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Abstract:

In this paper, a model of endogenous economic growth is developed with special focus on the interaction between unionized labour market and environmental pollution. We introduce a trade union; and use both 'Efficient Bargaining' model and 'Right to Manage' model to solve the negotiation problem. Environmental pollution is the result of production; and the labour union bargains not only for wage and employment but also for the protection of environment. We derive properties of optimum income tax policy while financing abatement expenditure; and also analyse the effects of unionization on the level of employment and on growth rate. It appears that the optimum rate of income tax varies inversely with the relative bargaining power of the labour union. An increase in the relative bargaining power of the labour union may enhance employment in 'Efficient Bargaining' model if the labour union is highly employment oriented. However, the union always forces the firm to raise the spending rate for environment protection. So, unionisation may raise the growth rate, even if the first effect is negative, but the second effect dominates the first effect.

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1. Introduction:

There exists a vast literature on endogenous growth theory; and a small set of models focuses on the effects of unionisation in the labour market on the long run rate of growth.[1] However, these models do not consider trade union's concern about workers' health and safety and environmental protection. Hence they cannot analyse how unionisation affects the growth rate of the economy through its positive role on environmental protection.

Various empirical works point out that many labour unions fight hard for protection of workers' health and working environment. For example, Gahan (2002) shows that workplace safety always remains in the set of priorities of the union. Khan et al. (2012a) presents evidences to show that labour unions struggle for environmental protection. Khan (2010) and Khan et al. (2012b) also justify trade unions' role to protect environment. Valenduc (2001) points out that labour unions in Belgium have environmental awareness projects. Kawakami et al. (2004) points out that trade unions in Asia organize training workshop to improve workers' safety and health. Stevis (2011) shows that, over the last two decades, labour unions have developed their environmental agendas consistent with their concerns about safety and health.

There are also evidences to establish that labour unions negotiate for workers' health and safety and for environment protection. Gray et al. (1998) studies many private-sector collective bargaining agreements in which health and safety provisions frequently appear. Magane et al. (1997) provides evidences of firm's switching to eco-friendly production techniques due to struggle of labour unions for health and safety. Trade unions force firms to spend for improvement in workers' health and safety condition in the workplace; and this, in turn, leads to improvement of the broader natural environment. Magane et al. (1997), Davies (1993) and Dembo et al. (1988) also think that workplace environment should be seen as part of the broader natural environment. The disasters of Thor Chemicals in South Africa, Union Carbide plant in Bhopal, India, Sandoz warehouse in Basel, Switzerland, Nuclear power plant in Chernobyl, Soviet Union etc. also support the link between the global natural environmental disasters and industrial environment problem in the workplace. These empirical findings and views motivate us to analyse whether trade unions' role to protect workers' health and safety has an impact on environmental quality as well as on economic

[1] The set includes the works of Bräuninger (2000), Lingens (2003a, 2003b), Irmen and Wigger (2002/2003), Sorensen (1997), Palokangas (1996), Chang et al. (2007), Lai and Wang (2010), Ramos-Parren˜o and Sa´nchez-Losada (2002) etc.

growth. To the best of our knowledge, this aspect has not received proper attention in the existing theoretical literature.

A few models analyse properties of optimal income tax used to finance the abatement expenditure in the presence of environmental pollution[2]. However, all these models assume competitive labour market with full employment equilibrium. So it is important to analyse whether optimal tax policy used to finance public abatement expenditure in a unionized economy differs from that in a competitive economy especially when the labour union can force the firm to spend for environmental development.

The present paper is an attempt to analyse both. On the one hand, it analyses the effect of unionisation in the labour market on the long run economic growth rate in the presence of environmental pollution; and, on the other hand, it analyses the properties of an optimum income tax policy designed to finance public abatement expenditure when labour unions bargain for workers' health and safety and for environment development. We consider two alternative bargaining models to analyse the negotiation problem – the 'Efficient bargaining model' of McDonald and Solow (1981) and the 'Right to manage model' of Nickell and Andrews (1983).

We derive interesting results from this model. First, growth rate maximising rate of income tax used to finance public abatement expenditure varies inversely with the relative bargaining power of the labour union. Secondly, how unionisation affects employment depends on the nature of bargaining. In the 'Efficient bargaining model', unionisation raises employment level only if the labour union is highly employment oriented. Otherwise, it always lowers the level of employment. Thirdly, the effect on economic growth depends partly on the employment effect and partly on the effect on employer's spending to protect environment; and this is valid for each of the two bargaining models. Since the environmental protection effect is always positive, it may outweigh the employment effect even if it is negative; and thus unionisation may have a positive effect on economic growth even when unions are wage oriented. Such a result cannot be obtained in Chang et al. (2007) because this positive environment development effect of unionisation does not exist in that model.

The paper is organized as follows. In section 2, we describe the basic model with an 'Efficient Bargaining' theory. Section 3 analyses the properties of the growth rate

[2] Gupta and Barman (2009, 2010, 2013), Barman and Gupta (2010), Economides and Philippopoulos (2008), Greiner (2005) etc.

maximising tax policy. Effects of unionisation on growth rate are analysed in section 4. These results are compared to the corresponding results obtained from the 'Right to Manage' model analysed in section 5. Section 6 concludes the paper.

2. The model:

2.1 Firms:

The representative competitive firm produces the final good, Y, using private capital, K, labour, L, average economy wide stock of capital, $\overline{K}$, and environmental quality, E.[3] The production function of the final good is given by[4]

$$Y = F(K, L, \overline{K}, E) = AK^{\alpha}L^{\beta}\overline{K}^{1-\alpha}E^{\delta} \tag{1}$$

satisfying $\alpha, \beta, \delta \in (0,1)$, $\alpha + \beta < 1$ and $A > 0$.

Existence of decreasing returns to private inputs leads to a positive profit if employers' association owns a positive degree of bargaining power. Following Chang et al. (2007), we assume that a fixed quantity of land is essential for a firm; and thus the number of firms is fixed even in the presence of positive profit.[5]

The firm maximises profit, π, defined as

$$\pi = Y - \gamma Y - wL - rK \quad . \tag{2}$$

Here γ is the rate of firm's expenditure incurred to protect environment. w and r represent the wage rate and rental rate on private capital respectively.

2.2 Capital market:

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[3] An improvement in environmental quality leads to an improvement of health capital of workers and an increase in efficiency of public capital. As Gupta and Barman (2009) writes "*There are various ways by which degradation of environmental quality reduces the effective benefit of public investment expenditure. For example, deforestation reduces rainfall; and this, in turn, reduces the efficiency of the public irrigation programme by reducing the canals' water flow and lowering the recharging rate of groundwater. Poor quality of natural resources (coal) and the lack of current in the river water negatively affect the generation of electricity. Global warming leads to natural disasters like floods, earthquakes, cyclones, etc.; and these, in turn, cause severe damage to infrastructural capitals like roads, electric lines, power plants, buildings, industrial plants, etc. Water pollution and air pollution cause various diseases; and hence the public health expenditure programme fails to provide the desired benefit to the workers which, in turn, lowers their efficiency.*". Gupta and Barman (2009, 2010, 2013), Barman and Gupta (2010), Economides and Philippopoulos (2008), Greiner (2005) etc. include environmental quality as an input in the production function.

[4] Chang et al. (2007) also assumes similar production function where average economy wide stock of capital enters as an input in the production function. However, they do not consider the productive role of environmental quality in the production process.

[5] Number of firms is normalised to unity.

Private capital market is perfectly competitive; and so the supply-demand equality determines the equilibrium value of the perfectly flexible rental rate on capital. Demand function for capital is derived from firms' profit maximizing behaviour; and the inverted demand function is given by

$$r = (1-\gamma)A\alpha K^{\alpha-1}L^{\beta}\overline{K}^{1-\alpha}E^{\delta} = \frac{(1-\gamma)\alpha Y}{K} \quad . \tag{3}$$

2.3 Environment:

Following Greiner (2005), we consider environmental quality, *E*, as a flow variable satisfying public input properties. Following Gupta and Barman (2010), Barman and Gupta (2010), Economides and Philippopoulos (2008) and Greiner (2005), we assume that production of the final good is the only source of emission[6]; and public abatement expenditure incurred by the government can improve environmental quality. However, we also consider firms' role to protect environment. Firms are forced to spend for environmental development by the bargaining power of the labour union. For example, firms may use costly eco-friendly techniques of production or may allocate resources to non-productive activities for the sake of workers' health and safety. This aspect has not been considered in the existing literature on labour unions. The environmental quality function is given by

$$E = E(\tau_E Y, \gamma Y, vY) \text{ with } E_1 > 0, \quad E_2 > 0 \text{ and } E_3 < 0 \quad . \tag{4}$$

Here τ_E is the rate of income tax used to finance public abatement expenditure and v is the emission - output coefficient. We specify a simple functional form given by

$$E = (\tau_E Y + \gamma Y)^{\mu}(vY)^{-\mu} = (\tau_E + \gamma)^{\mu} v^{-\mu} \quad . \tag{4.a}$$

Here $((\tau_E + \gamma)/v)$ represents the effective abatement activity per unit of pollution; and $\mu > 0$ is the elasticity of environmental quality with respect to this argument.

2.4 Labour union's utility function:

The labour union derives utility from three arguments: (i) the hike in the wage rate over the competitive wage rate,[7] (ii) level of employment and (iii) firm's spending rate to

[6] There may be many other sources, for example - consumption of pollution intensive goods, extracting natural resources etc.

[7] Some works assume that the difference between the bargained wage rate and the unemployment benefit is an argument in the labour union's utility function. Contrary to this, Irmen and Wigger (2003), Lingens (2003a) and Lai and Wang (2010) assume that the difference between the bargained wage rate and the competitive wage rate

protect environment. The third argument is generally not considered in the existing literature. The utility function is given by

$$u_T = (w - w_c)^{\varepsilon_1} L^{\varepsilon_2} \gamma^{\varepsilon_3} \text{ with } \varepsilon_j > 0 \text{ for } j = 1,2,3 \ . \tag{5}$$

Here u_T and w_c stand for the utility of the labour union and the competitive wage rate respectively. ε_1, ε_2 and ε_3 are three non-negative parameters representing degrees of orientation of the labour union towards those arguments. If $\varepsilon_j = Max\{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$, then the labour union is called *jth* argument oriented.

In a competitive labour market, perfectly flexible wage rate is equated to the marginal productivity of labour and labour is fully employed. The firm is not forced to spend for environmental development; and hence $\gamma = 0$. So, with labour endowment being normalized to unity, we have

$$w_c = \beta A K^{\alpha} \bar{K}^{1-\alpha} E^{\delta} \ . \tag{6}$$

2.5 The 'Efficient bargaining model':

In this section, we choose the 'Efficient bargaining model' where wage rate, level of employment and the rate of firm's spending to protect environment are determined jointly by the labour union and the employer's association; and they maximize the 'generalised Nash product' function given by

$$\psi = (u_T - \overline{u_T})^{\theta} (\pi - \bar{\pi})^{(1-\theta)} \ . \tag{7}$$

Here $\overline{u_T}$ and $\bar{\pi}$ symbolize the reservation utility level of the labour union and the reservation profit level of the firm respectively. Bargaining disagreement stops production and hence results into zero employment, which, in turn, implies that $\overline{u_T} = 0$ and $\bar{\pi} = 0$. $\theta \in (0,1)$ represents the relative bargaining power of the labour union. Unionisation in the labour market implies an exogenous increase in the value of θ.

Using equations (2) and (3), we obtain

$$\pi = (1 - \gamma)(1 - \alpha)Y - wL \ . \tag{8}$$

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is an argument in the labour union's utility function. Since the provision of unemployment benefit is not considered in this model, so we incorporate the difference between the bargained wage rate and the competitive wage rate as an argument in the labour union's utility function.

Finally, using equations (5), (7) and (8), we obtain

$$\psi = \{(w - w_c)^{\varepsilon_1} L^{\varepsilon_2} \gamma^{\varepsilon_3}\}^{\theta} \{(1-\gamma)(1-\alpha)Y - wL\}^{(1-\theta)} \quad . \tag{9}$$

Here ψ is to be maximised with respect to w, L and γ. Using equations (1) and (6), and the three first order conditions of optimisation, we solve for optimal w, L and γ. These are given by[8]

$$L^* = \left\{\frac{(1-\alpha)\{\beta(1-\theta+\theta\varepsilon_1) - \theta(\varepsilon_1 - \varepsilon_2)\}}{\beta\{(1-\theta+\theta\varepsilon_2) + \theta\varepsilon_3(1-\beta)\}}\right\}^{\frac{1}{1-\beta}} ; \tag{10}$$

$$w^* = \frac{\{\beta(1-\theta) + \theta\varepsilon_2\} w_c}{\{\beta(1-\theta) + \theta\varepsilon_2 - \theta\varepsilon_1(1-\beta)\}} \quad ; \tag{11}$$

and

$$\gamma^* = \frac{\theta\varepsilon_3(1-\beta)}{(1-\theta+\theta\varepsilon_2) + \theta\varepsilon_3(1-\beta)} \quad . \tag{12}$$

To ensure positive values of L^* and w^* and to ensure $L^* < 1$, we need a parametric restriction. This is given by

$$Condition\ A: \qquad -\ \beta(1-\theta+\theta\varepsilon_1) < \theta(\varepsilon_2 - \varepsilon_1) < \frac{\alpha\beta(1-\theta+\theta\varepsilon_1) + \theta\varepsilon_3\beta(1-\beta)}{(1-\alpha-\beta)} \ .$$

From equation (10), we obtain

$$\frac{\partial L^*}{\partial \theta} = \frac{[\varepsilon_2 - \varepsilon_1 - \beta\varepsilon_3] L^*}{\{(1-\theta+\theta\varepsilon_2) + \theta\varepsilon_3(1-\beta)\}\{\beta(1-\theta+\theta\varepsilon_1) - \theta(\varepsilon_1 - \varepsilon_2)\}} \quad . \tag{10.a}$$

Here the denominator in the R.H.S. of equation (10.a) is always positive. So $\frac{\partial L^*}{\partial \theta} > 0$ if and only if $\varepsilon_2 > \varepsilon_1 + \beta\varepsilon_3$. This means that an increase in union's relative bargaining power may raise the employment level if the union is highly employment oriented.

Chang et al. (2007) does not consider trade union's concern about environment development; and hence $\varepsilon_3 = 0$ there. So sign of $\left(\frac{\partial L^*}{\partial \theta}\right)$ depends solely on the sign of $(\varepsilon_2 - \varepsilon_1)$. However, in our analysis, $\varepsilon_3 > 0$ and the nature of the employment effect depends on the magnitude of ε_3.

[8] See Appendix A for derivation.

The intuition behind this can be explained as follows. The labour union wants to raise L with a rise in θ if it obtains a marginal utility of L higher than its marginal opportunity cost. In the case of Chang et al. (2007), i.e. in the absence of trade union's concern for environment development, opportunity cost of raising L is same as the loss in utility from not raising w. However, in the present model, this opportunity cost also includes the loss in utility from not raising γ. Hence ε_3 enters into the picture.

Equation (11) shows that the negotiated wage rate, w^*, exceeds the competitive equilibrium wage rate, w_c. From this equation, we obtain

$$\frac{\partial w^*}{\partial \theta} = \frac{\{\beta\varepsilon_1(1-\beta)\}w_c}{\{\beta(1-\theta)+\theta\varepsilon_2-\theta\varepsilon_1(1-\beta)\}^2} > 0 \quad . \tag{11.a}$$

So w^* varies positively with θ. This is obvious because the labour union always derives higher utility from a higher wage; and so it receives a higher wage with a greater bargaining power.

From equation (12), we obtain

$$\frac{\partial \gamma^*}{\partial \theta} = \frac{\varepsilon_3(1-\beta)}{\{(1-\theta+\theta\varepsilon_2)+\theta\varepsilon_3(1-\beta)\}^2} > 0 \quad . \tag{12.a}$$

An increase in θ forces the firm to spend a higher fraction of output on environment development because the labour union always derives higher utility from a higher value of γ.

Second order conditions of maximization of ψ are also satisfied[9]; and we now state the following proposition.

Proposition 1: *Unionisation defined as an increase in the relative bargaining power of the labour union always raises the wage rate as well as the firm's spending rate to protect environment but raises employment level only if the labour union is highly employment oriented.*

2.6 Households:

The representative household derives instantaneous utility only from consumption of the final good.[10] She maximises her discounted present value of instantaneous utility over the

[9] See Appendix A for derivation.

infinite time horizon subject to the intertemporal budget constraint. The household's problem is given by the following.

$$Max \int_0^\infty \frac{c^{1-\sigma}-1}{1-\sigma} e^{-\rho t}\, dt \tag{13}$$

subject to, $\dot{K} = (1-\tau_E)Y - c$; (14)

$K(0) = K_0$ (K_0 is historically given)

and $c \in [0, (1-\tau_E)Y]$.

Here c denotes the level of consumption of the representative household. c is the control variable and K is the state variable. σ and ρ are the elasticity of marginal utility with respect to consumption and the constant rate of discount of consumption respectively. Savings is always invested; and private capital does not depreciate.

Solving this dynamic optimisation problem, we obtain the growth rate of consumption given by

$$g = \frac{\dot{c}}{c} = \frac{(1-\tau_E)A\alpha\left(\frac{\bar{K}}{K}\right)^{1-\alpha} L^\beta E^\delta - \rho}{\sigma} . \tag{15}$$

2.8 Equilibrium:

At the symmetric equilibrium, $\bar{K} = K$; and hence, from equations (3), (6) and (15), we obtain

$$r = (1-\gamma)A\alpha L^\beta E^\delta \quad ; \tag{3.a}$$

$$w_c = \beta AKE^\delta \tag{6.a}$$

and

$$g = \frac{\dot{c}}{c} = \frac{(1-\tau_E)A\alpha L^\beta E^\delta - \rho}{\sigma} . \tag{15.a}$$

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[10] We assume that households supply constant amount of labour; and so labour - leisure choice of representative household is ruled out. We also do not consider environmental quality as an argument in household's utility function for the sake of simplicity even though we incorporate trade union's concern about environmental effects. This exclusion is a restrictive one.

It looks like an *AK* model and there is no transitional dynamics. At equilibrium, employment of labour, tax rate, rental rate of capital, environmental quality, γ^* all are time-independent. So the growth rate of consumption given by equation (15.a) is also time-independent. Capital stock, *K*, final output, *Y*, negotiated wage rate, w^*, firm's profit, *π,* also grow at that rate in the steady-state equilibrium.

3. Optimal tax rate:

We first derive the growth rate maximising income tax rate. Using equations (4.a) and (15.a), we obtain the growth rate maximising income tax rate, ${\tau_E}^*$, given by[11]

$$ {\tau_E}^* = \left(\frac{\mu\delta - \gamma^*}{1+\mu\delta}\right) \quad . \tag{16} $$

We assume the following parametric restriction to ensure that $0 < {\tau_E}^* < 1$.

$$ \mu\delta > \frac{\theta\varepsilon_3(1-\beta)}{\theta\varepsilon_3(1-\beta) + (1-\theta+\theta\varepsilon_2)} \quad . $$

Using equations (12) and (16), we obtain

$$ {\tau_E}^* = \left\{\frac{\mu\delta[(1-\theta+\theta\varepsilon_2)+\theta\varepsilon_3(1-\beta)] - \theta\varepsilon_3(1-\beta)}{(1+\mu\delta)[(1-\theta+\theta\varepsilon_2)+\theta\varepsilon_3(1-\beta)]}\right\} \quad . \tag{16.a} $$

From equation (16.a), we obtain

$$ \frac{\partial {\tau_E}^*}{\partial\theta} = -\frac{\varepsilon_3(1-\beta)}{\{(1-\theta+\theta\varepsilon_2)+\theta\varepsilon_3(1-\beta)\}^2(1+\mu\delta)} < 0 \quad . \tag{17} $$

Equation (17) implies that an increase in θ lowers ${\tau_E}^*$. This is so because this raises γ^* which, in turn, upgrades environmental quality and thus lowers the marginal benefit of public abatement expenditure.

Proposition 2: *Optimum rate of income tax used to finance public abatement expenditure varies inversely with the bargaining power of the labour union.*

Here we do not incorporate environmental quality in the household's utility function. So there exists a positive monotonic relationship between the growth rate and welfare level. So the growth rate maximising tax rate is identical to the welfare maximising tax rate.

[11] Second order condition of maximisation of the balanced growth rate is satisfied.

4. Growth effect of unionisation:

We now analyse the effect of an increase in θ on the endogenous growth rate. Using equations (4.a), (16.a) and (15.a) and putting $\gamma = \gamma^*$ and $L = L^*$, we obtain

$$g^* = \frac{A\alpha L^{*\beta}\left(\frac{\mu\delta}{v}\right)^{\mu\delta}\frac{(1+\gamma^*)^{\mu\delta+1}}{(1+\mu\delta)^{\mu\delta+1}} - \rho}{\sigma} \quad . \tag{18}$$

From equation (18), we have

$$\frac{\partial g^*}{\partial \theta} = \frac{A\alpha L^{*\beta}\left(\frac{\mu\delta}{v}\right)^{\mu\delta}\frac{(1+\gamma^*)^{\mu\delta+1}}{(1+\mu\delta)^{\mu\delta+1}}}{\sigma}\left[\beta\frac{\frac{\partial L^*}{\partial \theta}}{L^*} + (1+\mu\delta)\frac{\frac{\partial \gamma^*}{\partial \theta}}{(1+\gamma^*)}\right] \quad . \tag{19}$$

Equation (19) shows that the effect of an increase in θ on the growth rate, g^*, is ambiguous. The first term inside the bracket on the right hand side of equation (19) represents the employment effect on growth due to unionisation. Its sign is determined by the sign of $\frac{\partial L^*}{\partial \theta}$; and so it depends on the nature of labour union's orientation towards arguments in its utility function. However, the second term inside this bracket is definitely positive because equation (12.a) shows that $\frac{\partial \gamma^*}{\partial \theta} > 0$. This term represents the environment development effect on growth due to unionisation. So the effect of unionisation on the growth rate may be qualitatively different from its employment effect.

In Chang et al. (2007), the environment development effect on growth does not exist. Hence the growth effect of unionisation is qualitatively identical to the employment effect in that model; and hence its nature is determined by the nature of orientation of the labour union. So the growth effect is positive (negative) when the union is employment (wage) oriented. However, in the present model where environment development effect exists, nature of the orientation of the labour union alone cannot determine the nature of the growth effect. If the environment development effect dominates the employment effect, then unionisation always raises the growth rate regardless of the nature of orientation of the labour union. Growth effect may be positive even if the employment effect is negative, i.e., if the union is wage oriented.

We can establish the following proposition.

Proposition 3: *Unionisation in the labour market must (may) raise the endogenous growth rate if the labour union is employment (wage) oriented.*

Since there exists a positive monotonic relationship between growth rate and welfare, welfare effects of unionisation are identical to its growth effects.

5. The 'Right to manage model' case:

In the 'Right to manage model', two parties bargain over *w* and γ. The individual firm determines *L* from its labour demand function derived from its profit maximising behaviour; and it is given by

$$w = (1-\gamma)\beta A K^{\alpha}\bar{K}^{1-\alpha}E^{\delta}L^{\beta-1} = (1-\gamma)w_c L^{\beta-1} \quad . \tag{20}$$

Using equations (8) and (20), we have

$$\pi = (1-\gamma)(1-\alpha-\beta)Y \quad ; \tag{21}$$

and hence the 'generalised Nash product' function is obtained as follows.

$$\psi = \{(w-w_c)^{\varepsilon_1}L^{\varepsilon_2}\gamma^{\varepsilon_3}\}^{\theta}\{(1-\gamma)(1-\alpha-\beta)Y\}^{(1-\theta)} \quad . \tag{22}$$

In this model, ψ is to be maximised with respect to *w* and γ, subject to equation (20). Optimum values of *w* and γ are same as those obtained in the 'Efficient bargaining model'. However, employment level is different; and is given by[12]

$$L^{**} = \left\{\frac{(1-\theta+\theta\varepsilon_2)\{\beta(1-\theta+\theta\varepsilon_1)-\theta(\varepsilon_1-\varepsilon_2)\}}{\{\beta(1-\theta)+\theta\varepsilon_2\}[(1-\theta+\theta\varepsilon_2)+\theta\varepsilon_3(1-\beta)]}\right\}^{\frac{1}{1-\beta}} \quad . \tag{23}$$

Comparing equation (23) to equation (10) we find that $L^{**} \neq L^{*}$.

Condition A guarantees that $L^{**} > 0$; equation (23) clearly shows that $L^{**} < 1$ because $\beta(1-\theta+\theta\varepsilon_1) > \theta(\varepsilon_1-\varepsilon_2)$. Second order conditions of maximisation are also satisfied.

Both government's objective and representative household's objective are same in both the models. So equations and solutions derived are also same in these two models. Optimal tax rate is also identical.

From equation (23), we have

[12] See Appendix C for detailed derivations of the section 5.

$$\frac{\partial L^{**}}{\partial \theta} = -\Big[\frac{\varepsilon_3(1-\beta)}{(1-\theta+\theta\varepsilon_2)[(1-\theta+\theta\varepsilon_2)+\theta\varepsilon_3(1-\beta)]} + \frac{\beta\varepsilon_1(1-\beta)}{\{\beta(1-\theta+\theta\varepsilon_1)-\theta(\varepsilon_1-\varepsilon_2)\}\{\beta(1-\theta)+\theta\varepsilon_2\}}\Big] < 0 \quad . \tag{24}$$

So an increase in θ definitely lowers L^{**}; and this implies that employment effect of unionisation is negative. This result is contradictory to the result obtained in the 'Efficient bargaining model' where the nature of the employment effect depends on the mathematical sign of $(\varepsilon_2 - \varepsilon_1 - \beta\varepsilon_3)$.

Effects of unionisation on the wage rate, firm's environment development expenditure, and on optimum tax rates in this model are qualitatively similar to corresponding effects obtained in the previous model. Equation (18) is otherwise valid here except that L^* is replaced by L^{**}. So the effect of unionisation on the rate of growth is given by

$$\frac{\partial g^{**}}{\partial \theta} = \frac{A\alpha L^{**\beta}\left(\frac{\mu\delta}{v}\right)^{\mu\delta}\frac{(1+\gamma^*)^{\mu\delta+1}}{(1+\mu\delta)^{\mu\delta+1}}}{\sigma}\left[\beta\frac{\frac{\partial L^{**}}{\partial\theta}}{L^{**}} + (1+\mu\delta)\frac{\frac{\partial\gamma^*}{\partial\theta}}{(1+\gamma^*)}\right] \quad . \tag{25}$$

Here the first term and the second term of the right hand side of equation (25) are negative and positive respectively. So unionisation on the one hand lowers the growth rate through negative employment effect and on the other hand raises it through positive environment development effect; and the net effect depends on the relative strength of these two. Important results are summarized in the following proposition.

Proposition 4: *If the 'Right to manage model' of bargaining is introduced, unionisation in the labour market always lowers employment level regardless of its nature of orientation; and so unionisation raises the growth rate if and only if the positive environment development effect dominates the negative employment effect.*

6. Conclusions:

This paper, on the one hand, investigates the effect of unionisation in the labour market on the long run growth rate of an economy in the presence of environmental pollution and trade union's concern about environment development, and, on the other hand, derives properties of the optimum income tax policy designed to finance public abatement expenditure. Here we use two alternative versions of bargaining models – the 'Efficient bargaining model' of McDonald and Solow (1981) and the 'Right to manage model' of Nickell and Andrews (1983).

Our major findings are as follows. First, negotiated wage rate and firm's spending rate to protect environment varies positively with degree of unionisation in the labour market. Secondly, the optimum rate of income tax used to finance public abatement expenditure varies inversely with this degree of unionisation. Existing literature on unionisation and growth does not focus on the sensitivity of optimal tax policy to the degree of unionisation. These two results hold for both versions of bargaining models. Thirdly, effects of unionisation on employment level and on economic growth depend on the nature of the bargaining model considered. In the case of 'Efficient bargaining model' unionisation may raise employment level only if the labour union is highly employment oriented. Otherwise, it is always harmful for the level of employment. Effect of unionisation on the long run growth rate partly depends on its employment effect and partly on its environment development effect. If the positive second effect dominates the negative first effect, then unionisation produces net positive effect on growth and on welfare. The importance of this positive environment development effect is not discussed in the existing theoretical literature. However, results of Chang et al. (2007) appear to be special cases of our results obtained in the absence of environment development effect.

However, our model is abstract and fails to consider many aspects of reality. We rule out the possibility of human capital accumulation, population growth, technological progress etc. Hence the allocation of government's budget and household's income towards education, R&D etc. is not analysed here. We assume 'closed shop union' for simplicity and do not consider 'open shop union', which is more common in reality. We plan to do further research in future attempting to get rid of these limitations.

References

Barman, T. R., & Gupta, M. R. (2010). Public Expenditure, Environment, and Economic Growth. Journal of Public Economic Theory, 12(6) , 1109–1134.

Bräuninger, M. (2000). Wage Bargaining, Unemployment, and Growth. Journal of Institutional and Theoretical Economics, 156(4) , 646-660.

Chang, J.-j., Shaw, M.-f., & Lai, C.-c. (2007). A ‘‘Managerial’’ trade union and economic growth. European Economic Review, 51(2) , 365-384.

Davies, A. (1993). The workplace environment continuum. In D. Hallowes, Hidden faces: environment, development, justice: South Africa and the global context (pp. 70-73). Scottsville: Earthlife Africa, Scottsville.

Dembo, D., Dias, D., Kadwani, A., & Morehouse, W. (1988). Nothing to lose but our lives: empowerment to oppose industrial hazards in a transnational world. New Horizons Press, New York.

Economides, G., & Philippopoulos, A. (2008). Growth enhancing policy is the means to sustain the environment. Review of Economic Dynamics, 11(1) , 207–219.

Gahan, P. G. (2002). (What) do unions maximise? Evidence from survey data. Cambridge Journal of Economics, 26(3) , 279-297.

Gray, G. R., Myers, D. W., & Myers, P. S. (1998). Collective Bargaining Agreements: Safety and Health Provisions. Monthly Labor Review, 121, 13 .

Greiner, A. (2005). Fiscal Policy in an Endogenous Growth Model with Public Capital and Pollution. Japanese Economic Review, 56(1) , 67–84.

Gupta, M. R., & Barman, T. R. (2013). Endogenous growth with environmental pollution and depreciation of public capital : a theoretical note. Keio economic studies 49 , 93-103.

Gupta, M. R., & Barman, T. R. (2009). Fiscal policies, environmental pollution and economic growth. Economic Modelling, 26(5) , 1018–1028.

Gupta, M. R., & Barman, T. R. (2010). Health, infrastructure, environment and endogenous growth. Journal of Macroeconomics, 32(2) , 657–673.

Irmen, A., & Wigger, B. U. (2002/2003). Trade Union Objectives and Economic Growth. FinanzArchiv / Public Finance Analysis, 59(1) , 49-67.

Kawakami, T., Kogi, K., Toyama, N., & Yoshikawa, T. (2004). Participatory approaches to improving safety and health under trade union initiative--experiences of POSITIVE training program in Asia. Industrial Health, 42(2) , 196–206.

Khan, M. T. (2010). Role of Labor Unions Beyond Collective Bargaining. Far East Journal of Psychology and Business, 1(1) , 78-90.

Khan, M. T., Khan, N. A., Khan, T. M., & Khan, W. A. (2012b). Environmental Problem and Labor Unions’ Efforts for Environment Friendly Generation, Management and Use of Energy – A Literature Review. International Journal of Business and Behavioral Sciences, 2(5) , 32-40.

Khan, M. T., Khan, N. A., Khan, T. M., & Khan, W. A. (2012a). Labor Unions' Role for Environmental Protection - by Exerting Efforts for Legislation and Alliance Making With Environmental Activists. Far East Journal of Psychology and Business, 8(1) , 43-57.

Lai, C.-H., & Wang, V. (2010). The effects of unionization in an R&D growth model with (In)determinate equilibrium. MPRA Paper 27748. University Library of Munich, Germany.

Lingens, J. (2003a). The impact of a unionised labour market in a Schumpeterian growth model. Labour Economics, 10(1) , 91-104.

Lingens, J. (2003b). Unionisation, Growth and Endogenous Skill-Formation. Economics Discussion Papers No. 45/03. University of Kassel, Institute of Economics, Germany.

Magane, P., Miller, S., Goldblatt, M., & Bethlehem, L. (1997). Unions and Environment: Life, health and the pursuit of employment. In L. Bethlehem, & M. Goldblatt, The Bottom Line: Industry and the Environment in South Africa (pp. 176-193). UCT Press.

McDonald, I. M., & Solow, R. M. (1981). Wage Bargaining and Employment. The American Economic Review, 71(5) , 896-908.

Nickell, S. J., & Andrews, M. (1983). Unions, Real Wages and Employment in Britain 1951-79. Oxford Economic Papers, New Series, 35 , 183-206.

Palokangas, T. (1996). Endogenous growth and collective bargaining. Journal of Economic Dynamics and Control, 20(5) , 925-944.

Ramos Parreño, J. M., & Sánchez-Losada, F. (2002). The role of unions in an endogenous growth model with human capital. Journal of Macroeconomics, 24(2) , 171–192.

Sorensen, J. R. (1997). Do Trade Unions Actually Worsen Economic Performance? Working Paper No. 1997-6. School of Economics and Management, University of Aarhus.

Stevis, D. (2011). Unions and the Environment : Pathways to Global Labor Environmentalism . WorkingUSA, 14(2) , 145–159.

Valenduc, G. (2001). Trade unions as agents of environmental awareness : Outcomes from the RISE project. In E. Hildebrandt, B. Lorentzen, & E. Schmidt, Towards a Sustainable Worklife (pp. 55-66). Sigma, Berlin.

Appendix:

Appendix A: 'Efficient bargaining model'

First order conditions:

From equations (9) and (1), we have

$$log\psi = \theta\varepsilon_1\, log(w - w_c) + \theta\varepsilon_2\, logL + \theta\varepsilon_3\, log\gamma$$
$$+(1-\theta)\, log\{(1-\gamma)(1-\alpha)AK^{\alpha}L^{\beta}\bar{K}^{1-\alpha}E^{\delta} - wL\} \quad . \qquad (A.1)$$

The first order conditions of maximization of log ψ are given by the followings.

$$\frac{\theta\varepsilon_1}{w - w_c} + \frac{(1-\theta)(-L)}{\{(1-\gamma)(1-\alpha)AK^{\alpha}L^{\beta}\bar{K}^{1-\alpha}E^{\delta} - wL\}} = 0 \quad ; \qquad (A.2)$$

$$\frac{\theta\varepsilon_2}{L} + \frac{(1-\theta)\{(1-\gamma)(1-\alpha)\beta AK^{\alpha}L^{\beta-1}\bar{K}^{1-\alpha}E^{\delta} - w\}}{\{(1-\gamma)(1-\alpha)AK^{\alpha}L^{\beta}\bar{K}^{1-\alpha}E^{\delta} - wL\}} = 0 \quad ; \qquad (A.3)$$

and

$$\frac{\theta\varepsilon_3}{\gamma} + \frac{(1-\theta)\{-(1-\alpha)AK^{\alpha}L^{\beta}\bar{K}^{1-\alpha}E^{\delta}\}}{\{(1-\gamma)(1-\alpha)AK^{\alpha}L^{\beta}\bar{K}^{1-\alpha}E^{\delta} - wL\}} = 0 \quad . \qquad (A.4)$$

Now using equations (A.2), (A.3) and (6), we have

$$\Rightarrow \quad (\varepsilon_1 - \varepsilon_2)w = \varepsilon_1(1-\gamma)(1-\alpha)L^{\beta-1}w_c - \varepsilon_2 w_c \quad . \qquad (A.5)$$

From equations (A.2) and (6), we obtain

$$\theta\varepsilon_1\left\{(1-\gamma)(1-\alpha)L^{\beta-1}\frac{w_c}{\beta} - w\right\} = (1-\theta)(w - w_c) \quad . \qquad (A.6)$$

Using Equations (A.5) and (A.6), we obtain

$$L^{\beta-1} = \frac{\beta\theta\varepsilon_2 + \beta(1-\theta)}{(1-\gamma)(1-\alpha)\{\beta(1-\theta) + \beta\theta\varepsilon_1 - \theta(\varepsilon_1 - \varepsilon_2)\}} \quad . \qquad (A.7)$$

From equation (A.4), we obtain

$$\frac{\theta\varepsilon_3}{\gamma} = \frac{(1-\theta)\{(1-\alpha)AK^{\alpha}L^{\beta-1}\bar{K}^{1-\alpha}E^{\delta}\}}{\{(1-\gamma)(1-\alpha)AK^{\alpha}L^{\beta-1}\bar{K}^{1-\alpha}E^{\delta} - w\}} \quad . \qquad (A.8)$$

Using equations (A.5), (A.7), (A.8) and (6), we obtain

$$\Rightarrow \quad \gamma = \frac{\{1-\beta\}\theta\varepsilon_3}{\{\theta\varepsilon_2 + (1-\theta)\} + \theta\varepsilon_3\{1-\beta\}} \quad . \qquad (A.9)$$

Equation (A.9) is identical to equation (12) in the body of the paper.

Using equations (A.9) and (A.7), we have

$$L^{\beta-1} = \frac{\beta[\theta\varepsilon_2 + (1-\theta) + \theta\varepsilon_3\{1-\beta\}]}{(1-\alpha)\{\beta(1-\theta) + \beta\theta\varepsilon_1 - \theta(\varepsilon_1 - \varepsilon_2)\}} \quad . \qquad (A.10)$$

From equation (A.10), we obtain equation (10) in the body of the paper.

Using equations (A.7) and (A.5), we have

$$w = \frac{\beta(1-\theta) + \theta\varepsilon_2}{\{\beta(1-\theta) + \beta\theta\varepsilon_1 - \theta(\varepsilon_1 - \varepsilon_2)\}}w_c \quad . \qquad (A.11)$$

Equation (A.11) is identical to equation (11) in the body of the paper.

Derivation of Condition A:

We derive Condition A as follows. To ensure positive values of L^* and w^*, we need the following parametric restriction.

$Condition\ A_1$: $\quad \beta(1-\theta+\theta\varepsilon_1) > \theta(\varepsilon_1-\varepsilon_2)$[13] .

Again, labour employment has to be less than its endowment i.e., $L^* < 1$; and so the following parametric restriction is needed.

$Condition\ A_2$: $\quad (1-\alpha-\beta)\theta(\varepsilon_2-\varepsilon_1) < \alpha\beta(1-\theta+\theta\varepsilon_1) + \theta\varepsilon_3\beta(1-\beta)$.

Combining these two conditions A_1 and A_2, we obtain condition A given in the body of the paper.

Second order conditions:

Using equations (A.2) and (6), we have

$$\frac{\partial^2 Log(\psi)}{\partial w^2} = -\frac{\theta\varepsilon_1}{(w-w_c)^2} - \frac{(1-\theta)}{\left\{(1-\gamma)(1-\alpha)L^{\beta-1}\frac{w_c}{\beta} - w\right\}^2} < 0\ . \tag{A.12}$$

From equations (A.3), (6), (A.7) and (A.11), we have

$$\frac{\partial^2 Log(\psi)}{\partial L^2} = -\frac{\theta\varepsilon_2}{L^2} - (1-\theta)\ .\left[\frac{\left\{\begin{array}{c}\frac{(1-\gamma)(1-\alpha)(1-\beta)w_c L^{\beta-1}(1-\beta)(1-\theta)w_c}{\{\beta(1-\theta)+\beta\theta\varepsilon_1-\theta(\varepsilon_1-\varepsilon_2)\}} \\ +\left\{(1-\gamma)(1-\alpha)L^{\beta-1}w_c - w\right\}^2\end{array}\right\}}{\left\{(1-\gamma)(1-\alpha)L^{\beta}\frac{w_c}{\beta} - wL\right\}^2}\right] < 0\ . \tag{A.13}$$

From equations (A.4) and (6), we have

$$\frac{\partial^2 Log(\psi)}{\partial \gamma^2} = -\frac{\theta\varepsilon_3}{\gamma^2} - \frac{(1-\theta)\left\{(1-\alpha)L^{\beta-1}\frac{w_c}{\beta}\right\}^2}{\left\{(1-\gamma)(1-\alpha)L^{\beta-1}\frac{w_c}{\beta} - w\right\}^2} < 0\ . \tag{A.14}$$

Now, using equations (A.12), (A.11) and (A.7), we have

$$\frac{\partial^2 Log(\psi)}{\partial w^2} = -\frac{\{\beta(1-\theta)+\beta\theta\varepsilon_1-\theta(\varepsilon_1-\varepsilon_2)\}^2(1-\theta+\theta\varepsilon_1)}{{w_c}^2(1-\beta)^2\theta\varepsilon_1(1-\theta)}\ . \tag{A.15}$$

Using equations (A.13), (A.11) and (A.7), we have

$$\frac{\partial^2 Log(\psi)}{\partial L^2} = -\frac{(1-\theta+\theta\varepsilon_2)[\beta(1-\theta)+\theta\varepsilon_2]}{L^2(1-\theta)}\ . \tag{A.16}$$

Again using equations (A.14), (A.9), (A.11) and (A.7), we have

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[13] Condition A_1 also ensures that negotiated wage rate is higher than the competitive wage rate.

$$\frac{\partial^2 Log(\psi)}{\partial \gamma^2} = -\frac{[\{\theta\varepsilon_2 + (1-\theta)\} + \theta\varepsilon_3\{1-\beta\}]^2(1-\theta+\theta\varepsilon_3)}{(1-\beta)^2(1-\theta)\theta\varepsilon_3} \quad . \tag{A.17}$$

Now from equations (A.2), (A.7), (A.11) and (6), we have

$$\frac{\partial^2 Log(\psi)}{\partial L\, \partial \mathrm{w}} = -\frac{[1-\theta+\theta\varepsilon_2]\{\beta(1-\theta)+\beta\theta\varepsilon_1 - \theta(\varepsilon_1-\varepsilon_2)\}}{L(1-\theta)w_c(1-\beta)} \quad . \tag{A.18}$$

Using equations (A.15), (A.16) and (A.18), we have

$$\frac{\partial^2 Log(\psi)}{\partial L^2} . \frac{\partial^2 Log(\psi)}{\partial w^2} - \left[\frac{\partial^2 Log(\psi)}{\partial L\, \partial \mathrm{w}}\right]^2$$

$$= \frac{\{\beta(1-\theta)+\theta\varepsilon_2-\theta\varepsilon_1(1-\beta)\}^3[\theta\varepsilon_2+(1-\theta)]}{L^2 {w_c}^2(1-\beta)^2(1-\theta)\theta\varepsilon_1} > 0 \quad . \tag{A.19}$$

Again from equations (6), (A.3), (A.7), (A.10) and (A.11), we have

$$\frac{\partial^2 Log(\psi)}{\partial L\, \partial \gamma} = -\frac{[\beta(1-\theta)+\theta\varepsilon_2][\theta\varepsilon_2+(1-\theta)+\theta\varepsilon_3\{1-\beta\}]}{L(1-\theta)(1-\beta)} < 0 \quad ; \tag{A.20}$$

and from equations (6), (A.2), (A.7), (A.11), we have

$$\frac{\partial^2 Log(\psi)}{\partial \gamma \partial w} = \frac{\{\beta(1-\theta)+\theta\varepsilon_2-\theta\varepsilon_1(1-\beta)\}[\theta\varepsilon_2+(1-\theta)+\theta\varepsilon_3\{1-\beta\}]}{-(1-\theta)w_c\{1-\beta\}^2}$$

$$< 0 \ . \tag{A.21}$$

From equations (A.18), (A.15), (A.21) and (A.20), we have

$$\frac{\partial^2 Log(\psi)}{\partial L\, \partial \gamma} . \frac{\partial^2 Log(\psi)}{\partial w^2} - \frac{\partial^2 Log(\psi)}{\partial L\, \partial \mathrm{w}} . \frac{\partial^2 Log(\psi)}{\partial w \partial \gamma}$$

$$= \left\{\frac{\{\beta(1-\theta)+\theta\varepsilon_2-\theta\varepsilon_1(1-\beta)\}^3[\theta\varepsilon_2+(1-\theta)+\theta\varepsilon_3\{1-\beta\}]}{\theta\varepsilon_1(1-\theta)L(1-\beta)^3{w_c}^2}\right\} > 0 \quad . \tag{A.22}$$

From equations (A.18), (A.20), (A.16) and (A.21), we have

$$\frac{\partial^2 Log(\psi)}{\partial L\, \partial \mathrm{w}} . \frac{\partial^2 Log(\psi)}{\partial L \partial \gamma} - \frac{\partial^2 Log(\psi)}{\partial L^2} . \frac{\partial^2 Log(\psi)}{\partial w \partial \gamma} = 0 \quad . \tag{A.23}$$

So from equations (A.17), (A.19), (A.20), (A.22), (A.21) and (A.23), we have

$$\frac{\partial^2 Log(\psi)}{\partial \gamma^2}\left\{\left(\frac{\partial^2 Log(\psi)}{\partial w^2}\right) \cdot \left(\frac{\partial^2 Log(\psi)}{\partial L^2}\right) - \left(\frac{\partial^2 Log(\psi)}{\partial w\, \partial L}\right)^2\right\}$$

$$- \frac{\partial^2 Log(\psi)}{\partial \gamma\, \partial L}\left\{\left(\frac{\partial^2 Log(\psi)}{\partial w^2}\right) . \left(\frac{\partial^2 Log(\psi)}{\partial \gamma\, \partial L}\right) - \left(\frac{\partial^2 Log(\psi)}{\partial w \partial L}\right) . \left(\frac{\partial^2 Log(\psi)}{\partial \gamma \partial w}\right)\right\}$$

$$+ \frac{\partial^2 Log(\psi)}{\partial \gamma \partial w}\left\{\left(\frac{\partial^2 Log(\psi)}{\partial w \partial L}\right) . \left(\frac{\partial^2 Log(\psi)}{\partial \gamma\, \partial L}\right) - \left(\frac{\partial^2 Log(\psi)}{\partial L^2}\right) . \left(\frac{\partial^2 Log(\psi)}{\partial \gamma \partial w}\right)\right\}$$

$$= -\frac{[\theta\varepsilon_2+(1-\theta)+\theta\varepsilon_3\{1-\beta\}]^3\{\beta(1-\theta)+\theta\varepsilon_2-\theta\varepsilon_1(1-\beta)\}^3}{\theta^2\varepsilon_3\varepsilon_1{w_c}^2L^2(1-\beta)^4(1-\theta)} < 0. \tag{A.24}$$

Appendix B: Derivation of equations of section 5

Bargaining: First order conditions

From equations (1), (20) and (22), we obtain

$$\psi = (w - w_c)^{\theta\varepsilon_1}\left[\frac{(1-\gamma)w_c}{w}\right]^{\frac{\theta\varepsilon_2+\beta(1-\theta)}{1-\beta}}\gamma^{\theta\varepsilon_3}(1-\gamma)^{(1-\theta)}(1-\alpha-\beta)^{(1-\theta)}$$

$$\left[AK^{\alpha}\overline{K}^{1-\alpha}E^{\delta}\right]^{(1-\theta)} \quad . \tag{B.1}$$

The first order conditions of maximisation ψ with respect to w and γ are given by

$$\frac{\partial Log(\psi)}{\partial \mathrm{w}} = \frac{\theta\varepsilon_1}{(w-w_c)} - \frac{\theta\varepsilon_2+\beta(1-\theta)}{(1-\beta)w} = 0 \tag{B.2}$$

and

$$\frac{\partial Log(\psi)}{\partial \gamma} = -\frac{\theta\varepsilon_2+\beta(1-\theta)}{(1-\gamma)(1-\beta)} - \frac{(1-\theta)}{(1-\gamma)} + \frac{\theta\varepsilon_3}{\gamma} = 0 \tag{B.3}$$

From equation (B.2), we obtain

$$w = \frac{[\theta\varepsilon_2+\beta(1-\theta)]}{[\theta\varepsilon_2+\beta(1-\theta)-\theta\varepsilon_1(1-\beta)]}w_c \quad . \tag{B.4}$$

Equation (B.4) is identical to equation (11) in the body of the paper.

From equation (B.3), we obtain

$$\gamma = \frac{\theta\varepsilon_3(1-\beta)}{[\theta\varepsilon_2+(1-\theta)+\theta\varepsilon_3(1-\beta)]} \quad . \tag{B.5}$$

Equation (B.5) is same as equation (12) in the body of the paper.

Using equations (B.4), (B.5) and (20), we obtain

$$L = \left[\frac{[\theta\varepsilon_2+(1-\theta)][\theta\varepsilon_2+\beta(1-\theta)-\theta\varepsilon_1(1-\beta)]}{[\theta\varepsilon_2+(1-\theta)+\theta\varepsilon_3(1-\beta)][\theta\varepsilon_2+\beta(1-\theta)]}\right]^{\frac{1}{1-\beta}} \quad . \tag{B.6}$$

Equation (B.6) is same as equation (23) in the body of the paper.

Second order conditions:

From equations (B.2) and (B.4), we obtain

$$\frac{\partial^2 Log(\psi)}{\partial w^2} = -\frac{[\theta\varepsilon_2+\beta(1-\theta)-\theta\varepsilon_1(1-\beta)]^3}{{w_c}^2(1-\beta)^2\theta\varepsilon_1[\theta\varepsilon_2+\beta(1-\theta)]} < 0 \quad . \tag{B.7}$$

From equation (B.3), we obtain

$$\frac{\partial^2 Log(\psi)}{\partial \gamma^2} = -\frac{\theta\varepsilon_2+(1-\theta)}{(1-\gamma)^2(1-\beta)} - \frac{\theta\varepsilon_3}{\gamma^2} < 0 \quad . \tag{B.8}$$

and

$$\frac{\partial^2 Log(\psi)}{\partial \gamma w} = 0 \quad . \tag{B.9}$$

Using equations (B.7), (B.8) and (B.9), we have

$$\frac{\partial^2 Log(\psi)}{\partial w^2} \cdot \frac{\partial^2 Log(\psi)}{\partial \gamma^2} - \left[\frac{\partial^2 Log(\psi)}{\partial \gamma w}\right]^2$$

$$= \frac{[\theta\varepsilon_2 + \beta(1-\theta) - \theta\varepsilon_1(1-\beta)]^3}{{w_c}^2(1-\beta)^2\theta\varepsilon_1[\theta\varepsilon_2 + \beta(1-\theta)]}\left\{\frac{\theta\varepsilon_2 + (1-\theta)}{(1-\gamma)^2(1-\beta)} + \frac{\theta\varepsilon_3}{\gamma^2}\right\} > 0 \ . \quad (\mathrm{B}.10)$$

Employment effect:

From equation (23), we obtain

$$(1-\beta)\frac{\frac{\partial L^{**}}{\partial \theta}}{L^{**}} = \frac{\varepsilon_2 - 1}{[\theta\varepsilon_2 + (1-\theta)]} + \frac{\varepsilon_2 - \beta - \varepsilon_1(1-\beta)}{[\theta\varepsilon_2 + \beta(1-\theta) - \theta\varepsilon_1(1-\beta)]}$$

$$- \frac{\varepsilon_2 - \beta}{[\theta\varepsilon_2 + \beta(1-\theta)]} - \frac{\varepsilon_2 - 1 + \varepsilon_3(1-\beta)}{[\theta\varepsilon_2 + (1-\theta) + \theta\varepsilon_3(1-\beta)]}$$

$$\Rightarrow \quad \frac{\frac{\partial L^{**}}{\partial \theta}}{L^{**}} = \frac{-\beta\varepsilon_1}{[\theta\varepsilon_2 + \beta(1-\theta)][\theta\varepsilon_2 + \beta(1-\theta) - \theta\varepsilon_1(1-\beta)]}$$

$$- \frac{\varepsilon_3}{[\theta\varepsilon_2 + (1-\theta)][\theta\varepsilon_2 + (1-\theta) + \theta\varepsilon_3(1-\beta)]} \quad . \qquad (\mathrm{B}.11)$$

From equation (B.11), we obtain equation (24) in the body of the paper.

Derivations of other equations in this section are similar to those in the previous model.