Do Consumers' Preferences Really Matter? - A Note on Spatial Competition with Restricted Strategies

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Do Consumers' Preferences Really Matter? – A Note on Spatial Competition with Restricted Strategies

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Abstract

In the framework Hotelling-Downs competition two players can freely choose a position along a one-dimensional market. We introduce restrictions of feasible strategies and analyze the consequences for players and consumers. In equilibrium players may minimally differentiate away from the center of the market and even locate completely independently of consumers' preferences. We provide conditions for these novel cases as well as for the standard result that players locate on the median of the distribution of consumers. In addition to the short run, where restrictions are fixed, we elaborate on the long run by studying the players' choice of restrictions under (potential) market entry. In both settings, we find an inefficient outcome, in which a firm is capable of offering a product at the center of the market, but instead chooses a position that is worse for most of the consumers.

Keywords: duopoly; product differentiation; Hotelling-Downs; median voter; market entry

JEL classification: D43, D49, L13, P16

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1 Introduction

The design of new products is a key task for companies in virtually any industry. As an example, consider Samsung's tablet device called 'Galaxy' which is characterized by a certain size, shape, price, and functionality. The ongoing 'patent war' between Samsung and Apple (see Cusumano, 2013) illustrates that the choice of these product specifications cannot be seen independently of Samsung's technological capabilities, which are further restricted by several patents on the competitor product 'iPad'. Generally, product specification choices have to account for technological abilities and constraints as well as for the competitor's products and the preferences of the consumers. While economic models of product differentiation incorporate heterogeneous consumer preferences and strategic behavior in the face of competition, they usually abstract from the technological constraints. Consequently, these models focus on the effects of reducing price competition by increasing differentiation and on the effects of 'stealing' consumers of the competitor by decreasing differentiation, but they conceal that restrictions of the company's and competitor's abilities affect strategic product positioning. This is unsatisfying – not only because empirical evidence documents that strategy restrictions are present when choosing a product position, but because firms' behavior and consumers' surplus are substantially affected by strategy restrictions, as it turns out.

In particular, when standard models predict that two firms choose similar product specifications (minimal differentiation), then the chosen position must be at the median of the distribution of consumers, which is interpreted as the center of the market or 'where the demand is'. In this note, we introduce restrictions of feasible positions and observe that under various conditions, this conclusion does not hold true anymore. Specifically, we find equilibria where the center of the market would be a feasible product position for a company, but this company does not have an incentive to locate there, due to strategy restrictions of its main competitor. This suggests that the standardization of products, e.g. the similarity of Samsung's and Apple's tablets, does not inevitably result from consumers' tastes, but could also reflect a technological constraint of one of the two companies. In that case, strategy restrictions do not contribute to an increase in product variety and social welfare is even lower than predicted by the standard analysis.

It is empirically well-justified to consider strategy restrictions because they seem ubiquitous in applications of horizontal differentiation. Restrictions have several sources and take many forms. In addition to technological constraints and the legal environment (e.g. patents, regulations), overriding corporate strategies can reduce the set of feasible product positions for a company. Specific examples for spatial restrictions in the literal sense are territorial restrictions in distribution, such as in the case of franchising (Stern *et al.*, 1976), and zoning restrictions imposed by a government (Datta and Sudhir, 2013). In the application of flight scheduling in the airline market (e.g. Panzar, 1979; Schipper *et al.*, 2007) strategies are restricted, e.g. by local bans on nighttime flying. Moreover, there are generally constraints based on the brand perception of consumers, such that due to observable characteristics (e.g. the company's home country), perceived technical expertise and previous (marketing) activities, certain product positions cannot credibly be chosen by a company (Hauser, 1988).

Models of spatial competition commonly abstract from the fact that in the short term the set of feasible product positions is restricted. However, one approach to incorporate strategy restrictions is given by the framework of Prescott and Visscher (1977), which was further developed by Loertscher and Muehlheusser (2011). In this dynamic model players sequentially choose positions and are then fully restricted from relocating. More explicit accounts for restrictions of strategies are the works of Samuelson (1984), Hummel (2010), and Hauser (1988). Samuelson (1984) analyzes a multidimensional model of probabilistic voting and incorporates restricted strategies in the following way: Each candidate is endowed with an initial position and her choice of strategy is restricted within some convex compact set around this endowment. In the context of brand positions Hauser (1988) introduces a restriction on feasible positions by assuming that given positions cannot change their relative order. A similar assumption is made by Hummel (2010) in the context of political candidates since his model restricts political candidates from moving more than halfway into the direction of the opponent. This is motivated by the idea that a change in political position from a primary to a general election, so-called 'flip-flopping', undermines a candidate's credibility. Within the context of political competition there are additional approaches to incorporate restrictions of positions (see Samuelson, 1984, and the references therein). Even though these works use the important idea of strategy restrictions, they do not systematically discuss their effect on equilibrium locations and they focus on the application to political competition, but not to product differentiation.

In this note we address this gap in the literature by using the most basic model: Two players simultaneously choose a position on a line, while for each of them only some interval of positions is feasible. We thereby follow Samuelson's (1984) approach of how to restrict strategies. We use this simple yet powerful model as a benchmark to make the arguments clear. Our analysis of the model addresses the following research questions: How do strategy restrictions affect product positions of the firms (i.e. equilibrium locations)? What are the welfare implications of strategy restrictions? What is the optimal choice of restrictions for an incumbent firm, if there are potential entrants? We find that in the equilibrium analysis there are three cases to distinguish. In one case we obtain the standard result that both players locate on the median. In the second case the analysis resembles Bertrand competition, where instead of price undercutting, there is virtually an undercutting in the product position.[1] In the final case players minimally differentiate on the boundaries of their restrictions independently of the location of the median. This independence means that consumers' preferences do not influence players' decisions in any way. While equilibrium positions in each case can be characterized by some form of minimal differentiation, consumers' preferences only play a vital role in the first case. Thus, our results differ from previous findings in that preferences of consumers do not matter to the same extent as in the standard model.

Since strategy restrictions are only exogenous in the short term, we also elaborate on the choice of restrictions. In a simple market entry game, we find two types of equilibria in both of which the incumbent firm chooses an optimal level of flexibility at the center of the market, i.e. around the median. The threat of copying the entrant's product position is credible and thus either fully deters entry or leaves only niche positions to a potential entrant. In case of market entry, the median position is feasible for the incumbent, but would not be chosen, as it happens in the static (short-term) game. To our best knowledge, this outcome, two firms locating next to each other but not at the center of the market, is new to the literature. We assess welfare by aggregated transportation costs and show that this novel case is highly inefficient. Transportation costs are even larger than in the classic case of minimal differentiation.

This note contributes to the existing literature in several ways. First, we close a gap in the literature on spatial competition by *providing conditions* when and how strategy restrictions affect equilibrium outcomes. Already 30 years ago, Larry Samuelson pointed to this gap by noting that "the common assumption that candidates choose freely from the entire strategy space is an unrealistic one" (cf. Samuelson 1984, who gives credit to two articles which, however, do not make this statement as explicitly). However, to our knowledge, the effect of strategy restrictions on equilibrium locations has still not been systematically discussed. Second, we *demonstrate the significance* of restrictions of feasible strategies for horizontal product differentiation. We

[1] In our model we abstract from price competition. If we include price competition in the standard way, then we would receive a trivial form of maximal differentiation.

show that under restrictions qualitatively different strategies become prevalent, where not the consumer, but the competitor is the first interest. As a result, restrictions are an essential factor for market outcomes. Third, we *derive implications* for consumers and a social planner by assessing social welfare. For instance, our results imply that if a social planner were in the position to impose restrictions on one company, e.g. in order to increase product variety in the market, it should only do so if it can impose restrictions on the other company as well.

The Hotelling-Downs model of spatial competition is a widely used tool in the analysis of product differentiation and of political competition. Considering the vast amount of literature dealing with this approach, it is remarkable that the idea of restricting the companies' strategies plays hardly any role. Models of spatial competition can be organized into location-cum-prices models which study a game of spatial competition before price competition (e.g. Hotelling, 1929; d'Aspremont *et al.*, 1979; de Palma, 1985; Meagher and Zauner, 2004; Król, 2012) and purely spatial models (e.g. Downs, 1957b; Eaton and Lipsey, 1975; Prescott and Visscher, 1977; Loertscher and Muehlheusser, 2011) which abstract from endogenous price setting. A well-known result for location-cum-prices models is that firms maximally differentiate in a one-dimensional market, which holds for a uniform distribution of consumers with quadratic transportation costs (d'Aspremont *et al.*, 1979) and for some generalizations of quadratic transportation costs (Economides, 1986). In a multidimensional market, the same specification leads to maximal differentiation with respect to one dimension and to minimal differentiation with respect to all others (Irmen and Thisse, 1998). Moreover, using an evolutionary approach Hehenkamp and Wambach (2010) show that minimal differentiation with respect to all dimensions emerges in this setting. A caveat of location-cum-prices models is that they have to ensure existence of equilibria by assuming very simple distributions of consumers and specific functional forms.

This is not true to the same extent for pure spatial models. Pure spatial models are not only sensible in regulated markets where prices are fixed, but also apply to markets in which prices are not a dominant marketing instrument, such as a newspaper market (George and Waldfogel, 2006). Pure spatial models typically find minimal differentiation under general conditions (Downs, 1957b; Eaton and Lipsey, 1975), but not when an endogenous number of players sequentially chooses a position (Prescott and Visscher, 1977; Loertscher and Muehlheusser, 2011). In fact, the latter approach can be considered as an alternative way to introduce strategy restrictions because in this dynamic game firms once choose locations which are fixed thereafter. A main feature of this model is that firms agglomerate in densely populated areas (Loertscher and Muehlheusser, 2011), which is similar to locating 'where the demand is'. Further recent literature on product differentiation has focused on tying (Amelio and Jullien, 2012; Egli, 2007; Gilbert and Riordan, 2007) and on empirical evidence on minimal versus maximal differentiation (Barreda-Tarrazona *et al.*, 2011; Picone *et al.*, 2009).

Introducing strategy restriction typically does not qualitatively affect maximal differentiation results. For example, in the most common location-cum-price model (d'Aspremont *et al.*, 1979), it is easy to show that firms would choose the furthest feasible positions from each other. Minimal differentiation results, on the other hand, have to be carefully reconsidered, as we will show.

Our model of spatial competition with restricted strategies does not only apply to companies in a market but also to parties in a political competition. We do consider this application, although the terminology and arguments in the note are adapted to firms and consumers.

2 A Model with Restricted Strategies

We define a game between two players L and R, who compete in a one-dimensional market. The players simultaneously choose a product position in order to maximize their payoff.

There is a continuum of positions $X = [0, 1]$. Positions are ordered by the relation $\leq$ such that we can refer to distances ($|x - y|$), being closer ($|x - y| < |x - z|$) or being in between ($x < y < z$). A product position is interpreted as the mixture of all the criteria a consumer takes into account when deciding which product to buy.[2] We consider a unit mass of consumers who are distributed on X according to the cumulative distribution function $F : X \to \mathbb{R}_+$ with full support in the corresponding density function: $f(x) > 0$ for any $x \in X$. Let $q := F^{-1}(\frac{1}{2}) \in X$ be the *median* of the distribution which is also referred to as the 'median consumer' (Waldfogel, 2008, p. 568).[3] If the distribution satisfies symmetry, i.e. $F(x) = 1 - F(1 - x)$, then the median is $q = \frac{1}{2}$.

Consumers are distinguished from firms, or players, whom we denote by L

[2]Product positions are basically a perception or as Desarbo and Rao (1986) put it: "many combinations of product features and other marketing mix attributes may map into a specific perceptual product position."

[3]Depending on the application the much more common term 'median voter' can be used synonymously.

and R.[4] Because of technological or other constraints, players' strategies are restricted in the sense of Samuelson (1984), i.e. a strategy (product position) s^P for $P = L, R$, cannot be chosen freely from all the product positions in X but only from some compact convex subset.

Assumption 1 *We define the set of feasible product positions for each player as an interval within X, i.e. a player $P = L, R$ has the strategy set $S^P = [\underline{s}^P, \bar{s}^P]$.*

Let $S = S^L \times S^R$ denote the strategy space and let s^{-P} (S^{-P}) denote the strategy (set) of the player that is not P. Without loss of generality we let $\underline{s}^L \leq \underline{s}^R$.

We assume that the firms' costs are independent of the chosen product position and normalized to zero. Moreover, we abstract from price competition because in it would lead to trivial results (under the standard set-up), as discussed above.[5] Thus, profit maximization in this model equals the maximization of market share. Consumers are assumed to buy one unit at the firm that is closer to them. Let $\hat{x} := \frac{s^L+s^R}{2}$ be the position of an indifferent consumer. Then the players' payoffs for a strategy profile $s^L < s^R$ are as follows:

$$\begin{aligned} \pi^L(s) &= F(\hat{x}), \\ \pi^R(s) &= 1 - F(\hat{x}) \end{aligned}$$

and vice versa for $s^L > s^R$. For two equal positions $s^L = s^R$ we assume that the two firms split the market equally, i.e. $\pi^L(s) = \pi^R(s) = \frac{1}{2}$. We solve our model with the standard notion of Nash equilibrium. In one case there will be an open set problem very similar to Bertrand competition with constant but unequal marginal costs.[6] We tackle this issue by studying epsilon-equilibria as done by Radner (1980) and Dixon (1987).[7]

Definition 1 (cf. Dixon, 1987) *Let $\epsilon \geq 0$. $s \in S$ is a ϵ-equilibrium if for $P = L, R$, there does not exist a strategy $\tilde{s}^P \in S^P$ such that*

$$\pi^P(\tilde{s}^P, s^{-P}) - \pi^P(s) > \epsilon. \tag{1}$$

[4] As a convention we use the male form for consumers and the female form for players.

[5] Prices can be considered as exogenously fixed and equal.

[6] In Bertrand competition there does not exist a smallest price difference to undercut an opponent. In our model there will not exist a smallest unit of product differentiation.

[7] Other ways to handle the open set issue would not lead to qualitatively different results.

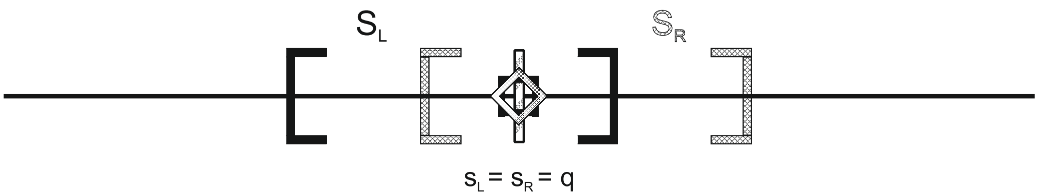

Figure 1: In Case (I), i.e. $q \in S^L \cap S^R$, both players choose the median as their product position: $s^L = s^R = q$. This result equals the result of the classic Hotelling-Downs model.

Clearly, for sufficiently large ϵ every strategy profile is an epsilon-equilibrium. Thus, this notion is sensible for small epsilon only. In particular, for $\epsilon = 0$ it coincides with the notion of Nash equilibrium.

3 Equilibrium Positions

To analyze our model we distinguish between three cases, which are distinct and exhaustive.

3.1 Case (I)

In Case (I) we analyze the model when the feasible strategies overlap and the median is part of this intersection, i.e. $q \in S^L \cap S^R$. An example for Case (I) is $S^L = S^R = X$, the classic model of unrestricted strategies. The result of Black (1948) on majority voting implies that the equilibrium outcome in this location game is minimal differentiation on the median as illustrated in Figure 1. Because this is an equilibrium in the classic model of unrestricted strategies, it must also be an equilibrium in our model, since the set of possible deviations has been reduced. Our first result establishes that there are no additional equilibria.

Proposition 1 *If $q \in S^L \cap S^R$, the unique Nash-equilibrium is that both players choose the median as their product position, i.e. $s^L = s^R = q$.*

Proof. By the definition of the median q we have $F(q) = 1 - F(q) = \frac{1}{2}$. Suppose s is such that no player chooses the median q. Let P be a player with $\pi^P(s) \leq \pi^{-P}(s)$. Deviating to $\tilde{s}^P = q$ leads to a payoff of $\pi^P(q, s^{-P}) > \frac{1}{2}$ while $\pi^P(s) \leq \frac{1}{2}$. Suppose s is such that exactly one player, P, chooses the median. Then $\pi^{-P}(s) < \frac{1}{2}$, while $\pi^P(q,q) = \frac{1}{2}$. Finally, for $s^L = s^R = q$ any deviation leads to a lower payoff. ■

In Case (I) we get the classic outcome of minimal differentiation (on the median), despite strategy restrictions. Minimal differentiation is known to be inefficient since the work of Hotelling (1929). However, under the constraint of minimal differentiation, i.e. if both have to locate on the same position, locating on the median is constrained efficient: The median is the position that minimizes the sum of distances to all consumers. In that sense the median incorporates the preferences of the consumers. Comparative-static changes in preferences or changes in the sample of consumers would affect the location of the median and thus the product positions taken by players, as long as the condition $q \in S^L \cap S^R$ still holds, i.e. as long as we are still in Case (I).

3.2 Case (II)

In Case (II) we examine the situation where the feasible strategies S^L and S^R overlap and the median is not part of the intersection, i.e. $S^L \cap S^R \neq \emptyset$ and $q \notin S^L \cap S^R$.

Definition 2 (More-Central Player) *If there is a player who can choose a product position which is strictly closer to the median, she is called the more-central player (MC); her opponent is called the less-central player (LC).*

In Case (II) we have to distinguish between two different subcases: In Subcase (II-a) a more-central player does not exist and in Subcase (II-b) a more-central player exists. Subcase (II-a) occurs if the players' feasible product positions share a boundary and the median is beyond this boundary, e.g. $q < \underline{s}^L = \underline{s}^R$. This case occurs when there is a technological or legal restriction that hinders both companies from offering at the center of the market. In Subcase (II-b) the median is not in the strategy set of the less-central player, e.g. $q < \underline{s}^{LC}$, while it might, but need not, be in the strategy set of the more-central player, i.e. $\underline{s}^{MC} < \underline{s}^{LC}$. A particular example for this subcase is given if one player is unrestricted, i.e. $S^P = X$, while the other player is restricted such that she cannot choose the median $q \notin S^{-P}$.

Figure 2 illustrates the two subcases and the corresponding equilibrium analysis. The strategic situation resembles Bertrand competition with constant marginal costs. In the Bertrand model players can improve by undercutting the opponent's prices for any price above the marginal costs; here players can improve by locating closer to the median for any strategy away from the restriction boundary. In the Bertrand model we have a unique Nash equilibrium for equal marginal costs and only epsilon-equilibria under unequal marginal costs. Here, we find a fully analogous result in Subcase

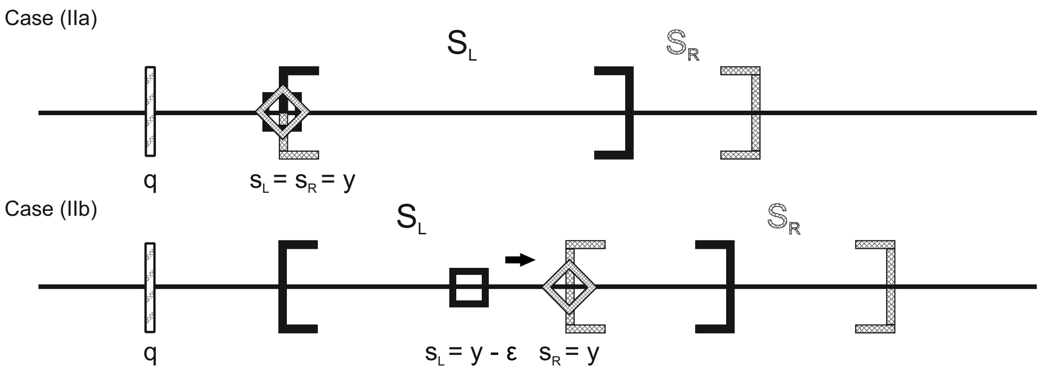

Figure 2: (IIa) if a more-central player does not exist, i.e. none of the players can choose a product position closer to the median than her opponent, both players take the same product position on the edge of their restriction toward the median. (IIb) if a more-central player exists (here: L), the less-central player (here: R) locates on the edge of her restriction toward the median, while the more-central player approaches this position from the side of the median (here: from the left).

(IIa) and an analogous, but unique type of epsilon-equilibrium in Subcase (IIb), as Proposition 2 shows.

Proposition 2 *Suppose $S^L \cap S^R \neq \emptyset$ and $q \notin S^L \cap S^R$.*

(i) If a more-central player does not exist, the unique Nash equilibrium is such that both players take a product position on the edge of their restriction toward the median, i.e. if $q < \underline{s}^L = \underline{s}^R$, then $s = (\underline{s}^L, \underline{s}^R)$ is the Nash equilibrium; if $q > \bar{s}^L = \bar{s}^R$, then $s = (\bar{s}^L, \bar{s}^R)$ is the Nash equilibrium.

(ii) If a more-central player exists, then for any $\epsilon > 0$, there is an ϵ-equilibrium such that the less-central player takes the product position on the edge of her restriction in direction of the median and the more-central player locates closely next to it in direction of the median, i.e. if $\underline{s}^{MC} < \underline{s}^{LC}$ and $q < \underline{s}^{LC}$, then $s^{LC} = \underline{s}^{LC}$ and $s^{MC} = \underline{s}^{LC} - \varepsilon$; if $\bar{s}^P > \bar{s}^{-P}$ and $q > \bar{s}^{-P}$, then $s^{-P} = \bar{s}^{-P}$ and $s^P = \bar{s}^{-P} + \varepsilon$, for some $\varepsilon > 0$.

Proof. (i) Let y be the feasible position which is closest to the median. No player can improve by relocating from $s^L = s^R = y$ since relocation has to be in the opposite direction of the median. Now let $\hat{s} \neq s$ be a strategy profile where not both players locate on y. Take a player P such that $\pi^P(\hat{s}) \leq \pi^{-P}(\hat{s})$ and relocate her to $\tilde{s}^P = y$. This is a strict improvement (irrespective of whether the other player $-P$ is also located on y).

(ii) Assume $\underline{s}^L < \underline{s}^R$ and $q < \underline{s}^R$ such that L is the more-central player. Suppose L locates within the feasible set of R, i.e. $s^L \in S^R$. If $\pi^L(s) \leq \frac{1}{2}$, L can improve by moving closer to the median; if $\pi^L(s) > \frac{1}{2}$, R can improve by choosing the same position as L. Thus, in equilibrium $s^L < \underline{s}^R$. Suppose $s^R \neq \underline{s}^R$, then R can improve by moving to $\underline{s}^R$. Thus, in equilibrium $s^R = \underline{s}^R$. The maximal possible payoff for L is then bounded from above by $F(\underline{s}^R)$. Choosing $s^L = \underline{s}^R - \varepsilon$ leads to a payoff of $F(\underline{s}^R - \frac{\varepsilon}{2})$. For small ε, any difference ϵ between these payoffs can be undercut. Thus, $(\underline{s}^R - \varepsilon, \underline{s}^R)$ is an ϵ-equilibrium for sufficiently small ε. If $\bar{s}^P > \bar{s}^{-P}$ and $q > \bar{s}^{-P}$, then the proof is in full analogy to above. ∎

In Subcase (IIa) both players locate as closely as possible to the median, which is qualitatively similar to the standard result of Case (I). In Subcase (IIb) this only holds for the less-central player. For the more-central player, however, it is possible that the median is a feasible product position, but this central position will never be chosen. Instead, the more-central player minimally differentiates to the opponent's strategy restriction. The practical implication of this scenario is that products are sold which do not fit to the median consumer's taste, although this would be a feasible position for one of two firms. This is highly inefficient, as we will see in Subsection 3.4.

3.3 Case (III)

In Case (III) we examine the model with restricted strategies when the feasible strategies do not overlap, i.e. $S^L \cap S^R = \emptyset$.

Proposition 3 *If $S^L \cap S^R = \emptyset$, then $s^L = \bar{s}^L$ and $s^R = \underline{s}^R$ are strictly dominant strategies.*

Proof. For any strategy profile with $s^L < \bar{s}^L$, changing to $\tilde{s}^L = \bar{s}^L$ is a strict improvement because it shifts the indifferent consumer $\hat{x}$ to the right. Analogously, for player R. ∎

Proposition 3 shows that in the unique Nash equilibrium both players locate at the edge of their strategy set in direction of the opponent as illustrated in Figure 3. While the result is trivial, it has a remarkable implication.

Unlike Case (I) and Case (II), equilibrium positions in Case (III) do not depend on the median. Even more generally, consumers preferences do not at all affect the position of the players, as noted in Remark 1.

Remark 1 *Equilibrium positions in Case (III) are* in*dependent of consumers' preferences (represented by F or q).*

Figure 3: Equilibrium positions in Case (III), i.e. if $S^L \cap S^R = \emptyset$. Both players locate as closely as possible to the opponent's strategy set. Equilibrium strategies are independent of the position of the median q and thus independent of consumers' preferences.

Thus, companies might offer products which do not respect consumers' needs. For instance, it is possible, as in Case (II), that the median position is available for one of the two players (such as in Figure 3), but she does not choose it.

3.4 Welfare Implications

In our model producers' surplus is constant because firms play a zero-sum-game for market share. Therefore, welfare effects can be discussed solely on the basis of the consumers' surplus. Welfare is now measured by the total transportations costs $TC(s^L, s^R)$, which is the sum of distances of each consumer to a closest player.[8] Formally, for $s^L \leq s^R$,

$$TC(s^L, s^R) := \int_0^{\hat{x}} |s^L - x| f(x)dx + \int_{\hat{x}}^1 |s^R - x| f(x)dx. \tag{2}$$

To ease the exposition, let us assume that consumers are uniformly distributed, i.e. $f(x) = 1$ for all x. Then (2) simplifies to $TC(s^L, s^R) = \frac{3}{4}(s^L)^2 + \frac{3}{4}(s^R)^2 - \frac{1}{2}s^L s^R - s^R + \frac{1}{2}$. The global optimum is attained for $s^L = \frac{1}{4}$ and $s^R = \frac{3}{4}$, which yields $TC(\frac{1}{4}, \frac{3}{4}) = \frac{1}{8}$. We will use this social optimum as a benchmark to assess efficiency of the three cases. In particular, to quantify inefficiency we report the so-called Price of Anarchy (henceforth: PoA), which is attained by dividing the "worst" equilibrium, i.e. the equilibrium strategy profile with maximal transportation costs, by the globally minimal

[8] In models where transportation costs are quadratic in distance it is common to use the sum of squared distances (e.g. d'Aspremont *et al.*, 1979; Meagher and Zauner, 2004; Król, 2012). Since our model is more general in this respect, we use linear transportation costs to measure efficiency. The choice of efficiency criterion, however, is not crucial for our discussion.

transportation costs (Koutsoupias and Papadimitriou, 2009).[9]

In Case (I) both players locate on the median $q(=\frac{1}{2})$. This yields total transportation costs of $TC(q,q)=\frac{1}{4}$ and, hence, a PoA of 2.

In Subcase (IIa) both players choose the same location, say y, which was their common boundary in direction of the median. Since in Subcase (IIb) players differentiate only by some ε, which is vanishingly small for a notion of epsilon-equilibrium close to the notion of Nash equilibrium, we can approximate welfare properties of this subcase also by equal positions $s^L = s^R = y$. In an extreme example strategies are restricted to be at an endpoint of the line, e.g. $S^L = S^R = \{0\}$, which yields the globally maximal transportation costs of $\frac{1}{2}$. More generally, transportation costs are U-shaped in the common location y and the closer y to the median, the lower the transportation costs. Thus, in Case (II) the transportation costs lie in the interval $(\frac{1}{4}, \frac{1}{2}]$, which yields a PoA in the interval $(2, 4]$. The intervals are open on one side because the median q cannot coincide with the common location y when we are in Case (II).

The unique equilibrium in Case (III) is that players choose $s^L = \bar{s}^L$ and $s^R = \underline{s}^R$. By coincidence this choice may be socially optimal (which happens when $\bar{s}^L = \frac{1}{4}$ and $\underline{s}^R = \frac{3}{4}$), but there are also examples with an almost maximal transportation costs, e.g. if $S^L = \{0\}$ and $S^R = \{0.001\}$. Therefore, in Case (III) the transportation costs lie in the interval $[\frac{1}{8}, \frac{1}{2})$ with a corresponding PoA in the interval $[1, 4)$. If we focus, however, on situations where the median is between the two strategy sets, i.e. $\bar{s}^L < q < \underline{s}^R$, e.g. because the players' restrictions are symmetric with respect to $q = \frac{1}{2}$, then inefficiency is bounded. The worst case example is then $S^L = \{0\}$ and $S^L = \{1\}$, which yields transportation costs of $\frac{1}{4}$ and a PoA of 2. Similarly, if we suppose that the difference between the two strategy sets are at least one third, i.e. $\underline{s}^R - \bar{s}^L \geq \frac{1}{3}$, the same conclusion holds. Table 1 summarizes the welfare properties of the three cases, where Case (III+) stands for Case (III) under the qualification that at least one of these two properties, either the median is between the restrictions or the restrictions differ by at least one third, holds. We observe that the three cases can be ranked according to the welfare they induce in equilibrium as follows:

$$\text{Case (III+)} \succeq \text{Case (I)} \succeq \text{Case (II)}.$$

This holds with respect to the cardinal criterion of total transportation costs. Considering the ordinal notion of Pareto efficiency, we come to a similar con-

[9]Likewise the Price of Stability refers to the "best" equilibrium in relation to the social optimum. There is no need to use both measures in our model since we have uniqueness of equilibria, up to small differences between a multitude of epsilon-equilibria in Subcase (IIb).

CASE	Transportation Costs	Price of Anarchy	Market Share
Case (I)	$\frac{1}{4}$	2	50%
Case (IIa)	$(\frac{1}{4}, \frac{1}{2}]$	$(2, 4]$	50%
Case (IIb)	$(\frac{1}{4}, \frac{1}{2}]$	$(2, 4]$	$(50\% - 100\%)$
Case (III)	$[\frac{1}{8}, \frac{1}{2})$	$[1, 4)$	$[50\% - 100\%)$
Case (III+)	$[\frac{1}{8}, \frac{1}{4}]$	$[1, 2]$	$[50\% - 75\%]$

Table 1: Summary of welfare properties of equilibria for different cases under the assumption of a uniform distribution of consumers. Market Share stands for the equilibrium market share of the larger competitor, which measures inequality between firms.

clusion. Equilibrium positions in Case (I) and Case (II) are not Pareto efficient, because one of the two minimally differentiated players could be relocated to shorten distances of some consumers without harming all others. On the other hand, equilibrium positions in Case (III) are Pareto efficient. We can summarize the welfare implications of strategy restrictions by comparing unrestricted strategies $S^P = X$ with restricted strategies $S^P \subset X$. Interestingly, unrestricted competition where all positions are feasible (which is a special example of Case (I)) leads to a worse outcome than Case (III+) that implies that restrictions are strong enough to exclude some positions from the feasible set of both firms. Moreover, if only one player is restricted, while the other can cover the entire spectrum (which is a special example of Case (II)), the outcome is even worse. Thus, if a social planner were in the position impose a restrictions of one firm, it should only do so if it can impose a restriction on the other firm as well.

Let us finally discuss the consequences for firms. As mentioned above, for firms there is no issue of inefficiency in our model, but we can discuss their inequality. Market shares are equal in equilibrium in Case (I) and in Subcase (IIa). In Subcase (IIb), the more-central player receives a larger proportion of the consumers. Finally, in Case (III) L receives a higher payoff if and only if $\bar{s}^L$ is closer to the median than $\underline{s}^R$. These observations are also indicated in Table 1, where the last column reports the equilibrium market share of the largest competitor. Moreover, it is easy to show that a player P receives at least half of all consumers

(a) if her strategy set includes the median, i.e. $q \in S^P$, or

(b) if her strategy set is a superset of the opponent's, i.e. $S^P \supseteq S^{-P}$.

In that sense, a strategy set, e.g. a technology, is particularly valuable if (a) it allows to serve the center of the market and if (b) it is more flexible than the opponent's. On the other hand, if strategy restrictions are not exogenous, flexibility can be assumed to be costly. Next, we analyze the optimal choice of strategy restrictions.

4 Analysis of the Long Run: Choice of Restrictions

In contrast to short-term, long-term competition provides other opportunities and the rules of competition can change (Porter, 1985). While in the short term restrictions must be considered as fixed, in the long run companies can influence their feasible positions. We consider a simple variant of the standard market entry game. Positions X and consumers are as specified above. There are now two periods $t = 0, 1$. In period 0 a firm F1 is a monopolist in the market. In period $t = 1$ either F1 stays monopolist or a second firm F2 enters. The sequence of actions is as follows. (i) F1 chooses a set of feasible strategies S^1 and some initial position $s_0^1 \in S^1$. (ii) F2 chooses whether to enter the market and if it enters it chooses some position $s^2 \in X$.[10] (iii) F1 chooses a position $s^1 \in S^1$ to compete against F2. In a monopoly situation F1 serves all consumers, in a duopoly with $s^1 \neq s^2$ consumers are split as in the static model. If $s^1 = s^2$, we assume that all consumers stay at the incumbent and do not switch to the entrant.[11] One interpretation for this assumption is that consumers face small switching costs which cause inertia. Let $\pi : [0, 1] \to \mathbb{R}_+$ be a continuously increasing function that assigns a profit to any mass of consumers. Moreover, let $0 < f^{entry} < \pi(\frac{1}{2})$ be the fixed costs of market entry. For F1, let $C : [0, 1] \to \mathbb{R}_+$ be an increasing function that represents the costs of flexibility. We assume that the larger the range $[\underline{s}^1, \bar{s}^1]$, the higher these costs. Moreover, let $\delta \in (0, 1]$ be F1's discount factor. The payoffs of F1 and F2 are then

$$\Pi^1, \Pi^2 = \begin{cases} \pi(1) - C(\bar{s}^1 - \underline{s}^1) + \delta\pi(1 - F(\hat{x})), \;\; \pi(F(\hat{x})) - f^{entry}, & \text{if } s^2 < s^1 \\ \pi(1) - C(\bar{s}^1 - \underline{s}^1) + \delta\pi(F(\hat{x})), \;\; \pi(1 - F(\hat{x})) - f^{entry}, & \text{if } s^2 > s^1 \\ \pi(1) - C(\bar{s}^1 - \underline{s}^1) + \delta\pi(1), \;\; \pi(0) - f^{entry}, & \text{if } s^2 = s^1 \\ \pi(1) - C(\bar{s}^1 - \underline{s}^1) + \delta\pi(1), \;\; \pi(0), & \text{if no-entry.} \end{cases}$$

[10] After observing F1's first move, there would be no incentive to build a strategy set that consists of more than one position.

[11] The result for the convention that the two firms split the market equally will be trivially that both firms choose the median. This observation stays true in the model variation, where firms first simultaneously choose feasible strategies and then simultaneously choose positions within their feasible set.

We now derive a subgame perfect Nash equilibrium (SPNE) by backward induction. Because of the open set issue this will be a "perfect epsilon-equilibrium" (Radner, 1980).

(iii) If F2 does not enter, then the choice $s^1 \in S^1$ is arbitrary. If F2 enters and $s^2 \in S^1$, then $s^1 = s^2$ is profit maximizing (because then F1 receives all consumers). If F2 enters and $s^2 \notin S^1$, then $s^1 = \underline{s}^1$ when $s^2 < \underline{s}^1$ and $s^1 = \bar{s}^1$ when $s^2 > \bar{s}^1$ is profit maximizing for F1.

(ii) Given optimal behavior of F1 in decision (iii), we receive the following payoffs for different decisions of F2 at stage (ii):

$$\Pi^2 = \begin{cases} \pi(0) - f^{entry}, & \text{if } s^2 \in S^1 \\ \pi(F(\underline{s}^1 - \frac{\varepsilon}{2})) - f^{entry}, & \text{if } s^2 = \underline{s}^1 - \varepsilon \\ \pi(1 - F(\bar{s}^1 + \frac{\varepsilon}{2})) - f^{entry}, & \text{if } s^2 = \bar{s}^1 + \varepsilon \\ \pi(0), & \text{if F2 does not enter} \end{cases}$$

for $\varepsilon > 0$. Choosing $s^2 \in S^1$ is strictly dominated by not entering. In the two central cases, the payoff of F2 is decreasing in ε. Thus, we have an open set problem as in Case (IIb) of the short-term analysis. The supremum here is $F(\underline{s}^1)$ respectively $1 - F(\bar{s}^1)$ and it can be approached by letting ε shrink. Therefore F2 enters if

$$\pi(\max\{F(\underline{s}^1), 1 - F(\bar{s}^1)\}) > f^{entry} \tag{3}$$

and chooses a sufficiently small ε. Otherwise, i.e. if Condition (3) does not hold, F2 does not enter.

(i) To derive the optimal behavior of F1 in stage (i), we distinguish between the best entry deterring and the best entry admitting choice. Anticipating the behavior in stage (ii) and (iii) a strategy set S^1 is entry deterring if $\pi(F(\underline{s}^1)) \leq f^{entry}$ and $\pi(1 - F(\bar{s}^1)) \leq f^{entry}$. Let $\underline{y} := F^{-1}(\pi^{-1}(f^{entry}))$, i.e. the rightmost position that still does not allow for profitable entry to the left and, similarly, $\bar{y} := F^{-1}(1 - \pi^{-1}(f^{entry}))$.[12] Then the best entry deterring choice is $S^1 = [\underline{y}, \bar{y}]$. Note that $\underline{y}$ is increasing in f^{entry}, i.e. the larger the entry costs, the smaller the necessary flexibility to deter entry.

The best choice of S^1 given that F2 enters is the solution to the following maximization problem:

$$\max_{\underline{s}^1, \bar{s}^1} \pi(1) - C(\bar{s}^1 - \underline{s}^1) + \delta\pi(1 - \max\{F(\underline{s}^1), 1 - F(\bar{s}^1)\}). \tag{4}$$

Since any choice such that $F(\underline{s}^1) \neq 1 - F(\bar{s}^1)$ is a "waste" of flexibility costs, we have in equilibrium $F(\underline{s}^1) = 1 - F(\bar{s}^1)$. Thus, we can substitute

[12] F and π are strictly increasing continuous functions such that they can be inverted.

$\underline{s}^1 = F^{-1}(1 - F(\bar{s}^1))$ to rewrite the maximization problem in dependence of one variable only:

$$\max_{\bar{s}^1 \in [q, \bar{y}]} \pi(1) - C(\bar{s}^1 - F^{-1}(1 - F(\bar{s}^1))) + \delta\pi(F(\bar{s}^1)) \tag{5}$$

A choice $\bar{s}^1 > \bar{y}$ is excluded by assumption because it deters entry and the last profit is the simplification of $\pi(1 - (1 - F(\bar{s}^1))$.

This maximization problem (5) incorporates the trade-off between leaving few consumers for a potential entrant (large $\bar{s}^1$) and saving flexibility costs (small $\bar{s}^1$). The solution to this problem depends on the specifications of the cost function C, of the entry costs f^{entry}, of the payoff function π, and of the distribution of consumers F, but it certainly exists because we maximize a continuous function over a compact set. Let $\bar{z}$ be a solution to this problem (5), be it an interior solution ($\bar{z} \in (q, \bar{y})$) or a boundary solution ($\bar{z} = q$ or $\bar{z} = \bar{y}$). Let $\underline{z} := F^{-1}(1 - F(\bar{z}))$. Then F1's profit maximizing behavior under entry and no-entry of F2 leads to the following payoffs:

$$\Pi^1 = \begin{cases} \pi(1) - C(\bar{y} - \underline{y}) + \delta\pi(1), & \text{if } S^1 = [\underline{y}, \bar{y}] \\ \pi(1) - C(\bar{z} - \underline{z}) + \delta\pi(F(\bar{z})), & \text{if } S^1 = [\underline{z}, \bar{z}] \end{cases}$$

The specific functional forms determine which choice leads to higher payoff and, hence, F1's choice in stage (i). Inspecting the two equilibrium payoffs above reveals that entry deterrence becomes relatively more attractive for lower costs of flexibility, for higher costs of entry, and for a larger discount factor. In Example 1 we illustrate how these model parameters determine the equilibrium path.

From the backward induction exercise we learn first of all that there always exists a subgame perfect epsilon-equilibrium. Moreover, there are two types of these equilibria, one entry admitting one entry deterring, which both satisfy the following two properties.

(a) $q \in S^1 \subseteq [\underline{y}, \bar{y}]$, i.e. F1 chooses a feasible set at the center of the market within certain boundaries and

(b) $F(\underline{s}^1) = 1 - F(\bar{s}^1)$, i.e. the 'niches' left for F2 at both sides of the center are of equal size.

In the entry deterring equilibrium, (i) F1 chooses $S^1 = [\underline{y}, \bar{y}]$, (ii) F2 does not enter, and (iii) F1's final position s^1 is arbitrary within S^1 because it acts as monopolist. An entry deterrent F1 gains $\pi(1) - C(\bar{y} - \underline{y}) + \delta\pi(1)$. Thus, it has the cost of flexibility $C(\bar{y} - \underline{y})$ to keep a threat to potential entrants. This is similar to a threat of a price war, but this threat is credible because after investments into flexibility have been made, a 'minimal differentiation war' is costless in our model. Welfare depends on the exact location of $s^1 \in S^1$ since the closer s^1 to the median, the smaller the total transportation costs. Thus, the size of the feasible set S^1 not only determines the cost of flexibility, but also provides an upper bound for the transportation costs. Since the size of F1's restriction is increasing in F2's costs of entry f^{entry}, entry barriers might even be considered as welfare enhancing.[13] Similarly, low marginal costs of flexibility increase the set of feasible positions S^1 and thus relax the upper bound of transportation costs. By property (a) this boundary for total transportation costs also applies to the entry admitting equilibrium.

The entry admitting equilibrium path is as follows: (i) F1 chooses $S^1 = [\underline{z}, \bar{z}]$ such that $\underline{z}$ solves (5), i.e. it optimizes the trade-off between low costs of flexibility and a large market share; (ii) F2 enters and chooses an adjacent position to F1's restriction, i.e. $s^2 = \underline{s}^1 - \varepsilon$, respectively $s^2 = \bar{s}^1 + \varepsilon$; and (iii) F1 reacts with choosing its restriction adjacent to s^2, i.e. $s^1 = \underline{z}$ or $s^1 = \bar{z}$. Observe that the outcome of this dynamic model corresponds to Case (IIb) of the static analysis, where F1 is in the role of the more-central player. We discussed in Subsection 3.4 that this is the case with potentially high inequality and low welfare. In given examples, the specific inequality and the total transportation costs are determined by the size of the interval S^1 such that we get the following comparative static effects. Both equality of firms' payoffs and welfare are increasing in F1's marginal costs of flexibility (called c in Example 1 below) and in F1's discount factor δ. In the worst situation, F1 values the second period highly ($\delta = 1$), while flexibility is relatively cheap. Then it chooses a large feasible set S^1 with only small niches left for F2 such that market shares are highly unequal, while consumers' transportation costs are large because two similar products away from the center of the market are offered. Of course, this can only be an entry admitting equilibrium if F2's costs of entry f^{entry} are sufficiently low.

To study how costs of market entry and other model parameters determine which equilibrium is played and to illustrate further comparative static effects, we use a specific example for which an explicit solution can be easily obtained.

Example 1 *Consider the special case of uniform distribution of consumers, i.e. $F(x) = x$, quadratic costs of flexibility, i.e. $C(r) = cr^2$ with cost parameter c, and linear payoff function, i.e. $\pi(a) = a$. From* (3) *we get that F2 enters if* $\max\{\underline{s}^1, 1 - \bar{s}^1\} > f^{entry}$. *Moreover, let $f^{entry} < \frac{3}{8}$, which in this case (π is the identity function) can be interpreted as the market share that*

[13]The intuition is that low costs of entry lead to costly investments into flexibility that allow the incumbent to offer products which are not close to the center of the market.

is necessary to make market entry profitable. F1 can optimally deter entry by choosing $\underline{s}^1 = \underline{y} = f^{entry}$ *and* $\bar{s}^1 = \bar{y} = 1 - f^{entry}$*. The optimal choice of F1 given that F2 enters is the solution to the maximization problem (cf.* (4)*), which simplifies to*

$$\max_{\bar{s}^1 \in [\frac{1}{2}, 1-f^{entry}]} 1 - c(2\bar{s}^1 - 1)^2 + \delta \bar{s}^1. \tag{6}$$

Analogous to Eq. (5)*, the main idea of the simplification is that best actions satisfy here* $\underline{s}^1 = 1 - \bar{s}^1$*. If* $f^{entry} > \frac{1}{2} - \frac{\delta}{8c}$*, then we have the boundary solution* $\bar{z} = \bar{y} = 1 - f^{entry}$ *and* $\underline{z} = \underline{y} = f^{entry}$*. In that case entry admission is never profitable and we have the entry deterring equilibrium. On the other hand, if* $f^{entry} \leq \frac{\delta}{8c}$*, then the unique solution to this maximization problem is* $\bar{z} = \frac{1}{2} + \frac{\delta}{8k}$*. In that case we have to compare the payoff of F1 under the optimal entry admitting choice* $S^1 = [\frac{1}{2} - \frac{\delta}{8k}, \frac{1}{2} + \frac{\delta}{8k}]$ *with the payoff of the optimal choice that deters entry* $S^1 = [f^{entry}, 1 - f^{entry}]$*. Low enough entry costs* f^{entry}*, high marginal costs of flexibility* c*, as well as low enough valuation of the future* δ*, make the entry admitting choice of restrictions more profitable than entry deterrence.*

For instance, for $c = 1$ *and* $\delta = 0.8$*, F1 prefers to admit entry of F2 if* $f^{entry} < \frac{1}{5}$*, i.e. if the required market share to make entry profitable is below* 20%*. In that case we get the following equilibrium path: (i) F1 chooses* $S^1 = [0.4, 0.6]$ *and* $s_0 \in S^1$ *arbitrary, e.g.* $s_0 = 0.5 = q$*. F2 enters with strategy* $s^2 = 0.4 - \varepsilon$ *(or with* $s^2 = 0.6 + \varepsilon$*) for some small* $\varepsilon > 0$*. F1 reacts with* $s^1 = 0.4$ *(respectively,* $s^1 = 0.6$*). The market share of F1 is approximately* 60%*, while F2 receives approximately* 40%*. The outcome is inefficient for two reasons. First costly investments into flexibility are not justified by some welfare benefit. Second, F1 locates at the position within* S^1 *that actually maximizes total transportation costs.*

There is an alternative interpretation for the model set-up of this section. Consider the incumbent's investment into flexibility as investment into patents that protect its initial product s^1_0. Specifically, the choice $S^1 = [\underline{s}^1, \bar{s}^1]$ can be interpreted as restricting the feasible strategies of a potential entrant, i.e. F2's strategy set is restricted to $X \setminus S^1 = [0, \underline{s}^1) \cup (\bar{s}^1, 1]$. The model results in an entry deterring or an entry admitting equilibrium as described above.

5 Discussion

We have introduced restrictions of feasible positions for two players who compete in a one-dimensional market. The most striking insight is probably that in the case where feasible positions do not overlap (Case III) short-run equilibrium choices are fully independent of consumers' preferences. Thus, in that case firms are predicted to ignore the consumers and base their product position on the competitor's strategy only. This analysis naturally applies to short-term competition where product positions underlie fixed restrictions. In a longer term companies can invest into changes of their feasible positions. In a very simple model of endogenous restrictions under market entry we show that there is an optimal choice of flexibility for an incumbent firm. The practical prediction of this model is that an incumbent adapts a position at the center of the market and keeps sufficient flexibility to quickly react to a new entrant by changing its product position closely to the entrant's. If costs of entry are large, then the flexibility of the incumbent prevents the other firm from entering at all. Otherwise, the two firms choose highly similar product specifications, which are far from an ideal product in the eyes of most of the consumers, even though such an ideal product would be feasible for the incumbent. This outcome equals the outcome of one case in the short-term analysis (Case IIb), which leads to highest transportation costs and thus to lowest welfare.

As many models of spatial competition, our model also applies to political competition. Let us now change the interpretation from firms and consumers to political candidates and voters. The left-right spectrum stands for possible political platforms from left-wing to right-wing positions. Political candidates also have a restricted set of feasible positions (Samuelson, 1984) for two reasons. First, a candidate has to 'maintain a loyalty' to its political party, whereas parties only cover some part of the political spectrum, they need sponsors and supporters, and are relatively ideologically immobile (Downs, 1957a). Second, the own history of public perception restricts the political positions that a candidate can credibly represent. A similar set of credibility restrictions is based on personal characteristics (Samuelson, 1984), e.g. a young candidate cannot credibly present himself as highly experienced.

In the classic model the main result is often summarized by the role of the so-called median voter.[14] At least two forms of the median voter theorem are popular (Congleton, 2002). The weak form of the median voter theorem claims that the median voter always casts his vote for the policy that is

[14] A median voter is a voter whose preference peak coincides with the median.

adopted. As it is easy to show, this is still true in the model with restricted strategies. The strong form of the median voter theorem says that the median voter always gets his most preferred policy. With restricted strategies this is obviously not possible if $q \notin S^L \cup S^R$. But even if the median position is available, it is not always chosen such as in Case (IIb) or in Case (III). Under unrestricted strategies this could not be an equilibrium because each candidate could beneficially deviate toward the median voter. If one candidate is incapable of such a move, however, the other candidate lacks incentive to do so. Thus, restrictions of political platforms do severely affect political campaigns and the outcome of two-party competition.

Our model uses several standard simplifications which have been relaxed for the classic model and are also worth studying under restricted strategies. First, the assumption of perfectly inelastic demand could be relaxed such as in Anderson and Glomm (1992) and George and Waldfogel (2006). Second, we have studied a one-dimensional market that summarizes all relevant product characteristics including the price.[15] A natural extension is to consider multi-dimensional product differentiation. In such a model Irmen and Thisse (1998) find that minimal differentiation is prevalent in all dimensions but one. While in each dimension with minimal differentiation both firms cluster on the median, this result would not generalize to restricted strategies. Moreover, larger classes of consumers', respectively voters', preferences can be considered. Our results do directly extend to single-peaked preferences on tree graphs (Demange, 1982), which is a much more general class of preferences. However, the extension to graphs that include cycles, e.g. grids or hypercubes, as they have been studied in Nehring and Puppe (2007) or Buechel and Roehl (2014), is an open problem. Finally, we have restricted attention to the classic case of two players. While for an exogenous number of players, the results under unconstrained competition resemble the two player case ("minimum clustering" Eaton and Lipsey, 1975; Buechel and Roehl, 2014), this is not true for an endogenous number of players (Prescott and Visscher, 1977; Loertscher and Muehlheusser, 2011). Extending our model into these directions is a worthwhile endeavor, but it clearly exceeds

[15] In many works, the price is used as an independent dimension which can be chosen after a product position (d'Aspremont *et al.*, 1979; de Palma, 1985; Meagher and Zauner, 2004; Król, 2012). Since the introduction of prices leads to non-existence of equilibria, attention is often restricted to an equal distribution of consumers who are all endowed with quadratic transportation costs. In that special case two firms maximally differentiate in a one-dimensional set-up (d'Aspremont *et al.*, 1979). Strategy restrictions would not change this result apart from the fact that firms can only maximally differentiate within their feasible sets.

the scope of this note.

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