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by

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Abstract

Two salient features of modern economic growth are the rise in aggregate savings rates and the steady increase in life expectancy. This paper links these processes, showing that under certain conditions economic theory supports the hypothesis that increased longevity leads to higher aggregate savings in steady state. The analysis is based on a lifecycle model with uncertain longevity in which individuals choose an optimum consumption path and a retirement age. Conditions on the age-specific pattern of improvements in survival probabilities are shown to ensure that individual savings rise with longevity and that aggregation preserves this result. Population theory (Coale (1972)) is used to link the steady-state age density function and the population's growth rate to individuals' survival probabilities. The importance of a competitive annuity market in avoiding unintended bequests is underscored.

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1 Introduction

A salient feature of modern economic growth is the increase in aggregate savings rates, reflected in the rise of capital-income ratios[1]. During 1820 - 1992, non-residential capital in the US grew at 4.1 percent per annum, compared with GDP growth of 3.6 percent (Kinugasa and Mason (2007)). Consequently, the ratio of gross non-residential capital to GDP increased more than fourfold, from .71 to 3.02 (Maddison (1995)). The experience in the UK and Japan is similar. Even more pronounced, over a shorter period, were the high savings rates and capital deepening in Asian countries (Lee, Mason and Miller (2001), Deaton and Paxson (2000)).

Parallel to the increase in savings rates there was a steady rise in life expectancy. Mortality has fallen substantially in the past hundred years: in 1900, about 2.5 people per hundred died in the US and the UK in a typical year. Today mortality is two-thirds lower. As Cutler (2004) points out, the trend of declining mortality has three distinct phases. Early in the twentieth century there has been a significant drop in infant mortality due to improved nutrition and improvement in health conditions. This was followed by a major reduction in mortality rates of adults due to infections diseases: "until the 1950's there was no evidence in any society of people reducing mortality from chronic diseases of old age... and then cardiovascular disease mortality started declining extremely rapidly" (Cutler (2004, page 8). In recent decades, the rise of longevity is concentrated in life lengthening of the old due to medical advances.

The objective of this paper is to analyze whether economic theory supports the hypothesis, suggested in a number of empirical studies (see below), that the rise in aggregate savings rates was largely driven by higher life expectancy. It is shown that under certain conditions on the pattern of survival improvements the answer is positive. The analysis is based on a lifecycle model with uncertain longevity and on explicit aggregation of individuals' response functions. Two effects are recognized and analyzed in detail:

[1]Naturally, aggregate savings in absolute terms are expected to increase with the growth of population (due, say, to higher longevity, birth rates or other reasons).

(a) Behavioral Effects

An increase in survival probabilities affects individuals' consumption and retirement decisions. One has to identify the patterns of mortality declines that lead individuals to increase their savings, taking into account the response of endogenously chosen retirement ages. For example, when survival probabilities increase mainly at older ages, individuals are expected to save more during their working years in order to support a longer retirement. Although in this case retirement age is also shown to rise with longevity, this compensates only partially for the need to decrease consumption. The opposite response can be expected when survival probabilities rise mainly for adults in their early life. Demonstrating the dependence of individuals' responses on the age related pattern of improvements in survival probabilities is particularly pertinent in view of the uneven history of age-specific declines in mortality rates outlined above.

(b) Age Composition Effects

An increase in survival probabilities changes the population's age density function. The direction of this change depends on which of two opposite effects dominates. First, an increase in survival rates raises the size of all age cohorts, some more than others depending on the specification of the age related improvements in survival probabilities. Second, with given age-specific birth rates, an increase in survival probabilities raises the population's growth rate. A higher growth rate, in turn, increases the relative weight of younger age groups. Since older ages are typically retirees who are dissavers while younger ages are workers who save towards retirement, the first effect tends to reduce aggregate savings while the second effect raises savings. Conditions provided below ensure that the latter effect is dominant.

The dynamics of demographic processes generated by a change in survival probabilities is quite complex. There exists, however, a well developed theory of the dependence of *steady-state age density distributions* on the underlying parameters (e.g. Coale (1972)). Building on this theory, we shall study the long-run effects of changes in longevity on aggregate savings, taking into account endogenous changes in the age density function.

Starting with Yaari's (1965) seminal work, a number of papers analyzed the effects of rising life expectancy on individual savings (e.g. Leung (1994), Bloom and Canning (2003), Zilcha and Friedman (1985), Kinugasa and Mason (2007)). These papers study the response of individuals to changes in longevity but none provides a general characterization of the pattern of age-specific improvements in survival probabilities that lead to an increase in individual savings. Furthermore, these works do not incorporate endogenous changes in the chosen retirement age due to changes in longevity.

The effects of a longevity increase on aggregate savings have been explored empirically or by simulations in a number of the above and other papers (e.g. Miles (1999), Deaton and Paxson (2000) and Lee, Mason and Miller (2001)). All these papers find a positive correlation between longevity and aggregate savings but, in the absence of explicit aggregation of individuals' response functions it is impossible to identify the underlying factors which determine the direction of the age composition effects.

This paper performs two tasks: first, it derives individual response functions based on a model of individual lifetime decisions about consumption and retirement in the presence of longevity risks with access to a competitive annuity market. Second, it aggregates individuals' response functions, linking their survival functions with the steady-state population age density function.

Existence of a competitive annuity market is crucial for individual decisions on savings and retirement. In the absence of this market, these decisions have to take into account the existence of *unintended bequests*, that is, assets left at death because individuals do not want to outlive their resources. In these circumstances, uncertain lifetime generates a random distribution of bequests which become initial endowments for a subsequent generation. A general analysis of the long-term effects of longevity changes on the ergodic distribution of these bequests and endowments is beyond the scope of this paper. Section 7, however, provides an example of savings and random bequests in the absence of annuities. Not surprisingly, in this case the direction of the effect of a rise in longevity cannot generally be ascertained.

2 A Simple Life-Cycle Model With Uncertain Survival

Consider a simple individual life-cycle model with uncertain survival. At age 0, the probability of surviving to age z is $F(z) : F(0) = 1$, and $F'(z) \leq 0$, $z \geq 0$. There may be a finite age $T > 0$ for which $F(T) = 0$, but this is not necessary[2].

Individuals derive instantaneous utility $u(c)$ ($u' > 0$, $u'' < 0$), independent of age, from consumption, c, and can decide to work or retire (disregarding the choice of labor intensity). Work is normalized to a level of unity.

Disutility from work at age z, $e(z) > 0$, is assumed to be independent of consumption and, in order to ensure that work precedes retirement, non-decreases with age ($e'(z) \geq 0$). In the absence of time-preference, expected lifetime utility, V, is therefore

$$V = \int_0^\infty u(c(z))F(z)dz - \int_0^R e(z)F(z)dz \tag{1}$$

where $c(z)$ is consumption at age z and R is the age of retirement.

Let $a(z)$ be the amount of annuities held by an age z individual[3]. Then the dynamic budget constraint is

$$\dot{a}(z) = r(z)a(z) + w(z) - c(z) \tag{2}$$

where $\dot{a}(z)$ is the amount of annuities purchased (> 0) or sold (< 0), $r(z)$ is the instantaneous rate of return on annuities and $w(z)$ is the wage rate ($w(z) = 0$ for $z > R$) of an age z individual. It is assumed that $w(z)$ non-increases in z. As shown below, this assumption ensures that the individual

[2] A commonly used function is $F(z) = \dfrac{e^{-\alpha z} - e^{-\alpha T}}{1 - e^{-\alpha T}}$ ($\alpha > 0$ constant) defined over $0 \leq z \leq T$. Its limit is the exponential, $\lim_{T\to\infty} F(z) = e^{-\alpha z}$.

[3] We know from Yaari (1965) that when longevity is the only uncertainty then rational individuals will annuitize all their assets. The modifications required when individuals have a positive time preference and/or there is a positive rate of interest on non-annuitized assets are well-known and have no impact on the following analysis.

will choose a working phase followed by a retirement phase. Solving (2) for a given consumption path, $c(z)$, the holdings for annuities at age z are

$$a(z) = e^{\int_0^z r(x)dx} \int_0^z e^{-\int_0^x r(h)dh} (w(x) - c(x))dx \tag{3}$$

with $a(0) = 0$.

In a competitive annuity market equilibrium, the rate of return on annuities is equal to the *Hazard-Rate*, the conditional probability of dying at age z

$$r(z) = -\frac{d\ln F(z)}{dz} = \frac{f(z)}{F(z)} \tag{4}$$

where $f(z) = -\dfrac{dF(z)}{dz}$ is the probability of dying at age z[4].

From (3), (4) and the transversality condition $\lim_{z\to\infty} a(z)\ e^{-\int_0^z r(x)dx} = 0$, we obtain the lifetime budget constraint

$$\int_0^\infty c(z)F(z)dz - \int_0^R w(z)F(z)dz = 0, \tag{5}$$

Thus, equilibrium condition (4) implies that expected consumption is equal to expected wages, that is, zero expected profits. Maximization of (1) s.t.(5) yields constant optimum consumption and an optimum retirement age which satisfy

$$c^* = \frac{\int_0^{R^*} w(z)F(z)dz}{\overline{z}} \tag{6}$$

$$u'(c^*)w(R^*) - e(R^*) = 0 \tag{7}$$

where $\overline{z} = \int_0^\infty F(z)dz$ is expected lifetime[5]. We denote the solution to (6) - (7) by $(c^*,\ R^*)$. Inserting (6) into (7), it is seen that the solution $(c^*,\ R^*)$ is unique

[4] See Sheshinski (2007).

[5] Integrating by parts, $\overline{z} = \int_0^\infty z f(z)dz$.

if the first term in (7) strictly decreases with R^*. For this, the assumption that $w'(z) \leq 0$ is a sufficient condition[6].

Condition (6) makes optimum consumption equal to expected wages divided by expected lifetime. Condition (7) equates the marginal benefits and costs of a small postponement of retirement.

Individual savings at age z, $s^*(z)$, are positive during the working phase and negative during retirement:

$$s^*(z) = \begin{cases} w(z) - c^*, & 0 \leq z \leq R^* \\ \quad\quad\; - c^*, & R^* < z < \infty \end{cases} \tag{8}$$

In the absence of a bequest motive, expected savings over the whole lifetime are zero: $\int_0^\infty s^*(z)F(z)dz = 0$.

3 Effects of Longevity Changes on Individual Decisions

Suppose that the survival function depends on a parameter denoted α, $F(z, \alpha)$, representing longevity. We take a decrease in α to cause an upward shift in survival probabilities, $\dfrac{\partial F(z,\alpha)}{\partial \alpha} < 0$, at all ages, $z > 0$[7]. Obviously, expected lifetime, $\bar{z}(\alpha) = \int_0^\infty F(z,\alpha)dz$, decreases with α.

Denote by $\mu(z, \alpha)$ the proportional change in the survival function at age z due to a change in α: $\mu(z,\alpha) = \dfrac{1}{F(z,\alpha)} \dfrac{\partial F(z,\alpha)}{\partial \alpha}$ (< 0). Differentiating (3) partially w.r.t. α, holding R^* constant, yields

$$\frac{1}{c^*}\frac{\partial c^*}{\partial \alpha} = \varphi(R^*, \alpha) \tag{9}$$

[6] For an interior solution when T is finite, it is sufficient to assume that $e(z)$ strictly increases from zero to ∞ as z rises from zero to T.

[7] Of course, $F(0, \alpha) = 1$ for any α. If the effect of a change in α on $F(z, \alpha)$ is continuous, the implication is that the effect of a change in α around $z = 0$ is small. See Assumption 1 below. When there is a finite T for which $F(T, \alpha) = 0$, T depends on α. In view of the rise in survival probabilities at very old ages, this is an expected outcome.

where

$$\varphi(R^*,\alpha)=\frac{\int\limits_0^{R^*}\mu(z,\alpha)F(z,\alpha)dz}{\int\limits_0^{R^*}F(z,\alpha)dz}-\frac{\int\limits_0^{\infty}\mu(z,\alpha)F(z,\alpha)dz}{\int\limits_0^{\infty}F(z,\alpha)dz} \tag{10}$$

Clearly, $\lim_{R^*\to\infty}\varphi(R^*,\ \alpha)=0$. Hence, when $\dfrac{\partial\varphi}{\partial R^*}(R^*,\ \alpha)\leq 0\ (\geq 0)$ (with strict inequality for some R^*), then $\varphi(R^*,\ \alpha)>0\ (<0)$ for all R^*.

We have

$$\frac{\partial\varphi(R^*,\ \alpha)}{\partial R^*}=\frac{F(R^*)}{\int\limits_0^{R^*}F(z,\alpha)dz}\int\limits_0^{R^*}[\mu(R^*,\alpha)-\mu(z,\alpha)]F(z,\alpha)dz \tag{11}$$

The following assumption ensures that (11) is negative:

Assumption 1. $\mu(z,\alpha)$ *non-increases in* z, $\dfrac{\partial\mu(z,\alpha)}{\partial z}\leq 0$, *for all* z.

This assumption has a straightforward interpretation: *improvements in survival rates are proportionately larger at later ages.* It is equivalent to assuming that an increase in α raises the *Hazard-Rate*[8].

It follows from (10) and (11) that under Assumption 1, $\dfrac{\partial c^*}{\partial\alpha}>0$. That is, an increase in longevity, holding retirement age constant, decreases consumption. Note that when $\mu(z,\alpha)$ non-decreases in z, then $\dfrac{\partial c^*}{\partial\alpha}<0$. When increases in survival probabilities are proportionately larger at early ages compared to later ages then, as could be expected, individuals increase consumption (and decrease savings).

The effect of a change in survival probabilities on optimum retirement is obtained by totally differentiating (6) – (7) w.r.t. α. In elasticity form:

[8] According to a standard definition of *Stochastic Dominance* (see Sheshinski (2007)), when this assumption is satisfied then a survival function with a lower α stochastically dominates any survival function with a higher α.

Note that the function in f.n. 2 above satisfies Assumption 1 (for any T).

$$\frac{\alpha}{R^*}\frac{dR^*}{d\alpha} = -\frac{\sigma \dfrac{\alpha}{c^*}\dfrac{\partial c^*}{\partial \alpha}}{\sigma \dfrac{R^*}{c^*}\dfrac{\partial c^*}{\partial R} + \dfrac{R^* e'(R^*)}{e(R^*)}} \tag{12}$$

where $\sigma = -\dfrac{u''(c^*)c^*}{u'(c^*)} > 0$ is the *coefficient of relative risk aversion.*

From (6), $\dfrac{R^*}{c^*}\dfrac{\partial c^*}{\partial R} = \dfrac{F(R^*,\alpha)R^*}{\int_0^{R^*} F(z,\alpha)dz}$. Since F non-increases in z, it is seen that

$0 < \dfrac{R^*}{c^*}\dfrac{\partial c^*}{\partial R} < 1$. Hence, $\dfrac{dR^*}{d\alpha} \lesseqgtr 0 \Leftrightarrow \dfrac{\partial c^*}{\partial \alpha} \gtreqless 0$.

The total change in consumption is, using (12),

$$\frac{dc^*}{d\alpha} = \frac{\partial c^*}{\partial R}\left(\frac{\alpha dR^*}{R^* d\alpha}\right) + \frac{\partial c^*}{\partial \alpha} = \left(\frac{\dfrac{R^* e'(R^*)}{e(R^*)}}{\sigma \dfrac{R^*}{c^*}\dfrac{\partial c^*}{\partial R} + \dfrac{R^* e'(R^*)}{e(R^*)}} \right) \frac{\partial c^*}{\partial \alpha}. \tag{13}$$

By Assumption 1, an increase in longevity increases the optimum retirement age, but this only partially compensates for the required decrease in consumption (and, correspondingly, the increase in savings) and hence, $\dfrac{dc^*}{d\alpha} > 0$.

We summarize the analysis so far:

Proposition 1 *Under Assumption 1, an increase in longevity increases optimum retirement,* $\dfrac{dR^*}{d\alpha} < 0$*, and decreases optimum consumption,* $\dfrac{dc^*}{d\alpha} > 0$*.*

It is of interest to find the effect of a change in α on optimum lifetime utility, $V^* = u(c^*)\overline{z} - \int_0^{R^*} e(z)F(z,\alpha)dz$.

By the envelope theorem, (3) – (4), (6) and (7),

$$\begin{aligned} \frac{dV^*}{d\alpha} &= \frac{\partial V^*}{\partial \alpha} = [u(c^*) - u'(c^*)c^*]\int_0^{\infty} \frac{\partial F(z,\alpha)}{\partial \alpha} dz + \\ &\quad + \int_0^{R^*} [e(R^*) - e(z)] \frac{\partial F(z,\alpha)}{\partial \alpha} dz \end{aligned} \tag{14}$$

Strict concavity of $u(c)$ and the assumption that $e'(z) \geq 0$ ensure that $\frac{dV^*}{d\alpha} < 0$. An increase in longevity always increases welfare[9].

4 Longevity Changes and Aggregate Savings

Suppose that the population grows at a constant rate, g. The *steady-state* age density function of the population, denoted $h(z, \alpha, g)$, is given by[10]

$$h(z, \alpha, g) = me^{-gz}F(z, \alpha) \tag{15}$$

where $m = \frac{1}{\int_0^\infty e^{-gz}F(z,\alpha)dz}$ is the birth rate.

The growth rate g, in turn, is determined by the second fundamental equation of stable population theory:

$$\int_0^\infty e^{-gz}F(z, \alpha)b(z)dz = 1 \tag{16}$$

where $b(z)$ is the age specific fertility function.

The effect on g of a change in α, can be determined by totally differentiating (16):

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[9] This result depends on our assumption that $u(c) > 0$ independent of age, compared to zero utility at death. In discussions of life extending treatments this assumption has at times been questioned.

[10] Equations (15) and (16) are derived as follows (see Coale (1972): let the current number of age z females be $n(z)$, while the total number is N. When population grows at a rate g, the number of females z periods ago was Ne^{-gz}. If m is the birth rate, then z periods ago mNe^{-gz} females were born. Given the survival function $F(z, \alpha)$,

$$h(z, \alpha, g) = \frac{n(z)}{N} = \frac{Ne^{-gz}mF(z,\alpha)}{N} = me^{-gz}F(z, \alpha).$$

Since $\int_0^\infty h(z, \alpha, g)dz = 1$ if follows that the birth rate m is equal to $m = 1 \Big/ \int_0^\infty e^{-gz}F(z,\alpha)dz$. This yields equation (15). By definition, $m = \int_0^\infty h(z, \alpha, g)b(z)dz$, where $b(z)$ is the specific *fertility* rate at age z. Substituting the above definition of $h(z, \alpha, g)$ we obtain (16).

$$\frac{dg}{d\alpha} = \frac{\int\limits_0^\infty e^{-gz} \frac{\partial F(z,\alpha)}{\partial \alpha} b(z) dz}{\int\limits_0^\infty e^{-gz} z F(z,\alpha) b(z) dz} < 0. \tag{17}$$

An increase in longevity raises the steady-state growth rate of the population. The magnitude of g depends implicitly on the form of the survival and fertility functions, $F(z,\ \alpha)$ and $b(z)$, respectively. It can be solved explicitly in some special cases. For example, with $F(z,\alpha) = e^{-\alpha z}$ and $b(z) = b > 0$, constant, for all $z \geq 0$, (16) yields $g = b - \alpha$. The population growth rate is equal to the difference between the birth rate and the mortality rate. Indeed, substituting $\frac{1}{F}\frac{\partial F}{\partial \alpha} = -z$ into (17), we obtain that in this case $\frac{dg}{d\alpha} = -1$.

Aggregate steady-state savings per capita, S, are

$$\begin{aligned} S &= \int\limits_0^\infty s^*(z,\alpha) h(z,\alpha,g) dz = \\ &\quad \text{from (8)} \\ &= \int\limits_0^{R^*} w(z) h(z,\alpha,g) dz - c^* = \\ &= \int\limits_0^{R^*} w(z) \left[\frac{e^{-gz}}{\int\limits_0^\infty e^{-gz} F(z,\alpha) dz} - \frac{1}{\int\limits_0^\infty F(z,\alpha) dz} \right] F(z,\alpha) dz. \end{aligned} \tag{18}$$

It is seen that $S = 0$ when $g = 0$. A stationary economy without population growth has no aggregate savings per capita, corresponding to zero personal lifetime savings. We shall now show that $S > 0$ when $g > 0$. Denote average life expectancy of the population below a certain age, R, by $\widetilde{z}(R)$. From (15),

$$\widetilde{z}(R) = \int_0^R e^{-gz} z F(z,\alpha) dz \Bigg/ \int_0^R e^{-gz} F(z,\alpha) dz \tag{19}$$

The average population age, $\widetilde{z}$, is

$$\widetilde{z} = \widetilde{z}(\infty) = \int_0^\infty e^{-gz} z F(z,\alpha) dz \Bigg/ \int_0^\infty e^{-gz} F(z,\alpha) dz. \tag{20}$$

Clearly, $\widetilde{z}(R) < \widetilde{z}$ for any R.

Differentiating (18) partially w.r.t. g,

$$\frac{\partial S}{\partial g} = \left(\int_0^{R^*} e^{-gz} F(z,\alpha) dz \Bigg/ \int_0^\infty e^{-gz} F(z,\alpha) dz \right) (\widetilde{z} - \widetilde{z}(R^*)) > 0 \tag{21}$$

A positive population growth rate, $g > 0$, entails positive aggregate steady-state savings per capita.

To examine the effect of a change in α on aggregate savings, differentiate (18) totally,

$$\frac{dS}{d\alpha} = w(R^*) h(R^*, \alpha, g) \frac{dR^*}{d\alpha} - \frac{dc^*}{d\alpha} + \int_0^{R^*} w(z) \frac{dh(z,\alpha,g)}{d\alpha} dz \tag{22}$$

We have seen that under Assumption 1, $\frac{dR^*}{d\alpha} < 0$ and $\frac{dc^*}{d\alpha} > 0$. Hence, when the last term in (22) is non-positive this ensures that $\frac{dS}{d\alpha} < 0$.

The sign of $\frac{dh(z,\alpha,g)}{d\alpha}$ reflects two opposite effects: an increase in longevity raises the survival function at all ages and, as shown above, also raises the population growth rate. The first effect raises h while the second decreases it. Since $\int_0^\infty \frac{dh(z,\alpha,g)}{d\alpha} dz = 0$, the crucial question is which of these effects is dominant at different ages. Since $w(z)$ non-increases in z, it can be seen that the last term in (22) is negative when $\frac{dh}{d\alpha}$ is negative for small z and positive for large z. The interpretation is straightforward: a rise in longevity which raises the population steady-state density in "working ages", when individuals save, and decreases the density in "retirement ages", when individuals dissave, tends to increase aggregate savings (and *vice-versa*). This is the Age Composition Effect.

We now provide conditions which ensure that, in steady-state, aggregate savings increase with longevity. These conditions further highlight the tension between the opposing effects discussed above.

Two additional assumptions are made:

Assumption 2 *The age specific birth rate, $b(z)$, non-increases with age,* $b'(z) \leq 0$.

Recall that we denote $z = 0$ as the age when individuals plan for their future. So this is a natural assumption, certainly at the more advanced ages.

Assumption 3 *The elasticity of* $\mu(z, \alpha)$ w.r.t. z does not exceed unity, $\frac{z}{\mu(z,\alpha)} \frac{\partial \mu(z,\alpha)}{\partial z} \leq 1$, for all z[11].

Recall that in order to determine that individuals increase their lifetime expected savings as survival probabilities rise, it was assumed that improvements in longevity are tilted towards older ages (Assumption 1). Taken by itself, this implies that the population's density function increases proportionately more at older ages. Higher longevity also raises the population's growth rate. As seen in (15), this leads to a steeper rate of decline of the population density with age, as the ratio of the size of any two successive age groups rises. Assumption 3, constraining the rate of increase of survival probabilities with age, ensures that between these two effects, the latter effect dominates.

We can now state our central result:

Proposition 2 *Under Assumptions 1, 2 and 3, aggregate steady-state savings rise with longevity,* $\frac{dS}{d\alpha} < 0$.

[11] Note that the limiting case which satisfies this assumption is the exponential function, $F(z, \alpha) = e^{-\alpha z}$, $0 \leq z \leq \infty$, where $\frac{z}{\mu} \frac{\partial \mu}{\partial z} = 1$.

Proof Appendix

It is worth noting that the assumptions underlying Proposition 2, whose empirical validity can be ascertained, are sufficient conditions and hence a positive relation between longevity and aggregate savings may be found (and empirically observed) in special cases which do not satisfy some of these assumptions. These conditions ensure, however, that the outcome pertains to a wide class of individual preferences and survival functions.

5 Example: Exponential Survival Function[12]

The above expressions can be solved explicitly for the particular survival function $F(z,\alpha) = e^{-\alpha z}$, $z \geq 0$, a constant wage rate, $w(z) = w$, and a constant age specific birth rate, $b(z) = b$.

Equation (6) becomes

$$c^* = w(1 - e^{-\alpha R^*}) \tag{23}$$

and (11) and (12) are (in elasticity form):

$$\frac{\alpha}{R^*}\frac{dR^*}{d\alpha} = -\frac{\sigma}{\sigma + \frac{R^* e'(R^*)}{e(R^*)}\left(\frac{e^{\alpha R^*} - 1}{\alpha R^*}\right)} \tag{24}$$

$$\frac{\alpha}{c^*}\frac{dc^*}{d\alpha} = \frac{\alpha R^*}{e^{\alpha R^*} - 1}\left(1 + \frac{\alpha}{R^*}\frac{dR^*}{d\alpha}\right) \tag{25}$$

Clearly, $-1 \leq \frac{\alpha}{R^*}\frac{dR^*}{d\alpha} \leq 0$ and $0 \leq \frac{\alpha}{c^*}\frac{dc^*}{d\alpha} \leq 1$.

The steady-state age density function, (15), is

$$h(z,\alpha,g) = (g+\alpha)e^{-(g+\alpha)z} \tag{26}$$

while the population growth rate, g, with constant birth rate, b, is solved from

[12]See Sheshinski (2006).

(16), $g = b - \alpha$. Hence, $\dfrac{dg}{d\alpha} = -1$.

Aggregate steady-state savings, (18), are

$$S = e^{-\alpha R^*}(1 - e^{-gR^*}) \tag{27}$$

Totally differentiating (27),

$$\frac{dS}{d\alpha} = -we^{-\alpha R^*}\left\{1 + \frac{\alpha}{R^*}\frac{dR^*}{d\alpha}\left[1 - \frac{be^{-gR^*}}{\alpha}\right]\right\} < 0 \tag{28}$$

6 No Annuitization

It was assumed that annuitization is available at all ages, which means that individuals can take full advantage of risk pooling. To demonstrate that this is a critical assumption, consider the case of no insurance[13]. The budget constraint (5) now becomes:

$$\int_0^\infty c(z)dz - \int_0^R w(z)dz = 0 \tag{29}$$

In the absence of insurance, there is also a constraint that assets must be non-negative at all ages (individuals cannot die with debt). Equating expected marginal utility across ages yields decreasing optimum consumption, whose shape reflects the individual's degree of risk aversion. To demonstrate that the effects of a change in longevity on savings and retirement are, in general, indeterminate, it suffices to take particular utility and survival functions. Thus, assume that $u(c) = \ln c$ and $F(z, \alpha) = e^{-\alpha z}$. For a constant wage $w(z) = w$, optimum consumption, $\hat{c}(z)$, now becomes (instead of (6)):

$$\hat{c}(z) = w\alpha\hat{R}e^{-\alpha z} \tag{30}$$

Accordingly, individual savings, (8), are now:

[13]Social Security systems provide such annuitization. Mandatory uniform formulas may, however, be inadequate for some individuals and excessive for others. See Sheshinski (2003, p. 27-54).

$$\hat{s}(z) = \begin{cases} w(1 - \alpha \hat{R}\, e^{-\alpha z}) & 0 \leq z \leq \hat{R} \\ \\ -w\alpha \hat{R}\, e^{-\alpha z} & \hat{R} \leq z \leq \infty \end{cases} \tag{31}$$

and optimum retirement is obtained from condition (7):

$$\frac{1}{\alpha \hat{R}} e^{\alpha \hat{R}} = e(\hat{R}). \tag{32}$$

For this condition to have a unique solution it is assumed that the L.H.S. of (32) strictly decreases with $\hat{R}$. This holds iff $\hat{R} < \frac{1}{\alpha}$, i.e. optimum retirement age is lower than expected lifetime, which is reasonable. When this condition holds then $\frac{d\hat{R}}{d\alpha} \leq 0$, that is, as before, an increase in longevity leads to an increase in retirement age.[14]

Aggregate steady-state savings, (14), now become:

$$S = w \left[1 - e^{-(g+\alpha)\hat{R}} - \frac{\alpha \hat{R}(g + \alpha)}{g + 2\alpha} \right] \tag{33}$$

Taking into account that $\frac{dg}{d\alpha} = -1$, it is seen that, holding $\hat{R}$ constant, a decrease in α affects S positively. However, when the change in $\hat{R}$ is also taken into account, the direction of the change in S is indeterminate, depending on parameter configuration.

7 Unintended Bequests

The analysis in the previous section disregards the fact that in the absence of full annuitization there are *unintended bequests* which affect individual behavior, in particular individual savings[15]. A general equilibrium analysis of

[14] The same condition ensures the non-negativity of assets at all ages ($S^*(0) = w(1 - \alpha R^*) > 0$).

[15] The empirical importance of bequests and intergenerational transfers is debated extensively, among the inconclusive issues is the separation of planned bequests from those due to lack of annuity markets.

See, for example, Kotlikoff and Summers (1981) and more recently Kopczuk and Lupton (2005).

longevity effects on aggregate savings has to take these intergenerational transfers into account.

In the absence of full annuitization, uncertain lifetime generates a distribution of bequests which depends on survival probabilities. A proper comparison of steady-states with and without annuitization requires derivation of the *ergodic*, *long-term*, *distribution of bequests* which, in turn, generates a distribution of individual and aggregate savings. A general analysis of this process is beyond the scope of this paper. The issue can, however, be clarified by means of a simple example.

Suppose that individuals live one period and with probability p, $0 \leq p \leq 1$, two periods. With no time preference, expected lifetime utility, V, is

$$V = u(c) + pu(c_1) \tag{34}$$

where c is first period consumption and c_1 is second period consumption. Without annuities and a zero interest rate, the budget constraint is

$$c + c_1 = w + b \tag{35}$$

where $w > 0$ is income and $b \geq 0$ is initial endowment. Let $u(c) = \ln c$. Then optimum consumption, $\hat{c}$ and $\hat{c}_1$, is

$$\hat{c}(b) = \frac{w+b}{1+p}, \qquad \hat{c}_1(b) = \frac{p(w+b)}{1+p} \tag{36}$$

Having no bequest motive, individuals who live two periods leave no bequest. Consequently, some individuals will have no initial endowments. Others will have positive endowments which depend on the history of parental survivals. In fact, the steady-state distribution of initial endowments is a *Renewal Process.*

Denote by $\hat{b}_k$ the initial endowment of an individual whose k previous generations of parents lived one period only. If p_0 is the probability of a zero endowment, then the probability of $\hat{b}_k$ is $(1-p)^k\ p_0$. Since $p_0 \sum_{k=0}^{\infty} (1-p)^k = 1$, it follows that $p_0 = p$. We can calculate $\hat{b}_k$ from (38):

$$\begin{aligned} \hat{b}_k &= w + \hat{b}_{k-1} - \hat{c}(\hat{b}_{k-1}) = \left[\frac{p}{1+p} + (\frac{p}{1+p})^2 + ... + (\frac{p}{1+p})^k \right] w = \\ &= p\left(1 - (\frac{p}{1+p})^{k-1}\right) w \qquad\qquad k = 1, 2, ... \end{aligned} \tag{37}$$

Thus, savings of an individual with endowment $\hat{b}_k$, $s(\hat{b}_k)$, is

$$s(\hat{b}_k) = w - \hat{c}(\hat{b}_k) = (\frac{p}{1+p})^{k+1}\ w \tag{38}$$

and expected total savings, S, is

$$S = p\sum_{k=1}^{\infty} s(\hat{b}_k)(1-p)^k = \frac{p^2}{1+p}\sum_{k=1}^{\infty}\left(\frac{p(1-p)}{1+p}\right)^k \tag{39}$$

While $S > 0$ for any $0 < p < 1$, the sign of the effect on S of an increase in the survival probability p is indeterminate.

Incorporating a positive birth rate would not change this conclusion: *in the absence of a competitive annuity market, the effect of increased longevity on steady-state aggregate savings is indeterminate.*

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Appendix

From (15),

$$\frac{dh(z,\alpha,g)}{d\alpha}\bigg/ h(z,\alpha,g) = \frac{1}{m}\frac{dm}{d\alpha} - h(z,\alpha,g)z\frac{dg}{d\alpha} + h(z,\alpha,g)\mu(z,\alpha). \tag{A.1}$$

Since $m = 1 \bigg/ \int_0^\infty e^{-gz}F(z,\alpha)dz$,

$$\frac{1}{m}\frac{dm}{d\alpha} = \left(\int_0^\infty h(z,\alpha,g)zdz\right)\frac{dg}{d\alpha} - \int_0^\infty h(z,\alpha,g)\mu(z,\alpha)dz \tag{A.2}$$

Substituting from (17), (A.2) can be rewritten

$$\frac{1}{m}\frac{dm}{d\alpha} = A\int_0^\infty b(z)\varphi(z,\alpha,g)dz \tag{A.3}$$

where

$$A = \frac{\left(\int_0^\infty h(z,\alpha,g)zdz\right)\left(\int_0^\infty h(z,\alpha,g)\mu(z,\alpha)dz\right)}{\int_0^\infty h(z,\alpha,g)zb(z)dz} < 0 \tag{A.4}$$

and

$$\varphi(z,\alpha,g) = \frac{h(z,\alpha,g)\mu(z,\alpha)}{\int_0^\infty h(z,\alpha,g)\mu(z,\alpha)dz} - \frac{h(z,\alpha,g)z}{\int_0^\infty h(z,\alpha,g)zdz} \tag{A.5}$$

Since $\int_0^\infty \varphi(z,\alpha,g)dz = 0$, φ changes sign at least once, say at $z = \tilde{z}$. At this point, by (A.5),

$$\frac{\mu(\tilde{z},\alpha)}{\int_0^\infty h(z,\alpha,g)\mu(z,\alpha)dz} = \frac{\tilde{z}}{\int_0^\infty h(z,\alpha,g)zdz} \tag{A.6}$$

Differentiating φ w.r.t. z is, at $\tilde{z}$,

$$\varphi'(\tilde{z},\alpha,g) = \frac{\dfrac{\partial\mu(\tilde{z},\alpha)}{\partial z}}{\int\limits_0^\infty h(z,\alpha,g)\mu(z,\alpha)dz} - \frac{1}{\int\limits_0^\infty h(z,\alpha,g)zdz} =$$

inserting from (A.6)

$$= \frac{\mu(\tilde{z},\alpha)}{\tilde{z}\int\limits_0^\infty h(z,\alpha,g)\mu(z,\alpha)dz}\left(\frac{\tilde{z}}{\mu(\tilde{z},\alpha)}\frac{\partial\mu(\tilde{z},\alpha)}{\partial z} - 1\right) \tag{A.7}$$

It follows from Assumption 3 that

$$\varphi'(\tilde{z},\alpha,g) \le 0. \tag{A.8}$$

With strict inequality, (A.8) implies that $\tilde{z}$ is unique and that

$$\varphi(\tilde{z},\alpha,g) \gtreqless 0 \;\text{ as }\; z \lesseqgtr \tilde{z} \tag{A.9}$$

By Assumption 2, $b'(z) \le 0$. Hence, by (A.9),

$$\int\limits_0^\infty b(z)\varphi(z,\alpha,g)dz \ge b(\tilde{z})\int\limits_0^\infty \varphi(z,\alpha,g)dz = 0 \tag{A.10}$$

In view of (A.3), we conclude that $\dfrac{1}{m}\dfrac{dm}{d\alpha} \le 0$.

Since $\int\limits_0^\infty \dfrac{dh(z,\alpha,g)}{d\alpha}dz = 0$, $\dfrac{dh}{d\alpha}$ is either 0 for all z or changes sign at least once, say at $\hat{z}$. From (A.1), at $\hat{z}$,

$$\frac{1}{m}\frac{dm}{d\alpha} - h(\hat{z},\alpha,g)(\hat{z}\frac{dg}{d\alpha} - \mu(\hat{z},\alpha)) = 0 \tag{A.11}$$

Since $\dfrac{1}{m}\dfrac{dm}{d\alpha} \le 0$, it follows that

$$\hat{z}\frac{dg}{d\alpha} - \mu(\hat{z},\alpha) \le 0 \tag{A.12}$$

Partially differentiating $h(z,\alpha,g)$ w.r.t. z at $z = \hat{z}$ is, by (A.1),

$$\frac{\partial}{\partial z}\left(\frac{dh(\hat{z},\alpha,g)}{d\alpha}\right) = -h(\hat{z},\alpha,g)\left(\frac{dg}{d\alpha} - \frac{\partial\mu(\hat{z},\alpha,g)}{\partial z}\right) \tag{A.13}$$

from (A.12) and Assumption 3,

$$\geq -\frac{h(\hat{z},\alpha,g)\mu(\hat{z},\alpha)}{\hat{z}}\left(1-\frac{\hat{z}}{\mu(\hat{z},\alpha)}\frac{\partial\mu(\hat{z},\alpha)}{\partial z}\right)\geq 0 \tag{A.14}$$

Hence, unless $\frac{dh}{d\alpha}=0$ for all z, $\hat{z}$ is unique and

$$\frac{dh(z,\alpha,g)}{d\alpha} \lesseqqgtr 0 \quad \text{as} \quad z \lesseqqgtr \hat{z} \tag{A.15}$$

Since $w(z)$ non-increases and $\int_0^\infty \frac{dh(z,\alpha,g)}{d\alpha}dz = 0$, it now follows from (A.15) that for any R^*,

$$\int_0^\infty w(z)\frac{dh(z,\alpha,g)}{d\alpha}dz \leq 0 \tag{A.16}$$

Recapitulating, by Assumption 1, $\frac{dR^*}{d\alpha} < 0$ and $\frac{dc^*}{d\alpha} > 0$. Going back to (22), we see that together with (A.16), this establishes that $\frac{dS}{d\alpha} < 0 \;\; \|$.