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Abstract: This paper proves that there is a similar Uzawa (1961) steady-state growth theorem in a Malthusian model: If that model possesses steady-state growth, then technical change must be purely land-augmenting and cannot include labor augmentation.

Keywords: Malthusian Model, Neoclassical Growth Model, Uzawa's Steady-State Theorem

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1. Introduction

Uzawa's theorem (1961) says that for a neoclassical growth model to exhibit steady-state growth, the technological progress must be Harrod-neutral (purely labor-augmenting). This result raises the question as to why technological progress cannot be, say, Hicks neutral or Solow neutral. Many have explicitly asked this question (see Fellner, 1961; Kennedy, 1964; Samuelson, 1965; Drandakis and Phelps, 1966; Acemoglu, 2003, 2009; Barro and Sala-i-Martin, 2004; Jones, 2005; Jones and Scrimgeour, 2008) without achieving a clear answer, leaving the issue to be a puzzle for the growth theory. However, the above literature only discussed the requirements for the neoclassical growth model concerning the direction of technical change. Does a Malthusian model (Malthus, 1798; Ricardo, 1817) also require limiting the direction of technical change along a steady-state growth path? While there seems to be no literature about this question, it is important not only for an in-depth understanding of the Malthusian model itself, but also for solving the question as to why the neoclassical growth model must limit the technical change to be Harrod-neutral along a steady-state growth path. Specifically, by comparing the two types of environments we can find out whether the restriction on the direction of technical change in steady-state growth is special to the neoclassical growth model, or is required in other models too.

Kremer (1993) constructs and empirically tests a model of long-run world population growth combining the idea that high population spurs technological change, as implied by many endogenous growth models, with the Malthusian assumption that technology limits population. Lucas (2002) restated the Malthusian model in a neoclassical framework and proved that even with technological progress and capital accumulation, sustained growth of per-capita income cannot be achieved in that environment. While these papers discussed the effects of technological progress in a Malthusian world, they did not ask whether a Malthusian model requires limiting the direction of technical change to generate steady-state growth. Irmen (2004) pointed out the structural similarities between the Malthusian and the Solow (Solow, 1956) models, but did not address the aforementioned question either. Different from the above literature, this paper focuses precisely on that question. To this end, by using the same method as Schlicht (2006), this paper proves that for a Malthusian model to exhibit steady-state growth, technical change must be purely land-augmenting and cannot include labor augmentation.

2. The Malthusian Model

Consider an economy with a neoclassical production function F. In particular, this function relates, at any point in time t, the quantity produced, Y(t), to labor input L(t) and land input T(t) and is characterized by constant returns to scale in these inputs. Due to technological progress, it shifts over time, and we write:

$$Y(t) = F[T(t), L(t), t] \tag{1}$$

with

$$F(\lambda T_t, \lambda L_t, t) = \lambda F(T_t, L_t, t), \quad \text{for all} \quad (T, L, t, \lambda) \in R_+^4 \tag{2}$$

Land input, T, grows exponentially at rate τ :

$$T(t) = e^{\tau t} T_0, \quad \tau \geq 0 \tag{3}$$

If τ =0, then the land is invariant. But even though τ >0, the key result of Malthusian model will still be valid.

The labor input, L, change over time according to the Malthusian assumption that population growth depends on the level of income per capita. The higher that level is, the higher is the birth rate and the lower the mortality rate, implying a higher rate of population growth. Let $n(t)$ denote the total population growth rate, and $b(t)$ and $d(t)$ the birth and mortality rate, respectively. Let per-capita income be given by $y(t) = Y(t)/L(t)$. Then the population growth function is defined as

$$n(t) = b[y(t)] - d[y(t)], \quad \partial b(t)/\partial y(t) > 0, \partial d(t)/\partial y(t) < 0 \tag{4}$$

From equation (4), it is obvious that

$$\frac{\partial n(t)}{\partial y(t)} = \frac{\partial b[y(t)]}{\partial y(t)} - \frac{\partial d[y(t)]}{\partial y(t)} > 0 \tag{5}$$

3 Steady-state Theorem in the Malthusian Model:

If the system (1)-(4) possesses a solution where Y(t) and L(t) are all nonnegative and grow at constant rates, g and n, respectively, then

$$F[T(t), L(t), t] = G[e^{(g-\tau)t} T(t), L(t)] \tag{6}$$

According to this theorem, exponential growth requires technological progress to be purely land-augmenting, with a rate of progress of g-τ.

***Proof*:** By assumption we have

$$Y(t) = Y_0 e^{gt} \tag{7}$$

$$L(t) = L_0 e^{nt} \tag{8}$$

From equation (4) and (8), we can obtain

$$n = b(y) - d(y) = b[y_0 e^{(g-n)t}] - d[y_0 e^{(g-n)t}] \tag{9}$$

Taking time derivatives yields

$$0 = b' y_0 e^{(g-n)t}(g-n) - d' y_0 e^{(g-n)t}(g-n) \tag{10}$$

which implies

$$(b' - d')(g-n) = 0 \tag{11}$$

According to the Malthusian assumption: $b' > 0, d' < 0$ so that $b' - d' > 0$. Therefore, we must have

$$g - n = 0 \tag{12}$$

Define

$$G(T, L) = G(T, L, 0) \tag{13}$$

As $Y_0 = G(L_0, T_0)$, $Y_t = Y_0 e^{gt}, L_0 = L_t e^{-nt}, T_0 = T_t e^{-\tau t}$, and G is linear homogeneous, we can write

$$Y_t = Y_0 e^{gt} = G[T_t e^{(g-\tau)t}, L_t e^{(g-n)t}] \tag{14}$$

As g=n, this proves the theorem.

4 Conclusion

This paper proves that there is a steady-state growth theorem in a Malthusian model which is analogous to Uzawa's in a neoclassical environment. In particular, for a Malthusian model to exhibit steady-state growth, technical change must be purely land-augmenting and cannot include labor-augmentation. The result shows that the restriction on the direction of technical change in steady-state growth is required not only for the neoclassical growth model but also for other models.

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