Optimal Policy to Influence Individual Choice Probabilities

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by

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Abstract

This paper presents a model in which government may affect outcomes by manipulating individual choice probabilities through the design of the domain of choice or the use of fiscal instruments. Such manipulations are ineffective when individuals are perfectly rational, provided all alternatives are permitted. However, even a small deviation from perfect rationality is shown to call for policy that substantially manipulates choice probabilities. This policy aims to lend weight to alternatives preferred by individuals who are prone, more than others, to make mistakes.

At very low levels of rationality, when choices are largely random, it is always socially optimal to entirely eliminate individual choice in order to prevent the errors generated by such choice. It is better to impose one alternative that is not the preferred one for some individuals instead of inducing a completely random draw by everybody.

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1 Introduction

Providing choice to individuals is a good thing if they can be assumed always to choose their preferred alternative, that is, when they are *'perfectly rational'*[1]. Individuals, however, do not always choose what is best for them (they are *'boundedly rational'*) and the probabilities of making errors may depend on the *domain of choice*. In particular, because of intrinsic limits on cognitive capacity, a larger number of alternatives may exacerbate the errors compared to selection from a smaller set. A government whose objective is to maximize a social welfare function which is an aggregate of expected utilities of a heterogeneous population may then find it optimal to constrain the choice set which individuals face. Such constraints may be detrimental to the welfare of some individuals but may increase the welfare of others who will make fewer mistakes. More generally, under bounded rationality, utilitarian governments will view certain policies aimed at manipulating individual choice probabilities as socially desirable. The entire elimination of some alternatives is an extreme case of such policy.

There is convincing evidence on the effects that policy design has on individual choice probabilities. Most striking are the effects attributed to the design of *default options*. For example, a recent study (Johnson and Goldstein (2003)) surveys the effects of *opt-in* vs. *opt-out* plans for organ donations after death. In the former, the presumption is that people agree to donate their organs, but may ask to be excluded. The later, in contrast, requires active consent to participate. Cross-country evidence (most European countries have opt-out plans, while the US, Israel and some others have opt-in designs) shows that on average the former design doubles the percent of participants. The same study surveys numerous other real-world situations, such as health care plans, privacy policies

[1] In a broader social context, provision of choice can be considered desirable in itself, as an expression of freedom. Also, choice can have positive effects on costs and prices through producer's competition.

and mandatory car insurance, in which people are assigned to a certain program but are given a chance to choose among a set of alternatives. In each of these cases, the assignment to a default has a substantial role in determining what is chosen. Another impressive recent study provides evidence on the dramatic effect that the design of 401(K) retirement programs in the US has on employees' participation rates (Choi, *et-al*, 2002).

When a government considers the design of policies that affect individual choice probabilities, three factors seem *a-priory* important: **(a)** The ability of individuals to comprehend and decipher the implications of the alternatives which they face ('degree of rationality'); **(b)** The distribution of preferences in the population. Governments are unable to identify directly individuals' preferences but can infer from their revealed self-selection the aggregate distribution of preferences. Policy should be designed to accommodate those alternatives which have a high density concentration of preferences; and **(c)** Government has to evaluate the *intensity* of preferences among various individuals. A utilitarian policy cannot avoid such cardinal comparisons among conflicting preferences.

This paper presents an asymmetric information model of probabilistic choice among a finite set of alternatives by boundedly rational individuals. Following Luce (1959), we adopt the *multinomial logit model* as the basis for specifing individual choice probabilities.

This model enables a quantification of the *precision* of individual choice, ranging from 'perfect rationality' to uniformly random choice. There is an underlying distribution of individual preferences over the alternatives that they face. The government may use policies that affect the probability that each alternative be chosen. Such policy is represented by weights assigned to the probability of each alternative independent of personal characteristics which are private information. The objective of the government is to find the weights that maximize the sum of individuals' expected utilities.

Our major conclusions are:

(a) At low degrees of rationality it is best to entirely eliminate individual choice. The single remaining alternative depends on the distribution of preferences and on their intensity;

(b) At high degrees of rationality, all alternatives should be assigned a positive probability;

(c) The optimal weight assigned to each alternative may not vary monotonically with the degree of rationality;

(d) While policy to shift choice probabilities becomes ineffective when individuals are perfect choosers, substantially shifting choice probabilities is called for even at high degrees of rationality, when errors of choice made by individuals are very small. Such policy aims at reducing differentially the larger errors made by individuals with less pronounced preferences and hence prone to make mistakes.

2 A Probabilistic Choice Model

Consider a population consisting of heterogeneous individuals, each characterized by a parameter θ ("individual θ"). Individuals choose one among a finite number, n, of alternatives, $i = 1, 2, ..., n$. Individual θ's utility of alternative i is denoted $u_i(\theta)$[2]. We follow Luce (1959) by postulating that the probability that individual θ chooses alternative i is given by the *multinomial logit* function:

$$p_i(q, \theta) = \frac{e^{qu_i(\theta)}}{\sum\limits_{j=1}^{n} e^{qu_j(\theta)}}, \quad i = 1, 2, ..., n \tag{1}$$

[2]Cost considerations can be incorporated by assuming that utility is linearly separable in a numeraire good and $u_i(\theta)$ is interpreted as the *net* (of cost) utility of alternative i.

where q is a positive constant representing the *precision of choice*. When $q = 0$, all alternatives have an equal probability to be chosen: $p_i(0, \theta) = \frac{1}{n}$, for all i and θ. Let $M(\theta) = \{i|u_i(\theta) \subseteq argmax\, [u_1(\theta), u_2(\theta), ..., u_n(\theta)]\}$, and suppose that the number of elements (number of ties) in $M(\theta)$ is $R(\theta)$. For all $i \in M(\theta)$, $p_i(q, \theta)$ strictly increases monotonically with q, approaching $\frac{1}{R(\theta)}$ as $q \to \infty$. Denote by $\overline{M}(\theta)$ the complementary set of $M(\theta)$. For all $i \in \overline{M}(\theta)$, $p_i(q, \theta)$ strictly decreases monotonically as q increases, approaching 0 as $q \to \infty$. With these properties, it is natural to call q the *'degree of rationality'* (with $q = \infty$ called *'perfect rationality'*).

Individuals' welfare is assumed to be represented by expected utility, $V(q, \theta)$,

$$V(q, \theta) = \sum_{i=1}^{n} p_i(q, \theta) u_i(\theta) \tag{2}$$

Clearly, $V(q, \theta)$ continuously increases in q (strictly, when not all u_i are equal), approaching the maximum utility, denoted $\overline{V}(\theta)$, where $\overline{V}(\theta) = u_i(\theta)$ for all $i \in M(\theta)$, as $q \to \infty$.

It is assumed that all individuals have the same q[3] and that social welfare, W, is utilitarian:

$$W(q) = \int V(q, \theta) dF(\theta) \tag{3}$$

where $F(\theta)$ is the distribution function of θ in the population.

[3] Generalizing to include gropes with different q's would not essentially change the results below. In a broader context, however, if individuals with low q's are aware of their tendency to make errors, they may attempt to mimick or delegate choice to others that are considered to have similar preferences, but are less prone to errors. This complex question of optimal strategies is not discussed here.

3 Optimal Policy to Affect Individual Choice

The individual characteristic θ is assumed to be private information (e.g. health or attitude towards work), the government having only information on it's distribution, $F(\theta)$. Hence, policy which depends on observables can discriminate between alternatives but cannot depend on θ.

The government has various ways to influence choice probabilities, including the design of the choice set itself, lending weight to default alternatives or by the use of fiscal instruments. The following is an example of such tax/subsidy policy but its representation can support other interpretations[4]. Consider a policy of imposing a tax/subsidy, t_i, on alternative i. The policy $t = (t_1, t_2, ..., t_n)$ affects the choice probabilities, (1), which are now written:

$$p_i(\mathbf{g}, q, \theta) = \frac{e^{q(u_i - t_i)}}{\sum\limits_{j=1}^{n} e^{q(u_j - t_j)}} = \frac{e^{qu_i} g_i}{\sum\limits_{j=1}^{n} e^{qu_j} g_j} \tag{4}$$

where $g_i = e^{-qt_i} \geq 0$, $i = 1, 2, ..., n$ and $\mathbf{g} = (g_1, g_2, ..., g_n)$.

The objective of the government's policy is to choose the weights, $\mathbf{g}$, that yield maximum social welfare:

[4] For example, a multi-stage choice process can be shown to change, in a predictable way, the imputed choice probability of each alternative. To demonstrate this, consider three alternatives with utilities (given θ) u_i, $i = 1, 2, 3$.. In a one-stage choice, the probability of choosing alternative i is $p_i = e^{qu_i} / \sum\limits_{j=1}^{3} e^{qu_j}$. Suppose that the choice is first between alternatives 1 and a 'package' of alternatives 2 and 3. The expected utility of the 'package', denoted $\widehat{u}_{23}$,, is $\widehat{u}_{23} = \widehat{p}_2 u_2 + (1 - \widehat{p}_2) u_3$ where $\widehat{p}_2 = e^{qu_2}/(e^{qu_2} + e^{qu_3})$. The probability of choosing alternative 1 in a two-round choice is $\widetilde{p}_1 = e^{qu_1}/(e^{qu_1} + e^{q\widehat{u}_{23}})$. It is easy to show that $\widetilde{p}_1 \leq p_1$, with strict inequality when there are no ties in the u_i's. Thus, the probabilities that each of the 'package' alternatives is chosen are raised in the two-stage process. This can be generalized to any finite numbers of alternatives and multi-stage process.

Another example is along the "Blue Bus - Red Bus Paradox" pointed out by Debreu (1960) when discussing the Luce model. He has shown that the introduction of additional alternatives raises the combined probabilities of similar alternatives (red and blue buses) and reduces the probabilities of others (private cars).

$$W(\mathbf{g}, q) = \int V(\mathbf{g}, q, \theta) dF(\theta) = \int \left[\sum_{i=1}^{n} p_i(\mathbf{g}, q, \theta) u_i(\theta) \right] dF(\theta) \tag{5}$$

Tax revenues are assumed to be reimbursed as lump-sum transfers to individuals. When utilities are (separably) linear in a numeraire good, the distribution of these lump-sum transfers does not affect the level of social welfare.

Note that $p_i(\mathbf{g}, q, \theta)$ is homogenous of degree zero in $\mathbf{g}$. As only relative weights matter, we normalize, $\sum_{i=1}^{n} g_i = 1$.

Suppose that all $g_i > 0$, $i = 1, 2, ..., n$ (no alternative is excluded). It is seen from (4) that

$$W(\mathbf{g}, \infty) = \lim_{q \to \infty} W(\mathbf{g}, q) = \int \overline{V}(\theta) dF(\theta) = \overline{W} \tag{6}$$

Since $\overline{V}(\theta)$ is independent of the specific weights, $\mathbf{g}$, so is $\overline{W}$. By continuity, it follow that for large q, the optimal policy is not to exclude any alternative. It does not follow, however, as we shall show below, that for large q (that is, with small deviations from 'perfect rationality') the government should abstain from manipulating probabilities.

At the other end, it is seen from (4) and (5) that

$$W(\mathbf{g}, 0) = \lim_{q \to 0} W(\mathbf{g}, q) = \frac{1}{n} \sum_{i=1}^{n} g_i W_i \tag{7}$$

where

$$W_i = \int u_i(\theta) dF(\theta), \quad i = 1, 2, ..., n \tag{8}$$

is the level of social welfare when **all** individuals choose alternative i.

Let

$$W_m \subseteq \arg\max[W_1, W_{2,}..., W_n] \tag{9}$$

It follows from (7) that when all alternatives have an equal probability to be chosen by all individuals the optimal policy is to assign a weight $g_m = 1$ and $g_i = 0$ to all $i \neq m$. Thus, the optimal policy at $q = 0$ is to e*liminate individual choice*[5].

Having identified the optimal policy at the two extremes, $q = \infty$ and $q = 0$, we wish to examine more closely the dependence of the optimal policy on the degree of rationality q.

For given q, maximization of W *w.r.t.* $\mathbf{g}$, yields F.O.C.

$$g_i \frac{\partial W}{\partial g_i} = \frac{1}{g_i} \int p_i(\mathbf{g}, q, \theta)[u_i(\theta) - V(\mathbf{g}, q, \theta)]dF(\theta) \leq 0 \qquad i = 1, 2, ..., n \tag{10}$$

with equality when $g_i > 0$.

Since, by definition, $\sum\limits_{i=1}^{n} p_i(\mathbf{g}, q, \theta)\,[u_i(\theta) - V(\mathbf{g}, q, \theta)] = 0$ for any $\mathbf{g}$, equations (10) have at most a rank of $n - 1$. Denote the solution to (10) by $\mathbf{g}^*(q) = (g_1^*(q), g_2^*(q),\ ..., g_n^*(q))$. Condition (10) states that increasing marginally the weight of any alternative relative to the mean cannot increase social welfare.

It can be shown that when preferences satisfy certain monotonicity conditions, second-order conditions for a local maximum of W at $\mathbf{g}^*$are satisfied. That is, the matrix $\left[\frac{\partial^2 W}{\partial g_i \partial g_j}(\mathbf{g}^*,\ q)\right]$, where

[5] When (9) contains more than one element, this conclusion applies to any of these alternatives.

$$\frac{\partial^2 W}{\partial g_i \partial g_j} = -\frac{1}{g_i^* g_j^*} \int p_i(\mathbf{g}^*, q, \theta)\, p_j(\mathbf{g}^*, q, \theta)[(u_i(\theta) - V(\mathbf{g}^*, q, \theta) + +(u_j(\theta) - V(\mathbf{g}^*, q, \theta))]\, dF(\theta) \tag{11}$$

is negative semi-definite (see Apendix).

From (5) we can derive:

$$\frac{dW}{dq}(\mathbf{g}^*(q), q) = \frac{\partial W}{\partial q}(\mathbf{g}^*(q),\, q) = =\sum_{i=1}^{n} \int p_i(\mathbf{g}^*, q, \theta)[u_i(\theta) - V(\mathbf{g}^*, q, \theta)]^2 dF(\theta) > 0 \tag{12}$$

It is seen from (12) that W strictly increases in q whenever there are at least two different alternatives with positive weight.

We summarize the results so far:

Proposition 1. (a) *There exists a positive number,* q_0*, and an index* m*, such that for all* $0 \leq q \leq q_0$*,* $g_m^* = 1$ $(g_i^* = 0,\, i \neq m,\, i = 1, 2, ..., n)$ *and hence* $p_m = 1$ $(p_i = 0,\, i \neq m,\, i = 1, 2, ..., n)$*,* $u_m(\theta) = V(g^*, q, \theta)$ *for all* θ *and* $W(g^*,\, q) = W_m$*.*; (b) *For* $q > q_0$*,* $W(g^*(q),\, q)$ *strictly increases with* q*, approaching* $\overline{W}$ *as* $q \to \infty$*.*; (c) *For large* q*,* $g_i^*(q) > 0$ *for all* $i = 1, 2, ..n$*.*

Figure 1 exhibits the relation between the optimal W and q.

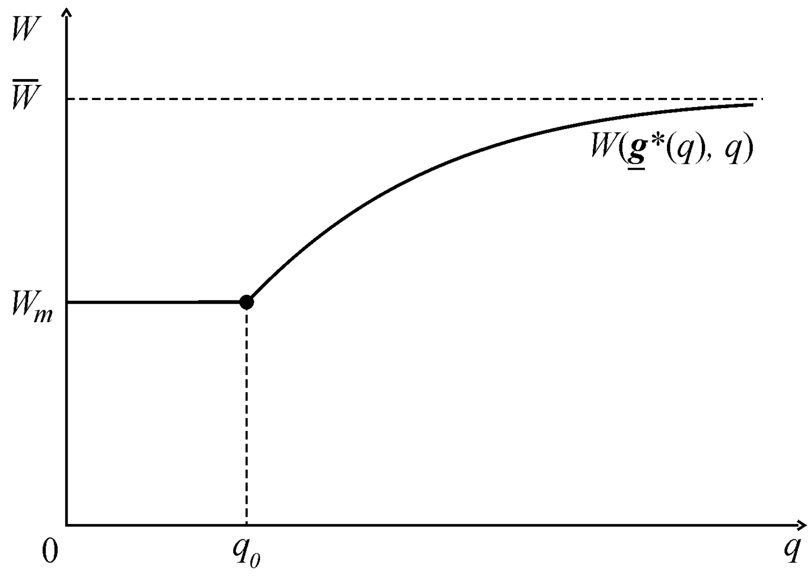

Figure 1.

4 An Example With Two Alternatives

Consider the case $n = 2$. Then $p = p_1(g, q, \theta) = e^{q\Delta(\theta)}g/((e^{q\Delta(\theta)} - 1)g + 1)$, where $\Delta(\theta) = u_1(\theta) - u_2(\theta)$ and $g = g_1 = 1 - g_2$. Condition (10) for an interior solution is now written:

$$g\frac{\partial W}{\partial g} = \int \frac{e^{q\Delta(\theta)}\Delta(\theta)}{[(e^{q\Delta(\theta)} - 1)g + 1]^2} dF(\theta) = 0 \tag{13}$$

It is seen that the second-order condition is satisfied:

$$\frac{\partial^2 W}{\partial g^2} = -2\int \frac{e^{q\Delta(\theta)}\Delta(\theta)(e^{q\Delta(\theta)} - 1)}{[(e^{q\Delta(\theta)} - 1)g + 1]^3} dF(\theta) < 0 \tag{14}$$

Denote the solution to (13) by g^*, $0 \leq g^* \leq 1$. By (14), g^* is unique. Clearly, from (12), a necessary condition for an interior solution is that $\Delta(\theta)$ changes sign at least once in the relevant range: some individuals prefer alternative one over two and some have the opposite preference.

Suppose that $\frac{\partial W}{\partial g}(0,0) < 0$. By (13), this occurs when $\int \Delta(\theta) dF(\theta) = W_1 - W_2 < 0$. At low q, it is best to eliminate alternative 1. If there exists an open interval of θ with a positive density for which $\Delta(\theta) > 0$ (some individuals prefer alternative 1), then, from (13), $\frac{\partial W}{\partial g}(0,q) = \int e^{q\Delta(\theta)} \Delta(\theta) dF(\theta)$ increases unboundedly with q. Hence, there exists a $q_0 > 0$, such that $\frac{\partial W}{\partial g}(0,q) \geq 0$ for all $q \geq q_0$. Consequently, $g^*(q) = 0$ for all $0 \leq q \leq q_0$ and $0 < g^*(q) < 1$ for all $q > q_0$.

Take the opposite case, $\frac{\partial W}{\partial g}(1,0) = \int \Delta(\theta) dF(\theta) = W_1 - W_2 > 0$. At low q, it is optimal to eliminate alternative 2. If there is an open interval of θ with a positive density for which $\Delta(\theta) < 0$ (some individuals prefer alternative 2), then, from (13), $\frac{\partial W}{\partial g}(1,q) = \int \frac{\Delta(\theta)}{e^{q\Delta(\theta)}} dF(\theta)$ decreases, approaching 0 as q increases. Hence, there exists a $q_0 > 0$, such that $\frac{\partial W}{\partial g}(1,q) \leq 0$ for all $q \geq q_0$. Consequently, $g^*(q) = 1$ for all $0 \leq q \leq q_0$, and $0 < g^*(q) < 1$ for all $q > q_0$.

Interestingly, the sign of $\frac{dg^*}{dq}$ is indeterminate, as can be seen by differentiating (13) totally (the sign of $\frac{\partial^2 W}{\partial g \partial q}$ can be positive or negative). This is verified in the 2×2 example below. The reason for this ambiguity is straightforward. When q increases, all individuals make fewer errors of choice. The extent of the reduction in errors, though, depends on the intensity of preferences, $\Delta(\theta)$. When those who prefer alternative 1, $\Delta(\theta) > 0$, display a more moderate intensity than the others and therefore are more prone to make mistakes, then an increase in g^* aimed at reducing the errors of this group is called for. Of course, the effect of a change in g^* on social welfare also depends on the number of individuals at different θ, i.e. on $F(\theta)$.

To obtain an explicit solution for g^*, consider the following 2×2 case. In the case with two alternatives, assume further that there are two individual types $i = 1, 2$ (corresponding to θ_1 and θ_2). Let the density of type 1 in the population be f, $0 < f < 1$. Condition (10) can now be written in a simple form:

$$\frac{p^1(1-p^1)}{p^2(1-p^2)} = \alpha^2 \tag{15}$$

where $p^i = p(g^*, q, \theta_i) = g^* e^{q\Delta^i} / \left[\left(e^{q\Delta^i} - 1\right) g^* + 1\right]$, $\Delta^i = u_1(\theta_i) - u_2(\theta_i)$, $i = 1, 2$, and $\alpha = \left[-\frac{\Delta^2(1-f)}{\Delta^1 f}\right]^{\frac{1}{2}}$.

We assume that $\Delta^1 > 0$, $\Delta^2 < 0$ (or the opposite) and hence the R.H.S. term in the α definition is positive. Substituting these definitions into (15), we obtain an explicit solution for g^*:

$$g^* = \frac{\alpha e^{q\frac{\Delta^2}{2}} - e^{q\frac{\Delta^1}{2}}}{e^{q\frac{\Delta^1}{2}}\left(e^{q\Delta^2} - 1\right) - \alpha e^{q\frac{\Delta^2}{2}}\left(e^{q\Delta^1} - 1\right)} \tag{16}$$

There are two basic cases:

(I) $\boldsymbol{\alpha} > \mathbf{1}$ (Figure 2(a)). From (16), $g^* = 0$ for all $0 \le q \le q_0$ where $q_0 = \frac{\ln \alpha}{\Delta^1 - \Delta^2} > 0$ while $0 < g^*(q) < 1$ for $q > q_0$. The limit of $g^*(q)$ as $q \to \infty$ is:

$$\lim_{q \to \infty} g^*(q) = \begin{cases} 1 & \text{when } \Delta^1 + \Delta^2 < 0 \\ \frac{1}{1+\alpha} & \text{when } \Delta^1 + \Delta^2 = 0 \\ 0 & \text{when } \Delta^1 + \Delta^2 > 0 \end{cases} \tag{17}$$

(II) $\boldsymbol{\alpha} < \mathbf{1}$ (Figure 2(b)). From (16), $g^* = 1$ for all $0 \le q \le q_0$, where

$q_0 = \dfrac{\ln \alpha}{\Delta^2 - \Delta^1} > 0$. For $q_0 < q$, $0 < g^* < 1$. As before, the limit of g^* as $q \to \infty$ is given by (17). In the singular case that $\alpha = 1$, $q_0 = 0$ (that is, no alternative is eliminated at small q's).

This example is instructive in a number of ways. As expected, the degree of rationality at which choice is eliminated, q_0, is negatively correlated with the intensity of choice, $\Delta^1 - \Delta^2$ (or $\Delta^2 - \Delta^1$).

Interestingly, with asymmetric preferences, for example when $\Delta^1 + \Delta^2 < 0$ (that is, group 1 prefers alternative 1 less intensely than group 2 prefers alternative 2), g^* rises with q, approaching 1 as $q \to \infty$. Thus, the optimal policy when individuals are close to 'perfect rationality' is to substantially increase the probability of choosing alternative 1. When individuals are close to being 'perfectly rational', shifting relative weights in choice probabilities has little effect, but it is *differentially* important for those who have 'weaker' preferences. In our example, group 1 is prone to make larger errors due to weaker preferences, and the increase in g^* helps this group to select it's preferred alternative. The opposite holds when $\Delta_1 + \Delta_2 > 0$. It is striking that the optimal weight, $g^*(q)$, is close to 1 or to 0 for large q. It would be interesting to find out how general this result is. Note also that the limit of g^* as $q \to \infty$ depends only on relative preferences and is independent of the relative size of the groups, $(1 - f)/f$.

5 Controlling Only the Number of Alternatives

It is reasonable to consider the case where the government does not have the broad set of instruments needed to fine-tune choice probabilities, but it may not permit certain alternatives. Let us examine in the previous 2×2 example with exogeneously given and equal probability weights the critical value of q below which one of the two alternatives is eliminated. When, for example, only alternative 1 is allowed, social welfare is

$$W_1 = u_1^1 f + u_1^2(1-f) \tag{18}$$

When individuals are allowed to choose between the two alternatives, but the weights in the choice probabilities are fixed and equal, $(g = 1-g = \frac{1}{2})$, social welfare is

$$W(q) = (\frac{e^{q\Delta^1} u_1^1 + u_2^1}{e^{q\Delta^1}+1})f + (\frac{e^{q\Delta^2} u_1^2 + u_2^2}{e^{q\Delta^2}+1})(1-f) \tag{19}$$

Equating $W_1 = W(\widehat{q})$, we obtain an implicit equation for the level of q, $\widehat{q}$, at which alternative 2 is eliminated:

$$\frac{1+e^{\widehat{q}\Delta^2}}{1+e^{\widehat{q}\Delta^1}} = \alpha^2 \tag{20}$$

where α has been defined in (15). For all $0 \le q \le \widehat{q}$, mandating alternative 1 is socially preferable to providing choice and **vice-versa**. It is easy to prove that with the same parameters, the elimination of alternative 2 occurs at a higher q than when the government can continuously manipulate probabilities, that is, $\widehat{q} > q_0$. It is interesting to calculate the type II errors of each group at $\widehat{q}$. Take, for example, $-\frac{\Delta_2}{\Delta_1} = \frac{1}{2}$ and $f = \frac{1}{2}$. The errors are then $1 - P^1(\widehat{q}) = .27$ and $P^2(\widehat{q}) = .38$, respectively. That is, 27 percent of type 1 individuals erroneously choose alternative 2 while 38 percent of type 2 individuals erroneously choose alternative 1.[6] These numbers, though, are quite sensitive to parameter values.

[6]The fact that group 2 errors are larger at the optimum is expected in view of their 'weaker' preferences.

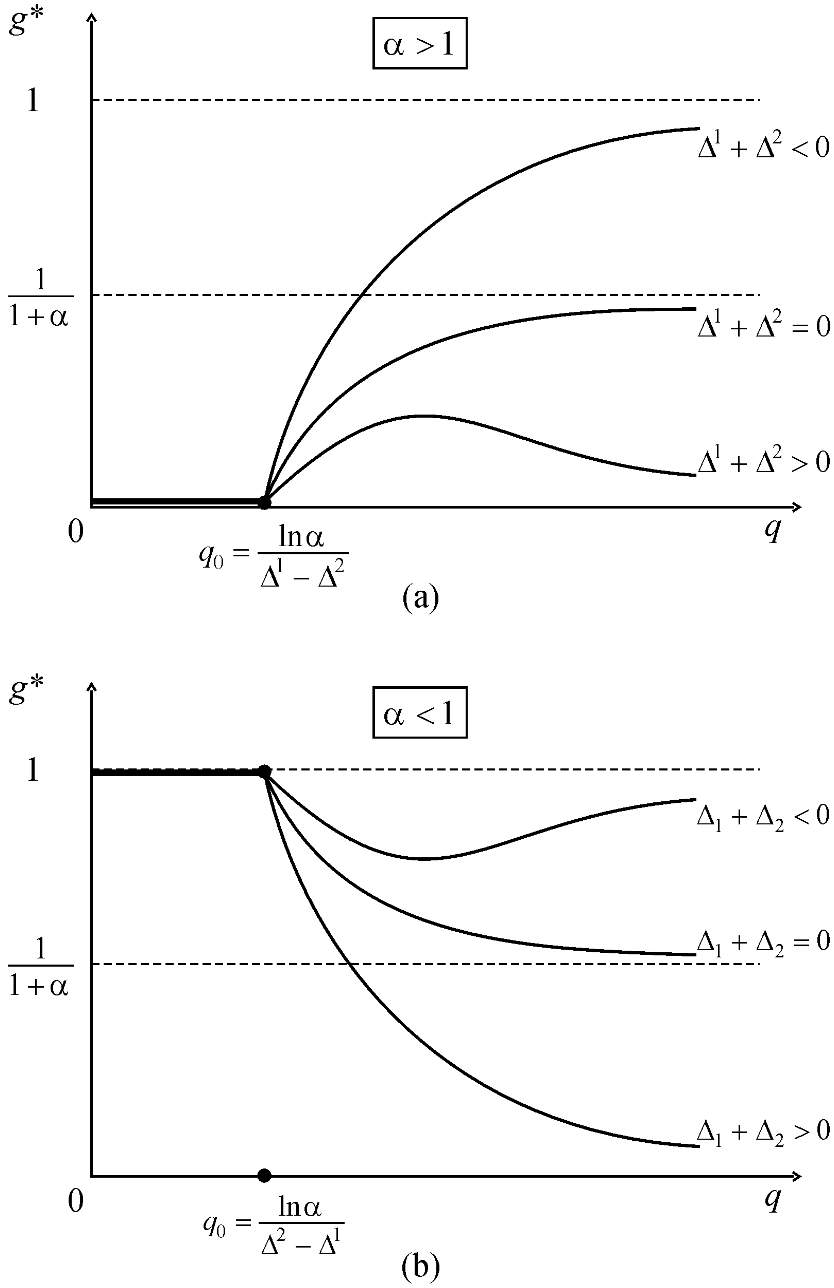
g*
α > 1
1
1/(1+α)
Δ1 + Δ2 < 0
Δ1 + Δ2 = 0
Δ1 + Δ2 > 0
0
q
q0 = lnα / (Δ1 − Δ2)
(a)
g*
α < 1
1
1/(1+α)
Δ1 + Δ2 < 0
Δ1 + Δ2 = 0
Δ1 + Δ2 > 0
0
q
q0 = lnα / (Δ2 − Δ1)
(b)

Figure 2.

References

[1] Choi, J., Laibson, D., Madrian, B. and Metrick, A. (2002), "Benign Paternalism and Active Decisions: A Natural Experiment in Savings", *N.B.E.R.*

[2] Debreu, G. (1960) "Review of R. D. Luce, Individual Choice Behavior: A Theoretical Analysis" *American Economic Review, 50,* 186-88

[3] Johnson, E. and Goldstein, D. (2003), "Do Defaults Save Lives?" *Center for Decision Science, Columbia University.*

[4] Luce, D. (1959), *Individual Choice Behavior: A Theoretical Analysis (Wiley).*

[5] Sheshinski, E. (2002), "Bounded Rationality and Socially Optimal Limits on Choice in a Self-Selection Model" *Department of Economics, The Hebrew University of Jerusalem.*

Appendix

For the case $n = 2$ we have seen in the text, eq. (14), that the solution g_1^*, $g_2^* = 1 - g_1^*$, is unique. For $n > 2$, a monotonicity condition on preferences has to be assumed. For example, from (11) we have that the diagonal terms of the matrix $\left[\dfrac{\partial^2 W}{\partial g_i \partial g_j}\right]$ are:

$$\frac{\partial^2 W}{\partial g_i^2} = \frac{2}{g_i^{*2}} \int p_i^2(\mathbf{g}^*, q, \theta)\Delta_i(\mathbf{g}^*, q, \theta)dF(\theta) \quad i = 1, 2, ..n \tag{A.1}$$

where $\Delta_i = \Delta_i(\mathbf{g}^*, q, \theta) = u_i(\theta) - V(\mathbf{g}^*, q, \theta)$ and $p_i(\mathbf{g}^*, q, \theta) = \dfrac{e^{q\Delta^i}}{\sum\limits_{j=1}^{n} e^{qu_j} g_j}$.

Suppose Δ_i is monotone in θ. Since p_i is strictly monotone increasing in Δ_i, it follows that $p_i\Delta_i$ is strictly monotone in θ. From (10) we now have that $p_i\Delta_i$ changes sign once.

Let $\widetilde{\theta}$ be the value of θ at which $p_i\Delta_i = 0$. Assume that Δ_i, and hence p_i, increases in θ.

Then, $p_i^2(\mathbf{g}^*, q, \theta)\Delta_i(\mathbf{g}^*, q, \theta) > p_i(\mathbf{g}^*, q, \widetilde{\theta})p_i(\mathbf{g}^*, q, \theta)\Delta_i(\mathbf{g}^*, q, \theta)$, for all θ. Multiplying both sides by $dF(\theta)$ and integrating, using (10),

$$\begin{aligned} &\textstyle\int p_i^2(\mathbf{g}^*, q, \theta)\Delta_i(\mathbf{g}^*, q, \theta)dF(\theta) > \\ &\quad > p_i(\mathbf{g}^*, q, \widetilde{\theta}) \textstyle\int p_i(\mathbf{g}^*, q, \theta)\Delta_i(\mathbf{g}^*, q, \theta)dF(\theta) = 0 \end{aligned} \tag{A.2}$$

This proves that the term in (A.1) is strictly negative. This or a broader monotonicity condition suffices to ensure that $\left[\dfrac{\partial^2 W}{\partial g_i \partial g_j}\right]$ is negative semi-definite.