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# Locational Signaling and Agglomeration\*

Marcus Berliant<sup>†</sup> and Chia-Ming Yu<sup>‡</sup>

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**Abstract:** Agglomeration can be caused by asymmetric information and a locational signaling effect: The location choice of workers signals their productivity to potential employers. The cost of a signal is the cost of housing at that location. When workers' marginal willingness to pay for housing is negatively correlated with their productivity, only the core-periphery (partially stratified) equilibria are stable. When workers' marginal willingness to pay for housing and their productivity are positively correlated, there is no core-periphery equilibrium. The urban wage premium is explained when there is core-periphery equilibrium. Furthermore, location can at best be an approximate rather than a precise sieve for high-skill workers. (*JEL Classifications:* D51; D82; R13)

**Keywords:** Agglomeration; Adverse Selection; Asymmetric Information; Locational Signaling

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# 1 Introduction

As shown in Baum-Snow and Pavan [2012], US wages were more than 30 percent higher in metropolitan areas with over 1.5 million inhabitants than in rural areas in the year 2000. Furthermore, their model indicates that ability sorting and returns to experience across locations are crucial elements in explaining the wage premium in large cities. Glaeser and Mare [2001] show that sorting on human capital accounts for about one-third of the city-size wage gap in the US. Moreover, Gould [2007] demonstrates that migration of high-skill workers is important in justifying the urban productivity premium, that is amplified by steeper experience profiles in urban areas. These analyses suggest that workers signal their skill and experience using their locations, which yields the urban wage premium.

Households' private information includes their productivity, which varies among individuals. When locations can possibly reveal workers' productivities, it is natural to ask why in practice some locations are attached to a signal for high productivity of workers, while others are not. For example, fashion designers in Milan, software programmers in Seattle, entertainers in Hollywood, financiers on Wall Street, or high-tech workers in Silicon Valley can be viewed as having a higher productivity than do workers in the same field in other locations. These observations could be due to learning from other workers, or interaction with R&D in these locations; however, they could also be due to a locational signaling effect. Many tools are used to signal workers' abilities since information about workers' skill is very important to firms and workers, for example: college diplomas, professional certificates, and academic alliance memberships.<sup>1</sup> It is interesting to examine how high-skill workers can use locational agglomeration to distinguish themselves from other workers, and how effective location can be as a reference for workers'

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<sup>1</sup>In urban economics, for example, there is the UEA.

productivity.<sup>2</sup>

Berliant and Kung [2010] analyze how asymmetric information causes agglomeration. Using a screening model, they show that workers can agglomerate and be sorted by skill in equilibrium due to asymmetric information in the labor market. Though it seems intuitive that both signaling and screening can explain sorting by human capital and the significant wage premium in large cities, one major difference between them is in the equilibrium sorting patterns: In the screening model, since contracts are offered first, separation of types by contract instead of location can occur, and thus, any distribution of workers constitutes an equilibrium. Even considering stability, equilibrium patterns are not narrowed down much. In contrast, for the signaling model, separation of types can only occur by choice of location, not by choice of contract. Thus, equilibrium narrows things down quite a bit. This paper answers the question: When there is asymmetric information, does stratification emerge in equilibrium due to the signaling value of the choice of location? The shadow cost of location, and thus of the signal, is the price of housing in a region.

Krugman [1991a] and New Economic Geography (NEG) models adopt increasing returns to scale to explain the agglomeration of manufacturing firms in one region. When transportation cost is decreased as transportation technology is improved, a core-periphery pattern is more likely in equilibrium. Many economic agglomeration phenomena in reality cannot be satisfactorily explained by increasing returns to scale. That is, there is a need to offer economic explanations other than increasing returns to scale in explaining the agglomeration of industries without increasing returns. A signaling incentive

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<sup>2</sup>McGuire [1974] advocates the importance of studies of economic reasons for voluntary segregation. Glaeser and Saiz [2004] also examine the incentives for people to agglomerate around high-skill workers. They find three reasons for agglomeration that they call the consumer city view, the information city view, and the reinvention city view. Our locational signaling viewpoint can be a fourth reason.

potentially fills this need. It is natural to ask: Is a core-periphery configuration more likely to constitute an equilibrium when there are no increasing returns to scale in production, but rather asymmetric information?

In contrast to aggregate uncertainty discussed in Berliant and Yu [2013], idiosyncratic uncertainty (individual-specific information) is the source of asymmetric information in this paper. A model with two regions and two types of workers, with high and low productivity, is analyzed. Workers are mobile across regions whereas differences in regional wages and housing rents determine their migration incentives. We examine under what conditions the equilibrium distribution of workers would be stratified. When workers' marginal willingness to pay for housing is negatively correlated with their productivity, there are at least three equilibria: a completely symmetric equilibrium where both types of workers are evenly distributed over both regions, and two partially stratified equilibria (or say core-periphery equilibria) where high-productivity workers are agglomerated in one region, but low skill workers are not. The completely symmetric equilibrium is unstable whereas the partially stratified equilibria are stable. In contrast, when workers' marginal willingness to pay for housing is positively correlated with workers' productivity, there always exists a completely symmetric equilibrium but there are no core-periphery equilibria. The completely symmetric equilibrium is stable when the difference in workers' productivities is not large. When the difference in productivities is very large, the completely symmetric equilibrium is unstable.

Though a higher wage for workers in the fashion industry in Milan attracts workers in an alternative region to migrate to Milan, due to a larger aggregate housing demand, in equilibrium there will be a higher housing rent in Milan to offset workers' migration incentives. When high-productivity workers have a lower marginal willingness to pay for housing than low-productivity

workers, the signaling cost for high-productivity workers is lower than that for low-productivity workers at the core-periphery equilibrium. Therefore, for a given wage premium in Milan, there is a long-run stratified equilibrium such that all the high-productivity workers agglomerate in Milan while the low-productivity workers reside in both Milan and the alternative region. When high-productivity workers have a higher marginal willingness to pay for housing than low-productivity workers, the signaling cost for high-productivity workers is higher than that for low-productivity workers under any core-periphery configuration. This intuition is verified in this paper, which suggests a potentially testable implication of our model, namely the prevalence of agglomeration of high-skill workers as a function of the correlation of skill and marginal willingness to pay for housing.

Notice that, in either a stratified or a symmetric equilibrium, no region is fully occupied by high-productivity workers alone. That is, there is no completely stratified equilibrium, but a semi-pooled equilibrium may exist.<sup>3</sup> Therefore, it is only possible to ensure that any worker who does not reside in Milan is a low-productivity worker. For every worker in Milan, it is impossible to guarantee that his/her productivity is high in any equilibrium. *This observation indicates that location is at best an approximate instead of a precise sieve for high-productivity workers.*

Furthermore, if we consider a continuous increase in high-skill workers' productivity relative to that of low-skill workers, a core-periphery equilibrium is present, even if there are no increasing returns to scale in production and knowledge spillovers. In other words, the agglomeration of high-productivity industries can be attributed to the existence of a locational signaling effect. Since, intuitively, increasing returns to scale in fashion design seems bizarre,

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<sup>3</sup>The core-periphery equilibrium in this paper corresponds to a semi-pooling equilibrium where some types of senders choose the same signal (location) and other types choose different signals (locations).

the agglomeration of fashion industries in Milan can be explained from a signaling viewpoint.<sup>4</sup>

Signaling cost in our model is determined by housing prices, and housing prices are different for different distributions of workers. In contrast with most signaling models where the marginal signaling cost is exogenous, i.e., Spence [1973], Wilson [1977], Grossman [1981], and Rothschild and Stiglitz [1976], the marginal signaling cost is endogenous in our paper. That is, signaling cost affects workers' migration incentives, and after their migration, the distribution of workers' types further influences the signaling cost.<sup>5</sup> We explore the question: Does the interaction between migration and marginal signaling cost yield a stratified equilibrium? The same type of endogeneity holds in cheap-talk models like Crawford and Sobel [1982] and Austen-Smith and Banks [2000].

In what follows, our model is introduced in Section 2. Moreover, necessary and sufficient conditions for the existence of stable core-periphery equilibria and for the stability of integrated equilibria are presented. Analytical equilibrium analysis and related welfare analyses are offered in Section 3. Conclusions are in Section 4. An appendix contains the proof of the main result.

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<sup>4</sup>We do not claim that all agglomerations of high skill workers result from signaling. Our view is much more modest, that signaling can be a contributing factor.

<sup>5</sup>The feature that the marginal signaling cost is endogenously determined by housing market clearing is in contrast to Fang's [2001] settings. This feature allows us to examine whether the market mechanism helps or hinders private information revelation. In addition, Fang adopts the Bayesian Nash equilibrium concept, whereas a stability concept is employed in our model (common in the spatial economics literature). Since we don't have an informational free-riding problem, the completely symmetric equilibrium is always Pareto optimal, whereas in Fang's model parameters are chosen so that an asymmetric equilibrium is efficient.

## 2 Model

There are two regions  $k \in K \equiv \{x, y\}$  with the same land endowment  $\bar{s}$ . There are two types of mobile workers  $i \in N \equiv \{H, L\}$  with exogenous populations  $n^H, n^L \in \mathbb{R}_{++}$ , respectively, where the productivity of  $H$ -type workers is higher than that of  $L$ -type workers.  $H$ -type ( $L$ -type) workers can be interpreted as high-skill (low-skill) workers, or can be interpreted as experienced (novice) workers. With the second interpretation, the appearance of a stratified equilibrium implies that returns to experience are important in explaining the city size wage premium.

Throughout this paper, workers' type is indexed by a superscript and location is indexed by a subscript. The (endogenous) population of  $i$ -type workers living in  $k$  is denoted by  $n_k^i$ , and the (exogenous) aggregate population in the model is  $n = n^H + n^L$ . Firms cannot recognize any worker's type directly; however, firms know the (equilibrium) distribution of workers' types over the two regions and can infer the probability of a worker's type using his/her location. Utility is quasilinear. Let  $s_k^i, z_k^i$  be each  $i$ -type worker's house size and the consumption of composite goods in region  $k, i \in N, k \in K$ , respectively. Let  $r_k$  denote the rent per unit of housing and let  $w_k$  denote the worker's wage in  $k, k \in K$ . Each worker is endowed with one unit of labor. There is no disutility from work, and composite good has price 1. The rents are collected and consumed by households, each of whom is endowed with  $e_k^i$  units of housing in  $k, i \in N, k \in K$ . Notice that  $n^H e_k^H + n^L e_k^L = \bar{s}, k \in K$ . Letting  $\varphi_k^i \equiv (s_k^i, z_k^i), i \in N, k \in K$ , the optimization problem for  $H$ -type

workers in region  $k$ ,  $k \in K$ , is<sup>6</sup>

$$\begin{aligned} \max \quad & u_k^H(\varphi_k^H) = z_k^H - \frac{\alpha}{s_k^H} \\ \text{s.t.} \quad & r_k s_k^H + z_k^H \leq w_k + r_x e_x^H + r_y e_y^H, \\ & s_k^H, z_k^H \in \mathbb{R}_+; \end{aligned} \tag{1}$$

whereas the optimization problem for  $L$ -type workers in  $k$  is

$$\begin{aligned} \max \quad & u_k^L(\varphi_k^L) = z_k^L - \frac{\beta}{s_k^L} \\ \text{s.t.} \quad & r_k s_k^L + z_k^L \leq w_k + r_x e_x^L + r_y e_y^L, \\ & s_k^L, z_k^L \in \mathbb{R}_+. \end{aligned} \tag{2}$$

Assume that  $\alpha, \beta > 0$ . Either  $\alpha > \beta$  holds, which implies that workers' demand for housing is positively correlated with productivity, or  $\alpha < \beta$  holds, implying that workers' demand for housing and productivity are negatively correlated, depicted in Figures 1 and 2.<sup>7</sup> This is the analog of the single crossing property that is used in signaling models.

To simplify the analysis, assume that each worker inelastically supplies one unit of labor, so we need not be concerned about monitoring and voluntary participation constraints. Every firm hires one worker at most. Each firm can adopt a high type technology together with a  $H$ -type labor to produce  $Y^H$ , or adopt a low type technology together with a  $L$ -type labor to produce  $Y^L$ , where  $0 < Y^L < Y^H$ . The corresponding profit in  $k$  is  $Y^H - w_k$

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<sup>6</sup>Except for asymmetric information, our model satisfies all the assumptions of Starrett's [1978] theorem. That is, asymmetric information is the only source of agglomeration in this model.

<sup>7</sup>When  $\alpha = \beta$ , the signaling cost is the same for both types of workers who thus have the same migration incentive. Then, one of two cases occur. Either  $H$ -type workers want to agglomerate in one region in equilibrium, in which case  $L$ -types want to agglomerate in the same region, and thus, the land market in the other region cannot clear. Or  $H$ -type workers do not want to agglomerate in any region, in which case for any given distribution of  $H$ -type workers over the two regions, there exists a distribution of  $L$ -type workers which can constitute an equilibrium. That is, given  $\alpha = \beta$ , either there are an infinite number of equilibria or there is no long-run equilibrium, which is not a case of interest.

and  $Y^L - w_k$ , respectively,  $k \in K$ . When any firm adopts a high type technology with a  $L$ -type worker, the output is zero. On the other hand, when a firm adopts a low type technology with a  $H$ -type worker, the output is  $Y^L$ , which is lower than  $Y^H$ . That is, no firm would prefer to adopt a technology that is incompatible with the type of the hired worker. Firms maximize their expected profit; their equilibrium behavior in choosing technology will be explained later. Every firm or worker is so small that he/she cannot influence competitive market prices. Furthermore, assume that there is free entry of firms, and thus, every firm earns zero expected profit in equilibrium. Finally, workers choose locations to maximize their utilities, including the consideration that firms can possibly learn about workers' types only from observing their locations.<sup>8</sup>

To extract the influence of signaling effects, assume that there is no commuting; that is, workers can work only in the place where they live. In other words, this is a regional, not city, model. However,  $H$ -type and  $L$ -type workers are allowed to migrate to earn a higher utility.<sup>9</sup> Denote  $\rho^H$  ( $\rho^L$ ) as the ratio of  $H$ -type ( $L$ -type) workers in the world living in  $x$ , and thus  $1 - \rho^H$  ( $1 - \rho^L$ ) is the ratio of all  $H$ -type ( $L$ -type) workers living in  $y$ . The population in  $x$  and  $y$ , given  $(\rho^H, \rho^L)$ , can be expressed as  $n_x \equiv \rho^H n^H + \rho^L n^L$  and  $n_y \equiv (1 - \rho^H) n^H + (1 - \rho^L) n^L$ , respectively.

To characterize locational signaling effects, the market process is given as follows. First, each firm hires a worker without knowing his/her productivity. Though firms do not know each worker's type, suppose that firms do not misperceive; that is, they know the actual equilibrium proportion of  $H$ -type workers in each region and thus have a common distribution over a worker's type conditional on his/her equilibrium location. Then, since there is free

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<sup>8</sup>Since agents are competitive in the housing market, they cannot do anything to attract workers and increase their housing rental income.

<sup>9</sup>When  $H$ -type workers are mobile but  $L$ -type workers are immobile, there are similar results.

entry of firms, each firm in a region pays its worker a wage according to the expected profit in the region. After learning the type of worker that the firm hires, the firm chooses its production technology to maximize ex post profit or minimize ex post loss. A mixed adoption of technology is assumed not available for firms. The above assumptions are standard in labor-market models of adverse selection, particularly that of Greenwald [1986].<sup>10</sup>

Note that given  $(\rho^H, \rho^L)$ , since there is free entry of firms, each firm earns zero expected profit. Thus, the wages for every worker in region  $x$  and  $y$  are<sup>11</sup>

$$w_x(\rho^H, \rho^L) = \frac{1}{n_x}(\rho^H n^H Y^H + \rho^L n^L Y^L), \quad (3)$$

$$w_y(\rho^H, \rho^L) = \frac{1}{n_y}[(1 - \rho^H)n^H Y^H + (1 - \rho^L)n^L Y^L]. \quad (4)$$

Let us temporarily leave workers' mobility aside. Short-run equilibrium is defined as a competitive market equilibrium, given a population distribution over the two regions.

**Definition 1 (*Short-Run Equilibrium*)**

$(\varphi_k^{H*}, \varphi_k^{L*}, w_k^*, r_k^*)_{k \in K}$  constitutes a short-run equilibrium if, given an arbitrary

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<sup>10</sup>Surely, changing the specified market process can change the results of our model. For example, when firms are assumed to choose their technology before knowing workers' type, the chosen technology must be the same for all firms in one region (since there is no difference between firms in the same region). Moreover, given workers' distribution is not completely symmetric, when the high technology is chosen in one region in equilibrium, the other region will choose the low technology. Since the  $H$ -type ( $L$ -type) workers can be hired only in the region adopting the high (low) technology, a core-periphery equilibrium is immediate for any not-completely symmetric initial distribution of workers. Actually, this setting is more like a screening model as analyzed in Berliant and Kung [2010], instead of a signaling model. In addition, when firms pay the wage after they know workers' type, there is no need for workers to use locational signaling. Therefore, the market process specified here is more appropriate in presenting a story for signaling effects than alternative assumptions.

<sup>11</sup>The main purpose of this paper is to characterize agglomeration across regions, instead of migration within one region; therefore, wage inequality within the same region is not considered here. Both inequality across and within regions can be explained by a variation of this model.

$(\rho^H, \rho^L)$ , workers choose optimal consumptions, firms make competitive wage offers for the distribution of workers, and the housing and the composite good markets in each region clear. That is:

(a)  $u_k^i(\varphi_k^{i*}) \geq u_k^i(\varphi_k^i)$ , for all  $\varphi_k^i \in \mathbb{R}_+^2$  satisfying  $r_k s_k^i + z_k^i \leq w_k + r_x e_x^i + r_y e_y^i$ ,  $\forall i \in N$ ,

$k \in K$ ;

(b)  $w_x^* = \frac{1}{n_x}(\rho^H n^H Y^H + \rho^L n^L Y^L)$ , and

$w_y^* = \frac{1}{n_y}[(1 - \rho^H)n^H Y^H + (1 - \rho^L)n^L Y^L]$ ;

(c)  $\rho^H n^H s_x^{H*} + \rho^L n^L s_x^{L*} = \bar{s}$ ,

$(1 - \rho^H)n^H s_y^{H*} + (1 - \rho^L)n^L s_y^{L*} = \bar{s}$ ,

$(\rho^H z_x^{H*} + (1 - \rho^H) z_y^{H*}) n^H + (\rho^L z_x^{L*} + (1 - \rho^L) z_y^{L*}) n^L$   
 $= n^H Y^H + n^L Y^L$ .

The short-run equilibrium, by Walras' law, is determined by conditions (a), (b), and the first two (or any two) equalities in (c). Recalling that  $n_x \equiv \rho^H n^H + \rho^L n^L$  and  $n_y \equiv (1 - \rho^H)n^H + (1 - \rho^L)n^L$ , and letting  $Y_x \equiv \rho^H n^H Y^H + \rho^L n^L Y^L$  and  $Y_y \equiv (1 - \rho^H)n^H Y^H + (1 - \rho^L)n^L Y^L$ , Theorem 1 shows that the short-run equilibrium exists and is unique.

**Theorem 1** For each  $(\rho^H, \rho^L) \in [0, 1] \times [0, 1]$ , there exists a unique short-run equilibrium, where

$$s_x^{H*} = \frac{\sqrt{\alpha \bar{s}}}{\sqrt{\alpha} \rho^H n^H + \sqrt{\beta} \rho^L n^L}, \quad s_y^{H*} = \frac{\sqrt{\alpha \bar{s}}}{\sqrt{\alpha} (1 - \rho^H) n^H + \sqrt{\beta} (1 - \rho^L) n^L}, \quad (5)$$

$$s_x^{L*} = \frac{\sqrt{\beta \bar{s}}}{\sqrt{\alpha} \rho^H n^H + \sqrt{\beta} \rho^L n^L}, \quad s_y^{L*} = \frac{\sqrt{\beta \bar{s}}}{\sqrt{\alpha} (1 - \rho^H) n^H + \sqrt{\beta} (1 - \rho^L) n^L}, \quad (6)$$

$$z_x^{H*} = \frac{e_x^H(\sqrt{\alpha}\rho^H n^H + \sqrt{\beta}\rho^L n^L)^2}{\bar{s}^2} + \frac{e_y^H(\sqrt{\alpha}(1-\rho^H)n^H + \sqrt{\beta}(1-\rho^L)n^L)^2}{\bar{s}^2} + \frac{Y_x}{n_x} - \frac{\alpha\rho^H n^H + \sqrt{\alpha\beta}\rho^L n^L}{\bar{s}}, \quad (7)$$

$$z_y^{H*} = \frac{e_x^H(\sqrt{\alpha}\rho^H n^H + \sqrt{\beta}\rho^L n^L)^2}{\bar{s}^2} + \frac{e_y^H(\sqrt{\alpha}(1-\rho^H)n^H + \sqrt{\beta}(1-\rho^L)n^L)^2}{\bar{s}^2} + \frac{Y_y}{n_y} - \frac{\alpha(1-\rho^H)n^H + \sqrt{\alpha\beta}(1-\rho^L)n^L}{\bar{s}}, \quad (8)$$

$$z_x^{L*} = \frac{e_x^H(\sqrt{\alpha}\rho^H n^H + \sqrt{\beta}\rho^L n^L)^2}{\bar{s}^2} + \frac{e_y^H(\sqrt{\alpha}(1-\rho^H)n^H + \sqrt{\beta}(1-\rho^L)n^L)^2}{\bar{s}^2} + \frac{Y_x}{n_x} - \frac{\beta\rho^L n^L + \sqrt{\alpha\beta}\rho^H n^H}{\bar{s}}, \quad (9)$$

$$z_y^{L*} = \frac{e_x^H(\sqrt{\alpha}\rho^H n^H + \sqrt{\beta}\rho^L n^L)^2}{\bar{s}^2} + \frac{e_y^H(\sqrt{\alpha}(1-\rho^H)n^H + \sqrt{\beta}(1-\rho^L)n^L)^2}{\bar{s}^2} + \frac{Y_y}{n_y} - \frac{\beta(1-\rho^L)n^L + \sqrt{\alpha\beta}(1-\rho^H)n^H}{\bar{s}}, \quad (10)$$

$$w_x^* = \frac{Y_x}{n_x}, \quad w_y^* = \frac{Y_y}{n_y}, \quad r_x^* = \left( \frac{\sqrt{\alpha}\rho^H n^H + \sqrt{\beta}\rho^L n^L}{\bar{s}} \right)^2, \quad \text{and} \quad (11)$$

$$r_y^* = \left( \frac{\sqrt{\alpha}(1-\rho^H)n^H + \sqrt{\beta}(1-\rho^L)n^L}{\bar{s}} \right)^2. \quad (12)$$

*Proof.* Firms' free-entry condition gives equilibrium wages. Substituting  $w_k^*$  into workers' utility maximization problems (1) and (2), workers' optimal consumptions are functions of  $(r_k)_{k \in K}$  and  $(\rho^H, \rho^L)$ ; the equilibrium housing prices can be solved by substituting demands into market clearing conditions. Finally, equilibrium consumption is found by substituting equilibrium prices into demand functions. *Q.E.D.*

Based on the indirect utility functions derived from the short-run equilibrium given above, the long-run equilibrium of this model and related welfare implications are analyzed in the next section.

### 3 Signaling Equilibrium and Welfare Analysis

When workers' mobility is considered, workers have to choose their optimal locations according to the utilities from living in the two regions. Since  $i$ -type workers' indirect utility from living in region  $k$  is  $u_k^i(\varphi_k^{i*})$ ,  $i \in N$ ,  $k \in K$ , the equilibrium condition for no further migration is

$$u_x^i(\varphi_x^{i*}) = u_y^i(\varphi_y^{i*}), \text{ if } \rho^{i*} \in (0, 1), \forall i \in N. \quad (13)$$

However, when all  $i$ -type workers are agglomerated in region  $k$ ,  $i \in N$ ,  $k \in K$ ,  $i$ -type workers' utility in the other region  $k'$ ,  $k' \in K$  where  $k' \neq k$ , is not defined. Following the literature, the potential wage and housing rent for  $i$ -type workers in  $k'$  is defined as the limit of the equilibrium wage and equilibrium rent in  $k'$  when the number of  $i$ -type workers in  $k'$  approaches zero. So the potential utility for  $i$ -type workers in  $k'$  is defined according to their potential wage and potential housing rent in  $k'$ . Given this setting, the signaling equilibrium concept is in fact defined by a pair  $(\rho^{H*}, \rho^{L*}) \in [0, 1] \times [0, 1]$ , and the corresponding  $(\varphi_k^{H*}, \varphi_k^{L*}, w_k^*, r_k^*)_{k \in K}$  that satisfy the following conditions.

**Definition 2 (Signaling Equilibrium)**

$((\varphi_k^{H*}, \varphi_k^{L*}, w_k^*, r_k^*)_{k \in K}, \rho^{H*}, \rho^{L*})$  constitutes a signaling equilibrium if and only if  $(\varphi_k^{H*}, \varphi_k^{L*}, w_k^*, r_k^*)_{k \in K}$  constitutes a short-run equilibrium for  $(\rho^{H*}, \rho^{L*})$ , and, in addition, no worker in any region has an incentive to migrate to the other region. That is, in addition to conditions (a)-(c) in Definition 1, it is required that<sup>12</sup>

$$(d) \ u_x^i(\varphi_x^{i*}) = u_y^i(\varphi_y^{i*}) \text{ if } \rho^{i*} \in (0, 1), \forall i \in N, k \in K;$$

$$u_x^H(\varphi_x^{H*}) > \lim_{\rho^H \rightarrow 1} u_y^H(\varphi_y^H[r_y(\rho^H, \rho^{L*}), w_y(\rho^H, \rho^{L*})]), \text{ if } \rho^{H*} = 1;$$

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<sup>12</sup>It is assumed that there is a small positive installation cost when a household is the first one to live in a region with no other resident. Therefore, when any inequality in condition (d) holds with equality, households still have an incentive not to migrate into an empty region.

$$\begin{aligned}
u_x^L(\varphi_x^{L*}) &> \lim_{\rho^L \rightarrow 1} u_y^L(\varphi_y^L[r_y(\rho^{H*}, \rho^L), w_y(\rho^{H*}, \rho^L)]), \quad \text{if } \rho^{L*} = 1; \\
u_y^H(\varphi_y^{H*}) &> \lim_{\rho^H \rightarrow 0} u_x^H(\varphi_x^H[r_x(\rho^H, \rho^{L*}), w_x(\rho^H, \rho^{L*})]), \quad \text{if } \rho^{H*} = 0; \\
u_y^L(\varphi_y^{L*}) &> \lim_{\rho^L \rightarrow 0} u_x^L(\varphi_x^L[r_x(\rho^{H*}, \rho^L), w_x(\rho^{H*}, \rho^L)]), \quad \text{if } \rho^{L*} = 0.
\end{aligned}$$

The long-run signaling equilibrium can be found as a solution to the system of equations including (a), (b), (d), and, by Walras' Law, the first two equations of condition (c) in Definition 1. More specifically, recall that the equilibrium consumption and prices are functions of  $(\rho^H, \rho^L)$  as shown in Theorem 1. Substituting equilibrium consumption into the utility functions, we have workers' difference in indirect utilities from living in the regions, given a distribution of workers. Letting  $u_k^{i*} = u_k^i(\varphi_k^{i*})$ , it can be checked that

$$u_x^{H*} - u_y^{H*} = w_x^* - w_y^* - 2\sqrt{\alpha}(\sqrt{r_x^*} - \sqrt{r_y^*}), \quad (14)$$

$$u_x^{L*} - u_y^{L*} = w_x^* - w_y^* - 2\sqrt{\beta}(\sqrt{r_x^*} - \sqrt{r_y^*}). \quad (15)$$

Notice that  $w_x^* - w_y^*$  is interpreted as a signaling gain (if it is positive), or signaling loss (if it is negative) from living in  $x$  compared to living in  $y$ , which is the same for both types of workers. On the other hand, the signaling cost of living in  $x$  relative to living in  $y$  is  $2\sqrt{\alpha}(\sqrt{r_x^*} - \sqrt{r_y^*})$  and  $2\sqrt{\beta}(\sqrt{r_x^*} - \sqrt{r_y^*})$  for  $H$ -type and  $L$ -type workers, respectively. When  $\alpha < \beta$ , if  $r_x^* > r_y^*$ , the signaling cost for high-skill workers is smaller than that for low-skill workers, indicating that there should exist stratified equilibria. On the other hand, when  $\alpha > \beta$  and  $r_x^* > r_y^*$ , there should exist no stratified equilibrium.

Signaling equilibrium is a solution to the system of simultaneous nonlinear equations (14) and (15). It is interesting to notice that if  $(\rho^{H*}, \rho^{L*}) = (\frac{1}{2}, \frac{1}{2})$  constitutes an equilibrium, the result is exactly the case where both types of workers are equally distributed over the two regions, which is called a completely symmetric equilibrium; whereas if either  $(\rho^{H*}, \rho^{L*}) = (1, 0)$  or  $(\rho^{H*}, \rho^{L*}) = (0, 1)$  in equilibrium, there is a completely stratified equilibrium.<sup>13</sup> Letting  $f \equiv u_x^{H*} - u_y^{H*}$  and  $g \equiv u_x^{L*} - u_y^{L*}$ , the following lemma

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<sup>13</sup>We are more interested in partially stratified equilibria.

ensures the existence of an interior equilibrium.

**Lemma 1** *Equal-dispersion*  $(\rho^{H*}, \rho^{L*}) = (\frac{1}{2}, \frac{1}{2})$  always constitutes a signaling equilibrium.

*Proof.* Given  $(\rho^H, \rho^L) = (\frac{1}{2}, \frac{1}{2})$ , it is obvious that  $w_x^* = w_y^*$  and  $r_x^* = r_y^*$ , which implies  $f = 0$  and  $g = 0$ . Therefore,  $(\rho^{H*}, \rho^{L*}) = (\frac{1}{2}, \frac{1}{2})$  is always one of the solutions to  $u_x^{H*} = u_y^{H*}$  and  $u_x^{L*} = u_y^{L*}$ . *Q.E.D.*

In addition to the existence of a signaling equilibrium, the stability of a long-run equilibrium should be examined. For a given  $(\rho^H, \rho^L) \in [0, 1] \times [0, 1]$  and the corresponding equilibrium utility levels  $(u_x^{i*}, u_y^{i*})$ ,  $i \in N$ , we consider standard dynamics with multiple types of workers. When  $u_x^{i*} > u_y^{i*}$  ( $u_x^{i*} < u_y^{i*}$ ),  $i \in N$ ,  $i$ -type workers in  $y$  ( $x$ ) surely have incentive to move to  $x$  ( $y$ ). In order to explore the stability of signaling equilibria, following Krugman [1991b], Fukao and Benabou [1993], and Forslid and Ottaviano [2003], for  $i \in N$ , let  $\dot{\rho}^i$  describe the *ad hoc* dynamics:

$$\dot{\rho}^i \equiv \frac{d\rho^i}{dt} = \begin{cases} \max\{0, \gamma(u_x^{i*} - u_y^{i*})\} & \text{if } \rho^i = 0, \\ \gamma(u_x^{i*} - u_y^{i*}) & \text{if } \rho^i \in (0, 1), \\ \min\{0, \gamma(u_x^{i*} - u_y^{i*})\} & \text{if } \rho^i = 1. \end{cases} \quad (16)$$

Notice that  $\gamma > 0$  represents a measure of the speed of adjustment in the ratio of  $i$ -type workers across regions,  $i \in N$  (as emphasized in Krugman [1991b], “ $\gamma$  is an inverse index of the cost of adjustment”). That is, when  $u_x^{i*} > u_y^{i*}$  ( $u_x^{i*} < u_y^{i*}$ ),  $i$ -type workers in  $y$  ( $x$ ) migrate to  $x$  ( $y$ ) with a speed of  $|\dot{\rho}^i|$ .

If there is a small perturbation such that in the Bayesian Nash equilibrium a new short-run equilibrium is attained where firms have rational expectations, then the signaling equilibrium is unstable. Otherwise, the signaling equilibrium is stable. The definition of stability of an equilibrium is formally given as follows.

**Definition 3 (Stability of Equilibrium)**

*For any small perturbation of workers from the equilibrium worker distribution, given that firms can only recognize a worker's type according to their beliefs in the new short-run equilibrium, if the utility difference from living in different locations drives the perturbed workers back to their equilibrium location, the equilibrium is stable; otherwise, the equilibrium is called unstable.*

Note that, given condition (d) in Definition 2, a core-periphery configuration (i.e.,  $\rho^{H*} = 0$  or  $\rho^{H*} = 1$ ) is always a stable equilibrium when it constitutes an equilibrium. However, a completely symmetric equilibrium can be stable or unstable. Intuitively, when  $\rho^H$  increases, fixing  $\rho^L$  and all parameters, since the population in  $x$  ( $y$ ) increases (decreases), the demand for and thus the equilibrium price of houses in  $x$  ( $y$ ) increases (decreases) and at the same time, the average productivity or wage of workers in  $x$  ( $y$ ) increases (decreases). Therefore,  $u_x^{i*} - u_y^{i*}$ ,  $i \in N$ , may not be a monotonic function of  $\rho^H$ . On the other hand, given  $\rho^H$  and parameters, when  $\rho^L$  increases, the demand for housing in  $x$  increases and the average productivity of workers in  $x$  decreases. That is, there is no benefit but only damage for any resident in  $x$  when there are low-skill migrants coming from  $y$ , so  $u_x^{i*} - u_y^{i*}$ ,  $i \in N$ , is monotonically decreasing in  $\rho^L$ . Notice that the signaling gain is the same for both types of workers in the same region. As illustrated in Figure 3, when the marginal willingness to pay for housing for  $H$ -type workers is smaller than that for  $L$ -type workers, the signaling cost for  $H$ -type workers is less than the signaling cost for  $L$ -type workers at the core-periphery equilibrium, and thus,  $H$ -type workers have a stronger incentive to migrate to the region with a higher wage, which causes an agglomeration of  $H$ -type workers in the *ex post* core region. By contrast, in Figure 4, when the marginal willingness to pay for housing for  $H$ -type workers is larger than that for  $L$ -type workers, the signaling cost for  $H$ -type workers is higher than the signaling cost for  $L$ -type workers. In this case, there is no equilibrium with an agglomeration

of any type of worker. Though there is no closed-form solution for the simultaneous equations  $u_x^{i*} = u_y^{i*}$ ,  $i \in N$ , in the interesting cases with  $n^H < n^L$ , the intuition above is verified by the following proposition and depicted in Figures 5 and 6.

**Theorem 2** *Given  $n^H < n^L$ , when  $\alpha < \beta$ , there always exist a symmetric equilibrium and two stable core-periphery equilibria with  $\rho^{H*} = 0$  or  $\rho^{H*} = 1$ ; when  $\alpha > \beta$ , there is no core-periphery equilibrium, but only a symmetric equilibrium which is stable if and only if  $Y^H \leq Y^L + \frac{\alpha n^2}{\bar{s} n^L}$ .*

*Proof.* See Appendix A.

The wage in the core is higher than that in the periphery in any core-periphery equilibrium. That is, the urban wage premium can be explained by locational signals in productivity.

The intuition behind why there is no completely separating equilibrium in the case where  $\beta > \alpha$  is as follows. If the high productivity types were separated from the low productivity types, then low productivity types will want to join them because wage is higher in their location whereas housing cost is lower. We conjecture that the idea that the low productivity workers chase the high productivity workers extends to more general models, with more types of workers and more locations.

Since there are no increasing returns to scale in production and no agglomeration spillovers, the agglomeration of any type of worker in this model contributes nothing to production. To see this, notice that with no private information, equilibrium is first best and features equal land rent in the two regions, implying equal marginal willingness to pay for land for all consumers. With private information, among the various equilibria, only completely symmetric equilibria feature equal land rents in the two regions, and thus only they can be first best (but might not be, due to the information asymmetries). That is, households' use of resources for signaling is unproductive and

distorts housing prices.

The core-periphery equilibrium is apparently an example of what Spence calls the “hybrid equilibrium.” However, it is interesting to notice that when the signaling cost is endogenized by the housing market, the completely separating equilibrium is ruled out. That is, this paper implies that considering market mechanisms may help in the selection from multiple equilibria.

Notice also that the belief of workers’ type is not arbitrarily given (for example, when  $\rho^H = 0$  and one  $H$ -type worker migrates to  $x$ , this migrant is recognized as an  $L$ -type worker with probability 1), so there is no off the equilibrium path beliefs to worry about in our model. Moreover, in all core-periphery equilibria, population in the core region (where the high-skill workers locate) is larger than the population in the periphery region. Our model predicts that in core-periphery equilibria the difference in the populations of different regions increases with the difference between  $Y^H$  and  $Y^L$ .<sup>14</sup> The divergent trends in urban and rural populations are confirmed by data in the U.S. Census Bureau [1990] (Table 1) which shows that in addition to the increasing difference in urban and rural population, the percentage of US urban population to total population is increasing over time, and the percentage of US rural population decreased from 1950 to 1990.

## 4 Conclusions

Even without any increasing returns to scale in production, our results illustrate that the agglomeration of high-skill labor, and thus the agglomera-

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<sup>14</sup>For example, when  $\alpha < \beta$  and  $H$ -type workers agglomerate in  $x$  ( $\rho^{H*} = 1$ ) in equilibrium,  $n_x^* - n_y^* = (-\mathcal{B} + \sqrt{\mathcal{A}})/(2\beta n^L)$ , where  $\mathcal{B} \equiv \sqrt{\beta} n^L (\sqrt{\alpha} n^H + \sqrt{\beta} n^L)$  and  $\mathcal{A} \equiv \beta (n^L)^2 [n^H (4\bar{s}(Y^H - Y^L) + \alpha n^H) - 2\sqrt{\alpha}\sqrt{\beta} n^H (2n^H + n^L) + \beta (2n^H + n^L)^2]$ . Since  $\mathcal{A} - \mathcal{B}^2 = 4\beta n^H (n^L)^2 [(Y^H - Y^L)\bar{s} + (\beta - \sqrt{\alpha}\beta)(n^H + n^L)] > 0$ , we have  $n_x^* - n_y^* > 0$ ,  $\forall 0 < n^H < n^L$ . Furthermore, given  $n^H, n^L, \beta, \bar{s} > 0$ ,  $n_x^* - n_y^*$  is strictly increasing in  $(Y^H - Y^L)$ . Therefore, these statements are valid even when  $n^H$  is extremely small relative to  $n^L$ .

tion of high-technology firms, can be caused by asymmetric information and locational signaling effects, even if regional housing cost (the endogenous signaling cost) is increasing in the high-skill population residing there.

When workers' marginal willingness to pay for housing is positively correlated with their productivity, no core-periphery equilibrium can be sustained. Though there always exists a completely symmetric equilibrium, it is stable only if the difference between high-skill and low-skill workers' productivity is not too large. On the other hand, when workers' marginal willingness to pay for housing is negatively correlated with their productivity, there exist stable core-periphery equilibria. In this case, sorting on skill occurs, which accounts for the city size wage premium. Therefore, a core-periphery equilibrium can be present under locational signaling effects to explain the urban wage premium.

In summary, though the appearance of a core region is not socially optimal, the conclusions of this paper shed light on the importance of path-dependence or policies that attract high-skill labor for the development of a region, even when there are no increasing returns to scale, knowledge spillovers, or externalities. Moreover, in any stratified equilibrium, the agglomeration of high-skill labor in one region is mixed with a portion of low-skill labor. This suggests that when location signals workers' productivity and the signaling cost is determined by the housing market at a location, location can at best be a reference for rather than a guarantee of workers' high productivity.

From an empirical point of view, firms learn gradually about their workers; see Alós-Ferrer and Prat [2012]. Thus, the location signal might be more valuable for employees fresh out of school than older workers. This leverage might be exploited to test our model.

The model might be more interesting if applied to firms rather than con-

sumers. For example, consumers cannot distinguish the quality of food served by restaurants. A continuum of restaurants might signal their quality, high or low, to their consumers by where they choose to locate, whereas the signaling cost in this case is the price of renting a commercial space. This is another example of the type of phenomenon this model can capture.

Many extensions of the ideas presented here come to mind, for example, adding further heterogeneity to workers and firms, or adding firm investment in physical capital. The techniques introduced here can be extended to models where firms have private information, or to models where both firms and workers have private information.

## Appendix A. Proof of Theorem 2

When  $\alpha < \beta$ , productivity and the marginal willingness to pay for housing are negatively correlated. In the phase diagram, from  $f \equiv u_x^{H*} - u_y^{H*}$  and  $g \equiv u_x^{L*} - u_y^{L*}$ , it can be checked that  $f < 0$  ( $f > 0$ ) for all  $(\rho^H, \rho^L)$ -points above (below) the curve  $\dot{\rho}^H = 0$ . In addition,  $g < 0$  ( $g > 0$ ) for all  $(\rho^H, \rho^L)$ -points above (below) the curve  $\dot{\rho}^L = 0$ .<sup>15</sup> Letting  $\phi^i(\rho^H) \equiv \{\rho^L | u_x^{i*}(\rho^H, \rho^L) = u_y^{i*}(\rho^H, \rho^L)\}$ ,  $i \in N$ ,  $\phi^i(\rho^H)$ ,  $i \in N$ , is single valued and non-empty for  $\rho^H \in [0, 1]$ . The phase diagram shows that a necessary and sufficient condition for a stable completely symmetric equilibrium is  $\phi^{H'}(\rho^H) \leq \phi^{L'}(\rho^H)$  and  $\phi^{H'}(\rho^H) \leq 0$  at  $\rho^H = \frac{1}{2}$ . A sufficient condition for the existence of a core-periphery equilibrium is  $\phi^L(\rho^H) < \phi^H(\rho^H)$  at  $\rho^H = 1$  or  $\phi^L(\rho^H) > \phi^H(\rho^H)$  at  $\rho^H = 0$ .

Whether a core-periphery configuration can constitute an equilibrium depends on the relative positions of  $\dot{\rho}^H = 0$  and  $\dot{\rho}^L = 0$  in the phase diagram. From

$$f - g = \frac{4(\sqrt{\alpha} - \sqrt{\beta})}{\bar{s}} \left( \sqrt{\alpha} \left( \frac{1}{2} - \rho^H \right) n^H + \sqrt{\beta} \left( \frac{1}{2} - \rho^L \right) n^L \right), \quad (17)$$

it can be checked that when  $\alpha < \beta$ ,  $f < g$  if and only if  $\rho^L < \frac{1}{2} + \frac{\sqrt{\alpha n^H}}{\sqrt{\beta n^L}} \left( \frac{1}{2} - \rho^H \right)$ .

Furthermore,

$$\begin{aligned} f = g &= \frac{1}{\Psi} [4(Y^H - Y^L) \sqrt{\beta} n^H (\sqrt{\alpha} n^H + \sqrt{\beta} n^L) \left( \frac{1}{2} - \rho^H \right)] \\ \text{for } \rho^L &= \frac{1}{2} + \frac{\sqrt{\alpha} n^H}{\sqrt{\beta} n^L} \left( \frac{1}{2} - \rho^H \right), \quad \rho^H \in [0, 1], \end{aligned} \quad (18)$$

where  $\Psi \equiv [(\alpha - 2\sqrt{\alpha\beta})(1 - 2\rho^H)^2 - 4\beta\rho^H(1 - \rho^H)](n^H)^2 - \beta n^L(2n^H + n^L) < 0$ , for all  $\rho^H \in [0, 1]$ . Therefore, for  $\rho^H < \frac{1}{2}$ ,  $f = g < 0$  on  $\rho^L = \frac{1}{2} + \frac{\sqrt{\alpha} n^H}{\sqrt{\beta} n^L} \left( \frac{1}{2} - \rho^H \right)$ ; and for  $\rho^H > \frac{1}{2}$ ,  $f = g > 0$  on  $\rho^L = \frac{1}{2} + \frac{\sqrt{\alpha} n^H}{\sqrt{\beta} n^L} \left( \frac{1}{2} - \rho^H \right)$ . That is, the curves

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<sup>15</sup>It can be proved that  $\frac{\partial f}{\partial \rho^L} = -n^L \left( \frac{4\sqrt{\alpha\beta}}{\bar{s}} + \frac{n^H(Y^H - Y^L)}{n_x^2 n_y^2} \right) \Phi$ , and  $\frac{\partial g}{\partial \rho^L} = -n^L \left( \frac{4\beta}{\bar{s}} + \frac{n^H(Y^H - Y^L)}{n_x^2 n_y^2} \right) \Phi$ , where  $\Phi \equiv (1 - \rho^H) \rho^H n^H (n^H + 2n^L) + [\rho^H + (\rho^L)^2 - 2\rho^H \rho^L] (n^L)^2 > 0$  since  $[\rho^H + (\rho^L)^2 - 2\rho^H \rho^L] > (\rho^H - \rho^L)^2 > 0$ .

$\dot{\rho}^H = 0$  and  $\dot{\rho}^L = 0$  are below (above) the line  $\rho^L = \frac{1}{2} + \frac{\sqrt{\alpha n^H}}{\sqrt{\beta n^L}}(\frac{1}{2} - \rho^H)$  for  $\rho^H < \frac{1}{2}$  ( $\rho^H > \frac{1}{2}$ ). Therefore, for  $\rho^H < \frac{1}{2}$ , any point on  $\dot{\rho}^L = 0$  must satisfy both  $g = 0$  and  $f < g$ , which implies  $f < 0$ ; and for  $\rho^H > \frac{1}{2}$ , any point on  $\dot{\rho}^L = 0$  satisfies  $f > 0$ . Finally, since  $\phi^L(\rho^H) \in (0, 1)$ , for  $\rho^H \in \{0, 1\}$ ,<sup>16</sup> from Definition 2 and Lemma 1, there always exist three equilibria at  $(0, \phi^L(0))$ ,  $(\frac{1}{2}, \frac{1}{2})$ , and  $(1, \phi^L(1))$ .

When  $\alpha > \beta$ , since  $f > g$  if and only if  $\rho^L < \frac{1}{2} + \frac{\sqrt{\alpha n^H}}{\sqrt{\beta n^L}}(\frac{1}{2} - \rho^H)$  and  $g < 0$  ( $g > 0$ ) for all  $\rho^L = \frac{1}{2} + \frac{\sqrt{\alpha n^H}}{\sqrt{\beta n^L}}(\frac{1}{2} - \rho^H)$  where  $\rho^H \in [0, \frac{1}{2})$  ( $\rho^H \in (\frac{1}{2}, 1]$ ), it follows that for  $\rho^H < \frac{1}{2}$ , any point on  $\dot{\rho}^L = 0$  satisfies  $f > g = 0$ , and for  $\rho^H > \frac{1}{2}$ , any point on  $\dot{\rho}^L = 0$  satisfies  $f < 0$ . Therefore, there is no core-periphery equilibrium, and from Lemma 1, the unique equilibrium is symmetric.<sup>17</sup> Since  $\alpha > \beta$  implies  $\phi^{H'}(\frac{1}{2}) < \phi^{L'}(\frac{1}{2})$  and

$$-\frac{\partial f / \partial \rho^H}{\partial f / \partial \rho^L} \Big|_{(\rho^H, \rho^L) = (\frac{1}{2}, \frac{1}{2})} = \frac{(Y^H - Y^L)\bar{s} - \alpha n^2 / n^L}{(Y^H - Y^L)\bar{s} + \sqrt{\alpha\beta} n^2 / n^H}, \quad (19)$$

when  $\alpha > \beta$ , the symmetric equilibrium is stable if and only if  $Y^H \leq Y^L + \frac{\alpha n^2}{\bar{s} n^L}$ .

Recalling that  $\phi^i(\rho^H) \equiv \{\rho^L | u_x^{i*}(\rho^H, \rho^L) = u_y^{i*}(\rho^H, \rho^L)\}$ ,  $i \in N$ , let  $Y^H(S)$  be the sustain point where a given core-periphery pattern can be sustained, i.e.,  $Y^H(S) \equiv \min\{Y^H | \phi^H(1) \geq \phi^L(1)\}$ , and let  $Y^H(B)$  be the break point where the symmetric equilibrium starts to become unstable, i.e.,  $Y^H(B) \equiv \max\{Y^H | \phi^{H'}(\frac{1}{2}) \leq \phi^{L'}(\frac{1}{2}) \text{ and } \phi^{H'}(\frac{1}{2}) = 0\}$ . Theorem 2 implies that when  $\alpha < \beta$ , the sustain point and the break point are both at  $Y^H(S) = Y^H(B) = 1$ . *Q.E.D.*

<sup>16</sup>For example, at  $\rho^H = 0$ , the largest  $\phi^L(\rho^H) = \frac{1}{2}(1 + \frac{n^H}{n^L} \sqrt{\frac{\alpha}{\beta}})$  is achieved when  $Y^L = Y^H$ , which is less than 1 for  $n^H < n^L$  and  $\alpha < \beta$ . The smallest  $\phi^L(\rho^H) = \frac{1}{2\beta(n^L)^2}(\Lambda - \sqrt{\Lambda^2 - 32\beta(n^L)^2 n(\beta n^L - \alpha n^H)}) > 0$  where  $\Lambda \equiv 2(\sqrt{\alpha}\sqrt{\beta} + 2\beta)n^H n^L + 6\beta(n^L)^2 > 0$ .

<sup>17</sup>Though in this case, the curves  $\dot{\rho}^H = 0$  and  $\dot{\rho}^L = 0$  may intersect the boundaries of  $\rho^L = 0$  and  $\rho^L = 1$  on some  $\rho^H \in (0, 1)$ , these intersection points cannot constitute core-periphery equilibria since any point on  $\dot{\rho}^H = 0$  for  $\rho^H \in [0, \frac{1}{2})$  ( $\rho^H \in (\frac{1}{2}, 1]$ ) satisfies  $g < f = 0$  ( $g > f = 0$ ).

## References

- [1] Alós-Ferrer, Carlos, and Prat, Julien, “Job Market Signaling and Employer Learning,” *Journal of Economic Theory*, CXLVII (2012), 1787–1817.
- [2] Austen-Smith, David, and Jeffrey S. Banks, “Cheap Talk and Burned Money,” *Journal of Economic Theory*, XCI (2000), 1–16.
- [3] Baum-Snow, Nathaniel, and Ronni Pavan, “Understanding the City Size Wage Gap,” *Review of Economic Studies*, LXXIX (2012), 88–127.
- [4] Berliant, Marcus, and Fan-Chin Kung, “Can Information Asymmetry Cause Stratification?” *Regional Science and Urban Economics*, XL (2010), 196–209.
- [5] Berliant, Marcus, and Charles C. Yu, “Rational Expectations in Urban Economics,” *Regional Science and Urban Economics*, XLIII (2013), 197–208.
- [6] Crawford, Vincent P., and Joel Sobel, “Strategic Information Transmission,” *Econometrica*, L (1982), 1431–1451.
- [7] Fang, Hanming, “Social Culture and Economic Performance,” *American Economic Review*, XCI (2001), 924–937.
- [8] Forslid, Rikard, and Gianmarco I.P. Ottaviano, “An Analytically Solvable Core-Periphery Model,” *Journal of Economic Geography*, III (2003), 229–240.
- [9] Fukao, Kyoji, and Roland J. Benabou, “History versus Expectations: A Comment,” *Quarterly Journal of Economics*, CVIII (1993), 535–542.
- [10] Glaeser, Edward L., and David Mare, “Cities and Skills,” *The Journal of Labor Economics*, XIX (2001), 316–342.

- [11] Glaeser, Edward L., and Albert Saiz, “The Rise of the Skilled City.” *Brookings-Wharton Papers on Urban Affairs*, V (2004), 47–94.
- [12] Gould, Eric, “Cities, Workers, and Wages: A Structural Analysis of the Urban Wage Premium,” *Review of Economic Studies*, LXXIV (2007), 477–506.
- [13] Greenwald, Bruce C., “Adverse Selection in the Labour Market,” *Review of Economic Studies*, LIII (1986), 325–347.
- [14] Grossman, Sanford J., “The Informational Role of Warranties and Private Disclosure about Product Quality,” *Journal of Law and Economics*, XXIV (1981), 461–483.
- [15] Krugman, Paul, “Increasing Returns and Economic Geography,” *Journal of Political Economy*, XCIX (1991a), 483–499.
- [16] Krugman, Paul, “History versus Expectations,” *Quarterly Journal of Economics*, CVI (1991b), 651–667.
- [17] McGuire, Martin, “Group Segregation and Optimal Jurisdiction,” *Journal of Political Economy*, LXXXII (1974), 112–132.
- [18] Rothschild, Michael, and Joseph E. Stiglitz, “Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information,” *Quarterly Journal of Economics*, XC (1976), 629–649.
- [19] Spence, A. Michael, “Job Market Signaling,” *The Quarterly Journal of Economics*, LXXXVII (1973), 355–374.
- [20] Starrett, David, “Market Allocations of Location Choice in a Model with Free Mobility,” *Journal of Economic Theory*, XVII (1978), 21–37.

- [21] U.S. Census Bureau, “1990 Population and Housing Unit Counts: United States,” *1990 Census of Population and Housing* (Washington, D.C.: U.S. Census Bureau, 1990)
- [22] Wilson, Charles, “A Model of Insurance Markets with Incomplete Information,” *Journal of Economic Theory*, XVI (1977), 167–207.

Year	Urban population (percent of total)	Rural population (percent of total)	The difference in urban and rural population
1950	96846817 (64.0%)	54478981 (36.0%)	42367836
1960	125268750 (69.9%)	54045425 (30.1%)	71223325
1970	149646617 (73.6%)	53565309 (26.4%)	96081308
1980	167050992 (73.7%)	59494813 (26.3%)	107556179
1990	187053487 (75.2%)	61656386 (24.8%)	125397101

Table 1: Source: U.S. Census Bureau [1990], (CPH-2).

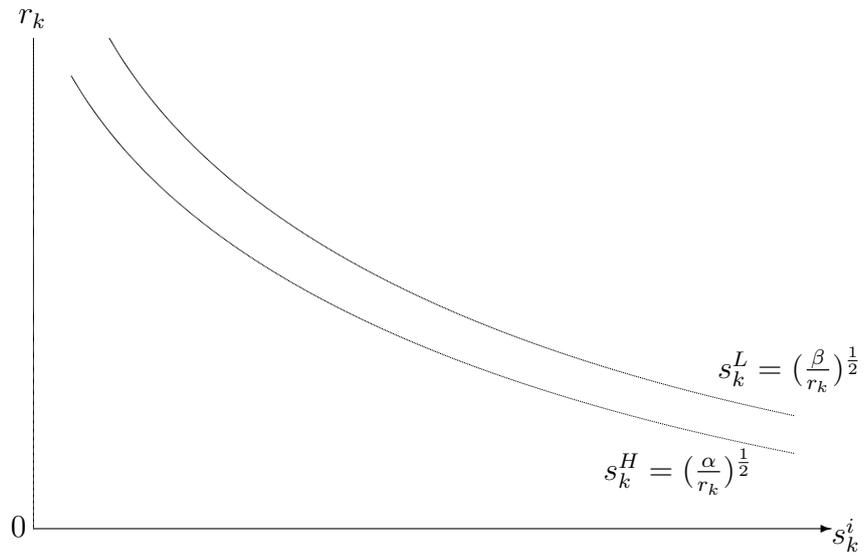


Figure 1: Households' demand curves when productivity and the marginal willingness to pay for housing are negatively correlated, i.e.,  $\alpha < \beta$ .

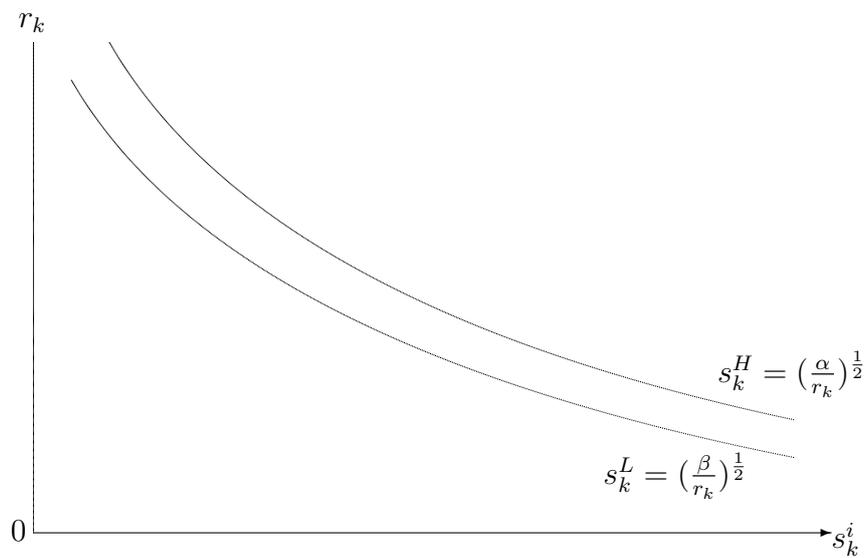


Figure 2: Households' demand curves when productivity and the marginal willingness to pay for housing are positively correlated, i.e.,  $\alpha > \beta$ .

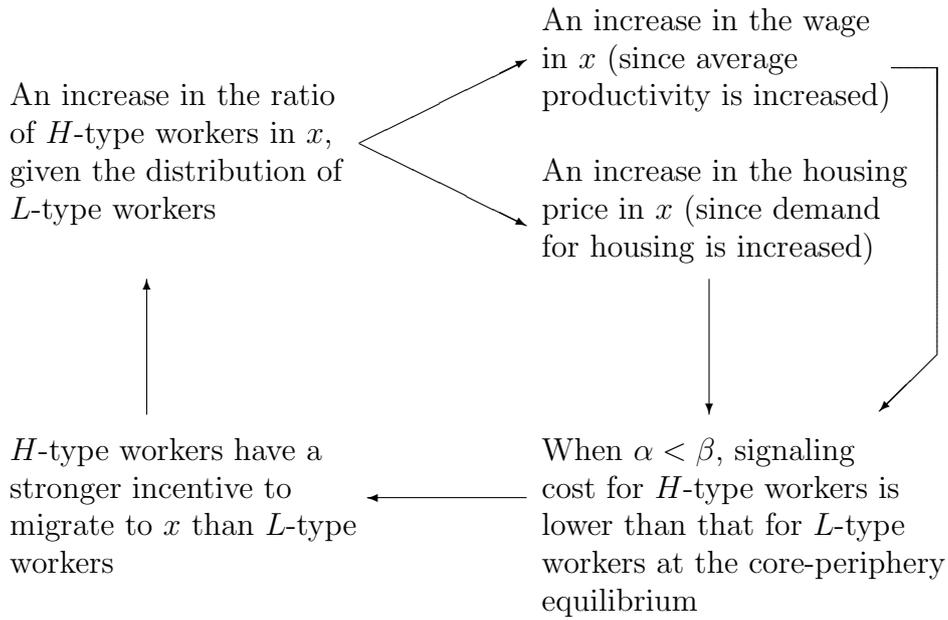


Figure 3: The logic and intuition for the existence of a core-periphery equilibrium when  $\alpha < \beta$ .

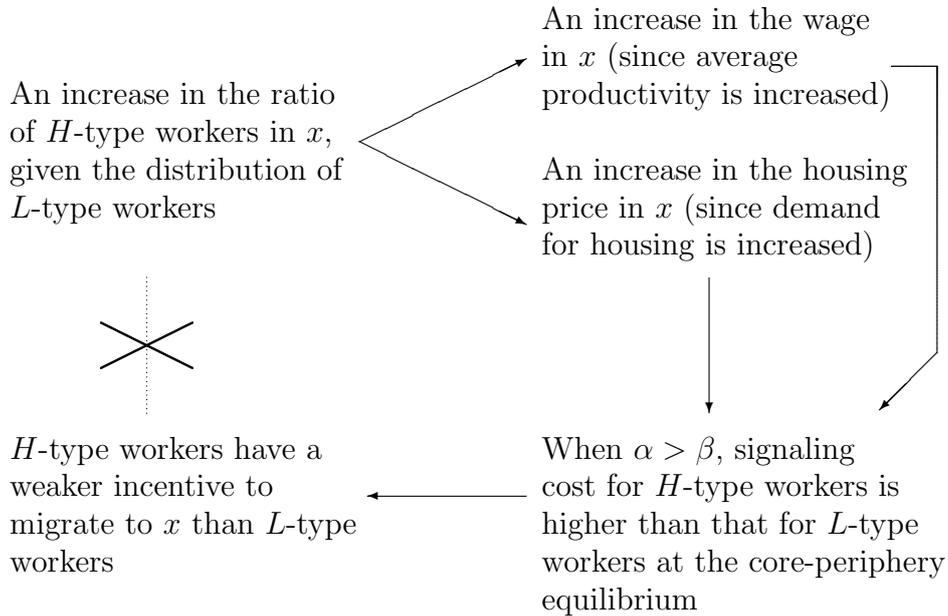


Figure 4: The logic and intuition for the non-existence of a core-periphery equilibrium when  $\alpha > \beta$ .

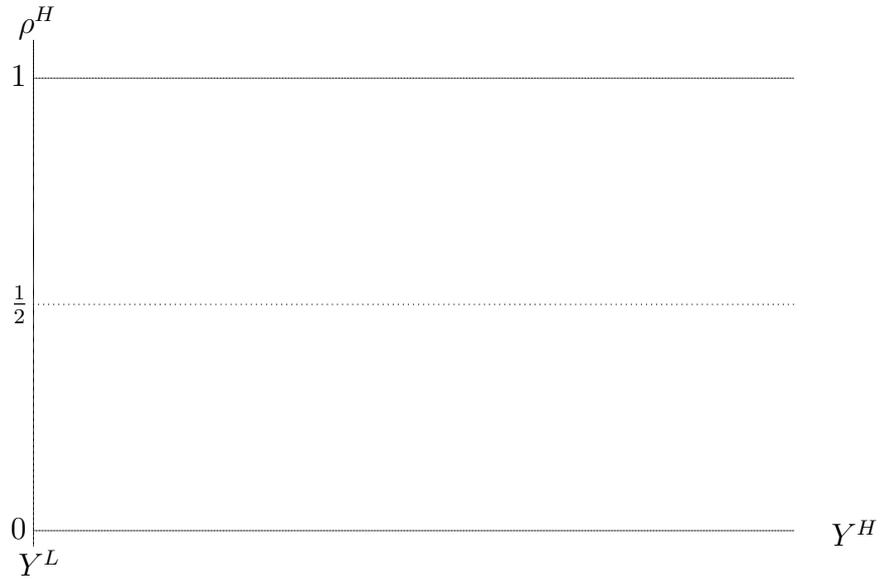


Figure 5: There are core-periphery equilibria when productivity and the marginal willingness to pay for housing are negatively correlated, i.e.,  $\alpha < \beta$ .

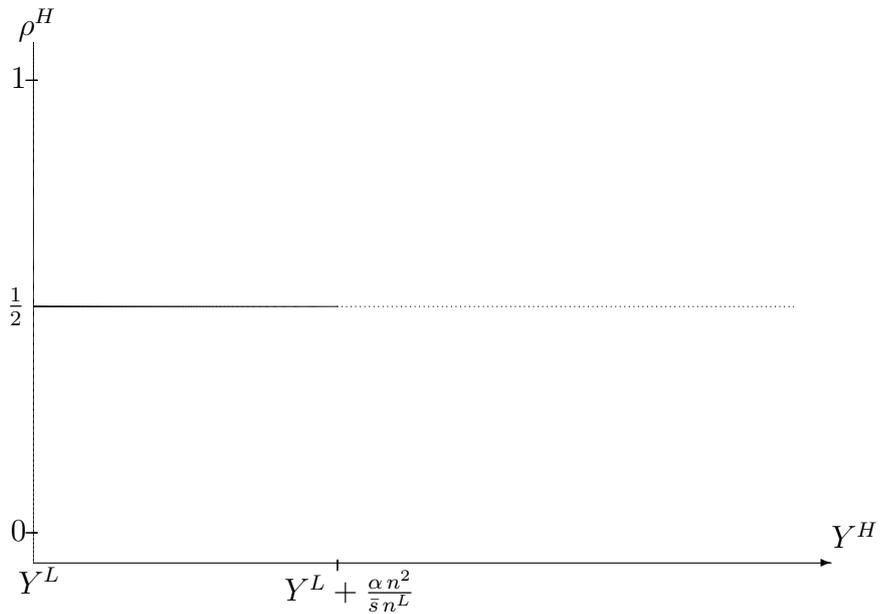


Figure 6: There is no core-periphery equilibrium when productivity and the marginal willingness to pay for housing are positively correlated, i.e.,  $\alpha > \beta$ .