General correcting formulae for forecasts

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The concept of unforeseen events is considered as a part of a hypothesis of uncertain future. The applications of the consequences of the hypothesis in utility and prospect theories are reviewed. Partially unforeseen events and their role in forecasting are analyzed. Preliminary preparations are shown to be able, under specified conditions, to quicken the revisions of forecasts and to hedge or diversify financial risks after partially unforeseen events have occurred. General correcting formulae for forecasts are proposed.

Keywords: forecast, uncertainty, risk, utility, Ellsberg paradox,

Contents

Introduction

Prehistory of the research

An absolutely exact knowledge is one of main aims of science. Surely, in many cases it can be attained for the past. But can it be attained for the future? Can forecasts be absolutely exact?

Many works have been devoted to accuracy and errors of forecasts (see, e.g., Chang, 2011, Morlidge, 2013, McAleer, 2008), including the influence of unforeseen, unanticipated events (see, e.g., Hendry and Mizon, 2013). There are works those analyze breaks and news impact surfaces (see., e.g., Clements and Hendry, 2006, Caporin and McAleer, 2011, Castle et al, 2012).

The unforeseen events can be either of the natural character as earthquakes or due to human activity. The possibilities of humankind technical power and technologies grow faster and faster. Due to this tendence, scientific discoveries, inventions and innovations become the growing sources of unforeseen events.

This article is devoted to partially unforeseen events and their influence on forecasting.

The considerations and formulae of the article can be used in various fields of human activity, including pure and applied science, economics and business, risk management, measuring the implicit risks, hedging financial risks, computing "capital charges that are required to cover unforeseen and extreme financial market fluctuations" (see Caporin and McAleer, 2010).

The importance of unforeseen events and partially unforeseen events cannot be overestimated. The unforeseen events can crucially and, sometimes, dramatically change situations. The "black swans" are an example of them. This work serves to smooth down or even turn to advantage the consequences of such events.

The article develops Harin (2004-2013) and, from one of possible general and abstract points of view, gives an initial (purely mathematical) outline of the influence of such unforeseen events on forecasts.

An example. Hiroshima 1945

Let us suppose, that in 1930-35, an imaginary estimate of risk was needed with respect to bombing for an underground factory, government bomb-proof shelter, etc. for the year 1945. Suppose, in 1930-35 the ideal forecast was made. The forecast should be based firstly, e.g., on the forecast of the maximal power of an aircraft bomb for 1945. The forecast should be based secondly, e.g., on the maximal weight that bombing aircraft can lift.

To 1945, due to the most optimistic forecasts, a bombing aircraft could lift a bombing weight much less than 20 tons and even less when calculating in trinitrotoluene equivalent. In 1945 Hiroshima was bombed by the 4-tons atomic bomb. But it was equal to 20000 tons in trinitrotoluene equivalent. So, the initial estimate of risk was catastrophically wrong.

The prerequisite of an atomic bomb (the division of uranium) was discovered in 1938. Naturally, in 1930-35 it was an unforeseen event. So, in this case the relative error, caused by the unforeseen event, is more than 1000 (more than 100000%).

If in 1938 the risk estimate was revised, then the plans and/or the realization of construction of such a factory, shelter, etc. were corrected and then their safety was saved.

1. Preliminary considerations

1.1. Partially unforeseen events

There is a wealth of sorts of unforeseen events. According to Caporin and McAleer (2010) these events may be represented by univariate and multivariate models depending on the numbers of the events. We may divide them also into two types: fully unforeseen events and partially unforeseen events.

Rigorously speaking, in the presence of fully unforeseen events, we cannot make any reliable forecast. In other words, "If anything can happen, then nothing can be predicted."

Let us consider further the partially unforeseen events.

1.2. About the continuity and differentiability of approximations

When choosing an adequately detailed time scale, the vast majority of macro-world phenomena are characterized by continuity in time. The discontinuity, the discreteness in time is observed only in quantum phenomena, for example at the birth of elementary particles. Therefore, the description of the phenomena of the macro-world by means of continuous functions is lawful.

Changes in the macro-world phenomena, that is, the acceleration in a particular dimension, requires physical movements, changes of electromagnetic fields, etc. That is, they are also characterized by continuity in time. Therefore, the description of differentiable functions is lawful for the description of the macro-world phenomena.

1.3. On the validity of the approximation forecasting

As for the macro-world phenomena the description by differentiable functions is lawful, then the forecasts of these phenomena in the form of approximations is lawful also. In this sense we can say that the future is a continuation of the present. And we can do calculations and estimates for the prediction of future events of macro-world on the basis of data on current status and rate of change of these phenomena. Naturally, accurate calculations are possible only for sufficiently small time intervals for which this approximation is correct. But an approximation approach is possible for longer intervals of time, as the basis for assessing the possible deviations.

Of course, except of the approximation approach, other approaches may be lawful also.

1.4. Frames of reference and transformations

From physics it is well known an event may be described in various frames of reference. Optimal choice of frame of reference is well known to be valuable. When one use various frames of reference, the expressions of transmission between various frames of reference are necessary.

Suppose a wheel rolls along the road. We should calculate the trajectory of a point of the rim. It is the cycloid which is a complex transcendental curve. But if we choose the frame of reference in the center of the rim, we obtain two simple trajectories: the trajectory of the center of the rim and the circular trajectory of the point of the rim around the rim.

Suppose a firm has a property, pays profit tax, pays turnover tax and speculates on the stock-exchange. If we do not know nothing about the firm except its total capital, then the dependence of the capital on the time is complex and obscure, incomprehensible. If we know the time points and the bases of the operations of the firm, then the dependence may be represented as the sum of the simple elementary dependences.

1.5. The piecewise smooth representation for univariate and multivariate models

Let us consider a pure mathematical case of infinitely differentiable analytical forecast functions. Consider a function $F(t) : F(t)$ is infinitely differentiable and analytic in a point t_{Base} of the timeline and on the semi-closed interval $[t_{Base}, t)$.

Let us denote the Taylor series of the forecast function $F(t)$ as $F(t_{Base}, t)$

$$F(t_{Base}, t) = F(t_{Base}) + \sum_{n=1}^{\infty} \frac{F^{(n)}(t_{Base})}{n!}(t - t_{Base})^n .$$

where $F^{(n)}(t_{Base})$ is the n-th derivative of $F(t)$ in the point t_{Base}.

Suppose there is a rupture of an n-th $: n \geq 1$, derivative of $F(t)$ in the point $t_{Corr,1} \equiv t_1$: $t_{Base} < t_1 < t$, $(t_{Corr,0} \equiv t_0 \equiv t_{Base})$, but $F(t)$ is infinitely differentiable and analytic on $(t_{Corr,1}, t)$. Then, for univariate models, we obtain

$$F(t_{Corr,1}, t) = F(t_{Corr,1}) + \sum_{n=1}^{\infty} \frac{F^{(n)}(t_{Corr,1})}{n!}(t - t_{Corr,1})^n .$$

where $F^{(n)}(t_{Corr,1})$ is the right-side limit of the n-th derivative of $F(t)$ in the point $t_{Corr,1}$.

By means of the identical transformation we obtain for $F(t)$

$$F(t) = F(t_{Corr,1},t) \equiv F(t_{Corr,1},t) + F(t_{Base},t) - F(t_{Base},t) =$$

$$F(t_{Base},t) + [F(t_{Corr,1},t) - F(t_{Base},t)] \equiv F(t_0,t) + [F(t_1,t) - F(t_0,t)].$$

Denoting the modification of the function $\Delta F(t_{Corr,r-1}, t_{Corr,r}, t) \equiv [F(t_{Corr,r}, t) - F(t_{Corr,r-1}, t)]$, we have

$$F(t) = F(t_{Base},t) + \Delta F(t_{Base},t_{Corr,1},t).$$

For $R : R<\infty$, rupture points $t_r : t_{Base} \equiv t_0$, $t_{r-1}<t_r<t : 1 \leq r \leq R$, (see also Castle et al, 2012) we obtain the general piecewise smooth representation for multivariate models

$$F(t) = F(t_0,t) + \sum_{r=1}^{R} \Delta F(t_{r-1},t_r,t).$$

Suppose a set of sub-functions $\{f_{1r}(t_{r-1}, t_r, t), \ldots, f_{sr}(t_{r-1}, t_r, t), \ldots, f_{Sr}(t_{r-1}, t_r, t)\} \equiv \{f_{sr}(t_{r-1}, t_r, t)\} : S<\infty$, $f_{sr}(t_{r-1}, t_r, t_r)=0$, of the modification of the function $\Delta F(t_{r-1}, t_r, t)$ exists such as $\Delta F(t_{r-1}, t_r, t)$ may be represented as $\Delta F(t_{r-1}, t_r, t) = \Delta F(\{f_{sr}(t_{r-1}, t_r, t)\})$ and $\Delta F(\{f_{sr}(t_{r-1}, t_r, t)\})$ is infinitely differentiable with respect to any $f_{sr}(t_{r-1}, t_r, t)$ and analytic on $(\{f_{sr}(t_{r-1}, t_r, t_r)\}, \{f_{sr}(t_{r-1}, t_r, t)\}]$. Let us denote a differential operator

$$T = f_{1r}(t_{r-1},t_r,t)\frac{\partial}{\partial f_{1r}(t_{r-1},t_r,t)} + \ldots + f_{Sr}(t_{r-1},t_r,t)\frac{\partial}{\partial f_{Sr}(t_{r-1},t_r,t)},$$

where the derivatives are the right-side limits in the point t_r. Then we have

$$\Delta F(t_{r-1},t_r,t) = \sum_{l=1}^{\infty} \frac{T^l \Delta F(\{f_{pr}(t_{r-1},t_r,t_r)\})}{l!}.$$

So, by means of the formal identical transformations we obtain for multivariate models

$$F(t) = F(t_0) + \sum_{n=1}^{\infty} \frac{F^{(n)}(t_0)}{n!}(t-t_0)^n + \sum_{r=1}^{R}\sum_{l=1}^{\infty} \frac{T^l \Delta F(\{f_{pr}(t_{r-1},t_r,t_r)\})}{l!}.$$

2. Particular formulae for univariate models

2.1. General notes

Let us further consider the case of the only rupture point, of the point of correction $t_1 \equiv t_{Corr,1} \equiv t_{Corr}$

$$F(t) = F(t_{Base}, t) + \Delta F(t_{Base}, t_{Corr}, t) .$$

Probably, the simplest sorts of partially unforeseen events are those having only unforeseen point of time or unforeseen magnitude and being represented by univariate models. Let us consider them further.

Let us suppose a partially unforeseen event with an unforeseen magnitude and/or an unforeseen point of time has taken place at t_{Corr}.

If we know the unit value $\delta_1 F(t_{r-1}, t_r, t)$ of modification of the function, which corresponds to the unit magnitude of the event, and if at $t > t_{Corr}$ we know the point of time t_{Corr} and we may determine the magnitude M, then we may denote $\Delta F(t_{r-1}, t_r, t) \equiv M * \delta_1 F(t_{r-1}, t_r, t)$.

If we have known the correction time point t_{Corr}, then we may express the modification of the function $F(t)$ also.

So, for the partially unforeseen events with the unforeseen magnitude and point of time, we may remain the form of the expression unchanged.

From physics it is well known an event may be described in various frames of reference. Optimal choice of frame of reference is well known to be valuable. When one use various frames of reference, the expressions of transformations between various frames of reference are necessary.

2.2. Low-order approximations by sub-functions

If a rupture of an n-th (where $n \geq 1$) derivative of $F^{(n)}(t)$ in a point t_{Corr} is caused by a foreseen event, then the series of the right-hand limits of the derivatives $F^{(n)}(t_{Corr})$ may be calculated in advance and the forecast may be corrected in advance also. If this rupture is caused by an unforeseen event, then sometimes the forecast correction should be performed extremely rapidly.

Let us consider a case of two preliminary conditions:

1) The calculation of the right-hand limits of the derivatives $F^{(n)}(t_{Corr})$ (or the explicit calculation of $F(t)$) is very complicated and needs too long time to be admissible.

2) The function $F(t)$ may be represented by means of a finite set of sub-functions $f_s(t)$ as

$$F(t) = F(\{f_1(t), ..., f_S(t)\}) \equiv F(\{f_s(t)\}) .$$

and the derivatives

$$\frac{\partial^n F(\{f_s(t)\})}{(\partial f_k(t))^n}$$

of ΔF or F may be calculated in advance.

Let us suppose, that after the partially unforeseen event have taken place, the following additional condition is true:

3. The derivatives

$$\frac{\partial^n F(\{f_s(t)\})}{(\partial f_k(t))^n}$$

are happened to do not essentially depend on this partially unforeseen event and the preliminarily calculated derivatives may be used (or they may be corrected during the admissible time).

Let us suppose, that the first few L terms give sufficient accuracy of approximation. Then

$$F(t) \approx F(t_{Base}) + \sum_{n=1}^{\infty} \frac{F^{(n)}(t_{Base})}{n!}(t - t_{Base})^n +$$
$$\sum_{l=1}^{L} \frac{T^l \Delta F(\{f_p(t_{Base}, t_{Corr}, t_{Corr})\})}{l!} \pm \Delta_{Error}$$

where $\varDelta_{Error}$ is the total error. Note, that $\varDelta_{Error+}$ errors can essentially differ from $\varDelta_{Error-}$. The impact of negative shocks can differ from that of positive shocks (see, e.g., Caporin and McAleer 2011).

For the first order approximation, the general formula may be easily simplified to

$$F(t) \approx F(t_{Base}) + \sum_{n=1}^{\infty} \frac{F^{(n)}(t_{Base})}{n!}(t - t_{Base})^n +$$
$$\sum_{k=1}^{S} \lim_{t_{Corr}+0 \leftarrow \tau} \left(\frac{\partial \Delta F(\{f_s(t_{Base}, t_{Corr}, \tau)\})}{\partial f_k(t_{Base}, t_{Corr}, \tau)}\right) f_k(t_{Base}, t_{Corr}, t_{Corr}) \pm \Delta_{Error}.$$

It may be expressed also as the derivative of a complex function

$$F(t) \approx F(t_{Base}) + \sum_{n=1}^{\infty} \frac{F^{(n)}(t_{Base})}{n!}(t - t_{Base})^n +$$
$$\sum_{k=1}^{S} \lim_{t_{Corr} \leftarrow \tau} \left(\frac{\partial \Delta F(\{f_s(t_{Base}, t_{Corr}, \tau)\})}{\partial f_k(t_{Base}, t_{Corr}, \tau)} \times \frac{\partial f_k(t_{Base}, t_{Corr}, \tau)}{\partial \tau}\right)(t - t_{Corr}) \pm \Delta_{Error}.$$

2.3. Additive-multiplicative formulae

Let us suppose that the modification of the forecast function $\Delta F(t_{Base}, t_{Corr}\ t)$ of an object may be exactly or approximately expressed in the form of explicit functions. These functions may be internal (relative to the object), external (relative to the object), periodic, etc, to specialize, specify unified and standardized forecasts to special, specific forecasting objects and situations. Then the modification $\Delta F(t_{Base}, t_{Corr}\ t)$ may be written in a general form as, for example,

$$\Delta F_{Corr}(t_{Base}, t_{Corr}, t) \approx \Delta F_{Corr}(\{f_{Internal,i}(t_{Corr}, t,)\}, \{f_{External,k}(t_{Corr}, t,)\},$$
$$\{f_{Periodic,l}(t_{Corr}, t,)\}, \{f_{Special,m}(t_{Corr}, t,)\}, \Delta_{Error}) .$$

where and further:

$\{f_{internal,i}\}$ - the set of internal (relative to the object) functions;
$\{f_{external,k}\}$ - the set of external (relative to the object) functions.
$\{f_{periodic,l}\}$ - the set of periodic functions;
$\{f_{special,m}\}$ - the set of specializing, specifying, adapting, concretizing functions to specialize, specify unified and standardized forecasts to special, specific forecasting objects and situations.

The operations of addition and multiplication are, probably, the most common and important ones as in practice so in the pure mathematics (see, e.g., Waerden van der, 1976).

Let us suppose that the partially unforeseen modification of the forecast function $\Delta F(t_{Base}, t_{Corr}\ t)$ may be exactly or approximately expressed by means of additive and multiplicative functions. Here, an additive function implies a function which additively contributes to the forecast. Here, a multiplicative function implies a function which additively contributes to the forecast.

Let us consider, as a heuristic hypothesis, the following formula

$$F(t_{Base}, t_{Corr}, t) \approx$$
$$[F_{Base}(t_{Base}, t) \times \prod_{m=1}^{M} K_{Multiplicat,m}(t_{Corr}, t) + \sum_{a=1}^{A} \Phi_{Addit,a}(t_{Corr}, t)] \times$$
$$[1 \pm \Delta_{Error}]$$

or, omitting the variables and indices,

$$F \approx [F_{Base} \times \prod K_{Multiplicat} + \sum \Phi_{Addit}] \times [1 \pm \Delta_{Error}],$$

where and further:

$F(t_{Base}, t_{Corr}, t)$ - the corrected forecast for the moment t;
$F_{Base}(t_{Base}, t)$ - the base forecast for the moment t;
$\prod K_{Multiplicat,m}$ - the product from 1 to M of the multiplicative (absolute) functions (coefficients) for partially unforeseen corrections;
$\sum \Phi_{Addit,a}$ - the sum from 1 to A of the additive (absolute) fu
nctions for partially unforeseen corrections;
Δ_{Error} - the total relative error.

For the cases when

$$F_{Base}(t_{Base},t)\times\prod_{m=1}^{M}K_{Multiplicat,m}(t_{Corr},t)\neq 0,$$

(preferentially for F~F_{base}) this formula may be written as

$$\begin{aligned}&F(t_{Base},t_{Corr},t)\approx F_{Base}(t_{Base},t)\times\\&[\prod_{m=1}^{M}(1+k_{Multiplicat,i}(t_{Corr},t))]\times\\&[1+\sum_{a=1}^{A}\varphi_{Addit,a}(t_{Corr},t)]\times\\&[1\pm\Delta_{Error}]\end{aligned},$$

or, omitting the variables and indices,

$$F\approx F_{Base}\times[\prod(1+k_{Multiplicat})]\times[1+\sum\varphi_{Addit}]\times[1\pm\Delta_{Error}],$$

where and further:
$\prod(1+k_{Multiplicat,m})$ - the product from 1 to M of the multiplicative (relative) functions (coefficients) for partially unforeseen corrections;
$\sum\varphi_{Addit,a}$ - the sum from 1 to A of the additive (relative) functions (normalized on $F_{Base}\times\prod K_{Multiplicat,m}$) for partially unforeseen (absolute) corrections.

2.4. Transformations

We may easily obtain the transformations between the versions of the formula.

For the multiplicative functions

$$K_{Multiplicat,m}=1+k_{Multiplicat,m}.$$

For the additive functions

$$\Phi_{Addit,a}=\varphi_{Addit,a}\times F_{Base}\times[\prod_{m=1}^{M}(1+k_{Multiplicat,m})].$$

3. Applications

3.1. Unforeseen events. Forecasts, perspectives and plans

Perspectives can be determined, ascertained by forecasts and estimates. Perspectives are guidelines for plans. The concept of unforeseen events and the hypothesis of uncertain future can put some questions about forecasts, perspectives and plans.

Let us remind that the first consequence of the hypothesis of uncertain future means that unforeseen events can occur. The second consequence of the hypothesis of uncertain future means that the greater the data dispersion (uncertainty), the smaller the probability of a future event near the probability $p \sim 1$, and the greater can be the probability of a future event near the probability $p \sim 0$.

Further, the terms short-term, medium-term, long-term and super long-term forecasting and planning will be sufficiently conditional and will be treated mainly with respect to the forecasting time intervals and changes of the projected objects.

Forecasts

Possibility of absolutely accurate forecasting. A question follows from the second consequence of the hypothesis of uncertain future: Is an absolutely accurate (and reasonably reliable) forecast possible?

Possibility of absolutely reliable forecasting. A question follows from the second consequence of the hypothesis of uncertain future: Is an absolutely reliable (reasonably accurate) forecast possible?

Possibility of mid-term quantitative forecasting. The term "the quantitative forecasting" will mean the accuracy of the forecasting not worse than, for example, 20% -30%. A question follows from the first consequence of the hypothesis of uncertain future: Is a medium-term quantitative forecasting possible?

Possibility of long-term holistic qualitative forecasting. By holistic we will mean qualitative forecasting possible to predict the impact of all aspects, etc., which exceed the quality threshold, for example, 30% -40 %. A question follows from the first consequence of the hypothesis of uncertain future: Is a long-term holistic qualitative forecast possible?

Possibility of super long-term qualitative forecasting. Under the super long-term qualitative forecasting we will mean the forecasting which is possible to predict the overall changes those exceed the qualitative threshold, for example 50%. A question follows from the second consequence of the hypothesis of uncertain future: Is a super long-term qualitative forecast possible?

Perspectives

So, forecasts can be affected by unforeseen events. Hence perspectives can be affected by unforeseen events also. So, unforeseen events can make existing perspectives more fuzzy and can give rise to new perspectives.

Plans

The need for a flexible medium-term planning. Under the flexible planning we will mean the planning with the presence of adjustments to previously approved plans. A question follows from the first and second consequences of the hypothesis of uncertain future: Is there a need for a flexible medium-term planning?

The need for a redirectable, reorientable long-term planning. Under the reorientable planning we will mean the planning with the presence of significant qualitative changes to previously approved plans. A question follows from the first consequence of the hypothesis of uncertain future: Is there a need for a reorientable long-term planning?

3.2. New resources and areas

Expansion of possibilities of forecasting. Currently, high-quality forecasting is a rather expensive service (such forecasting should take into account a large number of characteristics: from the individual characteristics of the customer to the global settings. In addition, in the case of unforeseen events, the forecast can largely lose its value. That is, the period of possible utilization of the forecast can be very short. Therefore, at present, only sufficiently large teams of specialists can develop high-quality forecasts. And high-quality forecasts can be ordered only by the government or sufficiently large and rich firms, corporations.

However, forecasting is an integral part of almost any management process. Therefore, forecasting is a service of mass demand, but its high price prevents its widespread dissemination.

The correcting formula can allow:

1) To significantly prolong the period of the use of forecasts. This will reduce the costs of forecasting for consumers forecasts.

2) To increase the degree of unification and standardization of forecasting. This will reduce the cost of forecasting for users of forecasts.

Consequently, the use of correcting formula for forecasts can expand the scope of forecasting.

The extension of possibilities of application of forecasting. The extension of possibilities of application of forecasting is due to lower development costs, falling costs of completion of the forecast for a particular customer and cost reductions on the use of the forecasts.

Tasks of small and medium business. The correcting formula for forecasts can essentially increase the possibilities of application of forecasting for medium and small business. In this case, apparently, it may be appropriate to start with the most mass and popular forms of business and for forecasting.

Government orders for the municipal needs. Government orders for the municipal needs are one of the most promising areas for application of the correcting formula for forecasts. Here, the combination of a wide market of forecasts, high-quality development of basic forecasts and standardization is possible. Especially useful it can be in municipal city-planning program.

Forecasting for individuals. The correcting formula for forecasts will make available orders for the needs of the individual forecasts of individuals, that is, it will make available the individual forecasting.

Here, apparently, it is advisable to start with a few, the most mass and popular kinds of tasks for individual forecasting.

The possibilities for expansion of forecasts development. Expansion of opportunities for the development of forecasts is due to lower cost of forecasts development, general decrease of costs for revision of the forecast for a particular customer and the considerable expansion of forecasts market.

Opportunities for small groups.

The correcting formula for forecasts will allow constructing and assemblage of forecasts from building blocks, adjustment of the standard forecasts for specific companies and their activities. Such works can perform not only large but also small groups of specialists.

Opportunities for private consultants. Application of the formula will allow prediction with little effort to adjust forecasts depending on the offensive (or non-occurrence) of certain events. Such adjustments can perform even private consultants-spotters.

4. Unforeseen events and a hypotheses of uncertain future

4.1. Unforeseen events and the hypothesis

4.1.1. Origins and formulation

The concept of unforeseen events is a part of a hypothesis of uncertain future (see, e.g., Harin, 2007a). The origins of the hypothesis of uncertain future are an incomplete knowledge and noises (those may be also treated as an incomplete knowledge). The incomplete knowledge prevents today to predict exactly what will happen tomorrow. The noises prevent to predict exactly what will happen a moment later.

The general hypothesis of uncertain future states: A future event contains an uncertainty.

The special hypothesis of uncertain future (hereinafter referred to as simply the hypothesis of uncertain future) states: The estimated probability of a future event contains an uncertainty. Or: At present, we cannot actually make an absolutely exact estimate of the probability of a future event (except imaginary cases).

4.1.2. Consequences of the hypothesis

The first (in the preceding works, see, e.g., Harin, 2007a), it was denoted as the second) consequence of the hypothesis: The present probability system of a future event is incomplete. Or: Unforeseen events can occur. More rigorously: At least one future unforeseen event can occur, such as, for the posterior future event, this future unforeseen event will lessen the total probability of the present probability system of this posterior future event.

The second (in the preceding works, see, e.g., Harin, 2007a), it was denoted as the first) consequence of the hypothesis: The greater the data dispersion (uncertainty), the smaller the probability of a future event near the probability $p \sim 1$,* and the greater can be** the probability of a future event near the probability $p \sim 0$.

* This consequence may be regarded as the rigorously proved mathematical statement of the existence theorem for non-zero restrictions on probability.

** Because of the first consequence, the magnitude of the low probability is decreased, but because of the second consequence, it is increased. So, it can be either increased or decreased or unchanged.

4.1.3. Foundations of the hypothesis. Heisenberg uncertainty principle

The general hypothesis of uncertain future can be formally supported by the Heisenberg uncertainty principle.

The Heisenberg uncertainty principle is one of the most distinctive aspects of quantum mechanics. It was devised by Werner Heisenberg at the Niels Bohr Institute in Copenhagen and introduced in Heisenberg (1927).

The Heisenberg's uncertainty principle states that one cannot simultaneously measure both impulse and position better than with uncertainty

$$\Delta p \times \Delta x \geq \frac{\hbar}{2},$$

where:

Δp - impulse uncertainty;

Δx - position uncertainty;

$\hbar$ - Planck's constant divided by 2π.

The Heisenberg's uncertainty principle is true for every physical object involved in an event, including future events. So, it supports the general hypothesis of uncertain future.

Existence theorems for restrictions

Let us suppose (see, e.g., Harin, 2012b), given a finite interval $X=[A, B]$: $0<Const_{AB}\leq(B-A)<\infty$, a set of points $x_k : k=1, 2, \ldots K : 2\leq K\leq\infty$, and a finite non-negative function $f_K(x_k)$ such that for $x_k<A$ and $x_k>B$ the statement $f_K(x_k)\equiv 0$ is true; for $A\leq x_k\leq B$ the statement $0\leq f_K(x_k)<\infty$ is true, and

$$\sum_{k=1}^{K} f_K(x_k) = W_K,$$

where W_K (the total weight of $f_K(x_k)$) is a constant such that

$$0 < W_K < \infty.$$

Without loss of generality, the function $f_K(x_k)$ may be normalized so that

$$W_K = 1.$$

The moduli of the central moments of the function $f_K(x_k)$ are not greater than

$$Max(|E(X-M)^n|) \leq$$
$$\leq (M-A)^n \frac{B-M}{B-A} + (B-M)^n \frac{M-A}{B-A} \cdot$$

General **lemma** about the tendency to zero for central moments. If, for the function $f_K(x_k)$, $M \equiv E(X)$ tends to A or to B, then, for $n : 2 \leq n < \infty$, $E(X-M)^n$ tends to zero.

Proof**.** For $M \to A$

$$
\begin{aligned}
&| E(X-M)^n | \leq \\
&\leq (M-A)^n \frac{B-M}{B-A} + (B-M)^n \frac{M-A}{B-A} = \\
&= [(M-A)^{n-1} + (B-M)^{n-1}] \frac{(M-A)(B-M)}{B-A} < \cdot \\
&< [(B-A)^{n-1} + (B-A)^{n-1}] \frac{(M-A)(B-M)}{B-A} \leq \\
&\leq 2(B-A)^{n-1}(M-A) \xrightarrow[M \to A]{} 0
\end{aligned}
$$

A more precise estimate states

$$| E(X-M)^n | \leq (B-A)^{n-1}(M-A) \xrightarrow[M \to A]{} 0 .$$

For $M \to B$, the proof is similar.

So, if $(B-A)$ and n are finite and $M \to A$ or $M \to B$, then $E(X-M)^n \to 0$.

General existence **theorem** for restrictions on the mean. If, for the finite non-negative discrete function $f_K(x_k)$ with the mean $M \equiv E(X)$ and the analog of an n-th $(2 \leq n < \infty)$ order central moment $E(X-M)^n$ of the function, a non-zero restriction on dispersion of the n-th order $r^n{}_{Disp.n} = Const_{Disp.n} > 0 : |E(X-M)^n| \geq r^n{}_{Disp.n}$, exists, then the non-zero restriction $r_{Mean} > 0$ on the mean $E(X)$ exists and $A < (A + r_{Mean}) \leq M \equiv E(X) \leq (B - r_{Mean}) < B$.

Proof. From the conditions of the theorem and from the preceding lemma, for $M \to A$, we have

$$0 < r^n{}_{Disp.n} \leq | E(X-M)^n | \leq (B-A)^{n-1}(M-A)$$

and

$$0 < \frac{r^n{}_{Disp.n}}{(B-A)^{n-1}} \leq (M-A) .$$

So,

$$(M-A) \geq r_{Mean} \equiv \frac{r^n{}_{Disp.n}}{(B-A)^{n-1}} > 0 .$$

For $M \to B$, the proof is similar.

So, as long as $(B-A)$ and n are finite and $r^n{}_{Disp.n} = Const_{Disp.n} > 0$, then $r_{Mean} = Const_M > 0$ and $A < (A + r_{Mean}) \leq M \leq (B - r_{Mean}) < B$.

This estimate is an ultra-reliable one. It is, in a sense, as ultra-reliable as the Chebyshev inequality. Preliminary calculations (see, e.g., Harin 2009b) which were performed for real cases, such as the normal, uniform and exponential distributions with the minimal values $\sigma^2{}_{Min}$ of the analog of the dispersion (in the particular sense), gave the restrictions r_{Mean} on the mean of the function, which are not worse than

$$r_{Mean} \geq \frac{\sigma_{Min}}{3} .$$

Lemma about the tendency to zero for the estimate probability. If a density $f(x)$ is as defined in the preceding sections, and either $E(X)\to 0$ or $E(X)\to 1$, then, for $1<n<\infty$, $E(X-M)^n\to 0$.

Proof. As long as the conditions of this lemma satisfy the conditions of the lemma about the tendency to zero for central moments, then the statement of this lemma is as true as the statement of the lemma about the tendency to zero for central moments.

Existence **theorem** for restrictions on the estimate probability. If: 1) a density $f(x)$ is defined in the lemma about the tendency to zero for the estimate probability, 2) there are n : $1<n<\infty$, and $r_{dispers}>0 : E(X-M)^n \geq r_{dispers}>0$, then, for the probability estimation, frequency $F\equiv M\equiv E(X)$, r_{expect} exists such that $0<r_{expect}\leq F\equiv M\equiv E(X)\leq(1-r_{expect})<1$.

Proof. As long as the conditions of this theorem satisfy the conditions of the general existence theorem for restrictions on the mean, then the statement of this theorem is as true as the statement of the general existence theorem for restrictions on the mean.

Existence **theorem** for restrictions on the probability. If, for the probability scale $[0; 1]$, a probability P and the probability estimation, frequency F_K, for a series of tests of number $K : K>>1$, are determined such that when the number of tests $K\to\infty$, the frequency F_K tends to the probability P, that is

$$P = \lim_{K\to\infty} F_K \ ,$$

and non-zero restrictions r_{mean} : $0<r_{mean}\leq F_K\leq(1-r_{mean})<1$ exist between the zone of the possible values of the frequency and every boundary of the probability scale, then the same non-zero restrictions r_{mean} : $0<r_{mean}\leq P\leq(1-r_{mean})<1$ exist between the zone of the possible values of P and every boundary of the probability scale.

Proof. Let us consider the left boundary 0 of the probability scale $[0; 1]$. F_K is not less than r_{mean}:

$$F_K \geq r_{mean} \ .$$

Hence, we obtain for P:

$$P = \lim_{K\to\infty} F_K \geq \lim_{K\to\infty} r_{mean} = r_{mean} \ .$$

So, $P\geq r_{mean}$. Note that this is true for both monotonous and dominated convergence. The reason is the fixation of the minimal value of all the F_K by the conditions of the theorem.

For the right boundary 1 of the probability scale the proof is similar to that above.

4.2. Illustrating examples and applications
4.2.1. Illustrating examples
Two points

Let us assume (see, e.g., Harin, 2012a) an interval $[A, B]$ (see Figure 1). Let us assume that two points are determined on this interval: a left point x_{Left} and a right point x_{Right} : $x_{Left}<x_{Right}$. The coordinates of the middle mean point may be calculated as $M=(x_{Left}+x_{Right})/2$.

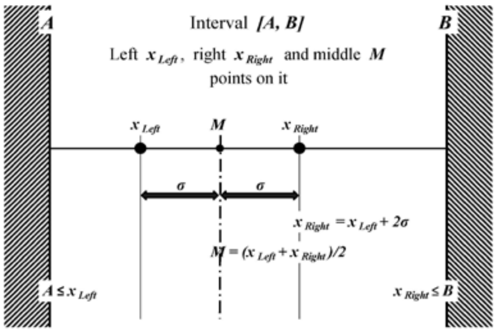

Figure 1. An interval $[A, B]$. Left x_{Left}, right x_{Right} and mean M points

Let us assume that $x_{Right}-x_{Left}\geq 2\sigma=2Const_{\sigma}>0$. So, of course, $x_{Right}\geq x_{Left}+2\sigma$ and $x_{Left}\leq x_{Right}-2\sigma$. For the sake of simplicity, Figures 1 to 3 represent the case of the equality $x_{Right}-x_{Left}=2\sigma$ and also, of course, $x_{Right}=x_{Left}+2\sigma$, $x_{Left}=x_{Right}-2\sigma$ and $M-x_{Left}=x_{Right}-M=\sigma=Const_{\sigma}>0$.

So, $M=x_{Left}+\sigma>x_{Left}$ and $M=x_{Right}-\sigma<x_{Right}$.

Suppose further that $x_{Left}\geq A$ and $x_{Right}\leq B$.

One can easily see that two types of zones for M can exist in the interval:

(1) The mean point M can be located only in the zone which will be referred to as "allowed" (see Figure 2).

(2) The mean point M cannot be located in the zones which will be referred to as "forbidden" (see Figure 3).

Allowed zone

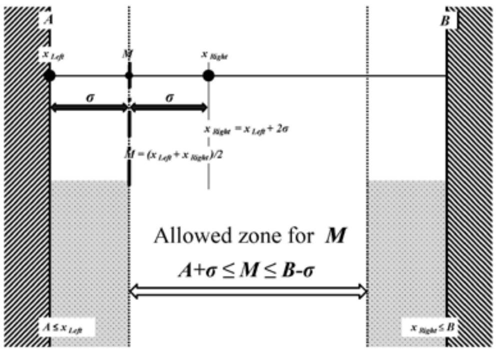

Figure 2. Allowed zone for M

The sample conditions mean that the left point x_{Left} may not be located further left than the left border of the interval $x_{Left} \geq A$ and the right point x_{Right} may not be located further right than the right border of the interval $x_{Right} \leq B$.

For M, we have $M = x_{Left} + \sigma \geq A + \sigma > A$ and $M = x_{Right} - \sigma \leq B - \sigma < B$ (see Figure 2).

The width of the allowed zone for M is equal to

$$B - \sigma - (A + \sigma) = (B - A) - 2\sigma$$

It is less than the width $(B-A)$ of the total interval $[A, B]$ by 2σ. Also, the allowed zone is a proper subset of the total interval.

If the distance 2σ between the left x_{Left} and right x_{Right} points is non-zero, then the difference between the width of the allowed zone and the width of the interval is non-zero also. If the distance is greater than 2σ, then the difference is greater than 2σ also.

So, the mean point M can be located only in the allowed zone of the interval.

Forbidden zones, restrictions

The value of a restriction or the width of a forbidden zone signifies the minimal possible distance between the mean and a border of the interval. For the sake of brevity, the term "the value of a restriction" may be shortened to "restriction."

If $A \leq x_{Left}$, $x_{Right} \leq B$ and $x_{Right} - x_{Left} = 2\sigma$, then restrictions, forbidden zones with the width of one sigma σ, exist between the mean point and the borders of the interval (see Figure 3). So there are two forbidden zones, located near the borders of the interval. The mean point M cannot be located in these forbidden zones.

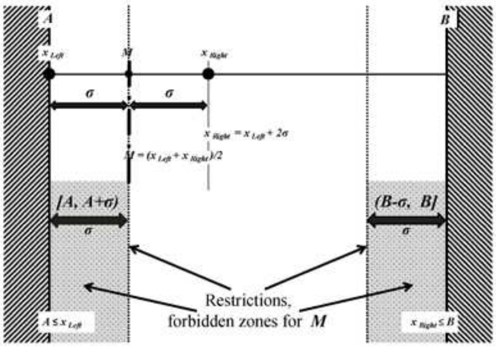

Figure 3. Forbidden zones, restrictions on M

The restrictions, the forbidden zones, are shown by two dotted lines and by painting in the bottom part of Figure 3.

As we can easily see, restrictions on the mean or forbidden zones exist between the allowed zone of the mean M and the borders A and B of the interval $[A; B]$. The value of the restriction, or, equivalently, the width of the forbidden zone, is equal to σ.

So, the restrictions of the value σ on the mean point M exist near the borders of the interval.

Restrictions on the probability

Let us consider (see, e.g., Harin, 2012a) a classical example: firing at a target. Suppose a round target (Figure 4) of diameter $2L$.

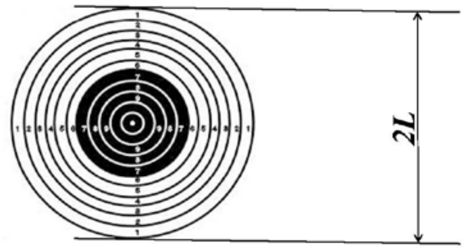

Figure 4. Firing target

Let us suppose (Figure 5) the dispersion of hits is uniformly (for the obviousness) distributed in a zone of diameter 2σ (see an example of the normal distribution, e.g., in Harin 2010a).

Let us consider two cases:

(1) The diameter $2\sigma_{Small}$ of the zone of dispersion of hits is considerably smaller than the diameter $2L$ of the target (small dispersion).

(2) The diameter $2\sigma_{Large}$ of the zone of dispersion of hits is considerably larger than the diameter $2L$ of the target (large dispersion).

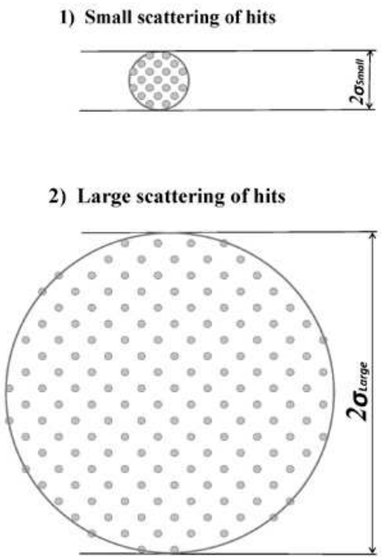

Figure 5. Dispersion of hits is uniformly distributed in a zone of diameter 2σ

Notes on the figure:

Note 1: This is only a simplified example (see an example of the normal distribution, e.g., in Harin 2010a).

Note 2: Case 1 represents a small diameter $2\sigma_{Small}$ of the zone of dispersion of hits.

Case 2 represents a large diameter $2\sigma_{Large}$ of the zone of dispersion of hits.

Suppose the aiming point varies between the center of the target and a point which is outside the target.

Small dispersion

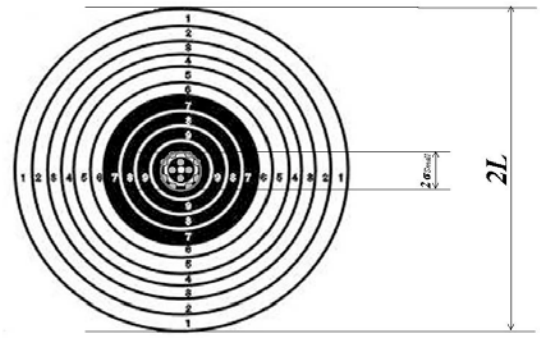

Figure 6. Small dispersion of hits

Small dispersion occurs when the diameter $2\sigma_{Small}$ of the zone of dispersion of hits is considerably smaller than the diameter $2L$ of the target, as drawn in Figure 6.

Notes:

The diameter $2\sigma_{Small}$ of the zone of dispersion of hits is considerably smaller than the diameter $2L$ of the target.

In the condition of the small dispersion of hits, the maximum possible probability of hitting the target can be equal to *one* (can reach the boundary of the probability scale).

When the point of aim varies between the center of the target and a point which is outside the target, the probability of hitting the target ranges from *one* to *zero*. There are no restrictions in the probability scale.

Large dispersion. Restrictions

The case when the diameter $2\sigma_{Large}$ of the zone of dispersion of hits is considerably larger than the diameter $2L$ of the target is drawn in Figure 7.

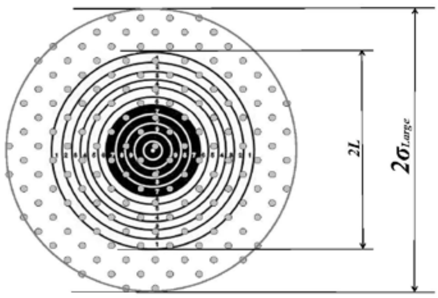

Figure 7. Large dispersion of hits

Note: The diameter $2\sigma_{Large}$ of the zone of dispersion of hits is considerably larger than the diameter $2L$ of the target.

At the condition of the large dispersion of hits (that is, when the diameter $2\sigma_{Large}$ of the zone of dispersion of hits is larger than the diameter $2L$ of a target), the maximum possible probability of hitting the target cannot be equal to *one*.

The probability for this case is shown in Figure 8.

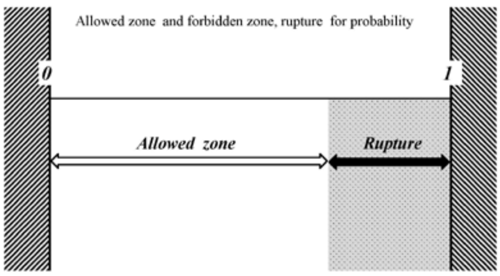

Figure 8. Restriction for the probability:
Allowed zone and forbidden zone

The value $P_{AllowedMax}$ of the maximal allowed probability of the allowed zone $[0, P_{AllowedMax}]$ may be estimated as the ratio of the mean number of the hits on the target to the total number of hits. In this particular case, when the distribution of hits is assumed to be uniform, this ratio equals the ratio of the area of scattered hits to the area of the target.

4.2.2. Applications of the hypothesis
4.2.2.1. Partial explanation of Ellsberg paradox
Experiment

Let us briefly review the application of the hypothesis to the Ellsberg paradox (in more detail see, e.g., Harin, 2008a).

The Ellsberg paradox (see Ellsberg, 1961) (here simplified and modified): the urn U1 (certain) contains red and black balls with certain proportion 1:1. the urn U2 (uncertain) contains red and black balls with unknown proportion. You will win \$100 if you draw a ball of the determined color from the urns U1 or U2. Most people state that they prefer the certain U1 to the uncertain U2 for both red and black balls.

The situation can be described as

$$P_{Red\_Uncertain} + P_{Black\_Uncertain} < 1 \ ,$$

or, more precisely,

$$P_{Red\_Uncertain} + P_{Black\_Uncertain} < P_{Red\_Certain} + P_{Black\_Certain} \ ,$$

where

$P_{Red\ Certain}$ - the probability of drawing a red ball from the certain urn U1;

$P_{Black\ Certain}$ - the probability of drawing a black ball from the certain urn U1;

$P_{Red\ Uncertain}$ - the probability of drawing a red ball from the uncertain urn U2;

$P_{Black\ Uncertain}$ - the probability of drawing a black ball from the uncertain urn U2.

Ideal, real and seeming cases

Let us suppose two types of cases:

(1) An ideal case (or an ideal point of view):

Unforeseen events cannot occur. The present probability system of a future event is complete. The total probability of the present probability system of a future event equals one (or, equivalently, 100%)

$$\sum P_{Present\_for\_Future} = 100\% \ ,$$

and

$$P_{Red\_Uncertain} + P_{Black\_Uncertain} =$$
$$= P_{Red\_Certain} + P_{Black\_Certain} = 100\% \ ,$$

where

$\sum P_{Present\_for\_Future}$ - the present sum of the probabilities of posterior future events.

The difference between the sum of the probabilities of unforeseen events for the uncertain and certain present situation is equal to zero.

(2) Real and seeming cases:
Unforeseen events can occur. At least one future unforeseen event can occur: this will lessen the total probability of the present probability system of a posterior future event. The total probability of the present probability system of a future event is less than 100%. Indeed, if

$$100\% = \sum P_{\Pr esent\_for\_Future} + \sum P_{Unforeseen}$$

and

$$\sum P_{Unforeseen} > 0$$

then

$$\sum P_{\Pr esent\_for\_Future} < 100\%$$

and

$$P_{\operatorname{Re} d\_Uncertain} + P_{Black\_Uncertain} < 100\%$$

and

$$P_{\operatorname{Re} d\_Certain} + P_{Black\_Certain} < 100\% \ ,$$

where

$\sum P_{Unforeseen}$ - the sum of the probabilities of unforeseen events.

Let us suppose a supplementary assumption: Suppose there are two present situations: a certain present situation and an uncertain one.

Let us assume that the sum of the probabilities of unforeseen events for the certain present situation is less than that for the uncertain present situation.

Because of this assumption, the total probability of the present probability system of a future event for the certain present situation is more than the total probability of the present probability system of the future event for the uncertain present situation. So a difference between the sum of the probabilities of unforeseen events for the uncertain and certain present situation can exist.

Human experience can reveal the existence of the non-zero sum of the probabilities of unforeseen events. So, humans can feel that there is the zero sum of the probabilities of unforeseen events. The whole of the preceding experience can lead to an averaged perceptible sum of the probabilities of unforeseen events. The sum of the probabilities of unforeseen events for a particular situation can differ from the averaged sum. So, one may bear in mind that there is the seeming sum of the probabilities of unforeseen events. The difference between the sum of the probabilities of unforeseen events for the uncertain and certain present situation can be both real and seeming. So, one may bear in mind either the real or seeming difference between the sum of the probabilities of unforeseen events for the uncertain and certain present situation.

Let us suppose that the real or seeming difference between the sum of the probabilities of unforeseen events for the uncertain and certain present situation is not less than, say, $\delta P_{Real\text{-}Ideal}=0.000001\%>0$.

Transformation and bias

Suppose a transformation from an ideal to a real case. This corresponds to the transformation from the ideal point of view to the point of view of the people.

The ideal probability P_{Ideal} is transformed to some real or seeming probability P_{Real}.

Because of the above assumption, the real or seeming difference between the sum of the probabilities of unforeseen events for the uncertain and certain present situation is increased from the ideal case of zero to the real case of some non-zero magnitude, say to $\delta P_{Real\text{-}Ideal}=0.000001\%>0$.

Therefore, for two types of cases we obtain:

In the ideal case (or from the ideal point of view), the difference between the sum of the probabilities of unforeseen events for the uncertain and certain present situation equals zero.

In the real case (or from the point of view of the people), the real or seeming difference between the sum of the probabilities of unforeseen events for the uncertain and certain present situation equals zero.

So, there is the non-zero bias of probability $\delta P_{Real\text{-}Ideal}>0$ between the real (or seeming) and ideal cases.

Because of the above bias, we obtain

$$\begin{aligned} &P_{Red\_Uncertain} + P_{Black\_Uncertain} = \\ &= P_{Red\_Certain} + P_{Black\_Certain} + \delta P_{Real-Ideal} \end{aligned} .$$

So, at

$$\delta P_{Real-Ideal} > 0 \ .$$

we obtain

$$P_{Red\_Uncertain} + P_{Black\_Uncertain} < P_{Red\_Certain} + P_{Black\_Certain} \ .$$

So, from the point of view of the first consequence of the hypothesis of uncertain future, the Ellsberg paradox is quite natural.

There may be other causes of the Ellsberg paradox but we can see that the first consequence of the hypothesis of uncertain future may, at least partially, explain it.

4.2.2.2. Probability weighting problems
Ideal, real and seeming cases

Let us briefly review the applications of the hypothesis to probability weighting problems (in more detail see, e.g., Harin, 2012b).

Let us suppose (see, e.g.,) two types of cases:

(1) An ideal case: There is no (or negligible) dispersion of data. Hence, the probability may be equal to any value near any boundary of the probability scale. Assume a value $P_{Ideal}=100\%-\delta$: $0<\delta<<100\%$ of the probability located near 100% in an ideal case with zero dispersion of data.

(2) Real and seeming cases: There is a non-zero dispersion of data. The non-zero dispersion of the data causes a non-zero restriction, say $r_{Restriction}\geq 3\%$, near any boundary of the probability scale.

The previous experience of people can lead to an averaged perceptible dispersion of data and to averaged restriction. This averaged restriction may differ from the real restriction for a particular situation. So, people may keep in mind the seeming dispersion and restriction.

From the ideal point of view, we may keep in mind no (or negligible) dispersion of data and we may propose probabilities that are very close to the boundary of the probability scale. If the real experience of people proves that the dispersion is usually large, then, contrary to the ideal point of view, they may keep in mind the real case of the large dispersion, namely of the dispersion that causes a non-zero restriction, say $r_{Restriction}\geq 3\%$.

Transformation

Suppose a transformation from an ideal to a real case. This corresponds to the transformation from the ideal point of view to the point of view of people.

The absence or negligible dispersion of data is transformed to non-zero dispersion of data. Zero value of the probability of the ideal case P_{Ideal} will be transformed to non-zero value of the real case P_{Real}.

Let a restriction in the probability scale be increased from the ideal case of zero to the real case of some non-zero magnitude, say to $r_{Restriction}=3\%$. The probability P_{Ideal} is transformed to some P_{Real}.

Bias of probability

Consider the probability near the right boundary, 100% of the probability scale [$0\%; 100\%$]. The probability cannot be located in the restriction. Hence, the probability P_{Real} cannot be more than $P_{Real} \leq 100\%-r_{Restriction}=100\%-3\%=97\%$. If the dispersion of data is increased to the extent that the restriction exceeds the difference between 100% and P_{Ideal}, that is, $r_{Restriction}>\delta$, then P_{Real} cannot be equal to (or more than) P_{Ideal}. The ideal case probability of, say, 98% cannot be located in the restriction and is biased to a position that is not more than 97%. Every ideal probability from $97.000...01\%$ to $99.999...99\%$ is also biased to a corresponding real position that is not more than 97%.

So, near 100%, P_{Real} is biased downward to the middle of the probability scale with respect to P_{Ideal}, that is, near 100%, $P_{Real}<P_{Ideal}$. The closer the probability P_{Ideal} is to 100%, the greater is the bias $P_{Ideal}-P_{Real}$. Conversely, for any non-zero restriction $r_{Restriction}>0$, a $P_{Ideal}=100\%-\delta : \delta>0$ will exist, such as $r_{Restriction}>\delta>0$, and, hence, $P_{Real}<P_{Ideal}$.

An analogous consideration may be performed for a probability located near 0% (keeping in mind the first consequence of the hypothesis).

So, the restrictions near the boundaries shift and bias the probability from the boundaries to the middle of the probability scale. The bias is directed to the middle and is maximal just near every boundary.

Therefore, in the ideal case (or from the ideal point of view), the probability is unbiased. In the real case (or from the point of view of people), the probability near every boundary is biased (in comparison with the ideal case) from the boundary to the middle of the probability scale. Taking into account the restrictions and the biases may help to overcome the influence of observation noise, and to refine the results of experiments.

Note that the bias may be assumed to exist not only in the zones of the restrictions but also beyond them and to vanish at the middle of the scale.

Note also that the signs of the biases of the probability are opposite for high and low probabilities. The sign of the bias is negative for high probabilities and positive for low probabilities. So, according to the mean value theorem, at some point in the middle of the scale the bias should be equal to zero.

Underweighting of high probabilities
Gain at high probabilities. Deposits

Let us assume that we offer a choice of two outcomes:

(A) a guaranteed gain of a prize of *$99* (with the probability *1* or *100%*)

or

(B) a probable gain of *$100* with the probability *0.99* (or *99%*), or nothing with the probability *0.01* (or *1%*).

For experimental accuracy, both *$99* and *$100* should be in *$1* banknotes, i.e. *99* and *100* banknotes of *$1*.

A real example: Despite the publicity and obvious advantages of bank deposits, people are not willing to use them as often and as much as predicted by the probability theory.

In the ideal case and from the ideal point of view, the probable gain has the probability *99%* and the mean values for the probable and guaranteed outcomes are

$$\$99 \times 100\% = \$99$$

and

$$\$100 \times 99\% = \$99 \ .$$

Here

$$\$100 \times 99\% = \$99 \ .$$

The mean value of obtaining the probable gain is evidently precisely equal to the mean value of obtaining the guaranteed gain.

The well-determined experimental fact, however, is this: in similar experiments for gains at high probabilities the overwhelming majority of people choose the guaranteed gain instead of the probable one (see, e.g., Tversky and Wakker, 1995, Di Mauro and Maffioletti, 2004). People underestimate probable outcomes and do not like risk.

In the real case and from the point of view of people, if the dispersion of real data leads to the restriction (near *100%*) that is more than *1%* and is equal to, say, *3%*, then the probability of the probable gain cannot be equal to *99%* and is not more than *97%*

$$\$99 \times 100\% = \$99$$

and

$$\$100 \times 97\% = \$97 \ .$$

Here

$$\$100 \times 97\% = \$97 \ .$$

The mean value of the probable gain is less than the mean value of the guaranteed gain.

So, in the real case and from the point of view of people, the mean of the probable gain is less than the mean of the guaranteed gain and the guaranteed outcome is preferable.

So, the paradox can be explained, at least partially, by taking into account the restriction and the bias of the mean result, which are caused by the second consequence of the hypothesis.

Overweighting of low probabilities
Gain at low probabilities. Lotteries

Let us assume that we offer a choice of two outcomes:

(A) a guaranteed gain of *$1* (with the probability *100%*)

or

(B) a probable gain of a prize of *$100* with the probability *0.01* (or *1%*), or nothing with the probability *0.99* (or *99%*).

A real example: Obviously the organizers of lotteries are paid from the lottery proceeds and people usually do not gain as much from lotteries as they pay in. Nevertheless, they are very willing to participate all the same.

In the ideal case and from the ideal point of view, the probable gain has the probability *1%* and the mean values for the probable and guaranteed outcomes are

$$\$1 \times 100\% = \$1$$

and

$$\$100 \times 1\% = \$1 \ .$$

Here

$$\$1 = \$1 \ .$$

The mean value for obtaining the probable gain is precisely equal to the mean value for obtaining the guaranteed gain.

The well-determined experimental fact is this, however: in similar experiments on gains at low probabilities the overwhelming majority of people choose the probable gain instead of the guaranteed one (see, e.g., Tversky and Wakker, 1995, Di Mauro and Maffioletti, 2004). People overestimate probable outcomes and do not like risk.

In the real or seeming cases and from the point of view of people, if the dispersion of real data leads to the restriction (near *0%*) that is more than *1%* and is equal to, say, *3%*, then the probability of the probable gain may be (keeping in mind the decreasing probability because of the first consequence of the hypothesis) *3%* or more

$$\$1 \times 100\% = \$1$$

and

$$\$100 \times 3\% = \$3 \ .$$

Here

$$\$3 > \$1 \ .$$

The mean value of the guaranteed gain may be less than the mean value of the probable gain.

So, in the real or seeming cases and from the point of view of people, the mean of the guaranteed gain may be less than the mean of the probable gain and the probable outcome is preferable.

So, the paradox can be partially explained by taking into account the restriction and the bias of the mean result, which are caused by the second consequence of the hypothesis.

Gains and losses
Loss at high probabilities. Defaults

Let us assume that we offer a choice of two outcomes:

(A) a guaranteed loss of *-$99* (with the probability *1* (or *100%*))

or

(B) a probable loss of *-$100* with the probability *0.99* (or *99%*), or no loss with the probability *0.01* (or *1%*).

A real example: It is clear that the earlier one declares a default, the less one will lose. Nevertheless, often people and even governments delay declaring a default.

In the ideal case and from the ideal point of view, the probable loss has the probability *0.99* (or *99%*) and the mean values for the probable and guaranteed outcomes are

$$-\$99 \times 100\% = -\$99$$

and

$$-\$100 \times 99\% = -\$99 \ .$$

Here

$$-\$99 = -\$99 \ .$$

The mean value of the probable loss is precisely equal to the mean value of the guaranteed loss.

The well-determined experimental fact, however, is this: in similar experiments for gains at high probabilities the overwhelming majority of people choose the probable loss instead of the guaranteed one (see, e.g., Tversky and Wakker, 1995, Di Mauro and Maffioletti, 2004). People overestimate probable outcomes and like risk.

In the real case and from the point of view of people, if the dispersion of real data leads to the restriction (near *100%*) that is more than *1%* and is equal to, say, *3%*, then the probability of the probable loss cannot be equal to *99%* and is not more than *97%*

$$-\$99 \times 100\% = -\$99$$

and

$$-\$100 \times 97\% = -\$97 \ .$$

Here

$$-\$99 < -\$97 \ .$$

The mean value of the probable gain is less than the mean value of the guaranteed gain.

So, in the real case and from the point of view of people, the mean of the guaranteed loss is less than the mean of the probable loss (it is less in terms of absolute value but more because of the negative sign of the loss). Hence, the probable outcome is preferable.

So, the paradox can be explained, at least partially, by taking into account the restriction and the bias of the mean result, which are caused by the second consequence of the hypothesis.

Conclusions

So, the unforeseen events can essentially modify forecasts and increase financial risks. But their negative influence can be lessened by the preliminary risk management in some cases of the partially unforeseen events. For example, if the influence of a partially unforeseen event could be and was preliminary (partially) estimated, then this estimate may be used just after this event has occurred.

At present, it is evident, that a forecast should manifestly contain errors' terms. A long-term forecast should manifestly contain unforeseen errors' terms (because the relative error, caused by an unforeseen event, can be much more than 100%). A long-use forecast should contain correcting terms. These terms may have the form of a framework for forecasts – a correcting formula for forecasts.

This correcting formula for forecasts may be used as a correcting tool for long-use forecasts and as an adapting tool in addition to unified forecasts to apply them to special situations.

Let us suppose that the modification $\Delta F(t_{Base}, t_{Corr}, t)$ of the forecast function may be exactly or approximately expressed in the form of explicit functions. The operations of addition and multiplication are, probably, the most common and important ones as in practice so in the pure mathematics (see, e.g., Waerden van der, 1976). If we suppose that the $\Delta F(t_{Base}, t_{Corr}, t)$ may be exactly or approximately expressed by means of additive and multiplicative functions, then the formula

$$F(t_{Base}, t_{Corr}, t) \approx$$
$$[F_{Base}(t_{Base}, t) \times \prod_{m=1}^{M} K_{Multiplicat,m}(t_{Corr}, t) + \sum_{a=1}^{A} \Phi_{Addit,a}(t_{Corr}, t)] \times [1 \pm \Delta_{Error}]$$

may be written, or, omitting the variables and indices, it may be written in the form

$$F \approx [F_{Base} \times \prod K_{Multiplicat} + \sum \Phi_{Addit}] \times [1 \pm \Delta_{Error}].$$

For the cases when

$$F_{Base}(t_{Base}, t) \times \prod_{m=1}^{M} K_{Multiplicat,m}(t_{Corr}, t) \neq 0,$$

(preferentially for $F \sim F_{base}$) this formula may be written as

$$F(t_{Base}, t_{Corr}, t) \approx F_{Base}(t_{Base}, t) \times$$
$$[\prod_{m=1}^{M} (1 + k_{Multiplicat,i}(t_{Corr}, t))] \times [1 + \sum_{a=1}^{A} \varphi_{Addit,a}(t_{Corr}, t)] \times [1 \pm \Delta_{Error}],$$

or, omitting the variables and indices,

$$F \approx F_{Base} \times [\prod (1 + k_{Multiplicat})] \times [1 + \sum \varphi_{Addit}] \times [1 \pm \Delta_{Error}].$$

We may easily obtain the transformations between the versions of the formula.

For multiplicative functions

$$K_{Multiplicat,m} = 1 + k_{Multiplicat,m}.$$

For additive functions

$$\Phi_{Addit,a} = \varphi_{Addit,a} \times F_{Base} \times [\prod_{m=1}^{M} (1 + k_{Multiplicat,m})].$$

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