# The Elasticity of Intertemporal Substitution Reconsidered

Dladla, Pholile and Malikane, Christopher and Ojah, Kalu

University of the Witwatersrand

26 April 2014

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Pholile Dladla and Christopher Malikane
Macro-Financial Analysis Group
School of Economic and Business Sciences
University of the Witwatersrand
1 Jan Smuts Avenue
Johannesburg
2050

Kalu Ojah[1]
Graduate School of Business Administration
University of the Witwatersrand
1 Jan Smuts Avenue
Johannesburg
2050

**Abstract**

The elasticity of intertemporal substitution is a crucial parameter in finance and macroeconomics, yet its estimation remains elusive. We show, based on Fisher's relation and the expectations theory of the term structure, that the EIS is the inverse of the product of the average term to maturity of debt instruments and the consumption-output ratio. Therefore, the EIS need not be estimated but can be calibrated from observable data.

Keywords: Elasticity of intertemporal substitution, Fisher's relation, expectations theory of the term structure.

## 1. Introduction

The elasticity of intertemporal substitution (EIS) is an important parameter in asset pricing theory. It measures the rate at which economic agents trade future consumption for current consumption. The EIS tells us by how much current consumption must be adjusted in response to a change in the rate of return of an asset. The EIS also features prominently in macroeconomic

[1]Email: kalu.ojah@wits.ac.za. Tel: +27-11-717-3764. Fax: +27-11-717-8081.

literature through the IS curve. The forward-looking IS curve in particular, establishes a direct link between macroeconomics and finance since it is a statement of consumption-based asset pricing theory (see Gali (2008, Chapter 2) and Woodford (2003, Chapter 4)). In pure forward-looking versions of the IS curve, the EIS is the only parameter of interest. An accurate estimation of the EIS is therefore important for asset pricing and for the quantitative determination of the effects of monetary policy on the real economy.

Despite estimation efforts to pin it down, the EIS remains elusive. Carroll (1997) argues that estimation of the EIS using the log-linear consumption Euler must be abandoned. The literature has not taken heed of Carroll's advice. Recently, Gomes and Paz (2013) show that estimates of the EIS using the rate of return from a synthetic mutual fund, which is representative of the households' asset portfolio, suggest that the EIS is significantly above zero. However, in line with the finding by Fuhrer and Rudebusch (2004), these authors also find that these estimates suffer from weak identification. Estimates using the treasury bill rate fall within an interval that contains zero, implying that the EIS may be close to zero. This result supports the finding by Hall (1988) that there is weak evidence that the EIS is positive, while Campbell (2003) and Yogo (2004) finds that the EIS is small and sometimes takes on negative values. On the other hand van Binsbergen et al.(2012) estimate the EIS to be substantially more than 1, under the assumption that households have Epstein-Zin recursive preferences.

Efforts to reconcile conflicting estimates of the EIS point to the importance of limited asset market participation. Based on an analysis of US consumer survey data, Vissing-J$\varnothing$rgensen (2002) finds that the EIS for stockholders is substantially less than 1 and the EIS of bondholders is close to 1. From the macroeconomic perspective, Guvenen (2006) finds that the EIS of stockholders is substantially higher than the EIS of agents who have no access to the stock market. From another angle, Ogaki and Reinhart (1998) find that the EIS is substantially above zero, ranging between 0.33 and 0.45, if intratemporal substitution between durable and non-durable consumption is taken into account. Furthermore, as pointed out by Gomes and Paz (2013), another line of research investigates whether the choice of instruments or the choice of asset returns matters in the estimation of the EIS. Gomes and Paz (2013) conclude their paper as follows: "At the end of the day, the final word on the true estimate of the EIS is still to come".

This paper contributes to the literature by pinning down the EIS through an alternative derivation of the Euler equation. A key assumption is that, since a significant component of aggregate consumption is made up of durable goods, it makes sense to drop the commonly-used assumption of a one-period debt instrument (see for example, Fuhrer and Rudebusch, 2004). We then use Fisher's relation and the expectations theory of the term structure to show that the standard interpretation of the interest rate parameter in the asset pricing equation as the EIS is not accurate. In fact, the EIS is the inverse of the average term to maturity of debt instruments and, under certain conditions, the EIS is the inverse of the average term to maturity scaled by the consumption-output ratio.

## 2. Theoretical framework

Standard derivations of the Euler equation, based on power utility, assume an economy in which there is a one-period bond that yields a nominal interest rate $i_t$. The representative agent maximises intertemporal utility, by choosing the optimal consumption level, subject to a dynamic resource constraint. It can be shown, e.g. Gali (2008: Chapter 2), that the first order condition of the household is:

$$\widehat{c}_t = E_t \widehat{c}_{t+1} - \psi \left( i_t - E_t \pi_{t+1} \right), \tag{1}$$

where $\psi > 0$ is the EIS, $\pi_t$ is the inflation rate and $\widehat{c}_t$ is the percentage deviation of consumption from the steady state. Macroeconomists sometimes express the Euler equation in terms of output to get the following relation:

$$\widehat{y}_t = E_t \widehat{y}_{t+1} - \alpha \psi \left( i_t - E_t \pi_{t+1} \right) + \zeta_t, \tag{2}$$

where $\zeta_t = -\left(1 - \alpha\right) E_t \Delta \widehat{d}_{t+1}$, $\alpha$ is the share of consumption in output and $\widehat{d}_t$ denotes other components of aggregate demand. The main weakness of eq.(1) is the assumption that the representative agent has only a one-period bond in which to invest. In practice there are many instruments to invest in, a point that has been taken up in the literature on the EIS (see Gomes and Paz (2013) for example). We therefore change this assumption accordingly and focus on debt instruments.

Assume an agent possesses an amount of money $P_t$, which purchases one unit of output. This agent has a choice of either investing in a $n$-period bond that yields a one-period gross nominal interest rate $(1 +^n i_t)$ or in a one-period project that produces output. At symmetric equilibrium each unit of output grows at the same rate as aggregate output. Under no arbitrage the following condition must hold:

$$E_t\left(\frac{Y_{t+1}}{Y_t}\right) = E_t\left(\frac{1 +^n i_t}{1 + \pi_{t+1}}\right), \tag{3}$$

where $Y_t$ denotes aggregate output. From the expectations theory of the term structure of interest rates we have:

$${}^n i_t = \frac{1}{n} i_t + \frac{1}{n} E_t \sum_{j=1}^{n} i_{t+j}, \tag{4}$$

Log-linearising eq.(3), applying the expectations theory of the term structure and then expressing the no-arbitrage condition in terms of a one-period interest rate we obtain:

$$\widehat{y}_t = E_t \widehat{y}_{t+1} - \frac{1}{n}\left(i_t - E_t \pi_{t+1}\right) + \zeta_t^{'} \tag{5}$$

where $\zeta_t^{'} = -\left(\frac{1}{n} E_t \sum_{j=1}^{n} i_{t+j} - \frac{n-1}{n} E_t \pi_{t+1}\right)$. Since the no-arbitrage must occur simultaneously with household optimisation, eqs.(5) and (2) are identical and so the $\psi = (\alpha n)^{-1}$. This implies that the EIS can be calibrated using observable data. Note that this result depends on whether the consumption deflator or the GDP deflator is used.

If the consumption deflator is used, we can invoke the fact that at equilibrium, consumption grows at the same rate as output. Therefore, the log-linearised Fisher relation eq.(5) can be expressed in terms of consumption, yielding an Euler-type equation similar to eq.(1).

$$\widehat{c}_t = E_t\widehat{c}_{t+1} - \frac{1}{n}\left(i_t - E_t\pi_{t+1}\right) + \zeta_t' \tag{6}$$

In this situation the EIS is the inverse of the average term to maturity $n^{-1}$. In this paper, we provide calibrations for both cases, i.e. where the output and consumption deflators are used to measure inflation.

## 3. Review of empirical evidence

The empirical studies that we use as benchmarks are Campbell (2003) and Yogo (2004). The novelty of these studies is that they provide international estimates of the EIS. We compare the estimates of these studies and our calibrated EIS. Beyond these two studies, we also comment on the relationship between our calibration and the estimates from new Keynesian macroeconomic literature (e.g. McCallum and Nelson, 1999). The data we use to calibrate our EIS is from the OECD, where average term to maturity of outstanding central government debts is reported. Furthermore, also from the OECD database, we calculate average consumption-output ratios over the period 1980–2010.

Table 1 reports the results by Campbell (2003) and Yogo (2004) for a number of developed economies, together with our calibrations of the EIS based on $(\alpha n)^{-1}$ and $n^{-1}$. In addition, we also report Yogo's preferred estimates of the EIS, which is the upper limit of his conditional LR 95% confidence interval. By definition, all our calibrations are positive whereas the Campbell-Yogo estimates have negative values.

Interestingly, in the case of the US our consumption-deflator based calibration is very close to the estimate by Hall (1988) and Yogo (2004), who find an EIS of around 0.2. Our calibrations that are based on the output deflator are close to Yogo's conditional LR estimates for Australia, France, Germany and the US. The calibrations based on the consumption deflator are close to Campbell's estimates for Australia. Similar results can be observed for Australia, where the calibration based on the output deflator is very close to Yogo's conditional LR estimate, while the calibration that is based on the consumption deflator is close to Campbell's estimate.

Turning to DSGE literature, very few studies estimate the pure forward-looking macro-model. However in the case of the US, McCallum and Nelson

(1999) obtain a similar value to our consumption-deflator calibration. Similarly, in the case of the UK, Kara and Nelson (2004) obtain a value similar to ours based on the consumption deflator, their estimate for Australia is lower than ours, at 0.05. Lastly, Dib (2007) estimates the EIS for Canada to be 0.30, which is close to our calibration based on the output deflator.

Overall, we conclude that the EIS can be calibrated with reasonable accuracy, based on historical data on the average term to maturity and the consumption-output ratios.

Table 1: Estimated and Calibrated EIS

| Country | Maturity | Cons-Output Ratio | EIS* | EIS** | Campbell | Yogo† | Yogo‡ |
|---|---|---|---|---|---|---|---|
| Australia | 6.86 | 0.57 | 0.26 | 0.15 | 0.11 | 0.03 | 0.27 |
| Canada | 5.79 | 0.56 | 0.31 | 0.17 | -0.09 | -0.34 | 0.00 |
| France | 6.38 | 0.57 | 0.27 | 0.16 | -0.10 | -0.08 | 0.33 |
| Germany | 5.48 | 0.57 | 0.32 | 0.18 | -0.10 | -0.44 | 0.28 |
| Italy | 5.21 | 0.59 | 0.33 | 0.19 | -0.06 | -0.07 | 0.12 |
| Japan | 5.56 | 0.58 | 0.31 | 0.18 | -0.11 | -0.05 | 0.45 |
| Netherlands | 6.43 | 0.49 | 0.32 | 0.16 | -0.16 | -0.14 | 0.48 |
| Sweden | 4.17 | 0.49 | 0.49 | 0.24 | 0.04 | 0.00 | 0.21 |
| Switzerland | 6.90 | 0.60 | 0.27 | 0.14 | 0.36 | -0.50 | 0.09 |
| United Kingdom | 12.0* | 0.63 | 0.13 | 0.08 | 0.30 | 0.16 | 0.43 |
| United States | 5.15 | 0.67 | 0.29 | 0.19 | 0.12 | 0.02 | 0.23 |

Notes: *Output deflator, **Consumption deflator. †LIML estimates, ‡Upper limit of 95% Cond. LR interval.

## 6. Conclusion

Recent literature continues to underline the considerable uncertainty regarding the estimation of the elasticity of intertemporal substitution (EIS). By assuming an economy that is characterised by debt instruments with an average term to maturity $n$, we have shown, based on Fisher's relations and the expectations theory of the term structure, that the EIS can be calibrated with reasonable accuracy without estimation.

In the case where a consumption deflator is used to measure inflation, we find that the EIS is just the inverse of the average term to maturity of debt instruments. In the case where the output deflator is used to measure inflation, the EIS is the inverse of the product of the average term to maturity and the consumption-output ratio. This finding allows us to ditch the estimation of the EIS as suggested by Carroll (1997), and to opt for calibration.

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