Optimal Policy to Influence Individual Choice Probabilities

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Note on Income Taxation and Occupational Choice

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Abstract

With varying aptitudes in different occupations, individuals typically maximize income by specializing in one occupation which promises the highest income. Due to numerous labor market imperfections and uncertainties, the choice of best occupation is accomplished with only partial success. We demonstrate that an income tax that reduces after-tax income differentials across occupations tends to exacerbate the errors of choice made by individuals. Following a model proposed by Tinbergen (1951) and developed by Houthakker (1974), we use Luce's (1959) multinomial logit approach to evaluate the magnitude of the distortions due to errors in occupational choice caused by income taxation. In an example, we show that the deadweight loss can be as high as a third of total income.

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1 Introduction

With varying aptitudes in different occupations, individuals typically maximize income by specializing in one occupation which promises the highest income. Due to numerous labor market imperfections and uncertainties, this is accomplished with only partial success. We demonstrate that an income tax that reduces the after-tax income differentials across occupations tends to exacerbate the errors of choice made by individuals.

Following a model proposed by Tinbergen (1951) and developed by Houthakker (1974)[1], we use Luce's (1959) multinominal logit approach to evaluate the magnitude of the distortions caused by income taxation. In an illustration with a specific example, we show that at high marginal tax rates these distortions can be in excess of a third of mean income.

[1] A related paper is Sheshinski (1983)

2 An Occupational Choice Model

Individuals are endowed with aptitudes in different occupations. These aptitudes are represented by a vector $(y_1, y_2, ..., y_n)$, where $y_i (\geq 0)$ is the value of the i-th commodity that the individual could produce in a given time period if he/she did nothing else. Since the y_i's are constants and all individuals have a given working time, value maximization implies that each individual will work all the time on the occupation for which y_i is greatest. Generally, there is only one such occupation. If there is more than one, the allocation is indeterminate.

In view of the many imperfections in the labor market, it is unrealistic to assume perfect income maximization. We shall follow the approach suggested by Luce (1959), that individuals maximize "imperfectly", the probability of choosing occupation i, p_i, being given by

$$p_i = p_i(y_1, y_2, ..., y_n) = \frac{e^{qy_i}}{\sum\limits_{j=1}^{n} e^{qy_i}}, \quad i = 1, 2, ..., n \tag{1}$$

where q is a positive constant, representing the 'precision' of choice. As $q \to \infty$, the probability p_i increases monotonically, approaching 1 if $y_i = \arg\max(y_1, y_2, ..., y_n)$ and decreases monotonically, approaching 0, otherwise. At the other end, as $q \to 0$, p_i approaches $\frac{1}{n}$ which means that all occupations have an equal probability of being chosen, irrespective of individual aptitudes. It is natural to call q the *"degree of rationality"* ($q = \infty$, *"perfect rationality"*).

Assume that the aptitude vector $(y_1,\, y_2, ..., \,y_n)$ varies randomly over the population with a continuous density function $f\,(y_1,\, y_2, ..., \,y_n)$. The distribution function $F\,(y_1,\, y_2, ..., \,y_n)$ is then also continuous (and differentiable). The marginal density functions in different occupations need not be independent.

Let $G(z)$ be the cumulative distribution function for labor incomes, and $g(z)$ the corresponding density function. It is seen that:

$$G(z) =$$

$$= \sum_{i=1}^{n} \int_0^{\infty} ... \int_0^{z} ... \int_0^{\infty} p_i(y_1, ...\, y_{i-1},\, x,\, y_{i+1}, .., \,y_n) f(y_1, ...\, y_{i-1},\, x,\, y_{i+1}, .., \,y_n) \qquad (2)$$

$$dy_1 ... dy_{i-1}\, dx\, dy_{i+1} ... dy_n$$

In subsequent discussion it will suffice to examine the case $n = 2$. For this case, (2) is written

$$\begin{aligned} G(z) = & \int_0^{z}\int_0^{\infty} p_1(y_1,\, y_2) f\,(y_1,\, y_2)\, dy_1\, dy_2 \\ & + \int_0^{\infty}\int_0^{z} p_2(y_1,\, y_2) f\,(y_1,\, x)\, dy_1\, dy_2 \\ & = \int_0^{z}\int_0^{\infty} \frac{e^{qy_1}}{e^{qy_1} + e^{qy_2}}\, f\,(y_1,\, y_2)\, dy_1\, dy_2 \\ & + \int_0^{\infty}\int_0^{z} \frac{e^{qy_2}}{e^{qy_1} + e^{qy_2}}\, f\,(y_1,\, y_2)\, dy_1\, dy_2 \end{aligned} \qquad (3)$$

3 An Example

Consider the bivariate exponential density function[2]

$$f\,(y_1,\, y_2) = \alpha_1\alpha_2 e^{-\alpha_1 y_1 - \alpha_2 y_2} \tag{4}$$

and the corresponding distribution function

$$F(y_1,\, y_2) = (1 - e^{-\alpha_1\, y_1})(1 - e^{-\alpha_2\, y_2}) \tag{5}$$

(a) **Perfect Rationality**

When $q = \infty$, $p_i(y_1,\, y_2)$ is 1 when $y_i \geq y_j$, $i,\, j = 1,\, 2$ and 0 otherwise. Hence, by (3) and (4),

$$\begin{aligned} G(z)_{q=\infty} = \; & \alpha_1\alpha_2 \int_0^z\!\!\int_0^{y_1} e^{-\alpha_1\, y_1 - \alpha_2\, y_2} dy_1 dy_2 \\ & + \alpha_1\alpha_2 \int_0^z\!\!\int_0^{y_2} e^{-\alpha_1\, y_1 - \alpha_2\, y_2} dy_1 dy_2 \\ & = (1 - e^{-\alpha_1 z})(1 - e^{-\alpha_2 z}) \\ & = F(z,\, z) \end{aligned} \tag{6}$$

The corresponding density function

$$g(z)_{q=\infty} = \alpha_1 e^{-\alpha_1 z} + \alpha_2\, e^{-\alpha_2 z} - (\alpha_1 + \alpha_2)\, e^{-(\alpha_1+\alpha_2)z} \tag{7}$$

has an interior mode and positive skewed shape as observed in empirical income distributions.

[2] This is the product of two univariate distributions. While not allowing for dependence, this is a simple illustrative case that has zero probability of ties (see below).

Expected income, $\overline{y}_{q=\infty}$, is

$$\overline{y}_{q=\infty} = \frac{1}{\alpha_1} + \frac{1}{\alpha_2} - \frac{1}{\alpha_1 + \alpha_2} \tag{8}$$

(b) **Uniformly Random Choice**

At the other extreme, when $q = 0$, $p_i(y_1,\, y_2) = \frac{1}{2}$ independent of $(y_1\,, y_2)$. By (3) and (4),

$$G(z)_{q=0} = 1 - \frac{1}{2}\,(e^{-\alpha_1 z} + e^{-\alpha_2 z}) \tag{9}$$

The corresponding density is

$$g(z)_{q=0} = \frac{1}{2}\,(\alpha_1 e^{-\alpha_1 z} + \alpha_2 e^{-\alpha_2 z}) \tag{10}$$

which, as expected, is the arithmetic mean of two univariate densities.

Mean income, $\overline{y}_{q=0}$, is now

$$\overline{y}_{q=0} = \frac{1}{2}(\frac{1}{\alpha_1} + \frac{1}{\alpha_2}). \tag{11}$$

It is not surprising that maximization of income by individuals leads to a larger mean income than when individuals choose occupations randomly[3]:

[3] The variances in these two cases are $\sigma^2_{q=\infty} = \frac{1}{\alpha_1^2} + \frac{1}{\alpha_2^2} - \frac{3}{(\alpha_1 + \alpha_2)}$, and $\sigma^2_{q=0} = \frac{3}{4}(\frac{1}{\alpha_1^2} + \frac{1}{\alpha_2^2}) - \frac{1}{2\alpha_1\alpha_2}$. The sign of the difference between these variances depends on parameter values.

$$\overline{y}_{q=\infty} - \overline{y}_{q=0} = \frac{1}{2}(\frac{1}{\alpha_1} + \frac{1}{\alpha_2}) - \frac{1}{\alpha_1 + \alpha_2} > 0.$$

The relative loss of income can be quite substantial. For illustration, take $\alpha_1 = .01$ and $\alpha_2 = .02$ (corresponding to mean abilities of 100 and 50, respectively). For these parameter values, the relative loss exceeds 36 percent of income!

It can be shown that the distribution function $G_{q=\infty}(z)$ s*tochastically dominates (in the 'first-degree')* the distribution $G_{q=0}(z)$[4]. That is, for any concave utility function, social welfare is higher under perfect rationality.

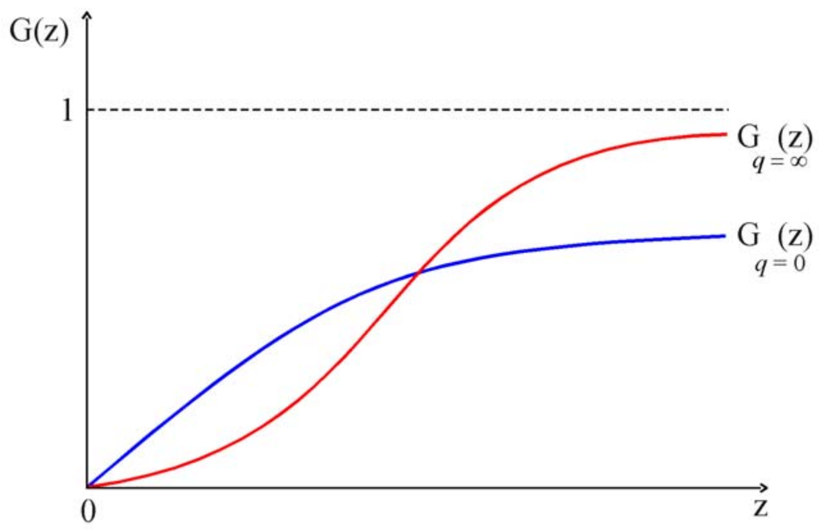

Figure 1

Calculations for intermediate cases, $0 < q < \infty$, turn out to be complex, yielding no explicit analytic solutions.

[4] $G_{q=\infty}(z)$ and $G_{q=0}(z)$ intersect once, with $G_{q=\infty}(z)$ steeper at the intersection point.

4 Effect of An Income Tax

The effect of a progressive income tax on mean income is the same as the effect of a reduction in the degree of rationality, q. For simplicity, suppose there is in place a linear income tax function $t(z) = -a + (1-b)z$, where the support level a, $a > 0$, and the after-tax rate b, $0 < b < 1$, are constants and z is before-tax income. After-tax income is $z - t(z) = a + bz$. The probabilities of individual choice, (1), now depend on after-tax income:

$$p_i = \frac{e^{q(a+by_i)}}{\sum_{j=1}^{n} e^{q(a+by_j)}} = \frac{e^{qby_i}}{\sum_{j=1}^{n} e^{qby_j}} \qquad i = 1, 2, ..., n \tag{12}$$

It is seen that q and b (the after-tax rate) are interchangeable. Except in the polar cases $q = \infty$ and $q = 0$, the marginal tax rate affects occupational choice and hence entails a deadweight loss in terms of mean income. We have seen before that this effect can be significant. Of course, more detailed calculations for alternative levels of q are required in order to evaluate the effect of marginal tax increases and a corresponding increase in the support level on mean income and on the distribution of after-tax incomes.

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