \tag{3}$$

$$s.t. \quad c_t + q(b_{t+1}, 0, y_t)b_{t+1} = y_t + b_t$$

and $V^D(b_t, 0, y_t; \alpha(b_t, 0, y_t), \phi(b_t, 1, y_t))$ is the value associated with default given with debt recovery schedule $\alpha(b_t, 0, y_t)$, and additional spread premia $\phi(b_t, 1, y_t)$ which will be determined at renegotiation after current default.

$$V^D\left(b_t, 0, y_t; \alpha(b_t, 0, y_t), \phi(b_t, 1, y_t)\right) = u\left((1 - \lambda_d)y_t + \alpha(b_t, 0, y_t)b_t\right) + \beta \int_Y V(0, 1, y_{t+1}) d\mu(y_{t+1}, y_t) \tag{4}$$

where $V(0, 1, y_{t+1})$ is value function next period with credit history $h_{t+1} = 1$ defined below in general cases with $h_t \geq 1$ and $-\alpha(b_t, 0, y_t)b_t$ is the amount of defaulted debt which the country repays at the debt negotiation and $\lambda_d y_t$ denotes output costs which the country suffers due to defaults.

Next we consider the problem with $h_t \geq 1$ expressing that the country has experienced the debt renegotiation at least once in the past.

For $b_t \geq 0$ ($h_t \geq 1$), the country has savings. The country receives payments from foreign investors and determines its next-period asset position ($b_t$) and its consumption ($c_t$) to maximize utility. Thus the value function is

$$V(b_t, h_t, y_t) = \max_{c_t, b_{t+1}} u(c_t) + \beta \int_Y V(b_{t+1}, h_t, y_{t+1}) d\mu(y_{t+1,} y_t) \tag{5}$$

$$s.t. \quad c_t + q(b_{t+1}, h_t, y_t)b_{t+1} = y_t + b_t$$

Note that credit history remains unchanged in next period $h_{t+1} = h_t$.

For $b_t < 0$ ($h_t \geq 1$), the country has the debt. The country can borrow money from the foreign investors, but the country needs to pay not only the risk-free interest rate ($r$), but also additional spread premia $\phi(b_t, h_t, y_t)$ which was agreed to by both the country and foreign investors at the time of previous debt renegotiations. Thus, the price of bonds $q(b_{t+1}, h_t, y_t)$ is different from the one with history $h_t = 0$, defined as $q(b_{t+1}, 0, y_t)$, as it incorporates the effects of additional default premia associated with deteriorated credit history. As in the case of history $h_t = 0$, the country chooses either to pay the debt or to default. The values are as before:

$$V(b_t, h_t, y_t) = \max\left[V^R(b_t, h_t, y_t), V^D\left(b_t, h_t, y_t; \alpha(b_t, h_t, y_t), \phi(b_t, h_t + 1, y_t)\right)\right] \tag{6}$$

where $V^R(b_t, h_t, y_t)$ is the value associated with paying debt with history $h_t \geq 1$,

$$V^R(b_t, h_t, y_t) = \max_{c_t, b_{t+1}} u(c_t) + \beta \begin{bmatrix} (1-\chi) \int\limits_Y V(b_{t+1}, h_t, y_{t+1}) d\mu(y_{t+1}, y_t) \\ +\chi \int\limits_Y V(b_{t+1}, h_t - 1, y_{t+1}) d\mu(y_{t+1}, y_t) \end{bmatrix} \tag{7}$$

$$s.t. \quad c_t + q(b_{t+1}, h_t, y_t) b_{t+1} = y_t + b_t$$

Note that with exogenous probability $\chi$, the country's credit history next period will revert due to limited memory of the investors as $h_{t+1} = h_t - 1$. Otherwise, it remains constant as $h_{t+1} = h_t$.

$V^D(b_t, h_t, y_t; \alpha(b_t, h_t, y_t), \phi(b_t, h_t + 1, y_t))$ is the value associated with default given with debt recovery schedule $\alpha(b_t, h_t, y_t)$, and additional spread premia agreed after current default $\phi(b_t, h_t{+}1, y_t)$ which are defined below:

$$V^D\left(b_t, h_t, y_t; \alpha(b_t, h_t, y_t), \phi(b_t, h_t + 1, y_t)\right) = u\left((1-\lambda_d) y_t + \alpha(b_t, h_t, y_t) b_t\right) + \beta \int\limits_Y V\left(0, h_t + 1, y_{t+1}\right) d\mu(y_{t+1}, y_t) \tag{8}$$

where $V(0, h_t + 1, y_{t+1})$ is the value function next period with credit history $h_{t+1} = h_t + 1$ and $-\alpha(b_t, h_t, y_t) b_t$ is amount of defaulted debt which the country recovers after negotiation.

Every time (at period $t$) the country defaults, its credit history records the current debt renegotiation $h_{t+1} = h_t + 1$. Thus, the credit condition, i.e. borrowing costs of the country after re-entry to the market depends on how much the country pays at the renegotiation. When the country issues new bonds after it defaults, it must pay returns based on the risk-free rate and the sum of additional spread premia, which are determined at the previous debt renegotiations.

The country's default policy can be characterized by default sets $D(b_t, h_t) \subset Y$, defined as the set of income shock $y$'s for which default is optimal given the debt position $b_t$, and credit history $h_t$.

$$D(b_t, h_t) = \{y_t \in Y : V^R(b_t, h_t, y_t) < V^D(b_t, h_t, y_t; \alpha(b_t, h_t, y_t), \phi(b_t, h_t + 1, y_t))\} \tag{9}$$

Furthermore, we define an indicator of non-defaulting given initial asset position ($b_t < 0$), credit history ($h_t$), and income level ($y_t$) as follows;

$$I(b_t, h_t, y_t) = \begin{Bmatrix} 1 & if & y_t \notin D(b_t, h_t) \\ 0 & if & y_t \in D(b_t, h_t) \end{Bmatrix}$$

Finally, based on the policy function of asset position derived above ($b_{t+1}(b_t, h_t, y_t)$) and non-defaulting indicator $I(b_t, h_t, y_t)$, we define discounted value of expected amount of debt which will be paid to investors next period as:

$$P(b_t, h_t, y_t) = \frac{1}{1+r} \int_Y I\left(b_{t+1}(b_t, h_t, y_t), h_t, y_{t+1}\right) b_{t+1}(b_t, h_t, y_t) d\mu(y_{t+1}, y_t) \tag{10}$$

Note that we use the discount factor for foreign investors ($\frac{1}{1+r}$), not the discount factor for the country ($\beta$).

### 5.2 Debt renegotiation problem

The debt renegotiation takes a form of generalized Nash bargaining game. Not only the recovery rate, but also additional spread premia are agreed to by both parties. This is because foreign investors will obtain interest returns every time the country issues new bonds after current default as long as the country does not default again. From the country's perspective, it has to pay interests on bonds every time it issues new bonds after renegotiation, unless it chooses to remain in the financial autarky permanently.

After debt renegotiation, the country pays a fraction $\alpha(b_t, h_t, y_t)$ of defaulted debt. The value of the country after the renegotiation is defined above;

$$V^D(b_t, h_t, y_t; \alpha(b_t, h_t, y_t), \phi(b_t, h_t + 1, y_t)) = u\left((1 - \lambda_d)y_t + \alpha(b_t, h_t, y_t)b_t\right) + \beta \int_Y V\left(0, h_t + 1, y_{t+1}\right) d\mu(y_{t+1}, y_t)$$

Needless to say, this value takes into account the impact of both debt reduction to $-\alpha(b_t, h_t, y_t)b_t$, and additional spread premia $\phi(b_t, h_t + 1, y_t)$ which will be agreed at current debt negotiation.

Foreign investors obtain the present value of the reduced debt $-\alpha(b_t, h_t, y_t)b_t$ and interests on newly issued bonds after debt negotiation. The present value of expected payment of bonds which investors receive in the future after the country's re-entry to the market, can be defined in the following recursive form:

$$R(b_t, h_t, y_t) = P(b_t, h_t, y_t) + \frac{1}{1+r} \int\limits_Y R(b_{t+1}, h_t, y_{t+1}) d\mu(y_{t+1}, y_t) \tag{11}$$

$$s.t. \quad b_{t+1} = b^*_{t+1}(b_t, h_t, y_t),$$

where $P(b_t, h_t, y_t)$ is the discounted value of expected amount of bonds which are returned in next period defined in equation (10) and $b^*_{t+1}(b_t, h_t, y_t)$ is policy function of the country if it chooses not to default ($h_{t+i} = h_t$).

We assume that debt negotiation takes place only once for each default event. The threat point of the bargaining game is that the country stays in permanent autarky and the foreign investors get nothing. Moreover, we assume that impose direct sanctions $\lambda_s y_t$ on the country, which is in addition to the defaulting country's direct output cost $\lambda_d y_t$ if the country chooses not to negotiate. The expected value of autarky for the country, $V^{AUT}(y_t)$ is given by following expression;

$$V^{AUT}(y_t) = u((1 - \lambda_s - \lambda_d)y_t) + \beta \int\limits_Y V^{AUT}(y_{t+1}) d\mu(y_{t+1,} y_t) \tag{12}$$

We consider one-round bargaining since one-round bargaining keeps the model tractable as there is no need to consider multiple rounds of bargaining or the debt arrears based on different reduction schedules.[23]

For any debt recovery rate $a_t$ and additional spread premia $sp_t$, we denote the country's surplus

[23] Bi (2008) and Benjamin and Wright (2009) analyze multi-round bargaining to consider delay in renegotiation. Based on the assumption that the lenders have an option to "pass" proposing to the debtor, Bi (2008) argues that both parties can be better off by waiting and dividing a larger "cake" as it takes time for the economy to recover. On contrary, Benjamin and Wright (2009) assume that the debtor and representative creditor randomly alternate in their ability to propose a bargaining outcome with a changes in the probability of making future proposals serving to capture changes in bargaining power. They find that both parties find it optimal to postpone renegotiation until future default risk is low since the debtor's ability to share the future surplus created by by a debt renegotiation is limited by future default risk.

in Nash bargaining by $\Delta^B(a_t, sp_t; b_t, h_t, y_t)$, which is the difference between the value of accepting a proposal of debt recovery rate $a_t$ and additional spread premia $sp_t$, and the value of rejecting it, given the country's debt level ($b_t$), credit history ($h_t$), and income level ($y_t$).

$$\Delta^B(a_t, sp_t; b_t, h_t, y_t) = V^D(b_t, h_t, y_t; \alpha(b_t, h_t, y_t), \phi(b_t, h_t + 1, y_t)) - V^{AUT}(y_t) \tag{13}$$

The surplus to the country comes from two sources. First, the country will be able to issue bonds again from the following period, though its credit history deteriorates. Also, the direct cost to output is smaller under renegotiations because no sanctions are imposed.

On contrary, the surplus to investors is the present value of the sum of recovered debt and interest returns on newly issued bonds after renegotiation:

$$\Delta^L(a_t, sp_t; b_t, h_t, y_t) = -a_t b_t - sp_t R(b_t, h_t + 1, y_t) \tag{14}$$

where interest returns are evaluated with expected payment incorporating the future default choices of the country as in equation (11).

We assume that the country has a bargaining power $\theta$ and foreign investors have a bargaining power $(1 - \theta)$. A bargaining power parameter $\theta$ summarizes the institutional arrangement of debt negotiation. To ensure that the bargaining problem is well defined, we define the bargaining power set $\Theta \subset [0, 1]$ such that for $\theta \in \Theta$ the negotiation surplus has a unique optimum for any asset position ($b_t < 0$) , its history ($h_t$), income level ($y_t$).

Given the country's asset level ($b_t < 0$), its credit history ($h_t$), and income level ($y_t$), recovery rates $\alpha(b_t, h_t, y_t)$ and additional spread premia $\phi(b_t, h_t + 1, y_t)$ solve the following bargaining problem:

$$\begin{Bmatrix} \alpha(b_t, h_t, y_t) \\ \phi(b_t, h_t + 1, y_t) \end{Bmatrix} = \arg\max_{a_t, sp_t} \left[ \left(\Delta^B\left(a_t, sp_t; b_t, h_t, y_t\right)\right)^{\theta} \left(\Delta^L\left(a_t, sp_t; b_t, h_t, y_t\right)\right)^{1-\theta} \right] \tag{15}$$

$$s.t. \quad \Delta^B(a_t, sp_t; b_t, h_t, y_t) \geq 0$$

$$s.t. \quad \Delta^L(a_t, sp_t; b_t, h_t, y_t) \geq 0$$

Note that $\phi(b_t, h_t{+}1, y_t)$ is a function specifying state-variant contracts depending on future streams of

$b_t$ and $y_t$.[24] Since the set of both debt recovery schedule and additional spread premia that maximize total negotiation surplus conditional on the country's asset level, credit history, and income level, negotiation outcome provides better insurance to the country in the case of default.

### 5.3 Foreign investors' problem

For the cases with $h_t \geq 1$, our derived bond price incorporates the effects of additional spread premia agreed at previous debt renegotiations, which are the new elements in our model. First we consider foreign investors' problem given the country's credit history $h_t = 0$.

With the country's credit history $h_t = 0$, taking the bond price function as given, foreign investors choose the amount of asset $(b_{t+1})$ that maximizes their expected profit $\pi(b_{t+1}, 0, y_t)$, given by

$$\pi(b_{t+1}, 0, y_t) = \left\{ \begin{array}{ll} q(b_{t+1}, 0, y_t)b_{t+1} - \frac{1}{1+r}b_{t+1} & if \quad b_{t+1} \geq 0 \\ \frac{[1-p(b_{t+1},0,y_t)+p(b_{t+1},0,y_t)\gamma(b_{t+1},0,y_t)]}{1+r}(-b_{t+1}) - q(b_{t+1}, 0, y_t)(-b_{t+1}) & otherwise \end{array} \right\} \tag{16}$$

where $p(b_{t+1}, 0, y_t)$ and $\gamma(b_{t+1}, 0, y_t)$ are the expected default probability and expected recovery rates respectively for country with asset position $(b_{t+1} < 0)$, credit history $(h_t = 0)$, income level $(y_t)$, and $r$ is risk-free rate.

Since we assume that the market for new sovereign bonds is completely competitive, foreign investors' expected profit is zero in equilibrium. Using the zero expected profit condition, we get

$$q(b_{t+1}, 0, y_t) = \left\{ \begin{array}{ll} \frac{1}{1+r} & if \quad b_{t+1} \geq 0 \\ \frac{[1-p(b_{t+1},0,y_t)+p(b_{t+1},0,y_t)\gamma(b_{t+1},0,y_t)]}{1+r} & otherwise \end{array} \right\} \tag{17}$$

When the country buys bonds from foreign investors $b_{t+1} \geq 0$, the sovereign bond price is equal to the price of risk-free bond, $\frac{1}{1+r}$. When the country issues bonds to foreign investors $b_{t+1} < 0$, there is default risk, and the bond is priced to compensate foreign investors for this. Since $0 \leq p(b_{t+1}, 0, y_t) \leq 1$ and $0 \leq \gamma(b_{t+1}, 0, y_t) \leq 1$, the bond price $q(b_{t+1}, 0, y_t)$ lies in $\left[0, \frac{1}{1+r}\right]$.

Next, we consider foreign investors' problem for general cases with the country's history $h_t \geq 1$. Note that the borrowing costs of the country is denoted by $1 + r + \phi(b_t, h_t, y_t)$ which include the additional spread premia agreed at the previous debt renegotiations. Given the borrowing costs, together

[24] As the credit history keeps track of timing of default and debt renegotiation and is reverted with exogenous probability, the spread premia are pinned down by both current level of debt $(b_t)$ and income $(y_t)$ together with credit history. Thus, value functions of sovereign do not need the interest rates as additional state.

with the bond price $q(b_{t+1}, h_t, y_t)$, foreign investors maximize their expected profit $\pi(b_{t+1}, h_t, y_t)$, given by

$$\pi(b_{t+1}, h_t, y_t) = \begin{Bmatrix} q(b_{t+1}, h_t, y_t)b_{t+1} - \frac{1}{1+r}b_{t+1} & if \quad b_{t+1} \geq 0 \\ \frac{[1-p(b_{t+1},h_t,y_t)+p(b_{t+1},h_t,y_t)\gamma(b_{t+1},h_t,y_t)]}{1+r+\phi(b_t,h_t,y_t)}(-b_{t+1}) - q(b_{t+1}, h_t, y_t)(-b_{t+1}) & otherwise \end{Bmatrix} \tag{18}$$

where $p(b_{t+1}, h_t, y_t)$ and $\gamma(b_{t+1}, h_t, y_t)$ are as above. Using the zero profit condition, we obtain

$$q(b_{t+1}, h_t, y_t) = \begin{Bmatrix} \frac{1}{1+r} & if \quad b_{t+1} \geq 0 \\ \frac{[1-p(b_{t+1},h_t,y_t)+p(b_{t+1},h_t,y_t)\gamma(b_{t+1},h_t,y_t)]}{1+r+\phi(b_t,h_t,y_t)} & otherwise \end{Bmatrix} \tag{19}$$

When the country issues bonds to foreign investors, the bond price $q(b_{t+1}, h_t, y_t)$ lies in $\left[0, \frac{1}{1+r+\phi(b_t,h_t,y_t)}\right]$ since $0 \leq p(b_{t+1}, h_t, y_t) \leq 1$ and $0 \leq \gamma(b_{t+1}, h_t, y_t) \leq 1$. Thus, the bond price incorporates the additional default premia $\phi(b_t, h_t, y_t)$ due to the previous debt renegotiations; the price of bonds decreases as additional spread premia increase.

Moreover, for any credit history $(h_t)$, interest rate on sovereign bonds is defined as follows; $r^S(b_{t+1}, h_t, y_t) = \frac{1}{q(b_{t+1},h_t,y_t)} - 1$. It is bounded below by the risk-free rate $(r)$. We define the country's total spreads which is a difference between country's interest rate and the risk-free rate,

$$s(b_{t+1}, h_t, y_t) = \frac{1}{q(b_{t+1}, h_t, y_t)} - 1 - r \tag{20}$$

### 5.4 Recursive equilibrium

We define a stationary recursive equilibrium of the model.

**Definition 1** *:A **recursive equilibrium** is a set of functions for, the country's value function $V^*(b_t, h_t, y_t)$ (together with $V^{*R}(b_t, h_t, y_t)$ and $V^{*D}(b_t, h_t, y_t)$), asset position $b^*_{t+1}(b_t, h_t, y_t)$, consumption $c^*_t(b_t, h_t, y_t)$, default set $D^*(b_t, h_t)$, discounted expected payment $P^*(b_t, h_t, y_t)$, recovery rate $\alpha^*(b_t, h_t, y_t)$, additional spread premia $\phi^*(b_t, h_t, y_t)$, bond price function $q^*(b_{t+1}, h_t, y_t)$, and total spread $s^*(b_{t+1}, h_t, y_t)$ such that*

[1]. *Given the bond price function $q^*(b_{t+1}, h_t, y_t)$, recovery rate $\alpha^*(b_t, h_t, y_t)$ and additional spread premia $\phi^*(b_t, h_t, y_t)$, the country's value function $V^*(b_t, h_t, y_t)$ (together with $V^{*R}(b_t, h_t, y_t)$ and $V^{*D}(b_t, h_t, y_t)$),*

*asset position $b^*_{t+1}(b_t, h_t, y_t)$, consumption $c^*_t(b_t, h_t, y_t)$, default set $D^*(b_t, h_t)$ satisfy the country's optimization problem (1)-(10).*

[2]. *Given the bond price function $q^*(b_{t+1}, h_t, y_t)$, the country's value function $V^*(b_t, h_t, y_t)$ (together with $V^{*R}(b_t, h_t, y_t)$ and $V^{*D}(b_t, h_t, y_t)$), discounted expected payment $P^*(b_t, h_t, y_t)$, the recovery rate $\alpha^*(b_t, h_t, y_t)$ and additional spread premia $\phi^*(b_t, h_t, y_t)$ solve debt renegotiation problem (15).*

[3]. *Given recovery rate $\alpha^*(b_t, h_t, y_t)$ and additional spread premia $\phi^*(b_t, h_t, y_t)$, the bond price function $q^*(b_{t+1}, h_t, y_t)$, total spread $s^*(b_{t+1}, h_t, y_t)$ and satisfy optimal conditions of foreign investors' problem (17), (19) and (20).*

In equilibrium, default probability $p^*(b_{t+1}, h_t, y_t)$ is defined by using the country's default decision:

$$p^*(b_{t+1}, h_t, y_t) = \int\limits_{D^*(b_{t+1}, h_t)} d\mu(y_{t+1}, y_t) \tag{21}$$

The expected recovery rate $\gamma^*(b_{t+1}, h_t, y_t)$ in equilibrium is given by

$$\begin{aligned}\gamma^*(b_{t+1}, h_t, y_t) &= \frac{\int\limits_{D^*(b_{t+1}, h_t)} \alpha^*(b_{t+1}, h_t, y_{t+1}) d\mu(y_{t+1}, y_t)}{\int\limits_{D^*(b_{t+1}, h_t)} d\mu(y_{t+1}, y_t)} \\ &= \frac{\int\limits_{D^*(b_{t+1}, h_t)} \alpha^*(b_{t+1}, h_t, y_{t+1}) d\mu(y_{t+1}, y_t)}{p^*(b_{t+1}, h_t, y_t)} \end{aligned} \tag{22}$$

The numerator is expected proportion of the debt which the country will repays at renegotiation, and the denominator is default probability.

# 6 Quantitative Analysis

This section provides quantitative analysis of the model. We set parameters and functional forms of the model and discuss equilibrium properties of the model. Simulation results based on equilibrium distribution of the model are presented in Section 6.3. We explore the impacts of additional spread premia in Section 6.4. Finally, we summarize main implications of quantitative analysis.

## 6.1 Parameters and functional forms

We use most of parameters and functional forms specified in Yue (2010). There are three new elements in our model: (1) the maximum level of additional spread premia, (2) the maximum level of credit history and (3) probability of upgrading in credit history. The rationale of the upper limits of both additional spread premia and credit history is to satisfy the stationarity of the model; if we do not set the upper limits, the country will face high borrowing costs and repeat defaults in short periods leading to higher spreads, and investors will not be able to receive spread payments. Reflecting the fact that the record of defaults remains on the country's credit history for only a finite number of years rather than infinite periods, we assume the probability of upgrading in credit history.

We define each period as a quarter. The following constant relative risk-aversion (CRRA) utility function is used in numerical simulations:

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} \tag{23}$$

where $\sigma$ expresses degree of risk aversion. We set $\sigma$ equal to 2, which is a common value used in real business cycle studies. Following Arellano (2008), the risk-free rate is equal to 1.7%. The baseline output loss parameter $\lambda_d$ is set to 2% based on Strurzeneger's (2002) estimate.

We follow the same stochastic process for output used in Yue (2010). She models the output growth rate as AR(1) process to capture the stochastic trend in GDP of Argentina as;

$$\log(y_t) = (1-\rho_g)\log(1+\mu_g) + \rho_g \log(y_{t-1}) + \epsilon_t^g \tag{24}$$

where growth rate is $g_t = \frac{y_t}{y_{t-1}}$, growth shock is $\epsilon_t^g \sim^{i.i.d.} N(0, \sigma_g^2)$, and $\log(1+\mu_g)$ is expected log gross growth rate of the country's endowment. We set $\mu_g = 0.0042$, $\sigma_g = 0.0253$, and $\rho_g = 0.41$, and approximate this stochastic process as a discrete Markov chain of 21 equally spaced grids by using the quadrature method in Tauchen (1986).

Since a realization of the growth shock permanently affects endowment and the model economy is nonstationary, we detrend the model by dividing by the lagged endowment level $y_{t-1}$. The detrended counterpart of the any variable $x_t$ is thus $\hat{x}_t = \frac{x_t}{x_{t-1}}$. The equilibrium value function, bond price function, recovery rate and interest spreads are evaluated based on the detrended variables.

Concerning time discount factor $\beta$ and baseline country's bargaining power $\theta$, we set $\beta = 0.75$, $\theta = 0.72$, to obtain its average default frequency 2.65% annually or 0.66% quarterly and recovery rate 31.3%. We target default probability 2.7% annually and the average recovery rate 33% for the 2005 international debt restructuring estimated by Sturzenegger and Zettelmeyer (2006, 2008). For interest spreads, we set the maximum level of additional spread premia ($\phi_{\max}$) corresponding to the evidence in Figure 2 that increase in spreads is less than 0.01 (100 basis points). Lastly, taking into account 3 defaults of Argentina in the period from 1901-2002 indicated in Reinhart, Rogoff, and Savastano (2003), we specify the maximum level of credit history ($h_{\max}$) as 3. The probability of upgrading $\chi$, which governs the average length of time that a recent default remains on the country's credit history is set to 0.025, reflecting that investors' memory lasts for 10 years.[25] This is also consistent with spreads dynamics in Argentina: an average of spreads for 2002Q1-2011Q4 is higher than one for pre-default period. Table 5 summarizes the model parameters. Our computation algorithm is shown in Appendix A.

[25] Chatterjee et al (2007) assume that creditors' memory lasts for 10 years in the case of consumer defaults.

Table 5: Model parameters

| Parameter | Value | Sources |
|---|---|---|
| Risk aversion | $\sigma = 2$ | RBC Literature |
| Risk-free interest rate | $r = 0.017$ | Arellano (2008) |
| Baseline output loss in default | $\lambda_d = 0.02$ | Sturzenegger (2002) |
| Average endowment growth | $\mu_g = 0.0042$ | Yue (2010) |
| Standard deviation of endowment growth shock | $\sigma_g = 0.0253$ | Yue (2010) |
| Endowment growth AR(1) coefficient | $\rho_g = 0.41$ | Yue (2010) |
| Discount factor | $\beta = 0.75$ | Computed |
| Baseline bargaining power | $\theta = 0.72$ | Computed |
| Maximum level of additional spread premia | $\phi_{\max} = 0.01$ | Computed |
| Maximum level of credit history | $h_{\max} = 3$ | Computed |
| Probability of upgrading in credit history | $\chi = 0.025$ | Computed and Chatterjee et al (2007) |

## 6.2 Numerical results on equilibrium properties

In this subsection, we cover the equilibrium properties of the model. Figure 4 shows the relationship between increase in interest spreads and recovery rates unconditional on income states.[26] As in Section 3, we define increase in spreads as the difference between spreads with defaults and those with non-defaults. We calculate spreads after default based on both expected recovery rates for next default and agreed additional spread premia, and spreads with non-defaults are measured with expected recovery rates for the current default. It is clear that there is a negative relationship between recovery rates and increase in interest spreads. If the increase in spreads is high, recovery rate is low and vice versa. One interpretation is that if the country repays a large fraction of its debt at the renegotiations, long-term borrowing costs will be small. In the case of Yue (2010), the slope of the contract curve is vertical as shown in Appendix C. A driving force which makes our results different from Yue (2010) is additional spread premia agreed at the debt restructurings.

[26] Figure A2 in Appendix D displays the relationship between increase in interest spreads and recovery rates conditional on income states.

Figure 4: Relationship between increase in interest spreads and recovery rates

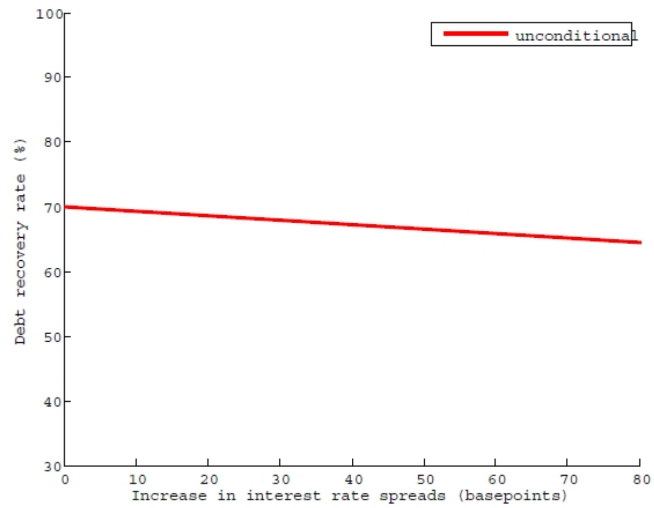



Figure 5: Default probability under baseline case

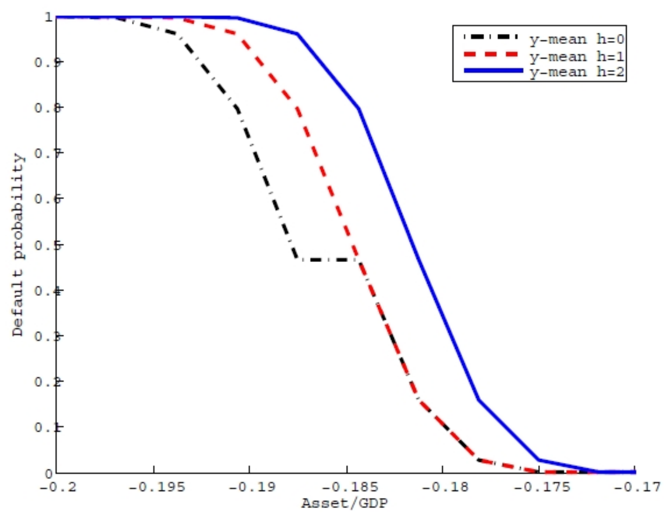
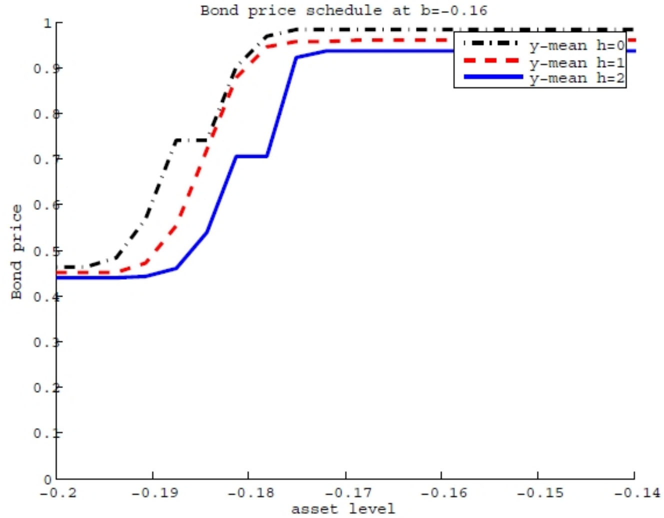



Figure 5 illustrates the baseline default probability at the mean income level. It is apparent that the default probability is weakly increasing with the credit history. At the higher level of credit history, additional increase in spreads on the newly issued bonds, determined at the previous debt renegotiation, leads to higher costs for the country to borrow from investors compared with credit history $h_t = 0$.

Figure 6 presents that bond price is also weakly decreasing with respect to the credit history. What play behind are additional spread premia agreed at the past debt renegotiations: as explained in detail in Section 6.4, these additional spread premia decrease the bond price both directly and indirectly through default probability.

Figure 6: Bond price schedule under baseline case ($b_{t+1} = -0.16$)




### 6.3 Simulation results

We conduct 1000 rounds of simulations with 2000 periods per round and then extract 80 observations before and 25 observations after each default event in stationary distribution to compute statistics.[27] Bond spreads are from the J.P. Morgan's Emerging Markets Bond Index Global (EMBIG) for Argentina for 1997Q1–2001Q4 and 2005Q3–2011Q3. Output data are seasonally adjusted from the MECON for 1980Q1–2001Q4 and 2005Q3–2011Q3. Consumption and trade balance data are also seasonally adjusted from the MECON for 1993Q1–2001Q4 and 2005Q3–2011Q3. Trade balance is calculated as ratio to real GDP. Argetina's external debt data are from the IMF WEO for 1980–2001 and 2005–2011. We compute two measures of the sovereign's indebtness: the first measure is the average external debt/GDP ratio. We also compute the ratio of the country's debt service (including short-term debt) to its GDP for Argentina. One advantage of our model compared with Yue (2010) or Aguiar and Gopinath (2006) is that we obtain the statistics for post-default periods.

As apparent from Table 6, the model matches the business cycle statistics in data. For pre-default periods, our model replicates volatile consumption and trade balance/GDP volatility, both of which are prominent features of emerging economies business cycle models In addition, it also generates the negative correlation between trade balance and output. However, a novelty of our model comes from the better match of statistics with data in post-default periods, particularly on consumption volatility

[27] We choose 80 observations prior to and 25 observations after a default event to compute the sample in the data for Argentina from 1980Q1 to 2001Q4, before default in 2002Q1 and from 2005Q3 to 2011Q3 after its completion of restructuring in 2005Q2. See also Arellano (2008) and Yue (2010) for this treatment of simulation.

and correlation of trade balance and output.

Table 6: Business Cycle Statistics

| | Data | Model | Yue (2010) | A. and G.(2006) |
|---|---|---|---|---|
| *Before Default* | | | | |
| Consumption Std/Output Std | 1.03 | 1.24 | 1.04 | 1.05 |
| Trade Balance/Output Std Dev. (%) | 1.23 | 3.71 | 2.81 | 0.95 |
| Corr (Trade Balance/GDP, Output) | -0.83 | -0.005 | -0.16 | -0.19 |
| *After Default* | | | | |
| Consumptionn Std/Output Std | 1.00 | 1.31 | - | - |
| Trade Balance/Output Std Dev. (%) | 1.03 | 4.20 | - | - |
| Corr (Trade Balance/GDP, Output) | -0.74 | -0.02 | - | - |

Source: Aguiar and Gopinath (2006), Datastream, IMF WEO, MECON, Yue (2010)

We move on to non-business cycle statistics of the model and data. First of all, in pre-default periods, the model creates a moderate level of debt relative to data statistics. In the data, the total debt service/GDP ratio is 10.2%. The model generates the average debt/GDP ratio of 10.4%. In addition, the model also shows the relation among bond spreads, debt/GDP ratio and outputs as in the data. Bonds spreads are possitively correlated with debt/GDP, but negatively correlated with output. This is because default probability is high and recovery rates are low in low income states resulting in high spreads. The average bond spreads is 3.1% in our simulations, lower than 7.4% reported in the data, but higher than in Yue (2010). The volatility of bond spreads is 1.9% in our simulation, close to the data (2.9%). The debt recovery rates are negatively correlated with default probability.

What makes our model more distinctive is the model accounts the regularities in the post-default periods. The average debt/GDP ratio is 12.3%, close to the debt service/GDP ratio of 13.2%. It is clear that the model explains one prominent feature of average debt/GDP ratio in both pre-default and post-default periods: the average debt/GDP ratio is higher in post-default period (12.3%) than in pre-default period (10.4%). What is driving behind is increase in borrowing costs which forces the sovereign to accumulate higher debts. Furthermore, our model provides the better match of the relation among bond spreads, debt/GDP ratio and output in post-default periods than in pre-default periods. Even in the same low income states, the sovereign tends to accumulate higher debts in post-default periods leading to higher spreads than in pre-default periods. This is also justified by the average bond spreads in post-default periods (3.9%) higher than one in pre-default periods (3.0%). It also shows an obvious improvement of the average spreads compared with Yue (2010). On contrary, the volatility of bond sreads in the post-default periods remains the same as in pre-crisis period.

Table 7: Model statistics for Argentina

| | Data | Model | Yue (2010) | A. and G.(2006) |
|---|---|---|---|---|
| **Target Statistics** | | | | |
| Default Probability | 2.7 | 2.65 | 2.67 | 0.92 |
| Average Recovery Rate (%) | 33 | 31.3 | 27.31 | 0 |
| **Non-target Statistics** | | | | |
| *Before Default**1 | | | | |
| Average Debt/GDP ratio*3 | 12.7 / 10.2 | 9.5 | 10.1 | 5.95 |
| Corr (Spreads, Output) | -0.86 | -0.19 | -0.11 | -0.29 |
| Average Bond Spreads (%) | 7.4 | 3.1 | 1.86 | 3.58 |
| Bond spreads Std Deviation (%) | 2.9 | 1.9 | 1.58 | 6.36 |
| Corr (Debt/GDP, spreads)*3 | 0.43 | 0.72 | - | - |
| *Debt Renegotiation**2 | | | | |
| Corr (Default Prob., Recovery Rates) | - | -0.31 | -0.26 | - |
| Corr (Defaulted Debt, Recovery Rates) | 0.33 | 0.31 | 0.31 | - |
| Average Exclusion (years) | 3.5 | 0.25 | 0.25 | 2.5 |
| *After Default**2 | | | | |
| Average Debt/GDP ratio*3 | 43.0 / 13.2 | 12.3 | - | - |
| Corr (Spreads, Output) | -0.43 | -0.32 | - | - |
| Average Bond Spreads (%) | 6.7 | 3.9 | - | - |
| Bond spreads Std Deviation (%) | 4.1 | 2.0 | - | - |
| Corr (Debt/GDP, spreads)*3 | 0.72 | 0.90 | - | - |
| Span between defaults (years) | - | 14.25 | - | - |

Source: Aguiar and Gopinath (2006), Datastream, IMF WEO, MECON, Yue (2010)

*[1]: Data statistics before default correspond to sample of 1980Q1-2001Q4 (output), 1990Q1-2001Q4 (trade balance and consumption), and 1997Q1-2001Q4 (spreads). *[2]: Data statistics during and after debt renegotiation correspond to samples of 2002Q1-2005Q2 and of 2005Q3-2011Q3 respectively. *[3]: Two measures are the average total debt service (interest and amortization paid) and the average short-term debt outstanding at year end. We use the second measure (short-tern debt outstanding) to calculate correlation.

Furthermore, we calculate the average time spans between defaults based on 2000 rounds of simulations by extracting the initial 200 periods of total 2000 periods per round. Table 8 reports that the average spans between defaults are weakly decreasing with respect to the number of past debt renegotiations. This feature is robust to extensions related with the upper limits of credit history.

Table 8: Average time spans between defaults (quarters)

| Data: group average (emerging countries) in 1824-2001 | | | | | | 64 | Default Probability | 2.7 |
|---|---|---|---|---|---|---|---|---|
| | 1st def. | 2nd def. | 3rd def. | 4th def. | 5th def. | 6th def. | | |
| $\mathbf{h_{max} = 3}$ | **57** | **19** | **8** | - | - | - | | **2.65** |
| $h_{max} = 4$ | 57 | 30 | 8 | 8 | - | - | | 2.99 |
| $h_{max} = 5$ | 59 | 27 | 16 | 14 | 8 | - | | 3.67 |
| $h_{max} = 6$ | 59 | 27 | 22 | 16 | 14 | 8 | | 4.47 |

### 6.4 Impacts of additional spread premia

In this subsection, we explain how additional spread premia agreed at past debt renegotiations lead to increase in spreads, which distinguishes this paper with the previous work. Based on equation (19) and (20), we can rewrite interest spreads for credit history $h_t \geq 1$ as follows.

$$s(b_{t+1}, h_t, y_t) = \begin{Bmatrix} 0 & if \quad b_{t+1} \geq 0 \\ \frac{1+r+\phi(b_t,h_t,y_t)}{[1-p(b_{t+1},h_t,y_t)+p(b_{t+1},h_t,y_t)\gamma(b_{t+1},h_t,y_t)]} - (1+r) & otherwise \end{Bmatrix} \tag{20a}$$

Given risk-free rate ($r$), total spreads can be decomposed into two factors:

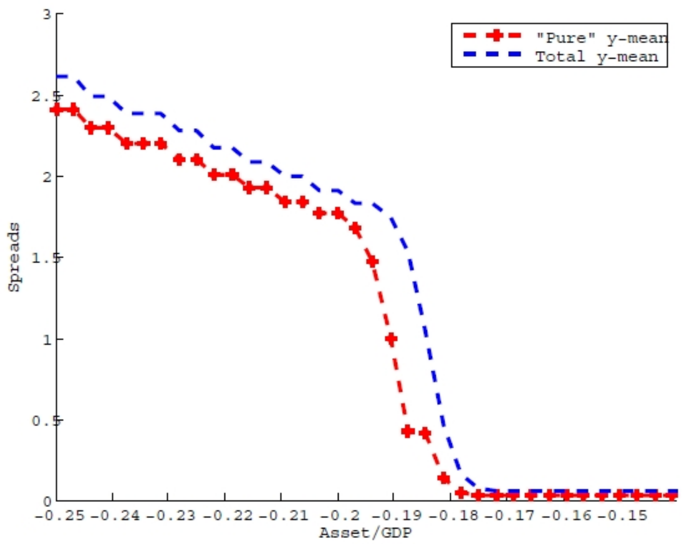
(A) spread components based on "pure" default probability,

(B) spread components based on impact of additional spread premia.

The former which is simply calculated based on "pure" probability of future defaults is totally irrelevant to the credit history. It is the measure of interest spreads used in Yue (2010). The latter is how much the term $\phi(b_t, h_t, y_t)$, increases total spreads. It can be regarded as spread components associated with the past default history.

Figure 7 displays both the total spreads and spread components measured with "pure" default probability. The spread components measured with "pure" default probability is equal to (A). The total spreads is defined by equation (20a). The difference between these two corresponds to (B), which can be interpreted as spread components associated with the past default history. It is clear that total spreads deviate from spread components measured with "pure" default probability when the debt-to-GDP ratio is above the threshold value 0.175 in the mean income state.

Figure 7: The total spreads and spreads based on "pure" default probability




## 6.5 A brief summary of quantitative analysis

Our major findings can be summarized as follows. First of all, by incorporating additional spread premia, the model accommodates an observed pattern of lower recovery rates associated with larger increases in yield spreads. Second, we show that default probability is weakly increasing with credit history, given the same debt-to-GDP ratio. Third, simulation exercises show that our model accounts both business cycle and non-business cycle regularities in the post-default periods, which differentiates this model from the previous work Finally, interest spreads in our model can be decomposed into two parts: spread components based on "pure" default probability, and spread components associated with impacts of additional spread premia due to past defaults.

# 7 Model implications

In this section, we explore the determinants of the slope of the contract curve. Moreover, we consider possible implications derived from the changes in length of creditors' memory and size of additional spread premia.

## 7.1 Determinants of the slope of the contract curve

We focus on factors which affect the value of the slope of the contract curve. Table 9 shows the values of the slope under different values for the discount factor, the maximum level of additional spread premia, output cost, risk-free rate and probability of upgrading in credit history.[28] The impacts of a change in one parameter, leaving all other parameters fixed are indicated respectively.

Table 9: Values of the slope of the contract curve under different parameter values

| | | | | | |
|---|---|---|---|---|---|
| Data | -0.62 | | | | |
| Discount factor | Slope | Maximum level of additional spread premia | Slope | Output cost | Slope |
| $\beta = 0.81$ | -0.03 | $\phi_{\max}= 0.025$ | -0.03 | $\lambda_d= 0.025$ | -0.10 |
| $\boldsymbol{\beta = 0.75}$ | **-0.07** | $\boldsymbol{\phi_{\max}= 0.01}$ | **-0.07** | $\boldsymbol{\lambda_d= 0.0225}$ | **-0.08** |
| | | $\phi_{\max}= 0.005$ | -0.12 | $\lambda_d= 0.02$ | -0.07 |
| Risk-free interest rate | | Probability of upgrading in credit history | | | |
| $r = 0.03$ | -0.08 | $\chi = 0$ | -0.07 | | |
| $\mathbf{r = 0.017}$ | **-0.07** | $\boldsymbol{\chi = 0.025}$ | **-0.07** | | |
| $r = 0.01$ | -0.05 | $\chi = 0.075$ | -0.07 | | |

Note: all the values are those at the default

First, the slope gets steeper as the discount factor decreases. From the country's perspective, the cost of paying to one additional unit of defaulted debt at the renegotiation relative to the cost of facing one additional unit of increase in spreads, gets smaller as the discount factor decreases. Next, when the maximum level of additional spread premia is reduced to 50 basis points ($\phi_{\max} = 0.005$), the absolute value of the slope increases. Since increase in spreads is limited to a lower level due to the lower maximum level of additional spread premia, paying one additional unit of defaulted debt at

[28] Changes in value for bargaining power has an ambitious impact on the slope of the contract curve. Rather than the slope, the intercept (levels of recovery rates at 0 basis point increase in spreads) will be influenced by changes in value of bargaining power.

the renegotiation is less costly relative to paying one additional unit of spread increases in the future period.

On contrary, an increase in output cost leads to an increase in the absolute value of the slope. As the cost of default is larger for the country, relative cost of paying one additional unit of defaulted debt at the renegotiation instead of facing one additional unit of increase in spreads decreases taking into account the cost of next default.

The absolute value of slope increases as the risk-free rate increases. Total size of increase in spreads gets larger associated with an increase in risk-free interest rate. Given the constant change in recovery rate, it makes the slope of the contract curve more flatter, indicating that from the country's point of view, paying one additional unit of defaulted debt at the renegotiation is less costly than paying one additional unit of spread returns in the future periods. Lastly, probability of upgrading in credit history does not affect the value of slope.

### 7.2 Duration and size of additional spread premia

Determination of both recovery rates and additional spread premia at the debt renegotiation plays an important role in our model. Probability of upgrading in credit history and maximum level of additional spread premia are two key parameters which specify the duration and size of deterioration in long-term credit. Table 10 reports how changes in these parameter values influence the non-business cycle statistics.[29]

Increase in probability of upgrading reduces the average debt/GDP ratio, average bond spreads and correlation between debt/GDP and spreads. As the probability of upgrading in credit history gets higher, length of deterioration in long-run credit gets shorter. The sovereign tends to have lower levels of debt and spreads, which also lead to lower correlation between debt/GDP ratio and spreads.

On the contrary, not only the average debt/GDP, average bond spreads and correlation between debt/GDP and spreads, but also the default probability increases as the upper limit of additional spread premia gets higher. The maximum level of additional spread premia identifies the size of deterioration in long-term credit, given the fixed duration. Associated with increase in borrowing costs, the sovereign accumulates more debts leading to increases in both spreads and probability in

[29] Changes in parameter values of both probability in credit history and maximum level of additional spread premia do not affect the business cycle statistics significantly.

default.

Table 10: Statistics for different levels of upgrading in credit history and additional spread premia

| | Probability of upgrading | | | Maximum level of additional spread premia | | |
|---|---|---|---|---|---|---|
| | $\chi = 0$ | $\boldsymbol{\chi} = \mathbf{0.025}$ | $\chi = 0.075$ | $\phi_{\max} = 0.005$ | $\boldsymbol{\phi}_{\max} = \mathbf{0.01}$ | $\phi_{\max} = 0.025$ |
| Default Probability | 2.67 | **2.65** | 2.65 | 2.55 | **2.65** | 3.04 |
| Average Recovery Rate (%) | 31.9 | **31.3** | 31.9 | 32.2 | **31.3** | 31.5 |
| Before Default*1 | | | | | | |
| Average Debt/GDP ratio*3 | 10.4 | **9.5** | 10.4 | 10.4 | **9.5** | 11.8 |
| Corr (Spreads, Output) | -0.08 | **-0.06** | -0.08 | -0.05 | **-0.06** | -0.13 |
| Average Bond Spreads (%) | 3.4 | **3.0** | 3.4 | 3.2 | **3.0** | 4.0 |
| Bond spreads Std Deviation (%) | 1.9 | **1.9** | 1.9 | 1.8 | **1.9** | 2.4 |
| Corr (Debt/GDP, spreads)*3 | 0.82 | **0.72** | 0.82 | 0.81 | **0.72** | 0.82 |
| After Default*2 | | | | | | |
| Average Debt/GDP ratio*3 | 10.9 | **12.3** | 10.9 | 10.9 | **12.3** | 12.4 |
| Corr (Spreads, Output) | -0.25 | **-0.41** | -0.27 | -0.29 | **-0.41** | -0.21 |
| Average Bond Spreads (%) | 3.9 | **3.9** | 3.9 | 3.6 | **3.9** | 4.6 |
| Bond spreads Std Deviation (%) | 2.0 | **2.0** | 2.0 | 1.9 | **2.0** | 2.6 |
| Corr (Debt/GDP, spreads)*3 | 0.90 | **0.90** | 0.90 | 0.88 | **0.90** | 0.89 |

## 8 Conclusion

Emerging countries that have defaulted on their debt repayment obligations in the past are more likely to default again in the future than are non-defaulters with the same debt-to-GDP ratio. This paper explains this stylized fact within a dynamic stochastic general equilibrium framework that explicitly models debt renegotiations between a defaulting country and its creditors. Specifically, the model extends the existing literature by allowing defaulters and creditors to bargain not just over recovery rates, but also over the rate of return offered on newly-issued debt. Quantitative analysis of the model reveals that the equilibrium probability of default for a given debt-to-GDP level is weakly increasing with the number of past defaults, consistent with empirical observations. The equilibrium of the model also corresponds with an additional observed trend: countries for which default terms require less than a 100 percent recovery rate tend to pay a higher rate of return (relative to the risk-free rate) on debt that is issued subsequently than do defaulting countries that agree to a full recovery rate. These findings are robust to extensions that allow for the negotiated outcome to be modeled more flexibly.

So far, we have considered the debt renegotiation under symmetric information between the country and investors. It might be possible that some of the information concerning the country's profile remains unrevealed to investors at the time of renegotiation, such as the country's government type as in Hachondo et al (2009) and D'Erasmo (2011), income process or actual level of output costs. On the other hand, degree of coordination among the creditors or creditor composition is uninformed to the country at the renegotaition. A comparison of renegotiation outcomes under two asymmetric information cases will be a potential research topic in the future.

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# A Computation Algorithm

Procedure to compute the equilibrium distribution of the model is the following. Note that the spread premia are pinned down by both current level of debt ($b_t$) and income ($y_t$) together with credit history ($h_t$) as the credit history keeps track of timing of default and debt renegotiation and is reverted with exogenous probability. Thus, value functions of sovereign do not need the interest rates as additional state.

(1) First, we set discrete grids on the space of credit history as $H = [0, 1, 2, 3]$ corresponding to $h_{\max} = 3$.

(2) Second, we set finite grids on the space of endowment and asset holdings as $B = [-0.3, ......., 0]$. The limits of asset space are set to ensure that the limits do not bind in equilibrium. The limits of endowment space are big enough to include large deviations from the average value of shocks. We approximate the stochastic income process given by equation (24) using a discrete Markov chain of 21 equally spaced grids. Moreover, we calculate the transition matrix based on the probability distribution $\mu(y_{t+1}|y_t)$.

(3) Third, we set finite grids on the space of recovery rate and additional spread premia. Limits of both recovery rates and additional spread premia are set to ensure that they do not bind in

equilibrium.

(4) Fourth, we set the initial values for equilibrium bond price, recovery rate, and interest spreads. We use the risk-free bond price ($q_1 = q^f = (1+r)^{-1}$) for the baseline value of equilibrium bond price. We use $\alpha_0 = 0.5$, and $\phi_0 = 0.01$ for the baseline recovery rate and additional spread premia.

(5) Fifth, given the baseline equilibrium bond price ($q_0 = q^f$), recovery rate ($\alpha_0 = 0.5$), and additional spread premia ($\phi_0 = 0.01$), we solve for the country's optimization problem for each credit history ($h_t = 0, 1, 2, ...$). This procedure finds the value function as well as the default decisions. We first guess the value function ($V^0$, $V^{D,0}, V^{R,0}$) and iterate it using the Bellman equation to find the fixed value ($V^*$, $V^{D*,}, V^{R*}$), given the baseline bond price, recovery rate, and spreads. By iterating the Bellman function, we also derive the optimal asset policy function for every value ($a'$, $a'^D, a'^R$). For each credit history, we also obtain choices of default, which requires comparison of the values of defaulting and non-defaulting. By comparing the these two values, we calculate the corresponding default set. Based on default set, we also evaluate the default probability using the transition matrix.

(6) Sixth, using the default set in step (5), and the zero profit condition for foreign investors, we compute the new price of discounted bond ($q_1$). Then we iterate step (5) to have fixed value of equilibrium bond price.

(7) Seventh, given the value functions ($V^*$, $V^{D*,}, V^{R*}$), value of autarky ($V^A$), the payment of bonds ($R^*$) derived from the iterations above and the price of discounted bond ($q^*$), we solve the bargaining problem and compute the new debt recovery schedule ($\alpha'$) and additional spread premia ($\phi'$) for every $(b, h, y)$. Then, we iterate step (5), (6) to have the fixed optimal debt recovery rate ($\alpha^*$), and the optimal additional spread premia ($\phi^*$).

# B Tables in Section 3

Table 1: Predicted Institutional Investor Ratings and Debt Intolerance Regions for Argentina and Malaysia

| | Argentina | | Malaysia | |
|---|---|---|---|---|
| External debt/GNP | Predicted *IIR* | Region | Predicted *IIR* | Region |
| 0 | 51.4 | **1** | 61.1 | **1** |
| 5 | 49.3 | **1** | 59.0 | **1** |
| 10 | 47.3 | **1** | 57.0 | **1** |
| 15 | 45.2 | **3** | 54.9 | **1** |
| 20 | 43.2 | **3** | 52.9 | **1** |
| 25 | 41.1 | **3** | 50.8 | **1** |
| 30 | 39.1 | **3** | 48.8 | **1** |
| 35 | 37.0 | **3** | 46.7 | **2** |
| 40 | 34.9 | **4** | 44.7 | **4** |
| 45 | 32.9 | **4** | 42.6 | **4** |

Source: Reinhart, Rogoff, and Savastano (2003)

Note: 1. The Institutional Investor Ratings (IIR) are compiled twice a year, are based on information provided by economists and sovereign risk analysts at leading global banks and securities firms. The ratings grade each country on a scale from 0 to 100, with a ratings of 100 given to those countries perceived as having the lowest chance of defaulting on their government debt obligations.

2. For countries in club B (24.2< IIR <67.7), the four regions (from least to most vulnerable) defined are : Least debt intolerant, Type **1** Region (45.9≤IIR≤67.7 and debt/GNP<35), quasi debt intolerant, Type **2** Region (45.9≤IIR≤67.7 and debt/GNP>35), quasi debt intolerant, Type **3** Region (25.2≤IIR≤45.9 and debt/GNP<35) and; most debt intolerant Type **4** Region (25.2≤IIR≤45.9 and debt/GNP>35).

Table 2: External Debt Defaults or Restructurings in 1824-2001

| | Number of default or restructuring episodes 1824-2001 | Number of years since last year in default or restructuring status |
|---|---|---|
| | *Emerging countries with at least one external default or restructuring since 1824* | |
| Argentina | 4 | 0 |
| Brazil | 7 | 7 |
| Chile | 3 | 17 |
| Colombia | 7 | 57 |
| Egypt | 2 | 17 |
| Mexico | 8 | 12 |
| Philippines | 1 | 10 |
| Turkey | 6 | 20 |
| Venezuela | 9 | 4 |
| *Group average* | *5.2* | *16* |
| | *Emerging countries with no external default history* | |
| India | 0 | n.a. |
| Korea | 0 | n.a. |
| Malaysia | 0 | n.a. |
| Singapore | 0 | n.a. |
| Thailand | 0 | n.a. |
| *Group average* | *0* | *n.a.* |

Source: Reinhart, Rogoff, and Savastano (2003)

Table 3: Stylized facts about sovereign debt renegotiations in 1998-2007[*1]

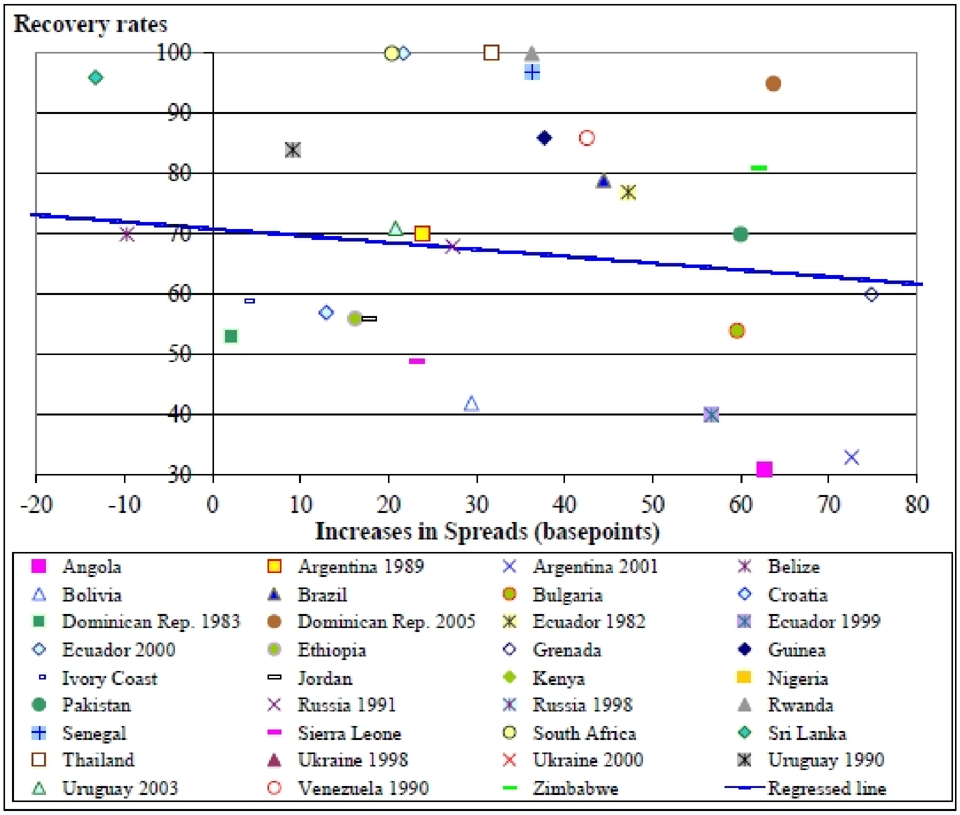
| Country | Year[*2] of default | Defaulted debt ($ billions) | Defaulted debt[*2] (of GDP) | Recovery[*3] rates(%) | Increases[*7] in spreads |
|---|---|---|---|---|---|
| **Expost-default** | | | | | |
| Russia | 1998 | 72.709 | 26.8% | 35% [*4] | 69.97 |
| Ecuador | 1999 | 6.604 | 39.6% | 40% [*4] | 7.73 |
| Ecuador | 2000 | 0.346 | 2.5% | 100% | 18.72 |
| Ivory Coast | 2000 | 15.6 | 148.3% | 59% | 16.84 [*8] |
| Argentina | 2001 | 82.268 | 30.6% | 33% [*4] | 20.30 |
| Grenada | 2004 | 0.297 | 68.0% | 60% [*5] | 33.27 |
| Moldova | 2004 | -0.145 | 9.8% | 42% [*6] | n.a. |
| **Preemptive** | | | | | |
| Pakistan | 1998 | 1.627 | 2.7% | 70% [*4] | 35.87 [*8] |
| Ukraine | 1998 | 1.271 | 3.9% | 72% [*4] | 34.05 [*8] |
| Ukraine | 2000 | 1.064 | 3.4% | 60% | 47.85 |
| Moldova | 2002 | 0.04 | 2.4% | 94% [*6] | n.a. |
| Dominica | 2003 | n.a. | n.a. | 71% | n.a. |
| Uruguay | 2003 | 5.744 | 51.3% | 71% [*4] | 11.54 |
| Dominican Republic | 2005 | 1.622 | 5.6% | 95% [*5] | 25.78 |
| Belize | 2006 | 0.242 | 19.9% | 76% [*5] | 2.59 [*8] |

Source: Bedford, Penalver and Salmon (2005), Benjamin and Wright (2009), Datastream, Finger and Mecagni (2007) Moody's (2007) and Sturzenegger and Zettelmeyer (2008).

Note: [*1]We list only export-default and preemptive renegotiation episodes in 1998-2007. We exclude the cases of swap agreement or delay in payment such as Venezuela in 1995, 1998 and 2005, Peru in 2000 and Paraguay in 2003. [*2]Data (year of default and defaulted debt) is from Moody's (2007). The debt is total amount of sovereign bonds which the government defaulted on and does not include the private debt. [*3]Data for recovery rate is from Benjamin and Wright (2009). [*4]Recovery rates for Russia, Ecuador, Argentina, Pakistan, Ukraine, and Uruguay are from Strurzeneger and Zettelmeyer (2008). [*5]Recovery rates for Grenada, Dominican Rep. and Belize are from Bedford, Penalver and Salmon (2005). [*6]Recovery rate for Moldova 2002, 2004 is from Finger and Mecagni (2007). [*7]Data (spreads) is from J.P. Morgan's Emerging

Market Bond Index (EMBI) on Datastream and we define "increases in spreads" as a difference in spreads between at the time of renegotiations and one with one year before the renegotiations. *[8]Spread data for Pakistan and Ukraine is measured at 6/2002 and at 9/2001 respectively. Spread data for Ivory Coast and Belize is one of African composite sovereign bonds.

Figure A1. Recovery rates and increase in spreads for 2-year window




Note: We define "increase in spreads" for 2-year window, such as the difference between one year before and after the renegotiation.

# C Features at the steady state distribution

Figure A2 shows the relationship between increase in interest spreads and recovery rates conditional on income realization..It is clear that there is a negative relationship between recovery rates and increase in interest spreads in the lowest, mean and highest mean income states. The slope of the contract curve in the lowest income state is steeper than ones in both the mean or the highest income states.

Furthermore, Figure A3 presents that the slope of the contract curve is vertical in the case of

Yue (2010). Since Yue (2010) does not consider any additional spread premia agreed at the debt renegotiation, there is no increase in spreads.

Figure A2. Relationship between increase in interest spreads and recovery rates

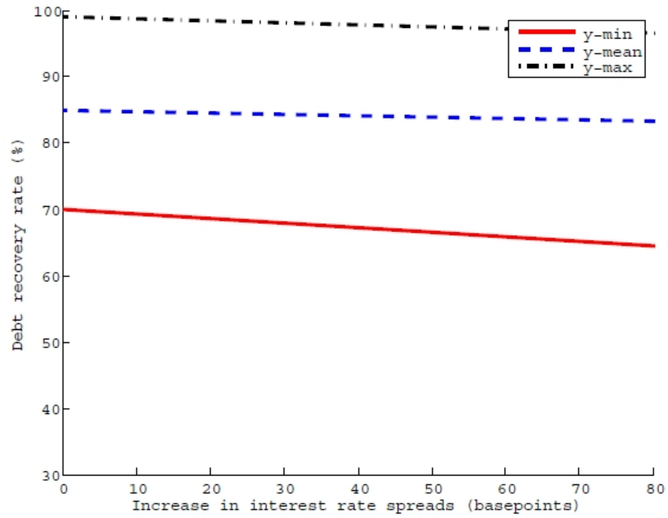



Figure A3: Relationship between increase in interest rates and recovery rates in Yue (2010)