Contagion of Sovereign Default Risk: the Role of Two Financial Frictions.

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Contagion of Sovereign Default Risk: the Role of Two Financial Frictions*

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Abstract

This paper develops a quantitative general equilibrium model of sovereign default with heterogeneous agents to account for spillover of default risk across countries. Borrowers (sovereign governments) and foreign lenders (investors) in the model face financial frictions, which endogenously determine each agent's credit condition. Due to lack of enforcement in sovereign debt, borrowing constraints for the governments are endogenous to incentives to default for the governments. On the other hand, investors who hold a portfolio of sovereign debts face a collateral constraint that limits their leverage of investment in sovereign debts. When the collateral constraint for investors binds due to a decrease in the value of collateral, triggered by a high default risk for one country, credit constrained investors ask for liquidity premiums even to countries in which there is no worsening of domestic fundamentals. This increase in the cost of borrowing, in turn, increases incentives to default for other countries with normal fundamentals, further constraining investors in obtaining credit through a decrease in the value of collateral. The interplay of each agent's credit condition generates a bad spiral through which we observe spread of default risk across countries. In a quantitative analysis, the model is calibrated to Greece and Spain, and predicts (1) that cross-county correlation in sovereign spreads between Greece and Spain increases significantly during a crisis period, and (2) that Spain's default rate, conditional on Greece' default, increases about three times compared to Spain's unconditional default rate. The model's predictions are consistent with the recent European debt crisis.

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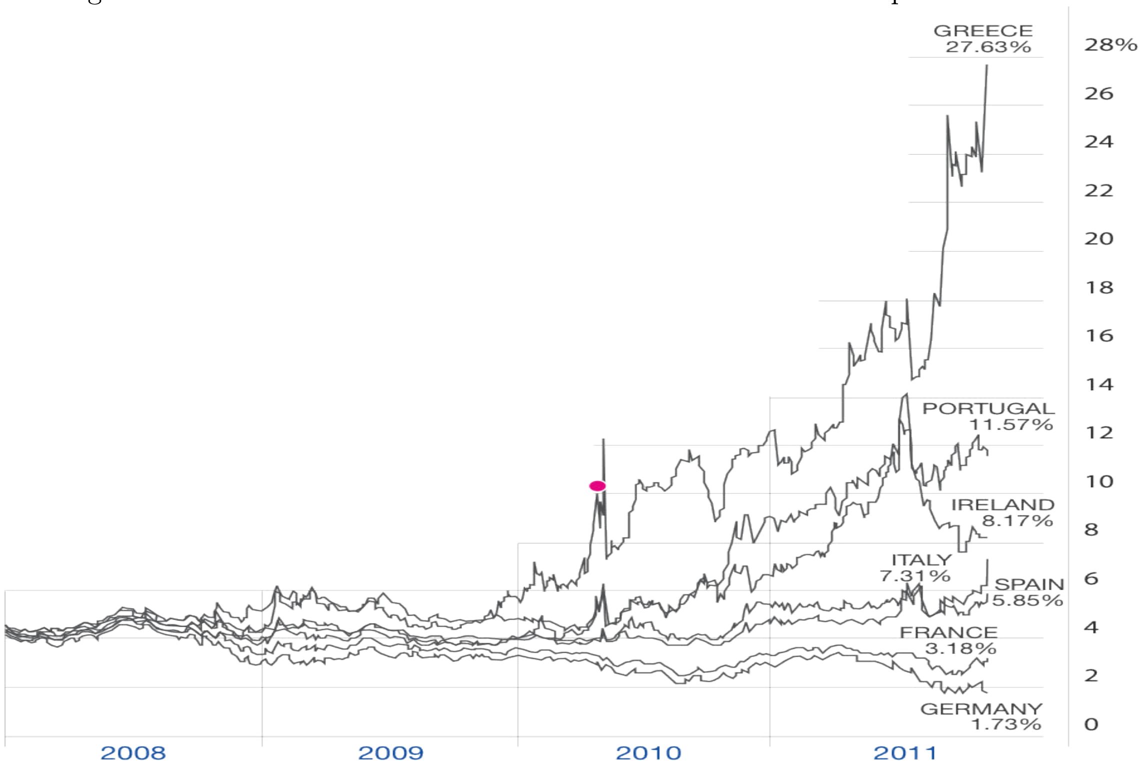
Figure 1.1: 10 Year Government Bond Yields across Several European Countries



Note: The period runs from January 1st in 2008 through 2011. Source:Bloomberg.

1 Introduction

The recent European debt crisis has ignited a debate about channels for contagion of sovereign default risk. Since the Asian financial crisis of 1997, there have been numerous studies about contagion of financial crises. However, relatively little research has been conducted on contagion of sovereign default risk. Figure 1 plots 10-year government bond yields for several European countries from the beginning of 2008 through the end of 2011. This figure shows that since April 23rd in 2010 when Greek government requested the first bail-out funds from European Central Bank and IMF, 10 year Greek government bond yields soared up, and along the way, government bond yields for several countries such as Ireland, Portugal, and Spain have substantially increased, suggesting a possibility that the default risk originating in Greece has spread to these countries. The contagion of sovereign default risk is, however, not unique to the recent European debt crisis. We have already observed that during Mexico's tequila crisis of 1994, Mexico's neighboring countries such as Argentina and Brazil also suffered from an increase in their sovereign bond yields.

Table 1: Pairwise Correlation in Sovereign Spreads for Selected European Countries with Greece during the Stable and Crisis Period

Country	Stable Period	Crisis Period
Ireland	0.07 (0.05)	0.65 (0.17)
Spain	0.23 (0.05)	0.68 (0.16)
Portugal	0.19 (0.05)	0.81(0.13)
Italy	0.71 (0.04)	0.97 (0.06)
France	0.55 (0.05)	0.63 (0.17)

Note: The stable period is from Jan.1 in 2009 to Apr 22th in 2010. The crisis period is a month starting on April 23rd in 2010. Source: Bloomberg. The frequency of the data is daily. Figures in parenthesis are standard errors of correlation coefficients.

Despite an extensive literature on financial contagion, there is not yet a uniform definition of what constitutes contagion.[1] However, the term contagion generally refers to the spread of bad market disturbances, from one country to another. Accordingly, most empirical studies on contagion focus on a significant increase in cross-market linkages after a shock to an individual country (Greece in the recent European debt crisis), which is observed through "excessive" co-movements in exchange rates, stock prices, and sovereign spreads. If contagion of sovereign default risk occurred during the recent European debt crisis, we would see a significant increase in cross-country correlations in sovereign spreads between Greece and other countries during the recent crisis period compared to the stable period before the crisis started.

Table 1 compares the pairwise cross-country correlations of sovereign spreads on 10 year government bonds for selected European countries with Greek government sovereign spreads between the stable and crisis period. The stable period is defined as the period from the beginning of 2009 through April 22nd in 2010, a day before Greek government requested the first bail-out funds from ECB and IMF.[2] The crisis period is one month starting on April 23rd in 2010.[3] The spread is calculated as the difference between 10 year government bond yields and German bond yields with identical bond structures. The European countries included in the table are Ireland, Spain, Portugal, Italy, and France. Sovereign spread data are daily and are from Bloomberg.

[1]Claessens, Stijn, Rudiger W. Dornbusch, and Yung Chul Park, 2001, Contagion: Why crises spread and how this can be stopped, in Stijn Claessens, and Kristin J Forbes, eds**:** *International Financial Contagion*(Kluwer Academic Publishers, Norwell, MA)

[2]Even though there are several candidates for the starting date of crisis, April 23rd in 2010 is considered by most media and policy makers to be the date in which Greece debt crisis started.

[3]Using one month for the crisis period is conventional in the empirical work on test for contagion. I do not use a longer length of time for the stable time because of the fact that the structural change might happen over the longer period.

We can see from Table 1 that during the crisis, there is a significant increase in cross-country correlations in sovereign spreads with Greece where the current crisis originates. Even if there are still a lot of ongoing debates on what constitutes contagion, this significant increase in cross-country correlations during the crisis period suggests that spread of default risk actually occurred during the recent European debt crisis.

This paper proposes a financial channel through which one country's sovereign default risk spreads to other countries in a quantitative dynamic general equilibrium model with heterogeneous agents. In analyzing the financial channel for contagion of sovereign default risk, I focus on the role of two financial frictions, one for sovereign borrowers (sovereign governments) and the other for lenders (sovereign debt specialists), and the interplay of credit conditions determined by these two financial frictions will be at the core of analysis, generating contagion as well as a vicious cycle leading up to a crisis. For sovereign borrowers, there is lack of enforcement in sovereign debt contracts due to sovereignty: in the event of default, no foreign creditors can enforce repayments nor seize any assets of sovereign debtors within its border. On the other hand, when foreign investors borrow from international financial markets in order to invest in sovereign debts, they face a collateral constraint that limits the amount of borrowing and that closely resembles a margin clause that most investment bankers face in the real world.

There has been plenty of research on the effect of a financial friction facing sovereign governments since Eaton and Gersovitz (1981). As first pointed out in Eaton and Gersovitz (1981), due to the lack of enforcement in sovereign debt, a default decision by the government depends on the *willingness to pay,* not on the *ability to pay.* That is, the sovereign government optimally chooses to default on its external debt whenever it is better off with default, even if the government can afford to repay. Due to this nature of sovereign default, one country's default decision is sensitive to a cost of borrowing. An increase in the cost of borrowing increases incentives to default for the government who needs to roll over external debts coming due, and this, in turn, increases the cost of borrowing for the sovereign government, because foreign lenders will ask for higher risk-premiums due to the increased likelihood of default of the country. If Greece's crisis increases the borrowing cost of other countries by negatively affecting investors' credit conditions, this will increase incentives to default for other countries by making their rolling-over of the current debt costly. At the same time, this increase in incentives to default will be reflected in an increase in spreads for those countries.

On the other hand, the financial friction facing foreign investors who specialize in lending to sovereign countries also plays a crucial role in generating contagion in this model. This is a new aspect of our model that has so far been little addressed in the literature of sovereign debt and default. The importance of the role of investors in explaining sovereign spreads is emphasized by a recent empirical study by Longstaff et al (2011). They find that there is commonality in sovereign spreads and that this commonality is related to foreign creditors' credit conditions. Moreover, empirical studies on financial contagion by Kaminsky and Reinhart (2000,2003) find that financial links through *common bank lenders* are among the most important contagion channels for financial contagion occured since 1990s. Given the fact that a majority of Greek government bonds are owned by European commercial banks[4], investigating the role of foreign investors in accounting for the recent European debt crisis is important.

The *risk-averse* investors in the model hold a portfolio of different sovereign bonds[5] and make a portfolio choice each period. They leverage their investment in sovereign bonds with funds (short term loans) obtained from international financial markets. However, foreign investors face a collateral constraint that restricts their borrowing from international financial markets: the amount of borrowing from international financial markets must not exceed a certain fraction of the value of the collateral. Moreover, the value of collateral depends on the market value of the investors' portfolio.

The interaction of credit conditions for sovereign borrowers and lenders associated with the two financial frictions is a key mechanism for contagion of sovereign default in the model. Contagion occurs in the following way. Suppose that a bad income shock hits a country (Greece), which leads to an increase in default risk for the country (Greece). This leads to a decrease in the price of the country's bond in the investors' portfolio, hence lowering the market value of the investors' portfolio. The decrease in the value of the portfolio also lowers the value of collateral, constraining investors in borrowing from international financial markets. Credit (liquidity) constrained investors, in turn, ask for liquidity premiums on other sovereign bonds whose domestic fundamentals are not deteriorating. Higher liquidity premiums translate into a higher cost of borrowing, thus increasing incentives to default for countries even with normal fundamentals. Then, a vicious cycle is set in motion. In this paper, crises similar to the European debt crisis start when the investors' collateral constraint binds

[4]A recent survey by Barclays Capital shows that around 70% of Greek government debt is owned by European financial institutions including public institutions such as ECB and central banks in Eurozone.

[5]Hence, countries in investors' portfolio are linked through common lenders.

(i.e., margin call). One country's default risk affects credit conditions for investors, which, in turn, affects the cost of borrowing for other countries. And this again increases the incentive to default for other countries, making investors further credit-constrained through a decrease in the value of collateral. This is the main financial channel for contagion in this paper.

The model is calibrated to Greece and Spain, which are considered to be among the countries hit hardest by the recent European debt crisis. However, the two countries are different in terms of the cost of default. This is a main source of heterogeneity in the model, which generates different default rates and mean spreads in simulations. In quantitative studies, the model predicts a significant increase in cross-country correlations in sovereign spreads between the two coutries during a crisis period, which is in line with the one observed during the recent European crisis. Moreover, the model predicts that Spain's default rate, conditional on Greece's default, increases about three times compared to Spain's unconditional default rate. A counterfactual analysis in which investors are assumed to be risk-neutral and not credit constrained as in most of quantitative sovereign default studies, shows that the financial channel in this paper plays an important role in generating contagion.

Related Literature

With the exception of a few recent papers, the sovereign debt literature has not paid much attention to contagion of sovereign default risk. Since Aguiar and Gopinath (2006) and Arellano (2008), most quantitative sovereign default studies[6] have focused on the role of sovereign default risk in generating unique business cycle dynamics of emerging economies, while being silent about how sovereign default risk spreads to other countries, even though most sovereign defaults occurred in bunches in history. Another strand of research on sovereign default has focused on the issue of crisis resolution, bailouts, and debt restructuring. While this literature can give important lessons for the resolution of the European debt crisis, it does not provide a good understanding of a financial mechanism for contagion of sovereign default, which is necessary for understanding the current European debt crisis.

On the other hand, there has been a growing number of studies on contagion of *financial* crises

[6] A short list includes Aguiar and Gopinath (2006), Hatchondo, Martinez and Sapriza (2008), and Mendoza and Yue (2011)

since the Asian crisis of 1997. A number of plausible explanations for contagion and spread of financial crises have been proposed, but most of these explanations have not utilized the unique feature of sovereign debt crises, *lack of enforcement* in sovereign debt, which plays a crucial role in generating contagion in this paper. However, the closest contagion channel in the literature of financial crises to this paper is the portfolio recomposition channel, the channel studied by Choueri (1999) and Schinasi and Smith (1999) among others. Using theoretical frameworks, they show that contagion can occur if investors are rebalancing their portfolio by liquidating their assets in responding to a negative shock to returns on specific assets. Liquidating assets puts a downward pressure on the price of other assets, which do not suffer from any negative shocks, thus generating a spread. Following this idea, Lizarazo (2010) develops a quantitative model of contagion of sovereign default on the framework of Eaton and Gersovitz (1981) in which the crucial mechanism of contagion is that after making a loss due to a country's default, a foreign lender with preferences that exhibit decreasing absolute risk aversion (DARA) asks for higher risk-premiums on other countries' sovereign debt. However, in our paper, the key mechanism is the interaction of credit constraints for investors and sovereign borrowers, not the contagion channel due to the wealth effect. In our model, the key factor generating higher spreads during crisis is liquidity premiums that credit constrained investors demand during a crisis. Arellano and Bai (2012) extends Lizarazo (2010) by introducing a debt renegotiation problem to study the contagion channel of sovereign default risk. In their model, the key channel for contagion comes from strategic interactions of borrowers in Cournot competition.

This paper is also related to another strand of literature that studies the effects of "occasionally binding" borrowing constraints on macroeconomic behaviors of small open economies. By extending Aiyagari and Gertler (1999) to an open economy setting, Mendoza and Smith (2006) shows the collateral constraint whose value of collateral is linked to a small open economy's domestic fundamental can generate volatile economic dynamics in line with ones observed in the crises of emerging market economies of 20th century. However, their studies do not consider sovereign default crises, which are the main focus of our paper.

Bolton and Jeanne (2011) also studies contagion of sovereign default risk in financially integrated economies under the assumption that domestic banks use sovereign bonds as collateral for interbank loans. In their model, changes in the value of a portfolio responding to changes in default risk play a key role in generating contagion of a crisis. In a simple three period model, they show that domestic

Figure 2.1: Two Groups of Small Open Economies

banks' incentive to diversify a portfolio of sovereign debt generates risk diversification benefits ex ante, but it generates contagion ex post. However, their model does not feature endogenous government default decisions as in Eaton and Gersovitz (1981).

The rest of the paper is organized as follows. Section 2 presents the economic environment and the theoretical framework. Section 3 analyzes the quantitative implications of the benchmark model and conducts a counterfactual analysis. Section 4 concludes.

2 Model

This section describes the model and defines the recursive equilibrium of the economy. Time is discrete and infinite $t = 0, 1, \ldots$. There are two groups of small open economies in this model. In each group, there is a continuum of identical small open countries in each of which there are a representative household and a sovereign government. A representative household in each country in both groups receives a stochastic stream of identical tradable consumption goods each period. A benevolent government in each country maximizes the welfare of its representative household by making one of two choices: borrowing from foreign investors or defaulting on its debt. There is a sovereign debt market which is populated by a large number of *risk-averse* investors who trade one period *non-contingent* bonds with each government. When lending to each government (i.e., investing in government bonds), investors can obtain funds from international financial markets at a risk-free rate to leverage its investment in government debts. The sovereign debt market is assumed to be competitive. There is no borrowing or lending between sovereign governments; sovereign governments trade only with investors in sovereign debt markets. Both borrowers (sovereign governments) and lenders (foreign investors) have perfect information regarding each country's endowment process.

The asset market is both incomplete and imperfect: *incomplete* due to the fact that the asset that borrowers (sovereign governments) and lenders (foreign investors) trade is restricted to one period non-contingent bonds. *Imperfect* due to the two financial frictions: (1) Sovereign debt contracts are not enforceable, and (2) foreign investors are facing a collateral constraint (liquidity constraint) which limits their leverage in sovereign debt investment.

In what follows, the script i denotes variables corresponding to the Group i. A lowercase letter indicates an individual state variable for a small open economy in each group whereas a uppercase letter an aggregate state variable.

2.1 State Variables

Let S be a vector of aggregate state variables characterizing the state of the world. There is a tuple of state variables $S^i = (Y_t^i, B_t^i)$ specific to Group $i \in \{1, 2\}$: a current endowment Y_t^i and a current bond holding B_t^i for group i. There is also a state variable specific to the investor $S^{inv} = X_t$, which is the amount of funds that investors obtained last period from international financial markets. Then, S can be written as follows:

$$S = S^1 \times S^2 \times S^{inv} \tag{2.1}$$

2.2 Timeline

The timing of actions within each period is :

- At the beginning of the period, the state of the world S is observed by sovereign governments in both groups and investors.

- Investors offer a menu of bond prices (a bond price schedule) to each sovereign government in both groups, and taking this menu as given, each sovereign government decides to default or not.

- If a government decides not to default, the government trades one-period bonds with investors in a sovereign debt market after repaying its current debt.

- When a government defaults on its debt, the debt is erased from budget constraints for the economy and investors. In the default period, however, the defaulting economy incurs an output cost associated with default. The next period, the government resumes trading with investors with zero debt.

2.3 Sovereign Governments and Households

I follow Arellano (2008) in specifying the economic environment regarding a sovereign government who makes a borrowing and default decision. Each country in group i [7]is populated by one government and a representative household who receives a stochastic endowment at each period Y_t^i, which is a tradable consumption good.[8] The representative household in this country has the preference given by

$$E_0 \sum_{t=0}^{\infty} \beta_i^t u(c_t^i) \tag{2.2}$$

where c_t^i denotes consumption at period t, $\beta_i \in (0,1)$ the time discount factor for households in group i. The utility function $u(\cdot)$ is strictly increasing, strictly concave, and satisfies Inada conditions. The endowment Y_t^i is assumed to have a compact support and to be a Markov process.

The government is benevolent and its objective is to maximize the expected discounted utility of the household. At the beginning of each period, each government in group i decides whether to default or not on its debt, after comparing the value of default and not-default. However, since countries in each group are small, sovereign governments in each group act as if their decisions regarding default and borrowing do not affect a law of motion for aggregate state variables, which determine current and future prices for their borrowing.[9] As we see later in more detail, however, the decisions of an individual government in group i coincides with the decisions of the group i in equilbrium (aggregate consistency condition). Besides a vector of aggregate state variables S, there is an additional individual state variable for a country in group i : the current bond holding (debt) b_t^i.[10]

[7] Since countries in each group are identical, we can think of this setting as the case in which there is one representative small open economy in each group but the government in each representative small open economy takes prices and motions for aggregate variables as given.

[8] Tradable goods for both groups are identical, so the relative price between two goods is one.

[9] If we assume that each government can internalize the effect of their default and borrowing decisions on the law of motion for aggregate state varaibles, which determines the country's credit condition, they will exploit this opportunity by behaving strategically as in a game theoretical setting. Arellano and Bai (2012) study this strategic behaviors of multiple sovereign governments to account for contagion of sovereign default risk. To prevent this strategic interaction, this model features *"Big B and little b" trick*, which is very similar to so called *"Big K and little k"* trick, which is used to compute a competitive allocation in a closed economy setting.

[10] $b_t^i > 0$ means that the government i holds a positive bond holding, whereas $b_t^i < 0$ means that the government i

The value of the option to default for the government in group i is given by:

$$V_o^i(b_t^i, S_t) = \max_{nd,d}\{V_{nd}^i(b_t^i, S_t), V_d^i(b_t^i, S_t)\}, \tag{2.3}$$

where V_{nd}^i denotes the value of repaying the debt (not default) and V_d^i the value of default. When the government decides to default, its default decision function $d^i(b_t^i, S_t)$ takes on the value of one, and zero otherwise.

Value of Repayment

When the government decides to repay its debt, the government trades one-period discount bonds b_{t+1}^i with investors in a sovereign debt market, taking the bond price function (bond price schedule) $q^i(b_{t+1}^i, S_t)$ as given. The bond price function shows how costly or lenient external borrowing is to each sovereign government and determines endogenous *credit limits* for the government. Note that the bond price function $q^i(b_{t+1}^i, S_t)$ depends not only on b_{t+1}^i but also on the aggregate state of the world S_t, which includes state variables for the other group S^{-i} as well as investors' credit condition S^{inv}.

The government rebates back to households all the proceedings from its trading with investors in a lump sum fashion. Hence, when the government pays back its debt, the resource constraint for the country i reads

$$c_t^i = Y_t^i + b_t^i - q_t^i(b_{t+1}^i, S_t)b_{t+1}^i \tag{2.4}$$

Hence, the sovereign government rolls over its current debt b_t^i with a new borrowing from investors, which is $-q_t^i(b_{t+1}^i, S_t)b_{t+1}^i$. A value function of no-default (repayment) for the country i is written in a recursive way:

$$V_{nd}^i(b_t^i, B_t^1, B_t^2, Y_t^1, Y_t^2, X_t) = \max_{b_{t+1}^i, c_t^i} u(c_t^i) + \beta^i E_t[V_o(b_{t+1}^i, B_{t+1}^1, B_{t+1}^2, Y_{t+1}^1, Y_{t+1}^2, X_{t+1})] \tag{2.5}$$

subject to the resource constraint equaion (2.4).

is indebted.

Value of Default

When the government defaults on its debt, the government is not allowed to trade with investors. In addition, the economy suffers an output cost of default $h^i(Y_t^i) \geq 0$ that lowers the economy's income to $Y_{i,t}^{def} \equiv (Y_t^i - h^i(Y_t^i))$. The economy in default must consume $Y_{i,t}^{def}$, which is less than or equal to her income shock Y_t^i. That is,

$$c_{i,t}^{def} = Y_{i,t}^{def} \tag{2.6}$$

This output cost of default is standard in quantitative studies of sovereign default based on Eaton and Gersovitz (1981) and is related to the negative effects of sovereign default on the domestic output, usually through disruption in economic activity caused by collapse of domestic financial systems after a government's default.[11]

Unlike most quantitative models of sovereign default, I assume that the defaulting government resumes trading with investors in the period following default, with all the debt written off from the economy's budget constraint.[12] This assumption is related to the structure of the sovereign debt market in the model: the sovereign debt market is *competitive*. In a competitive sovereign debt market, it is hard to justify that investors are able to collude to exclude defaulting governments from credit markets for a certain period as a punishment. Several empirical studies including Wright (2005) suggest that sovereign debt markets have become more competitive over the last three decades.

Two groups of economies differ in the output cost of default $h^i(\cdot)$, which is a main source of heterogeneity across groups in the model. This heterogeneity can be thought of as reflecting structural differences among countries including political instability. Countries with a high cost of default would be less likely to default than those with a low cost of default.

[11] It has been well documented in many empirical studies that sovereign default is, for most of the time, accompanied with a huge decrease in the economy's output. For example, Argentina's GDP decreased by 10% in 2001 when Argentina's government defaulted on her sovereign debt.

[12] Most papers in quantitative studies of sovereign default assume exclusion of sovereign government from credit markets for a certain period of time as an additional punishment of default. See, for example, Aguiar and Gopinath(2006) and Arellano(2008), among others. However, Hatchondo and Martinez(2009) assume no exclusion from credit markets in case of default.

The value of default for government i can be written recursively:

$$V_d^i(b_t^i, B_t^1, B_t^2, Y_t^1, Y_t^2, X_t) = u(Y_t^i - h^i(Y_t^i)) + \beta^i E[V_o^i(0, B_{t+1}^1, B_{t+1}^2, Y_t^1, Y_t^2, X_{t+1})] \tag{2.7}$$

Note that when a government decides to default, the bond holding for the next period is zero, meaning that the current debt is written off upon defaulting.

When making a default or borrowing choice from investors, the atomistic government in each group takes as given the following law of motion for aggregate state variables as well as default decisions for each group of countries:

$$B_{t+1}^1 = \Gamma_{B^1}(S_t) \tag{2.8}$$

$$B_{t+1}^2 = \Gamma_{B^2}(S_t) \tag{2.9}$$

$$X_{t+1} = \Gamma_X(S_t) \tag{2.10}$$

$$D_t^1 = \Gamma_1(S_t) \tag{2.11}$$

$$D_t^2 = \Gamma_2(S_t) \tag{2.12}$$

The solution to a government's problem provides two decision functions for (1) default $d_t^i(b_t^i, S_t)$ and (2) future borrowing $b_{t+1}^i = b_{t+1}^i(b_t^i, S_t)$.

Discussion of the Government's Default Decision

Due to *lack of enforcement* in sovereign debt, the sovereign government's default decision depends on the government incentives to repay debt, meaning that each period, the government compares the value of default and repayment even if the country has assets with which to pay back the debt (i.e., the government is solvent). Despite the output cost of default, the government decides to default, because at a certain bad state of the world S, the government can not find any financial contract $\{q^i(b_{t+1}^i, S_t), b_{t+1}^i\}$ that increases current consumption by rolling over its current debt b_t. Recall that the consumption in case of repayment is:

$$c_t^i = e_t^i + b_t^i - q^i(b_{t+1}^i, S_t)b_{t+1}^i \tag{2.13}$$

The government rolls over its current debt b_t^i with new borrowing of $-q^i(b_{t+1}^i, S_t)b_{t+1}^i$. If the cost of borrowing represented by $q^i(\cdot)$ is too high (a low bond price) for the country in group i, the value of repayment would be lower, because borrowing from investors does not help much to finance households' consumption, especially in the face of adverse income shocks. Hence, if the government finds it very costly to borrow from sovereign debt markets, the government rather chooses to default, which makes the country better off even with the output cost of default. Since the bond price function depends on S^i as well as the other group's economic conditions S^{-i}, crises such as the other group's default can translate into a high borrowing cost of countries by adversely affecting investors' credit condtions, which, in turn, will increase incentives to default for the countries, even if there is no worsening of economic fundamentals for the countries in group i. However, this increased incentives to default for the countries in group i will be reflected in a higher cost of borrowing for the countries, further increasing incentives to default and possibly leading to contagion. In the next subsection, we will see that another financial friction facing investors reinforces incentives to default for countries in each group.

2.4 Investors

In a sovereign debt market, there are a large number of identical *risk-averse* investors who specialize in investing in sovereign debt. Their preferences are defined over a stream of consumption goods $\{c_t^{Inv}\}_{t=0,\infty}$, given by

$$E_0 \sum_{t=0}^{\infty} \beta_{inv}^t u(c_t^{Inv}) \quad , \tag{2.14}$$

where β_{inv} denotes the time discount factor for investors, c_t^{Inv} consumption at period t. β_{inv} is assumed to be greater than $\{\beta_i\}_{i=1,2}$, the time discount factor for households in group i, in order to induce sovereign governments in each group to borrow from investors in equilibrium. Investors receive a constant amount of tradable consumption goods ϕ each period. In addition, they can obtain one-period loans x_{t+1} from international financial markets with an exogenous world risk-free interest rate of r_f, to finance its investments in sovereign debt and their consumption. Finally, we assume that sovereign debt markets are competitive.

At each period, investors decide how much to consume, how much to invest in sovereign bonds from each group (i.e., making a portfolio choice), and how much to borrow from international financial markets. The representative investor is subject to a financial friction that limits his borrowing from international financial markets: he must pledge a certain fraction of his assets as collateral in order to borrow from international financial markets. Given this collateral constraint, the investor makes a portfolio choice on different sovereign bonds.

A representative investor's flow budget constraint is :

$$c_t^{Inv} + \frac{1}{1+r_f} x_{t+1} + \sum_i (1-D_t^i) Q_t^i \theta_{t+1}^i = \phi + x_t + \sum_i (1-D_t^i)\theta_t^i \quad , \tag{2.15}$$

where D_t^i is an indicator variable, which takes on the value of one if group i defaults, and zero otherwise. In addition, when the group i defaults, investors do not lend to countries in group i. This condition is given by

$$\theta_{t+1}^i = 0 \tag{2.16}$$

if $D_t^i = 1$ for $i = 1, 2$.

The left hand side in equation (2.15) is expenditure of the investor: c_t^{Inv} is investors' consumption, x_{t+1} is the amount of one period funds obtained from international financial markets, θ_{t+1}^i is the amount of group $i's$ bond holdings for the next period t+1, and Q_t^i is the price of group $i's$ sovereign bond, which atomistic investors take as given. The right hand side is investors' current wealth: ϕ is investors' current endowment, x_t is the amount of funds carried over from the last period t-1, and θ_t^i is the amount of group i's bond holdings carried over from t-1. Note that if $D_t^i = 1$, group i's bond is erased from the investor's budget constraint. In addition to this flow budget constraint, the representative investor faces the following collateral constraint.

$$\frac{x_{t+1}}{1+r_f} \geq -\kappa\left(\sum_i (1-D_t^i) Q_t^i \theta_{t+1}^i\right) \quad , \tag{2.17}$$

where κ is assumed to be in the range $(0, 1)$.

The collateral constraint implies that the total value of loans can not exceed a fraction κ of the market value of the investor's portfolio of sovereign bonds. That is, when borrowing from international financial markets, investors must pledge collateral whose value depends on the value of the portfolio. This constraint restricts the leverage ratio, defined as the ratio of the total asset to the net worth of investors, not to exceed $\frac{1}{1-\kappa}$. This collateral constraint resembles most directly a contract with a margin clause that investment banks face in the real world. Unlike sovereign governments, investors are subject to domestic law, which allows creditors to confiscate investors' valuable assets in the case of investors' default, but to a limited extent ($\kappa < 1$) mainly due to informational and institutional frictions. As shown in other macro models with endogenous credit constraints (e.g., Aiyagari and Gertler (1999), Kiyotaki and Moore (1997), Kocherlakota (2000), and Mendoza (2010)), this constraint constitutes a financial friction that induces distortions via credit-channel effects, which is a main financial friction on the investor's part in this model that has potential to lead to contagion.

The representative investor maximizes the present discounted value of lifetime utility subject to equation (2.15) and equation (2.17), taking the bond price for each group i, Q_t^i, and default decision D_t^i as given. The maximization problem yields the following Euler equations for loans from the international financial market x_{t+1} and each sovereign bond i whose group is not in default, θ_{t+1}^i

$$[x_{t+1}] : u'(c_t^{Inv}) = \beta_{inv}(1 + r_f)E_t[u'(c_{t+1}^{Inv})] + \mu_t \tag{2.18}$$

$$[\theta_{t+1}^i] : Q_t^i(u'(c_t^{Inv}) - \kappa\mu_t) = \beta_{inv}E[u'(c_{t+1}^{Inv})(1 - D_{t+1}^i)] \tag{2.19}$$

μ_t denotes a Lagrange multiplier for the collateral constraint equation (2.17). Equation (2.19) can be rewritten as $q_t^i = E_t[\beta \frac{u'(c_{t+1}^{Inv})}{(u'(c_t^{Inv}) - \mu_t\kappa)}(1 - D_{t+1}^i)]$. With the collateral constraint, the investor's effective stochastic discount factor is defined as $m_{t+1} \equiv \beta \frac{u'(c_{t+1})}{(u'(c_t) - \mu\kappa)}$. Then we have a typical asset pricing function for the sovereign bond i:

$$Q_t^i = E_t[m_{t+1}(1 - D_{t+1}^i)] \tag{2.20}$$

where $(1 - D_{t+1}^i)$ is a payoff of the sovereign bond i in the next period.

External Financing Premium on Debt

If the collateral constraint (equation (2.17)) does not bind, the Euler equations characterizing investors' optimality conditions collapse to the Euler equations as in standard asset pricing models. However, when the collateral constraint binds, $u'(c_t^{Inv}) > \beta^{inv}(1+r_f)E_t[u'(c_{t+1}^{Inv})]$, implying that the shadow value of additional funds (liquidity) from international financial markets increases compared to the case in which the constraint does not bind, and that this increase is measured by a positive Lagrange multiplier μ_t. Let $E_t[\frac{u'(c_t^{Inv})}{\beta_{inv}u'(c_{t+1}^{inv})}]$ be the effective real interest rate denoted by R_{t+1}^{inv} and $R_f \equiv 1 + r_f$. Then, from the equation (2.18) we have:

$$R_{t+1}^{inv} - R_f = \frac{\mu_t}{E_t[\beta_{inv}u'(c_{t+1}^{inv})]} \tag{2.21}$$

Again, if the collateral constraint does not bind, the effective real interest rate represented by the intertemporal marginal rate of substitution in consumption is equal to the gross risk-free rate set at the international financial market. However, when it binds, the real effective interest rate is greater than the gross risk-free interest rate by $\frac{\mu_t}{E_t[\beta_{inv}u'(c_{t+1}^{inv})]}$. This term $\frac{\mu_t}{E_t[\beta_{inv}u'(c_{t+1}^{inv})]}$ is called *the external financing premium on debt* in the literature.

In determining the external financing premium, not only whether the collateral constraint binds or not (whether μ_t is zero or not) matters, but also the degree to which the collateral constraint binds (how large μ_t is) matters. We can think of two cases to see the degree to which the collateral constraint binds: (1) A bad income shock hits countries in group i, so incentives to default for the countries in group i increase, but they do not default. In this case, a bad income shock lowers the bond price Q_t^i (i.e., a higher interest rate), so that it constrains investors' borrowing through a decrease in the value of collateral. Even if this makes the collateral constraint binding, the degree to which the collateral constraint binds is not high. (2) However, if a bad income shock leads to the group i's default, then this inflicts a loss to investors' net worth, and the investors want to borrow more to smooth their consumption. But due to the binding collateral constraint, investors are not able to borrow as much as they want. In this case (2) , the collateral constraint is more binding than the case (1). The external financing premium on debt becomes larger (i.e., μ_t becomes larger), so that credit constrained investors would ask for higher liquidity premiums on risky assets compared to the case in which his collateral constraint is not very binding (case (1)).

To see the effect of the external financing premium on bond prices for each government in both groups, we now look at an expected excess return on each bond. If we define the expected excess return on the sovereign bond i as $R^{ER} \equiv E_t(R_{t+1}^i - R_f)$, where $R_{t+1}^i \equiv (1 - D_{t+1}^i)\frac{1}{Q_t}$, an ex-post gross return on sovereign bonds i, then we have the following expression for $R^{i,ER}$:

$$R^{i,ER} = \frac{\mu_t(1-\kappa)}{(u'(c_t) - \mu_t\kappa)E_t(m_{t+1})} - \frac{cov_t(m_{t+1}, R_{t+1}^i)}{E_t(m_{t+1})} \tag{2.22}$$

The first term on the right hand side of the equation (2.22) shows the direct effect of the binding of the collateral constraint on the excess return on sovereign bond i, as first shown in Aiyagari and Gertler (1999) and extended to an open economy setting by Mendoza and Smith (2006). When the collateral constraint binds, the first term on the right hand becomes positive due to the positive μ_t, increasing the excess return on the sovereign bond i, which means a decrease in the bond price Q_t^i (i.e., an increase in the cost of borrowing for countries in group i) due to the external financing premium. Moreover, when the collateral constraint binds, $Cov_t(m_{t+1}, R_{t+1}^i)$ in the second term on the right hand side would be more negative because of the limited ability of investors to smooth their consumption.

The equation (2.22) shows that the expected excess returns on sovereign bonds for both groups have one thing in common when the collateral constraint binds: the liquidity premium term

$\frac{\mu_t(1-\kappa)}{(u'(c_t)-\mu_t\kappa)E_t(m_{t+1})}$. Hence, if group i's default or bad economic conditions make the investors credit (liquidity) constrained by making the collateral constraint binding, investors ask for liquidity premiums on group i's bonds as well as the other group $-i$'s bonds , thereby increasing the cost of borrowings for the countries in group $-i$. This is the main financial channel on the lender's side through which a sovereign default risk spreads to other countries even when their domestic fundamentals have not worsened. Note that in this model, bad income shocks to countries not only make sovereign governments credit constrained through an increase in the cost of borrowing but also make investors credit or liquidity constrained because of the collateral constraint.

The investors' problem can be rewritten recursively:

$$V^{Inv}(\theta_t^1, \theta_t^2, x_t, S_t) = \max_{c_t^{Inv}, \theta_{t+1}^1, \theta_{t+1}^2, x_{t+1}} u(c_t^{Inv}) + \beta^{inv} E_t[V^{Inv}(\theta_{t+1}^1, \theta_{t+1}^2, x_{t+1}, S_{t+1})] \quad , \tag{2.23}$$

subject to the following constraints:

$$c_t^{Inv} + \frac{1}{1+r_f} x_{t+1} + \sum_{i=1,2} Q^i(S_t)(1 - D_t^i(S_t))\theta_{t+1}^i = \phi + x_t + \sum_{i=1,2} (1 - D_t^i(S_t))\theta_t^i \tag{2.24}$$

$$\frac{x_{t+1}}{1+r_f} \geq -\sum_{i=1,2} \kappa(1 - D_t^i(S_t)) Q^i(S_t)\theta_{t+1}^i \tag{2.25}$$

$$\theta_{t+1}^i = 0 \quad if \quad D_t^i(S_t) = 1 \quad for \quad i = 1,2 \quad , \tag{2.26}$$

where $Q^i(S_t) = q^i(B_{t+1}^i(S_t), S_t)$, where B_{t+1}^i is an aggregate bond holding for group i the next period.

Investors also take as given a law of motion for the aggregate state variables S_t and default decision functions for each group D_t^i:

$$S_{t+1} = \Gamma(S_t) \tag{2.27}$$

$$D_t^1 = \Gamma_1(S_t) \tag{2.28}$$

$$D_t^2 = \Gamma_2(S_t) \tag{2.29}$$

The bond price function for each country in group i must be consistent with the Euler equation for the asset i, equation (2.19) and each government's default function $d^i(\cdot)$. Then, the bond price function $q^i(b_{t+1}^i, S_t)$ that each government in group i faces is given by

$$q^i(b^i_{t+1}, S_t) = E_t[\beta_{inv}\frac{u'(c^{Inv}_{t+1}(S_{t+1}))}{u'(c^{Inv}_t(S_t)) - \kappa\mu_t(S_t)}(1 - d(b^i_{t+1}, B^1_{t+1}, B^2_{t+1}, Y^1_{t+1}, Y^2_{t+1}, X_{t+1}))] \qquad (2.30)$$

Discussion of Contagion Mechanism

The financial friction that each agent faces determines the endogenous credit limits for each agent. Due to the possibility of default associated with the lack of enforcement in sovereign debt, a sovereign government pays higher interest rates, which reflect the likelihood of its future default. On the other hand, the capacity of investors to obtain funds from international financial markets is restricted by the collateral constraint. In this model, two endogenous credit conditions interact, finally generating contagion when a bad fundamental shock triggers a crisis. If a bad income shock to country **A** triggers the collateral constraint to bind for the investors (i.e., margin call), this adversely affects borrowing costs for the other country **B**. This, in turn, will increase incentives to default for the other country **B,** which will, in turn, be reflected in an increase in sovereign spreads for the country **B**. Furthermore, if a country **A** defaults on its debt due to a very bad income shock, this will make the investors very credit constrained (which is associated with a high positive value of the Lagrange multiplier μ_t), and then they will ask for high liquidity premiums on other sovereign governments. In the face of the high cost of borrowing, the other sovereign government **B** finds the value of repayment or borrowing from investors low compared to the value of default, so that it decides to default. This, in turn, makes investors more credit constrained, generating a vicious spiral. This financial channel that arises from two financial frictions is a source of *amplification* of fundamental shocks, generating a contagion of sovereign default risks across countries.

2.5 Equilibrium

The equilibrium concept in this economy is a recursive Markov perfect equilibrium. That is, the equilibrium default and borrowing decisions for governments as well as the decision functions for investors depend only on payoff relevant state variables.[13] In the following definition of the equilib-

[13] A Markov perfect equilibrium is necessary to prevent time-inconsistency problems of governments' decisions.

rium, x and x' denote any variable x at period t and t+1, respectively.

A recursive Markov perfect equilibrium for this economy consists of

1. *Policy and value functions for governments and households in each group* i : consumpition for households $c^i(b^i,S)$, debt choices $b_i^{'}(b^i,S)$, default decision $d_i(b^i,S)$, and the value functions $v_{nd}^i(b^i,S)$, $v_d^i(b^i,S)$, and $v_o^i(b^i,S)$ for each group $i=1,2$.

2. *Policy and value functions for investors*: consumption $c^{Inv}(\theta_1,\theta_2,x,S)$, asset choices $\theta_i'(\theta_1,\theta_2,x,S) \quad for \quad i = 1,2$, loans from international financial markets, $x'(\theta_1,\theta_2,x,S)$, and the value function $V^{Inv}(\theta_1,\theta_2,x,S)$

3. *Bond Price functions*: the bond price function that each sovereign government in group i faces $q^i(b_i,S)$ and the equilibrium bond prices $Q^i(S)$ that atomistic investors take as given.

4. *Law of motion for aggregate state variables*: $X'(S)$, $D^i(S)$,and $B_i^{'}(S)$ for $i=1,2$.

5. such that

   (a) Taking as given the bond price function $q^i(b_i^{'},S)$ and the law of motion for aggregate state variables, $c^i(b_i,S)$, $b_i'(b_i,S)$, $d_i(b_i,S)$ solves value functions $v_{nd}^i(b^i,S)$, $v_d^i(b^i,S)$, and $v_o^i(b^i,S)$.

   (b) Taking as given the bond price $Q^i(S)$ and the law of motion for aggregate state variables, $c^{Inv}(\theta_1,\theta_2,x,S)$, $\theta_i'(\theta_1,\theta_2,x,S)$, and $x'(\theta_1,\theta_2,x,S)$ solves the value function $V^{Inv}(\theta_1,\theta_2,x,S)$.

   (c) The bond price function for bond i $q^i(b_i',S)$ satisfies the equation (2.30).

   (d) Market Clearing Condition: $\theta_i'(-B^1,-B^2,X,S) = -B_i^{'}(S)$ and $Q^i(S) = q^i(B_i'(S),S)$ for $i=1,2$.

   (e) Aggregate Consistency:

$$B_i' = b_i^{'}(B^i,B^1,B^2,Y^1,Y^2,X_t) \quad if \quad D_t^i = 0 \quad for \quad i=1,2 \tag{2.31}$$

$$D^i = d^i(B^i,B^1,B^2,Y^1,Y^2,X) \quad for\, i=1,2 \tag{2.32}$$

$$X' = x'(-B^1,-B^2,X,B^1,B^2,Y^1,Y^2,X) \tag{2.33}$$

3 Quantitative Analysis

3.1 Solution Method

The model framework developed in the previous section is solved numerically to examine quantitative predictions of the model, with a focus on whether the model can generate excessive comovements of sovereign spreads between two countries similar to the ones observed during the recent European debt crisis. As a representative small open economy in each group, Spain and Greece are chosen and the model is calibrated to the two countries. Due to a fair number of state variables in the model, parallel computing using 16 CPUs is used to solve the model. The algorithm for the numerical solution combines features of the solution method by Heaton and Lucas (1996), Menodza and Smith (2006), Arellano (2008), and Hatchondo, Martinez, and Sapriza (2010). A sovereign government's problem in each group is solved using value function iteration with linear interpolation similar to Hatchondo, Martinez, and Sapriza (2010).[14] The investors' problem is solved using policy function iteration on investors' Euler equation with linear interpolation. Appendix 1 contains a detailed algorithm for the numerical solution.

3.2 Parameterization

For the quantitative work, the following specific functional forms are used.

- Endowment process for Greece and Spain:

$$log(Y^i_{t+1}) = (1-\rho^i)\mu^i_e + \rho^i_y log(Y^i_t) + \epsilon^i_{t+1} \tag{3.1}$$

  where $0 < \rho^i < 1$ and $\epsilon^i_t \sim N(0, \sigma^2_{\epsilon,i})$ and i={Greece, Spain}.

- Utility Function: $u(c^i) = c^{i,1-\gamma}/(1-\gamma)$ for i=Greece, Spain, and investors.

- The output cost of default in the event of default is given by:

$$h^i(Y) = max\{0, d_0 Y + d^i_1 Y^2\}, d^i_1 \geq 0. \tag{3.2}$$

[14] As Pouzo and Presno(2012) argues, there is convergence problem in numerical solutions associated with solving the quantitative model of default with risk-averse investors using Discrete State Space(DSS) method. However, Hatchondo, Martinez, and Sapriza(2010) use a spline function instead of linear interpolation.

The specification for the output cost of default $h^i(Y)$ is taken from Chaterjee and Eyigungor (2011). Table 2 and 3 report the parameter values used for quantitative analysis in this paper. Table 2 reports the values of the parameters that have counterparts in the data or are taken from the previous studies. Table 3 reports the values of parameters to be calibrated to match default rates for Greece and Spain.

Parameters Calibrated Outside the Model:

A period in the model is a quarter. The parameters of the endowment process for Greece and Spain are separately estimated on quadratic detrended quartely real GDP data for each country from 1980:1-2011:1.[15] Both country's real mean GDP per capita is normalized to be one by setting $\mu_e^i = -\frac{1}{2}\sigma_{\epsilon,i}^2$. The estimated value of ρ_e for Greece (Spain) is 0.87 (0.89), respectively, and the value of σ_ϵ^i is 2.44% (2.38%). An income shock process for each country is discretized with 9 shocks following Tauchen and Hussey (1991). The risk-free rate r_f is set at 0.01, which corresponds to a quarterly interest rate of 1% and is standard in the literature. Risk aversion parameter γ is set equal to 2 for both lenders and borrowers and is also standard in the literature. Lenders' time discount factor β^{inv} is set to be 0.98 such that $\beta^{inv}(1 + r_f) < 1$, to have a well defined stochastic steady state for x_t.[16] Each country's time discount factor is set to be 0.9[17] and is from Arellano and Bai (2012). The parameter that determines the leverage ratio κ is set to be 0.75 and is from Aiyagari and Gertler (1999). Investors' period income is not easy to pin down. However, it is set to be five, meaning that foreign investors' period income is five times higher than households' income in each country. Hence, more patient ($\beta^{Inv} > \beta_i$) and wealthier ($\phi > E(Y^i)$) investors lend to sovereign governments in this model.

Parameters Calibrated to Match Target Moments:

The parameters determining the output cost of default (d_0, d_1) are calibrated to match the default rate for Greece and Spain in the last 100 years. Since a sovereign default is a rare event, it is very

[15] The real GDP data are deseasonalized and is taken from Eurostat.

[16] If $\beta(1 + r_f) \geq 1$, the asset for investors x_{t+1} will diverge to infinity in equilibrium by Doob's supermartingale convergence theorem.

[17] The value of β in the quantitative sovereign default literature ranges from 0.8 to 0.95.

Table 2: **Parameters Calibrated outside the Model**

Investor	**Notation**	**value**
Time discount factor	β^{inv}	0.98
Risk Aversion	γ	2.00
Quarterly Risk Free rate	r_f	0.01
Margin Requirement	κ	0.75
Endowment	ϕ	5

Borrowers	**Notation**	**Greece**	**Spain**
Time discount factor	β^i	0.90	0.90
Risk Aversion	γ	2	2
Endowment Shock persistence	ρ_y^i	0.87	0.89
Endowment Shock s.d	σ_ϵ^i	2.44%	2.38%
Mean log endowment	μ_y^i	$-\frac{1}{2}\sigma_{\epsilon,i}^2$	$-\frac{1}{2}\sigma_{\epsilon,i}^2$

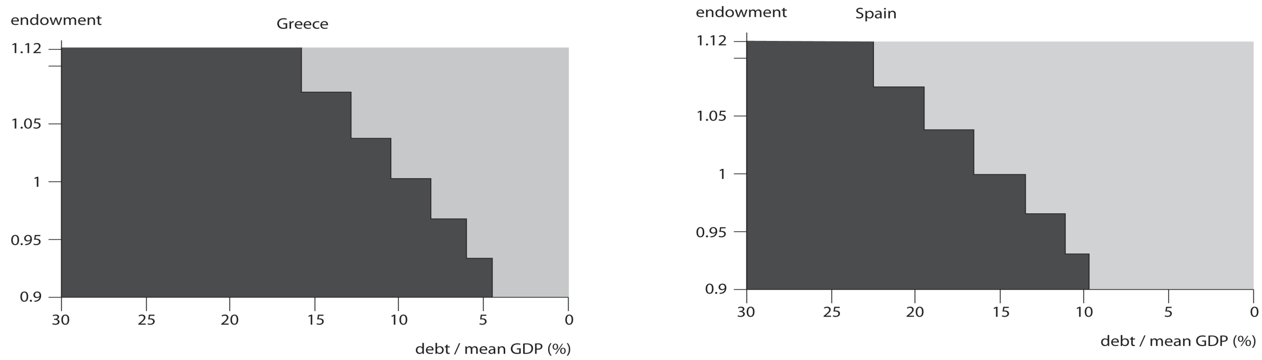
Table 3: **Parameters Calibrated to Match Target Moments**

Output Cost	Greece	Spain	Target Moment
d_0	-0.30	-0.30	Greece's default rate:2%
d_1^i	0.40	0.45	Spain's default rate: 1%

difficult to estimate default rates for each country. However, in the last 100 years, Greece defaulted twice including the current debt crisis, and Spain defaulted once.[18] Using a sample of 68 countries since 1970, Cruces and Trebesch (2011) find an annual frequency of 6.6 defaults. Because Spain is one of high income countries and Greece is one of mid-income countries, the default rate of one percent for Spain and two percent for Greece may be justified.[19]

Since we have two target moments but there are two parameters determining each country's output cost of default (i.e., there are a total of four parameters to be calibrated), the same value is used for d_o for both countries, but d_1^i is chosen to be different for each country to match the target moments.

Figure 3.1: **Default Region for Greece and Spain** $(D^i(B_t^1, B_t^2, Y_t^1, Y_t^2, X_t)$



3.3 Quantitative Result

Default Region

Figure 3 shows default regions for Greece and Spain in endowment and debt/GDP space. The darkly shaded region in the left panel represents combinations of endowment realizations and a debt/mean GDP ratio for which Greek government optimally decides to default, with Spain's debt and endowment realization, and funds obtained last period X_t for investors at a mean level in simulations. The lightly shaded region is accordingly the non-default region in which the Greek government decides to repay the debt. Likewise, the right panel in Figure 2 represents the default and non-default region for the Spanish government.

For a given realization of the endowment, each government is more likely to default with more debt. Along the other dimension, for a given level of debt, each government is less likely to default with higher endowment realizations. This prediction of the model is consistent with empirical evidence that sovereign default mostly occurs with bad income states and high indebtedness of countries. It is worth noting that the default region for Greece is larger than and covers the default region for Spain, because the model is calibrated to generate a higher default rate for Greece than for Spain.

[18] Greek government's last default occured in 1932, and Spain's last default occured in 1936 according to Reinhart and Rogoff (2010).

[19] There are many empirical studies that suggest a strongly negative correlation between default rates and income levels for countries.

Figure 3.2: Spain's Default Regions in the Case of Greece's Default

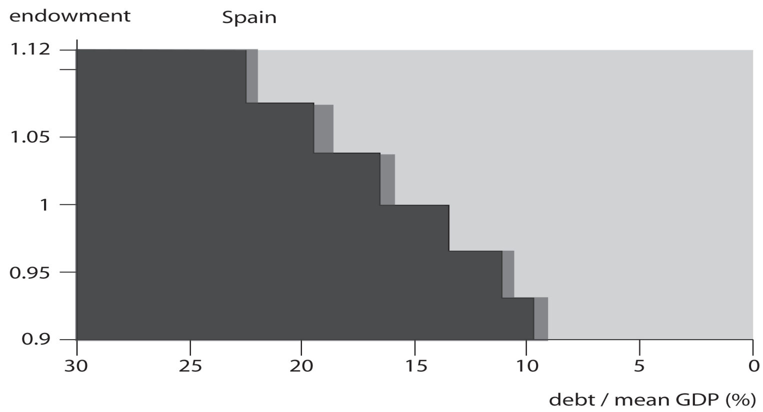
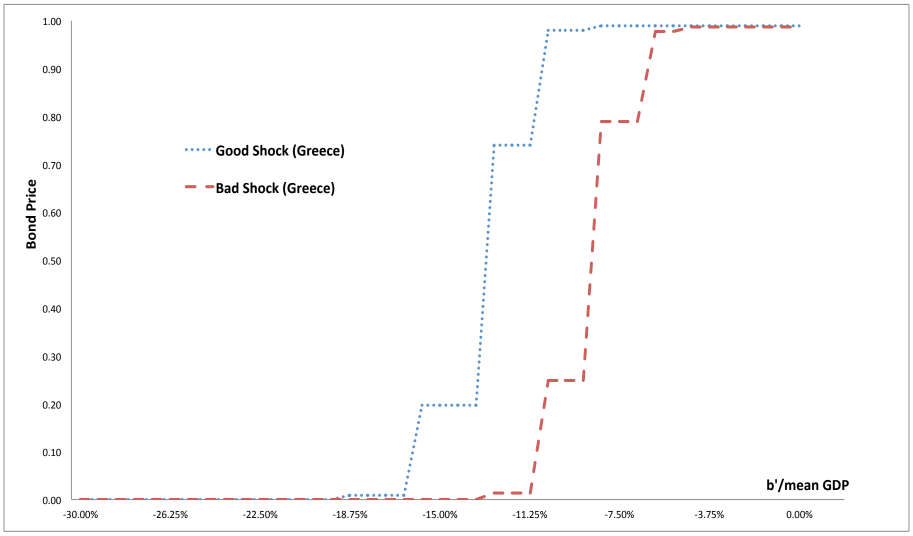


Figure 4 compares default regions for Spain in which Greece defaults and does not default. Two default regions for Spain overlap in the same endowment and debt space. A grey default region that is larger than and covers a darkly shaded region is the one for the case of Greece's default. As we have seen in the previous section, if Greece's default makes investors more credit-constrained (i.e., the collateral constraint binds), investors ask for higher borrowing costs to Spain, and this increase in borrowing costs for Spain reinforces incentives to default for Spain. The fact that Spain's default region for the case of Greece's default is larger than and covers the default region in which Greece does not default, shows that the increase in the cost of borrowing for Spain caused by Greece's default increases incentives to default for Spain through investors' credit conditions. This is consistent with the contagion channel discussed in the previous section.

Bond Price Function

Figure 5 plots bond price functions ($q^i(b^i_{t+1}, B^1_t, B^2_t, Y^1_t, Y^2_t, X_t)$) for Greece as a function of its borrowing choice for the next period for the good and bad realization of the endowment, fixing the current debt for Group 1 and Group 2 (B^1 and B^2) and X_t at the mean level in the simulation. The bond price is the inverse of the gross interest rate on the bonds (debt). The good shock is a realization that is 0.5 standard deviation above the unconditional mean of the endowment shock, and the bad shock is -0.5 standard deviation below the unconditional mean of the endowment shock for Greece. As seen in the default region in Figure 3, the country is more likely to default with

Figure 3.3: Bond Price Function(Greece): $q^i(b^i_{t+1}, B^1_t, B^2_t, Y^1_t, Y^2_t, X_t)$

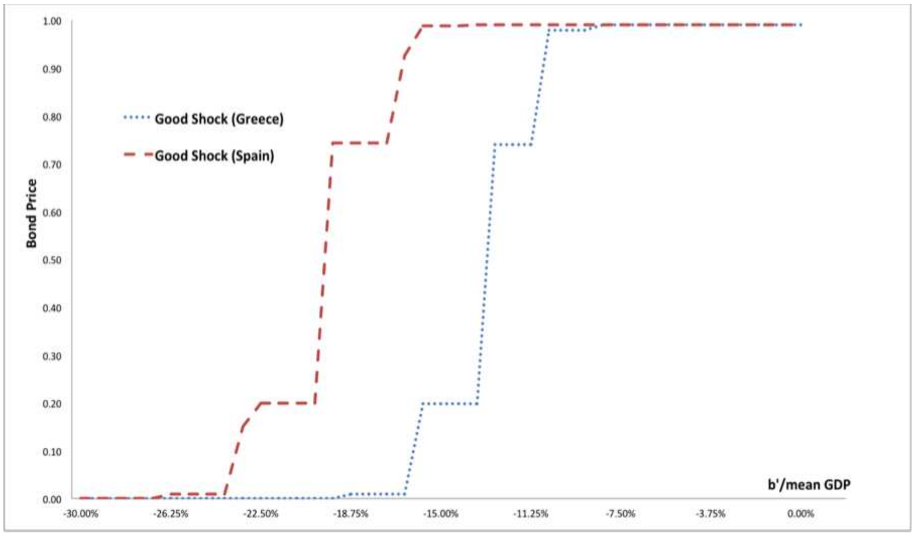


more debt. Taking this into account, investors charge higher interest rates when debt is larger, so that bond prices decrease in the amount of borrowing for the next period. At any given level of debt choice, Greece faces a higher bond price (low interest rates) for the good shock than for the bad shock. This result is related to persistence in the endowment shock process. With a high persistence ($\rho_e^{Greece} = 0.87$) of the shock process, today's good shock implies that Greece is likely to receive a good shock tomorrow. Hence, investors expect a low probability of default for Greece for the next period. Because of the low likelihood of default the next period, investors accordingly charge a lower interest rate on Greek bonds compared to the case of a bad shock today. Hence, in the face of a good shock, Greece can borrow more at almost risk-free rates than in the face of a bad shock today, implying that the likelihood of default risk endogenously determines Greece's credit limit each period.

Figure 6 compares the bond price functions for Greece and Spain in the case when both countries receive a good income shock (0.5 s.d above the unconditional mean). Since the model is calibrated so that Greece defaults twice as often as Spain in simulations, at any given level of its debt choice, Spain faces more lenient interest rates than Greece.

Now see how Spain's borrowing condition is affected by the investors' credit conditions. Figure 7 compares bond price functions for Spain between the cases when the collateral constraint for investors binds and does not bind. In both cases, Spain receives a bad income shock at the current

Figure 3.4: Bond Price functions for Greece and Spain

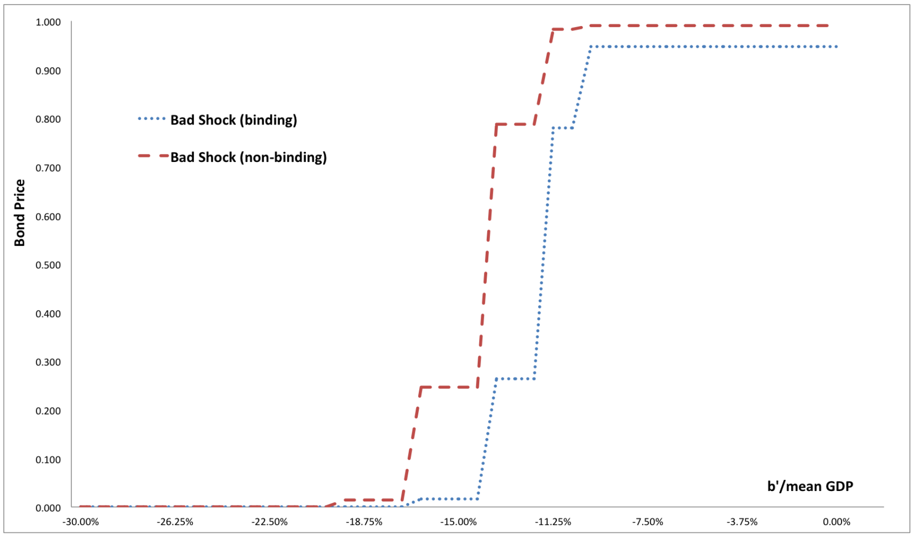


state of the world. The case when the collateral constraint binds refers to the situation when Greece's default induces investors to become credit constrained. The bond price schedule for the case in which the collateral constraint binds is strictly lower than for the no-binding case over all the range of debt levels. This implies that when investors become credit constrained, they ask for higher interest rates to the Spanish government, and this, in turn, reinforces incentives to default for Spanish government. As we have seen in Section 3, when the collateral constraint binds, credit constrained (liquidity constrained) investors ask for an additional liquidity premium on all the bonds in his portfolio, thereby increasing the cost of borrowing for Spain. Figure 7 shows a financial channel in this model through which Greece's default risk spreads to Spain, by worsening investors' credit conditions. In equilibrium, the credit condition for each agent (borrowers and lenders) is mutually reinforcing each other to generate contagion.

Event Study

To see whether the model can generate "excessive" comovements of sovereign spreads between Greece and Spain during the crisis, the event study techniques to simulated time series data are applied. Figure 8 plots the model's average path of annualized spreads in percentage terms for Greece and Spain. Spreads are defined as each country's interest rates minus a gross risk free interest rate $(1 + r_f)$. The event window covers 16 quarters up to and including Greece's default date, with

Figure 3.5: Bond Price Functions for Spain (with a Bad shock and Greece's Default)

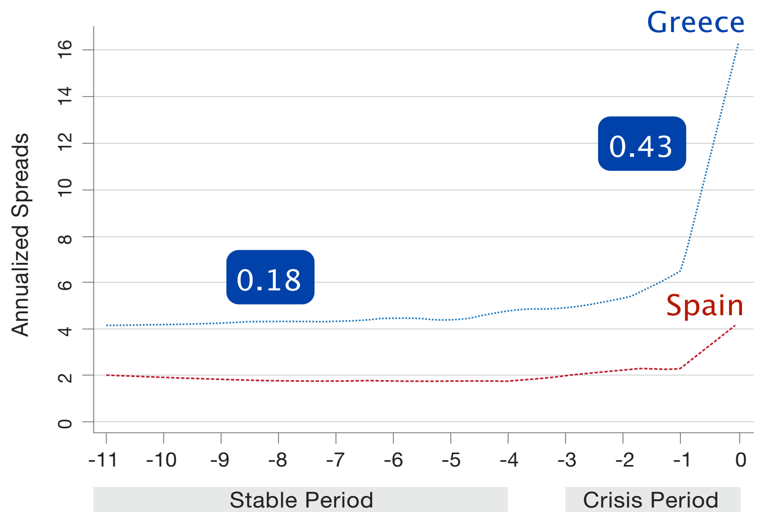


Greece's default dates normalized to date 0 in the figure. I conduct 1000 simulations with each simulation of 400 periods (100 years). A sequence of each country's income shocks is fed into the model, and when Greece's default is detected, a pair of sequences of simulated spreads for Greece and Spain within the event window are selected. At each pair of sequences in each simulation, I compute a cross-country correlation in spreads between Greece and Spain and take an average of correlations over 1000 simulations. The crisis period in this event study is defined as one year up to and including Greece's default (date 0), and the stable period is defined as 3 years before the crisis period. Hence the simulated spread line for each country represents the average behavior of spreads around the default in the model.

This event study shows that the model can generate a significant increase in a cross-country correlations between Greece and Spain during the crisis period. On average, the cross-country correlation during the stable period (-15 to -4) is 0.18, while the cross-country correlation during the crisis period (-3 to 0) is 0.43. And this increase is statistically significant with a t-value of 4.21 in a t-test for a difference in means for two samples. This result is consistent with the one observed in the data: the cross-country correlation between Greece and Spain increased from 0.23 to 0.68 during the crisis period in the real data as shown in Table 1 in Introduction.[20] Moreover, Spain's default probability conditional on Greece's default in the simulation increases from 1% (the

[20]It must be noted, however, that the time window and data frequency for the simulation and the empirical data are different so that these two numbers are not directly comparable.

Figure 3.6: Average Spread Dynamics around Greece's default

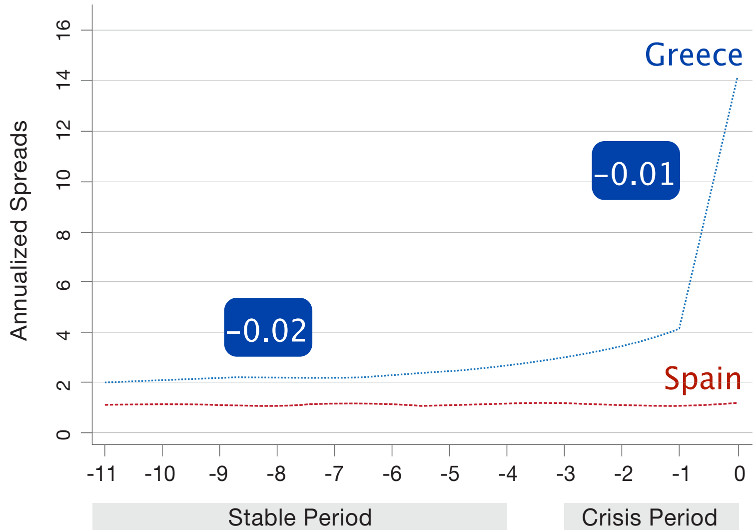


unconditional default rate for Spain in the simulation) to 3.2%, which suggests that Greece's default induces Spain's default by increasing the cost of borrowing for Spain through the financial channel linked to the investors' credit conditions.

Due to the persistence of the income shock process, an initial bad income shock to Greece, which finally leads to her default at date 0, starts at dates around -2 or -3. This initial bad shock makes the collateral constraint midly binding through a decrease in Greek bond prices. Hence, the common liquidity premium represented by the first term on the right hand side in the equation (2.22) is shared by both Greek and Spanish bonds even before Greece's default at date 0. However, when a very bad shock hits Greece at date 0, the Greek government defaults on its debt, and this makes the collateral constraint for investors very binding, so that investors ask for a high liquidity premium on Spanish bonds. Facing high borrowing costs, Spain is more likely to default than when Greece does not default. Note that even when Spain does not default at date 0, Spain faces a higher cost of borrowing, which is shown by a sudden increase in sovereign spreads for Spain at date 0 in Figure 8. Hence, the liquidity premium shared by both countries in this model is a main driver of comovements of sovereign spreads during a crisis period.

A counterfactual analysis is conducted to see how crucial the financial channel proposed in this paper is for generating contagion. With the same parameter values in the benchmark model, I assume that investors are risk-neutral and are not subject to a collateral constraint, and conduct

Figure 3.7: Average Spread Dynamics with Risk-Neutral Investors and No Collateral Constraint



the same event studies as in the Figure 8. This assumptions regarding foreign investors in the counterfactual analysis are common in quantitative studies of sovereign default built upon Eaton and Gersovitz (1981). Figure 9 shows the average path of sovereign spreads for Greece and Spain under these assumptions. As can be seen from the figure, there is no increase in the cross-country correlations in sovereign spreads between Greece and Spain, even when Greece defaulted at date 0. The correlation coefficient during the stable period is -0.02 and during the crisis period is -0.01. The difference in correlation coefficients is not statistically significant with a t-value of approximately zero. This shows that the financial friction on the part of investors plays an important role in generating contagion in my model. Besides, the average spreads for both countries in Figure 9 over the entire period are lower than those in Figure 8. This reflects the fact that with risk-averse investors, there are additional risk-premiums attached to bond prices.

4 Conclusions

This paper proposes a new financial channel through which one country's default risk spills over to other countries. Unlike other studies on financial contagion, this paper focuses on a unique feature of sovereign default in explaining contagion of sovereign default: lack of enforcement in sovereign debt contracts. Unlike other studies on sovereign debt and default, this paper put more structures on foreign creditors: foreign creditors are risk-averse and face a financial friction that determines their credit conditions. Furthermore, this paper shows that the interaction of each agent's credit condition associated with financial frictions is the key to generating contagion similar to the one observed in the recent European debt crisis.

Theoretically, I develop a model in which liquidity premiums demanded by credit constrained investors are a main driver of comovements in sovereign spreads across countries during a crisis. In the model, a crisis starts with a bad fundamental shock to one country, but this bad shock is amplified through the financial channel, affecting credit conditions for both lenders and other sovereign borrowers and possibly generating a bad spiral along which we see contagion. Quantitatively, the model is calibrated to Greece and Spain to examine the model's predictions. Quantitative results well match empirical facts regarding the recent European crisis: cross-country correlation in sovereign spreads between Greece and Spain increases significantly during a crisis period. Spain's default rate, conditional on Greece's default, increases around three times compared to Spain's unconditional default rate.

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Appendix: Computational Algorithm

1. Start with an initial set of parameters.

2. Conjecture an initial guess of a bond price function for each group $q_0^i(b_{t+1}^i, S_t)$, a law of motion for aggregate state variables $B_{0,t+1}^i$, decision functions $D_{0,t}^i$, and $X_{0,t+1}$. Subscript 0 denotes the number of iteration.

3. Solve an individual government problem in each group:

   (a) Taking as given the law of motion for aggregate state variables and aggregate decision functions as well as $q_o^i(b_{t+1}^i, S_t)$, solve for a government's policy functions in each group i $\{b_{j,t+1}^i(b_t^i, B_t^1, B_t^2, Y_t^1, Y_t^2, X_t), d_{j,t}^i(b_t^i, B_t^1, B_t^2, Y_t^1, Y_t^2, X_t)\}$ via value function iteration in the Bellman equations (equations 2.5, 2.7) with linear interpolation similar to Hatchondo, Martinez, and Sapriza (2010). Again, subscript j denotes the number of iteration.

   (b) Using the policy functions in the previous step, update the effective law of motions for aggregate variables and decision functions: $B_{j+1,t+1}^i = b_{j,t+1}^i(B_t^i, B_t^1, B_t^2, Y_t^1, Y_t^2, X_t)$ and $D_{j+1,t}^i = d_{j,t}^i(B_t^i, B_t^1, B_t^2, Y_t^1, Y_t^2, X_t)$ for $i = 1, 2$.

4. Solve investors' problem:

   (a) After solving the government's problem, we have each group's default decision function $D^i(S_t)$, policy functions for bond holdings $B_{t+1}^i(S_t)$, and the bond price $Q^i(B_{t+1}^i(S_t), S_t)$ for each group. Imposing these conditions on the Euler equation (2.18), solve for investors' decision functions $C^{Inv}(\cdot)$, $X_{t+1}(S_t)$, and a Lagrange multiplier $\mu_t(S_t)$ by policy function iteration with linear interpolation. When solving the investors' Euler equation, the collateral constraint is assumed to be non-binding first, but if the resulting $X_{t+1}(S_t)$ violates the collateral constraint equation (2.17), the collateral constraint is imposed with equaility.

   (b) Construct a stochastic discount factor $\beta^{Inv} \frac{u'(c_{t+1}^{Inv}(S_{t+1}))}{u'(c_t^{Inv}(S_t)) - \kappa\mu_t(S_t)}$ using the optimal consumption functions for investors and the Lagrange multiplier, and update bond price functions for each group as in eq (2.30).

   (c) Check the convergence conditions:

i. $||q^i_{j+1}(b^i_{t+1}, S_t)$-$q^i_j(b^i_{t+1}, S_t)|| < \epsilon$, for all$i$

ii. $||B^i_{j+1,t+1} - B^i_{j,t+1}|| < \epsilon$, for all i.

iii. $||D^i_{j+1,t+1} - D^i_{j,t+1}|| < \epsilon$, for all i.

iv. $||X_{j,t+1} - X_{j,t+1}|| < \epsilon$

(d) if (i,ii,iii, iv) are satisfied, then go to step (5). Otherwise, go back to step (3-(a)) with the updated bond price functions $q^i_{j+1}$and updated decision functions $B^i_{j+1,t+1}$ and $X_{j+1,t+1}$. When updating these functions, Gauss-Siedel algorithm is used to prevent the well-known problem of unstable “hog-cycles”.

5. Get the target moments by simulating the model with a given set of parameters. If the simulated moments are equal to the target moments, stop. Otherwise, go back to step (1) with a modifed set of parameters.