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# Indirect taxes in oligopoly in presence of licensing opportunities

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## Abstract

*This paper considers the relative efficiency of unit tax and ad valorem tax in Cournot duopoly in the presence of licensing opportunities after the announcement of the tax rates by the government. Anderson et al. (2001) shows that in such a case ad valorem tax welfare dominates the unit tax. However, it ignores the licensing possibilities. Interestingly, it is shown in the present paper that in case of fixed-fee licensing unit tax sometimes dominates ad valorem tax. However, unit tax and ad valorem tax are equally efficient in case of royalty licensing.*

Keywords: Unit tax, Ad valorem tax, Cournot Competition, Licensing  
JEL Classification: D 43, L 13, L 24

## 1. Introduction

Taxes that are levied on the production or consumption of goods and services or on imports and exports are generally known as indirect taxes. Examples of indirect taxes are sales taxes, value-added taxes (VAT), taxes

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on any aspect of manufacturing or production, taxes on legal transactions, and customs or import duties. Among these unit taxes are levied on per unit of output produced or sold and ad valorem taxes are collected on the value of the production or transaction. An extensive literature in economic theory, is devoted to the study of welfare consequences of various taxations under alternative market structures.<sup>4</sup> In a perfectly competitive market structure unit taxes and ad-valorem taxes are symmetrical in terms of welfare. Both these types of taxes, generate an identical equilibrium with the same output, when they are imposed to extract equal tax revenues<sup>5</sup>. This implies that if government is willing to extract a certain amount of tax revenue, it can go for either of the two taxation schemes as both of these schemes are equally efficient. However in case of imperfectly competitive markets, the story gets more nuanced. Suits and Musgrave (1953) proves the superiority of advalorem taxes over unit taxes in a general monopoly setting. Schroder (2004) shows unit taxes are less efficient than ad valorem taxes in a market characterized by monopolistic competition. This is because unit taxes distort prices while ad valorem taxation, since it is levied on profits, only reduces the number of firms operating in the market. Myles (1996) and Hamilton (1999) argues that tax/subsidy policies can be used to improve welfare by reducing the distortion due to product market imperfection. Though in a homogeneous product oligopoly with a fixed number of firms, both specific and ad valorem taxes reduce industry output and lead to efficiency loss (See Seade, 1985 and Stern, 1987), one may be less distortionary than the other. Delipalla and Keen (1992) and Anderson et al. (2001) show the superiority of ad valorem tax in case homogeneous Cournot oligopoly if the firms have symmetric costs. It has been argued that for any tax revenue, there always exists an ad valorem tax which provides the same tax yields with higher social surplus as compared to unit taxes. Denicolo and Matteuzzi (2000) and Anderson et al. (2001) also extends this result by considering asymmetric firms.

A parallel literature in industrial organisation deals with the issue of technology licensing among firms in a oligopolistic market structure. In the present era of globalization and economic integration technology transfer between firms has become more common than ever (See Vishwasrao, 2007

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<sup>4</sup>There has been substantial reform in the tax policies in various countries in the last century. Ahmed and Stern (1984) addresses these issues in the Indian context.

<sup>5</sup>Governments generally impose taxes to generate revenue.

and Hu et al., 2005). The most common forms of licensing are by either fixed-fee or royalty. Rostocker (1984) for example shows that royalty alone is used for 39 percent of time, fixed-fee alone for 13 percent and both instruments together for 46 percent. Shapiro (1985) discusses the possibilities of licensing in Cournot duopoly market where firms produce homogeneous product via fixed-fee, royalty and two-part tariff. It points out that the licensor cannot charge a per-unit royalty such that the effective unit cost of the licensee is greater than in case of no-licensing, as this type of agreements are abandoned by the anti-trust law.<sup>6</sup> Marjit (1990) discuss the possibilities of licensing by fixed-fee in a Cournot duopoly model and shows that technology is licensed if the initial cost difference is low. Gallini and Winter (1990) and Wang (1998) also considers licensing by royalty. Wang (1998) and Fauli-Oller and Sandonis (2002) show that in case of royalty licensing technology is always licensed. Sen and Tauman (2007) however provides a general licensing schemes to explore the implications of licensing for both the outside and the inside innovator. Sen (2014) studies the role of licensing in affecting the innovation decisions of the firms. Mukherjee and Tsai (2013) also examines the role of government policy in technology licensing decision.

The present paper, is the first attempt to incorporate the possibility of technology licensing, while government imposes ad valorem and unit taxation. In a duopoly market firms, with asymmetric costs of production are assumed to engage in Cournot (quantity) competition. After government announces the possible tax scheme, firms may engage in technology licensing.

It is shown that under unit taxation, if technology is licensed by fixed-fee, tax revenue, consumer surplus and industry profit increases after transfer. On the other hand if technology is licensed when an ad valorem tax has been levied, the tax revenue reduces but consumer surplus and industry profit increases. This implies that under ad valorem taxation, for higher tax revenue the government may set the advalorem tax rate as high as possible such that technology is not licensed<sup>7</sup>. It is shown that in case of licensing by fixed-fee if technology is licensed in both the tax schemes then ad valorem tax still dominates the unit tax. However in contrast to Anderson et al. (2001) even

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<sup>6</sup>In case of royalty licensing the present paper considers this type of restrictions in framing the licensing contracts.

<sup>7</sup>That is when the government is interested in enhancing the revenue and not welfare.

in homogeneous Cournot model unit tax can dominate ad valorem tax. This is because if the ad valorem tax rate is such that technology is not licensed, then there exists a unit tax such that technology is licensed and social surplus is more than in the case of ad valorem tax.

Technology licensing will always take place if the mode of licensing is per unit royalty, independent of the mode of taxation. Tax revenue and the consumer surplus remains unchanged after licensing, while only the industry profit increases. Hence interestingly, under *royalty licensing the two tax schemes are equally efficient*. Under royalty licensing, thus for any ad valorem tax rate there exists an unit tax rate which ensures equal equilibrium output in the two tax schemes, which entails equal consumer surplus and equal gross industry profit under the two tax schemes. This ensures same social surplus under the two tax schemes. However, if the government choose ad valorem tax in such a situation then after licensing the government tax revenue will be more and the net industry profit will be less in ad valorem tax than in case of unit tax.

The first section is the benchmark case where technology could not be transferred in presence of any kind of indirect taxation. The next two sections discuss the effect of taxation in presence of fixed-fee and royalty licensing respectively. Finally we conclude.

## 2. The Benchmark Case

This section is based on Anderson et al. (2001). They show that in a Cournot duopoly market ad valorem taxation are more efficient than unit taxes. However, the purpose of the present model is to understand how this dominance goes through when inter firm technology licensing opportunities are present. To facilitate comparison we discuss the model of Anderson et al. (2001) briefly, assuming a linear market demand.

Consider a Cournot duopoly where each firm produces a homogeneous product. The (inverse) market demand is given by  $P = a - q$ , where  $q = q_1 + q_2$ , and  $q$ ,  $q_1$  and  $q_2$  are the output produced by the industry, firm 1 and firm 2 respectively. Firm 1 and firm 2 respectively produce output,  $q_1$  and  $q_2$ , at constant unit production cost  $c_1$  and  $c_2$  respectively ( $c_1 > c_2$ ).  $P$  is the market price;  $a > 0$ . Further assume that  $c_1 < \bar{c}_1 = \frac{a+c_2}{2}$ , as for  $c_1 \geq \bar{c}_1$ , firm 2 is the monopolist in absence of any tax. In the equilibrium,

output and profit of firm  $i$  in absence of any tax are

$$q_i^* = \frac{(a - 2c_i + c_j)}{3} \quad \text{and} \quad \Pi_i^* = \frac{(a - 2c_i + c_j)^2}{9}$$

respectively, where  $i, j = (1, 2)$  and  $i \neq j$ .

Without any loss of generality assume  $c_2 = 0$  throughout. Under a **unit tax**,  $t (> 0)$  per unit of output sold, the effective unit cost of the firms increases by  $t$ . For firm 1 and firm 2 the effective unit cost under unit tax are  $c_1 + t$  and  $t$  respectively. In this context the profit function of firm  $i$  is

$$\Pi_i = [P - (c_i + t)]q_i.$$

Therefore in the equilibrium outputs of firm 1 and firm 2 in presence of unit tax of level  $t$  are

$$q_1^u = \frac{a - 2(c_1 + t) + t}{3} \quad \text{and} \quad q_2^u = \frac{a - 2t + (c_1 + t)}{3}. \quad (1)$$

Similarly, the profits of firm 1 and firm 2 are<sup>8</sup>

$$\Pi_1^u = \frac{[a - 2(c_1 + t) + t]^2}{9} \quad \text{and} \quad \Pi_2^u = \frac{[a - 2t + (c_1 + t)]^2}{9}. \quad (2)$$

Therefore the tax revenue collected under unit tax of level  $t$  is

$$R_u = t(q_1^u + q_2^u) = t \left[ \frac{2a - (c_1 + t) - t}{3} \right]. \quad (3)$$

However, under an **ad valorem tax**, the producer price is  $(1 - \tau)P$ , where  $\tau \in [0, 1]$  is the advalorem tax rate. Therefore the profit function of firm  $i$  is

$$\Pi_i = [(1 - \tau)P - c_i]q_i = (1 - \tau) \left[ P - \frac{c_i}{1 - \tau} \right] q_i.$$

Since  $(1 - \tau)$  is constant it acts as a pure profit tax. Hence in the equilibrium outputs of firm 1 and firm 2 in presence of an ad valorem tax of level  $\tau$  are

$$q_1^a = \frac{a - \frac{2c_1}{1 - \tau}}{3} \quad \text{and} \quad q_2^a = \frac{a + \frac{c_1}{1 - \tau}}{3}. \quad (4)$$

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<sup>8</sup>It is assumed that  $t$  is not so high such that firm 2 is the monopolist.

Similarly, the profits of firm 1 and firm 2 in case of an ad valorem tax of level  $\tau$  are <sup>9</sup>

$$\Pi_1^a = \frac{(1-\tau)[a - \frac{2c_1}{1-\tau}]^2}{9} \quad \text{and} \quad \Pi_2^a = \frac{(1-\tau)[a + \frac{c_1}{1-\tau}]^2}{9}. \quad (5)$$

Hence, comparing equation (1) and (4) it can be said that the industry output is same in the two tax schedule, i.e.  $q_1^u + q_2^u = q_1^a + q_2^a$ , if

$$t = \frac{\tau}{1-\tau} \frac{c_1}{2}. \quad (6)$$

This also implies that given relation (6), the equilibrium price is also same in the two tax regime. Thus it can be said as in Anderson et al. (2001) that “any equilibrium under the unit tax is also an equilibrium under the ad valorem tax (and vice versa)”, given relation (6). Therefore the tax revenue collected under the ad valorem tax of level  $\tau$  is

$$R_a = \tau P^a (q_1^a + q_2^a) = \tau \left[ \frac{a + \frac{c_1}{1-\tau}}{3} \right] \left[ \frac{2a - \frac{c_1}{1-\tau}}{3} \right], \quad (7)$$

where  $P^a = \frac{(a + \frac{c_1}{1-\tau})}{3}$  is the price and  $q_1^a + q_2^a = \frac{2a - \frac{c_1}{1-\tau}}{3}$  is the industry output under ad valorem tax.

Comparing the tax revenues under the two different tax schemes, given relation (6), it is found that

$$\begin{aligned} R_a - R_u &= (q_1^a + q_2^a) [\tau P^a - t] = (q_1^a + q_2^a) \left[ \tau \frac{(a + \frac{c_1}{1-\tau})}{3} - \frac{\tau}{1-\tau} \frac{c_1}{2} \right] \\ &= (q_1^a + q_2^a) \frac{\tau}{6} \left[ 2a - \frac{c_1}{1-\tau} \right] > 0. \end{aligned} \quad (8)$$

*This implies that as  $R_a - R_u > 0$ , for any given unit tax  $t$ , there exists an ad valorem tax  $\tau$  with a higher tax revenue.* Since the industry output is same in the two tax scheme, therefore the consumer surplus and price are also same. This implies that the total industry revenue is also same under the two tax schemes. Therefore to compare the relative efficiencies<sup>10</sup>

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<sup>9</sup>As in case of unit tax it is assumed that  $\tau$  is not so high such that firm 2 is the monopolist.

<sup>10</sup>Efficiency is measured in terms of Social Surplus, as in Anderson et al. (2001), where Social Surplus comprises of consumer surplus, net industry profit (profit after payment of tax) and tax revenue of the government.

of the two tax schemes, evaluating the total industry cost and finding out which taxation minimizes it is sufficient. The total industry cost in case of unit tax is  $TC_u = c_1 q_1^u$  and in case of ad valorem tax it is  $TC_a = c_1 q_1^a$ , as the unit cost of firm 2 is zero. Using relation (6) it can be showed that  $TC_u - TC_a = tc_1 > 0$ . This implies that industry profit (gross, without paying tax) is more under ad valorem tax than under unit tax. Therefore as shown in Anderson et al. (2001), the ad valorem tax is more efficient than the unit tax.

*The Benchmark Result: In a homogeneous Cournot market the ad valorem tax is more efficient than the unit tax in absence of licensing opportunities.*

In other words it can be said that if the objective of the government is to extract revenue of level say  $\bar{R}$  by unit tax, then there always exists an ad valorem tax such that tax revenue is more than  $\bar{R}$ . This not only ensures higher tax revenues for the government but also higher social surplus with same consumer surplus.

### 3. Taxation in presence of fixed-fee licensing

This section incorporates the licensing opportunities in the benchmark case to observe whether the ad valorem tax still dominates the unit tax. This is important because after the announcement of tax rates by the government the firms may decide for licensing of technology before it produces the output.

Let us first consider fixed-fee licensing in the context of *unit tax*. If  $t(> 0)$  per unit of output sold is levied, the effective unit cost of firm 1 and firm 2 are  $c_1 + t$  and  $t$  respectively. As considered in the literature, e.g. Marjit (1990), Wang (1998), if firm 2 licenses its technology to firm 1 then the unit cost of firm 1 becomes  $c_2$  or zero. As in the present context a unit tax of level  $t$  is imposed by the government, licensing of technology leads to the reduction of the effective unit cost of firm 1 from  $c_1 + t$  to  $t$ .

If technology is not licensed then the profits of firm 1 and firm 2 are  $\Pi_1^u$  and  $\Pi_2^u$  respectively as defined in (2). However, if firm 2 licenses its technology at a fixed-fee ( $F^u$ ), then the profits of firm 1 and firm 2 are

$$\Pi_1^{uf} = \frac{(a-t)^2}{9} - F^u \quad \text{and} \quad \Pi_2^{uf} = \frac{(a-t)^2}{9} + F^u \quad (9)$$



respectively. In this regard firm 2 will set  $F^u$  as high as possible such that,  $\Pi_1^{uf} = \frac{(a-t)^2}{9} - \bar{F}^u = \Pi_1^u$ . Therefore technology will be licensed if

$$\Pi_2^{uf} = \frac{(a-t)^2}{9} + \bar{F}^u = \frac{2(a-t)^2}{9} - \Pi_1^u \geq \Pi_2^u. \quad (10)$$

This also implies that technology is licensed if  $\frac{2(a-t)^2}{9} \geq \Pi_1^u + \Pi_2^u$ , i.e. the industry profit must increase after licensing. This happens only when

$$c_1 + t \leq \frac{2a + 3t}{5}. \quad (11)$$

If technology is licensed then not only the industry profit (net of tax) increases but also the consumer surplus and government tax revenue  $R_u$ , as licensing leads to higher industry output. Using (11) it can be said that if  $t$  is marginally higher such that technology is not licensed, then it is optimal for the government to reduce  $t$  slightly such that technology is licensed. It will not only lead to higher revenue for the government but also higher social surplus (consumer surplus plus industry profit plus tax revenue).

**Proposition 1.** *If technology is licensed then under unit taxation tax revenue, consumer surplus and industry profit increases after transfer.*

Similarly in case of **ad valorem tax** of level  $\tau$ , as  $\Pi_i = (1-\tau)[P - \frac{c_i}{1-\tau}]q_i$ , the effective unit cost (ignoring the constant term  $1-\tau$  in the profit function) of firm 1 and firm 2 can be considered as  $\frac{c_1}{1-\tau}$  and 0 respectively. In this regard if firm 2 licenses its technology to firm 1 then the actual unit cost of firm 1 becomes  $c_2$  or zero. However the effective unit cost of firm 1 also reduces from  $\frac{c_1}{1-\tau}$  to 0.

As defined in (5) in the presence of ad valorem tax if technology is not licensed then the profits of firm 1 and firm 2 are  $\Pi_1^a$  and  $\Pi_2^a$  respectively. On the other hand, if firm 2 licenses its technology at a fixed-fee ( $F^a$ ), then the profits of firm 1 and firm 2 are

$$\Pi_1^{af} = \frac{(1-\tau)a^2}{9} - F^a \quad \text{and} \quad \Pi_2^{af} = \frac{(1-\tau)a^2}{9} + F^a \quad (12)$$

respectively. In this regard firm 2 will set  $F^a = \bar{F}^a$  as high as possible such that firm 1 remains indifferent between licensing and no-licensing or  $\Pi_1^{af} = \frac{(1-\tau)a^2}{9} - \bar{F}^a = \Pi_1^a$ . Therefore technology will be licensed if

$$\Pi_2^{af} = \frac{(1-\tau)a^2}{9} + \bar{F}^a = \frac{2(1-\tau)a^2}{9} - \Pi_1^a \geq \Pi_2^a. \quad (13)$$

The above condition implies that technology is licensed if  $\frac{2(1-\tau)a^2}{9} \geq \Pi_1^a + \Pi_2^a$ , the industry profit increase after licensing or

$$\frac{c_1}{1-\tau} \leq \frac{2a}{5}. \quad (14)$$

As in case of unit tax if technology is licensed then not only the industry profit (net of tax) increases but also the consumer surplus. However, government tax revenue before licensing is  $R_a$ , as defined in (7), while after licensing it is  $R_{af} = \tau \frac{2a^2}{9}$ . Moreover,  $R_a - R_{af} = \frac{c_1\tau}{9(1-\tau)}(a - \frac{c_1}{1-\tau}) > 0$ . This is true for any values of  $c_1$ .<sup>11</sup> Hence, the tax revenue for  $\frac{c_1}{1-\tau} \leq \frac{2a}{5}$  is always less than when it is not.

**Proposition 2.** *If technology is licensed then under ad valorem taxation the tax revenue reduces but consumer surplus and industry profit increases.*

This implies that if technology is licensed the government revenue will definitely fall. In case of unit taxation, tax revenue is  $tq$ . As after licensing industry output expands it leads to increase in the tax revenue under unit taxation for any tax rate  $t$ . However, in case of ad valorem tax, as tax is levied on the value of the industry output, the tax revenue is  $\tau Pq$ . After the expansion in the industry output due to licensing, the fall in price dominates the expansion in industry output and hence leads to fall in the tax revenue for the government under ad valorem taxation.<sup>12</sup>

**Proposition 3.** *In case of ad valorem taxation, for higher tax revenue the government will set  $\tau$  as high as possible such that technology is not licensed if  $c_1 < \frac{2a}{5}$ .*

It can be construed from the above proposition that licensing has a negative impact on the government earnings and therefore for ensuring higher public expenditure the government must restrict licensing by charging higher  $\tau$ .<sup>13</sup>

Let government set a ad valorem tax  $\tau$ , such that the revenue is more in ad valorem tax than in unit tax with same industry output in both tax

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<sup>11</sup>It is assumed that  $a - \frac{c_1}{1-\tau} > 0$ , as otherwise firm 2 will become the monopolist, which is not possible. It is also important to note that  $R_a$  always increases in  $\tau$ .

<sup>12</sup>In general the effect of licensing on tax revenue under ad valorem taxation is ambiguous, as it depends on the elasticity of the demand.

<sup>13</sup>It can be assumed that the objective of the government is to extract higher tax revenues, in that context under ad valorem tax it will charge a high  $\tau$  such that technology is not licensed. Otherwise licensing will fetch a lower tax revenue to the government.

schemes. Substituting  $t = \frac{\tau}{1-\tau} \frac{c_1}{2}$  as defined earlier, which is the level of  $t$  given  $\tau$  such that industry output is same (in absence of licensing), in relation (11), it is found that if such  $t$  is set by the government technology is licensed under unit tax if

$$\frac{c_1}{1-\tau} \leq \frac{2a}{5-4\tau}. \quad (15)$$

Now comparing relation (14) and (15) it can be said that the possibility of licensing (with same industry output in the two tax scheme) is more in case of unit tax than in ad valorem tax as  $\frac{2a}{5-4\tau} > \frac{2a}{5}$ . *In case of fixed-fee licensing if the government set  $t$  and  $\tau$  such that the industry output is same in the two tax scheme (in absence of licensing), then the possibility of transfer is more in unit tax than in ad valorem tax.*

### 3.1. Superiority of unit tax

It is established in the literature by Delipalla and Keen (1992) and Anderson et al. (2001) that ad valorem tax is relatively more efficient than unit tax in homogeneous Cournot oligopoly in absence of licensing possibilities. However, licensing may change this welfare ranking. To fix ideas, we concentrate on a particular situation.

Suppose  $c_1 < \frac{2a}{5}$ , such that technology is licensed in absence of taxation. Moreover assume  $\tau$  is high such that technology is not transferred via fixed-fee as relation (14) is not satisfied. It can be assumed that the government charges a higher  $\tau$  such that technology is not transferred as it reduces the tax revenue as discussed earlier (See Proposition 2).

**Proposition 4.** *If the ad valorem tax rate is such that technology is not licensed, such that government gets higher tax revenues, then there exists a unit tax such that technology is licensed and social surplus is more than in case of ad valorem tax.*

**Proof.** Assume  $\tau$  much high such that technology is not transferred via fixed-fee as relation (14) is not satisfied, this implies that industry output in such a situation will be

$$q_1^a + q_2^a = \frac{2a - \frac{c_1}{1-\tau}}{3}. \quad (16)$$

Contrarily assume that  $t$  is set such that technology is transferred as relation (11) holds. Then under unit tax industry output will be

$$q_1^{uf} + q_2^{uf} = \frac{2(a-t)}{3}. \quad (17)$$

From equation (16) and (17) it can be said that if

$$t = \frac{c_1}{2(1 - \tau)}, \quad (18)$$

then industry output will be same under the two tax schemes. Putting the value of  $t$  from the above equation in relation (11) such that technology is licensed in case of unit tax ensures that  $\tau$  must be much greater than 0.2 such that relation (11) holds and relation (14) does not. If relation (14) holds, then technology will also be licensed in case of ad valorem tax. Here it is assumed that  $\tau$  is set by the government such that technology is not licensed. This will ensure that consumer surplus as well as total industry revenue will also be same under the two tax schemes. Therefore to compare the relative efficiencies of the two tax schemes, as in the benchmark case evaluating the total industry cost and finding out which taxation minimizes it is sufficient. The total industry cost in case of unit tax is  $TC_{uf} = 0$ , as technology is licensed and in case of ad valorem tax it is  $TC_a = c_1 q_1^a$ . Therefore  $TC_u < TC_a$ . This implies that industry profit (gross, without paying tax) is more under unit tax than under ad valorem tax. Therefore contrary to Anderson et al. (2001), the ad valorem tax is less efficient than the unit tax. Moreover, if it is assumed that  $c_1$  limits to  $\frac{2a(1-\tau)}{5}$  (slightly greater), then  $R_{uf} > R_a$ . Hence the social surplus is more in unit tax than in the ad valorem tax. ■

For an alternative proof of this proposition refer to Appendix A.1.

### 3.2. Superiority of ad valorem tax

Reconsider the situation that  $c_1 < \frac{2a}{5}$ . If  $c_1$  is very low then relation (11) and (14) will be satisfied, ensuring licensing of technology both under ad valorem and unit tax schemes. In this case as technology is licensed in both the tax schemes the revenues of the government under unit tax and ad valorem tax are

$$R_{uf} = \frac{2t(a-t)}{3} \quad \text{and} \quad R_{af} = \frac{2a^2\tau}{9}. \quad (19)$$

If  $R_{uf} = R_{af}$ , such that the two tax schemes yields equal revenue, then

$$\tau = \frac{3t(a-t)}{a^2}. \quad (20)$$

Under such tax rates the consumer surplus ( $CS$ ) and industry profit in unit tax is always less than ad valorem tax,

$$\begin{aligned} CS_{uf} &= \frac{2(a-t)^2}{9} < CS_{af} = \frac{2a^2}{9} \\ \Pi_1^{uf} + \Pi_2^{uf} &= \frac{2(a-t)^2}{9} < \Pi_1^{af} + \Pi_2^{af} = \frac{2a^2(1-\tau)}{9}. \end{aligned}$$

This implies that under fixed-fee it technology is licensed under both the schemes, then ad valorem tax will dominate unit tax.

**Proposition 5.** *Under fixed-fee licensing if  $c_1 < \frac{2a}{5}$ , and tax rates are such that technology is licensed under both the schemes then ad valorem tax will dominate unit tax.*

However if  $c_1 > \frac{2a}{5}$ , this implies technology is never licensed as relation (11) and (14) do not hold. In this context as in “The Benchmark Result” it can be said that the ad valorem tax is efficient than the unit tax. Therefore, if  $c_1 > \frac{2a}{5}$ , technology will not be licensed via fixed-fee in both the tax schemes and hence ad valorem tax will dominate the unit tax as in the benchmark case.

#### 4. Taxation in presence of royalty licensing

This section incorporates the royalty licensing in the benchmark case to observe whether the ad valorem tax still dominates the unit tax. Let us begin with *unit tax*. If technology is not licensed then the profits of firm 1 and firm 2 are  $\Pi_1^u$  and  $\Pi_2^u$  respectively as defined in (2). However, if firm 2 licenses its technology at a per-unit royalty ( $r^u$ ), then the profits of firm 1 and firm 2 are

$$\begin{aligned} \Pi_1^{ur} &= \frac{[a - 2(r^u + t) + t]^2}{9} \quad \text{and} \\ \Pi_2^{ur} &= \frac{[a - 2t + (r^u + t)]^2}{9} + \frac{r^u[a - 2(r^u + t) + t]}{3} \end{aligned} \quad (21)$$

respectively. In this regard firm 2 will set  $r^u$  as high as possible such that,  $\Pi_1^{ur} = \frac{[a - 2(r^u + t) + t]^2}{9} = \Pi_1^u$  or  $r^u = c_1$ , as post licensing profit of firm 2 increases in  $r^u$ . The licensor cannot charge  $r^u > c_1$ , as this will entail paying bribe to the licensee, violating the antitrust law as discussed in Shapiro (1985). Therefore technology is licensed always as

$$\Pi_2^{ur} = \frac{[a - 2t + (c_1 + t)]^2}{9} + \frac{c_1[a - 2(c_1 + t) + t]}{3} \geq \Pi_2^u. \quad (22)$$

Unlike fixed-fee licensing, the consumer surplus and government tax revenue remains unchanged when licensing takes place via royalty payments. Fauli-Oller and Sandonis (2002) and Wang (1998) discusses that a royalty licensing scheme keeps the industry output unaltered and thus no possible channel exists through which consumer surplus and tax revenues can change. It is to be noted that competition remains unchanged after licensing of technology, as the effective unit cost of firm 1 remains unchanged. Only the profit of the firm 2 increases, leading to increase in the industry profit by the amount of the cost reduction produced by the use of the new technology of firm 2.

**Proposition 6.** *Under unit taxation technology is always licensed. The tax revenue and the consumer surplus remain unchanged, while the industry profit increases after transfer of technology.*

Let us now consider the effect of royalty licensing in the presence of **ad valorem tax**. As in equation (5) technology is not licensed and the profits of firm 1 and firm 2 are  $\Pi_1^a$  and  $\Pi_2^a$  respectively. On the other hand, if firm 2 licenses its technology at a royalty rate ( $r^a$ ), then the profits of firm 1 and firm 2 are

$$\Pi_1^{ar} = \frac{(1-\tau)[a - 2\frac{r^a}{1-\tau}]^2}{9} \quad \text{and} \quad \Pi_2^{ar} = \frac{(1-\tau)[a + \frac{r^a}{1-\tau}]^2}{9} + \frac{r^a[a - 2\frac{r^a}{1-\tau}]}{3} \quad (23)$$

respectively. As in case of unit tax, here too firm 2 will set  $r^a$  as high as possible such that,  $\Pi_1^{ar} = \frac{(1-\tau)[a - 2\frac{r^a}{1-\tau}]^2}{9} = \Pi_1^a$  or  $r^a = c_1$ , as post licensing profit of firm 2 increases in  $r^a$ . Therefore technology is licensed always as

$$\Pi_2^{ar} = \frac{(1-\tau)[a + \frac{c_1}{1-\tau}]^2}{9} + \frac{c_1[a - 2\frac{c_1}{1-\tau}]}{3} \geq \Pi_2^a. \quad (24)$$

It can be discussed as before that the consumer surplus and government tax revenue remains unchanged. However, the profit of the firm 2 increases, leading to increase in the industry profit.

**Proposition 7.** *Under ad valorem taxation via royalty technology is always licensed. The tax revenue and the consumer surplus remain unchanged, while the industry profit increases after transfer of technology.*

In order to evaluate the efficiency of the two tax schemes it is important to compare the increase in the profit of firm 2 in the two tax schemes. In case of unit tax from equation (22) it can be said that the increase in the profit of firm 2 after licensing is

$$\Pi_2^{ur} - \Pi_2^u = \frac{c_1}{3}[a - 2c_1 - t]. \quad (25)$$

However, from equation (24), the increase in the profit of firm 2 in case of ad valorem tax is

$$\Pi_2^{ar} - \Pi_2^a = \frac{c_1}{3} \left[ a - \frac{2c_1}{1-\tau} \right]. \quad (26)$$

Now comparing these increase in profits it can be said that

$$\Pi_2^{ur} - \Pi_2^u > \Pi_2^{ar} - \Pi_2^a \text{ if } \frac{2c_1\tau}{1-\tau} > t. \quad (27)$$

Moreover as  $R_{af} - R_{uf} = R_a - R_u > 0$ , (which is discussed in the Benchmark case), for any given unit tax  $t$ , there exists an ad valorem tax  $\tau$  with a higher tax revenue. Since the industry output is same in the two tax scheme after licensing, if it is assumed that  $t = \frac{\tau}{1-\tau} \frac{c_1}{2}$  (See the section “The Benchmark case” equation (6)), then the consumer surplus and price are also same. This in turn implies that the total revenue remains equal in both tax schemes after licensing. Therefore to consider the efficiency of the ad valorem tax scheme, evaluating the total industry cost is meaningful to construe the industry profit after licensing. The total industry cost in case of unit tax after licensing is  $TC_{ur} = 0$  and in case of ad valorem tax it is  $TC_{ar} = 0$ , as technology is always licensed and the unit cost of firm 2 is zero. Contrary to Anderson et al. (2001) the social surplus after licensing is same under the two tax schemes.

**Proposition 8.** *As technology is always licensed under unit tax and ad valorem tax via per unit royalty, there always exists a unit tax  $t$  for any given ad valorem tax  $\tau$  such that after licensing equilibrium output and total industry revenue are same under the two tax schemes. The social surplus remains same in both these tax schemes after licensing. However if the government wants to acquire higher tax revenue it will choose ad valorem taxation.*

The above proposition also implies that in such a case as the consumer surplus is same under the two tax schemes, if the government chooses ad valorem tax (unit tax) schemes, the government revenue will be higher (lower) and the net industry profit will be lower (higher) than unit tax (ad valorem tax) schemes.

## Conclusion

This paper considers the relative efficiency of unit tax and ad valorem tax in Cournot duopoly in the presence of licensing opportunities after the announcement of the tax rates by the government. If technology is licensed by fixed-fee, tax revenue, consumer surplus and industry profit increases

after transfer under unit taxation. However, the tax revenue reduces but consumer surplus and industry profit increases after fixed-fee licensing under ad valorem taxation. This implies that under ad valorem taxation, for higher tax revenue the government may set the advalorem tax rate as high as possible such that technology is not licensed. In case of licensing by fixed-fee if technology is licensed in both the tax schemes then ad valorem dominates the unit tax. However, if the ad valorem tax rate is such that technology is not licensed, then there exists a unit tax such that technology is licensed and social surplus is more than in the case of ad valorem tax.

Both under unit tax and ad valorem taxation technology is always licensed via per unit royalty. In both of the tax schemes tax revenue and the consumer surplus remains unchanged after licensing and only the industry profit increases. This implies that there always exists a unit tax rate for any given ad valorem tax rate such that after licensing equilibrium output and total industry revenue are same under the two tax schemes. Therefore under royalty licensing the two tax schemes are equally efficient. Thus, royalty licensing ensures equal consumer surplus and equal gross industry profit under the two tax schemes. This ensures same social surplus under the two tax schemes, however the government gets higher tax revenues under the ad valorem taxation. Wang (1998), Fauli-Oller and Sandonis (2002) and Sen (2014) show that in homogeneous goods Cournot model with constant unit cost optimal two-part tariff licensing entails to only positive per-unit royalty with zero fixed-fee. Thus we desist from discussing two part tariffs in the present model.

There are a number of directions in which this analysis can be extended. Anderson et al. (2001a) and Wang and Zhao (2009) considers indirect taxes when oligopolistic firms produce differentiated goods. Anderson et al. (2001a) studies the incidence of these tax schemes with differentiated products and price-setting firms. Wang and Zhao (2009) shows that unit taxation can be welfare superior to ad valorem taxation in asymmetric and differentiated oligopolies. The present paper can be extended to consider the effect of licensing in differentiated goods Cournot and Bertrand duopoly with asymmetric cost structures to evaluate the efficiency of the tax schemes. As the present paper considers each type of taxes separately, a possible future research agenda can be to explore licensing possibilities when a combination of these taxes are adopted (See Myles (1996)).



## Appendix A.

### Appendix A.1.

Suppose,  $c_1 < \frac{2a}{5}$  and suppose  $\frac{c_1}{1-\tau} > \frac{2a}{5}$  (limitingly greater), this implies that *technology is not licensed in case of ad valorem tax*. It can be assumed that the government charges a higher  $\tau$  such that technology is not transferred as it reduces the tax revenue as discussed earlier (See Proposition 2). In this context if the government imposes a unit tax  $t$ , such that  $c_1 + t \leq \frac{2a}{5}$  (where  $t = \frac{\tau}{1-\tau} \frac{c_1}{2}$ ), then *technology will be transferred as relation (15) is satisfied*. In this context the government tax revenues under ad valorem tax and unit tax are respectively

$$R_a = \tau \left[ \frac{a + \frac{c_1}{1-\tau}}{3} \right] \left[ \frac{2a - \frac{c_1}{1-\tau}}{3} \right] \quad \text{and} \quad R_{uf} = \frac{2t(a-t)}{3}. \quad (\text{A.1})$$

Moreover as it is been assumed that  $t = \frac{\tau}{1-\tau} \frac{c_1}{2}$ , therefore  $R_{uf} = \frac{c_1 \tau}{3(1-\tau)} \left[ a - \frac{c_1 \tau}{2(1-\tau)} \right]$ . In can be further said that as  $c_1$  limits to  $\frac{2a(1-\tau)}{5}$  (slightly greater) then

$$R_a = \frac{56a^2\tau}{225} > R_{uf} = \frac{2a\tau}{15} \left[ a - \frac{a\tau}{5} \right]. \quad (\text{A.2})$$

Further the industry profits in case of ad valorem tax and unit tax are respectively

$$\Pi_1^a + \Pi_2^a = \frac{(1-\tau) \left[ a - \frac{2c_1}{1-\tau} \right]^2}{9} + \frac{(1-\tau) \left[ a + \frac{c_1}{1-\tau} \right]^2}{9} \quad (\text{A.3})$$

and

$$\Pi_1^{uf} + \Pi_2^{uf} = \frac{2(a-t)^2}{9} = \frac{2 \left( a - \frac{\tau}{1-\tau} \frac{c_1}{2} \right)^2}{9}. \quad (\text{A.4})$$

In can be further said as before that if  $c_1$  limits to  $\frac{2a(1-\tau)}{5}$  (slightly greater) then the industry profit is always greater in case of unit tax as

$$\Pi_1^{uf} + \Pi_2^{uf} = \frac{2a^2}{9} \left[ 1 - \frac{\tau}{5} \right]^2 > \Pi_1^a + \Pi_2^a = \frac{2a^2(1-\tau)}{9}. \quad (\text{A.5})$$

Similarly, the consumer surplus<sup>14</sup> ( $CS$ ) is greater in unit tax

$$CS_{uf} = \frac{2a^2}{9} \left[ 1 - \frac{\tau}{5} \right]^2 > CS_a = \frac{8a^2}{225}. \quad (\text{A.6})$$

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<sup>14</sup>From the demand function it can be showed that if the equilibrium output is  $q$ , consumer surplus will be  $\frac{q^2}{2}$ .

Finally the difference in the Social surplus<sup>15</sup> ( $SS$ ) are

$$SS_{uf} - SS_a = \frac{2a^2}{225}[6 - 8\tau - \tau^2]. \quad (\text{A.7})$$

Therefore  $SS_{uf} - SS_a > 0$  if  $\tau$  is less or  $c_1$  slightly lower than  $\frac{2a}{5}$ . The present discussion therefore proves that if the government charges an ad valorem tax  $\tau$  such that technology is not transferred for acquiring higher tax revenue, then there exists a unit tax  $t = \frac{\tau}{1-\tau} \frac{c_1}{2}$ , such that technology is licensed. In that case the consumer surplus and industry profit, if the government charges such a unit tax, is higher than in ad valorem tax. The social surplus will also be higher in unit tax than in case of ad valorem tax.

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<sup>15</sup>*Social Surplus = Consumer Surplus + Industry Profit + Tax revenue.*

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