The Overlooked Assumption Behind the New Keynesian Phillips Curve

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28 April 2014

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April 29, 2014

Abstract

The New Keynesian Phillips Curve rests on an assumption not mentioned in the literature. Specifically, firms that are price constrained align their production along the demand curve, ignoring the effects of marginal cost on supply. This paper investigates what happens when the relationship between marginal cost and pricing conforms instead to standard microeconomic theory. It shows that the New Keynesian Phillips Curve is invalid and prices are not procyclical, but acyclical in this case. Therefore, if the assumption in question is necessary to the model, it should be acknowledged for the sake of transparency.
Keywords: New Keynesian Phillips Curve, micro-foundations, price rigidity, marginal cost.
JEL: E31, D43.

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1 Introduction

Dynamic Stochastic General Equilibrium (DSGE) models have risen in popularity over the last fifteen years with the New Keynesian Phillips Curve (NKPC) as its backbone. Though few know the particulars, both the NKPC and DSGE models are familiar to every economist.

But a not-talked-about assumption, made in the middle of constructing the curve, is unusual. This paper challenges the NPKC manipulations on setting the production of constrained firms (those that cannot reset price in the NKPC specification) along the product's demand curve. According to marginal behavior, firms that cannot reset their price should choose production levels so marginal costs are coherent with the imposed prices. Therefore, firms will treat a binding price constraint as a price ceiling or a price floor. Specifically, as price takers, they should equal marginal cost to their imposed price, unless that leads to unsold surpluses. These firms would reduce production if prices were too low instead of increasing them, while unconstrained firms keep reacting to relative prices. Therefore, this type of price rigidity will not produce procyclical inflation.

Furthermore, the paper investigates whether different forms for the marginal cost function can serve to fix the issue. Specifically, one alternative specification poses that somehow marginal cost leads demand to equal supply for constrained firms, while the other specification poses constant marginal cost. These specifications fail, though results are informative.

This paper is to shows through simple algebra that standard microeconomic assumptions result in an intractable model with uninteresting results. Consequently, price rigidity relies on an unrealistic assumption. To my knowledge, no work or textbook on the NKPC mentions this assumption. The paper fills this gap, and makes a case for transparency: most economists using the NKPC are unaware that the model's producers actually bypass their own supply curves.

The structure of the paper permits readers familiar with the NKPC to skip ahead to Section 2 without loss of information. The next subsection provides some background on the NKPC for a general audience. Section 2 constructs the first order conditions for the consumer and poses the problem of the producers, then moves to three subsections. Subsection 2.1 presents the usual manipulations behind the NKPC. Subsection 2.2 considers the implications of constrained firms acting as price takers and Subsection 2.3 investigates whether different assumptions on marginal cost can resolve the issue. Finally, Section 3 concludes.

1.1 Background

First born of a simple statistical relationship with Keynesian justifications, the Phillips Curve eventually evolved to an equation where the output gap drives inflation. As the justifications became shaky, it became a simple reduced form model of a relation noticed two hundred years ago between the boom-bust cycles and inflation-deflation.

The NKPC supplanted the Phillips Curve in response to Lucas Critique. New Keynesian research had advanced, from the late seventies, concepts of price rigidity that are coherent with rational expectations and that culminated with the NKPC. Originating with Roberts (1995) and Yun (1996), it uses marginal cost instead of the output gap as the driver of inflation (though the output gap can be used as a proxy for marginal cost). Now, helped by the popularity of DSGE models, the NKPC dominates as the standard model of inflation in modern macroeconomics. DSGE models came to the forefront of macroeconomics starting with Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003). Academia uses them to test theories while central banks and international institutions rely on them as forecast models.[1] Consequently, the NKPC, in the original or the hybrid form, is a central part and permanent fixture of these models.

The NKPC combines monopolistic competition and price rigidities to create a structural model where marginal costs drive inflation. Dixit and Stiglitz (1977) aggregation implements monopolistic competition and Calvo (1983) contracts implements price rigidities. As a result, firms individually set their prices as a function of marginal costs and of the probability of being able to change them in the future; aggregation and log-linearization do the rest. The resulting equation is the NKPC,

$$\pi_t = \gamma c_t + \beta \mathrm{E}_t \pi_{t+1},$$

where π represents inflation, c, marginal cost, β, the discount factor and $\gamma = (1-\omega)(1-\beta\omega)/\omega$, where ω represents a firm's probability of not being able to reset price in a given period.

[1] Sims (2012) and Christiano, Trabandt and Walentin (2010) discuss the historical development and current importance of DSGE modeling in academic work, while Dotsey (2013), Smets, Christoffel, Coenen et al. (2010) and Botman, Rose, Laxton et al. (2007) discuss its use by central banks and international institutions.

Common critiques of the NKPC say it does not capture the persistence of inflation or that marginal cost measures do not drive inflation. Specifically, Fuhrer (1997) and Estrella and Fuhrer (2002) discuss the approach' empirical grounds for the persistence problem, while Rudd and Whelan (2005, 2007) and Lawless and Whelan (2011) discuss the empirical fit of the labor share proxy for marginal cost. In response to the inflation-persistence issue, Galí and Gertler (1999) add a share of irrational, or rule-of-thumb, producers to create the hybrid-NKPC. As for the labor share, as seen with Galí, Gertler and López-Salido (2005), the point is not yet settled.

Furthermore, the NKPC raises two theoretical issues. First, marginal cost measures seem arbitrary; it can be the output gap, the labor share or the derivative of all production inputs.[2] That some capital cannot be considered marginal cost, nor can some labor, complicate the issue; in other words, marginal cost may not be identifiable. Second, the numéraire always disappear in the microeconomic models we learned in school, how come it still matters in this model? Are not real variables invariant to the unit of account? The reason: basic microeconomic models are static; the survival of the numéraire comes from the dynamic implementation. But this comes from a series of modeling decisions that may not pass Occam's razor.[3]

This paper's critique of the NKPC is the subject of the next section.

2 How producers set supply

This section starts with a standard, if incomplete, derivation of the NKPC. It details the construction of the first order conditions on the consumer side and poses the problem on the producer side. Then, Subsection 2.1 shows the usual manipulations. In Subsection 2.2, I depart from standard manipulations by imposing that constrained firms try to set their production to make marginal costs equal to the imposed prices. Figure 1 illustrates the difference between Subsection 2.1 and Subsection 2.2. Finally, Subsection 2.3 investigates whether restrictions on marginal costs can fix the problem.

Here are the assumptions of the model as one can make them out:

[2] See Rotemberg and Woodford (1999) for a discussion of different marginal cost measures.

[3] Incidentally, the Occam's razor argument was the most important one raised against monopolistic competition in the first place, as noted in the Introduction of Brakman and Heijdra (2004, p. 10).

Assumption 1 *There is one representative consumer who values products with a constant elasticity of scale (CES) function with an elasticity parameter, ϵ.*

Assumption 2 *Time preferences are represented by a constant relative risk aversion (CRRA) function with a risk-aversion coefficient, ρ.*

Assumption 3 *All firms are ultimately owned by the consumer whose stochastic discount factor determines profit discounting.*

Assumption 4 *All firms are symmetrical and each produces a differentiated and perishable product.*

Assumption 5 *All firms have a constant probability ω of not being able to reset their price at the next period.*

Assumption 6 *All production goes into consumption, there is no capital.*

Assumption 7 *All firms have an increasing-in-production marginal cost function.*

These assumptions are intentionally too restrictive. Assumptions 2 and Assumptions 3 can be replaced by posing another reasonable stochastic discount factor, the probabilities can be generalized in Assumption 5 and the Assumption 6 can also be generalized. Those restrictions are standard and meant to simplify exposition. Additionally, Subsection 2.3 will relax Assumptions 7.

Before the next subsections, I present the non-contentious part of the NKPC derivation: the first order conditions for the consumer and the problem of the producers.

In the familiar consumer side equations, consumers optimize using cost minimization subject to a fixed utility constraint, or

$$\min_{y_{j,t}} \int_0^1 p_{j,t} y_{j,t} \mathrm{d}j \quad \text{subject to} \quad Y_t = \left(\int_0^1 y_{j,t}^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}j \right)^{\frac{\epsilon}{\epsilon-1}},$$

where $p_{j,t}$ and $y_{j,t}$ are respectively the price and production of firm j at time t and Y_t represents the aggregate production measure at time t. The constraint is a CES utility function for a continuum of products. The CES utility function is standard in macroeconomic models, first as a homogeneous function, second as it aggregates into a CRRA utility function, also a standard of macroeconomic models.

The minimization results in the first order conditions for each product j which consist of the utility constraint and

$$p_{j,t} - P_t Y_t^{\frac{1}{\epsilon}} y_{j,t}^{-\frac{1}{\epsilon}} = 0, \tag{1}$$

where P_t is the Lagrange multiplier, but also represents the price index, i.e. the marginal utility of an additional unit of aggregate production.

The intertemporal profit maximization problem of the firm j is

$$\max_{p_{j,t}} \left\{ \mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{Y_{t+i}}{Y_t} \right)^{-\rho} \left(\frac{p_{j,t}}{P_{t+i}} y_{j,t+i} - C_{t+i} \left(y_{j,t+i} \right) \right) \right\}, \tag{2}$$

where C represents real total cost as a function of production and common to all firms and ω^i represents the probability, uniform across firms, for a firm of not being able to reset price until time $t+i$. Notice the use of the CRRA stochastic discount factor for discounting profits, with β, the ordinary discount factor, and ρ, the relative-risk-aversion coefficient. This stochastic discount factor comes from production entirely going into consumption.

2.1 Supply on demand

This subsection replicates the manipulations behind the NKPC as seen in the literature. It goes from the producer first order condition to the resulting price index and price aggregation equation. The subsection shows that those manipulations rely on producers having a supply-on-demand business model. Calculations in this subsection stop short of deriving the NKPC as soon as the link between inflation and marginal cost becomes obvious.

The first order condition comes after inserting $y_{j,t}$ from equation (1) into equation (2), yielding

$$\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{Y_{t+i}}{Y_t} \right)^{1-\rho} \left(\frac{P_{t+i}}{P_t} \right)^{\epsilon} \left((1-\epsilon) \frac{p_{j,t}}{P_{t+i}} + \epsilon c_{t+i}(y_{j,t+i}) \right) = 0. \tag{3}$$

where c represents the real marginal cost function. Essentially, the insertion of equation (1) means constrained firms choose production level along the consumer demand curve yielding supply on demand where the demand function, equation (1), always applies. This insertion is the point of contention of the paper; firms ignore there own supply incentives. In simple terms, the insertion of future demand supposes some kind of commitment possible to keep firms from behaving as rational price takers in the future, but the literature offers no such commitment argument.

With supply on demand, the demand function applies to every product, yielding a simple price index,

$$P_t = \left(\int_0^1 p_{j,t}^{1-\epsilon} \mathrm{d}j \right)^{\frac{1}{1-\epsilon}}, \tag{4}$$

from a straightforward insertion of the demand functions into the utility constraint. In turn, because of the symmetry of firms, the price index solves recursively to

$$P_t^{1-\epsilon} = (1-\omega)(p_t^*)^{1-\epsilon} + \omega P_{t-1}^{1-\epsilon}, \tag{5}$$

where ω also acts as the share of constrained firms and p_t^* represents the optimal price common to unconstrained firms. Lagged prices appear because every firm has a uniform probability, ω, of not being able to reset its price, thus creating a mirror image of the price index.

The NKPC therefore relies on the equation (5) to yield contemporary inflation through P_{t-1} by combining the equation with equation (3) after manipulations involving log-linearization. Note that the level of the P's is unidentified in equation (3), a homogeneous function in only contemporary or future (not fixed in advance) prices. Past prices in equation (5) and marginal costs in equation (3) produce an inflation equation, the NKPC. This is straightforward, the reader can be spared the subsequent manipulations.

2.2 Supply at salable marginal cost

This subsection uses standard microeconomic behavior instead of supply on demand used in the previous subsection. It results in a different production schedule for constrained firms as illustrated in Figure 1. Furthermore, the resulting price index does not generate a well behaved price aggregation equation and prices stop being procyclical.

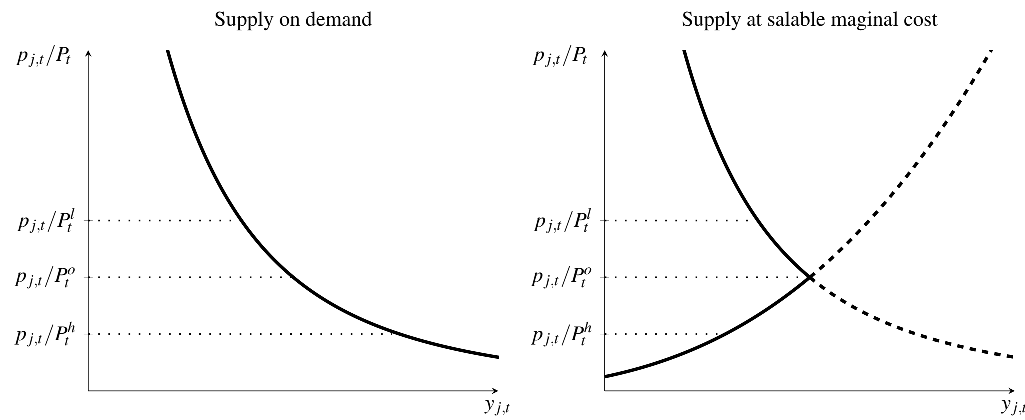
Since constrained firms are constrained in their price, not their production, they choose a production level along the consumer demand curve only if the relative price they face is too high. Firms produce only what can be sold with a relative price too high for a perishable product.

Otherwise, constrained firms choose a production level along the marginal cost curve if the relative price they face is too low. Maximization of profits under a fixed price results in the price taker setting nominal marginal cost to equal price, as they choose $y_{j,t}$ to maximize $p_{j,t}y_{j,t} - P_t C_t(y_{j,t})$. But because the product is perishable, these firms will instead set production to the minimal production level from the two curves. Thus, this type of firm behavior yields supply at salable marginal cost for constrained firms, or

$$y_{j,t} = \min\left\{c_t^{-1}\left(\frac{p_{j,t}}{P_t}\right), \left(\frac{p_{j,t}}{P_t}\right)^{-\epsilon} Y_t\right\},\ \forall j \in \{\text{constrained firms}\},$$

where c^{-1} represents the inverse of the marginal cost function.

Figure 1: Production determination for a constrained firm



Note: A low relative price can mean an increase or a decrease in production depending on how producers set supply.

Figure 1 illustrates both supply on demand and supply at salable marginal cost. In this figure, P^o represents the optimal price index from the standpoint of the constrained firm, while P^l and P^h represent price indexes that are respectively lower and higher than P^o. The supply-on-demand business model, from standard NKPC manipulations, associates P^h with increased production yielding a model where firms forced to sell at low relative prices produce more. Using standard microeconomic behavior instead, any deviation from P^o lowers production. A surprise rise in the price index does not raise production; in fact, any surprise lowers production.

Furthermore, supply at salable marginal cost does not produce a well-behaved price index, but

$$P_t = \left(\int_0^{z_t} \left(\frac{c_t^{-1} \left(\frac{p_{j,t}}{P_t} \right)}{P_t^{\epsilon} Y_t} \right)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}j + \int_{z_t}^{1} p_{j,t}^{1-\epsilon} \mathrm{d}j \right)^{\frac{1}{1-\epsilon}} .$$

where z_t expresses the proportion of firms that set price equal to marginal cost at time t. In addition, the proportion of firms that are both constrained and set production along the demand curve is $(\omega - z_t)$, while the proportion of unconstrained firms is $(1-\omega)$. Moreover, z_t depends on the other variables. This proportion varies through time according to the movement of that very price index, a price index that makes it impossible to generate the aggregation in equation (5).

Of course, generating the aggregation equation is not the point. The point is, the resulting inflation equation, if an inflation equation is possible, will not imply procyclical inflation. The rest of this paper will not determine what this inflation equation looks like, because the equation would not be interesting. A certain result, procyclical inflation, justified the assumptions behind the NKPC; finding an inflation equation free of that result is of little value.

2.3 Alternative marginal costs

The previous subsection relied on the standard assumption of an increasing marginal cost curve. This subsection investigates two other specifications. The first supposes that somehow there exists a macroeconomic argument restricting the constrained firms' product equilibrium always in the intersection between the demand curve and the marginal cost (supply) curve. This specification's analysis shows the model creates a proportionality between prices and nominal marginal costs that makes the NKPC vanish and a model of inflation impossible. The second specification supposes that every firm's marginal cost is constant. That specification's analysis shows some firms refuse to produce and the NKPC does not fare better.

The only way for the production of constrained firms to lie along the demand curve would be that prices also equal nominal marginal costs for every constrained firm. Then, going back to the first order condition, equation (3) yields, when future nominal marginal costs equal prices,

$$(1-\epsilon)\frac{p_{j,t}}{P_t}+\epsilon c_t(y_{j,t})+\mathrm{E}_t\sum_{i=1}^{\infty}\omega^i\beta^i\left(\frac{Y_{t+i}}{Y_t}\right)^{1-\rho}\left(\frac{P_{t+i}}{P_t}\right)^{\epsilon-1}\frac{p_{j,t}}{P_t}=0.$$

Therefore, firms set marginal costs according to

$$p_{j,t}=\begin{cases} P_t c_t(y_{j,t}) & \text{if } j\in[0,\omega] \\ \mu_t P_t c_t(y_{j,t}) & \text{if } j\in(\omega,1]\end{cases}, \tag{6}$$

with a markup, μ_t, common to firms that can reset price, of

$$\mu_t\equiv\epsilon\left(\epsilon-1-\mathrm{E}_t\sum_{i=1}^{\infty}\omega^i\beta^i\left(\frac{Y_{t+i}}{Y_t}\right)^{1-\rho}\left(\frac{P_{t+i}}{P_t}\right)^{\epsilon-1}\right)^{-1}.$$

Note that without price rigidity ($\omega=0$), the static monopolistic competition price, or Lerner formula, determines the markup, $\mu_t=\epsilon/(\epsilon-1)$. More importantly, the equation for μ_t puts everything in terms of growth, so even if P_t and Y_t appear, the equation, like equation (3) is entirely prospective, as P_t serves only to express future prices in relative terms.

As a result of equation (6), the price index, equation (4), becomes

$$P_t=\left(\int_0^{\omega}(P_t c_t(y_{j,t}))^{1-\epsilon}\mathrm{d}j+\int_{\omega}^{1}(\mu_t P_t c_t(y_{j,t}))^{1-\epsilon}\mathrm{d}j\right)^{\frac{1}{1-\epsilon}}.$$

Meaning, nominal marginal cost and the price index are proportional, or simply put,

$$\int_0^{\omega}c_t(y_{j,t})^{1-\epsilon}\mathrm{d}j+\mu_t^{1-\epsilon}\int_{\omega}^{1}c_t(y_{j,t})^{1-\epsilon}\mathrm{d}j=1.$$

This equation shows that the index applied to real marginal cost is invariant and consequently, because of the symmetry between firms, the price level does not affect production. In other words, for whichever direction constrained firms have to move their marginal cost, unconstrained firms will move theirs in the opposite direction so the price index moves independently of real marginal cost. This result holds not only when some firms are constrained in the price they set, but also when those that can set their price have to consider its effects on future profits. Although still technically valid, equation (5) is irrelevant because the optimal price, p_t^*, compensates for the old price, P_{t-1}, by way of unconstrained firms adjusting production, and prices, to respond to the adjustment made by constrained firms.

Ergo, equilibrium between supply and demand for every firm leads to the price index disappearing and the NKPC not existing. The disappearance of the price index, essentially a numéraire, appears a logical consequence of posing equilibrium, because the numéraire always disappear in ordinary microeconomic models. This specification fails, and that leads us to the other specification.

Finally, the other specification poses that marginal costs are constant. But with marginal costs constant, constrained firms refuse to produce when prices are under marginal costs. The problem admits only a border solution,

$$y_{j,t} = \begin{cases} 0 & \text{if } p_{j,s} < P_t \bar{c}_t \\ \left(\frac{p_{j,t}}{P_t}\right)^{-\epsilon} Y_t & \text{if } p_{j,s} \geq P_t \bar{c}_t \end{cases},$$

where $\bar{c}_t$ represents the common marginal cost. Low relative prices still means decreased production, more producers halting production to be exact, and the price index,

$$P_t = \left(\int_{[p_{j,t} \geq P_t \bar{c}_t]} p_{j,t}^{1-\epsilon} \mathrm{d}j \right)^{\frac{1}{1-\epsilon}},$$

is still not well behaved and makes it impossible to generate the aggregation in equation (5). Specifically, the integral's support depends on the price index as it more or less counts the number of $p_{j,t}$ above the common nominal marginal cost.

In conclusion, this subsection showed that the assumption of supply on demand cannot realistically be replaced by an assumption on the form of the marginal cost function.

3 Conclusion

As seen in Subsection 2.2, posing standard microeconomic behavior for how prices affect marginal costs invalidates the NKPC. As seen in Subsection 2.3, tinkering with marginal costs does not help. Therefore, if this paper does not aim to invalidate the NKPC; it aims to point to an assumption made unknowingly. Keynesian researchers can — and should — choose to ignore the issue for practical reasons, but be forthcoming about it by acknowledging the problem, and mentioning, as an assumption, this supply-on-demand business model that producers have.

So, herein lies the dilemma: standard microeconomics yields a complicated model with uninteresting results, and the NKPC relies on an unrealistic assumption. It may not be the case that price rigidity is a dead end; it is certainly the case the assumption should be public.

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A Additional material intended for referees

The following calculations are excessively detailed.

A.1 Calculation of equation (1)

Take the Lagrangian,

$$\mathcal{L} = \int_0^1 p_{j,t} y_{j,t} \mathrm{d}j - P_t \left(\left(\int_0^1 y_{j,t}^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}j \right)^{\frac{\epsilon}{\epsilon-1}} - Y_t \right),$$

with a first derivative of

$$p_{j,t} - P_t \frac{\epsilon}{\epsilon - 1} \left(\int_0^1 y_{j,t}^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}j \right)^{\frac{\epsilon}{\epsilon-1} - 1} \frac{\epsilon - 1}{\epsilon} y_{j,t}^{\frac{\epsilon-1}{\epsilon} - 1} = 0,$$

which simplifies to

$$p_{j,t} - P_t \left(\int_0^1 y_{j,t}^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}j \right)^{\frac{\epsilon}{\epsilon-1} \frac{1}{\epsilon}} y_{j,t}^{-\frac{1}{\epsilon}} = 0,$$

then, inserting the utility constraint, yields equation (1),

$$p_{j,t} - P_t Y_t^{\frac{1}{\epsilon}} y_{j,t}^{-\frac{1}{\epsilon}} = 0.$$

A.2 Calculation of equation (3)

Start with equation,

$$\max_{p_{j,t}} \left\{ \mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{Y_{t+i}}{Y_t}\right)^{-\rho} \left(\frac{p_{j,t}}{P_{t+i}} y_{j,t+i} - C_{t+i}\left(y_{j,t+i}\right)\right)\right\},$$

replace $y_{j,t+i}$ with its demand function,

$$\max_{p_{j,t}} \left\{ \mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{Y_{t+i}}{Y_t}\right)^{-\rho} \left(\left(\frac{p_{j,t}}{P_{t+i}}\right)^{1-\epsilon} Y_{t+i} - C_{t+i}\left(\left(\frac{p_{j,t}}{P_{t+i}}\right)^{-\epsilon} Y_{t+i}\right)\right)\right\},$$

then perform maximization to yield

$$\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{Y_{t+i}}{Y_t}\right)^{-\rho} \left((1-\epsilon)\left(\frac{p_{j,t}}{P_{t+i}}\right)^{-\epsilon} \frac{Y_{t+i}}{P_{t+i}} + \epsilon c_{t+i}(y_{j,t+i})\left(\frac{p_{j,t}}{P_{t+i}}\right)^{-\epsilon-1} \frac{Y_{t+i}}{P_{t+i}}\right) = 0,$$

then, by rearranging the terms, get

$$\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{Y_{t+i}}{Y_t}\right)^{-\rho} \left(\frac{P_{t+i}}{p_{j,t}}\right)^{\epsilon} Y_{t+i} \left((1-\epsilon)\frac{1}{P_{t+i}} + \epsilon c_{t+i}(y_{j,t+i})\frac{1}{p_{j,t}}\right) = 0.$$

Then, multiplying by $p_{j,t}$ and dividing by Y_t, it yields

$$\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{Y_{t+i}}{Y_t}\right)^{1-\rho} \left(\frac{P_{t+i}}{p_{j,t}}\right)^{\epsilon} \left((1-\epsilon)\frac{p_{j,t}}{P_{t+i}} + \epsilon c_{t+i}(y_{j,t+i})\right) = 0.$$

Finally, by multiplying by $p_{j,t}^{\epsilon}$ and dividing by P_t^{ϵ}, it yields equation (3),

$$\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{Y_{t+i}}{Y_t}\right)^{1-\rho} \left(\frac{P_{t+i}}{P_t}\right)^{\epsilon} \left((1-\epsilon)\frac{p_{j,t}}{P_{t+i}} + \epsilon c_{t+i}(y_{j,t+i})\right) = 0.$$

A.3 Calculation of equation (4)

Take equation,

$$p_{j,t} - P_t Y_t^{\frac{1}{\epsilon}} y_{j,t}^{-\frac{1}{\epsilon}} = 0,$$

rewrite it as

$$y_{j,t} = P_t^{\epsilon} Y_t p_{j,t}^{-\epsilon},$$

insert in the utility constraint to get

$$Y_t = \left(\int_0^1 \left(P_t^{\epsilon} Y_t p_{j,t}^{-\epsilon} \right)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}j \right)^{\frac{\epsilon}{\epsilon-1}} = Y_t P_t^{\epsilon} \left(\int_0^1 p_{j,t}^{1-\epsilon} \mathrm{d}j \right)^{\frac{\epsilon}{\epsilon-1}}.$$

Isolating P_t yields equation (4).

$$P_t = \left(\int_0^1 p_{j,t}^{1-\epsilon} \mathrm{d}j \right)^{\frac{1}{1-\epsilon}}$$

A.4 Calculation of the last equation of Section 2.2

Start with

$$y_{j,t} = \min\left\{c_t^{-1}\left(\frac{p_{j,t}}{P_t}\right), \left(\frac{p_{j,t}}{P_t}\right)^{-\epsilon} Y_t\right\},$$

and insert it in the utility constraint after defining z as the proportion of firms that are both constrained and set marginal cost equal to price,

$$Y_t = \left(\int_0^{z_t} c_t^{-1}\left(\frac{p_{j,t}}{P_t}\right)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}j + \int_{z_t}^{\omega}\left(\left(\frac{p_{j,t}}{P_t}\right)^{-\epsilon} Y_t\right)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}j + \int_{\omega}^{1}\left(\left(\frac{p_{j,t}}{P_t}\right)^{-\epsilon} Y_t\right)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}j\right)^{\frac{\epsilon}{\epsilon-1}},$$

which can be written

$$Y_t = \left(\int_0^{z_t} c_t^{-1}\left(\frac{p_{j,t}}{P_t}\right)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}j + \int_{z_t}^{1}\left(\left(\frac{p_{j,t}}{P_t}\right)^{-\epsilon} Y_t\right)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}j\right)^{\frac{\epsilon}{\epsilon-1}},$$

and simplifies as

$$1 = \int_0^{z_t}\left(\frac{c_t^{-1}\left(\frac{p_{j,t}}{P_t}\right)}{Y_t}\right)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}j + \int_{z_t}^{1}\left(\frac{p_{j,t}}{P_t}\right)^{1-\epsilon} \mathrm{d}j,$$

then, multiplying all terms by $P_t^{1-\epsilon}$,

$$P_t^{1-\epsilon} = \int_0^{z_t}\left(\frac{c_t^{-1}\left(\frac{p_{j,t}}{P_t}\right)}{P_t^{\epsilon} Y_t}\right)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}j + \int_{z_t}^{1} p_{j,t}^{1-\epsilon}\mathrm{d}j,$$

which yields the last equation of Section 2.2,

$$P_t = \left(\int_0^{z_t}\left(\frac{c_t^{-1}\left(\frac{p_{j,t}}{P_t}\right)}{P_t^{\epsilon} Y_t}\right)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}j + \int_{z_t}^{1} p_{j,t}^{1-\epsilon}\mathrm{d}j\right)^{\frac{1}{1-\epsilon}}.$$

A.5 Calculation of the first equation of Section 2.3

Start with equation (3),

$$\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{Y_{t+i}}{Y_t}\right)^{1-\rho} \left(\frac{P_{t+i}}{P_t}\right)^{\epsilon} \left((1-\epsilon)\frac{p_{j,t}}{P_{t+i}} + \epsilon c_{t+i}(y_{j,t+i})\right) = 0,$$

and impose $p_{j,t} = P_t c_{t+i}(y_{j,t}),\ \forall i > 0$ (notice the summation will now start at 1),

$$\begin{aligned} &\omega^0 \beta^0 \left(\tfrac{Y_t}{Y_t}\right)^{1-\rho} \left(\tfrac{P_t}{P_t}\right)^{\epsilon} \left((1-\epsilon)\tfrac{p_{j,t}}{P_t} + \epsilon c_t(y_{j,t})\right) \\ &+ \mathrm{E}_t \textstyle\sum_{i=1}^{\infty} \omega^i \beta^i \left(\tfrac{Y_{t+i}}{Y_t}\right)^{1-\rho} \left(\tfrac{P_{t+i}}{P_t}\right)^{\epsilon} \left((1-\epsilon)\tfrac{p_{j,t}}{P_{t+i}} + \epsilon \tfrac{p_{j,t}}{P_{t+i}}\right) = 0, \end{aligned}$$

which simplifies to

$$(1-\epsilon)\frac{p_{j,t}}{P_t} + \epsilon c_t(y_{j,t}) + \mathrm{E}_t \sum_{i=1}^{\infty} \omega^i \beta^i \left(\frac{Y_{t+i}}{Y_t}\right)^{1-\rho} \left(\frac{P_{t+i}}{P_t}\right)^{\epsilon} \frac{p_{j,t}}{P_{t+i}} = 0.$$

which can be written as the first equation of Section 2.3,

$$(1-\epsilon)\frac{p_{j,t}}{P_t} + \epsilon c_t(y_{j,t}) + \mathrm{E}_t \sum_{i=1}^{\infty} \omega^i \beta^i \left(\frac{Y_{t+i}}{Y_t}\right)^{1-\rho} \left(\frac{P_{t+i}}{P_t}\right)^{\epsilon-1} \frac{p_{j,t}}{P_t} = 0.$$

A.6 Calculation of the markup in Section 2.3

Start with the previous equation,

$$(1-\epsilon)\frac{p_{j,t}}{P_t} + \epsilon c_t(y_{j,t}) + \mathrm{E}_t \sum_{i=1}^{\infty} \omega^i \beta^i \left(\frac{Y_{t+i}}{Y_t}\right)^{1-\rho} \left(\frac{P_{t+i}}{P_t}\right)^{\epsilon-1} \frac{p_{j,t}}{P_t} = 0,$$

and multiply by P_t,

$$(1-\epsilon)p_{j,t} + \epsilon P_t c_t(y_{j,t}) + \mathrm{E}_t \sum_{i=1}^{\infty} \omega^i \beta^i \left(\frac{Y_{t+i}}{Y_t}\right)^{1-\rho} \left(\frac{P_{t+i}}{P_t}\right)^{\epsilon-1} p_{j,t} = 0,$$

then regroup $p_{j,t}$ and $P_t c_t(y_{j,t})$,

$$\left(1-\epsilon + \mathrm{E}_t \sum_{i=1}^{\infty} \omega^i \beta^i \left(\frac{Y_{t+i}}{Y_t}\right)^{1-\rho} \left(\frac{P_{t+i}}{P_t}\right)^{\epsilon-1}\right) p_{j,t} = -\epsilon P_t c_t(y_{j,t}),$$

then, $p_{j,t} = \mu_t P_t c_t(y_{j,t})$ means

$$\mu_t = -\epsilon \left(1-\epsilon + \mathrm{E}_t \sum_{i=1}^{\infty} \omega^i \beta^i \left(\frac{Y_{t+i}}{Y_t}\right)^{1-\rho} \left(\frac{P_{t+i}}{P_t}\right)^{\epsilon-1}\right)^{-1},$$

or,

$$\mu_t = \epsilon \left(\epsilon - 1 - \mathrm{E}_t \sum_{i=1}^{\infty} \omega^i \beta^i \left(\frac{Y_{t+i}}{Y_t}\right)^{1-\rho} \left(\frac{P_{t+i}}{P_t}\right)^{\epsilon-1}\right)^{-1}.$$

A.7 Calculation of the last equation of Section 2.3

Start with

$$y_{j,t} = \begin{cases} 0 & \text{if } p_{j,s} < P_t \bar{c}_t \\ \left(\frac{p_{j,t}}{P_t}\right)^{-\epsilon} Y_t & \text{if } p_{j,s} \geq P_t \bar{c}_t \end{cases},$$

insert in the utility constraint equation,

$$Y_t = \left(\int_{[p_{j,t} < P_t \bar{c}_t]} 0 \, \mathrm{d}j + \int_{[p_{j,t} \geq P_t \bar{c}_t]} \left(P_t^{\epsilon} Y_t p_{j,t}^{-\epsilon} \right)^{\frac{\epsilon - 1}{\epsilon}} \mathrm{d}j \right)^{\frac{\epsilon}{\epsilon - 1}},$$

which simplifies to

$$Y_t = Y_t P_t^{\epsilon} \left(\int_{[p_{j,t} \geq P_t \bar{c}_t]} p_{j,t}^{1-\epsilon} \mathrm{d}j \right)^{\frac{\epsilon}{\epsilon - 1}},$$

that, isolating P_t yields the last equation of Section 2.3,

$$P_t = \left(\int_{[p_{j,t} \geq P_t \bar{c}_t]} p_{j,t}^{1-\epsilon} \mathrm{d}j \right)^{\frac{1}{1-\epsilon}}.$$