Optimization issues of sectoral outputs in economic output

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Abstract

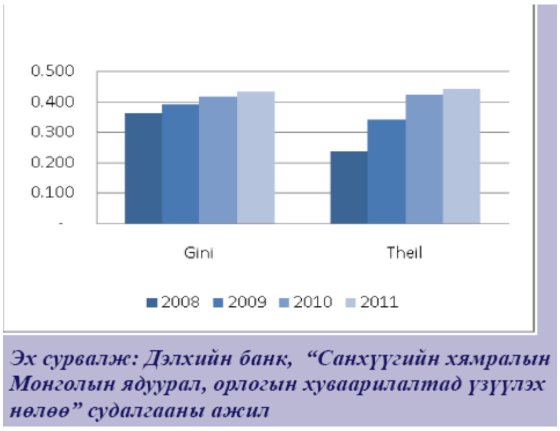
The traditional methodology uses the real GDP growth as a proxy for the economic growth. Unfortunately this way of calculating economic growth is not taking into account of the sectoral inequality in the economy (especially in the economy with high degree of natural resource dependency). Therefore, this paper proposes the new approach to optimize the share of sectoral outputs in the economy which take into account of the inequality.

CONTENTS

Introduction

In 2011, Mongolian economy grew 17.5% in real and 31% in nominal term which was not seen in the last 20 year while the GINI coefficient of Mongolia had increased in 2008 and 2011 according to the World Bank Study (Figure 1). This shows that the income inequality has been increasing regardless of the high economic growth. However, people demand more inclusive and sustainable growth, support for the vulnerable, to be able to participate the process of economic and enjoy the fruits of their own contribution for the increased in productivity.

Figure 1. GDP and GINI coefficient (Mongol Bank 2013, World Bank 2012)



Therefore, search to explore the new approach to account the economic growth by the sector growth was initiated. In this attempt, I developed a theoretical model which can be utilized to answer the following two questions:

1. Which sector is to be further developed and which is to be limited?

2. Is there a methodology to find the optimal ratios of different economic sectors?

Methodology

1. Ratios of sector outputs in the total economic output, which is important?

In this model we assume Cobb-Douglas production function:

$$X = AK^{\phi}L^{\varphi} \qquad (1.1)$$

$K, L-$ capital and labour consumed in the production process, $A-$innovation. Equation (1.1) is the production function.

For the sector i

$$X_i = A_i K_i^{\phi_i} L_i^{\varphi_i} \qquad (1.2)$$

K_i, L_i- capital and labour consumed in the sector i, A_i-innovation of the sector i. If we divide the both sides of the equation by the total output, we will find the ration of sector output as following.

$$\frac{X_i}{X} = \frac{A_i K_i^{\phi_i} L_i^{\varphi_i}}{AK^{\phi}L^{\varphi}} = \frac{A_i}{A} \cdot \left(\frac{K_i^{1/\phi}}{K^{1/\phi_i}}\right)^{\phi_i \cdot \phi} \cdot \left(\frac{L_i^{1/\varphi}}{L^{1/\varphi_i}}\right)^{\varphi_i \cdot \varphi} \qquad (1.3)$$

If we make the following notations

$$\frac{K_i^{1/\phi}}{K^{1/\phi_i}} = k_i \ ; \quad \phi_i \cdot \phi = \gamma_i \ ; \qquad \frac{X_i}{X} = x_i$$

$$\frac{L_i^{1/\varphi}}{L^{1/\varphi_i}} = l_i \ ; \quad \varphi_i \cdot \varphi = \eta_i \ ; \qquad \frac{A_i}{A} = a_i$$

and substitute these in the equation (1.3) we will find the ration of sector output as following

$$x_i = a_i \cdot k_i^{\gamma_i} \cdot l_i^{\eta_i} \qquad (1.4)$$

Equation (4) shows the ratio of sector i's output in the total economic output. k_i, l_i, a_i- are ratios of sector i's capital, labor and innovation in total capital, labor and innovation of the economy. Thus following will be true:

$$0 < x_i < 1$$
$$0 < k_i < 1 \qquad (1.5)$$
$$0 < l_i < 1$$

If we assume n sectors in the economy, there will be n equations such as (1.4) with following condition.

$$x_1 + x_2 + \ldots + x_n = 1 \qquad (1.6)$$

If we take the product of all n equations like (1.4), the following function, the measure of an economic output which takes into account the share of sector outputs in the total economic output, will be created.

$$S = x_1 \cdot x_2 \cdot \ldots \cdot x_n \qquad (1.7)$$

But sectoral impacts on the total output differ according to the level of innovations. Equation (1.7) has following deficiencies even though this can show whether the all sector shares in the economy output:

1. If the share of one sector in the economy reduces the measure of an economic output will reduce. However, it does not take into account the heterogeneous effects of heterogeneous sectors' output on the economic output.
2. Heterogeneous sectors' output shares in the total economic output will not have different effects on the measure of an economic output which takes into account the share of sector outputs in the total economic output.
3. In order to maximize the measure of an economic output which takes into account the share of sector outputs in the total economic output should be the same. This condition does not allow the country to use its comparative advantage.

Thus the elasticity of the economic output with respect to the shares of sectors outputs in the total economic output are included in the equation (1.7).

$$U = x_1^{\alpha_1} \cdot x_2^{\alpha_2} \cdot \ldots \cdot x_n^{\alpha_n} \qquad (1.8)$$

Subject to

$$x_1 + x_2 + \ldots + x_n = 1 \qquad (1.9)$$

$$0 < x_i < 1, \quad i = \overline{1,n} \qquad (1.10)$$

$U -$ Measure of an economic output which takes into account the sectors' outputs shares in the total economic output.

$n -$ total number of sectors, $\alpha_i -$ elasticity of economic output with respect to the sector *i*'s output shares in the total economic output.

The objective of the above problem is to explore the optimal level of sector participation in the economy that maximize the total economic output for the given level of the elasticity of the economic output with respect to the sector output share in the total economic output .

2. Optimal level of sector participation

Objective function:

$$U = y_1^{\alpha_1} \cdot y_2^{\alpha_2} \cdot \ldots \cdot y_n^{\alpha_n} \Rightarrow \max \qquad (2.1)$$

Subject to:

$$0 < y_i < 1 \quad i = \overline{1,n} \qquad (2.2)$$

$$y_1 + y_2 + \ldots + y_n = 1 \qquad (2.3)$$

In order to maximize the (2.1) subject to (2.2) and (2.3), we have write the Lagrangian function as follows:

$$L(y_i, \lambda_i, \gamma_i, \eta) = \prod_{i=1}^{n} y_i^{\alpha_i} + \sum_{i=1}^{n} \lambda_i y_i + \sum_{i=1}^{n} \gamma_i (1 - y_i) + \eta (1 - \sum_{i=1}^{n} y_i) \qquad (2.4)$$

$$i = \overline{1,n}$$

The FOC given by the Kuhn-Tucker conditions which is used to find the saddle points is:

$$\begin{cases} \dfrac{\partial L}{\partial y_i} = \alpha_i \cdot \prod\limits_{\substack{j=1,\\ i\neq j}}^{n} y_j^{\alpha_j} \cdot y_i^{\alpha_i - 1} + \lambda_i - \gamma_i - \eta \le 0, & y_i \dfrac{\partial L}{\partial y_i} = 0 \\ \dfrac{\partial L}{\partial \lambda_i} = y_i \ge 0, & \lambda_i \dfrac{\partial L}{\partial \lambda_i} = 0 \\ \dfrac{\partial L}{\partial \gamma_i} = \gamma_i (1 - y_i) \ge 0, & \gamma_i \dfrac{\partial L}{\partial \gamma_i} = 0 \\ \dfrac{\partial L}{\partial \eta} = \left(1 - (\sum\limits_{i=1}^{n} y_i)\right) \ge 0, & \eta \dfrac{\partial L}{\partial \eta} = 0 \end{cases}, \qquad i = \overline{1,n} \qquad (2.5)$$

Solution from (2.5) should be the saddle points for (2.1).

Since $0 < y_i < 1 \quad i = \overline{1,n}$, following coefficients will be $\lambda_i = \gamma_i = 0$, $i = \overline{1,n}$.

For η, we two possible cases:

- If $\eta = 0$ following should be true

  $$\left(1 - (\sum_{i=1}^{n} y_i)\right) \neq 0$$

  This conflicts with the condition (2.3) of the problem.

- If $\eta \neq 0$ following should be true

  $$\left(1 - (\sum_{i=1}^{n} y_i)\right) = 0$$

  This does not conflict with the condition (2.3) of the problem. Thus we have to solve the following system of $n+1$ equations with $n+1$ unknowns.

$$\begin{cases} \dfrac{\partial L}{\partial y_i} = \dfrac{\alpha_i}{y_i} \cdot \left(\prod\limits_{i=1}^{n} y_i^{\alpha_i} \right) - \eta \\ \dfrac{\partial L}{\partial \eta} = 1 - \left(\sum\limits_{i=1}^{n} y_i \right) \end{cases}, \qquad i = \overline{1,n} \qquad (2.6a)$$

(2.6a) can be written in a following form:

$$\begin{cases} y_i \dfrac{\partial L}{\partial y_i} = \left(\alpha_i \cdot \prod\limits_{i=1}^{n} y_i^{\alpha_i} - \eta \cdot y_i \right) = 0, \\ \eta \dfrac{\partial L}{\partial \eta} = \eta \left(1 - (\sum\limits_{i=1}^{n} y_i) \right) = 0, \end{cases}, \qquad i = \overline{1,n} \qquad (2.6b)$$

If we add the first n equations of (2.6b) we reach the following equation:

$$\left(\sum_{i=1}^{n}\alpha_i\right)\cdot\left(\prod_{i=1}^{n}y_i^{\alpha_i}\right)-\eta\cdot\left(\sum_{i=1}^{n}y_i\right)=0, \qquad i=\overline{1,n} \qquad (2.6c)$$

using $\left(\sum_{i=1}^{n}y_i\right)=1$ condition, the critical value of η will be found as:

$$\eta^*=\left(\sum_{i=1}^{n}\alpha_i\right)\cdot\left(\prod_{i=1}^{n}y_i^{\alpha_i}\right), \qquad i=\overline{1,n} \qquad (2.6d)$$

From the first *n* equations of (2.6b), the share of sector output in the total economic output y_i -will be found as (2.6e).

$$y_i=\frac{\alpha_i\cdot\left(\prod_{i=1}^{n}y_i^{\alpha_i}\right)}{\eta}, \qquad i=\overline{1,n} \qquad (2.6e)$$

Substituting (2.6e) into (2.6d) will find the critical value of the share of sector output in the total economic output y_i^* -as in (2.7) :

$$y_i^*=\frac{\alpha_i\cdot\left(\prod_{i=1}^{n}y_i^{\alpha_i}\right)}{\left(\sum_{i=1}^{n}\alpha_i\right)\cdot\left(\prod_{i=1}^{n}y_i^{\alpha_i}\right)}=\frac{\alpha_i}{\left(\sum_{i=1}^{n}\alpha_i\right)}, \qquad i=\overline{1,n} \qquad (2.7)$$

In order to test whether this critical value maximizes the objective function (2.1), second order condition has to be checked. According to the Sylvester's criterion, if the principal minors of the Lagrangian function are alternating between negative and positive the matrix is negative-definite, and the function has a *maximum* in the point (ChOU, 2006).

$$\left|H_2(y_i^*)\right|<0, \quad \left|H_3(y_i^*)\right|>0, \quad \left|H_4(y_i^*)\right|<0 \quad \ldots$$

Prior to testing the SOC, we need to pay attention to the following condition which shows that the elasticity is positive:

$$0<y_i<1 \quad\Rightarrow\quad 0<\frac{\alpha_i}{\sum_{i=1}^{n}\alpha_i}<1 \quad\Rightarrow\quad 0<\alpha_i<\sum_{i=1}^{n}\alpha_i \qquad (2.8)$$

Then the SOC will be as follows:

$$|H_{n+1}| = \begin{vmatrix} \frac{\partial L}{\partial\eta\partial\eta} & \frac{\partial L}{\partial y_1\partial\eta} & \frac{\partial L}{\partial y_2\partial\eta} & \frac{\partial L}{\partial y_3\partial\eta} & \cdots & \frac{\partial L}{\partial y_n\partial\eta} \\ \frac{\partial L}{\partial y_1\partial\eta} & \frac{\partial L}{\partial y_1\partial y_1} & \frac{\partial L}{\partial y_1\partial y_2} & \frac{\partial L}{\partial y_1\partial y_3} & \cdots & \frac{\partial L}{\partial y_1\partial y_n} \\ \frac{\partial L}{\partial y_2\partial\eta} & \frac{\partial L}{\partial y_2\partial y_1} & \frac{\partial L}{\partial y_2\partial y_2} & \frac{\partial L}{\partial y_2\partial y_3} & \cdots & \frac{\partial L}{\partial y_2\partial y_n} \\ \frac{\partial L}{\partial y_3\partial\eta} & \frac{\partial L}{\partial y_3\partial y_1} & \frac{\partial L}{\partial y_3\partial y_2} & \frac{\partial L}{\partial y_3\partial y_3} & \cdots & \frac{\partial L}{\partial y_3\partial y_n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial L}{\partial y_n\partial\eta} & \frac{\partial L}{\partial y_n\partial y_1} & \frac{\partial L}{\partial y_n\partial y_2} & \frac{\partial L}{\partial y_n\partial y_3} & \cdots & \frac{\partial L}{\partial y_n\partial y_n} \end{vmatrix} \quad (2.9a)$$

$$|H_{n+1}| = \begin{vmatrix} 0 & -1 & -1 & -1 & \cdots & -1 \\ -1 & \frac{\alpha_1(\alpha_1-1)}{y_1\cdot y_1}\cdot\left(\prod_{i=1}^n y_i^{\alpha_i}\right) & \frac{\alpha_1\alpha_2}{y_1\cdot y_2}\cdot\left(\prod_{i=1}^n y_i^{\alpha_i}\right) & \frac{\alpha_1\alpha_3}{y_1\cdot y_3}\cdot\left(\prod_{i=1}^n y_i^{\alpha_i}\right) & \cdots & \frac{\alpha_1\alpha_n}{y_1\cdot y_n}\cdot\left(\prod_{i=1}^n y_i^{\alpha_i}\right) \\ -1 & \frac{\alpha_2\alpha_1}{y_2\cdot y_1}\cdot\left(\prod_{i=1}^n y_i^{\alpha_i}\right) & \frac{\alpha_2(\alpha_2-1)}{y_2\cdot y_1}\cdot\left(\prod_{i=1}^n y_i^{\alpha_i}\right) & \frac{\alpha_2\alpha_3}{y_2\cdot y_3}\cdot\left(\prod_{i=1}^n y_i^{\alpha_i}\right) & \cdots & \frac{\alpha_2\alpha_n}{y_2\cdot y_n}\cdot\left(\prod_{i=1}^n y_i^{\alpha_i}\right) \\ -1 & \frac{\alpha_3\alpha_1}{y_3\cdot y_1}\cdot\left(\prod_{i=1}^n y_i^{\alpha_i}\right) & \frac{\alpha_3\alpha_2}{y_3\cdot y_2}\cdot\left(\prod_{i=1}^n y_i^{\alpha_i}\right) & \frac{\alpha_3(\alpha_3-1)}{y_3\cdot y_3}\cdot\left(\prod_{i=1}^n y_i^{\alpha_i}\right) & \cdots & \frac{\alpha_3\alpha_n}{y_3\cdot y_n}\cdot\left(\prod_{i=1}^n y_i^{\alpha_i}\right) \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & \frac{\alpha_n\alpha_1}{y_n\cdot y_1}\cdot\left(\prod_{i=1}^n y_i^{\alpha_i}\right) & \frac{\alpha_n\alpha_2}{y_n\cdot y_2}\cdot\left(\prod_{i=1}^n y_i^{\alpha_i}\right) & \frac{\alpha_n\alpha_3}{y_n\cdot y_3}\cdot\left(\prod_{i=1}^n y_i^{\alpha_i}\right) & \cdots & \frac{\alpha_n(\alpha_n-1)}{y_n\cdot y_n}\cdot\left(\prod_{i=1}^n y_i^{\alpha_i}\right) \end{vmatrix} \quad (2.9b)$$

If we substitute the critical values found by (2.7) into the (2.9b) following matrix (2.10a) will be found. Here $U^* = \left(\prod_{i=1}^n (y_i^*)^{\alpha_i}\right)$.

$$|H_{n+1}(y_i^*)| = \begin{vmatrix} 0 & -1 & -1 & -1 & \cdots & -1 \\ -1 & \frac{(\alpha_1-1)}{\alpha_1}\left(\sum_{i=1}^n\alpha_i\right)^2\cdot U^* & \left(\sum_{i=1}^n\alpha_i\right)^2\cdot U^* & \left(\sum_{i=1}^n\alpha_i\right)^2\cdot U^* & \cdots & \left(\sum_{i=1}^n\alpha_i\right)^2\cdot U^* \\ -1 & \left(\sum_{i=1}^n\alpha_i\right)^2\cdot U^* & \frac{(\alpha_2-1)}{\alpha_2}\left(\sum_{i=1}^n\alpha_i\right)^2\cdot U^* & \left(\sum_{i=1}^n\alpha_i\right)^2\cdot U^* & \cdots & \left(\sum_{i=1}^n\alpha_i\right)^2\cdot U^* \\ -1 & \left(\sum_{i=1}^n\alpha_i\right)^2\cdot U^* & \left(\sum_{i=1}^n\alpha_i\right)^2\cdot U^* & \frac{(\alpha_3-1)}{\alpha_3}\left(\sum_{i=1}^n\alpha_i\right)^2\cdot U^* & \cdots & \left(\sum_{i=1}^n\alpha_i\right)^2\cdot U^* \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & \left(\sum_{i=1}^n\alpha_i\right)^2\cdot U^* & \left(\sum_{i=1}^n\alpha_i\right)^2\cdot U^* & \left(\sum_{i=1}^n\alpha_i\right)^2\cdot U^* & \cdots & \frac{(\alpha_n-1)}{\alpha_n}\left(\sum_{i=1}^n\alpha_i\right)^2\cdot U^* \end{vmatrix} \quad (2.10a)$$

Since, $y_i^* > 0 \;\Rightarrow\; U^* > 0$.

If note $S^* = \left(\sum_{i=1}^n \alpha_i\right)^2 \cdot U^*$, (2.10a) will be in following form.

$$
\left| H_{n+1}(y_i^*) \right| = \begin{vmatrix} 0 & -1 & -1 & -1 & \dots & -1 \\ -1 & \frac{(\alpha_1 - 1)}{\alpha_1} S^* & S^* & S^* & \dots & S^* \\ -1 & S^* & \frac{(\alpha_2 - 1)}{\alpha_2} S^* & S^* & \dots & S^* \\ -1 & S^* & S^* & \frac{(\alpha_3 - 1)}{\alpha_3} S^* & \dots & S^* \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & S^* & S^* & S^* & \dots & \frac{(\alpha_n - 1)}{\alpha_n} S^* \end{vmatrix} \qquad \text{(2.10b)}
$$

Since $U^* > 0 \quad \Rightarrow \quad S^* > 0$.

Multiplying the first row of (2.10b) by S^* and adding on the remaining *n* rows, following will be found.

$$
\left| H_{n+1}(y_i^*) \right| = \begin{vmatrix} 0 & -1 & -1 & -1 & \dots & -1 \\ -1 & \frac{-1}{\alpha_1} S^* & 0 & 0 & \dots & 0 \\ -1 & 0 & \frac{-1}{\alpha_2} S^* & 0 & \dots & 0 \\ -1 & 0 & 0 & \frac{-1}{\alpha_3} S^* & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & 0 & 0 & 0 & \dots & \frac{-1}{\alpha_n} S^* \end{vmatrix} \qquad \text{(2.11)}
$$

The principal minors of (2.12) will be found as following:

$$
\left| H_2(y_i^*) \right| = \begin{vmatrix} 0 & -1 \\ -1 & \frac{-1}{\alpha_1} S^* \end{vmatrix} < 0
$$

$$
\left| H_3(y_i^*) \right| = \begin{vmatrix} 0 & -1 & -1 \\ -1 & \frac{-1}{\alpha_1} S^* & 0 \\ -1 & 0 & \frac{-1}{\alpha_2} S^* \end{vmatrix} = (-1)^{2+1}(-1) \begin{vmatrix} -1 & -1 \\ 0 & \frac{-1}{\alpha_2} S^* \end{vmatrix} + (-1)^{2+2} (\frac{-1}{\alpha_1} S^*) \begin{vmatrix} 0 & -1 \\ -1 & \frac{-1}{\alpha_2} S^* \end{vmatrix} =
$$

$$
= (-1)^{3+1} \left[(-1)(\frac{-1}{\alpha_2} S^*) + (\frac{-1}{\alpha_1} S^*)(-1) \right] = (-1)^{3+1} \left[\frac{\alpha_1 + \alpha_2}{\alpha_1 \alpha_2} \right] S^* > 0
$$

$$\left|H_4(y_i^*)\right| = \begin{vmatrix} 0 & -1 & -1 & -1 \\ -1 & \frac{-1}{\alpha_1}S^* & 0 & 0 \\ -1 & 0 & \frac{-1}{\alpha_2}S^* & 0 \\ -1 & 0 & 0 & \frac{-1}{\alpha_3}S^* \end{vmatrix} = (-1)^{2+1}(-1)\begin{vmatrix} -1 & -1 & -1 \\ 0 & \frac{-1}{\alpha_2}S^* & 0 \\ 0 & 0 & \frac{-1}{\alpha_3}S^* \end{vmatrix} + (-1)^{2+2}(\frac{-1}{\alpha_1}S^*)\begin{vmatrix} 0 & -1 & -1 \\ -1 & \frac{-1}{\alpha_2}S^* & 0 \\ -1 & 0 & \frac{-1}{\alpha_3}S^* \end{vmatrix} =$$

$$= (-1)^{4+1}\left[\frac{1}{\alpha_2}S^*\frac{1}{\alpha_3}S^* + \frac{1}{\alpha_1}S^*\left[\frac{\alpha_2+\alpha_3}{\alpha_2\alpha_3}\right]S^*\right] = (-1)^{4+1}(S^*)^2\left[\frac{\alpha_1+\alpha_2+\alpha_3}{\alpha_1\alpha_2\alpha_3}\right] < 0$$

k th principal minor of (2.11) will be as following:

$$\left|H_k(y_i^*)\right| = (-1)^{k+1}(S^*)^{k-2}\left[\frac{\sum_{i=1}^{k-1}\alpha_i}{\prod_{i=1}^{k-1}\alpha_i}\right], \qquad k = \overline{2, n+1} \tag{2.12}$$

(2.12) shows that the principal minors are altering in sign since $S^* > 0$ and $\alpha_i > 0$. Therefore, y^*_i will be giving the maximum point of (2.1).

3. Calibration of the model

If we consider the economy with two sectors, elasticity of the total economic output with respect to the share of sector output (sectoral participation), following scenarios can to be tested:

1. $\alpha_1 > 0, \quad \alpha_2 > 0$
2. $\alpha_1 < 0, \quad \alpha_2 > 0$
3. $\alpha_1 > 0, \quad \alpha_2 < 0$
4. $\alpha_1 < 0, \quad \alpha_2 < 0$

Equation (2.1) will have to following form

$$U(x, y) = Ax^{\alpha_1}y^{\alpha_2} \tag{3.1}$$

Subject to:

$$
\begin{aligned}
&0 < x < 1 \\
&0 < y < 1 \qquad (3.2) \\
&x + y = 1
\end{aligned}
$$

Scenario 1:

Figure 3.1a shows that when $\alpha_1 = 1.5 > 0, \quad \alpha_2 = 1.2 > 0, \quad A = 1$, (3.1) subject to (3.2) gives the following optimal level of sector participations:

$x^* = 0.55782759$, $y^* = 0.44217241$

At this level of sector participation, $U^*(x^*, y^*) = 0.15648040$

Scenario 2:

Figure 3.1b shows that when $\alpha_1 = -1.8 < 0, \quad \alpha_2 = 1.4 > 0, \quad A = 1$ (3.1) subject to (3.2) gives the following optimal level of sector participations: $x^* = 0.00010000$, $y^* = 0.99990000$

At this level of sector participation, $U^*(x^*, y^*) = 15846713.11851960$

Scenario 3:

Figure 3.1c shows that when $\alpha_1 = 0.5 > 0, \quad \alpha_2 = -0.9 < 0, \quad A = 1$,(3.1) subject to (3.2) gives the following optimal level of sector participations: $x^* = 0.99999000$, $y^* = 0.00001000$

At this level of sector participation, $U^*(x^*, y^*) = 31622.61848722$

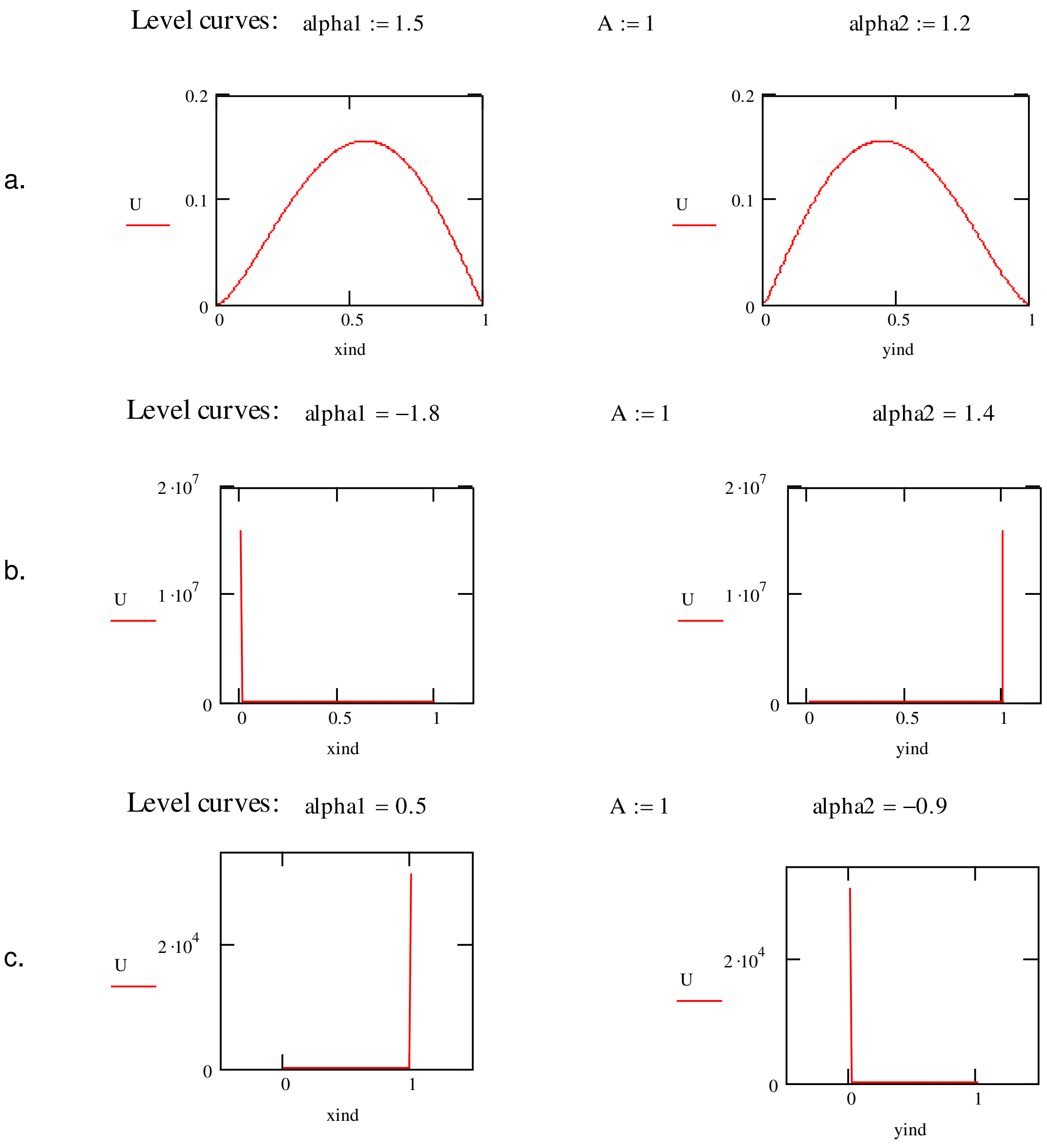
Scenario 4:

Figure 3.1d shows that when ,(3.1) subject to (3.2) gives the following optimal level of sector participations: $x^* = 0.99999000$, $y^* = 0.00001000$

At this level of sector participation $U^*(x^*, y^*) = 3162325094.88685000$

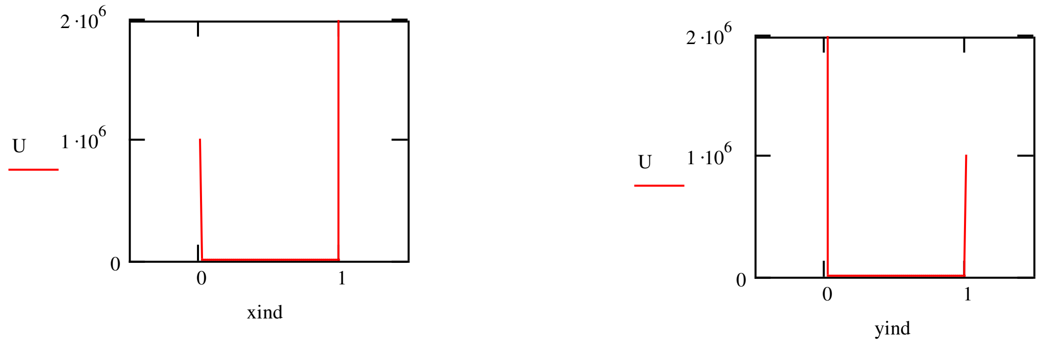
From Figure 3.1 we can see that when the elasticity of the total economic output with respect to the share of sector output is positive for both sector, the sector with the most impact is participating more than the other sector. Furthermore, the elasticity of the total economic output with respect to the share of sector output is negative, that sector is completely shut down since it is damaging the economy. When all the sectors are damaging the sector, the sector with the least damage is in operation.

Figure 3.1. Effect of varying levels of elasticity of total economic output respect to the sector output level



Level curves: alpha1 = −1.5 A := 1 alpha2 = −1.9

d.



Discussion

The model I am proposing in this paper optimizes the sector participation such that it maximizes the total output given the level elasticity of the total output with respect to the share of the sector output in the total output. Thus if the sector participation is above the optimal level, it is required to reduce the participation and increase the participation of other sectors which is below the optimal level.

If we compare the results with respect to the traditional measure of economic growth calculated by the GDP, we can make the following comparisons (Table 1)

Table 1. Comparison of economic output calculated by the sector outputs and the sector output shares

№	Sector participation	GDP
1.	If the sector participation is not optimal the total economic output will reduce	Increase in sector output will increase the economic output
2.	Increase in any sector participation will reduce the participation of other sectors in the economic activity	High growth of one sector will result in high economic growth even the other sector shrink
3.	Possible to see the difference between the maximum economic output and the actual ouput	There is no maximum output
4.	Economic growth will reduce the GINI coefficient (assumption)	Economic growth will may not reduce the GINI coefficient (assumption)
5.	Depending on the elasticity of the total output with respect to the share of the sector output in the total output, the sector which should be further developed can be selected	It is not clear which sector to be developed further.

Conclusion

The economic growth proxied by the GDP results in the following:

1. No effect on the Income inequality
2. Rapid expansion of one sector is seen as the economic growth
3. Optimal level sector participation in the economy is not considered

The model suggested in this paper have shows that the economic growth will only be observed if all sectors grew simultaneously or sector participations approach the optimal level of participations . Also the economy will not grow as much as the rapid expansion of any sector. Furthermore, it is possible to take into account the heterogeneous effects of heterogeneous sectors’ growth on the economic growth and in order sustain the economic growth sector participations should be constrained. Production function illustrates that allocation of resources (capital, labor, etc.) to the each sector will have heterogeneous effects on the total economic output. Another advantage of the model is that it can be used to identify the sector that has negative impact on the economic output.

Reference

ChOU, C.-F. (2006). *Lectures Notes on Mathematics for Economists.* Retrieved from http://homepage.ntu.edu.tw/~econman/faculty/cfchou/MathEco1.pdf

World_Bank. (2012). *Санхүүгийн хямралын Монголын ядуурал, орлогын хуваарилалтад үзүүлэх нөлөө.*

Монголбанк. (2013). *Төрөөс мөнгөний бодлогын талаар 2013 онд баримтлах үндсэн чиглэлийн төсөл.* Улаанбаатар.