On the architecture of the rings of Saturn: An "identity" theory of the distribution of gaps within rings

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On the Architecture of the Rings of Saturn:

An "Identity" Theory of the Distribution of Gaps within Rings

by Scott A. Albers[1]

Abstract: In the physical world the "identity" of something is taken generally as a given; an apple is an apple; *this* apple is *this* apple. When dealing with planetary structure and extension into space, however, the problem of the planet's "identity" in the surrounding cosmos is writ large. What does a planet's "identity" imply? What functions must it take on? What internal logic holds it together as a functional "being" in the universe?

A model of long-wave economic activity and crisis in the United States – "the Political Economy wave" – portrays the quest for social and economic "identity" with three simple curves: a sine curve over 56 years (20,454 days), a damping cosine curve of one-half the period of the sine curve, and the addition of these two in a "Political Economy Wave." The logic of this wave is a recurring structure which shapes social "identity" over time.

This paper compares the main peaks, intersections and troughs of the Political Economy wave for the United States, 1800-to-present, with the structure of the rings of Saturn, one of the most confounding structures known to science. At the present time gaps appear between rings which are unexplained; dynamism within the rings which should disperse the rings does not do so; edges of the rings are not diffuse but well defined; satellites between rings appear to have an impact but this is uncertain.

Comparing these gaps with the various "crises" which predictably impact the "self identity," the self-understanding, of society, this paper explores the possibility that the mathematics of identity may assist in the understanding of astrophysics, and possibly vice versa.

JEL classification: B41, B5, C01, C02, C50, C63, E00, E01, E10, E19, E30, N00, N01, N11, Z10, Z13

Keywords: Rings of Saturn, Real GNP, Golden Mean, Phi, Kondratiev Wave, Global Financial Crisis, American Economic History, GNP Spiral, Okun's Law, Revolution, Kaluza, Fifth Dimension, General Relativity, Astronomy

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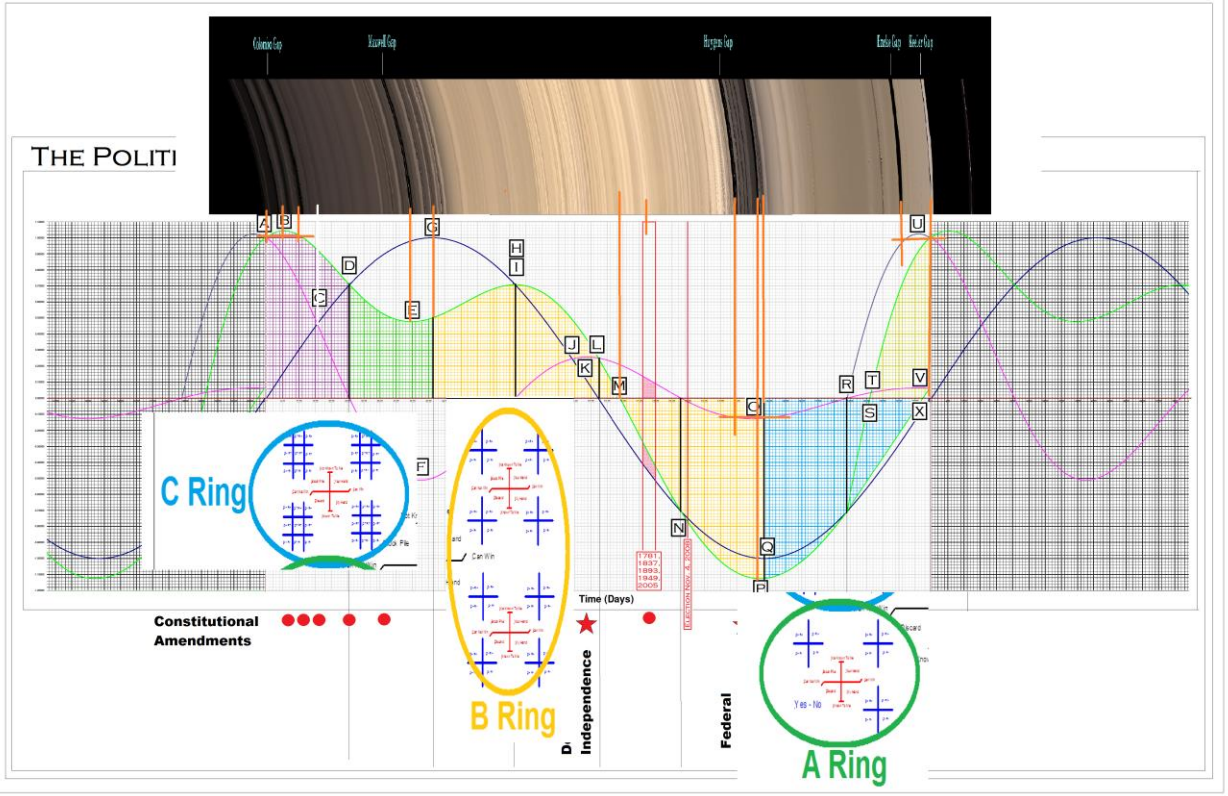
Introduction: Three Forms of "Identity"

Below is placed a photograph of the Rings of Saturn, a model of the economic history of the United States, and a graph of the logic of the three essential parts of the two-handed card game "Gin Rummy." This paper argues that these three disparate topices are connected via the topic of "identity": the identity of a physical planet, the identity of an historic economic-social system, and the identity of an evolving game of cards. Our ability to see in these three separate topics specific comparisons permits us to understand each better.

Chart 1: The Political Economy Wave and the Rings of Saturn



A brief description of the development of the models of the American economy and the card game Gin Rummy is placed in the Appendix. The connections between these first two – the Rings of Saturn and the "Political Economy wave" of the economy of the United States – is the basis of this paper.

Hypothesis

The Political Economy Wave of U.S. macroeconomic history organizes the Rings of Saturn.

Method

American economic history can be shown to be extremely periodic. This periodicity is tracked by "the Political Economy wave." The derivation and significance of this is explained briefly in the appendix.

Construction of the Political Economy wave begins with a sine wave with a maximum of "1" subdivided into 20,454 cells in an Excel spreadsheet. Each cell represents a single "day" in an exactly 56-year economic cycle of American economic history. Next to the cells representing the sine wave is constructed a damping cosine wave with a height of "1" at the y-axis, but with a period one-half of the sine wave and extending over the same length of time.

The Political Economy wave is the addition of these two. Because the damping cosine wave exceeds "1" prior to its y-axis intercept, additional Excel columns were constructed to investigate the significance of this fact, both prior to and subsequent to the main period of the PE wave. In Chart One these additional periods are placed in grey to the left and right.

The equation used to create this spreadsheet is as follows.

$$g(y) = 2^{\frac{1}{2} - \frac{2\left(y + \frac{1}{4}\pi\right)}{\pi}} \sin\left(2y + \frac{1}{2}\pi\right) + \sin(y)$$

DAMPING COSINE CURVE SINE CURVE

This set of curves easily translates into a number of mathematic points of intersection, peaks, troughs, etc. These are set out in Chart One, supra. The Rings of Saturn were placed upon it in a fashion which seemed most likely to render associations between the data.

The question was whether there exists some way to test the accuracy of these associations which are made on the basis of visual observation alone.

The calculations of this graphs were taken to five decimal places. Because of the long period of time and the extended Excel spreadsheet necessary to create this graph, the peaks and troughs of the graph frequently extended over periods of days. This is not unlike the features of the Rings of Saturn which have gaps varying from 10 km to 300 km.

The innermost, midpoint, and outermost points of both the PE wave and the Rings of Saturn were determined. Multiples were then figured which would lead, in that particular case, to a perfect alignment between the features.

These multiples were then compared and placed in **bold red ink** to permit easy association between them.

Data

The Cassini project of NASA has provided measures of various features of Saturn's rings as placed in a Wikipedia article on "Rings of Saturn." This includes the following photograph...

and the following data http://planetarynames.wr.usgs.gov/Page/Rings. After consulting a number of sources for the radial measurement of the features of the rings the Wikipedia article on "Rings of Saturn" was found to be the most current. These are copied here:

Major subdivisions of the rings

Name[3]	Distance from Saturn (from center, in km)[4]	Width (km)[4]	Named after
D Ring	66,900 – 74,510	7,500	
C Ring	74,658 – 92,000	17,500	
B Ring	92,000 – 117,580	25,500	
Cassini Division	117,580 – 122,170	4,700	Giovanni Cassini
A Ring	122,170 – 136,775	14,600	
Roche Division	136,775 – 139,380	2,600	Édouard Roche
F Ring	140,180 [1]	30 – 500	
Janus/Epimetheus Ring[2]	149,000 – 154,000	5,000	Janus and Epimetheus
G Ring	166,000 – 175,000	9,000	
Methone Ring Arc[2]	194,230	?	Methone
Anthe Ring Arc[2]	197,665	?	Anthe
Pallene Ring[2]	211,000 – 213,500	2,500	Pallene
E Ring	180,000 – 480,000	300,000	
Phoebe Ring	~4,000,000 – >13,000,000		Phoebe

Structures within the C Ring

Name[3]	Distance from Saturn's center (km)[4]	Width (km)[4]	Named after
Colombo Gap	77,870 [1]	150	Giuseppe "Bepi" Colombo
Titan Ringlet	77,870 [1]	25	Titan, moon of Saturn
Maxwell Gap	87,491 [1]	270	James Clerk Maxwell
Maxwell Ringlet	87,491 [1]	64	James Clerk Maxwell
Bond Gap	88,700 [1]	30	William Cranch Bond and George Phillips Bond
1.470R_S Ringlet	88,716 [1]	16	its radius
1.495R_S Ringlet	90,171 [1]	62	its radius
Dawes Gap	90,210 [1]	20	William Rutter Dawes

Structures within the Cassini Division

Name[3]	Distance from Saturn's center (km)[4]	Width (km)[4]	Named after
Huygens Gap	117,680 [1]	285–400	Christiaan Huygens
Huygens Ringlet	117,848 [1]	~17	Christiaan Huygens
Herschel Gap	118,234 [1]	102	William Herschel
Russell Gap	118,614 [1]	33	Henry Norris Russell
Jeffreys Gap	118,950 [1]	38	Harold Jeffreys
Kuiper Gap	119,405 [1]	3	Gerard Kuiper
Laplace Gap	119,967 [1]	238	Pierre-Simon Laplace
Bessel Gap	120,241 [1]	10	Friedrich Bessel
Barnard Gap	120,312 [1]	13	Edward Emerson Barnard

Structures within the A Ring

Name[3]	Distance from Saturn's center (km)[4]	Width (km)[4]	Named after
Encke Gap	133,589 [1]	325	Johann Encke
Keeler Gap	136,505 [1]	35	James Keeler

These have been compared with Chapter 13, “The Structure of Saturn’s Rings” by J. E. Colwell, P. D. Nicholson, M. S. Tiscareno, C. D. Murray, R. G French, and E. A. Marouf; and Hedman, et al, 2009, at p. 232.

Part One. Procedure

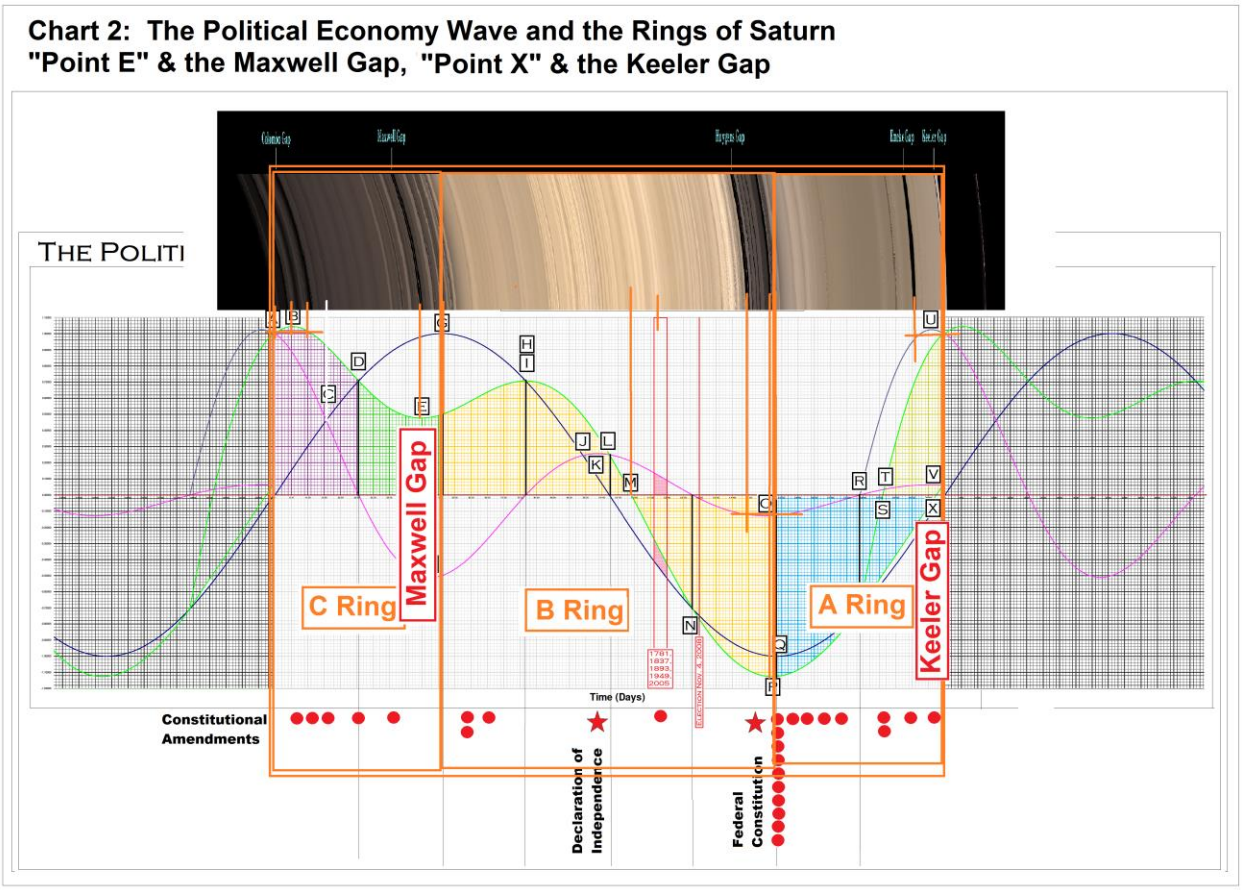
The "Political Economy wave" is the sum of a Sine curve and a Damping Cosine curve of half the period of the sine curve and with a y-intercept of "y = 1." The development of this curve is stated in the Appendix. In this section we compare directly the major points of the PE wave with the major features of Saturn's Rings.

The Maxwell Gap (Point E) and the Keeler Gap (Point X)

The Political Economy wave aligns with the C, B and A Rings, moving left to right. The C Ring is generally dark, the B Ring quite bright, and the A Ring more neutral in tone. These divisions generally align with the first quarter, the middle two quarters, and the final quarter of the Political Economy Wave, respectively. Because the Political Economy Wave originates as a model of acoustics, two possible features appeared useful in associating this economic model directly with Saturn's Rings.

The first of these was the Maxwell Gap. This gap appears toward the outer edge of the C Ring and is found above "Point E" of the Political Economy Wave.

Chart 2: The Political Economy Wave and the Rings of Saturn
"Point E" & the Maxwell Gap, "Point X" & the Keeler Gap



PE Wave	**Point E: (First depth of the PE wave)**			
	Inner	Midpoint	Outer	
Days	4,463	4,473	4,485	22 days
Y-value	+0.47704	+0.47704	+0.47704	
Saturn Rings	**Maxwell Gap:**			
	Inner	Midpoint	Outer	
	87,500	87,635	87,770	270 km
Minus				
Inner C Ring	74,658	74,658	74,658	
	12,842	12,977	13,112	
Divided by				
No. of Days	4,463	4,473	4,485	
	2.877	**2.901**	**2.923**	

The second feature which immediately seems pertinent is the Keeler Gap. This gap is found at the very outer edge of the A Ring, and appears to align directly with "Point X" of the Political Economy Wave.

PE Wave	**Point X: (The PE wave crosses "y=0" at the end of the series)**			
	Inner	Midpoint	Outer	
Days	20,246		20,247	1 day
Y-Value	-0.00009		+0.00021	
Saturn Rings	**Keeler Gap:**			
	Inner	Midpoint	Outer	
	136,530	136,547	136,565	35 km
Minus				
Inner C Ring	74,658	74,658	74,658	
	61,872	61,889	61,907	
Divided by				
No. of Days	20,246	20,246.5	20,247	
	3.056	**3.056**	**3.057**	

It was encouraging that two prominent gaps, located approximately 50,000 kilometers apart and joined by no obvious force, were within an approximate multiples of 2.9 to 3.0 for each midpoint calculation.[2]

[2] It must be added as well that the distance between the D Ring ending and the C Ring beginning (74,658 – 74,510 = 148 km), as compared to the distance between the Keeler Gap and the end of the A Ring (136,775 – 136,565 = 210 km) is but 62 km. This distance seems to be analogous in light of the mirror image of the Columbo Gap and the F Ring, and the beginning of the D Ring and the R' point of the Damping Cosine curve, see infra. Measuring the outer edge of the A Ring to the inner edge of the C ring yields a distance of 62,117 km for the range of the main part of the rings (136,775 – 74,658 = 62,117 km), a multiple of 0.001 of the variance.

A Tuning Fork Approach

This correlation between the Maxwell Gap and the Keeler Gap permits us to use these two as a form of tuning fork for the whole array. In the preceding example we considered multiples which link two features of Saturn's Rings against the two analogous features of the PE wave. We may also compare these features to the entire body of Saturn's Rings and the PE wave.

Midpoint to Midpoint

The midpoint of the Maxwell Gap lies at 12,977 km from the beginning of the C Ring, and the midpoint of the Keeler Gap lies at 61,889 km of the C Ring. This means that a span of 61,889 – 12,977 = 48,912 km lies between these two positions in the Rings of Saturn.

The midpoint of "Point E" of the PE wave occurs at Day 4,473 and the midpoint of "Point X" occurs at Day 20,246. This means that a span of 20,246 – 4,463 = 15,783 days lies between midpoints on the PE wave.

48,912 / 15,783 = **3.099** as a multiple between these two points.

Nearest to one another

The outer edges of the Maxwell Gap lies at 13,122 km from the beginning of the C Ring, and the inner edge of the of the Keeler Gap lies at 61,872 km of the C Ring. This means that a span of 61,872 – 13,122 = 48,750 km between these two positions in the Rings of Saturn.

The greatest point of "Point E" of the PE wave occurs at Day 4,485 and the least point of "Point X" occurs at Day 20,246. This means that a span of 20,246 – 4,485 = 15,761 days lies between these nearest points on the PE wave.

48,750 / 15,761 = **3.093** as multiple between these two points.

Furthest from one another

The inner edge of the Maxell Gap lies at 12,842 km from the beginning of the C Ring, and the outer edge of the Keeler Gap lies at 61,907 km of the C Ring. This means that a span of 61,907 – 12,842 = 49,065 km lies between these two positions in the Rings of Saturn.

The least point of "Point E" of the PE wave occurs at Day 4,463 and the greatest point of "Point X" occurs at Day 20,247. This means that a span of 20,247 – 4,463 = 15,784 days lies between the furthest points of the PE wave.

49,065 / 15,784 = **3.108** as a multiple between these two points.

Entire range

These figures might be compared to the distance between the inner edge of the C Ring (74,658 km) and the outer rim of the A Ring (137,775 km). This distance is 137,775 – 74,658 = 63,117 km.

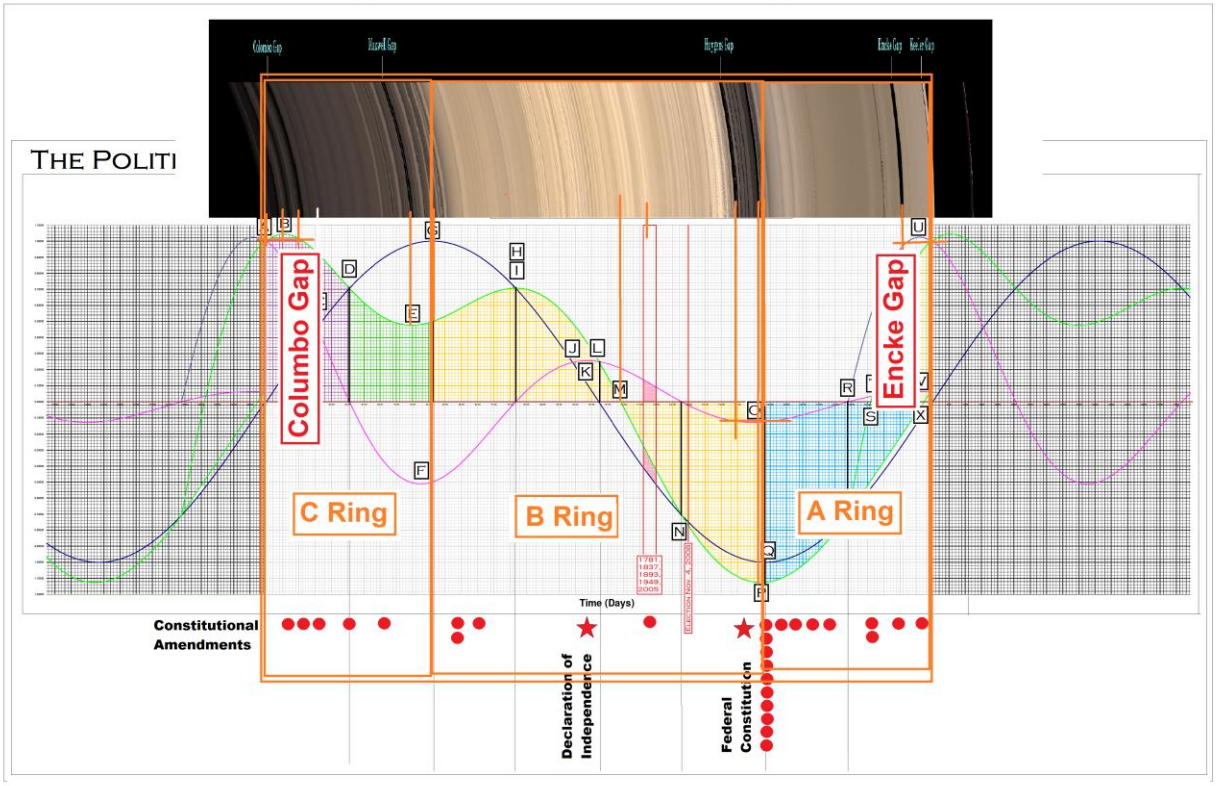
63,117 / 20,454 = **3.085** as a multiple between these two points.

These multiples may be kept in mind as the findings of the rest of the paper progress.

The Encke Gap (Point U') and the Columbo Gap (Point B')

It was noticed that whenever any of the waves which make up the Political Economy Wave or the Damping Cosine Wave exceed "y = 1" a point exists to test the relationship between this wave and the Rings of Saturn. This led to an consideration of the Encke Gap (toward the outer edge of the A Ring) and the Columbo Gap (at the inner edge of the C Ring).

Chart 3: The Political Economy Wave and the Rings of Saturn
"Point U' " & the Encke Gap, "Point B' " & the Columbo Gap



PE Wave	**Point U': (Damping Cosine curve passes "y = 1")**			
	Inner	Midpoint	Outer	
Days	19,759			1 day
Y-Value	+1.00014			
Saturn Rings	**Encke Gap:**			
	Inner	Midpoint	Outer	
	133,570	133,732	133,895	325 km
Minus				
Inner C Ring	74,658	74,658	74,658	
	58,912	59,074	59,237	
Divided by				
No. of Days	19,759	19,759	19,759	
	2.981	**2.989**	**2.997**	

Next let us consider the Columbo Gap in the C Ring, which requires the determination of a Point B' in the PE wave.

PE Wave	**Point B': (The PE wave, having reached a maximum at "B" descends and crosses the "y = 1" threshold at " B' ".)**			
	Inner	Midpoint	Outer	
Days	1,127			
Y-Value	+1.00000			1 day
Saturn Rings	**Columbo Gap:**			
	Inner	Midpoint	Outer	
	77,800	77,850	77,900	100 km
Minus				
Inner C Ring	74,658	74,658	74,658	
	3,142	3,192	3,242	
Divided by				
No. of Days	1,127	1,127	1,127	
	2.787	**2.832**	**2.876**	

Alternative: Because these multiples are outside the range of the previous 2.9-3.0 multiple considered previously, an alternative calculation was considered as generating the Columbo Gap.

If we take the number of days from Point A (the point which begins this analysis), to Point B (the peak of the PE wave and a date of significant crisis), and then double this range we obtain a point in time retreating from the crisis equal to the time preceding it.

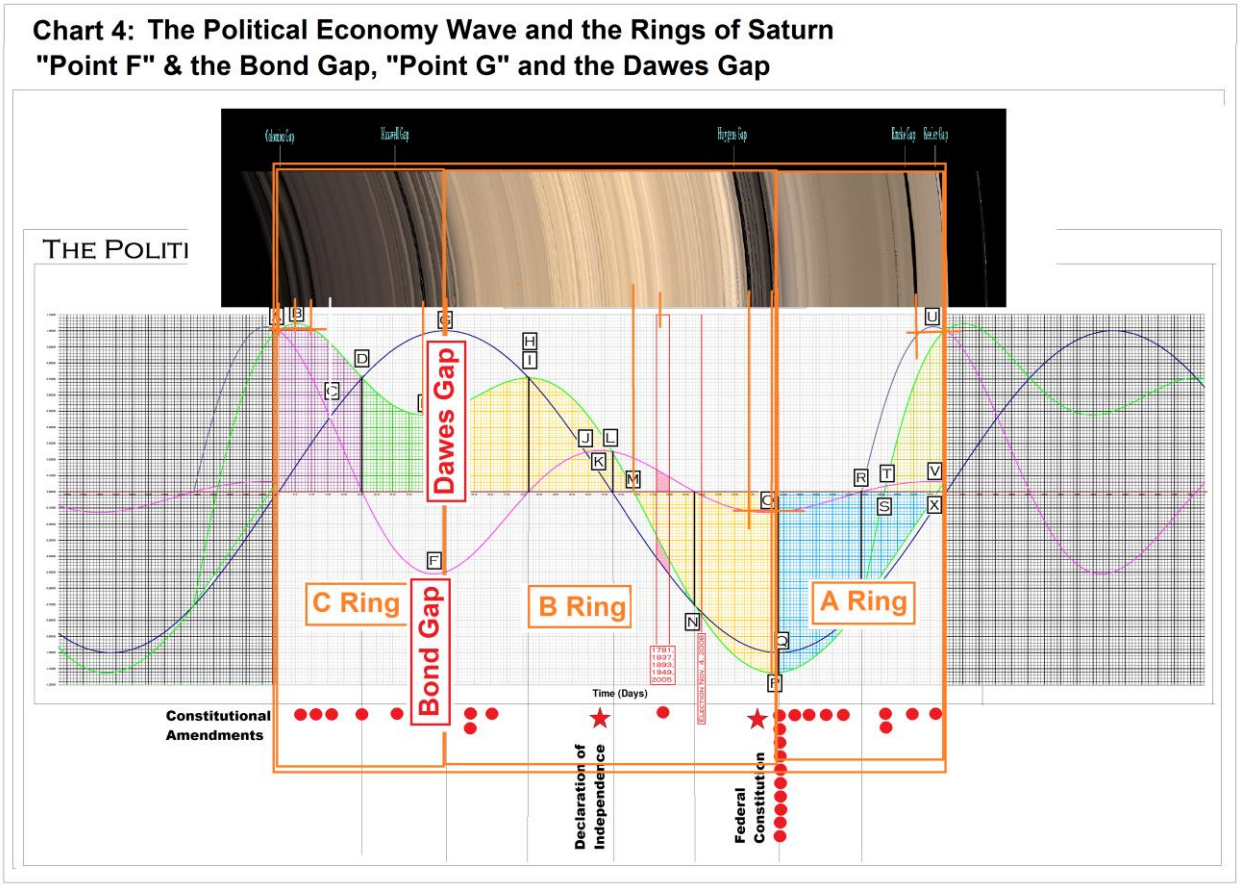
In this case the peak of B occurred during days 525-540 at a upper most point of 1.04386. Innermost, midpoint and outermost points of Point B' therefore are 525 days x 2 = 1050 days; 532.5 days x 2 = 1065 days; and 540 days x 2 = 1,080 days respectively. The points of the Columbo Gap would then be divided by this number instead of the point where the PE wave crosses the "y = 1" threshold.

Saturn Rings	**Columbo Gap:**			
	Inner	Midpoint	Outer	
	77,800	77,850	77,900	100 km
Minus				
Inner C Ring	74,658	74,658	74,658	
	3,142	3,192	3,242	
Divided by				
No. of Days	1,050	1,065	1,080	
	2.992	**2.997**	**3.001**	

This set of multiples is more consistent with the first set, but the "right" approach is not clear.

The Bond Gap (Point F) and the Dawes Gap (Point G)

This brought up the possibility of calculating the multiple implied in figuring the Bond Gap (as aligned with "Point F") and the Dawes Gap (as aligned with "Point G,") both found at the outer edge of the C Ring.



PE Wave	**Point F: (First descent of the Damping Cosine curve)**			
	Inner	Midpoint	Outer	
Days	4,753	4,760	4,767	14 days
Y-Value	-0.51222			
Saturn Rings	**Bond Gap:**			
	Inner	Midpoint	Outer	
	88,700	88,715	88,730	30 km
Minus				
Inner C Ring	74,658	74,658	74,658	
	14,042	14,057	14,072	
Divided by				
No. of Days	4,753	4,760	4,767	
	2.954	**2.953**	**2.951**	

PE Wave	**Point G: (Height of Sine curve)**			
	Inner	Midpoint	Outer	
Days	5,104	5,113	5,123	19 days
Y-Value	= +1.00000			
Saturn Rings	**Dawes Gap:**			
	Inner	Midpoint	Outer	
	90,200	90,210	90,220	20 km
Minus				
Inner C Ring	74,658	74,658	74,658	
	15,542	15,552	15,562	
Divided by				
No. of Days	5,104	5,113	5,123	
	3.045	**3.041**	**3.037**	

Initial Averages of Multiples

Simply taking the average of the figures for the Inner, Midpoint and Outer calculations so far we have:

	Inner	Midpoint	Outer
Major Gaps:			
"E" & Maxwell Gap	2.877	2.901	2.923
"X" & Keeler Gap	3.056	3.056	3.057
"B' " & Columbo Gap	2.787	2.832	2.876
"U' " & Encke Gap	2.981	2.989	2.997
"F" & Bond Gap	2.954	2.953	2.951
"G" & Dawes Gap	3.045	3.041	3.037
	17.7	17.772	17.841
Divided by	6	6	6
	2.950	**2.962**	**2.973**
compare:			
Alternative Columbo Gap	2.992	2.997	3.001

compare Maxwell Gap to Keeler Gap comparisons:

inner Maxwell Gap to outer Keeler Gap	**3.093**
midpoint Maxwell Gap to midpoint Keeler Gap	**3.099**
outer Maxwell Gap to inner Keeler Gap	**3.108**
compare entire system multiple:	**3.085**

The Dawes Gap as an Alternative Division Line between the C Ring and the B Ring

An issue which might be raised at this juncture is the appropriate characterization of the Dawes Gap, a thin gap of but 20 km.

If the Dawes Gap was taken as the terminal outer edge of the C Ring and the beginning edge of the B Ring, we would have a clear separation of the C Ring from the B Ring at Point G, i.e. the height of the Sine Curve.

At present the B Ring is deemed to begin at 92,000 km from the center of Saturn, or 92,000 – 74,658 = 17,342 km after the beginning of the C Ring. The midpoint of "Point G" is Day 5113. Dividing 17,342 / 5113 = **3.3917**, a multiple quite out of line with the association of "Point G" with the present denomination of the beginning of the B Ring.

Conversely the Dawes Gap presents a very clear possible alternative at 90,210 km from the center of Saturn, or 15,552 km from the beginning of the C Ring. The midpoint multiple for this association was **3.041**, a number much closer to the other multiples.

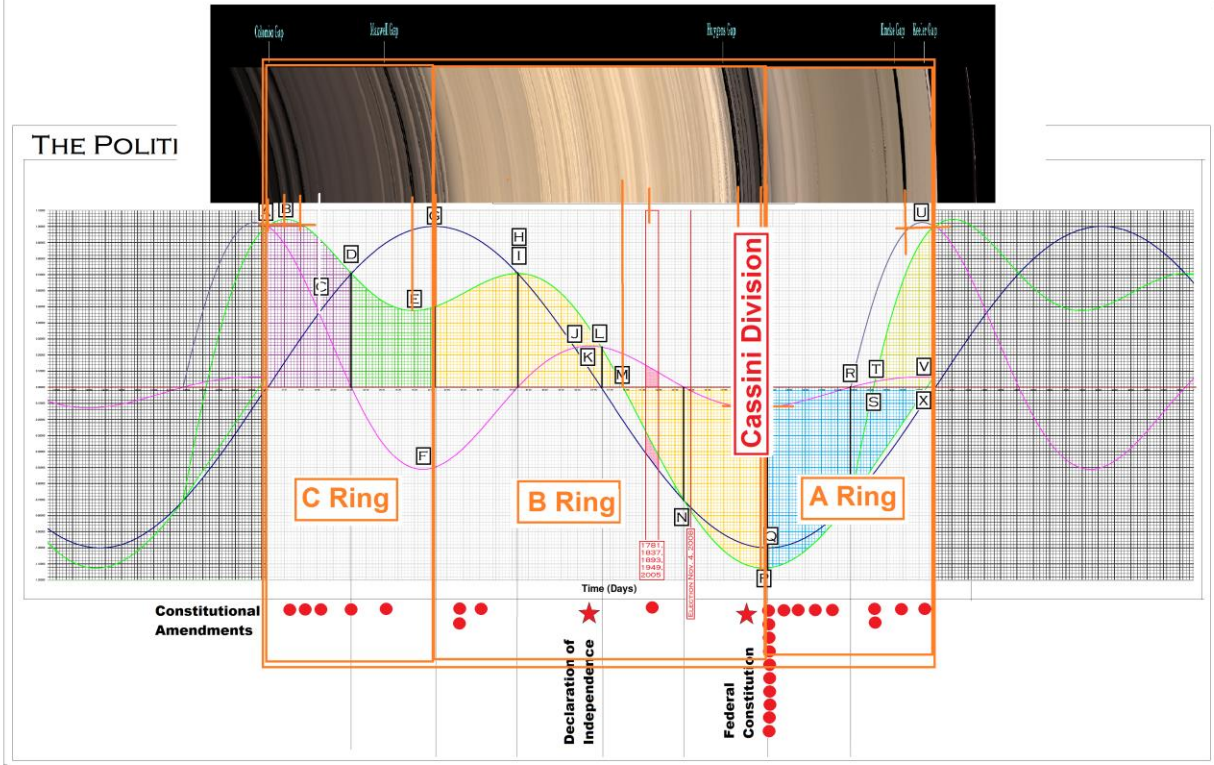
If there is no obvious reason that the next 1,790 km of the C Ring past the Dawes Gap to be designated as part of the B Ring, this alternative might be considered.

This matter will be raised again with "Point Q" and the Barnard Gap. The midpoint of the Barnard Gap is found at 120,305 km from the center of Saturn, or 45,647 km from the inner edge of the C Ring. The midpoint for the PE wave "Point Q" is Day 15,340 for a multiple of **2.975**. This is another thin gap of 13 km, found at the depth of the Sine Curve.

The Cassini Division

This brought forward an investigation of the Cassini Division. Notice first that each of the three waves which are considered – the Sine curve, the Damping Cosine curve and the PE wave – (1) are negative, (2) are relatively flat for long periods of time, and (3) are not synchronous to one another. This means that a large number of days is necessary to actually chart the curve at these points. This means as well that each of the curves reach their deepest negative values at different points in time.

Chart 5: The Political Economy Wave and the Rings of Saturn
The Cassini Division



Saturn Rings	**Cassini Division:**			
	Inner	Midpoint	Outer	
	117,500	119,835	122,170	4,670 km
Minus				
Inner C Ring	74,658	74,658	74,658	
	42,842	45,177	47,512	

There are two Gaps within the Cassini Division which exceed 200 km. These are (1) the Huygens Gap (400 km) and (2) the Laplace Gap (238 km). The full set of Gaps is as follows, with the possible associations to the Political Economy wave.

End present B Ring 117,580 km
Cassini Division:

Name(3)		Distance from Saturn's center (km)(4)	Width (km)
Point O	Huygens Gap	117,680 (1)	285–400
	Huygens Ringlet	117,848 (1)	~17
Point O?	Herschel Gap	118,234 (1)	102
	Russell Gap	118,614 (1)	33
	Jeffreys Gap	118,950 (1)	38
	Kuiper Gap	119,405 (1)	3
Point P	Laplace Gap	119,967 (1)	238
	Bessel Gap	120,241 (1)	10
Point Q	Barnard Gap	120,312 (1)	13

Begin present A Ring 122,170 km

PE Wave	**Point O: (Second depth of Damping Cosine curve)**			
	Inner	Midpoint	Outer	
Days	14,968	14,987	15006	38 days
Y-Value	-0.12805	-0.12805(*)	-0.12805	
Saturn Rings	**Huygens Gap: (within Cassini Division)**			
	Inner	Midpoint	Outer	
	117,680	117,880	118,080	400 km
Minus				
Inner C Ring	74,658	74,658	74,658	
	43,022	43,222	43,422	
Divided by				
No. of Days	14,968	14,987	15,006	
	2.874	**2.883**	**2.893**	
Saturn Rings	**Herschel Gap: (within Cassini Division)**			
	Inner	Midpoint	Outer	
	118,234	118,285	118,336	102 km
Minus				
Inner C Ring	74,658	74,658	74,658	
	43,576	43,627	43,678	
Divided by				
No. of Days	14,968	14,987	15,006	
	2.911	**2.910**	**2.910**	

PE Wave	**Point P: (greatest depth of PE wave)**			
	Inner	Midpoint	Outer	
Days	15,199	15,200	15,202	3 days
Y-Value	-1.12600	-1.12600	-1.12600	
Saturn Rings	**Laplace Gap: (within Cassini Division)**			
	Inner	Midpoint	Outer	
	119,848	119,967	120,086	238 km
Minus				
Inner C Ring	74,658	74,658	74,658	
	45,190	45,309	45,428	
Divided by				
No. of Days	15,199	15,200	15,202	
	2.973	**2.980**	**2.988**	

The Cassini Division: the Dividing Line between the B Ring and the A Ring (Point Q)

Point Q might easily be associated with the present line dividing the B Ring from the A ring at 122,170 km. In this case the multiple necessary for a perfect alignment between the two is between 3.095 and 3.099.

PE Wave	**Point Q: (Depth of Sine curve)**			
	Inner	Midpoint	Outer	
Days	15,331	15,340	15,350	19 days
Y-Value	-1.00000	-1.00000	-1.00000	
Saturn Rings	**Begin "A Ring"**			
	Inner	Midpoint	Outer	
	122,170	122,170	122,170	
Minus				
Inner C Ring	74,658	74,658	74,658	
	47,512	47,512	47,512	
Divided by				
No. of Days	15,331	15,340	15,350	
	3.099	**3.097**	**3.095**	

The Barnard Gap as an Alternative Division Line between the B Ring and the A Ring

While the above set of multiples is within the range of those we have come across, there is at least one other possibility. "Point Q," the depth of the Sine Curve, is in a similar position to "Point G" and the Dawes Gap at the height of the Sine Curve. If the midpoint of "Point Q," which is 15,340 days, would be associated with the Barnard Gap at 120,305 km from the center of Saturn (1865 km prior to the existing demarcation for the A Ring) the following calculation would apply:

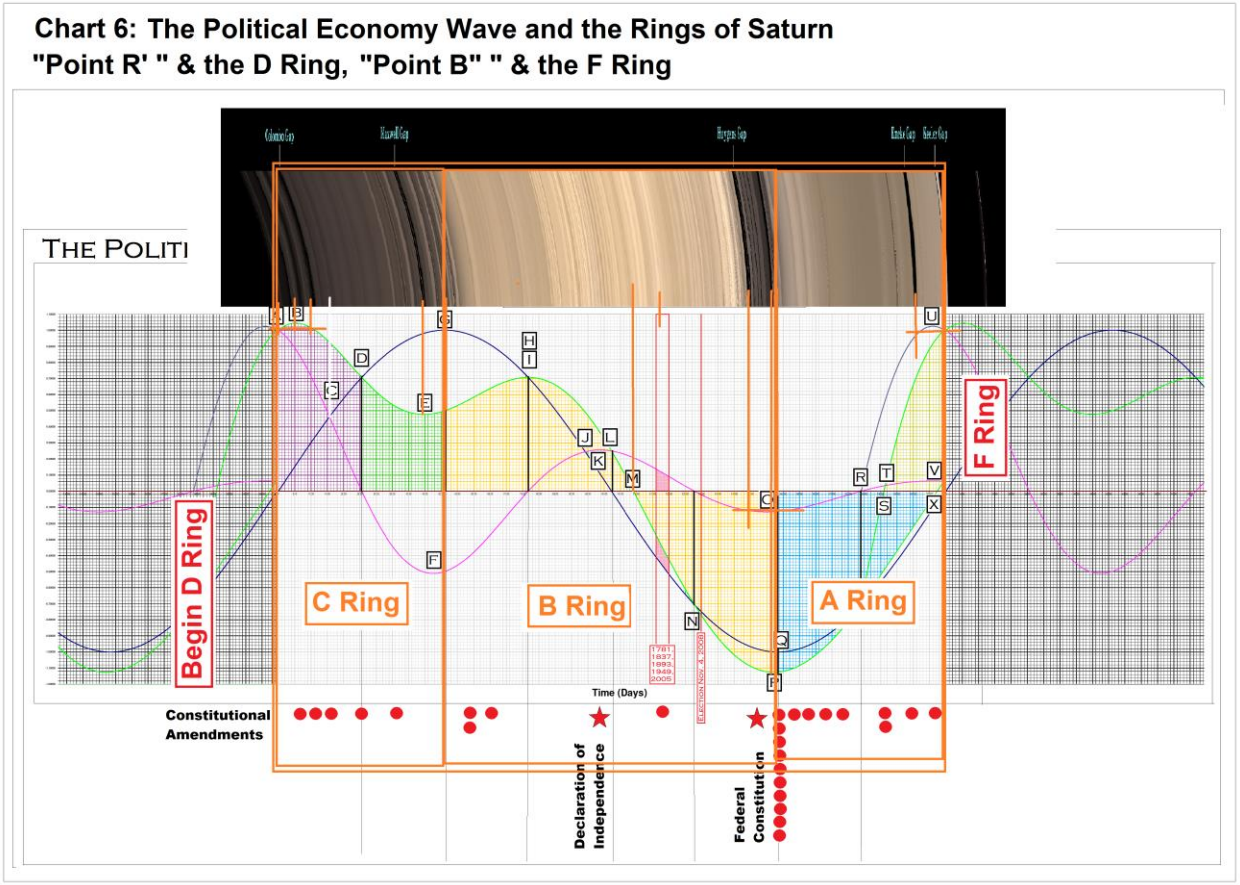
PE Wave	**Point Q: (Depth of Sine curve)**			
	Inner	Midpoint	Outer	
Days	15,331	15,340	15,350	19 days
Y-Value	-1.00000	-1.00000	-1.00000	
Saturn Rings	**Barnard Gap:**			
	Inner	Midpoint	Outer	
	120,312	120,318	120,325	13 km
Minus				
Inner C Ring	74,658	74,658	74,658	
	45,654	45,660	45,667	
Divided by				
No. of Days	15,331	15,340	15,350	
	2.977	**2.976**	**2.975**	

This second muliple is more typical of those we have seen. There are, moreover, several considerations which might be made in favor of using the Dawes Gap as the dividing line between the B Ring and the A Ring. See "Clues as to Causation," infra.

The Prelude and Postlude Rings: Rings D (Point R') and F (Point B")

The foregoing considerations led to the possibilty that the D Ring and the F Ring might be part of an extension of the economic model, as would be required to fully state the model itself.

Chart 6: The Political Economy Wave and the Rings of Saturn
"Point R' " & the D Ring, "Point B" " & the F Ring



As to the beginning of the D Ring, "Point R" and "Point S", taken from the middle of the A Ring, stood out as possibilities.

"Point R," which is the beginning point of the Damping Cosine wave as it leaves the x-axis and makes its way to the beginning "Point A" of the entire series, could be taken as the beginning point of the D Ring. To do this we simply figuring its distance to the end of the cycle, and then take this distance as preceding "Point A," which begins the PE wave. This gives us a simple way to work backwards to a new point of consideration, "Point R' ".

"Point S," which is the point at which the PE wave crosses the x-axis and makes its way to Point A, could also be taken as the beginning point of the D Ring, by the same method.

These occur as single points crossing the X-axis at "Point R" = 17,898 and "Point S" = 18,602. From these numbers we may subtract the length of the entire series, 20,454 days. This gives us Point R' " = -2556 and "Point S' " = -1852 respectively.

The D Ring commences at 66,900 km. The C Ring commences at 74,658 km. This gives the measured distance of the D Ring at 74,658 – 66,900 = 7,758 km.

We begin with the fact that the D Ring commences 74,658 – 66,900 = 7,758 km prior to the C Ring.

	Point R' precedes:		Point S' precedes:
	7,758		7,758
divided by number of days which "Point R" or "Point S" "Point A"	2,556	vs.	1,852
	3.035		4.188

Considing consistency with the rest of the multiples, it would appear that the D Ring is a manifestation of "Point R' ", the Damping Cosine wave as it leaves the X-axis at "y = 0" to join the Y-axis in this model at "Point A".

As to the F Ring, the most obvious guess is that it is a repetition of the B' Columbo Gap. This Gap appeared when the Political Economy wave returned to a value of "1", after exceeding it at Point B. This occurred on Day 1,127. (See similar discussion of the Encke Gap, *supra*.)

The F Ring occurs between 140,224 km and 140,724 km of the center of Saturn.

PE Wave	**Point B' ': (PE wave is less than "y = 1")**			
	Inner	Midpoint	Outer	
Days	20,454	20,454	20,454	
	+1,127	+1,127	+1,127	
	21,581	21,581	21,581	1 day
Y-Value	-1.00000	-1.00000	-1.00000	
Saturn Rings	**F Ring: (Extension of a new descent to "1".)**			
	Inner	Midpoint	Outer	
	140,224	140,494	140,724	500 km
Minus				
Inner C Ring	74,658	74,658	74,658	
	65,566	65,836	66,066	
Divided by				
No. of Days	21,581	21,581	21,581	
	3.038	**3.050**	**3.061**	

Using the "Alternative Approach to the Columbo Gap" we have a different point of comparison, i.e. an equivalent number of days both before and after the PE wave reaches "Point B". In this case the 20,454 days would have added to previous days, i.e. **1,050**, **1,065** and **1,080** for inner, midpoint and outermost points respectively. Notice that because we have added an entire cycle of 20,454 days and the collective span of the C, B and A rings, these multiples are virtually identical to those above. This operation has the effect of making the F Ring (width = 500 km) the mirror image of the Columbo Gap (width = 100 km).

PE Wave	**Point B' ': (Depth of sine curve)**			
	Inner	Midpoint	Outer	
Days	20,454	20,454	20,454	
	+1,050	**+1,065**	**+1,080**	
	21,504	**21,519**	**21,534**	1 day
Y-Value	-1.00000	-1.00000	-1.00000	
Saturn Rings	**F Ring: (Extension of a new descent to "1".)**			
	Inner	Midpoint	Outer	
	140,224	140,494	140,724	500 km
Minus				
Inner C Ring	74,658	74,658	74,658	
	65,566	65,836	66,066	
Divided by				
No. of Days	**21,504**	**21,519**	**21,534**	
	3.049	**3.059**	**3.067**	

Summary of Multiples

The collected average multiples to form a perfect alignment with the economic model are as follows. One can see that on average each day of the economic model (out of 20,454) equates with between 2.950 and 3.046 kilometers of distance in the radial span of Saturn's Rings. Taking the midpoint between these two extremes we have a general multiple of 3.005 with an average 3.2% variance from this midpoint.

	Inner	Midpoint	Outer
Major Gaps:			
"E" & Maxwell Gap	2.877	2.901	2.923
"X" & Keeler Gap	3.056	3.056	3.057
"B' " & Columbo Gap	2.787	2.832	2.876
"U' " & Encke Gap	2.981	2.989	2.997
"F" & Bond Gap	2.954	2.953	2.951
"G" & Dawes Gap	3.045	3.041	3.037
	17.7	17.772	17.841
Divided by	6	6	6
	2.950	**2.962**	**2.973**
Cassini Division:			
"O" & Huygens Gap	2.874	2.883	2.893
"P" & Laplace Gap	2.973	2.980	2.988
"Q" & Begin A Ring	3.099	3.097	3.095
	8.946	8.960	8.976
Divided by	3	3	3
	2.982	**2.986**	**2.992**
External Rings:			
"S' " and D Ring begins	3.032	3.032	3.032
"B' ' " and F Ring	3.038	3.050	3.061
	6.070	6.082	6.093
Divided by	2	2	2
	3.035	**3.041**	**3.046**
compare:			
Alternative Columbo Gap	2.992	2.997	3.001
Alternative F Ring	3.049	3.059	3.067
compare:			
Dawes Gap as beginning of B Ring at "Point G":		**3.045**	
Barnard Gap as beginning of A Ring at "Point Q"		**2.976**	

Arranging these in sequence, from the beginning of the D Ring to the end of the F Ring, we have the following

	Inner	Midpoint	Outer
"R' " and D Ring begins	3.035	3.035	3.035
"B' " & Columbo Gap	2.787	2.832	2.876
Alternative Columbo Gap	2.992	2.997	3.001
"E" & Maxwell Gap	2.877	2.901	2.923
"F" & Bond Gap	2.954	2.953	2.951
"G" & Dawes Gap	3.045	3.041	3.037
"G" & Dawes Gap begin B Ring	3.045	3.045	3.045
"O" & Huygens Gap	2.874	2.883	2.893
"P" & Laplace Gap	2.973	2.980	2.988
"Q" & Barnard Gap begin A Ring	2.975	2.976	2.977
"U' " & Encke Gap	2.981	2.989	2.997
"X" & Keeler Gap	3.056	3.056	3.057
"B' ' " and F Ring	3.038	3.050	3.061
Alternative F Ring	3.049	3.059	3.067
as contrasted with:			
"G" begin B Ring		3.391	
"Q" begin A Ring	3.099	3.097	3.095

The least multiple above is 2.787 and the greatest is 3.067, excluding the "G" and "Q" figures which are not associated with the suggested beginnings of the B Ring and the A Ring. Their average is 2.927 with approximately 5% spread either way in multiples.

Several explanations may be given for the lack of complete uniformity. These include:

(1) the rings may be in the process of evolution and although "anchored" by the locations they are still subject to fluctuation;

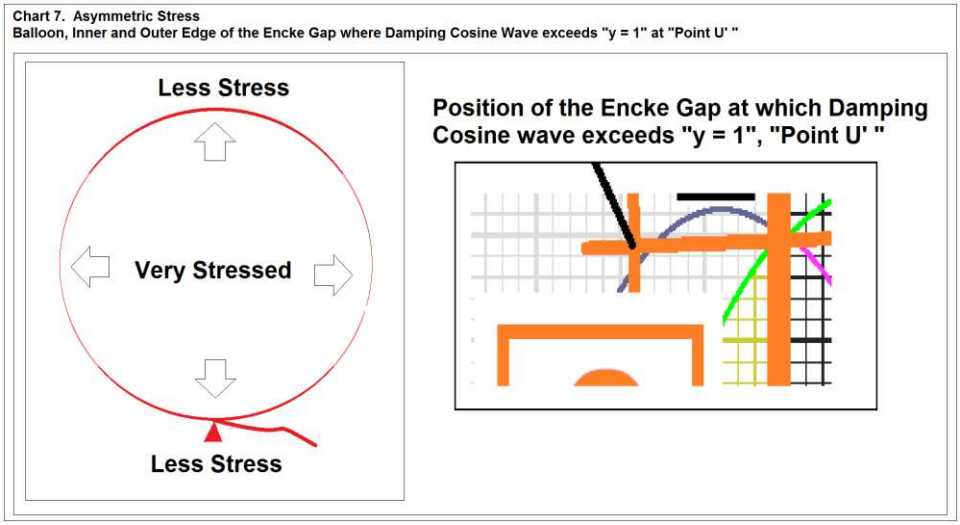
(2) the rings themselves may not be stationary and therefore remain affected by outside influences, including the stability of the other rings; and

(3) the data may be incomplete.

Clues as to Causation

Given the close range within which these multiples occur, one may suggest that a possible form of causation for these gaps might be a "tearing" of the fabric of "consciousness," or its more localized twin "identity," resulting in asymettric points of stress. For example, if a globe-shaped balloon is marked with similar lines in ink, the equatorial circumference will be far more stretched than the polar circumference. Moreover the side of the circumference nearest the equator will be more obviously stretched than the more relaxed side closest to the pole.

The following photographs of the inner and outer edge of the Encke Gap may support this proposition. The inner edge of the Encke Gap appears to be far more stressed and torn than the outer edge, given the nature of the stress placed upon it.



This differentiation between the inner and outer edges of the Encke Gap is below.

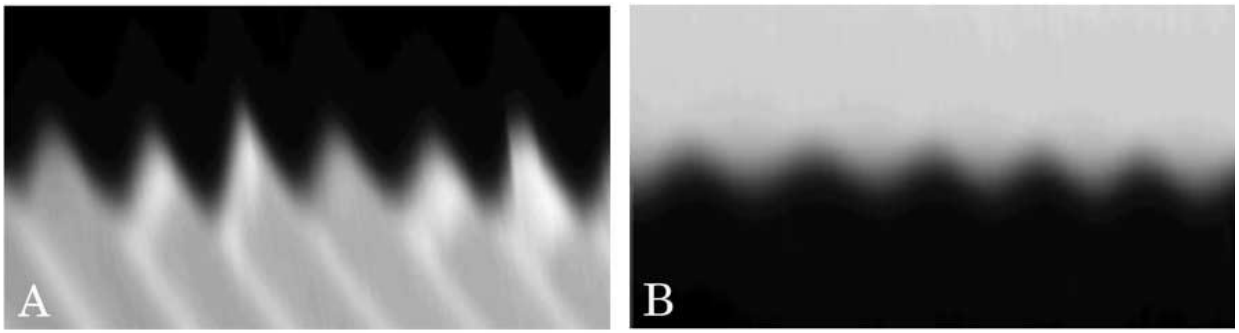


Fig. 9. Encke Gap. (A) Inner and (B) outer edges of the Encke gap as seen in Fig. 7C, mapped into a longitude-radius system, enhanced in contrast and brightness and radially stretched by a factor of 20.
As taken from p. 1235, Porco, c. et al, (2004) "Cassini Imaging Science: Initial Results on Saturn's Rings and Small Satellites," 22 February 2005, Vol. 307, Science, www.sciencemag.org, and http://www.ciclops.org/sci/docs/RingsSatsPaper.pdf pp. 1234-1236. Public Domain.

As to these strange gap edges of the Encke Gap, let us consider three points.

1. If the Dawes Gap is taken to be the endpoint of the C Ring and the beginning point of the B Ring, then it is significant that the Bond Gap precedes it in relation to Saturn.

2. Similarly if the Barnard Gap is taken to be the endpoint of the B Ring and the beginning of the A Ring, then it is significant that the Cassini Division precedes it with eight gaps preceding the Barnard Gap.

3. Another important consistency arguing in favor of using the Dawes Gap and the Barnard Gaps as demarcation for the beginning and the end of the B Ring is that, besides being preceded by closely associated gaps, no gaps follow them subsequently, at least not in close proximity.

Stress and "Identity"

These prior gaps, coming just before the +1 and -1 of the Sine Wave, suggest that the stress originates with Saturn. In short, the tearing of the fabric of consciousness, the identity of Saturn, has an origin, and it is Saturn itself.

Moreover there is a significant distinction between the stress placed upon the rings as between the Sine wave and the Damping Cosine wave. As can be seen below, the Sine wave brings about relatively minor tears (the Dawes Gap of 20 km and the Barnard Gap of 13 km) while the Damping Cosine wave, or its combination in the PE wave, initiates quite severe tears. These distances are as follows.

Begin C Ring	Damping Cosine wave hits Y axis, Sine = 0	
Columbo Gap	PE wave "Point B' "	150 km
Maxwell Gap	PE wave "Point E," first trough	220 km
Bond Gap	Damping Cosine wave, "Point F," first trough	30 km
Huygens Gap	Damping Cosine wave, "Point O," second trough	285 - 400 km
Herschel Gap	Second Depth of Damping Cosine curve	102 km
Laplace Gap	Greatest Depth of PE wave	238 km
Encke Gap	Damping Cosine exceeds "y = 1", "Point U' "	325 km
Keeler Gap	PE wave crosses "y = 0", "Point X"	35 km

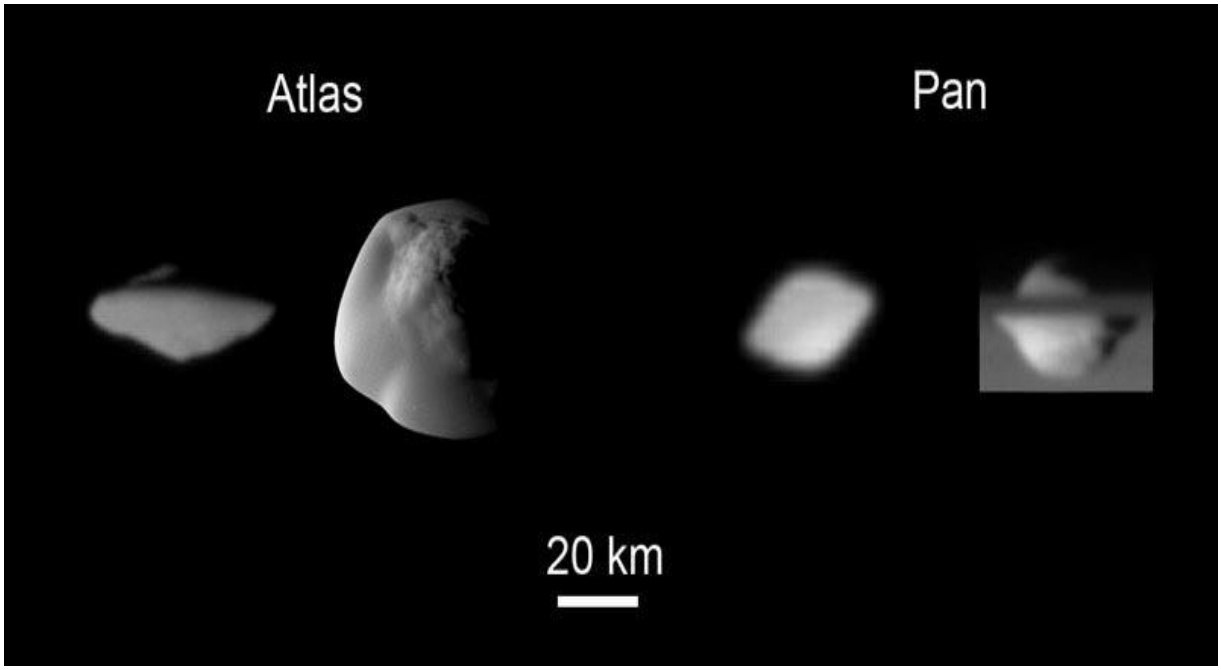
as opposed to:

Dawes Gap	Sine wave = +1	20 km
Barnard Gap[3]	Sine wave = -1	13 km

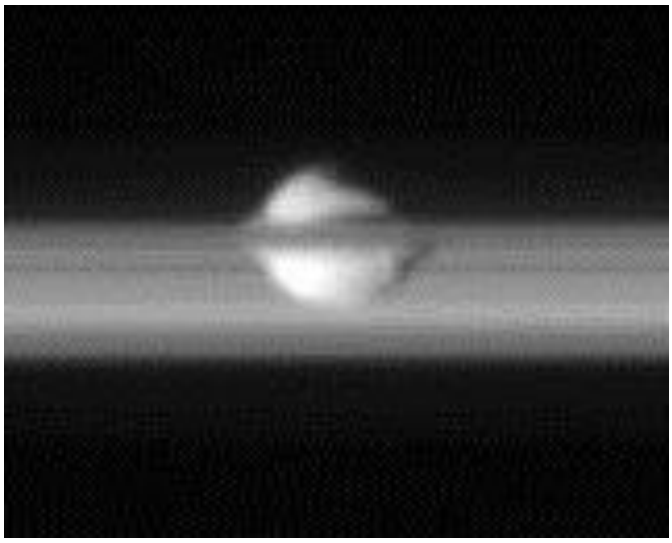
[3] Recall that the inner edge of the Huygens Gap is the outer edge of the B Ring, as presently understood. See Hedman (2009). "The Barnard Gap inner edge is a special case because it is the only inner edge of a gap in the Cassini Division besides the B-ring edge that cannot be fit to a simple eccentric model. All the other non-circular, non-eccentric edges are either on ringlets within the gaps (Herschel and Laplace) or at the outer edges of gaps containing such ringlets (Huygens and Herschel). Furthermore, the mean radius of the Barnard Gap's inner edge is 120,304 km, which is very close to the predicted location of the Prometheus 5:4 inner Lindblad resonance (ILR) at 120,303.7 km."

If the gaps of the Rings of Saturn are caused by stresses in the fifth dimension, Pan's behavior is likely more akin to a marble rolling in the track of tree bark, a small ball of contiguous matter falling into Saturn's gravitational pull yet remaining whole based upon its electrodynamic integrity, caught in the cracks between blocks of concrete sidewalk.

As the stresses which create these rings operate upon what may have been a bubble of lava within a hardened shell, weaknesses were created in alignment with the plane, and the lava oozed out forming a disk parallel to the plane itself. One can see the effects of Saturn's equator "tearing" at both Pan in the Encke Gap as it aligns with the A Ring.



in the Public Domain, see http://www.nasa.gov/mission_pages/cassini/multimedia/pia08405.html



in the Public Domain, see http://commons.wikimedia.org/wiki/File:Pan_side_view.jpg

A New Arrangement of the Rings

The following Chart 8 displays the differences in interpretation of the data as to the proposed and existing architecture of the Rings of Saturn.

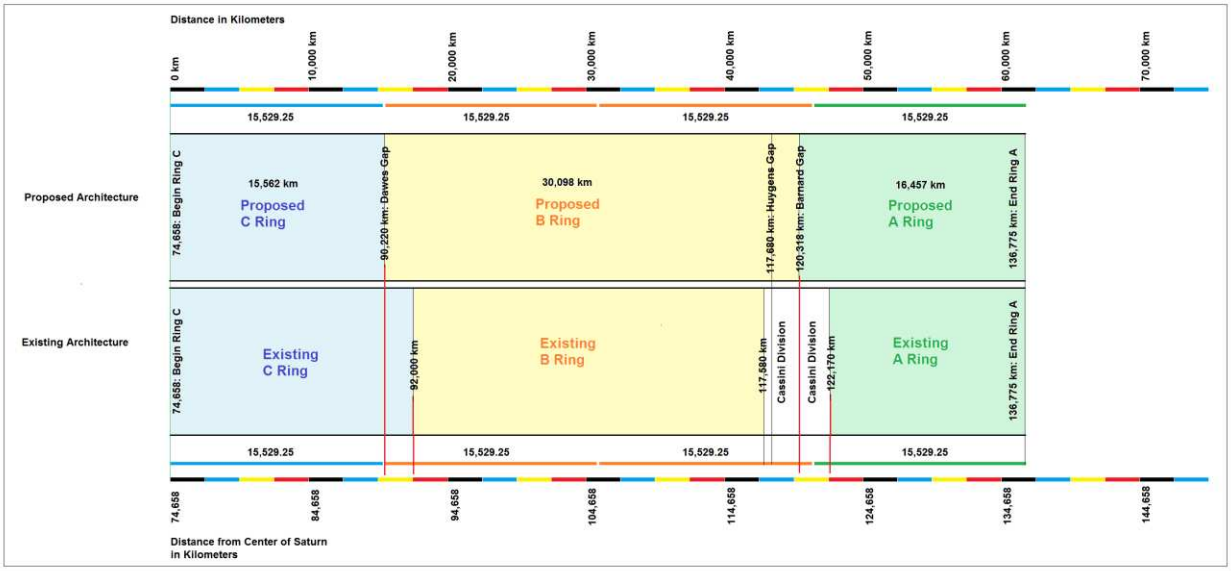
The B Ring is extended toward Saturn, beginning with the Dawes Gap.

The Proposed Architecture eliminates the existing demarcations for the "Cassini Division" as a separate group. In its place the series of eight gaps are placed at the end of the B Ring as a formal subdivision of the B Ring, and a formal division between the B Ring and the A Ring declared at the Barnard Gap.

Similary the alignment of the B Ring with the two most wide-apart gaps in the series aligns the B Ring virtualy exactly with the sine curve of the PE wave. This division is indicated by the two orange horizontal lines above the B Ring, as distinguished from the first quarter of the sine curve, indicated by a horizontal line in blue; and by the fourth quarter of the sine curve, indicated by a horizontal line in green.

Chart 8: Proposed Architecture of the Rings of Saturn based upon Stress in the 5th Dimension

Total Distance = 62,117 km $\quad \frac{\text{Total Distance}}{4} = 15{,}529.25 \quad \text{Radius} = \frac{62{,}117}{2\pi} = \frac{19{,}772.4719}{2} = \frac{31{,}058.5}{\pi} = 9{,}886.2359$ km

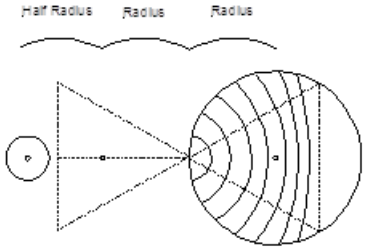


In short, it seems reasonable to suggest that the architecture supporting the Rings of Saturn may be better understood by reference to the macro-economic history of the United States as a form of "identity" or a "fifth dimension of consciousness" resident within the unification of gravity and electromagnetism.

An Alternative Derivation of the Roche Limit

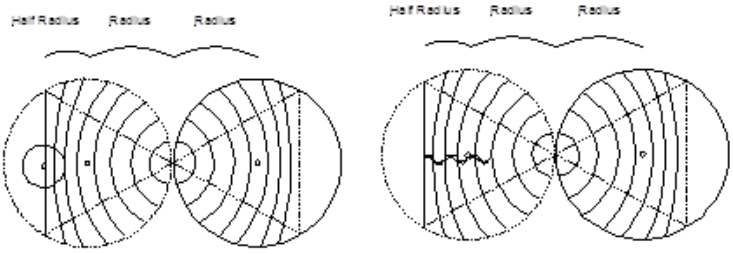
The idea that a Sine curve, and a Damping Cosine curve, when added together create a wave which structures the Rings of Saturn suggests that the "Identity" of Saturn is connected to these curves and their resulting wave. These waves must both push the outer boundary of Saturn's "identity" while being anchored firmly in the center of Saturn, thereby resulting in a "tearing" of the fabric of this "identity," this consciousness" as displayed at a fundamental level of reality. Moreover if the Universe "blinks" on an off after the fashion of a three-dimensional cinematic movie, (see Appendix), then this blinking and intermingled consciousness might be best seen if we look at very massive bodies and investigate their behavior.

If a circular pizza pan is filled with water, and then tapped gently at one point on the circumference, this tapping represents the recurring "blink" of the moon as it nears the larger planet, and as the planet as it attracts the moon. The wave then flattens out in an equilateral triangle with the point creating the wave as one of the angles of the triangle.



If the "ontologic" approach of the moon to Saturn (its "blinking" motion) is mirrored by the "epistemologic" identity of Saturn itself, then a "reflective" wave representing the "identity" of the larger planet could be proposed to exist. This reflective "identity wave" would equate with the Sine curve, the Damping Cosine curve, and their addition in the PE wave described to this point. If this is the case, then it serves to reason that when the moon approaches within this boundary its "moon" identity will be lost to the larger and more powerful "planet" identity at some point.

If this wave become orthogonal to the moon's approach at 2.5 radians of the larger planet, then a disintegration of the moon based upon the loss of its identity as a gravitational body could be suggested as the dramatic tearing, top to bottom, of the smaller planet takes place. This means that a moon disintegrates in the gravitational attraction of Saturn at 2.5 radians of the larger planet and that the wave characteristics of the larger planet play out against the space around it as a violin humming in a crowded room.

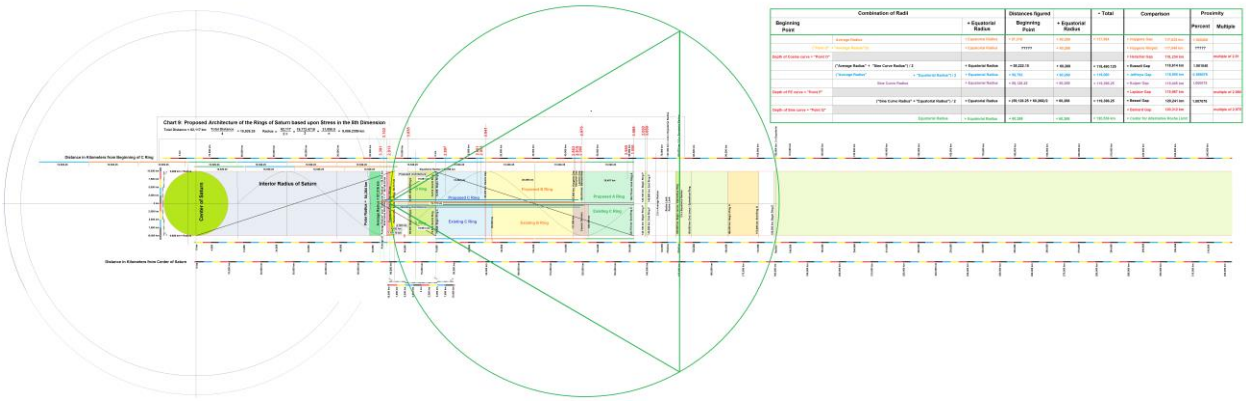


At the present time the Roche Limit defines this gravitational decimation as 2.44 radians of the larger planet. http://en.wikipedia.org/wiki/Roche_limit The 2.5 limit is presented here, with the additional caveat that the extension of an imaged planet 3 radians into space should be quite noticeable.

Part Two:

Seven Rules for the Construction of Saturn's Rings and Gaps

Taking the foregoing into consideration the remaining gaps in the Rings of Saturn were drawn carefully on a map of the rings.



The first horizontal scale toward the top of the chart is marked in units of 10,000 km; it begins at the innermost point of the C Ring, as the "x = 0" origin of the scale in both a positive (away from the center of Saturn) and negative direction (toward the center of Saturn). Each small colored rectangle in the sequence – black, blue, yellow, red –indicates a unit of 2,500 km.

The second horizontal scale begins with the center of Saturn, but makes its major notation marks in units of 10,000 km beginning with 74,658 – 70,000 = 4,658 km. The reason for this is to aid in finding more quickly the placement of any point in the rings from the innermost point of the C Ring at 74,658 km.

The third horizontal scale begins with the center of Saturn, and marks the distance from this central point in the typical fashion, uniformly positive and moving away from the center of Saturn, in units of 10,000 km.

In addition, two sine curves are charted, one using the dimensions of the PE wave as they move toward the center of Saturn, the other using the same dimensions as they begin at the center of Saturn and move outward. These assist in locating points which might be important in our understanding of the dynamics underlying the Rings of Saturn.

Toward the upper right of the chart each of the eight gaps of the Cassini Division is interpreted as resulting from the methods employed in this paper. The proximity between these estimates and the existing data is given in the multiples necessary to give the adjustment. These multiples fall within a range of between 0.999 to 1.007 of the values given by NASA. Although correlation to the data does not, in and of itself, prove causation, nevertheless the close poximity between the data and the theory suggests that there may be merit to the theory.

It was found that the radius of Saturn has an important impact upon the gaps, both as a structure and as individual features of the rings. There are three definitions of "radius" which will apply to this analysis.

The "Equatoral Radius" of Saturn is the radius of the planet as it spins upon its polar axis. This length is 60,268 km.

The "Average Radius" of Saturn is the (polar radius + the equatorial radius) / 2. This length is 57,316 km. This radius equates with the radius of Saturn as a stationary ball of gas which is not rotating.

The average of these two, ("Average Radius" + "Equatorial Radius") / 2. This length is the 58,792 km. This represents the half-way mark between two equally important physical features of Saturn.

Based upon the previous analysis of the PE wave and its relation to several of the gaps in the Rings, seven rules might be given for the construction of the rings themselves.

1) The Rings are based upon a Sine curve.

2) The amplitude of the Sine curve is "1."

3) The period of the Sine curve is "2π." (the standard unit circle)

4) The "x = 0" origin of the Sine curve begins at the innermost point of the C Ring.

5) The distance from the x-axis intecept on the "Average Radius," non-rotating sphere to the point +1, and to -1, of the amplitude measure, and between +1 and -1, will all equal one another. (i.e. = 2)

6) The $1/4^{th}$ Sine curve preceding "x = 0" (a trough) will begin at the depth of the Sine curve at:

$$\frac{\text{(radius of non-rotating sphere) + (radius of rotating sphere)}}{2}$$

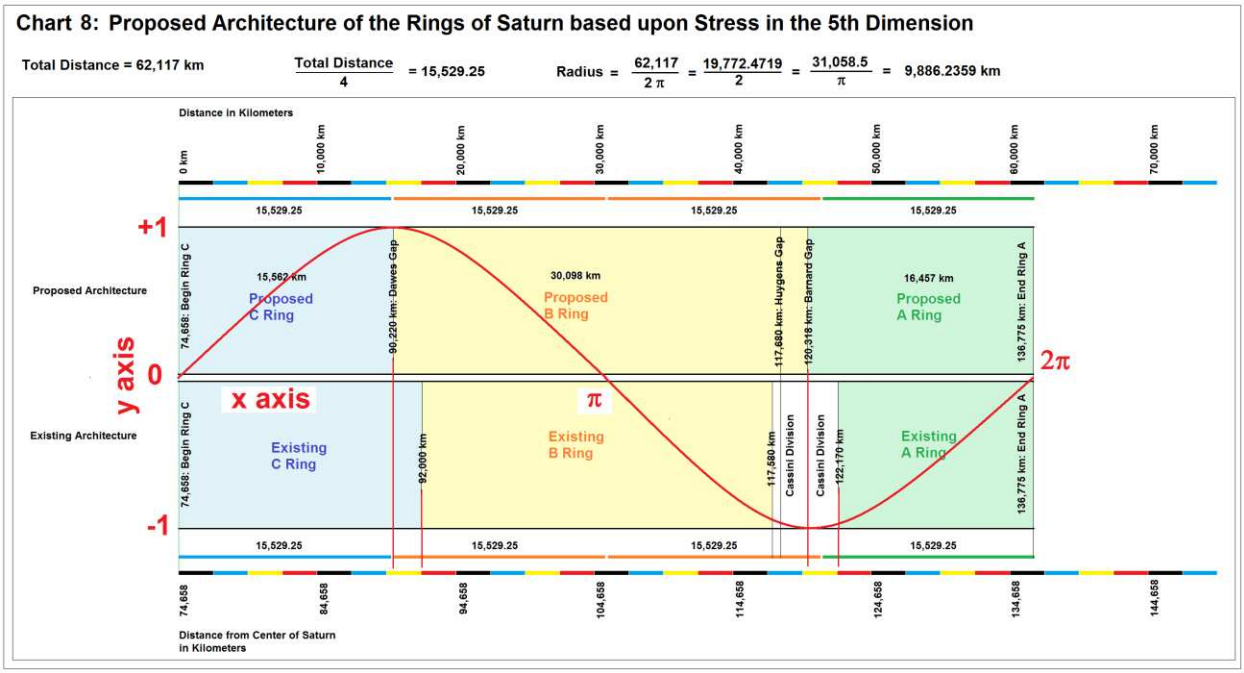
7) The distance between the first trough to the second trough will always be less than the length of the rotating radius.

Let us examine how these rules dictate the dimensions and activity in the Rings of Saturn.

The Main Sine Wave

The precise delineation of the innermost point of the C Ring (74,658 km) through the outermost point of the A Ring (136,775 km), and its association with a Sine curve of that length, permits a form of trigonometry to be considered as foundational to the entire series of rings.

In this manner the proposed architecture of the rings follows an independent gauge, one which is closely connected with the entire set of gaps within the rings. (See Part I)

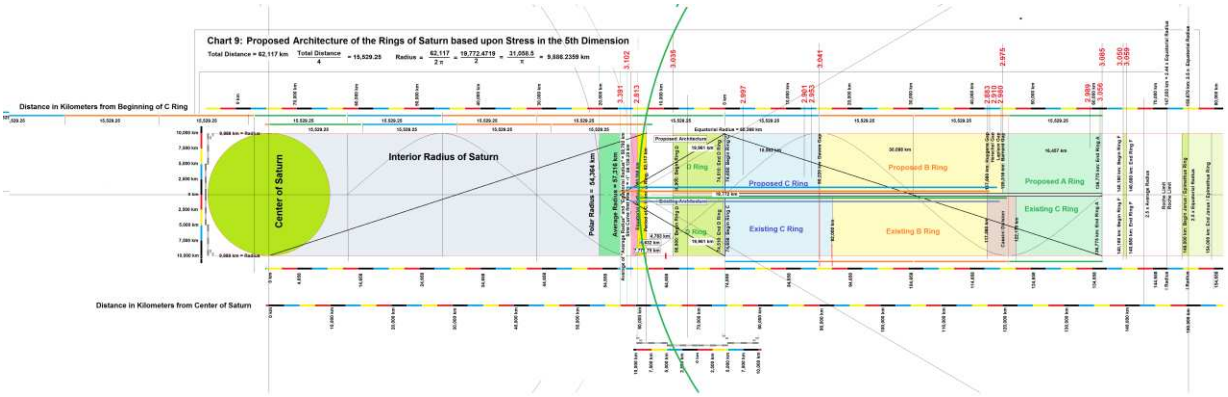


The Unit Circle

If we accept that the inner edge of the C Ring begins the "x = 0" origin of an x-axis Sine curve, then the wave itself must be proportional to the radius of the unit circle leading to its creation.

To determine the length of a radius of the series, we take the distance from the beginning of the C Ring to the outer edge of the A Ring, and divide by 2π. This equation is (136,775 – 74,658) / 2pi = 62,117 / 6.28318 = 9,886 km. This distance represents the y-axis distance from the x-axis when Sine y = 0, the beginning of the inner edge of the C Ring.

The sine curve in question is drawn below, along with an angle sloping negatively at a rate of 1:π. Twice the distance, 9,886 x 2 = 19,772 km, represents the diameter of a circle which generates the Sine curve of this graph. This circle is placed in green at the far left of the chart below to give a sense of the unit circle generating the Sine curve which underlies the structure of the C, B and A rings as a single unit. This diameter is used as the basic rectangle organizing our approach to Saturn's Rings.



This leads to an interest as to where these dimensions, and particularly the radius and/or circumference of this unit circle, might be found within the architecture of Saturn. In as much as they appear to emanate from Saturn itself, several possibilities were considered. These, as marked below, are:

(1) the Polar Radius (54,364 km),

(2) the average between the Polar Radius and the Equatorial Radius, a distance representing the radius of Saturn as a stationary sphere and not rotating, simply named here the 'Average Radius" (57,316 km),

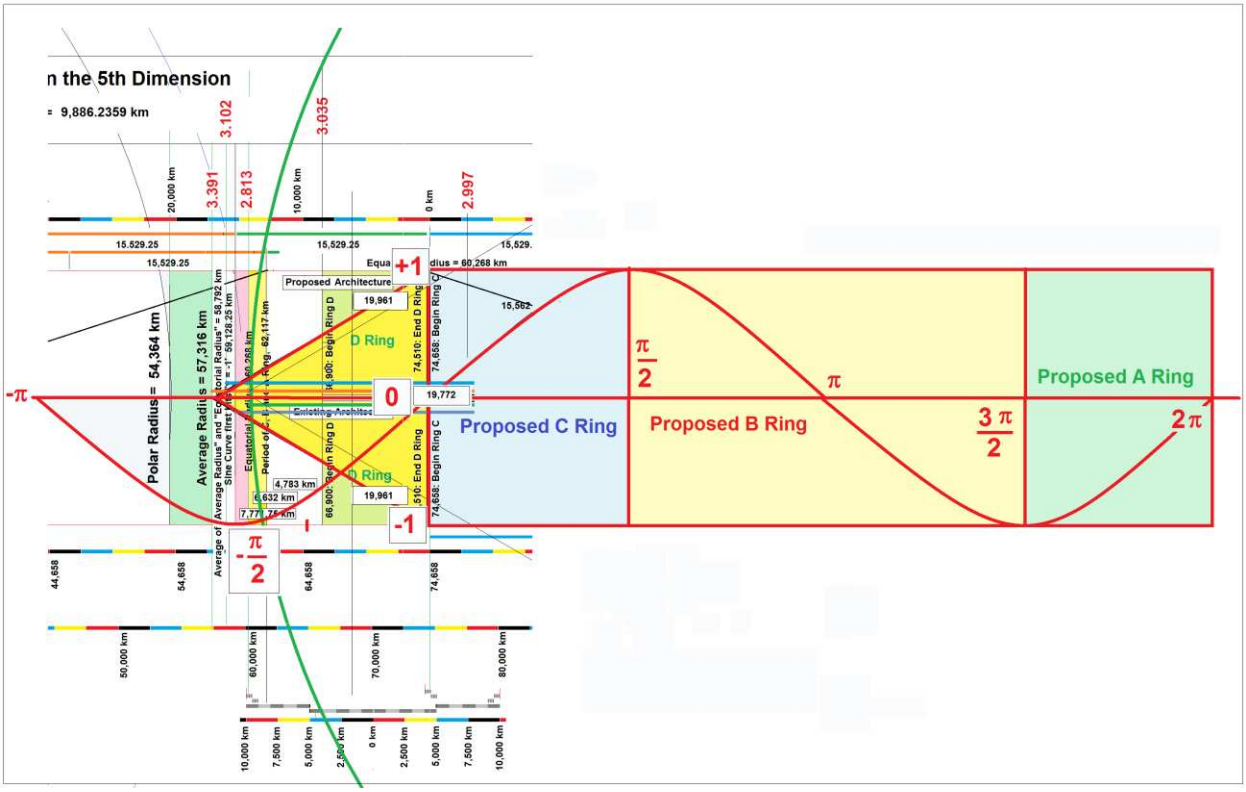
(3) the average between the "Average Radius" (Saturn not rotating) and the "Equatorial Radius" (Saturn rotating) (58,792 km), and

(4) the "Equatorial Radius," i.e. the radius of Saturn at the equator which is significantly enlarged due to the rapid rotation of Saturn upon its polar axis (60,268 km).

The Retreating Sine Curve

It was found that the Sine curve, when traced back to the surface of Saturn (59,128.25 km) came closest to (3) above, i.e. the average of the "Average Radius" (non-rotating) and the "Equatorial Radius (rotating) (58,792 km). A multiple of 3.102 is stated between the economically derived PE wave and the kilometric distance implied by the same point in the architecture of Saturn's Rings. This multiple is generally within the range of the multiples given in Part One, and certainly close to the multiples of the others given for the peak and trough of the Sine curve throughout the C, B and A Rings, these being: 3.035, 3.041, 2.975, and 3.085, the average of which is 3.034.

It was also found that an equalatoral triangle begins at the edge of the "Average Radius" (non-rotating) and extends to the y-axis of the C Ring, from whence the sine curve supporting the C, B and A Rings is generated. The dimensions of the triangle are 19,961 km, 19,961 km and 19,772 km, a distance between these numbers of 189 km, or 1.009 multiple.



If this dynamic holds true, then the faster Saturn spins, the gaps will expand, the period of the rings will lengthen, the identity of Saturn will move out across the equatorial plane, and the rings will become more charged.

The Equatorial Radius as Basic Unit of Length

It was found that the distance from the center of Saturn to the equatorial radius may be used as a measuring rod. Applying this fixed measurement to each of the previously mentioned points leads to a direct association with the gaps in the Cassini Division, as follows. The description of the gaps already considered in Part One is added as well, with the multiples used by way of comparison.

	Combination of Radii		Distances figured		= Total	Comparison	Proximity			
	Beginning Point	+ Equatorial Radius	Beginning Point	+ Equatorial Radius			Percent	Inner Multiple	Middle Multiple	Outer Multiple
D Ring	Damping Cosine curve leaves x-axis = "Point R' "									
	PE curve intercepts x-axis = "Point S' "									
	Damping Cosine curve intercepts PE curve = "Point T' "									
	Height of Damping Cosine curve = "Point U' "									
	Height of Damping Cosine curve = "Point V' "									
	Point W									
	PE curve intercepts x-axis = "Point X' "									
	Point Y									
	Point Z									
C Ring	Begin Sine curve = "Point A"					= Begin D Ring 68,900 km			3.035	
	Height of PE curve = "Point B"					= Columbo Gap 77,600 km		2.992	2.997	3.001
	Sine curve meets Damping Cosine curve = "Point C"									
	Sine curve meets PE curve = "Point D"									
	First trough of PE curve = "Point E"					= Maxwell Gap		2.677	2.901	2.923
	First depth of Damping Cosine curve = "Point F"					= Bond Gap		2.954	2.953	2.951
	Height of Sine curve = "Point G"					= Dawes Gap		3.045	3.041	3.037
B Ring	Sine curve intercepts PE curve = "Point H"									
	Second height of PE curve = "Point I"									
	Sine curve intercepts Damping Cosine curve = "Point J"									
	Second height of Damping Cosine curve = "Point K"									
	Damping Cosine curve intercepts PE curve = "Point L"									
	PE curve intercepts x-axis = "Point M"									
	Negative Sine curve intercepts Negative PE curve = "Point N"									
		Average Radius	+ Equatorial Radius	= 57,316	+ 60,268	= 117,584	= Huygens Gap 117,632 km	1.000490			
	("Point O" + | Average Radius) / 2	+ Equatorial Radius	?????	+ 60,268	?????	= Huygens Ringlet ?????	?????	?????	?????	?????
	Depth of Damping Cosine curve = "Point O"						= Herschel Gap 118,234 km		2.911	2.910	2.910
	((Point O") +	("Average Radius" + "Sine Curve Radius") / 2) / 2	+ Equatorial Radius	?????	+ 60,268	?????	= Herschel Ringlet ?????	?????	?????	?????	?????
		("Average Radius" + "Sine Curve Radius") / 2	+ Equatorial Radius	= 58,222.15	+ 60,268	= 118,490	= Russell Gap 118,614 km	1.001045			
		("Average Radius" + "Equatorial Radius") / 2	+ Equatorial Radius	= 58,792	+ 60,268	= 119,060	= Jeffreys Gap 118,950 km	0.999076			
		Sine Curve Radius	+ Equatorial Radius	= 59,128.25	+ 60,268	= 119,396.25	= Kuiper Gap 119,405 km	1.000075			
	Depth of PE curve = "Point P"						= Laplace Gap 119,967 km		2.973	2.980	2.988
	("Point P" + | Sine Curve Radius) / 2	+ Equatorial Radius	?????	+ 60,268	?????	= Laplace Ringlet ?????	?????	?????	?????	?????
		("Sine Curve Radius" + "Equatorial Radius") / 2	+ Equatorial Radius	= (59,128.25 + 60,268) / 2	+ 60,268	= 119,396.25	= Bessel Gap 120,241 km	1.007075			
A Ring	Depth of Sine curve = "Point Q"						= Barnard Gap 120,312 km			2.975	
		Equatorial Radius	+ Equatorial Radius	= 60,268	+ 60,268	= 120,536 km	= Center for Alternative Roche Limit				
	Damping Cosine curve leaves x-axis = "Point R"										
	PE curve intercepts x-axis = "Point S"										
	Damping Cosine curve intercepts PE curve = "Point T"										
	Height of Damping Cosine curve = "Point U"										
	Height of Damping Cosine curve = "Point V"										
	= "Point W"										
	PE curve intercepts x-axis = "Point X"						= Keeler Gap 136,547 km		3.056	3.056	3.057
	= "Point Y"										
	= "Point Z"										
	End of Sine curve = "Point A' "						= End of A Ring 137,775 km			3.085	
F Ring	Repeat B = "Point B' "						= F Ring		3.049	3.059	3.067

In summary of the manner in which these gaps are scheduled is as follows, using midpoints.

Proximities	Equatorial Addition	PE wave comparison
Begin D Ring		3.035
Begin C Ring		3.085 (for entire series)
Columbo Gap		2.997
Maxwell Gap		2.901
Bond Gap		2.953
Dawes Gap		3.041
Begin B Ring		
Huygens Gap	1.000408	
Huygens Ringlet	???	
Herschel Gap		2.910
Herschel Ringlet		???
Russell Gap	1.001045	
Jeffrey's Gap	0.999076	
Kuiper Gap	1.000075	
Laplace Gap		2.980
Laplace Ringlet		???
Bessel Gap	1.007075	
Barnard Gap		2.975
Begin A Ring		
Encke Gap		2.989
Keeler Gap		3.056
F Ring		3.059

Analysis

In the above set of numbers, those in red reveal a process directly connected to the PE wave and the non-rotating "Average Radius" which begins the equilateral triangle, which in turn sets up the length of the PE wave in the rings. If Saturn was non-rotating, or rotating very slowly and with an equilateral radius equal to the polar radius, the retreating Sine curve would fall so closely to the "Average Radius" that its effect upon the Rings might go unnoticed. This in turn may cause the gaps to be so small, or so slightly charged, that the rings would go unnoticed entirely.

The numbers in black represent a process directly connected to the rapid rotation of Saturn, its oblate shape, its shortened "Polar Radius" and its elongated "Equatorial Radius." The rapid rotation of Saturn creates thereby a fundamental unit of length which affects the Rings. This process is directly responsible for the creation of the majority – but not all – of the gaps in

the Cassini Division. As this "Equatorial Radius" adds itself to the "Average Radius," the "average of the 'Average Radius' and the 'Equatorial Radius' " and etc., it causes new stresses and strains on the Rings, and these in turn result in many – but not all – of the gaps within the Cassini Division. It is probably responsible also for the mountainous waves in the rings just preceding the Huygens Gap, the place where the "Average Radius" and the "Equatorial Radius" meet as parts of a sum for the first time.

In addition the nature of the ringlets and the gaps themselves may be suggested. If the gaps represent a "tearing" of "consciousness," then the seam of this tear may be capable of collecting material from the ring, creating a ringlet. Moreover if the material pulled into the tear exceeds the depth of the seam, this material may slide in and around the seam as apparently is the case with the "Strange" ringlet in the Huygens Gap.

The age of the Rings might be interpreted in a number of ways. First, because the dynamics of the rings are so largely dependent upon the size and speed of rotation of Saturn, it would be important to estimate the causes of this spin. The existence of this mathematics must occur with every planetary body held together by gravity. It appears that the speed of Saturn's spin exacerbates this mechanism thereby charging the fifth dimension with an additional attractive power.

The Cassini Division is interesting because both (1) the "Average Radius" and the PE wave, as well as (2) the "Equatorial Radius" and its addition, play a part in the rings. The interaction of these two processes indicate that the "identity" of Saturn may be considered as a basic idea of this approach.

The Big Slam

If we collect these ideas into a single proposal the following chronology of the Rings is possible.

1. The PE wave, as constructed from the Sine curve and Damping Cosine curve as these are constructed from the "Average" (non-rotating) radius of Saturn, are three dimensional structures without reference to the spin of Saturn, and inhabit all the space around the planet itself, activating a 2.5 "identity" in every direction.
2. A large moon enters within this 2.5 radius of the non-rotating Saturn and begins its spiral descent into the depth of Saturn itself, gathering speed while at the same time disintegrating along the way.
3. The speckled dust of this falling-apart moon creates a large spiral in a single plane, but one which is organized quickly by Saturn's "identity," i.e. the PE wave, the Sine curve and the Damping Cosine curve the non-rotating (or slowly rotating) Saturn creates.
4. This disintegrated moon then slams into a single, particular spot of Saturn, tremendously compressing the gas in this spot, and creating a much faster spin through the force of this blow.
5. The debris in the plane, which has settled into place within the existing "identity" pattern of the PE wave, the Sine curve and the Damping Cosine curve, is suddenly jolted by the force of the change which comes suddenly over the equatorial radius of Saturn. As this Average Radius, which previously had added with itself to create the "identity" of 2.5 Radians of Saturn, is suddenly extended to an entirely different 2.5 times the quickly spinning Equatorial Radius.
6. The force of this blow jolts the far end of the C Ring immediately, into the "land of mountains" recently described.

7. The tear which occurs in the PE wave, the Sine curve and Damping Cosine curve suddenly affect each of these various points as now a "Gap," a place where the old must give rise to the new expanded addition of the equatorial Radius.

8. As the Equatorial Radius is quickly added to the "Average Radius," the "average of the 'Average Radius' and 'Equatorial Radius'," the "Equatorial Radius" the majority of gaps in the Cassini Division open up. These, however, are not "tears" in the fabric of consciousness. Rather they are extensions of it, and therefore do not open "seams" in the fabric to be filled with ringlets or moons. Rather they are empty gaps brought on by a process different from the tearing of the other gaps.

The main exception to this is the Huygens Ringlet, which is brought about by the direct force of the "Equatorial Radius" striking the "Average Radius." The force of this shocks the Huygens Gap sufficiently to create the Huygens Ringlet AND the Strange Ringlet at the same time.

9. The striking of the moon into Saturn sends shock waves throughout its "identity," and these radiate from Saturn outward.

10. Conversely waves at the outer end of this ring system send back to Saturn the reflective nature of these waves.

11. And the tidal pull of Saturn's suddenly increased rotation pulls on each inch of thread around it as a ball of yarn tugs as it is tightened.

Alternatively, the seam may create a track in which the small "shepherding moons" travel. This approach to Saturn's Rings would not anticipate that these moons affect the rings as bodies which carve out the gaps themselves. Rather the gaps take place based upon the more fundamental dynamic of Saturn and the speed of its spin, and small satellites are trapped within the seams thereby created.

12. If the "Land of the Mountains" equates with the moment in time when Saturn's Equatorial Radius first – and suddenly – slammed into the construction of the Rings themselves, then this moment would be marked at a different spot on the exterior of the Rings. If we could find this spot on the outside of the Rings, and then rewind time to the point at which these were both existing at the same moment, then we would have an idea of the impact date which first created the Rings as we know them.

13. It would have created a sudden bulge in what we now know as the Huygen's Gap.

14. This would also have created all the eccentricities of the various ringlets.

15. Therefore figuring the date at which this "slam" into the side of Saturn took place would equate with that date on which all the eccentricities align with the Land of Mountains. In essence, this event would have been the equivalent of setting a clock, which we can now unwind to determine the date on which the rings were created. Investigating the core of Saturn at this radius should lead us to see within the density structure of Saturn some unevenness, and this would be the shadowy remnant of the disintegrated moon whose substance has given us the Rings.

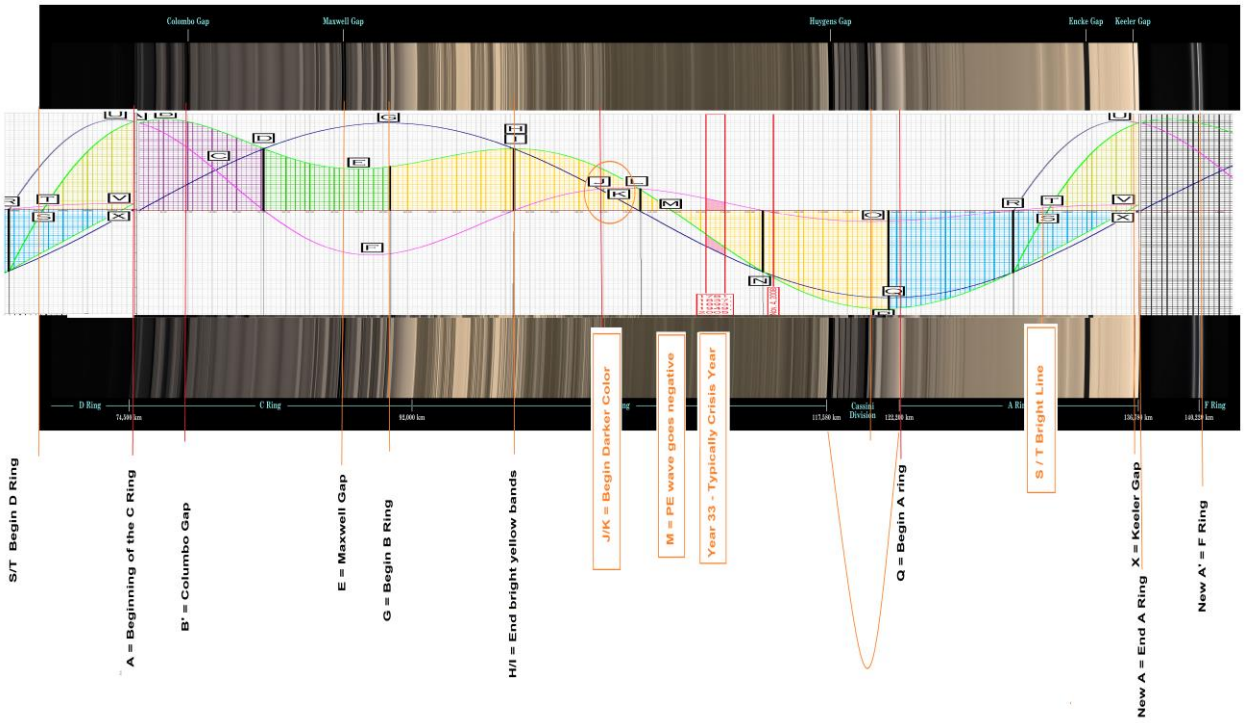
16. Once the equatorial spin of Saturn became a fundamental feature of its "identiy" further additions to its mass, as long as they are relatively insubstantial, simply act to increase the mass spinning in the direction of the prior spin, the way children on a merry go round pull into the same direction of the plane, despite their size or the position of their horse.

17. Once this spin took over as a mass of rotating gas, its pressure at the poles decreasing and its pressure at the equator increasing, a magneto-sphere also took existence and began to "light up" the rings in a fashion similar to the lighting of a fluorescent bulb.

Anticipated Further Proof

Given the relatively close association between the multiples necessary for a perfect alignment with the PE wave, it seems at least plausible that the PE wave is descriptive of a similar wave which underlies the architecture of these rings. Using this as the model, the B Ring begins at the Dawes Gap and Ends at the Barnard Gap. It contains as one of its central features the Cassini Division. The logic of the negative values in the damping cosine wave, the PE wave and finally the midpoint of the depth of the Sine Wave are all contained within this projected B Ring. The A Ring then commences with a steady upsweep to the outer edge of the A Ring. This analysis is possible because we have a mathematic statement of the rings as a unit, each part of which is shaped by reference to the whole.

In addition to aiding in the investigation of recognized phenomena, this approach also permits the researcher to look for heretofore unnoticed events in the architecture of Saturns Rings. For example the following photograph elongates the Political Economy wave.



Notice that Point C, the intersection of the Sine wave with the Damping Cosine wave midway through the C Ring, seems to be without obvious connection to the Rings of Saturn. This may be completely illusory. If so, the effect of this association should be felt as a relationship to Day 1,565. Figuring this point at a multiple of 2.927 this area of the C Ring should occur at 74,658 + (1,565 x 2.927) = **79,238 km** from the center of Saturn.

Notice that Point H/I the second peak of the PE wave, beings a markedly different color in the series. This occurs at Days 7665 through 7675. Figuring this period at a multiple of 2.927 this area of the B Ring should occur at 74,658 + (7665 x 2.927) = **97,093 km through** 74,658 + (7675 x 2.927) = **97,122 km** from the center of Saturn.

Notice that at "Point J" there is a marked difference in color in this photograph of the B Ring. This occurs at Day 9,420 where the Sine curve at Sin = 0.24537 meets the Damping Cosine curve at Cos = 0.24532. Figuring this at a multiple of 2.927, this change should take place at 74,658 + (9,420 x 2.927) = **102,230 km** from the center of Saturn.

Notice that Point K, the intersection of the PE wave with the Damping Cosine wave, occurs at the same Day 10,227 as Point L, the point at which the Sine wave becomes less than "y = 0." Figuring this point as a multiple of 2.927 this area of the B Ring should occur at 74,658 + (10,227 x 2.927) = **104,592 km.**

Notice that two particularly remarkable periods of crisis – "Point M" (Day 10,909) and Year 33 (Days 365 x 33 = 12,045 through Day 365 x 34 = 12,410) – are at the center of the dark grey band witin the second half of the B Ring. One would anticipate that the logic and cohesion of the ring system would change dramatically at these point. Figuring these at a multiple of 2.927 these areas may be anticipated to appear as unusual features of the B Ring at Point M = 74,658 + (10,909 x 2.927) = **106,588 km,** and Year 33 = 74,658 + (12,045 x 2.927) through 74,658 + (12,410 x 2.927) = **109,913 km through 110,982 km.**

Notice that the remaining gaps in the Cassini Division may represent the stress of a variety of types, each of which relates to the nature of the waves interacting. Particularly interesting is the Herschel Gap. The Hershel Gap aligns more with Point O than does the Huygens Gap. In addition the much smaller gaps may take their clues from other unnoticed aspects of the rings or the effects of multiple negative curves simultaneously interacting.

Notice that a particularly bright ring at either "Point S" (Day 18,601) or "Point T" (Day 18,641) or perhaps lying between them, alligns with the PE wave as it increases to more than "y = 0." Figuring this line at a multiple of 2.927 this line should occur at Point S = 74,658 + (18,601 x 2.927) through Point T = 74,658 + (18,641 x 2.927) = **129,103 km through 129,220 km.**

Note that if the gaps between rings results from stress in the fifth dimension, an alternative understanding is possible of the orbit of Pan, a small object found in the Encke Gap. At present this object is referred to as a "shepherding moon" and is understood to create the Encke Gap by gravitational attraction.

The Prime Number Series

It is quite clear that the 14-year spread of the PE wave, from whence it is derived, is closely connected to the Golden Mean. One might wonder whether there is a Golden Mean ratio resident in the Rings of Saturn.

It seems possible that the 20,454 days of the PE wave equate to 3 km, exactly, of the Rings of Saturn. The number 20,454 does not include the final, connecting day of the circuit, i.e. the 20,455th day. If this was taken into account, the number of days of the PE wave would be the product of two prime numbers, i.e. 5 x 4091 = 20,455. This means that a five pointed star, the archetype of the Golden Mean, would be available to organize the Rings still further.

Conclusion

The approach taken by this paper has several advantages:

(1) the difference between the boundary calculated herein and the historic Roche limit runs between 3,262 (polar radii compared) and 3,617 kilometers (equatorial radii compared). A planet degrading prior to the 2.44 limit, but within the 2.5 limit, would support this theory. This geographic distance between the two limits should be sufficient to observe and test;

(2) it suggests a way in which to include the larger, amorphous G ring as within 3 times the radius of the larger planet;

(3) it explains why the E ring commences at 180,000 km from the center of Saturn at three times the equatorial radius of Saturn;

(4) it describes the nature and position of the "gaps" within the rings as consistent with an understanding of a new "fifth dimension" which is quite subject to investigation, i.e. macroeconomics; and

(5) only this theory supports a limit wide enough to hold the nearest 1,670 km of the Janus/Epimetheus Ring at 149,000 to 154,000 km or to explain its "surprising" existence http://saturn.jpl.nasa.gov/photos/imagedetails/index.cfm?imageId=2277 outside the existing Roche limit.

A more extensive discussion of the Political Economy Wave follows in the Appendix.

Scott Albers
Great Falls, Montana
April 29, 2014

Appendix.

An investigation of the "Fifth Dimension" through social data

This paper suggests that the following five dimensions are necessary to describe any event. If one imagines the information necessary to schedule a meeting we have:

1)	1st dimension, X-axis,	(latitude)	Central Avenue, running in an east-west direction,
2)	2nd dimensions, Y-axis,	(longitude)	Fourth Street, running in a north-south direction
3)	3rd dimension, Z-axis,	(altitude)	Sixth Floor, running in an up-down direction,
4)	4th dimension, Time,	(time)	At 2:30 p.m.,
5)	5^{th} dimension, Consciousness,		With Jones, running in an in-out direction, the meeting's purpose.

The person "Jones" introduces of a form of "consciousness" into our description of the universe, an "in-out" spatial dimension. The "out" characteristic of this fifth dimension is the ontologic fact of the meeting, its "being," the fact that it is supposed to take place in "reality" as an existing thing. The "in" characteristic of this fifth dimension is the epistemologic understanding one derives from the meeting, its "awareness" or "understanding," the knowledge or perspective obtained from the meeting.

If this is the case, then the "consciousness" exhibited within the meeting may build – through collective amalgamation – into a larger model of the consciousness in the economy.

And this, in turn, will be useful in examining the same dimension, at the physical level.

In order to understand more concretely the manner in which this fifth dimension may be useful in ordering the rings of Saturn, the following analogy may be helpful. Through this analogy we may understand in a broader context the psychology of these dimensions and the manner in which the three dimensions of space and one of time (our present understanding of reality) unite with a fifth dimension of consciousness.

Five Dimensions

The logical necessity for five dimensions, or for any number of dimensions, or for the relative position of these dimensions vis-à-vis the other dimensions, must be considered.

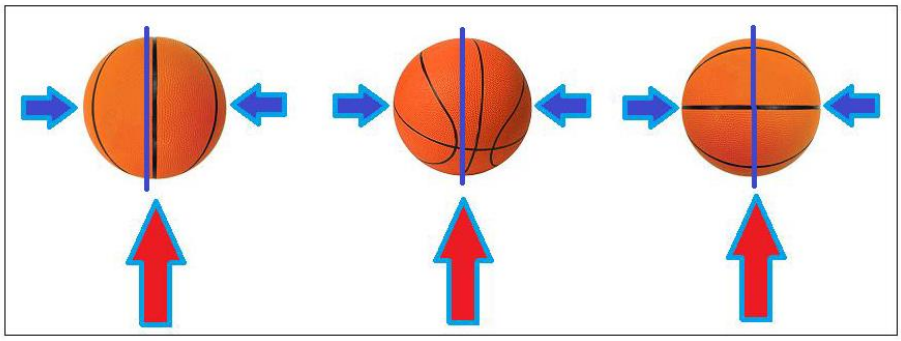
Imagine a basketball floating in a pool of water. If the basketball is spun in the water as one holds the ball at two fixed points (blue arrows below), it is clear that the image of the equator of the ball will be quite different depending upon the poles chosen.

Referring to the basketball on the left, the basketball's equator (in black) will appear to be a steady, straight line, one which is parallel to the equator of spin (in blue).

Referring to the basketball in the middle, the basketball's equator (in black) will appear to wobble back and forth, forming a wave around the equator of spin (in blue).

Referring to the basketball on the right, the basketballs' equator will appear to be a set of flashing, solid lines (in black) perpendicular to the equator of spin (in blue).

In this way one may say that the equator of spin of the basketball is *relative* to the points chosen at which to hold the ball (poles, longitude) as it spins. Similarly one may say that the poles of the ball (longitude) are *relative* to the direction of the spin itself (latitude).



On the other hand the diameter of the ball is fixed at all times, and the area of the surface of the ball is directly related to the diameter. We may contrast the fixed nature of these properties by referring to them as *absolute.* The ball does not change its diameter or surface area based upon the points arbitrarily chosen as its poles, nor based upon the equator of spin.

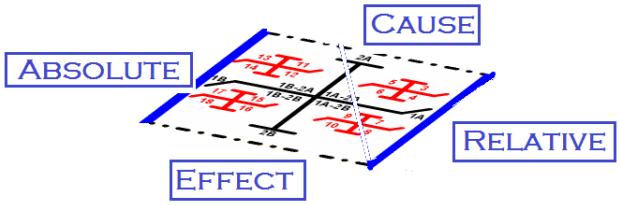
The logic of this set of relationships may be used to explain why four dimensions are essential to our understanding of reality. This logic may be expanded to provide a place for a fifth dimension – a dimension of "in - out" – as the fifth dimension of consciousness.

This effort is dependent upon two terms:

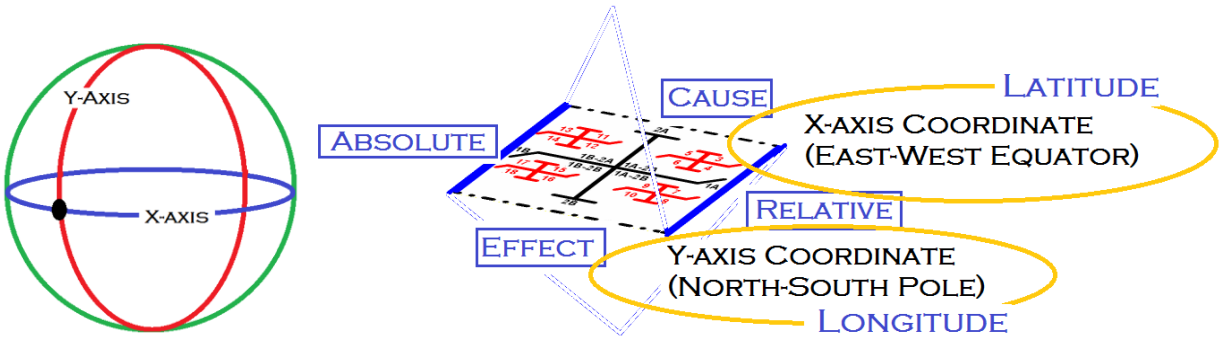
(1) A "Primary Opposition" is an idea which creates an absolute division between two things, "true," "not true."

(2) A "Secondary Opposition" is an idea which permits a gradation of two opposing things, "grey," "not grey."

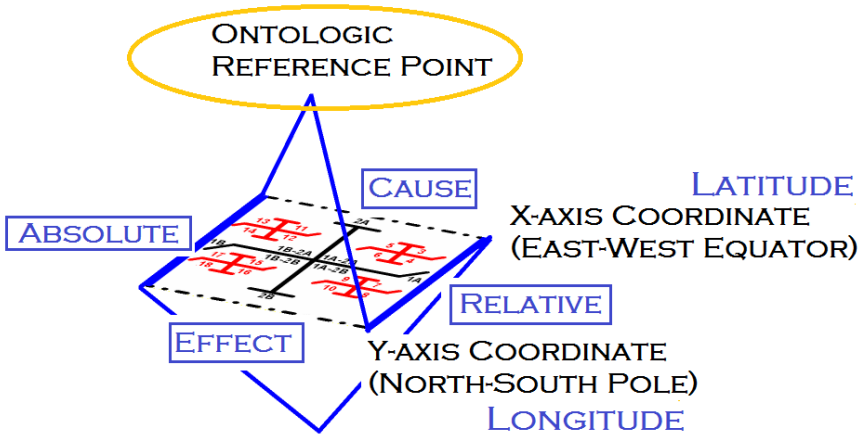
(1) The Primary Opposition of our understanding of physics is the distinction between (1) that which is Relative vs. (2) that which Absolute. The Secondary Opposition is the distinction between (3) Cause and (4) Effect. These are in blue.



(2) Taking the matter of four dimensions from the point of view of a person walking on a street, the street itself located on a spherical earth, we have the equator as an east-west "x-axis" (latitude, a geometric figure with no endpoints) and the poles as a north-south "y-axis" (longitude, a geometric figure with two endpoints). Using these relative positions every point on the surface of the sphere can be located.

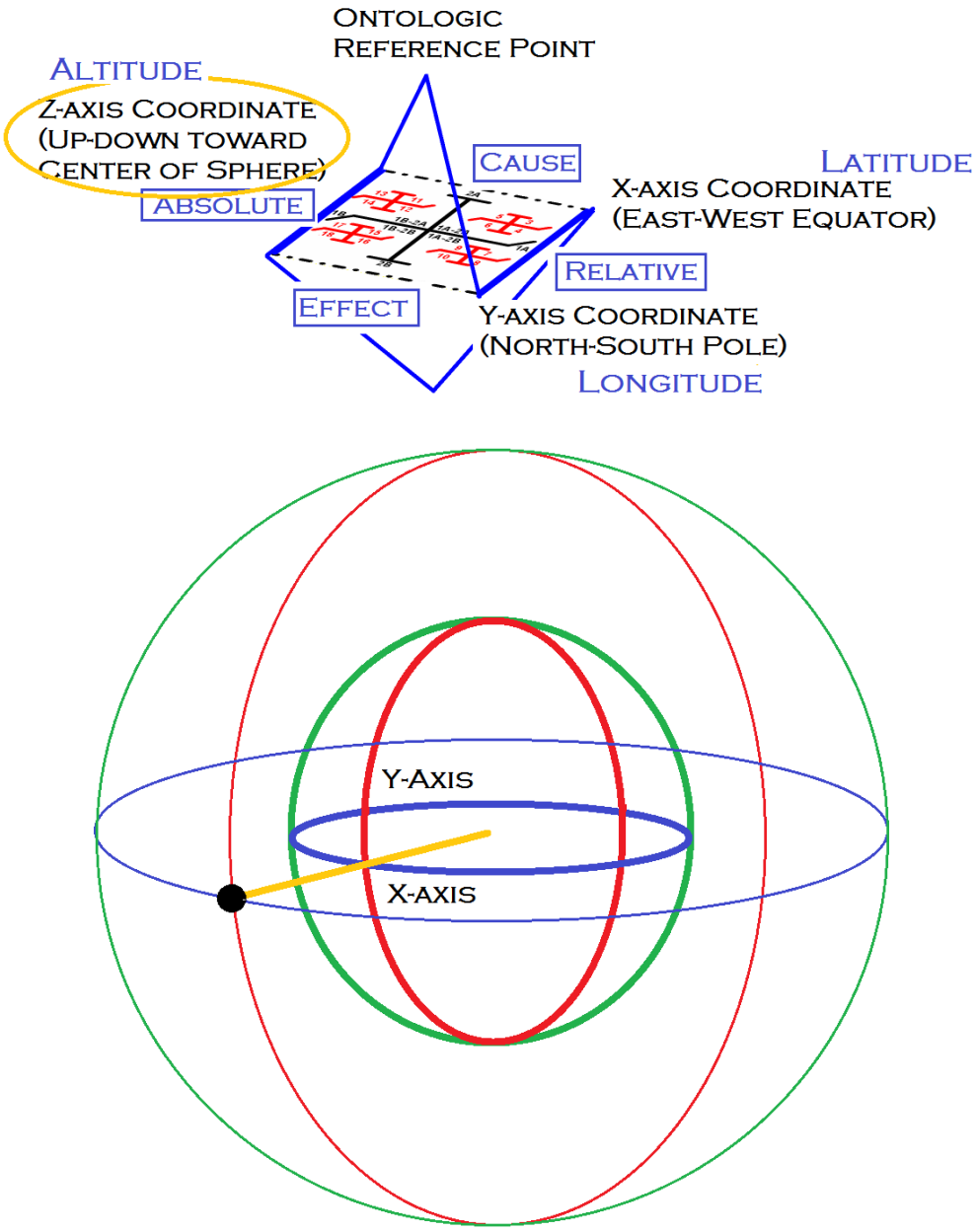


(3) These two positions state a Relative Cause (x-axis coordinate, one point, the equator) and a Relative Effect (y-axis coordinate, two points north and south) which, in combination with each other, create the initial "point" of reference for the sphere. This is an "Ontologic" event, a determination that a particular point along the surface of the sphere exists. I use the term "relative" because the sphere may use any circumference as an equator and thereby indicate an infinite number of point-pairs as north-south poles.

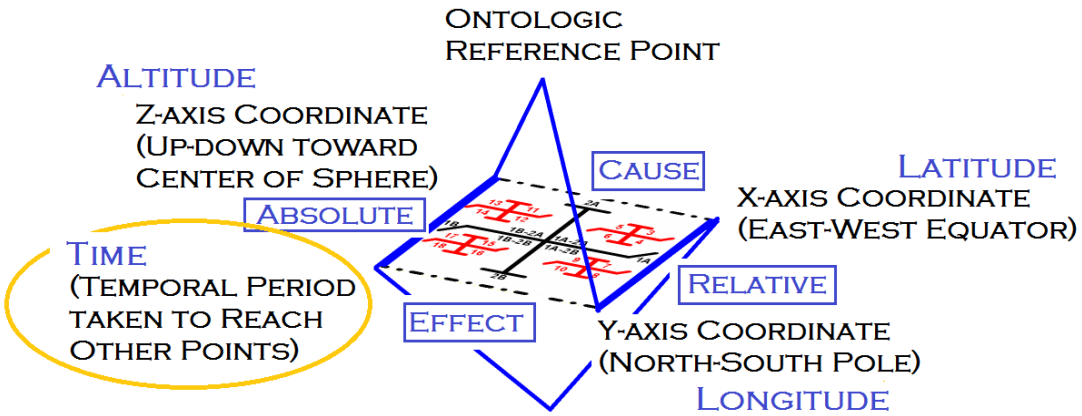


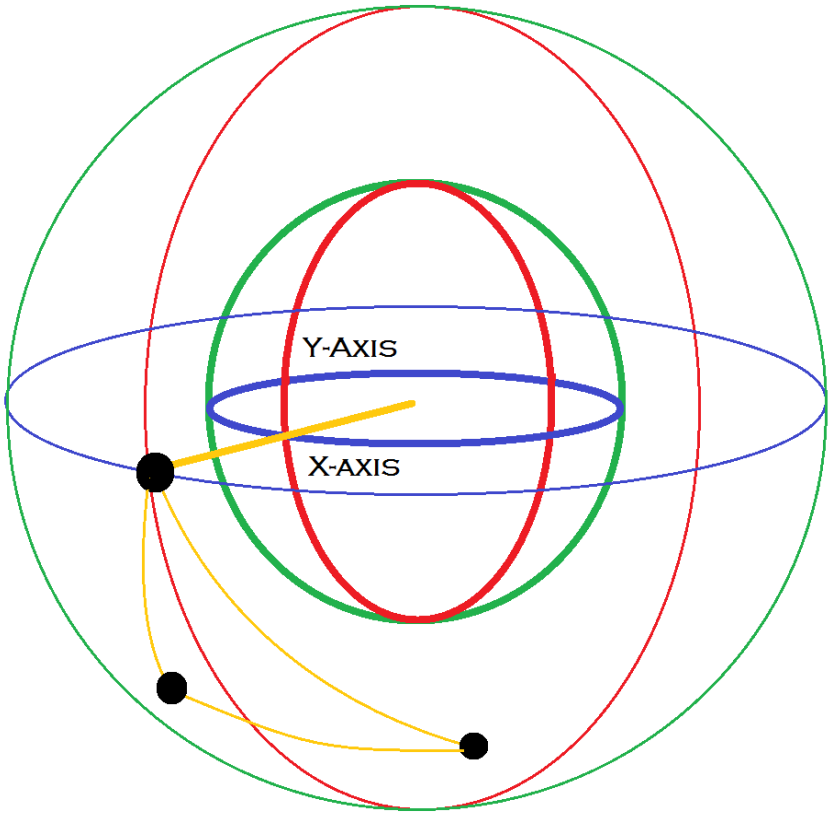
(4) Oppositional Analysis next requires an "Objective Cause," this being the distance from the point in question to the center of the sphere. I refer to this as "Objective" because there is a single point – one and only one point - from which this distance is measured (altitude, the distance from the center of the sphere).

The difference between the relative dimensions of latitude and longitude, and the absolute dimension of altitude, should be considered carefully. One may choose any number of circumferences as an equator, as then related to an infinite number of two-point pairs serving as the north-south pole of these coordinates. This is unlike the fixed, absolute distance of any point from the center of the sphere.

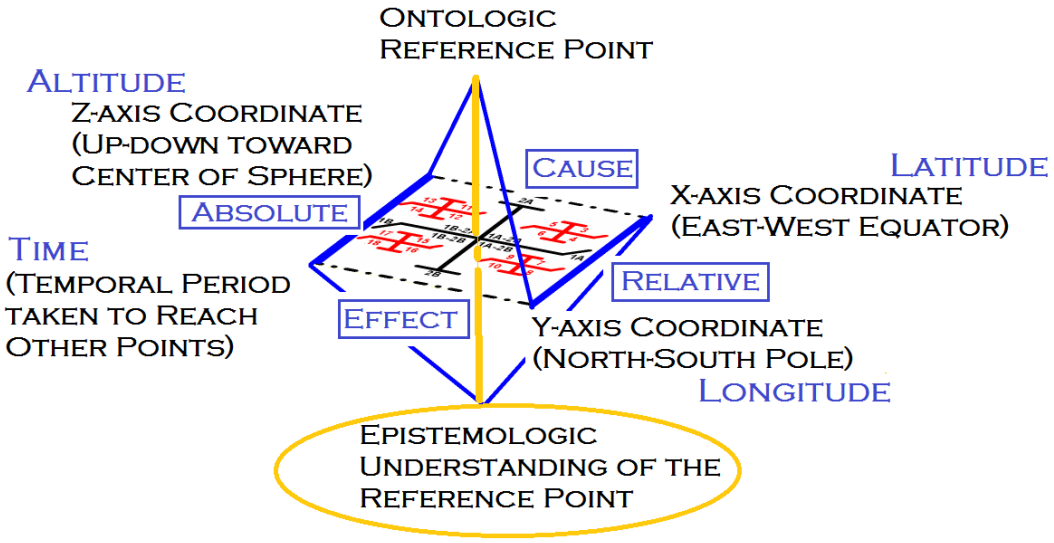


(5) Once we have an "up-down" dimension stated on this sphere, we come to the Objective Effect of this three-dimensional system (time). The uniqueness of this point vis-à-vis all other points, i.e. the time spent moving from one place in a three-dimensional system to any other point in the system, represents a new and previously unknown dimension. In this fashion the "time-coordinate" of a four-dimensional system is logically required to make the system make sense.





(6) Once we have stated each of these four dimensions, we have created an epistemologic statement of the previously ontologic point given initially.



(7) This difference between that which is Ontologically Real vs. that which Epistemologically Known creates a "Fifth Dimension." This might be termed a "spatial dimension" of "in-out," i.e. the difference between the exterior world which is viewed and the interior world in which these experiences are registered as meaning something.

The oscillating, back-and-forth dynamic between the "in" of epistemology and the "out" of ontological reality is reduced in these essays to the Fibonacci series, 1, 1, 2, 3, 5, 8, 13, 21, etc. The fractions which result from this series, 1/1, 2/1, 3/2, 5/3, 8/5, 13/8, etc. have the Golden Mean, 1.6180... = φ as their asymptote.

If the ontology / epistemology dichotomy of the universe is clearly resident within the perceptions of human beings, then it seems reasonable to suggest that the largest conglomeration of human beings – macroeconomics – should provide clues as to the operation of this fifth, in-and-out dimension and its applicability to the Rings of Saturn.

The Fifth Dimension and Zeno's Paradox

In order to describe the nature of the "fifth dimension" envisioned in this paper, let us begin with a straight-forward description of a well-known mathematic puzzle, Zeno's Paradox, as taken from the Encyclopedia Britannica in its article on "Philosophy."

> (O)f very great importance in the history of epistemology was Zeno of Elea (flourished mid-5th century), a younger friend of Parmenides. Parmenides had, of course, been severely criticized because of the strange consequences of his doctrine that in reality there is no motion and no plurality either because there is just one solid being. To support him, however, Zeno tried to show that the assumption that there is motion and plurality leads to consequences that are no less strange. This he did by means of his famous paradoxes, saying that the flying arrow rests since it can neither move in the place in which it is nor in a place in which it is not and that Achilles cannot outrun a turtle because when he has reached its starting point, the turtle will have moved to a further point, and so on ad infinitum - *that, in fact, he cannot even start running, for, before traversing the stretch to the starting point of the turtle, he will have to traverse half of it and again half of that and so on ad infinitum.*
>
> The article sums up the present situation as follows.
>
> All of these paradoxes are derived from the problem of the continuum. Although they have often been dismissed as logical nonsense, many attempts have also been made to dispose of them by means of mathematical theorems, such as the theory of convergent series or the theory of sets. *In the end, however, the difficulties inherent in his arguments have always come back with a vengeance, for the human mind is so constructed that it has two ways of looking at a continuum that are not quite reconcilable.*[4] (emphasis added)

---

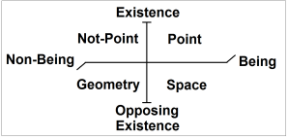
[4] One way of stating Zeno's Paradox in numeric form might be to recognize that the number "1" in Base Two is "1," and that fraction ½ in Base Two is 0.1, and that fraction ¼ in Base Two is 0.01, and that the fraction 1/16 in Base Two is 0.001, etc. Simply stating the number "0.000…0001 in Base Two" should be sufficient to indicate that the infinite number of zeros which can be placed between the first ".0" and the final "1" describes an infinitely small distance from the number zero on the positive side of the number line, thereby "proving" the validity of Zeno's Paradox. Since this small, positive number cannot be equal to zero or have the properties of zero, the number can never equal zero. Therefore the number may serve as the denominator of a fraction, because in using it one does not "divide by zero," an act prohibited. Perhaps this idea condenses Zeno's Paradox in numeric form, i.e. that Achilles "cannot even start running, for, before traversing the stretch to the starting point of the turtle, he will have to traverse half of it and again half of that and so on ad infinitum."

And yet, according to very accepted mathematic proofs, the inverse of the above number - "0.111111111… in Base Two" - is always deemed to equal 1 exactly despite the very different appearance of the numbers themselves. So it would seem that the first number "0.000…0001 in Base Two" is not recognized as a legitimate number, and the second number "0.111… in Base Two" is set equal to the very number intended to be set off as distinct.

Zeno claims that our perception of motion is in error,[5] for nothing can traverse the infinity of points between ANY two points.[6] Motion does not exist; reality is motion-less.[7]

---

5 If one concedes that "A thing can not 'Be' and 'Not Be' at the same time in the same way" (The Principle of Non-Contradiction); and also concedes that "In order for a thing to 'exist' it must possess an 'opposite' which also exists" (The Principle of Contradiction); one may offer a "proof" of Zeno's Paradox, as follows.



In the above description, the "Point" which subdivides the line always requires the "Being" of "Space" to ensure that the "Point" itself exists. Because this Space is as necessary to the existence of the Point as the Point is itself, Zeno's Paradox represents a valid bar to our present understanding of the continuum. See *A Theory of Reality*, at http://www.scribd.com/scott_albers_1 , for more on this topic.

6 The rich credentials of this paradox begin in antiquity. Two approaches to the problem of motion as presented by Zeno might be mentioned by way of comparison, to wit: Plato and Aristotle. (Torretti)

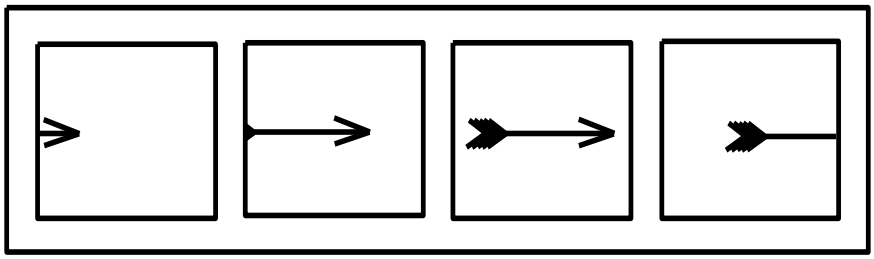
> ...Plato obviously countenances a purely mathematical theory of motion, which it would be appropriate to call kinematics or phoronomy. He conceives it quite broadly. "Motion- he says- presents not just one, but many forms. Someone truly wise might list them all, but there are two which are manifest to us." (ft 35) One is that which is imperfectly illustrated by celestial motions.
>
> The other is the "musical motion" (enarmonics phore), studied by Pythagorean acoustics. This science, says Plato, has been justly regarded as astronomy's "sister science." Exact observation- not to mention experiment-is completely out of place here too. Plato pours ridicule on "those gentlemen who tease and torture the strings and rack them on the pegs of the instrument." (ft. 36) Generally speaking, "if anyone attempts to learn anything about the objects of sense, I do not care whether he looks upwards with mouth gaping or down-wards with mouth shut, he will never, I maintain, acquire knowledge, because nothing of this sort can be the object of a science."
>
> ... Aristotle's solution of Zeno's paradoxes depends essentially on the premise that, even though a point can always be determined which divides a given segment into two parts in any assigned proportion, such a point need not exist before it is actually constructed. (ft. 25) ...
>
> Aristotle was well aware that his finite universe might appear to be incompatible with geometry. But, in his opinion, it was not. "Our account does not rob the mathematicians of their science," he writes "by disproving the actual existence of the infinite in the direction of increase. ... In point of fact they do not need the infinite and do not use it. They postulate only that the finite straight line may be produced as far as they wish. It is possible to have divided in the same ratio as the largest quantity another magnitude of any size you like. Hence, for the purposes of proof, it will make no difference to them to have such an infinite instead, while its existence will be in the sphere of real magnitudes. (Aristotle, Phys., 207b 27-34)

7 Newton discusses concepts surrounding Zeno's Paradox in the opening chapter of his Mathematical Principles of Natural Philosophy, Book One, Section I, Lemma 1.

> Lemma 1
>
> Quantities, and the ratios of quantities, which in any finite time converge continually to equality, and before the end of that time approach nearer to each other than by any given difference, become ultimately equal.
>
> If you deny it, suppose them to be ultimately unequal, and let D be their ultimate difference. Therefore they cannot approach nearer to equality than by that difference D; which is contrary to the supposition.

The answer that time and space might best be thought of as separate, discrete entities does not entirely solve the problem. (see Rucker):

> The basic intuition about an Absolutely Continuous line is that such a line cannot be conceived of as a set of points. Zeno expresses this intuition in his paradox of the arrow. The paradox of the arrow seems to constitute a proof that space is not made of points. For, Zeno argues, consider an arrow that flies from the bow to the target. If space is made up of points, then the flight of the arrow can be decomposed into an infinite set of frozen movements, movements where the tip of the arrow successively occupies each of the points between bow and target. The problem is that while the arrow is at any one fixed point, say the halfway point, the arrow is motionless. How can the flight of the arrow be a sequence of motionless stills? Where did the motion go?



> A movie of an arrow's flight is, of course, a sequence of motionless stills. But this does not disturb us, as we realize that the arrow moves in between the pictures. The problem Zeno raises is that if space is made of points, and if a still is taken at each point, then there is no possibility of "moving between the pictures" ... *because there is nothing between the pictures.* [8]

Einstein similarly ignores Zeno's Paradox. (Einstein, 1961)

> The surface of a marble table is spread out in front of me. I can get from any one point on this table to any other point by passing continuously from one point to a "neighboring" one, and repeating this process a (large) number of times, or, in other words, by going from point to point without executing "jumps." I am sure the reader will appreciate with sufficient clearness what I mean here by "neighboring" and by "jumps" (if he is not too pedantic). We express this property of the surface by describing the latter as a continuum.

[8] The approach taken herein bears some similarity to the understanding of reality expressed by Bishop George Berkeley, (1685-1753). Berkeley's criticism (The Analyst, 1734) of Newton's invention of "fluxions" to obtain the desired ends of calculus despite the challenge of Zeno's Paradox ends is famous.

> It must, indeed, be acknowledged, that (Newton) used Fluxions, like the Scaffold of a building, as things to be laid aside or got rid of as soon as finite Lines were found proportional to them. But then these finite Exponents are found by the help of Fluxions. Whatever therefore is got by such Exponents and Proportions is to be ascribed to Fluxions: which must therefore be previously understood. And what are these Fluxions? The Velocities of evanescent increments? And what are these same evanescent increments? They are neither finite Quantities nor Quantities infinitely small, nor yet nothing. May we not call them the Ghosts of departed Quantities?

The idea that "there is nothing between the pictures" forces us to consider the nature of motion itself. *If the observation of motion around us is, in reality, akin to watching a movie, then the "consciousness" or "belief pattern" of the individuals watching the movie is an essential component of the nature of motion itself.* Rucker mentions Parmenides in this context.

> Zeno's way of the paradox is to deny that space is really made up of points. As a Parmenidean monist, Zeno viewed space as an undivided whole that cannot really be broken down into parts. We can find scattered locations in space, but space is always more than the sum of these isolated points. One can pick out higher and higher infinities from an Absolutely Continuous tract of space, but there will always be a residue of leftover space, of continuous little pieces, infinitesimal intervals over which the actual motion takes place.

Various mathematicians have come to the conclusion that the line segment as described is a set of distinct points between which an infinite number of additional points might be plotted.

> This view of space has been held by several philosophers since Zeno, notably C. S. Pierce and, perhaps, Kurt Godel. Godel distinguishes between the set of points described in set theoretic analysis and the continuous line of space intuition: 'According to this intuitive concept, summing up all the points, we still do not get the line; rather the points form some kind of scaffold on the line.'
>
> Pierce goes further than this. According to him, a truly continuous line is so richly packed with points that no conceivable set, no matter how large, can exhaust the line. There should not just be one point between all of 1/2, 2/3, 3/4, 4/5, 5/6, ... and 1. There should be ∞ points, $\aleph$ points, Absolutely Infinitely many![9]

---

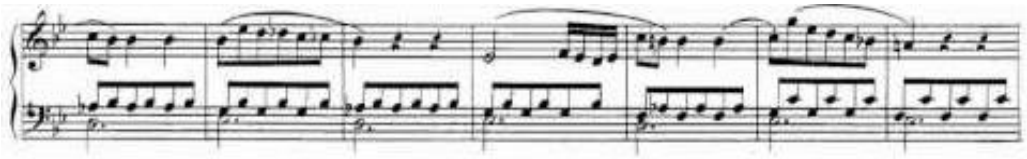
[9] The main problem with an acceptance of the significance of Zeno's Paradox is that, if motion does not exist, what are we to make of the reality of the motion around us? On the centrality of this fundamental point see also Davis and Hersch.

> The infinite is that which is without end. It is the eternal, the immortal, the self-renewable, the aperion of the Greeks, the ein-sof of the Kabbalah, the cosmic eye of the mystics which observes us and energizes us from the godhead.
>
> Observe the equation "1/2 + 1/4 + 1/8 + 1/16 + ... = 1."
>
> ... On the left-hand side we seem to have incompleteness, infinite striving. On the right-hand side we have finitude, completion. There is a tension between the two sides which is a source of power and paradox. There is an overwhelming mathematical desire to bridge the gap between the finite and the infinite. We want to complete the incomplete, to catch it, to cage it, to tame it.
>
> Mathematics thinks it has succeeded in doing this. The unnamable is named, operated on, tamed, exploited, finitized, and ultimately trivialized. Is, then, the mathematical infinite a fraud? Does it represent something that is not really infinite at all?

Identity and Infinity

In an effort to restore motion to the universe I propose that motion must reside in the conscious perception of motion, as this consciousness ties together the separate frames of existence.

This compels some understanding of motion under a scaffold of points. I propose that a musical score contains such a scaffold, the individual notes being "tied together" by the melody they create.

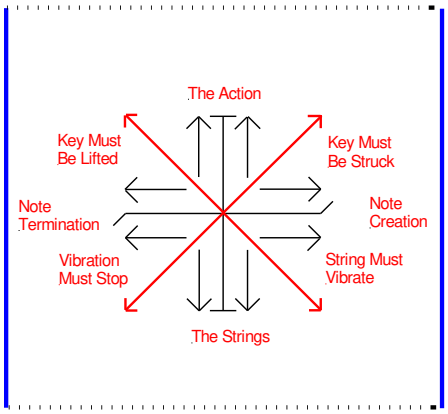


A systematic understanding of this scaffold of points can be constructed by way of an analogy to musical notation. The analogy is as follows.

There are four essential aspects of playing a musical note on the piano. These are:

1. The key must be struck.
2. The string must vibrate.
3. The key must be lifted.
4. The string's vibration must stop.

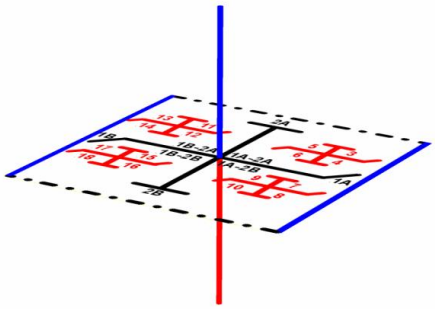
The formulation of these four points is presented as follows, and the central terms of the figure perhaps can be anticipated by the reader. "The Action" refers to the keyboard itself and the mechanisms used to control the strings. "The Strings" refers to the stretched wire making the sounds of the piano. The dotted lines represent "A Primary Opposition," i.e. an opposition between aspects of something which is absolute, an opposition with no middle grounds possible. The blue lines represent "A Secondary Opposition," i.e. an opposition of grades or degrees, something which has intermediate values between the two extremes.



On the right hand we have a vertical blue line representing the tension which exists between the key being struck and the string vibrating. On the left hand we have a vertical blue line representing the tension between a key being lifted and the vibration stopping.

An “Image Axis” may be added to this model, one which contrasts the “sound” of vibrating strings with a finished “note” of music as terminated by the performer at a particular time.[10]

THE SOUND



THE NOTE

In this regard it must be emphasized that the termination of a sound by the performer is of equal significance as the creation of the note itself. The introductory bars of Beethovan’s Fifth Symphony are meaningless without the swift repetition of three notes, each short, followed by a sustained fourth note a major third lower in pitch.

Consequently three dichotomies are conjoined in the above model:

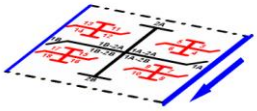
(1) creation	vs.	termination,
(2) action	vs.	vibrating strings, and
(3) sound	vs.	finished note.

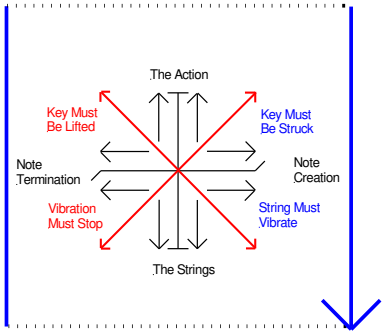
The “scaffold” upon which notes are built (and which by analogy points along a line must possess as well) is as follows.

[10] Kaluza proposed a “cylinder condition” as the premise of his “fifth dimension,” i.e. a dimension which did not affect any part of the physical world of three dimensions of space and one of time, and yet was resident within them. If one imagines that the square created above between the Primary Opposition and Secondary Opposition to be, instead, a circle serving as but one thing slice or segment of a cylinder, the “Image Axis” resembles the relationship which Kaluza had in mind when he presented the fifth dimension and its relationship to the more obvious four dimensions of physical reality.

Each note in a piece of music is connected by way of a similar underlying circuit.

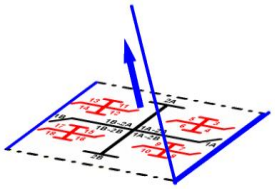
1) The key/hammer strikes the string, which then vibrates.





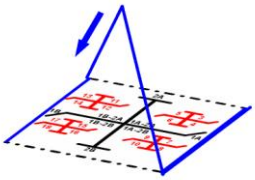
2) This vibration creates the sound of piano performance.

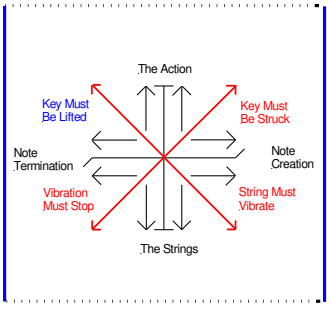
THE SOUND



3) A note - a sound which has ended on time - is created by lifting of the key…

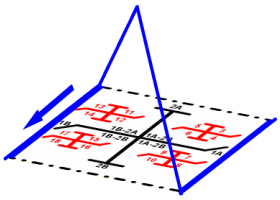
THE SOUND

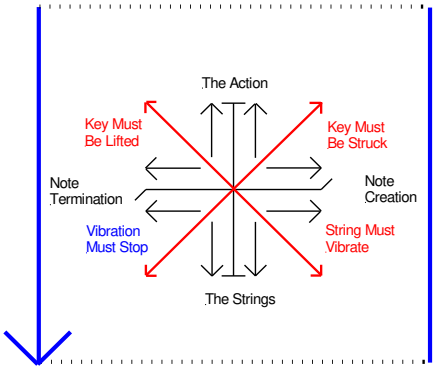




4) … Which permits the damper to fall onto the string, thereby stopping the vibration and the sound at a particular time in the piece of music.

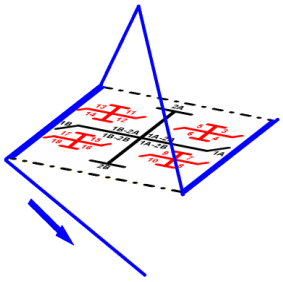
THE SOUND





5) This in turn terminates the duration of the note and provides a basis to understand the relationship of the note to the rest of the piece of music,

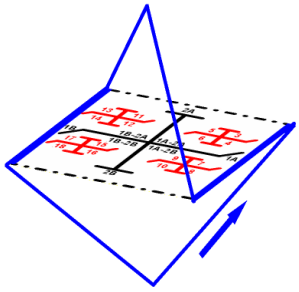
THE SOUND



THE NOTE

6) … Which in turn gives rise to a new understanding of the note, i.e. it is now a *particular* note in the context of the rest of the piece.

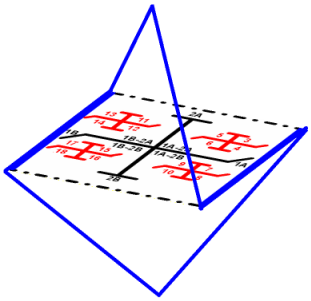
THE SOUND



THE NOTE

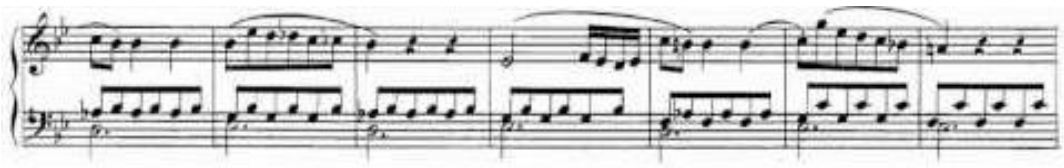
7) Each note of music represents a circuit of relationships, as follows.

THE SOUND



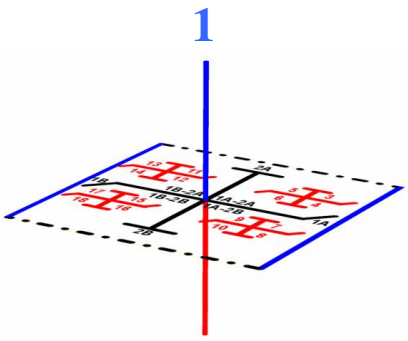
THE NOTE

The melody below by Mozart is a sequence of such notes.

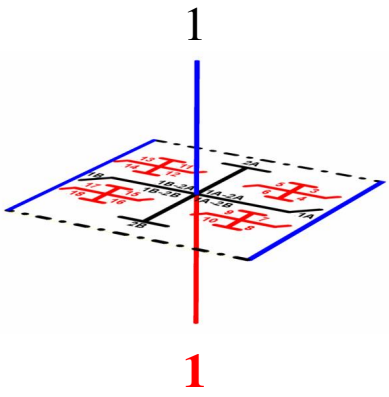


We may unify the notion of Sound and Note mathematically if we use this “Image Axis” to generate a series of numbers which can be used to join these two poles. For example, if we give the number “one” to the first vibration of the piece

First string vibrates.



… we can see that the “note” itself revolves entirely upon the sense that the sound has ended.

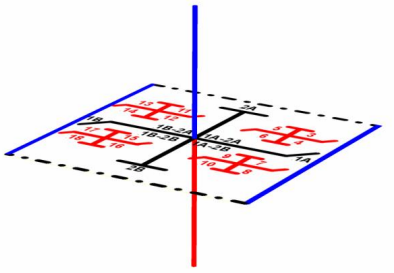


First note is heard.

Once this “note” has ended, a new sound in the melody follows:

Second string vibrates.

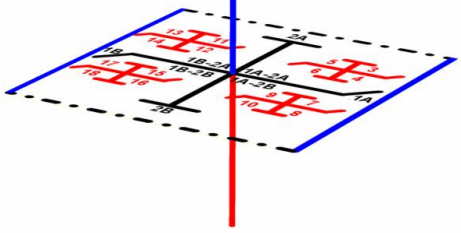
1 + 1 = 2



1

The combination of the second sound with the first note is something other than a simple addition of notes. Once the second sound terminates, it creates a new sense of both notes. It brings about a new understanding of the note previously heard in combination with itself.

2



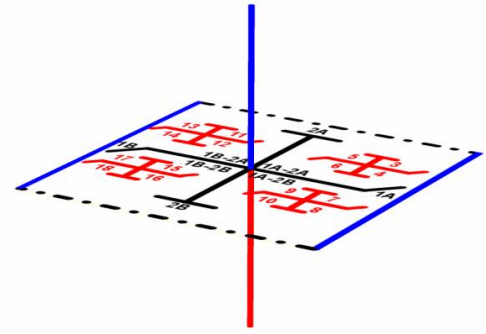
2 + 1 = 3

Second note is heard.

And then a third sound becomes part of the melody,

Third string vibrates.

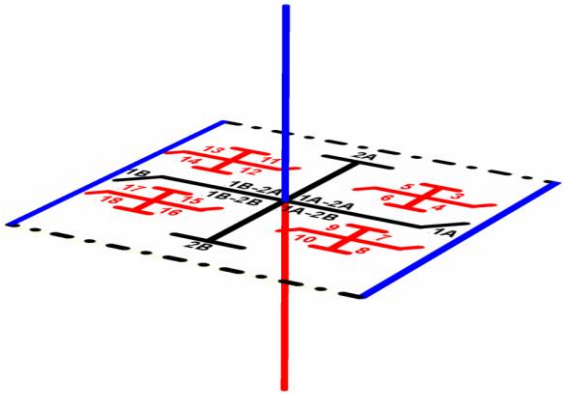
2 + 3 = 5



3

And followed, upon its termination, by the understanding of the three notes collectively:

5



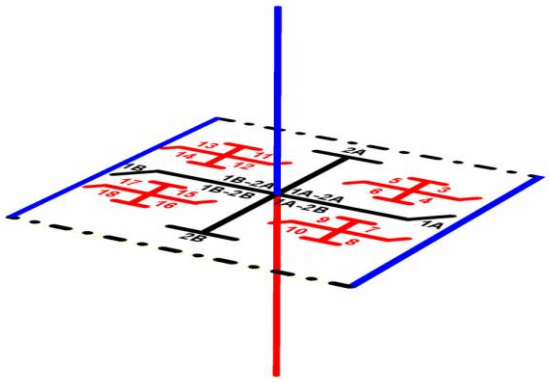
3 + 5 = 8

Third note is heard.

And then a fourth note…

Fourth string vibrates.

5 + 8 = 13



8

By extending this series of sums into a pattern we have the Fibonacci series, to wit:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, etc.

Joining these numbers into fractions in an effort to determine a common ratio, we have:

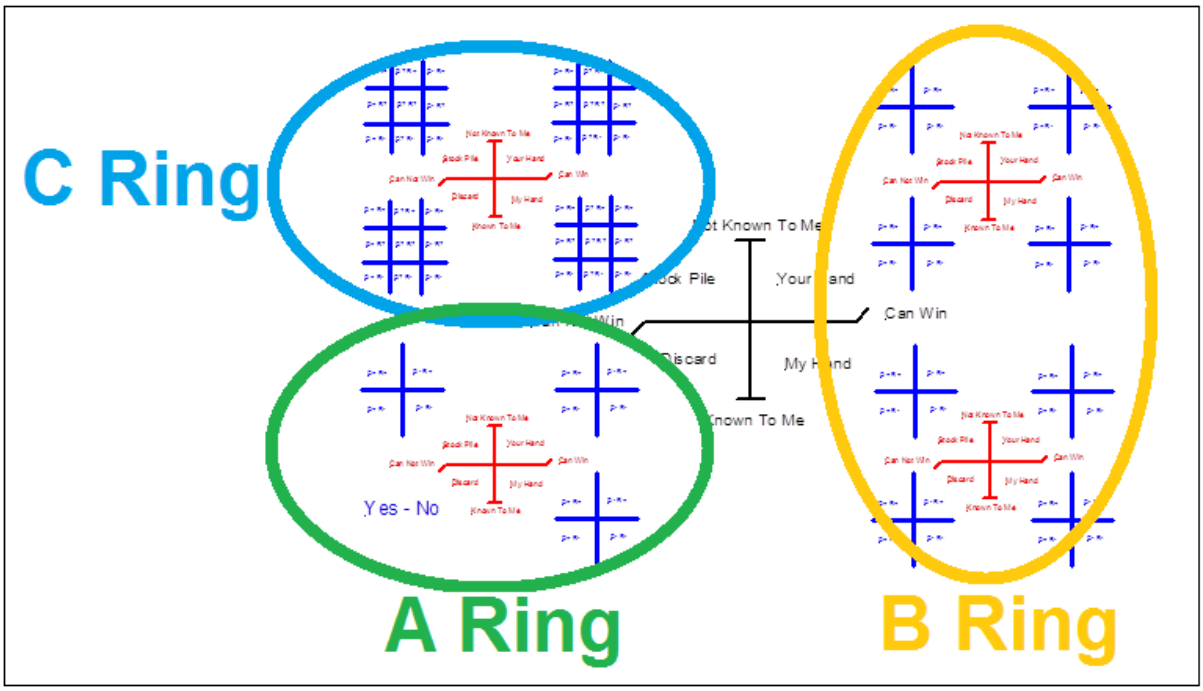
$$\frac{1}{1} \quad \frac{2}{1} \quad \frac{3}{2} \quad \frac{5}{3} \quad \frac{8}{5} \quad \frac{13}{8} \quad \frac{21}{13} \quad \frac{34}{21} \quad \frac{55}{34} \quad \frac{89}{55} \quad \ldots$$

This continuing series renders the constant "phi" $\varphi = 1.6180...$

This leads directly to the question as to whether it is possible to find this mathematic constant in the social statistics of groups of people, thereby confirming the utility of this approach to consciousness as a Fifth Dimension. To the evaluation of social data in a game of cards we now turn.

A Game of Cards

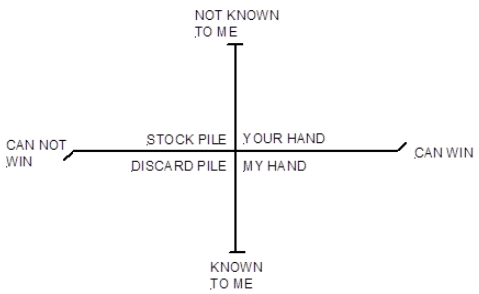
To consider more closely the manner in which "self identity" might be given to inanimate, unliving things, let us begin with the "identity" of a game of cards as associated with the Rings of Saturn.



The Plane of Definition

In the game of Gin Rummy four different groups of cards are constructed: Your Hand, My Hand, The Draw Pile and the Top Card of the Throw Away Pile. As the two players compete to win the game, each in turn takes a card from either the Throw Away Pile or from the unknown Top Draw Card, and then throws a single card away. The "identity" is the set of decisions made by both players.

Placing the cards which will ultimately win the game on the right (Your Hand, My Hand) and those that will not win the game on the left (Throw Away Pile, Top Draw Card), and placing those cards which I do not know at the top (Your Hand, Top Draw Card) and those I do know at the bottom (My Hand, Throw Away Pile) we have the following arrangement.

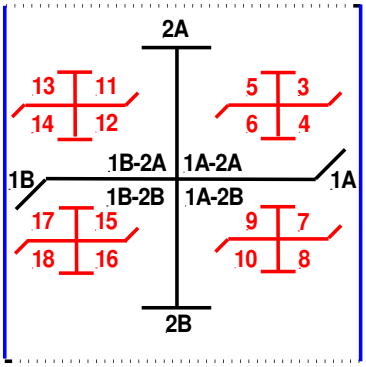


The Plane of Relationship

The relationships between the cards are part of the system whereby each player seeks to win the game. As each player attempts to win the game, each decision must be based upon (1) the categories of cards set up by the game and (2) the relationships which these four categories of cards have to one another.

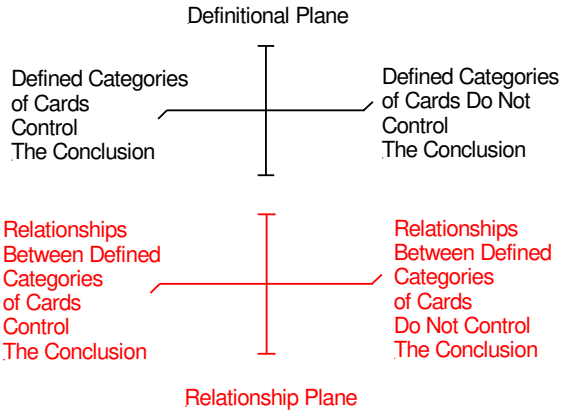
My understanding of these groups of cards and their interrelationships is divided between the two groups in competition TO WIN the game (Your Hand, My Hand) as opposed to the two groups which CAN NOT WIN the game (the Stock Pile, the Discard Pile).

If we number these quadrants in order of appearance in the square previously described, we have the following:

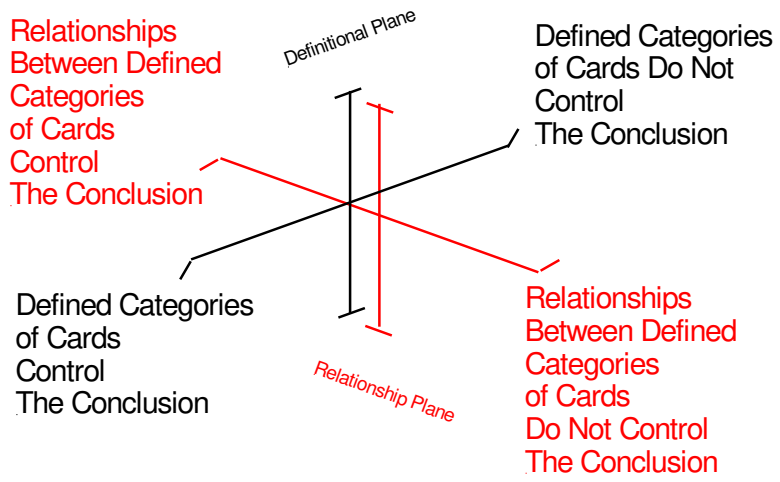


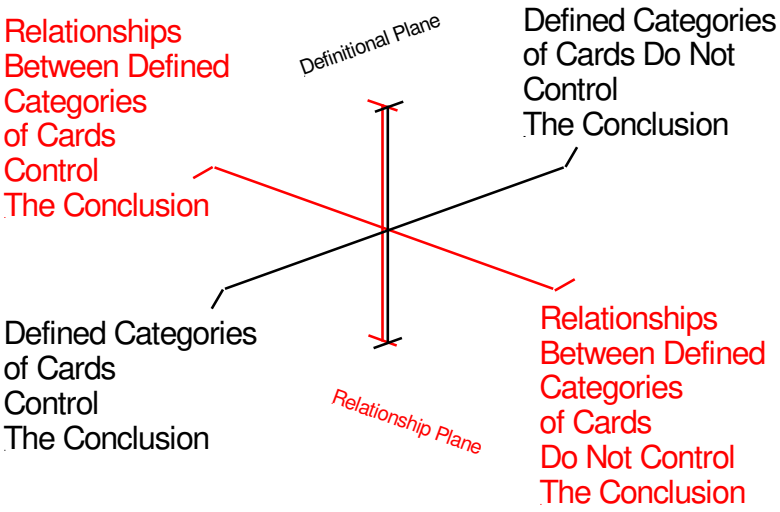
The Plane of Conclusion

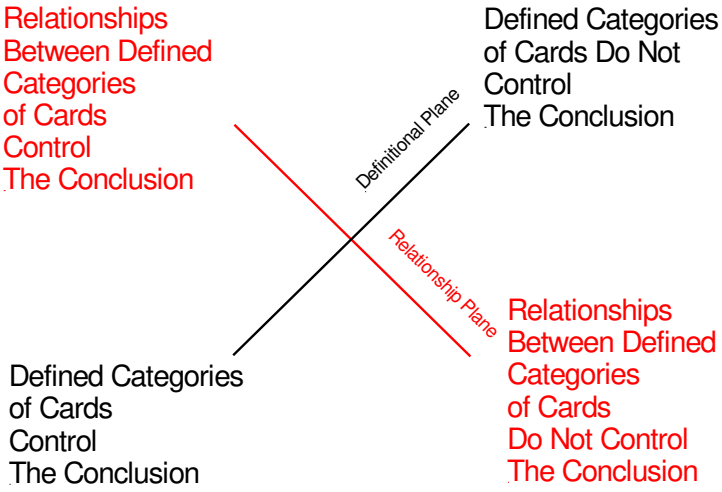
To imagine the development of a “Plane Of Conclusion” we might state the "Plane of Definition" with the "Plane of Relationship" separately, and then align them along a common axis.



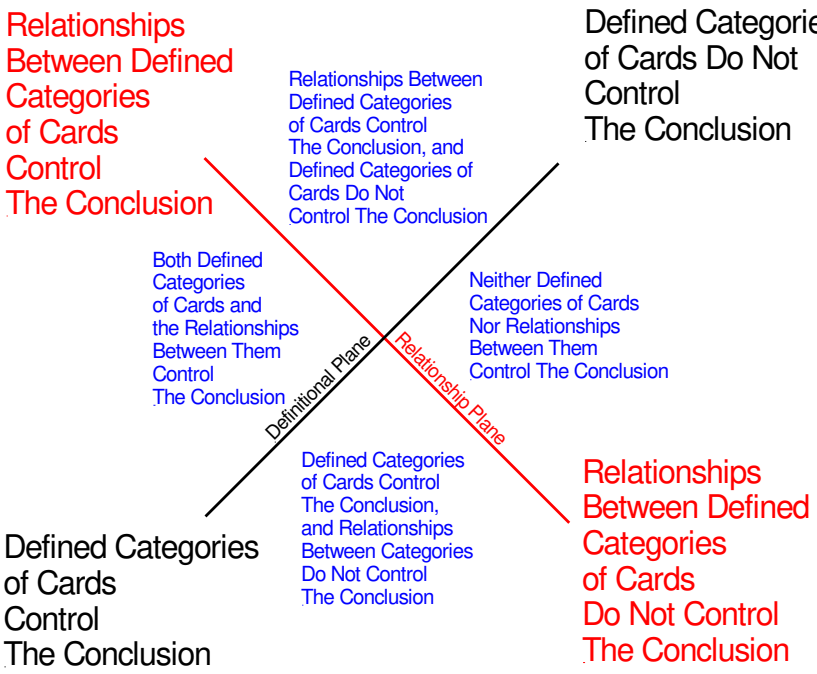
To answer any question in the game of Gin Rummy, either the definitions proposed, or the interrelationships specified, must CONTROL the answer. The player must reach any specified conclusion based upon the adequacy of the definition of a given set of cards, or upon the relationships mandated by the definitions adopted. The alignment of these concepts can be imagined as follows:





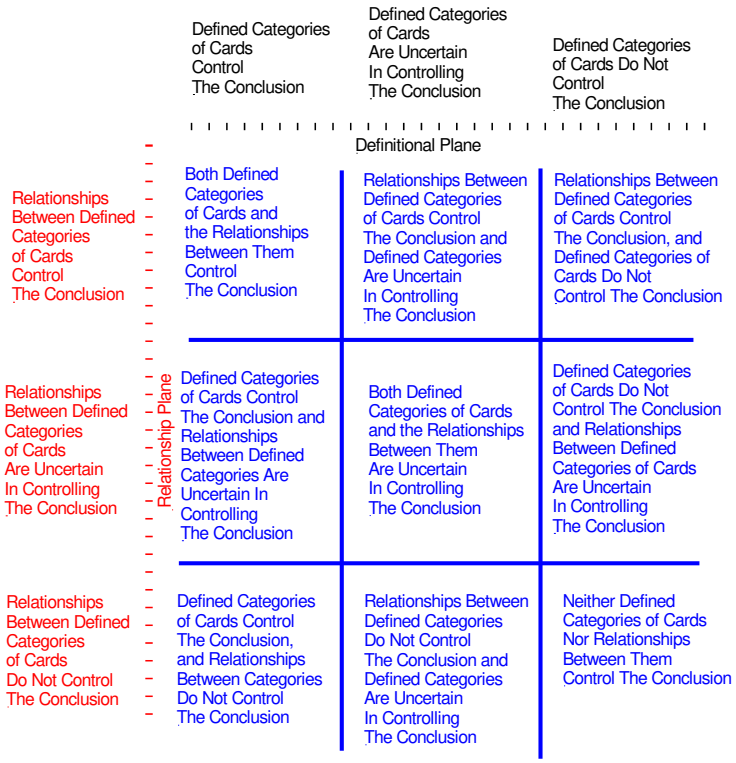


The card player now is expected to come to conclusions regarding the interplay between the definitions proposed and the interrelationships inherent in the definitions proposed. We should treat the concept of "defining" in Gin Rummy, and of interpreting "relationships" between defined entities, as two separate aspects which "control" our answer to any decision in the game. As this pertains to Gin Rummy, we would then have the following "conclusion" plane:

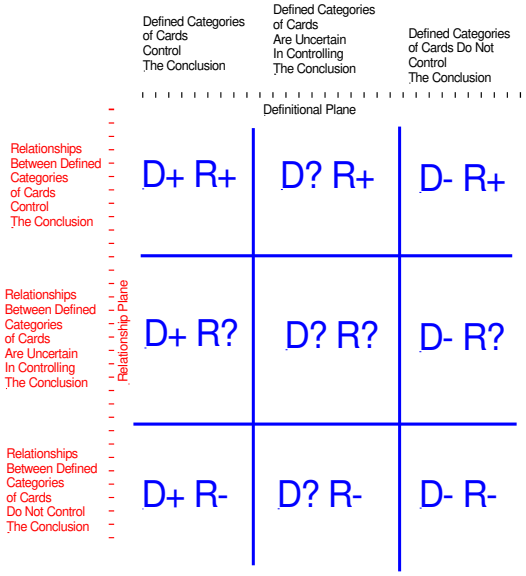


A fundamental expectation of every game is that one of these quadrants will render an appropriate answer.

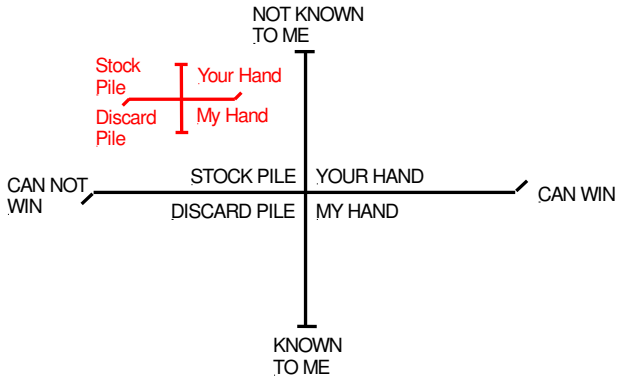
The uncertain usefulness of *both* Definitions and Relationships together renders one final possibility for a Conclusion. The checkerboard of possibilities below represents the uncertainty *inherent* in the unknown identity of the top card in the Stock Pile. To determine whether the top stock pile card should be chosen I face nine separate possibilities:



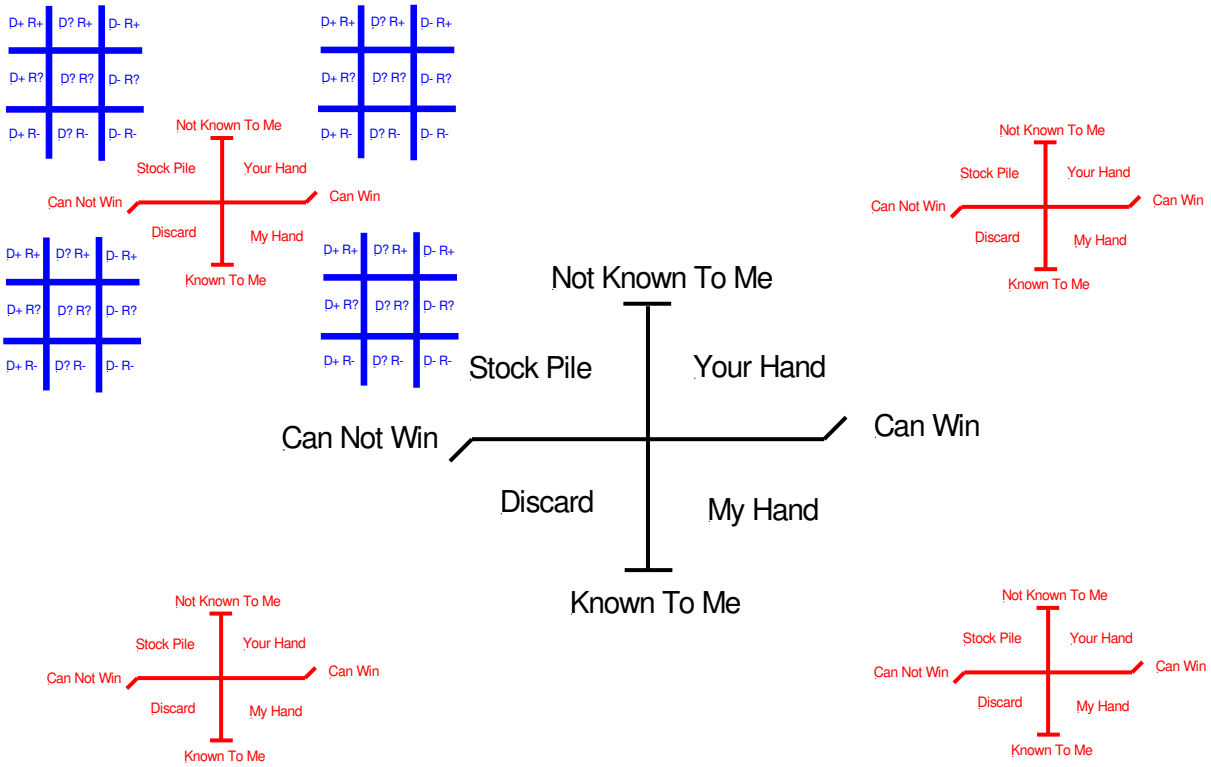
… or in simplified notation:



In other words, while the top card in the Stock Pile obviously has *relationships* with the groups of cards *defined* by the game …



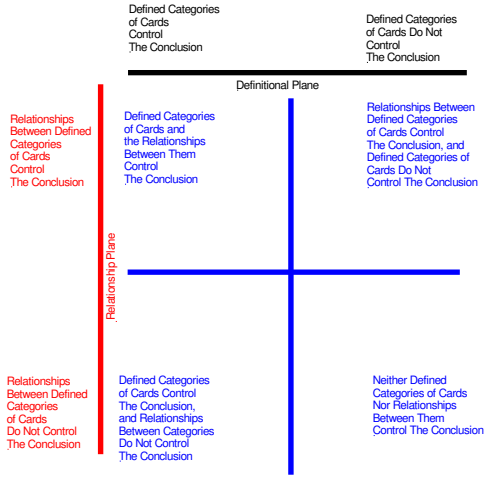
… the *utility* of these definitions and relationships, the ability to use the definitions and relationships to reach *conclusions* about whether or not to choose the top card of the Stock Pile and incorporate it into one's hand of cards, must remain an *inherently* uncertain part of the game itself.



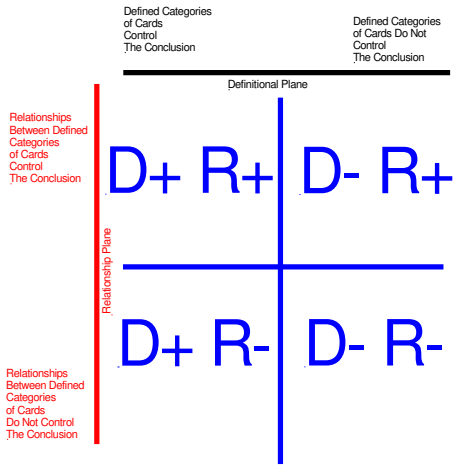
Conclusions Regarding Categories of Cards Which CAN WIN the Game
(Your Hand, My Hand)

We might expect that this same uncertainty would ruin any chance of a decent card game. We might expect that these same nine quadrants of uncertainty would "infect" every other category of cards – Your Hand, My Hand, the Discard Pile.

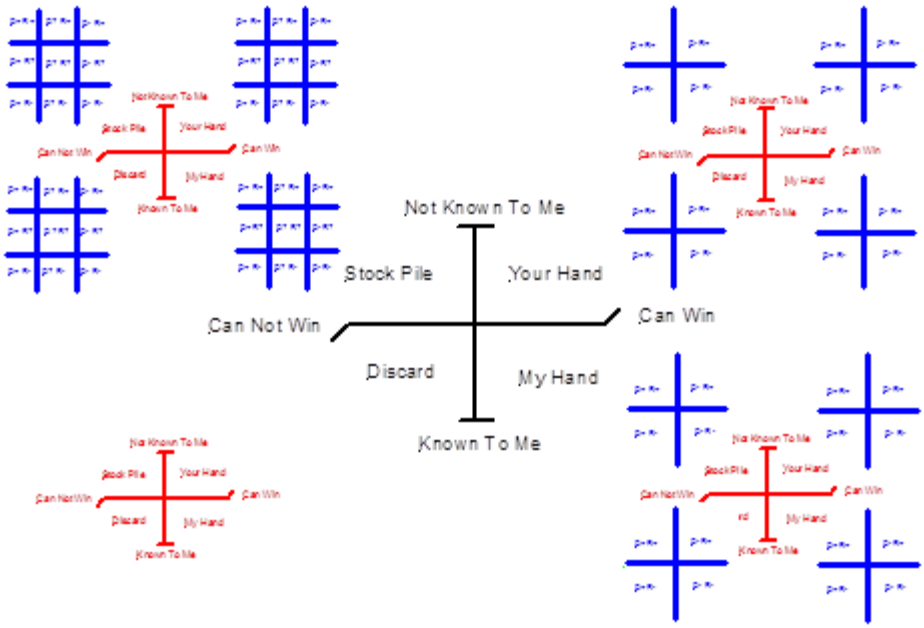
This, however, is not the case. If I ask you to reveal the identity of the cards in Your Hand, you could tell me. This would assist me greatly in my effort to win the game. The identity of the cards in Your Hand are not in question, nor are they inherently unknown; they are simply unknown to me. For this reason *neither* the Definitions nor the Relationships are *inherently* uncertain when it comes to Conclusions regarding Your Hand. Because we have no *inherent* uncertainty or contradiction in Your Hand, we may return to a simple and straightforward placement of planes. This simple blue cross indicates that only four possibilities of conclusion are possible. Once again, these are:



or,…



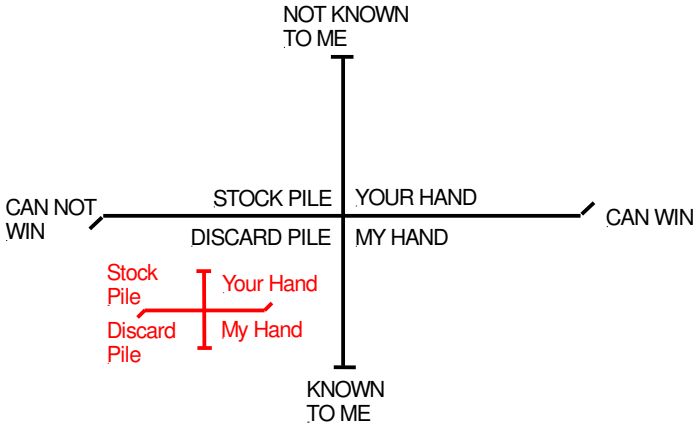
If we ask what Conclusions may be drawn between the cards in My Hand and the cards in the Discard Pile, there is no *inherent* uncertainty. I see all of these cards and develop my strategy accordingly.



The definitions and relationships specified for both quadrants of the right hand side - Your Hand, My Hand, categories of cards which "CAN win" - are clear. There is no *inherent* contradiction in dealing with these sets because each is a part of the game which CAN win. Both Your Hand and My Hand are the focus of a personal intellect which arranges the cards, oversees them, and controls them. There is no *inherent* uncertainty in their identity, although this identity may be withheld from the opposing player for the purposes of advancing the game.

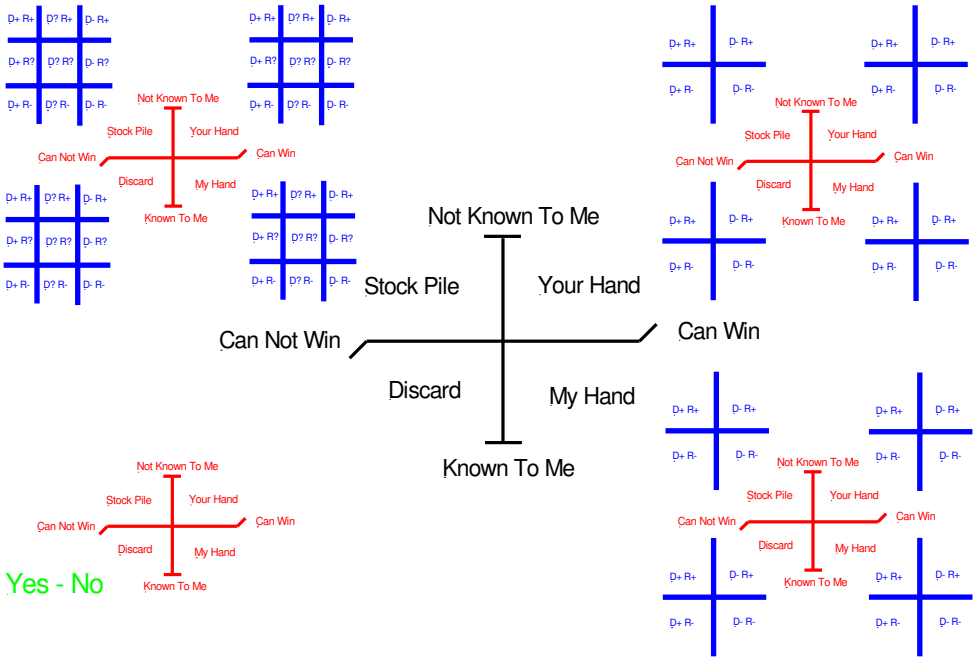
Conclusions regarding Discard Pile

We have already specified that there exist "relationships" between the cards in the Discard Pile.

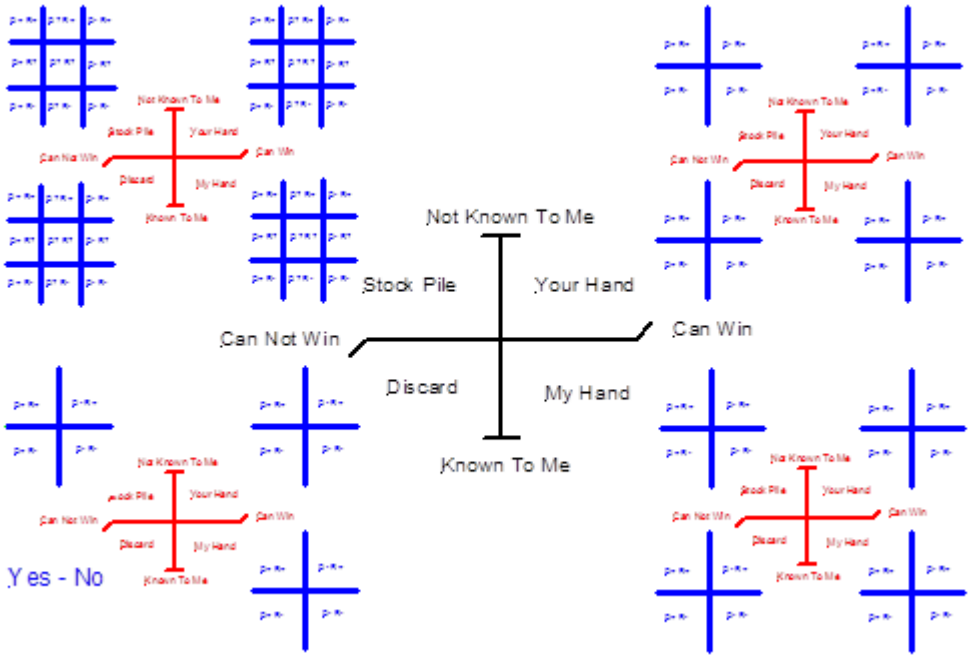


When we move to the Plane of Conclusion however the issue is no longer what "relationship" exists between the cards of the Discard Pile. Rather the question becomes "Do I want the card?" The identity of the card is known, there is no uncertainty about the card itself nor in its relationship to the rest of the cards. We may answer this question simply, Yes or No.

In this case, unlike the top card of the Stock Pile, the top card of the Discard Pile is not inherently uncertain. Here the answer must be "Yes-or-No." Although there may be a strategic choice involved based upon the alignment of the cards in the rest of the game, there is no built-in preference either way. Let us simply mark this card as a Yes-No choice.



Once the answer is determined, the conclusion reached in this question is not subject to any *inherent* uncertainty or contradiction. The simple blue cross is in tact, although the specific answer given relies entirely on the issue presented in the first question.



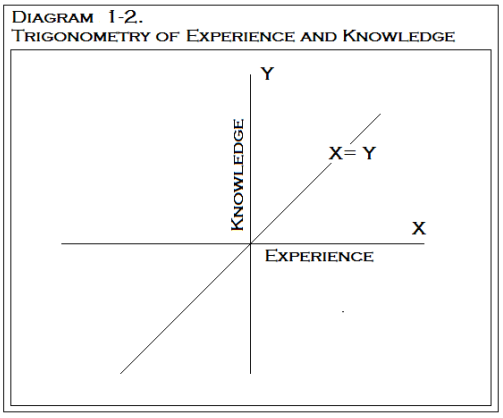
The above describes four different categories of logic. These may be found at work in the dynamics of the economic history of the United States as it develops, and then re-develops, its identity over time.

The Theory

For the purposes of this essay we will take as an axiomatic truth that all human life is based upon the presumed equivalence between that which we experience through the senses and that which we know to be real.[11] If "that which we experience" is given the variable "X" and "that which we know to be real" is given the variable "Y", we may state this equivalence as:

$$X = Y.$$

If we place this equation in a Cartesian coordinate system, we have the following 45 degree angle line, beginning at x = 0, y = 0 and extending on toward and infinite number of associations.



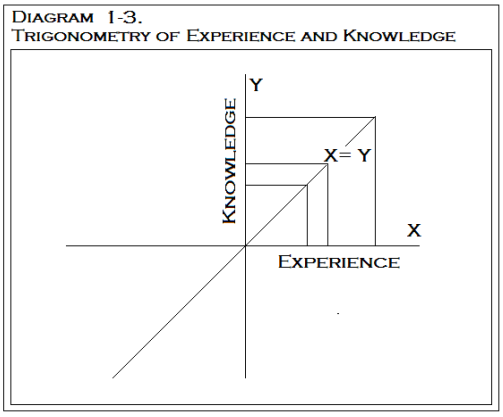


Diagram 1-2 is, in reality, the outcome of an infinite number of squares, wherein each corner point has a specific meaning. "X" represents our experience of something, "Y" represents our knowledge of the thing experienced, the point "(X, Y)" represents the interaction between our experience of the thing itself and our knowledge of the thing itself, and the origin of the graph "(0, 0)" represents the beginning association we make between experience and knowledge as fundamental assumptions of all inquiry.[12]

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[11] For a famous example of the meaning of this sentence, see Boswell, J. (1820). "After we came out of the church, we stood talking for some time together of Bishop Berkeley's ingenious sophistry to prove the nonexistence of matter, and that every thing in the universe is merely ideal. I observed, that though we are satisfied his doctrine is not true, it is impossible to refute it. I never shall forget the alacrity with which (Samuel) Johnson answered, striking his foot with mighty force against a large stone, till he rebounded from it -- "I refute it *thus*."

One might assert that the experience of reading a book and enjoying the imaginary world conveyed is not the same as "experiencing" or "knowing" anything about the world imagined.

Our point here is far more modest and direct. The "experience" referred in this essay is simply that of "reading the book" and the knowledge considered is simply that the person reading knows that he or she is reading a book. The equivalence understood between the *experience* of reading the book, and the *knowledge* that one *is* reading a book, is the equivalence with which we begin this analysis.

[12] See Ornstein, at 63: "In 1268, Roger Bacon, one of the founders of modern science, wrote (in his *Opus Maius....*), 'There are two modes of knowing, through argument and experience. Argument brings conclusion and compels us to concede them, but does not cause certainty nor remove doubts in order that the mind may remain at rest in truth, unless this is provided by experience.' These two modes are complementary (both are "right"), and together form the basis for the complete human consciousness."

Extension to the Jury Trial of a Criminal Case

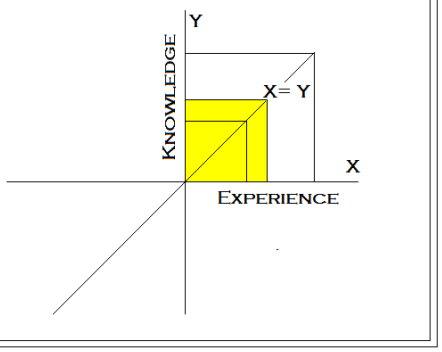
In the United States the jury trial of a case is premised on this same equation "X = Y," "experience" and "knowledge," taken to the next higher social level of the jury. The jury's reception and consideration of the evidence presented[13] indicates that this small group is the expansion of the smaller individual and included minds. In the jury's deliberation the jury demonstrates itself as being the larger, expanded, copied and congruent larger "fractal" of the individual mind.

Specifically, the jury's personal *experience* of the evidence as presented in trial represents the "X" of a trial proceeding.

The jury's evaluation of this evidence as understood through the prism of their own life experiences is the "Y" of the trial proceeding, their collective *knowledge* of the facts presented.

The final verdict given by the jury states its evaluation of the association between the "X" of the trial (the evidence presented) with the "Y" of the trial (the jury's evaluation of this evidence).

DIAGRAM 1-4.
TRIGONOMETRY OF EXPERIENCE AND KNOWLEDGE



This simple model may be expanded upon.

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[13] The law of evidence is an important branch of law within the United States. See Thayer 1898. "One who would state the law of evidence truly must allow himself to grow intimately acquainted with the working of the jury system and its long history." As taken from page 267, footnote 1 he states:

"At once, when a man raises his eyes from the common-law system of evidence, and looks at foreign methods, he is struck with the fact that our system is radically peculiar. Here, a great mass of evidential matter, logically important and probative, is shut out from the view of the judicial tribunals by an imperative rule, while the same matter is not thus excluded anywhere else. English-speaking countries have what we call a "Law of Evidence;" but no other country has it; we alone have generated and evolved this large, elaborate, and difficult doctrine. We have done it, not by direct legislation, but, almost wholly, by the slowly accumulated rulings of judges, made in the trying of causes during the last two or three centuries, - rulings which at first were not preserved in print but in the practice and tradition of the trial courts; and only during the last half or two-thirds of this period have they been revised, reasoned upon, and generalized by the courts in banc.

When one has come to perceive these striking facts, he is not long in finding the reason for them. … It is this institution of the jury which accounts for the common-law system of evidence, - an institution which English-speaking people have had and used, in one or another department of their public affairs, ever since the Conquest. Other peoples have had it only in quite recent times, unless, indeed they may belong to those who began with it centuries ago, and then allowed it to become obsolete and forgotten. England alone kept it, and, in a strange fashion, has developed it. "

The criminal law of the United States is based upon a dichotomy between the criminal act alleged to have been committed – (the *actus reus* of the offense[14]) – and the mental intent – (the *mens rea* of the offense[15]) – associated with the crime. For example, the act of killing someone is a homicide if done with the intent to kill the individual. If the killing was the result of recklessly driving in a crowded street, the crime is less because the evil of the intent to harm was less. Differences in the consequence to the Defendant can be quite significant, depending upon the nature of the criminal act and mental intent found by the jury.

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[14] The significance of an actual *act* in violtion of the law was highlighted in the case of *Robinson v. California*, 370 U.S. 660 (1962). In this case the U.S. Supreme Court ruled that a California law making it illegal to be a drug addict was unconstitutional because the mere status of being a drug addict was not an *act* and thus not criminal. The Court held:

"It is unlikely that any State at this moment in history would attempt to make it a criminal offense for a person to be mentally ill, or a leper, or to be afflicted with a venereal disease. A State might determine that the general health and welfare require that the victims of these and other human afflictions be dealt with by compulsory treatment, involving quarantine, confinement, or sequestration. But, in the light of contemporary human knowledge, a law which made a criminal offense of such a disease would doubtless be universally thought to be an infliction of cruel and unusual punishment in violation of the Eight and Fourteenth Amendments. ...

"We cannot but consider the statute before us as of the same category. In this Court counsel for the State recognized that narcotic addiction is an illness. Indeed, it is apparently an illness which may be contracted innocently or involuntarily. We hold that a state law which imprisons a person thus afflicted as a criminal, even though he has never touched any narcotic drug within the State or been guilty of any irregular behavior there, inflicts a cruel and unusual punishment in violation of the Fourteenth Amendment."

[15] The Model Penal Code has provided a general scheme for mens rea in criminal cases since its promulgation in 1957. These levels of intent are:

Strict liability: the actor engaged in conduct and his mental state is irrelevant. Under Model Penal Code Section 2.05, this mens rea may only be applied where the forbidden conduct is a mere violation, i.e. a civil infraction.

Negligently: a "reasonable person" would be aware of a "substantial and unjustifiable risk" that his conduct is of a prohibited nature, will lead to a prohibited result, and/or is under prohibited attendant circumstances, and the actor was not so aware but should have been.

Recklessly: the actor consciously disregards a "substantial and unjustifiable risk" that his conduct is of a prohibited nature, will lead to a prohibited result, and/or is of a prohibited nature.

Knowingly: the actor is practically certain that his conduct will lead to the result, or is aware to a high probability that his conduct is of a prohibited nature, or is aware to a high probability that the attendant circumstances exist.

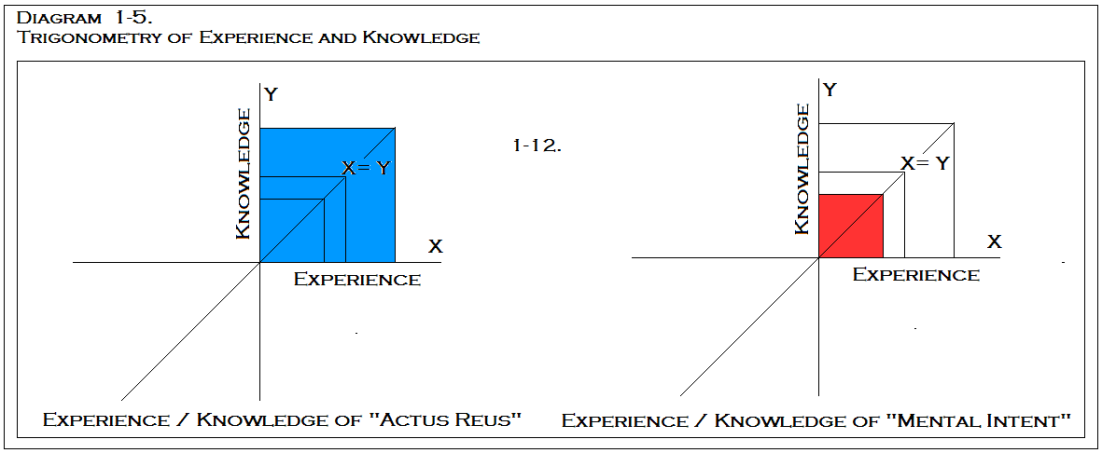
Purposefully: the actor has the "conscious object" of engaging in conduct and believes or hopes that the attendant circumstances exist.

Except for strict liability, these classes of mens rea are defined in Section 2.02(2) of the MPC.

The significance of these levels of mental intent and the actions to which they apply is well illustrated in the case of State of *Montana vs. Rothacher*, 901 P.2d 82, 86-87 (1995). In this case the court's prior decisions had left open the possibility that a homicide might be charged based upon a mens rea going simply to the act which created the crime, rather than the intent to commit the crime itself. The Montana Supreme Court reversed itself, as follows: "It is time to clear up this misperception of the state of mind which must be proven to establish deliberate or mitigated deliberate homicide before a significant injustice results. Our prior construction is clearly contrary to the plain language in the homicide statute and may, in the future, lead to serious and unjust perversion of its purpose. For these reasons, we conclude that the District Court erred when it instructed the jury that the State merely needed to prove that Rothacher acted purposely, without regard to the result that he intended. To the extent that our prior decisions in Sigler, McKimmie, and Byers are inconsistent with this opinion, they are overruled. District courts should not give a similar instruction in the future."

If we let the "actus reus" of any given offense equal a particular number – for example, 5 – then the jury's experience with the evidence presented as to the criminal act (X = 5) and the jury's understanding of that evidence (Y = 5) may be given as a square, in blue below.

Similarly, if we let the "mens rea" of the same offense equal a different number – for example, 3 – then the jury's experience with the evidence presented as to mental intent (X = 3) and the jury's understanding of that evidence (Y = 3) may be given as the red square below.[16]



The idea of giving physical "size" to the jury's experience in court with the evidence may be explained by comparing these experiences. One may readily imagine that prosecutor Jones, an obsessive-compulsive sort, might spend three days developing the actus reus of the case, replete with victim and expert testimony, etc. This is considerably different than might be the case put on by Prosecutor Smith who casually places before the evidence of the same charge a much lesser quantum of evidence, spending the bare minimum of time necessary to establish that a criminal act has occurred. As the jury experiences these differences in court, the outcome of the verdict will shift.

Likewise should Prosecutor Smith neglect to prove that a criminal mental state existed at the time of the alleged offense, it is possible that the proof of the crime as to mens rea may fail entirely. On the other hand, should the prosecutor Jones present proof of mens rea which includes confessions, eye-witness testimony, the testimony of co-conspirators, etc. the experience of the jury with this enlarged quantum of evidence will be fuller than with Smith.

The comparison of these different experiences with the evidence may be depicted by ever larger lengths along the x and y axis as to both the actus reus and mens rea of the charge. The point here is not to propose an absolute scale of proof but rather to suggest that there are very different quanta of proof going into these two essential elements of every criminal case. These

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16 The basic architecture underlying personal choice may be accessed through reference to the common law, an ongoing system of social, political and economic thought all of which is directed toward the maintenance of social order and progress. The central place of the American jury in the legal system of the United States provides a constant connection between the circumstances faced by the people and the laws governing the people. The central ideas of the common law in criminal cases – actus reus, mens rea – are profoundly important to economics because they state the fundamental social basis of common American understandings of human motivation and social judgment, much of which directly applies to very important matters of business, finance, morality and economics, as evolved over tens of thousands of jury trials. This wealth of information as to social and personal behavior is included in this model. It has proven to be both illustratively useful as well as mathematically helpful.

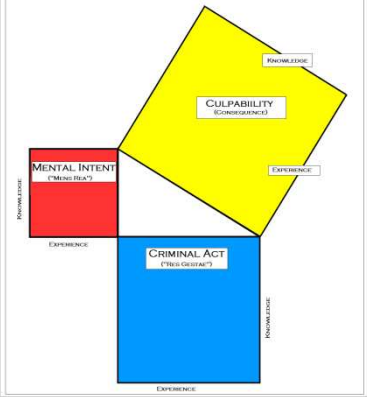
quanta are separate as to actus reus and mens rea but they are joined together in the jury's evaluation of the weight of the case against the Defendant.

The culpability, if any, of the Defendant for a crime is given in accordance with the sum of these two elements of proof. The full experience and knowledge summarized by the case will equal the sum of these two squares. Stating the jury's experience with the evidence of a criminal act as a positive distance "A" and the jury's experience with the evidence of mental intent as a positive distance "B", then the experience / knowledge represented by Culpability (C) associated with the verdict should equal the sum of these two things, or :

$$A^2 + B^2 = C^2$$

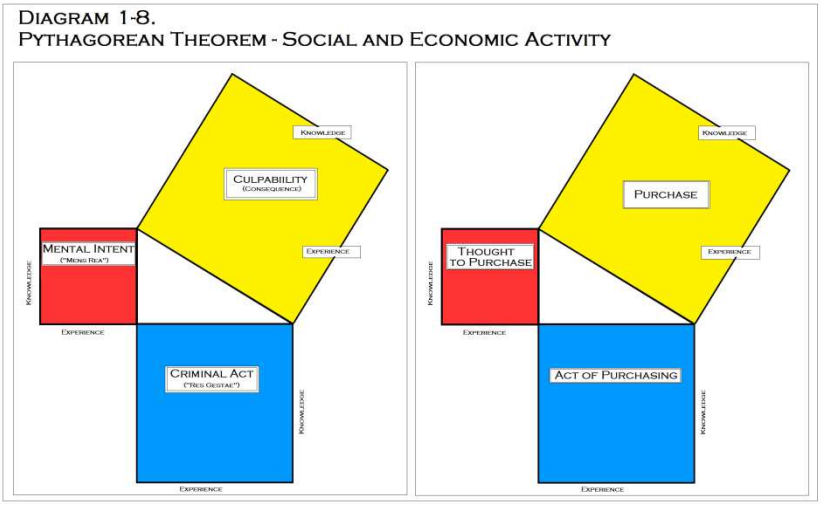
Geometrically, this equation may be portrayed with the proportions of the Pythagorean Theorem as follows.

DIAGRAM 1-6.
PYTHAGOREAN THEOREM OF SOCIAL CONDUCT



From the economic point of view, there is no difference between stating that "John purchased x" and "John is guilty of purchasing x." The relationship between the act and the thought which motivates the act, speaking economically, is the same as that of the court considering such an act criminally.

DIAGRAM 1-8.
PYTHAGOREAN THEOREM - SOCIAL AND ECONOMIC ACTIVITY



Micro-economics: The "Chooser – Available Choice" Model

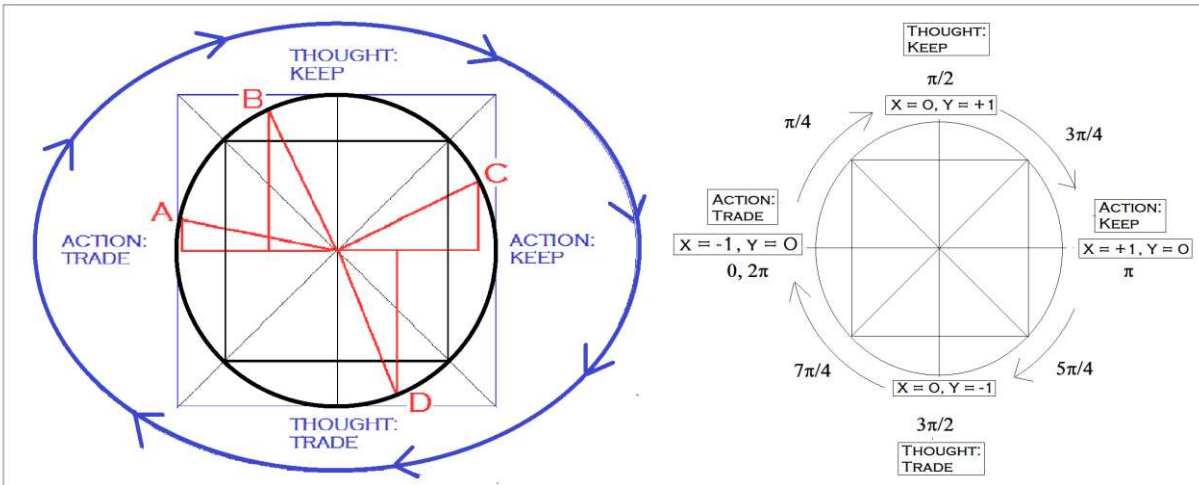
Each of the points within the plane of an indifference curve – both those on the curves and those outside the curve – represents a given decision to trade or to keep various properties. If we contrast the actions of *trading* a good versus *keeping* that same good, a set of dichotomies may be constructed which may be used to structure our understanding of economic development.

The first dichotomy – action, as comparable to the "actus reus" of criminal law – represents a tension between "Keeping" a particular good vs. "Trading" the good for something else. This is indicated in the circle below by the opposition of "Keep" at 3 o'clock and "Trade" at 9 o'clock. All economic life stems from the core principle that one may *act* freely in choosing either to keep a given property or to trade it for some other piece of property and that these transactions clearly affect the status of the property so owned or traded.

This is contrasted with a secondary dichotomy – thoughts, as comparable to the "mens rea" of criminal law – which represents a tension between one's mental "thoughts in favor of keeping" and "thoughts in favor of trading" a particular property, located at 12 o'clock and 6 o'clock respectively in the circle below. These are the mental pre-dispositions of every owner towards keeping or trading a given piece of property for something else.

Using the Pythagorean Theorem to structure the sum total of possible permutations between the "Action" aspect of a purchase, and the "Thought" aspect of a decision to Purchase, we may structure every possible balancing of these two with the "Purchase" itself.[17]

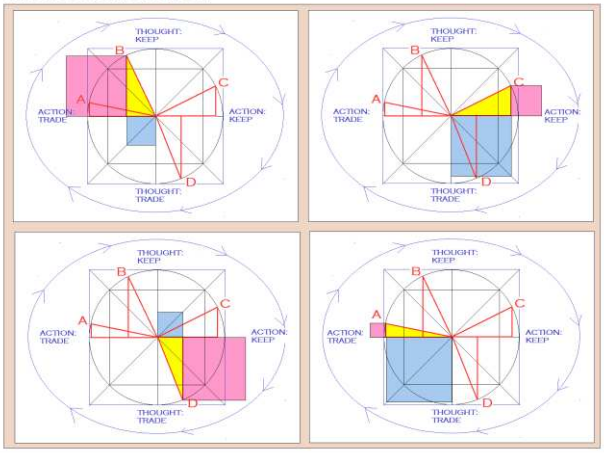
DIAGRAM 1-10.
TRIGONOMETRY OF TRADING VS. KEEPING PROPERTY



[17] The "clock-wise" direction of movement around the unit circle and the "9:00 o'clock" place of beginning the analysis as used in these essays are opposite that taken in most trigonometry textbooks. This approach does not alter the trigonometric identities considered in the slightest and provides an approach to the measurement of time which is consistent with the sense of the hands of a clock.

The Pythagorean relationships inherent in the association of Action and Thought as expressed previously create around the unit circle an infinite set of mathematic relationships wherein the actual possibility of a Purchase is set as the sum of some combination of Action and Thought.

DIAGRAM 1-11.
TRIGONOMETRIC RATIOS.



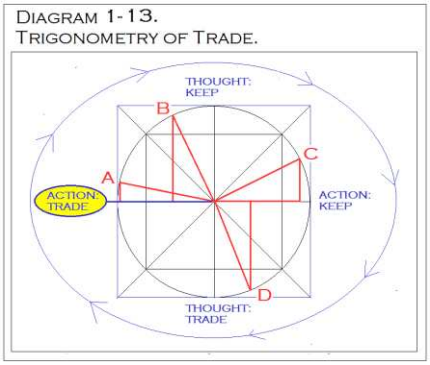
The unity of the underlying ego which selects these various points may be associated with the radius of this circle. If we give this radius the number "1" it represents the "unity" of the ego as a balancing radius between these two dichotomies of Action ("Trading" vs. Keeping") and Thoughts ("Thoughts related to Trading the property," "Thoughts related to Keeping the property"). An internal angle is thus constructed at the origin of the coordinate system.

The Significance of Trading

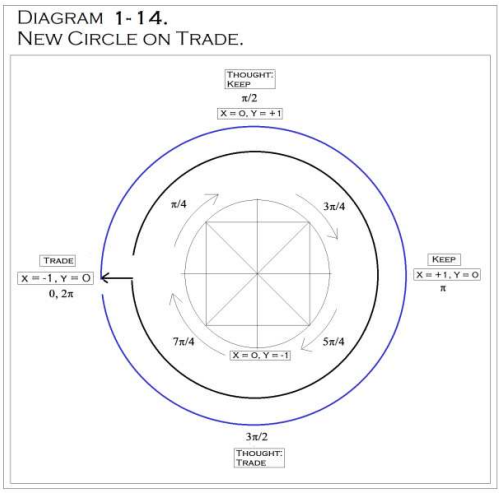
There is only one point along the Unit Circle where Action is wholly aligned with Trading, i.e. the point at 9:00. All other points along the unit circle are similar to one another in that there is some "Y" component connected to some mental aspect of trading and/ or keeping the object in question. This mental aspect must include some possibility of cancelling the action contemplated. Consequently only at 9:00 o'clock is the possibility of a "Trade" wholly equivalent with Action; at this point "Thought" is Zero and the Action "Trading" occurs.

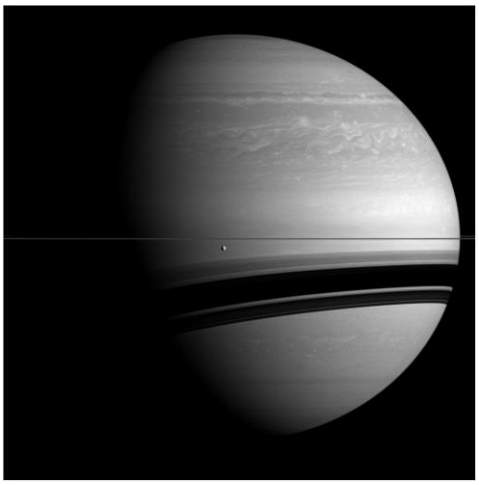
Conversely at 3:00 o'clock the Action undertaken is to "Keep" the property in question and the status quo is actively continued.

DIAGRAM 1-13.
TRIGONOMETRY OF TRADE.



The unique aspect of this point at 9:00 o'clock creates an unavoidable change in the overall unit circle. The break which is presented at (x = -1, y = 0) creates a new and unknown element in the unit circle itself. Once the trade is made, the situation is no longer the way it was. Something new has taken place.[18] In contrast, when the x-axis is directed toward "Keeping" a particular good, the point at which Thought = 0 will be in favor of the status quo.[19]





The model will be referred to as the "chooser – available choice" model, as a way of presenting the unit circle and its radius of "1" – representing the "chooser" – and the number π – representing the "choices available" – in a simple and direct fashion.

The photograph of Saturn, taken with the rings as a horizontal line, is intended to keep in mind the trigonometric similarity between these ideas.

In other words, one cannot simultaneously trade a good and keep the same good, or vice versa. The possible choices which *are* available toward any particular goal are those which are not directly undermining of whatever goal is chosen. The choices which are *not* available are those which are in some negative value, or opposite position, from this chosen goal. This same dynamic applies to any point of psychological consideration along the unit circle.

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[18] There is an analogy here to quantum mechanics in the "Schrodinger's Cat Thought Experiment." The second half of the third postulate of quantum mechanics states, roughly speaking, that observation changes the physical system. http://vergil.chemistry.gatech.edu/notes/quantrev/node20.html A physical system exists in as many state as possible until it is observed. Once the observation has been made, it changes into another state, one which can be unique or not.

Until one opens the box, the cat is both dead and alive. Opening the box (observing the state of the cat), indicates which state it is, and so changes the state of the physical system. In this essay, trading equates with the observation. By analogy, stating that with trade "something new has happened" one would indicate that the wave function describing the state of the cat has changed.

[19] As this relates to the use of indifference curves, at least in their original design by Pareto, see Lenfant 2012:119: "Pareto's own construction and discussion of indifference curves are developed in the Manual. ... Pareto (1900), 2008) already argued that indifference curves could be obtained through experiments or statistical studies. As long as statisticians have not established lines of indifference, 'for lack of more precise notion, the sciences possesses only some general data suggested by crude and everyday observations of facts.' ... So the final methodological position of Pareto is that the theoretical possibility of an empirical construction of indifference curves is at least enough for the foundation of the theory of choice. Eventually, when he comes to a precise description of indifference curves, Pareto appeals to "every day experience" and to introspection to discuss the shape of indifference curves."

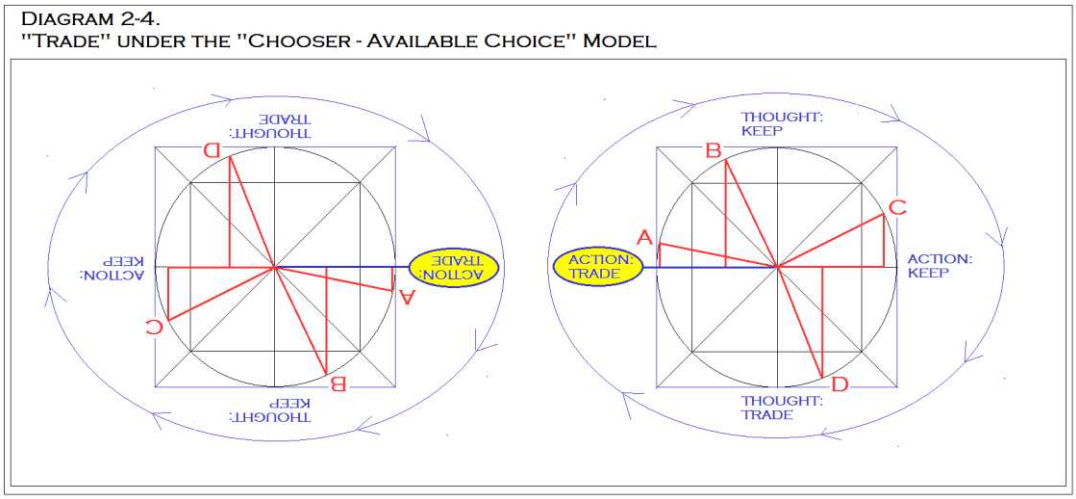
Macro-economics: The "Chooser - Available Choice" model in aggregate

The "chooser – available choice" model is the central point of departure for this model. If we invert this model such that the willingness to "trade" of one person meets the willingness to "trade" of a trading partner, we have a connection between two people indicating a mutual willingness to exchange goods or services with one another. (See discussion of Pareto efficiency *supra* and the inverted Edgeworth "box") The willingness and ability of persons to trade goods and their services with one another is the foundation for the entire economy.

Let us begin with a proposed willingness of Farmer Jones to part with two cows in return for three horses. This willingness is met by Farmer Smith who is willing to trade three specific horses which he owns in return for two specific cows belonging to Farmer Jones.

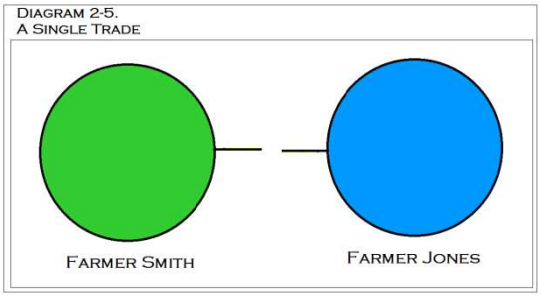
The fact that these two farmers have met with a match which in their minds is favorable to both is indicated by the fact that both have extended the 9:00 axis "Action : Trade" towards one another. As a result of this trade, Farmer Jones' two cows will be handed over to Farmer Smith, and Farmer Smith's three horses will be handed over to Farmer Jones.

DIAGRAM 2-4.
"TRADE" UNDER THE "CHOOSER - AVAILABLE CHOICE" MODEL



The following two circles simplify the basic ideas going into the above trade. Note that the early barter of horses for cows suggested by the circles below depicts trading at its most elementary level. Note that the trade itself must in some fashion state an improvement in the lives of the trading partners. Consequently the act of trading makes more efficient and useful the sum total of property within society because those who own the property are seeking ever more agreeable collections of that property by trading what they have for things which they desire but do not possess.

DIAGRAM 2-5.
A SINGLE TRADE

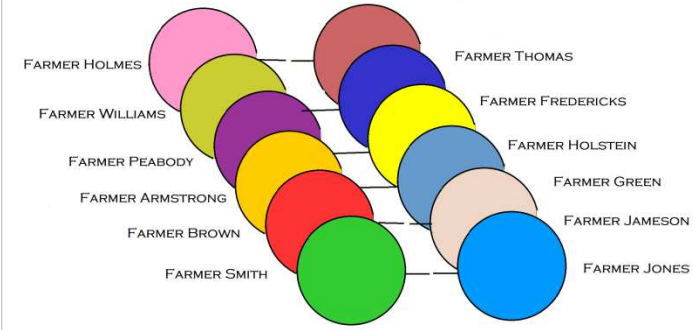


These trades represent a re-arrangement of property amongst those owning property. There is no "expansion" of the economy based upon this trade. However the usefulness of the property exchanged, in combination with the improved efficiency brought about by the trade, suggests that the natural rate of increase in any biologic organism – a farm, a household, a local market – will result from the full set of trades engaged in by all persons.

In short, the same property and the same traders exist after as well as before the trade. However the straight forward exchange of one set of property for another is conveyed by the model above.

There is no limit to the number of such trades which can be done over the course of any particular period of time. We may imagine two pipes running parallel, each suggesting the desire of one of two trading partners to enter into trade. Each trade may be listed in chronologic order and depicted as below.[20]

DIAGRAM 2-6.
THE STREAM OF TRADE



[20] The stream of trade considered in this paper is "Gross National Product" (GNP). This figure adds to Gross Domestic Product (GDP) the income receipts from the rest of the world minus payments to the rest of the world. The United States Bureau of Economic Analysis published the following table for these figures. (as taken from BEA 13-13, Table 9, http://www.bea.gov/newsreleases/national/gdp/2013/pdf/gdp4q12_3rd.pdf) Note that the difference between these is a multiple of (in billions of dollars) GNP = $16,130.8 / GDP = $15,864.1 = 1.016, or 1.6%, roughly $266 billion.

Table 9. Relation of Gross Domestic Product, Gross National Product, and National Income

[Billions of dollars]

Line		2010	2011	2012r	Seasonally adjusted at annual rates					Line
					2011	2012				
					IV	I	II	III	IVr	
1	**Gross domestic product**	**14,498.9**	**15,075.7**	**15,684.8**	**15,321.0**	**15,478.3**	**15,585.6**	**15,811.0**	**15,864.1**	1
2	Plus: Income receipts from the rest of the world	716.5	783.7	782.3	787.1	769.6	775.1	775.8	808.5	2
3	Less: Income payments to the rest of the world	507.2	531.8	539.3	523.1	554.7	527.8	532.7	541.8	3
4	**Equals: Gross national product**	**14,708.2**	**15,327.5**	**15,927.8**	**15,585.0**	**15,693.2**	**15,832.9**	**16,054.2**	**16,130.8**	4
5	Less: Consumption of fixed capital	1,873.4	1,936.8	2,011.7	1,966.6	1,984.9	2,004.8	2,019.8	2,037.4	5
6	Less: Statistical discrepancy	23.3	31.9	67.2	70.3	1.1	77.7	138.5	51.7	6
7	**Equals: National income**	**12,811.4**	**13,358.9**	**13,848.8**	**13,548.1**	**13,707.2**	**13,750.5**	**13,895.9**	**14,041.7**	7
8	Compensation of employees	7,970.0	8,295.2	8,565.8	8,340.1	8,495.7	8,527.7	8,577.6	8,662.1	8
9	Wage and salary accruals	6,404.6	6,661.3	6,880.7	6,692.4	6,825.9	6,849.2	6,888.5	6,959.3	9
10	Supplements to wages and salaries	1,565.4	1,633.9	1,685.1	1,647.7	1,669.8	1,678.5	1,689.1	1,702.8	10
11	Proprietors' income with inventory valuation and capital consumption adjustments	1,103.4	1,157.3	1,202.3	1,165.3	1,184.3	1,194.9	1,205.4	1,224.7	11
12	Rental income of persons with capital consumption adjustment	349.2	409.7	462.6	430.3	445.3	452.8	471.0	481.5	12
13	Corporate profits with inventory valuation and capital consumption adjustments	1,702.4	1,827.0	1,950.6	1,953.1	1,900.1	1,921.9	1,967.6	2,013.0	13
14	Net interest and miscellaneous payments	567.9	527.4	504.3	515.9	515.6	489.5	518.2	493.8	14
15	Taxes on production and imports less subsidies	998.0	1,036.2	1,069.2	1,047.1	1,067.7	1,069.8	1,067.8	1,071.3	15
16	Business current transfer payments (net)	140.0	132.6	128.0	127.4	130.5	127.9	123.8	129.7	16
17	Current surplus of government enterprises	–19.5	–26.5	–34.0	–31.1	–32.0	–34.1	–35.5	–34.5	17
	Addendum:									
18	Gross domestic income	14,475.6	15,043.8	15,617.5	15,250.7	15,477.1	15,507.9	15,672.6	15,812.5	18

r Revised

As reliable currency enters into circulation[21] persons engaged in trading have the further ability to makes trades of much greater complexity that a straight-forward barter. By saving the money obtained from prior trades people are able to amass a trading ability to trade which far exceeds the more clumsy and complicated trade of physical objects, herds of cattle or flocks of geese, etc.

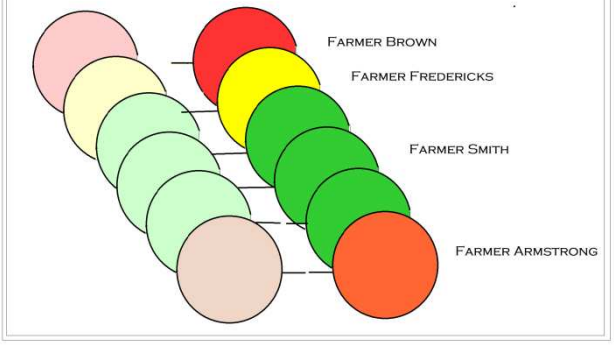
The ability to trade goods and services for currency permits the evaluation of the worth of the trade itself in relative terms visa vi all other trades, however subjective. A trade of $50 might represent an acre of land, a pair of mules, a suit of fine clothes or a suite of furniture. By "mirroring" the value of these various goods (or services), currency permits a much broader extent of trading and trading partners.

The pastel coloration below of the thing traded – money – is available to give a relative value to all the trades of an economy. These "trades" now become "sales," i.e. the surrender of something in return for currency.

The chronology of the trade is given be the difference in color, the red trade being first, the yellow being second, the green third, the orange fourth, etc. The pastel coloration indicates that in this case Farmer Jones did not trade goods for goods but rather money for goods (or services).

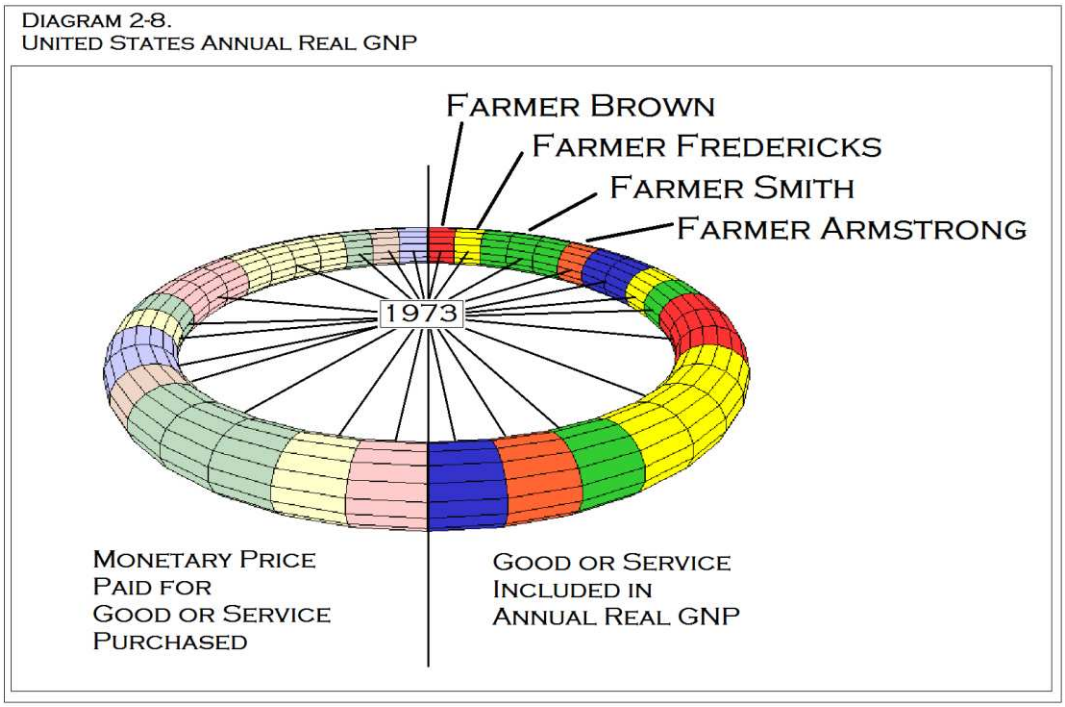
The size of the trade in question, its monetary value, is indicated by the number of circles used. For example Farmer Smith's trade of goods or services for money (three green circles) is three times as valuable in monetary terms as Farmer Brown's trade of goods and serves for money (one red circle), Farmer Frederick's trade of goods or services for money (one yellow circle) and Farmer Armstrong's trade of goods or services for money (one orange circle).

DIAGRAM 2-7.
CHRONOLOGIC SALES OF GOODS AND SERVICES



[21] See Penson and Webb (1981) on the importance of including capital into the determination of Okun's law. "The procedures used by the CEA (Council of Economic Advisors) assume that only the availability of labor and its productivity determine potential GNP. As Perry notes, however, 'it is hard to argue that capital should not be included in estimating potential output because everyone knows it belongs in the calculation.' Okun, in fact, also recognized that capital should be incorporated into the measurement of potential GNP when he stated 'I shall feel much more satisfied in the estimation of potential output when our data and our analysis have advanced to the point where ... the capital factor can be explicitly taken in into account.' ... All the procedures for estimating potential GNP, therefore either explicitly ignore the role of the current capital stock in the economy or implicitly assume the input shares for capital and labor are the same in each production sector of the economy. ... In measuring GNP at full employment, it is not enough to account only for the physical production process. One must also account for the changes in the relative prices of products and resources as the economy moves from current GNP to full employment GNP, and for the effects these price changes will have upon the economic decisions of producers and consumers'."

If we set an arbitrary division of the stream of trade at a single 365-day year, we can place the monetary and the "real" aspects of these sales of goods and services as oppositions antipodal from one another. The result is a circle of such sales. The length of half the circle indicates the monetary value of each of the sales of goods or services included in the year. If the size of these transactions is copied into the length of the circuit itself, we have the following. Because the connection of any particular sale of a good or service to the year "1973" is no greater than any other trade, we draw here a circle, i.e. that geometric construct in which all points in a plane lie equidistant from a single point. [22]



The development of currency and its association with trade given above suggests that the "work" necessary for Farmer Smith or Farmer Jones to possess "trade-able items" has now become the "employment" of Farmer Smith and Farmer Jones as engaged "sales" of these items in a money-based, capitalistic society. In this fashion the use of currency which has turned "trades" into "sales" is in a direct relationship to the rate of employment, i.e. that employment necessary to sustain the full scope of sales given above.

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22 The 2010 real GNP for the United States was $2.27 trillion dollars in 1958 dollars with a population in the same year of 308,745,538 residents, for a GNP per capita of $7,355 per resident in 1958 prices. (See Essay Three, Data Set One, for figures as to real GNP. See 2010 Census for population figures.)

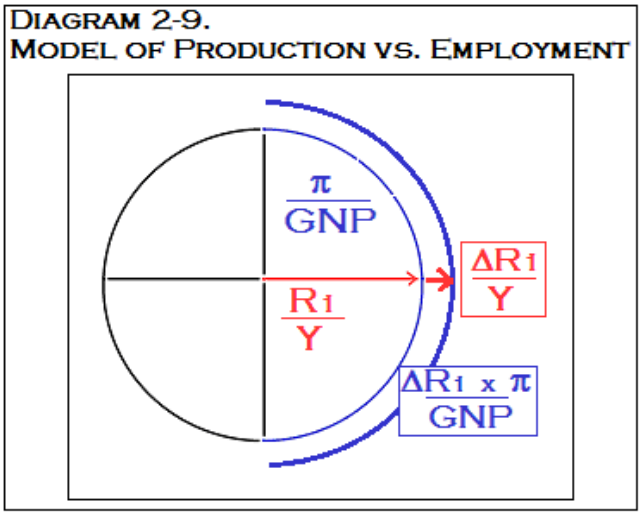
One might picture the relative size of these relationships by noting that if GNP per capita was set as the one inch radius of a pipe and the length of pipe set equal to U.S. real GNP, the pipe would run 406 miles (25,728,794 inches), roughly the distance from Chicago to Kansas City. To bend this pipe into the shape of half a circle would require a radius of 129 miles, roughly the distance from Washington D.C. to Philadelphia.

These proportions might be taken on a smaller scale. If a length of string representing 2010 real GNP was set equal to the length of a football field (3600 inches), the equivalent proportional thickness of the string would measure 0.00014 inches in a radial thickness. Spider silk measurements vary from 0.00012 to 0.00032 inches in diameter. The radius would run from the goal line to the 31.8 yard line.

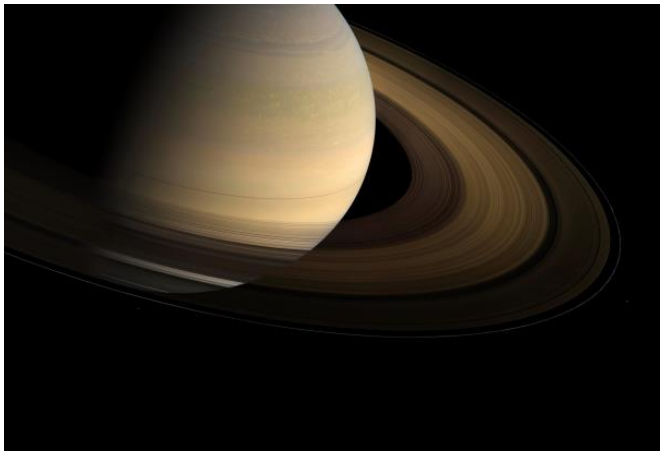
In the above diagram 2-8 we have used the GNP per capita of the United States as a radius "r" of the generating circle and the rate of unemployment as the radius "R" generating the torus which swings the smaller circle in an arc around the center point "1973."

If this relationship is stated geometrically, it would appear necessary that an increase in the rate of employment from one year to the next (R = the radius of the circle = 1) will correlate geometrically to a necessary increase in the size of GNP (Y = half circumference = π) at the necessary ratio of 1 : π, as follows.

DIAGRAM 2-9.
MODEL OF PRODUCTION VS. EMPLOYMENT



The above diagram is therefore the basis for an understanding of <u>why</u> Okun's Law works. The π:1 ratio (3.14159:1 ratio) given above between "Percent Change in real GNP" and "Percent Change in the Rate of Employment" is a trigonometric outcome of necessary and straightforward social realities of longstanding duration within the economic history of the United States.



Again the photograph of Saturn and its Rings is presented to keep in mind the trigonometric similarity between these structures.

Annual levels of GNP may be analyzed by way of spreadsheets wherein a current year is divided by a previous year By way of example, the real GNP of the United States for 2005 divided by that of 1995 represents the division of a numerator by a denominator both of which are stated in the billions of dollars, resulting in a quotient which is the final result of this simple mathematic operation. The term "ratio" suggests a proportion between these two numbers which, no matter how large, over time governs the general existence of the numbers themselves.

A typical Excel spread sheet with this data is as follows:

DIAGRAM 1:
SAMPLE SPREAD SHEET

12 YEAR RATIOS BASED ON ANNUAL REAL GNP; MULTIPLE 5.962552

		1		2		3		4		5		6		7		8		9		10		11		A	B	C	D	E	F	G
		YEAR	GNP	YEAR	GNP	YEAR	GNP	YEAR	GNP	YEAR	GNP	YEAR	GNP	YEAR	GNP	YEAR	GNP	YEAR	GNP	YEAR	GNP			Maximum Ratio of Row	Minimum Ratio of Row	Spread	Mid-Range Ratio of Row	Median Ratio of Row	Average Ratio of Row	Median Average
1	Year	1880	42.4000	1892	60.4000	1904	89.7000	1916	134.4000	1928	190.9000	1940	227.2000	1952	395.1000	1964	581.1000	1976	879.3138	1988	1303.1774	2002	1957.1959							
	14	1868	23.1000	1880	42.4000	1892	60.4000	1904	89.7000	1916	134.4000	1928	190.9000	1940	227.2000	1952	395.1000	1964	581.1000	1976	879.3138	1990	1351.3622							
	Ratio		1.8354978		1.4245283		1.4850991		1.4983278		1.4203869		1.19015191		1.7389964		1.4707669		1.51318844		1.4820396		1.448113329	1.835498	1.190152	0.645346	1.512825	1.482039	1.505898	1.493960
2	Year	1881	42.4000	1893	57.5000	1905	96.3000	1917	135.2000	1929	203.6000	1941	263.7000	1953	412.8000	1965	617.8000	1977	922.6690	1991	1360.3512	2005	2151.0247							
	14	1869	23.1000	1881	42.4000	1893	57.5000	1905	96.3000	1917	135.2000	1929	203.6000	1941	263.7000	1953	412.8000	1965	617.8000	1979	1001.7304	1993	1454.1409							
	Ratio		1.8354978		1.3561321		1.6747826		1.403946		1.5059172		1.29518664		1.56541524		1.49660827		1.49347523		1.358001315		1.479240904	1.835498	1.295187	0.540311	1.565342	1.493475	1.498436	1.495986
3	Year	1882	42.4000	1894	55.9000	1906	107.5000	1918	151.8000	1930	183.5000	1942	297.8000	1954	407.0000	1966	658.1000	1978	985.8821	1992	1418.0149	2006	2201.9891							
	14	1870	23.1000	1882	42.4000	1894	55.9000	1906	107.5000	1918	151.8000	1930	183.5000	1942	297.8000	1954	407.0000	1966	658.1000	1980	996.8309	1994	1514.3943							
	Ratio		1.8354978		1.3183962		1.9230769		1.412093		1.2088274		1.62288828		1.36668905		1.6169593317		1.49807139		1.422523018		1.45403948	1.923077	1.208827	0.714250	1.565952	1.422523	1.522502	1.472512
4	Year	1883	42.4000	1895	62.6000	1907	109.2000	1919	146.4000	1931	169.3000	1943	337.1000	1955	438.0000	1967	675.2000	1979	1001.7304	1993	1454.1409	2007	2272.2615							
	14	1871	23.1000	1883	42.4000	1895	62.6000	1907	109.2000	1919	146.4000	1931	169.3000	1943	337.1000	1955	438.0000	1967	675.2000	1981	1010.8394	1995	1546.7308							
	Ratio		1.8354978		1.4764151		1.7844089		1.3406593		1.1564208		1.99113995		1.2993171		1.561552511		1.48360545		1.438547904		1.469073675	1.991140	1.156421	0.834719	1.573780	1.476415	1.530759	1.503586
5	Year	1884	42.4000	1896	61.3000	1908	100.2000	1920	140.0000	1932	144.2000	1944	361.3000	1956	446.1000	1968	706.6000	1980	996.8309	1994	1514.3943	2008	2198.6295							
	14	1872	23.1000	1884	42.4000	1896	61.3000	1908	100.2000	1920	140.0000	1932	144.2000	1944	361.3000	1956	446.1000	1968	706.6000	1982	995.1411	1996	1615.0033							
	Ratio		1.8354978		1.4457547		1.6345886		1.3972056		1.03		2.5059478		1.2347081		1.583949787		1.41074283		1.521788518		1.361177214	2.505548	1.030000	1.475548	1.767778	1.445755	1.559978	1.502866
6	Year	1885	42.4000	1897	67.1000	1909	116.8000	1921	127.8000	1933	141.5000	1945	355.2000	1957	452.5000	1969	725.6000	1981	1010.8394	1995	1546.7308	2009	2208.7984							
	14	1873	23.1000	1885	42.4000	1897	67.1000	1909	116.8000	1921	127.8000	1933	141.5000	1945	355.2000	1957	452.5000	1969	725.6000	1983	1072.5727	1997	1681.8760							
	Ratio		1.8354978		1.5825472		1.7406855		1.0941781		1.1071987		2.51024735		1.27393018		1.603535912		1.39110837		1.442075488		1.31329444	2.510247	1.094178	1.416069	1.802213	1.442075	1.558300	1.500189
7	Year	1886	42.4000	1898	68.6000	1910	120.1000	1922	148.0000	1934	154.3000	1946	312.6000	1958	447.3000	1970	722.5000	1982	995.1411	1996	1615.0033	2010	2270.9907							
	14	1874	23.1000	1886	42.4000	1898	68.6000	1910	120.1000	1922	148.0000	1934	154.3000	1946	312.6000	1958	447.3000	1970	722.5000	1984	1129.4464	1998	1764.5370							
	Ratio		1.8354978		1.6179245		1.7507289		1.2323064		1.0425676		2.02592353		1.43090211		1.615247038		1.37735792		1.429906988		1.287017884	2.025926	1.042568	0.983358	1.534246	1.430902	1.535836	1.483369
8	Year	1887	42.4000	1899	74.8000	1911	123.2000	1923	165.9000	1935	169.5000	1947	309.9000	1959	475.9000	1971	751.2051	1983	1072.5727	1997	1681.8760									
	14	1875	23.1000	1887	42.4000	1899	74.8000	1911	123.2000	1923	165.9000	1935	169.5000	1947	309.9000	1959	475.9000	1971	751.2051	1985	1174.0716									
	Ratio		1.8354978		1.7641509		1.6470588		1.3465909		1.0216998		1.82831858		1.53565668		1.578493591		1.42780274		1.432515			1.835498	1.021700	0.813798	1.428599	1.535657	1.541779	1.538718
9	Year	1888	42.4000	1900	76.9000	1912	130.2000	1924	165.5000	1936	193.0000	1948	323.7000	1960	487.7000	1972	803.4814	1984	1129.4464	1998	1764.5370									
	14	1876	23.1000	1888	42.4000	1900	76.9000	1912	130.2000	1924	165.5000	1936	193.0000	1948	323.7000	1960	487.7000	1972	803.4814	1986	1203.2684									
	Ratio		1.8354978		1.8336792		1.6931079		1.2712214		1.1661631		1.6772020		1.50664159		1.647491081		1.40569879		1.46645337			1.835498	1.166163	0.669335	1.500830	1.506642	1.548309	1.527476
10	Year	1889	49.1000	1901	85.7000	1913	131.4000	1925	179.4000	1937	203.2000	1949	324.1000	1961	497.2000	1973	839.4182	1985	1174.0716	1999	1854.0672									
	14	1877	23.1000	1889	49.1000	1901	85.7000	1913	131.4000	1925	179.4000	1937	203.2000	1949	324.1000	1961	497.2000	1973	839.4182	1987	1256.1826									
	Ratio		2.1255411		1.7454175		1.5332555		1.3652968		1.1326644		1.59498031		1.53409442		1.688290829		1.39867303		1.475953576			2.125541	1.132664	0.992877	1.629103	1.533256	1.559417	1.546338
11	Year	1890	52.7000	1902	86.5000	1914	125.6000	1926	190.0000	1938	192.9000	1950	355.3000	1962	529.5000	1974	821.7401	1986	1203.2684	2000	1911.3209									
	14	1878	42.4000	1890	52.7000	1902	86.5000	1914	125.6000	1926	190.0000	1938	192.9000	1950	355.3000	1962	529.5000	1974	821.7401	1988	1303.1774									
	Ratio		1.2429245		1.6413662		1.4520231		1.5127389		1.0152632		1.84188695		1.4902899		1.551917052		1.46429315		1.466662098			1.841887	1.015263	0.826624	1.428575	1.490256	1.467937	1.479113
12	Year	1891	55.1000	1903	90.8000	1915	124.5000	1927	189.9000	1939	209.4000	1951	383.4000	1963	551.0000	1975	843.0778	1987	1256.1826	2001	1925.1794									
	14	1879	42.4000	1891	55.1000	1903	90.8000	1915	124.5000	1927	189.9000	1939	209.4000	1951	383.4000	1963	551.0000	1975	843.0778	1989	1340.0434									
	Ratio		1.2995283		1.6479129		1.3711454		1.5253012		1.1026856		1.83094556		1.43714137		1.530086751		1.48999606		1.436654514			1.830946	1.102688	0.728260	1.466818	1.489996	1.467140	1.478568
A	Maximum Ratio of Column		2.125541		1.813679		1.923077		1.525301		1.505917		2.510247		1.738996		1.688291		1.513188		1.521785									
B	Minimum Ratio of Column		1.242925		1.318396		1.371145		1.094178		1.015263		1.190152		1.234708		1.470767		1.377358		1.358001			Max. of F - Rows	Min. of F - Rows	Mid-Range of F - Rows	Median of F - Rows	Avg. of F - Rows	Mid-Range + Average/2	Median + Average/2
C	Spread		0.882617		0.495283		0.551932		0.431123		0.490654		1.320093		0.504288		0.217524		0.135831		0.163787			2.510247	1.015263	1.762755	1.533296	1.524695	1.643725	1.528996
D	Range Ratio of Column		1.684233		1.566038		1.647111		1.309740		1.260590		1.850200		1.486852		1.579529		1.445273		1.439895									
E	Median Ratio of Column		1.835498		1.582547		1.647059		1.397206		1.132684		1.828319		1.490290		1.578494		1.464293		1.442075			Max. of F - Columns	Min. of F - Columns	Mid-Range of F - Columns	Median of F - Columns	Avg. of F - Columns	Mid-Range + Average/2	Median + Average/2
F	Average Ratio of Column		1.765623		1.569519		1.637496		1.366647		1.159150		1.826202		1.451149		1.577079		1.446334		1.447760			2.510247	1.015263	1.762755	1.510334	1.524695	1.643725	1.517515
G	Median Average		1.800560		1.576033		1.642278		1.381926		1.145907		1.827260		1.470719		1.577784		1.455314		1.444918									

By way of example let us consider Column Four Row One of the 12 year spread. (See Diagram 1, Sample Spread Sheet.) This GNP ratio is 1916 / 1904, representing a spread of 12 years between the numerator and the denominator of the ratio. The US real GNP values for this fraction are 134.4 / 89.7 with a result of 1.49833. This ratio is placed in Column Four Row One in the 12-year spread spreadsheet.

The next ratio in the series, 1917 / 1905, or 135.2 / 96.3, gives the result of 1.40395. This is placed in Column Four Row Two of the 12-year spread spreadsheet.

This continues on for a period of 12 years, i.e. from 1916 through 1927. The final fraction in Column Four Row Twelve is 1927/1915, or 189.9 / 124.5, for a result of 1.5253. This result is placed in Column Four Row Twelve and the series continues on to the next column.

The next column, Column Five, begins in Row One with the ratio 1928 / 1916, for a ratio of 190.9 / 134.4 and a result of 1.42039. This is placed in Column Five Row One and the process continues. Notice that the numerator of the cell in Column Four Row One ("1916 = 134.4") becomes the denominator of the cell immediately to the right, Column Five Row One.

An Excel spread sheet may be generated for any given spread of years using "Data Base 2 – U.S. Real GNP" as its foundation.

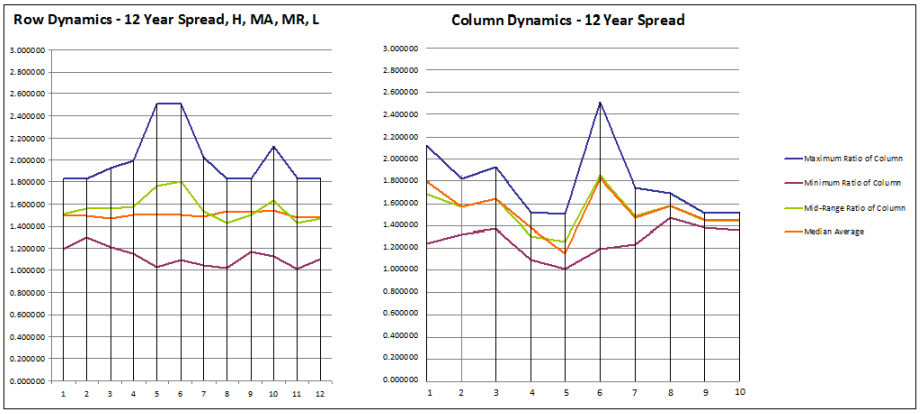
For every Row and for every Column in every spread sheet there exists a High Ratio and a Low Ratio. For example, in the Columns and Rows mentioned previously regarding the 12-year spread, we have the following:

12-year Spread,	High		
Row One	1880/1868	= 42.4/23.1	= 1.8354978
Row Two	1881/1869	= 42.4/23.1	= 1.8354978
Row Twelve	1951/1939	= 383.4/209.4	= 1.8309455
Column Four	1927/1915	= 189.9/124.5	= 1.5253012
Column Five	1928/1916	= 190.9/134.3	= 1.4203869
12-year Spread,	Low		
Row One	1940/1928	= 227.2/190.9	= 1.1901519
Row Two	1941/1929	= 263.7/203.6	= 1.2951866
Row Twelve	1939/1927	= 209.4/189.9	= 1.1026856
Column Four	1921/1909	= 127.8/116.8	= 1.0941781
Column Five	1938/1926	= 192.9/190.0	= 1.0152632

We noticed that High Averages represent ratios which contrast a very dynamic year of growth in the numerator with a previous year of very slow or depressed growth in the denominator. Conversely Low Averages contrast a year of slow or depressed growth in the numerator with a previous year of growth in the denominator.

The full range of these contrasts is as follows as to the 12-year spread.

DIAGRAM 2.
ROW AND COLUMN DYNAMICS FOR 12-YEAR SPREAD



From the above charts it becomes clear that these spread sheets are characterized by "Row Dynamics" and "Column Dynamics." From these dynamics we have calculated four additional points within both the Rows and the Columns of all spreadsheets. These are:

The "Mid-Range." The mid-range is the mid-point lying between the high and low ratios in the sample, i.e. the average of the highest and lowest numbers in the set: "(H + L) / 2".

The "Average" or "Arithmetic Mean." The sample mean is the sum of all the observations divided by the number of observations.

The "Median." The median is that number for which half the data is larger than it, and half the data is smaller. It is also called the 50th percentile. If the data has an odd number of members, the median will be the number in the center of these members; if an even number of members, the median will be the mid-point between the two numbers closest to the center.

The "Median Average." The Median Average is the mid-point between the Median and the Average (Arithmetic Mean). It is figured as: "(Median + Average) / 2" and is the approximation used throughout this paper – in conjunction with the Midrange – as the best estimate of the dynamics within Rows and Columns.

We then compared the High, Midrange, Median Average and Low of Row Dynamics for each Excel spread sheet. The following points are made as to this approach.

1) In every Row there exists a Highest Average of the possible averages in the Row. This Highest Average represents the greatest margin of growth over decline for the time period of that spread for that Row. Conversely the Lowest Average represents the greatest depth of decline over growth for the time period of the spread for that Row.

2) We noted that the Midrange between the Highest Average and the Lowest Average is simply the arithmetic division of the distance between these two. It lies half-way between them in any given row. The Midrange represents the arbitrary balance between these two extremes for that Row in any given spread of years. The Midrange is completely independent of, and unconnected to, the Median Average of the Row, other than the fact that they both include the Highest Average and the Lowest Average in their calculus.

3) The Median Average states the accumulated "weight" of all the ratios in the row. It is unconnected to the Highest Average and the Lowest Average other than it includes both of them as a part of its calculation. It is completely independent of, and unconnected to, the Midrange value and does not take it directly into account in its calculus.

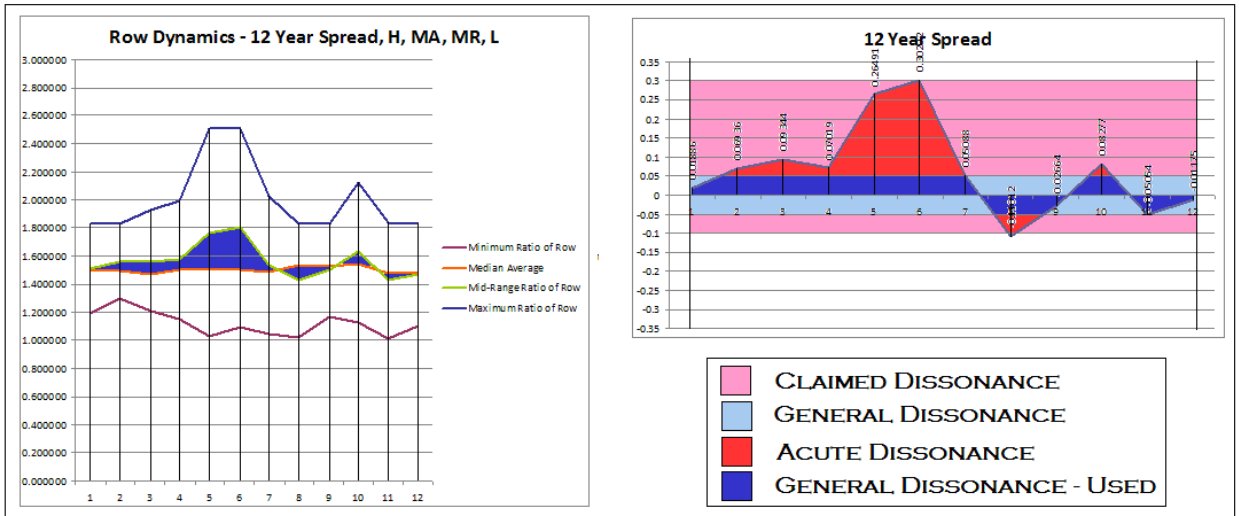
4) When a particular spread of years generates Rows which contain Midrange values and the Median Average values which are quite close to one another, the spread has established a relationship between the most basic ratios of the economy which is balanced and uniform. In the context of our search herein, we use the term "harmonic" to indicate this balance.

5) When a particular spread of years generates Rows which contain Midrange values and Median Average values which are at relatively great distances from one another, the spread has failed to establish a relationship between these basic ratios of the economy. By comparison to the other spreads, the particular spread in question is relatively unbalanced and not uniform. In the context of our search herein, we use the term "dissonant" to indicate this discord, turbulence or lack of harmony.

6) The implication is that when a given spread of years generates Midrange and Median Average values which are proximate to one another and therefore "harmonious" or "balanced," some underlying pattern or overriding logic may be at work to create this harmony as opposed to a random and disconnected set of processes and their resulting discordant and dissonant variables.

Diagram 2, left side, presents the Row Dynamics for the 12-year spread shown in Diagram 1. The x-axis indicates the row of the spreadsheet under consideration. The y-axis represents the figure presented by that row as its High, Low, Midrange or Median Average ratio.

DIAGRAM 3.
DISSONANCE BETWEEN MIDRANGE AND MEDIAN AVERAGE



1. MIDRANGE MINUS MEDIAN AVERAGE = AMOUNT OF DISSONANCE
2. DISSONANCE OUTSIDE +0.05 = "ACUTE DISSONANCE" (POSITIVE)
3. DISSONANCE OUTISDE -0.05 = "ACUTE DISSONANCE" (NEGATIVE)

Diagram 2, right side, presents the graph of the

x-axis = Row of the Spread
y axis = Midrange minus Median Average

When the Median Average is greater than the Midrange, the score is negative; when the Median Average is less than the Midrange, the score is positive. The number along the x-axis again indicates the row of the spread sheet under consideration. The number along the y-axis represents an amount of difference between Midrange and Median Average as found in that row.

The effort to compare systematically the common characteristics of different spreads led us to invent four new terms. Referring to Diagram 2 above these are:

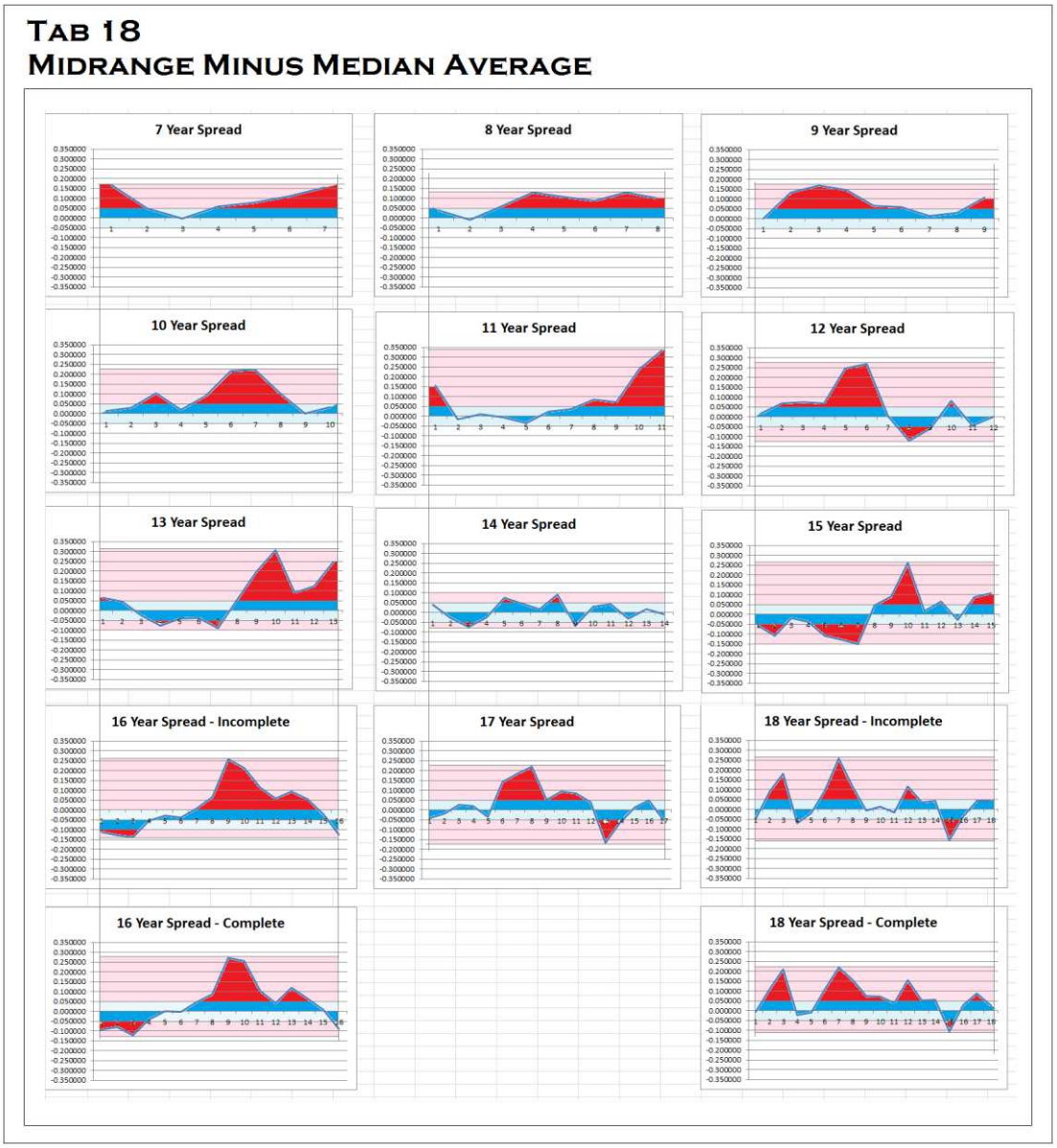
"General Dissonance." The pale blue area running as a ribbon from left to right represents the notion of a "General Dissonance," i.e. an arbitrary, acceptable distance between Median-Average and Midpoint. When a row possesses a Midrange and a Median Average which are in close proximity to one another, the distance between them will be found within the space designated by pale blue, "General Dissonance." After reviewing all spreads of years, this number has been set at +/- 0.05 in as much as it appears applicable to all spreads of years as general field of activity.

"Used General Dissonance." The amount of dark blue is termed "Used General Dissonance," i.e. that portion of "General Dissonance" which is actually used by the given row in stating the distance between the Midrange and the Median Average, either as a positive or negative amount surrounding $y = 0$.

"Acute Dissonance." The portion in red represents an "Acute Dissonance." When the distance between Midrange and Median Average falls outside the arbitrarily stated "General Dissonance" the excess is given in red shading. If the distance between the Midrange and the Median Average of a row is great, the "Acute Dissonance" so stated will be signified by large areas of red shading. Lesser amounts of "Acute Dissonance" generate less red shading.

"Claimed Dissonance." The pink portion running as a ribbon from left to right is "Claimed Dissonance," i.e. that volume of spread between the high point of "Acute Dissonance" and the low point of "Acute Dissonance." This is the range of values necessary to accommodate the entire spectrum of variation between these two extreme points.

In the chart below the number of years in the spread is equalized by stretching the horizontal frame so that all spreads between a 7-year and an 18-year spread take up the same total horizontal space. This balances large spreads (large number of rows, relatively few columns) with the smaller spreads (small number of rows, large number of columns).



One may notice above that some spreads have distinctly lower profiles as to claimed dissonance than the other spreads. We examined this finding in more detail by comparing the numbers generated by these different spreads and associating them with one another in a more systematic way.

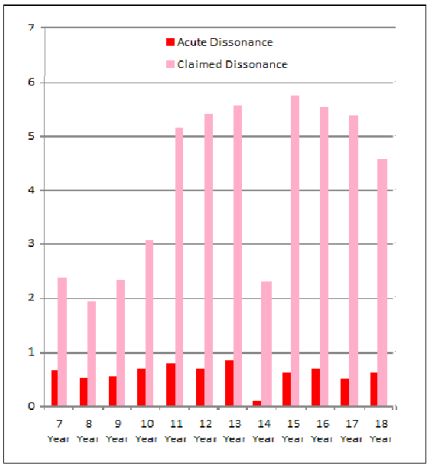
Each value given as the sum or difference for equation "Midrange Minus Median Average" may be divided into two parts, i.e. positive and negative values. These parts are further sub-divided by those values for this number which fall close to the y = 0 axis and inside the range of +/- 0.05. This range is referred to as "General Dissonance." Values which fall outside this range are referred to as "Acute Dissonance."

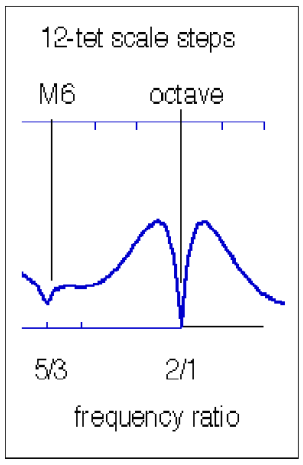
"Claimed Dissonance" locates the High and the Low extremes of the "Midrange Minus Median Average" for a given Row. Once we locate the point at which the Midrange most exceeds the Median Average (High), and the point at which the Midrange is most exceeded by the Median Average (Low), we may draw the y-axis distance between these two extremes (column 13). This is then taken as the boundary of a pink ribbon denoting "Claimed Dissonance" against the y-axis for the entire spread.

"Claimed Dissonance" is a measurement of the extent to which any given spread of years generates turbulence and discord between the Midrange and the Median Average. Like harmonies with discord between them, a high value for Claimed Dissonance indicates that the GNP ratio in question would not function well as a fundamental building block for an economic system, whereas low values for Claimed Dissonance provide the underlying balance necessary.

As demonstrated below, a remarkable and unexpected result occurs when a ratio of real GNP possesses a numerator and the denominator separated by 14 years. At this span of time, the level of Acute Dissonance is the least of all ratios (0.10682793) and the level of Claimed Dissonance is second-to-least (2.32355220). In addition, the spreads of three years before (11, 12, 13) and after (15, 16, 17) the 14-year spread generate the greatest amount of Claimed Dissonance, more than double that of the 14-year spread.

Diagram 4b. Taken from the Appendix: Tab 19b.
"Acute Dissonance" and "Claimed Dissonance" with Musical Octave





- RED	+ RED		Acute Dissonance	Claimed Dissonance
0.00000000	0.64710419	7 Year	0.64710419	2.38636208
0.00000000	0.54461218	8 Year	0.54461218	1.95758914
0.00000000	0.58384718	9 Year	0.58384718	2.36041529
0.00000000	0.68095806	10 Year	0.68095806	3.08490106
0.00000000	0.80581411	11 Year	0.80581411	5.16578384
-0.09646376	0.59136602	12 Year	0.68782978	5.42828771
-0.07244166	0.78931276	13 Year	0.86175442	5.55129001
-0.04090523	0.06592270	14 Year	0.10682793	2.32355220
-0.28312582	0.33837347	15 Year	0.62149929	5.77038878
-0.16258156	0.52695711	16 Year	0.68953867	5.54327070
-0.10784916	0.39431580	17 Year	0.50216496	5.40060912
-0.04361834	0.58559699	18 Year	0.62921532	4.58001012

The suggestion is that just as an octave[23] is created by the equal division of a vibrating string into two harmonic parts, and just as a slight variation from this even division between the perfect center of the vibrating string results in intolerable out-of-tune sense of dis-harmony, so does the use of a 14-year interval between years when measuring GNP values result in great sympathy and proximity between Midrange and Median Average values for the entire economy, unlike every other spread of years. And also like the vibrating string, the most out-of-tune dissonance occurs immediately surrounding the perfect division of the string, while tapering off as one takes distances further from the center.

This "piling on" of Claimed Dissonance immediately before and after the 14-year spread is the origin of our selection of the term "dissonant," i.e. the sense that at the 14-year spread an almost acoustic "octave" is sounded against an underlying reality.

The similarity of "Claimed Dissonance" to the "octave" of musical relationships will be central to the remainder of these papers. The technique and spreadsheets used to obtain this graph are presented at length in the Appendix.

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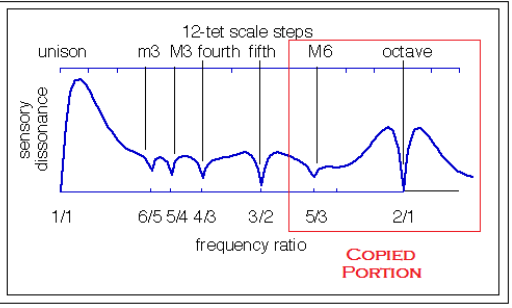
[23] See e.g. William Sethares, Relating Tuning and Timbre, *Experimental Musical Instruments*: "To explain perceptions of musical intervals, Plomp and Levelt note that most traditional musical tones have a spectrum consisting of a root or fundamental frequency, and a series of sine wave partials that occur at integer multiples of the fundamental. Figure 2 depicts one such timbre. If this timbre is sounded at various intervals, the dissonance of the intervals can be calculated by adding up all of the dissonances between all pairs of partials. Carrying out this calculation for a range of intervals leads to the dissonance curve. For example, the dissonance curve formed by the timbre of figure 2 is shown below in figure 3.

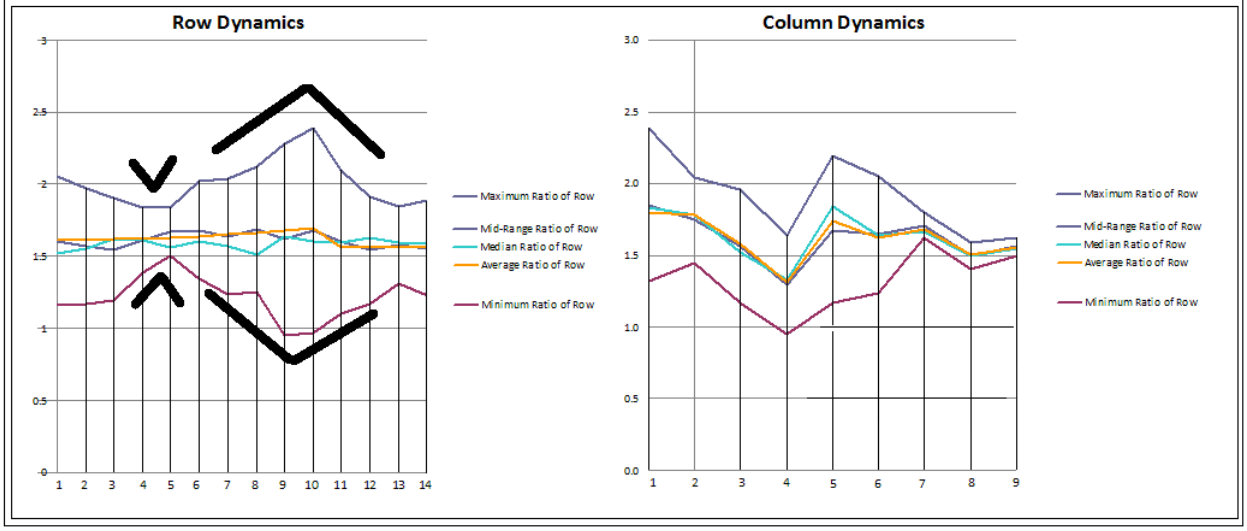
DIAGRAM 3.12. MUSICAL DISSONANCE AND HARMONY



Observe that this curve contains major dips at many of the intervals of the 12 tone equal tempered scale. The most consonant interval is the unison, followed closely by the octave. Next is the fifth, followed by the fourth, the major third, the major sixth, and the minor third. These agree with standard musical usage and experience. Looking at the data more closely shows that the minima do not occur at exactly the scale steps of the 12 tone equal tempered scale. Rather, they occur at the "nearby" simple ratios 1:1, 2:1, 3:2, 4:3, 5:4, and 5:3 respectively, which are exactly the locations of notes in the "justly intoned" scales (see Wilkinson). Thus an argument based on tonal consonance is consistent with the use of just intonation (scales based on intervals with simple integer ratios), at least for harmonic timbres."

When the measurement of the economy takes into account the underlying biology of the economy, a picture of American economy history may be developed which is in accord with both the biology of the individual member as well as the larger and encompassing biology of the economy.[24]

DIAGRAM 3-23.
ROW AND COLUMN DYNAMICS: 14-YEAR SPREAD



[24] The significance of a 14-year spread between years as a defining characteristic of the American economy finds at least tentative support in spectral analysis. See e.g. Korotayev and Tsirel, 2007:10. Note that in both charts provided, the 14-year span is the most significant point of balance between the two charts, no matter how adjusted. (as taken from)

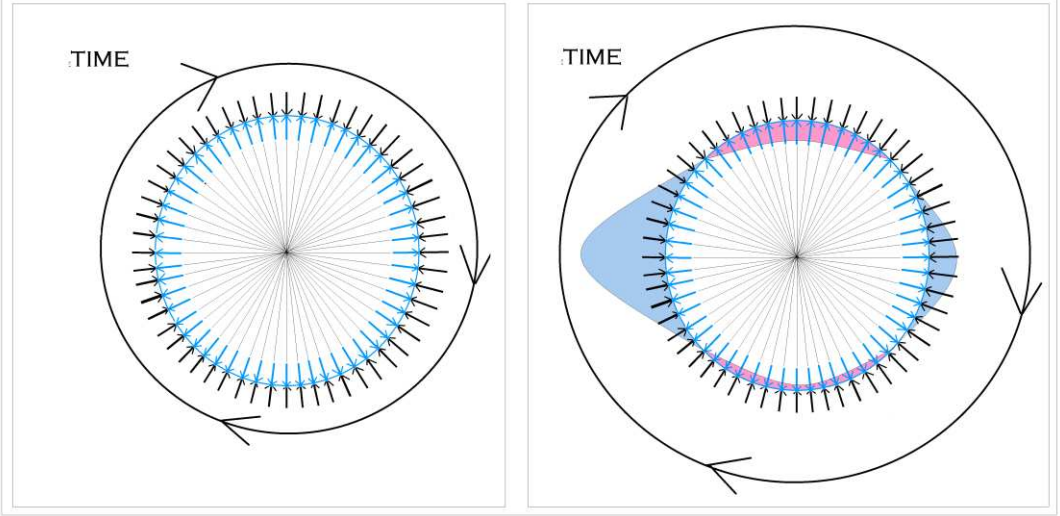
Evaluate Period of Long Wave

Having established that a 14-year sub-period may be important in the evaluation of the Kondratiev wave, we examined the price indexes for the United States between 1800 and 1994. The figures from "Data Set 1 – Prices" are stated below (1) in 7-year running averages (red line, top graph, semi-logarithmic scale), and (2) the change between a given year's seven-year average as divided by the average itself (blue line, bottom graph). The lower graph permits us to see the increasingly large inflationary price index values of later years (post-1966) as placed in a more consistent relationship with the preceding values of the series.

We noted that a 56 year period (14 x 4 = 56) between peaks at 1861 through 1917 suggests the possibility that similar periods of time might connect other peak points of inflation. If a 14-year span (blue rectangles above) is drawn around the years 1805, 1861, 1917 and 1973 (each of which is separated by periods of 56 years), virtually all inflationary peaks are contained in a single model.

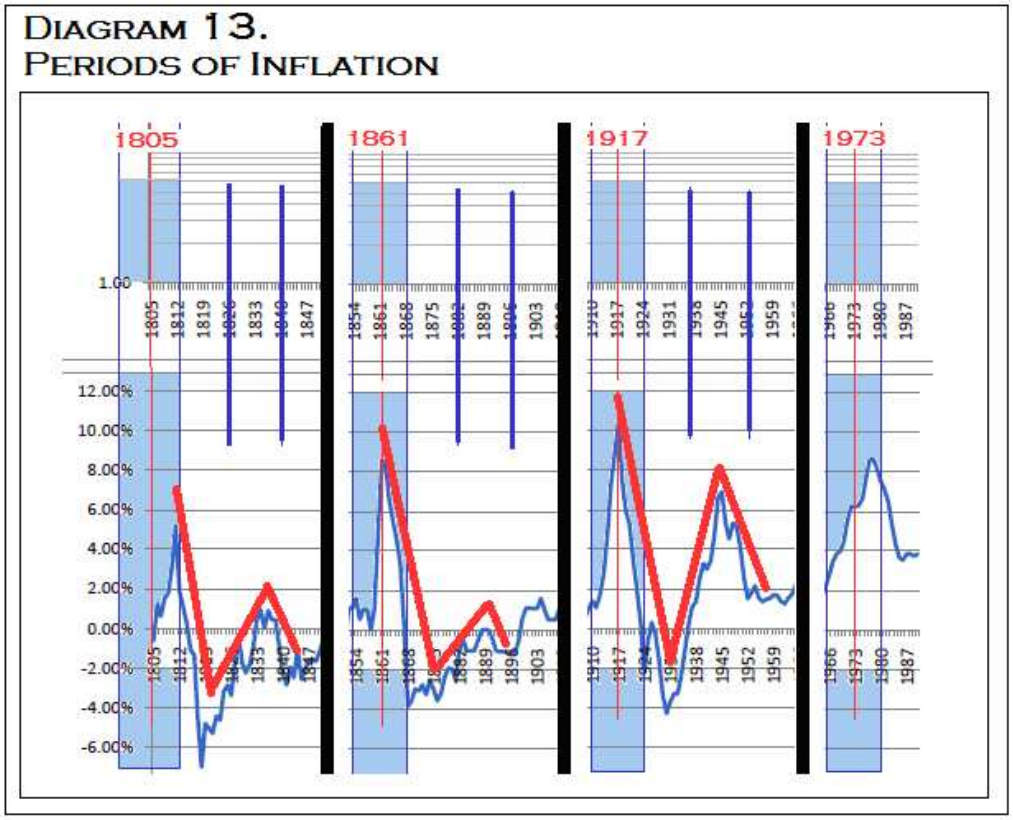
As this relates to the productive capacity represented by US real GNP, if we divide a circle into 56-year rays, all things being equal, as the arrows of production move outward to meet the expectation of GNP per year (arrows of radii moving out from the center of the circle) this production should be met by uniform resistances (arrows moving toward the center of the circle) which balance the natural increase of production exactly.

DIAGRAM 11.
STEADY STATE RATE OF GROWTH VS. WAVES OF GROWTH



However if a particular period of time fails to offer uniform resistance to production, or if the strength of production for some reason is particularly strong, the inherent productivity of the citizenry will create a bulge in productivity which must then be balanced out by a depression at some other time in the course of the circuit. Only in this fashion can a constant of growth be maintained in the face of unequal strengths of production and resistance to production. A wave must then develop over time during which this bulge will even out as time goes on until the next unexpected opportunity for unusual productivity.

The damping wave has been noticed three times in the course of American economic history in consideration of prices.



Regarding the above chart, and as mentioned at the beginning of the paper, we concern ourselves here exclusively with the United States and the discovery of strong evidence that a Kondratiev Wave appears to have significant impact upon the US economy. A long-standing issue regarding Kondratiev Waves is the causation of the wave itself. This debate centers largely upon the "exogenous" vs. "endogenous" nature of the cycle. (see footnotes 6, 7 and 11)

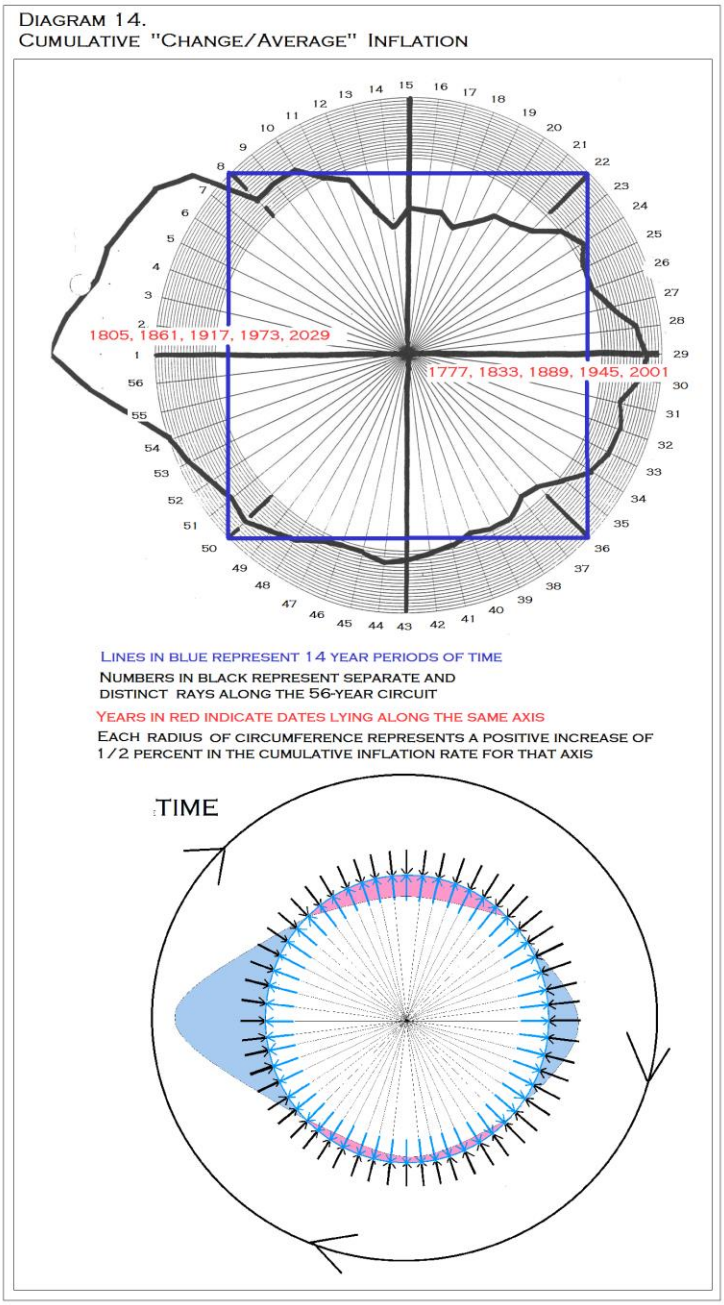
From the "exogenous" point of view, it is difficult to understand how events which occur with an apparently chaotic randomness outside the United States can affect the American economy with dependable regularity.

From the "endogenous" point of view, although a form of biologic regularity might be granted to the American economy, it remains difficult to explain how such internal developments might affect with the same regularity international events over which the United States has no control whatsoever.

There can be no question that political events in Europe and throughout the world have had much to do with the inauguration of these cycles. Nor can there be serious question that the relationship between the economic development of the United States and that of Europe must be explored. The problem appears to be that two distinct yet interacting levels of economic life must be considered, one national (American) and one European. These concerns are dealt with in our separate paper entitled "*On Revolution and the Cultural Development of Europe: Toward a European "System of Movement* ." (unpublished at this time)

The circumference of each circle represents a positive increase in the cumulative change/average figure of 1/2 percent (for example, a change/average cumulative amount of 1805 + 1861 + 1917 + 1973 lying directly at 9 o'clock). Points found within the interior of the smallest circumference represent negative figures by a comparable amount.

The blue square below represents the four 14-year segments of time set forth in Diagrams 10 and 13. The blue rectangles (previously given) are represented by the vertical left line segment (below). Taken together 4 x 14 periods of time create the 56 year circuit of time of this model. Note that the Great Depression of 1929-1940 is part of the deep indentation between axis 8 and 22, i.e. at the top horizontal of the blue square and interior to the smallest radii.

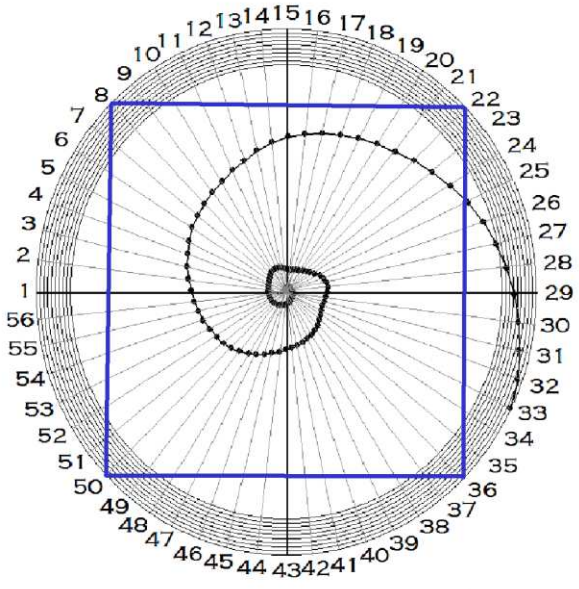


The fundamental average of the set

We then placed the U.S. real GNP figures given in "Data Set 2 – U.S. Real GNP" in a 56 year circuit, with the four 14-year quarter cycles indicated in blue, to create the spiral below. The center of the spiral, beginning at axis 9 = 1869, represents the real Gross National Product for that year of 23.10 billion dollars in 1958 prices. The Gross National Product for subsequent years in real terms are given along each axis respectively, with each circle of circumference representing ten billion dollars of real GNP in 1958 prices. Each row of the 14-year spreadsheet is represented by a "cross" within the spiral, beginning with Row 1 at the diagonal of the square, and moving to Row 8 at the horizontal and vertical axes of the square. The ratios of the spread sheet are simply the relative distances from the center of different points along the spiral as they relate to other points along the cross within the spiral.

DIAGRAM 15. THE "GNP SPIRAL"

	Date	Date	Date	Date	Date	Date
14	1762	1818	1874	1930	1986	
13	1761	1817	1873	1929	1985	
12	1760	1816	1872	1928	1984	
11	1769	1815	1871	1927	1983	
10	1768	1814	1870	1926	1982	
9	1767	1813	1869	1925	1981	
8	1766	1812	1868	1924	1980	2036
7	1765	1811	1867	1923	1979	2035
6	1764	1810	1866	1922	1978	2034
5	1763	1809	1865	1921	1977	2033
4	1762	1808	1864	1920	1976	2032
3	1761	1807	1863	1919	1975	2031
2	1760	1806	1862	1918	1974	2030
1	1759	1805	1861	1917	1973	2029
56		1804	1860	1916	1972	2028
55		1803	1859	1915	1971	2027
54		1802	1858	1914	1970	2026
53		1801	1857	1913	1969	2025
52		1800	1856	1912	1968	2024
51		1799	1855	1911	1967	2023
50		1798	1854	1910	1966	2022
49		1797	1853	1909	1965	2021
48		1796	1852	1908	1964	2020
47		1795	1851	1907	1963	2019
46		1794	1850	1906	1962	2018
45		1793	1849	1905	1961	2017
44		1792	1848	1904	1960	2016
43		1791	1847	1903	1959	2015



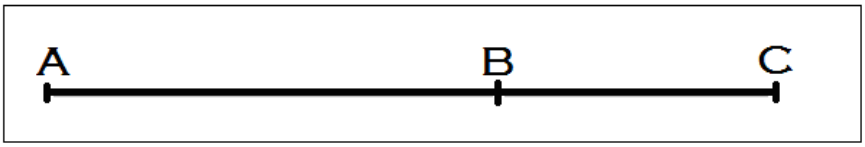
	Date	Date	Date	Date	Date
15	1763	1819	1875	1931	1987
16	1764	1820	1876	1932	1988
17	1765	1821	1877	1933	1989
18	1766	1822	1878	1934	1990
19	1767	1823	1879	1935	1991
20	1768	1824	1880	1936	1992
21	1769	1825	1881	1937	1993
22	1770	1826	1882	1938	1994
23	1771	1827	1883	1939	1995
24	1772	1828	1884	1940	1996
25	1773	1829	1885	1941	1997
26	1774	1830	1886	1942	1998
27	1775	1831	1887	1943	1999
28	1776	1832	1888	1944	2000
29	1777	1833	1889	1945	2001
30	1778	1834	1890	1946	2002
31	1779	1835	1891	1947	2003
32	1780	1856	1892	1948	2004
33	1781	1837	1893	1949	2005
34	1782	1838	1894	1950	2006
35	1783	1839	1895	1951	2007
36	1784	1840	1896	1952	2008
37	1785	1841	1897	1953	2009
38	1786	1842	1898	1954	2010
39	1787	1843	1899	1955	2011
40	1788	1844	1900	1956	2012
41	1789	1845	1901	1957	2013
42	1790	1846	1902	1958	2014

As can be seen from the following enlargement of the 14-year spreadsheet, we then:

(1) figured the average for each row of the spreadsheet for a total of 14 averages (Column F),
(2) figured the Median (1.617735) and Average (1.619446) of Column F, and
(3) figured a final Median Average for the entire spreadsheet of 1.618590.

In all spreadsheets this set of calculations is termed a "circle analysis." This nomenclature refers to the arrangement of Row Averages as points along the circumference of a circle, each one counted equally and but once toward a final Median Average of the spreadsheet.

This number 1.618590, the final Median Average of rows[25], is 0.034% greater than the constant phi, 1.6180339… This constant, sometimes referred to as "the Golden Mean," "the Golden Ratio" or "the Golden Section," was defined circa 300 b.c. by Euclid of Alexandria, as follows:



> A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the lesser.[26], [27]

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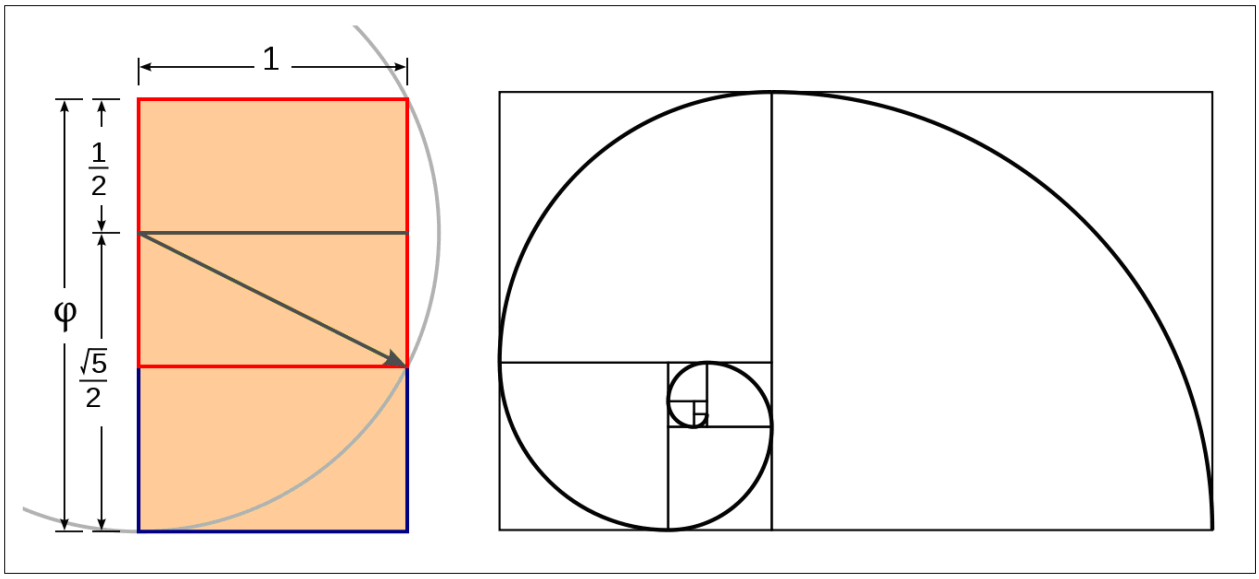
[25] As mentioned in the text, a "circle analysis" counts each average of rows (column F) a single time toward a final Median Average for the entire spreadsheet. A "square analysis" counts the first row twice, and arrives at a slightly different number, one which is 0.0053% in proximity to the Golden Mean. A further discussion of the rationales underlying "circle analysis" and "square analysis" is placed in the Second Post-script to this article.

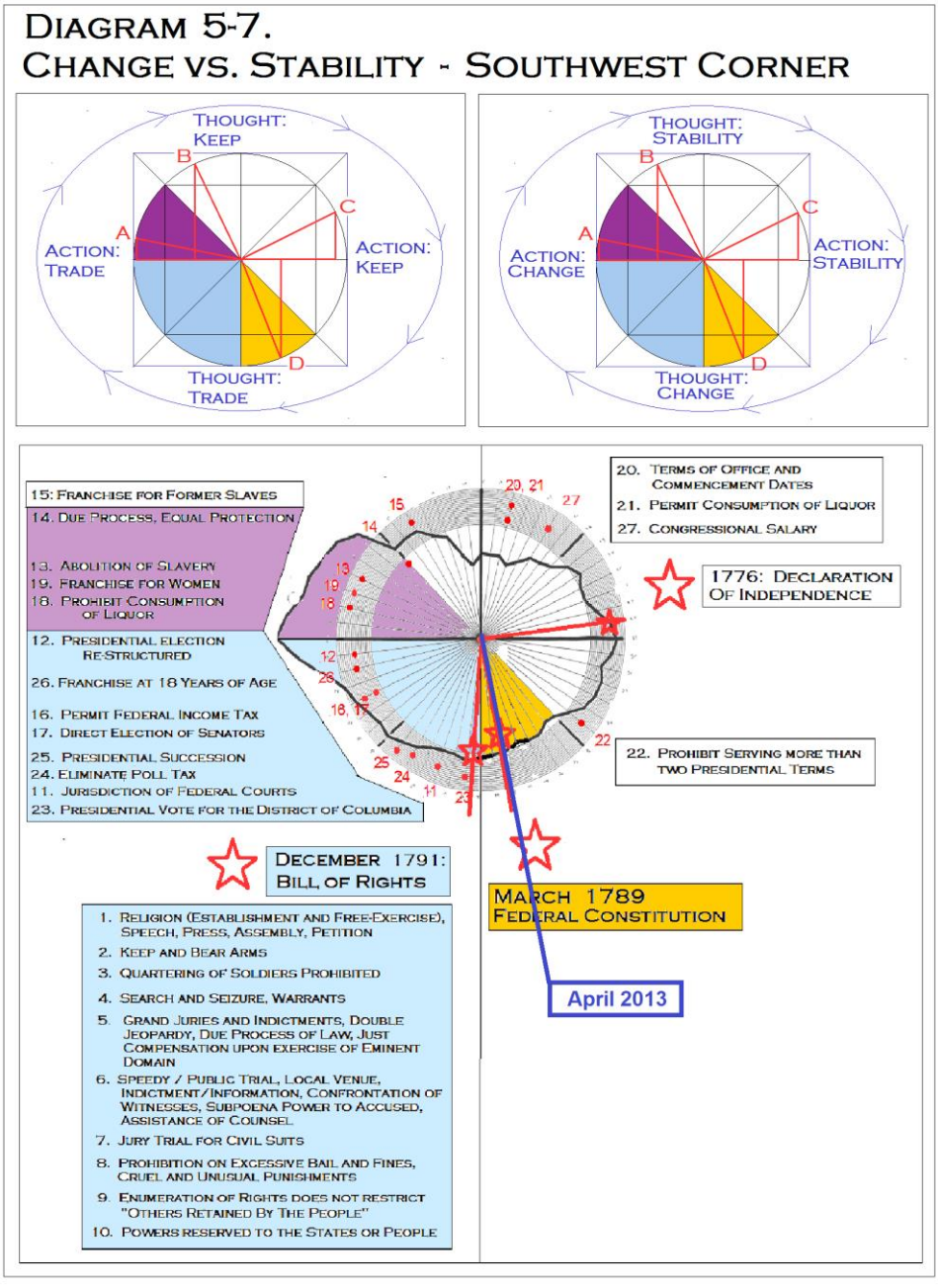
[26] Euclid of Alexandria, Elements, Book VI, Definition 3, circa 300 b.c.. A broad array of texts may be suggested describing the well-known associations between the Golden Mean and patterns discovered in Nature. See e.g. Livio, 2002; Skinner, 2006; Hemenway, 2005.

[27] Geometrically, the proportion of 1: φ may be created by the following construction. A spiral may be obtained from this construction as follows. This spiral and its relationship to the economy of the United States has been one of the central points of this paper.

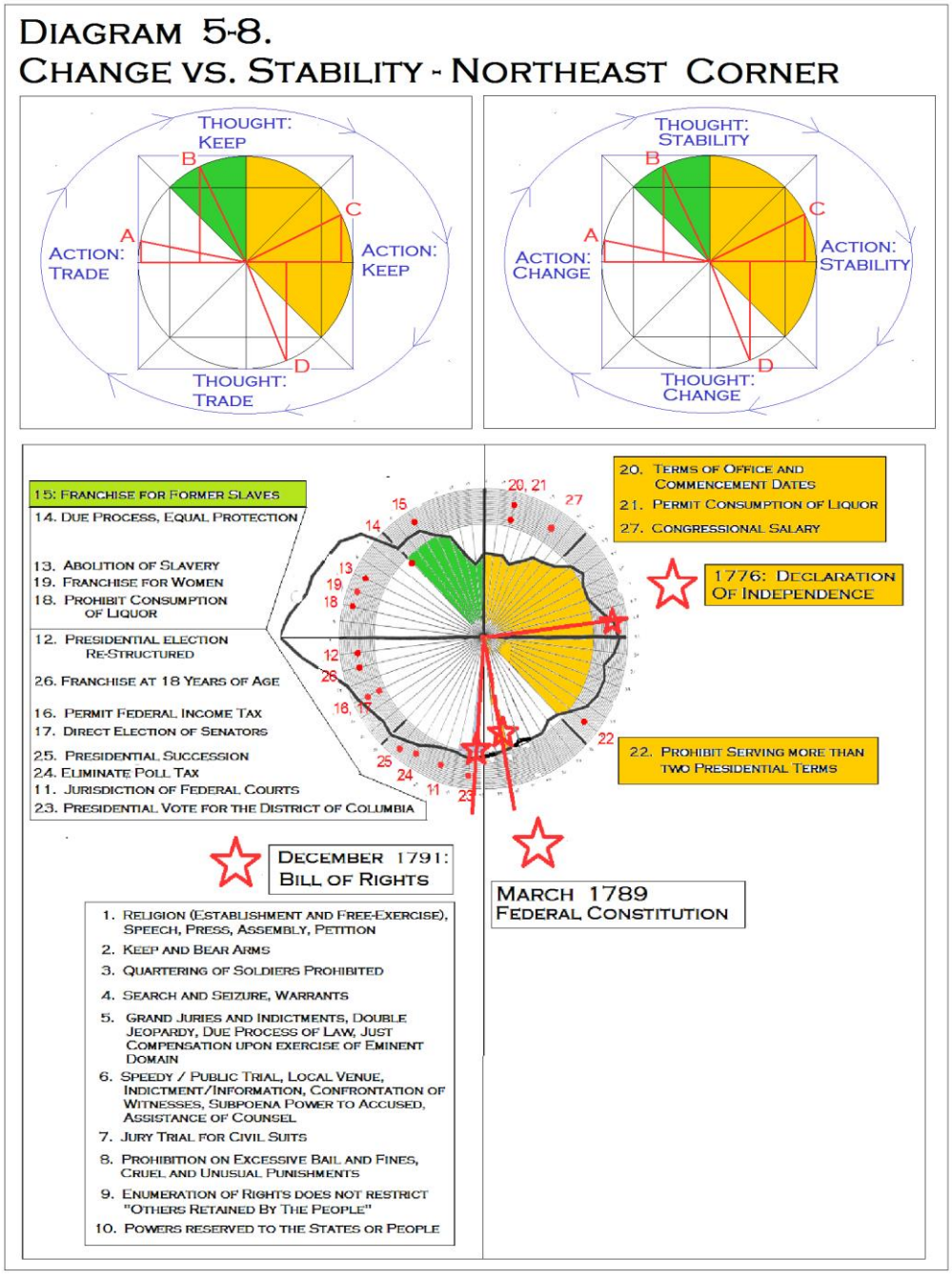


The Kondratiev Wave

The GNP Spiral gives rise to an evaluation of the economic history of the United States. This may be stated in a circuit of 56 years, i.e. eight sections of 7 years each. (Albers & Albers 2013) The political and economic emphasis of these different periods has economic impact upon Okun's law. At the present time, we face a sea change in political attitudes. (April 2013) During comparable historic periods frustration with the political status quo has led to significant and enduring constitutional change. These changes alter the ground rules of economic engagement and permit the capitalistic enterprise to move forward. The following constitutional amendments are associated with historically comparable periods of time.



The period of time which we are leaving is one of tremendous conservatism, a period during which the rules previously laid down are made permanent to the satisfaction of a newly empowered political elite. Historically comparable periods of time are associated with the Articles of Confederation, the rise of slavery in the South and the westward expansion of the United States, the Gilded Age and power of the Robber Barons, and the international dominance of the United States post-World War II.



The 56-Year Kondratiev Wave

This 56-year period may in turn be used to arrange real GNP figures in a spiral, the basic ratio of which is 1:1.618..., the Golden Mean, or 1:φ. We wrote:

> We then placed the U.S. real GNP figures given in "Data Set 2 – U.S. Real GNP" in a 56 year circuit, with the four 14-year quarter cycles indicated in blue, to create the spiral below. The center of the spiral, beginning at axis 9 = 1869, represents the real Gross National Product for that year of 23.10 billion dollars in 1958 prices. The Gross National Product for subsequent years in real terms are given along each axis respectively, with each circle of circumference representing ten billion dollars of real GNP in 1958 prices. ... (Albers & Albers 2014)

DIAGRAM 15. THE "GNP SPIRAL"

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51		1799	1855	1911	1967	2023
50		1798	1854	1910	1966	2022
49		1797	1853	1909	1965	2021
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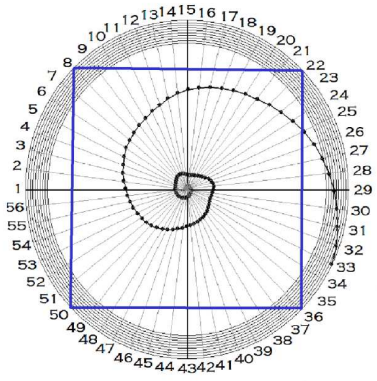
47		1795	1851	1907	1963	2019
46		1794	1850	1906	1962	2018
45		1793	1849	1905	1961	2017
44		1792	1848	1904	1960	2016
43		1791	1847	1903	1959	2015



	Date	Date	Date	Date	Date
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24	1772	1828	1884	1940	1996
25	1773	1829	1885	1941	1997
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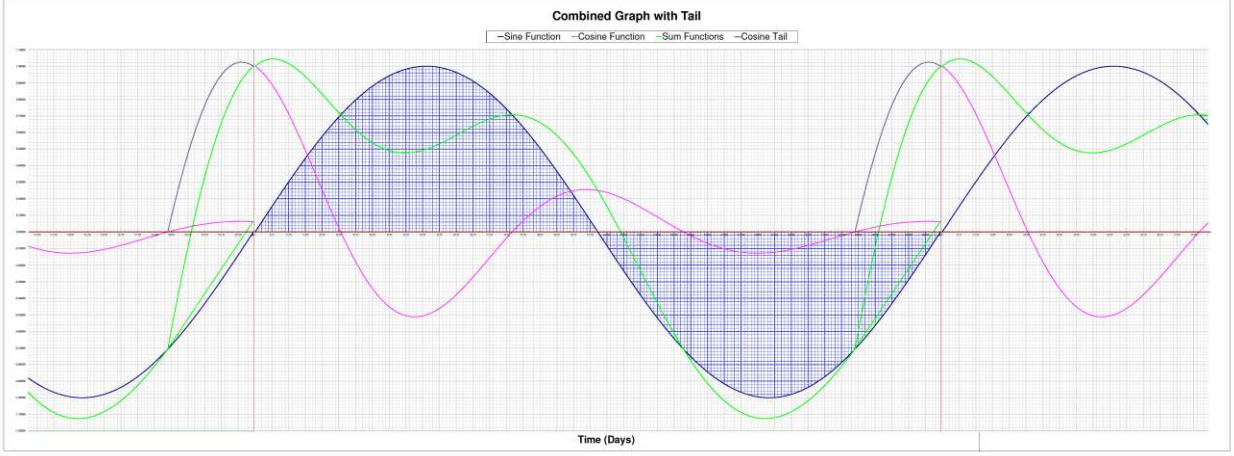
27	1775	1831	1887	1943	1999
28	1776	1832	1888	1944	2000
29	1777	1833	1889	1945	2001
30	1778	1834	1890	1946	2002
31	1779	1835	1891	1947	2003
32	1780	1856	1892	1948	2004
33	1781	1837	1893	1949	2005
34	1782	1838	1894	1950	2006
35	1783	1839	1895	1951	2007
36	1784	1840	1896	1952	2008
37	1785	1841	1897	1953	2009
38	1786	1842	1898	1954	2010
39	1787	1843	1899	1955	2011
40	1788	1844	1900	1956	2012
41	1789	1845	1901	1957	2013
42	1790	1846	1902	1958	2014

Placing this sine curve in blue, as superimposed upon the red damping curve, we have the following.

DIAGRAM 5-13.
SINE CURVE

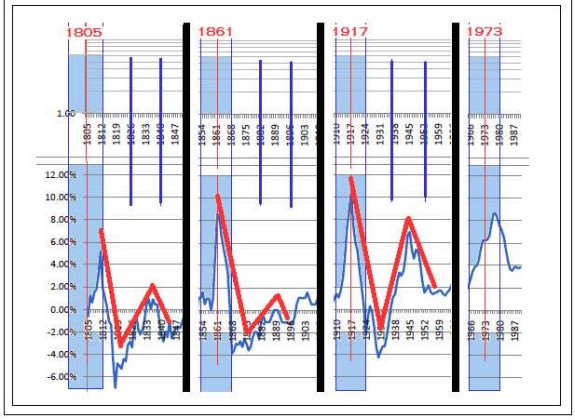


The 28-Year Damping Price Wave

Using the 14-year octave American economic history can be divided neatly into four periods of 14 years, using the price index as a damping 28-year price wave over the course of 56 years. We wrote:

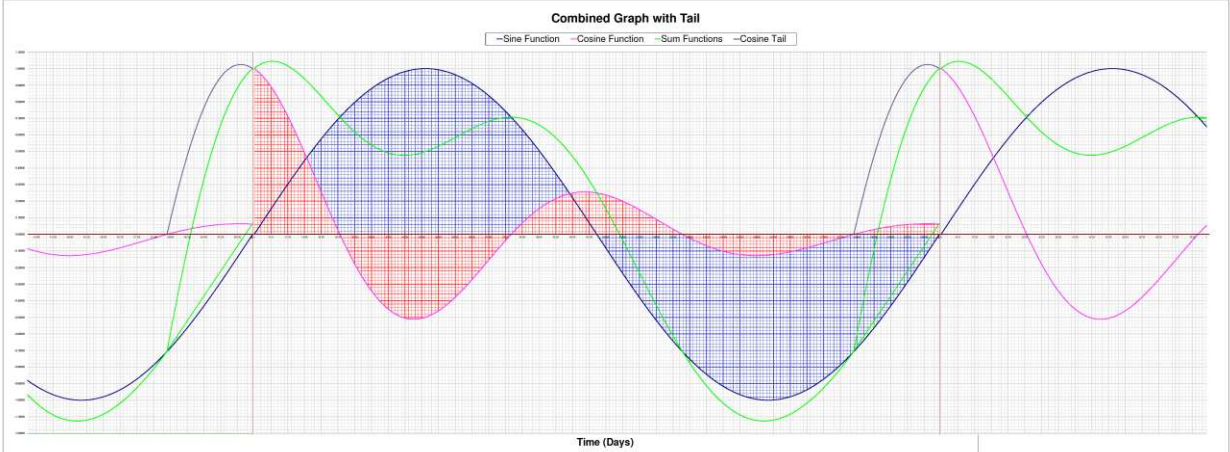
> We noted in the above that the 56 year period (14 x 4 = 56) between peaks at 1861 through 1917 suggests the possibility that similar periods of time might connect other peak points of inflation. If a 14-year span (blue rectangles above) is drawn around the years 1805, 1861, 1917 and 1973 (each of which is separated by periods of 56 years), virtually all inflationary peaks are contained in a single model. (Albers & Albers 2014)

DIAGRAM 13.
PERIODS OF INFLATION



A damping cosine curve may be constructed as follows to fit this pattern.

DIAGRAM 5-13.
DAMPING COSINE CURVE



The Political Economy Wave

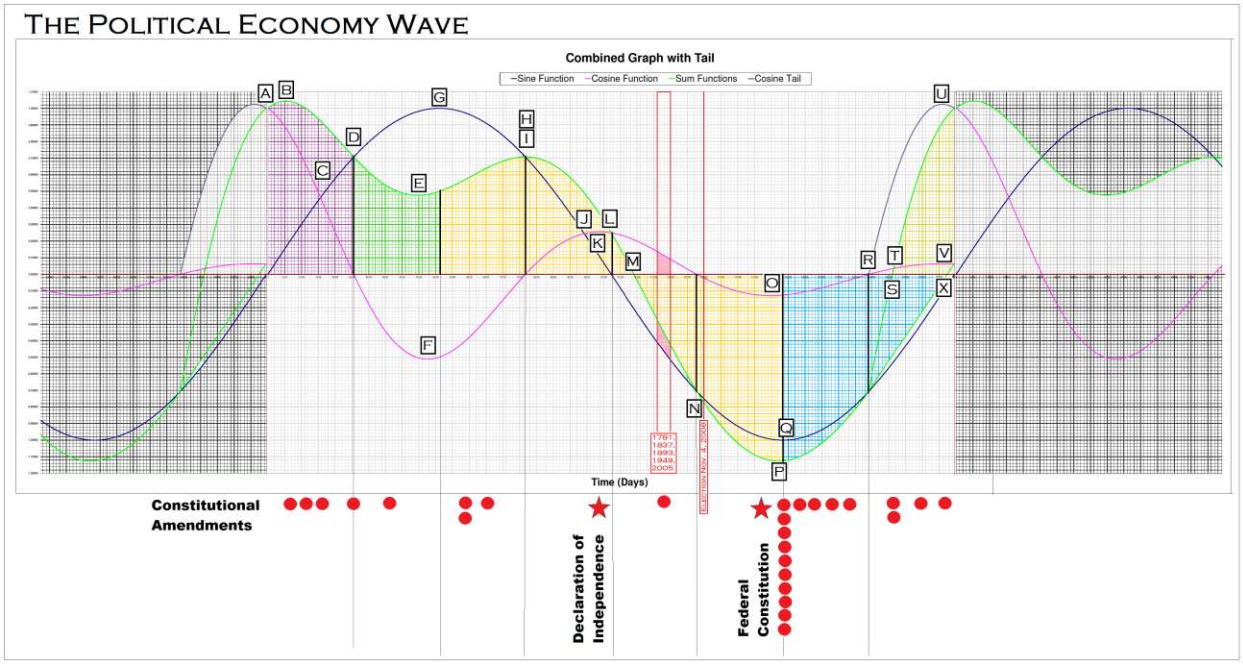
Keeping the peak of the damping cosine wave at the same level as that of the original sine wave ("1") we may graph the damping to occur by halves. The equation which adds the previous sine curve together with this damping cosine curve is as follows:

$$g(y) = 2^{\frac{1}{2} - \frac{2\left(y + \frac{1}{4}\pi\right)}{\pi}} \sin\left(2y + \frac{1}{2}\pi\right) + \sin(y)$$

DAMPING COSINE CURVE — SINE CURVE

To test whether any part of this equation is capable of anticipating dates of economi and / or social crisis, I have plotted the 28-year "economy wave" (damping cosine curve) and a 56-year "political wave" or "Kondratiev Wave" into the 56-year Political Economy Wave by charting a 56-year (20,454 day) period of changes beginning on April 9, 1805, 1861, 1917 and 1973.

Using the same color scheme given for the previous models we will examine in these essays the creation of the following mathematic wave running through American economic and social history.



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