Failure Risk and Quality Cost Management in Single versus Multiple Sourcing Decision

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ABSTRACT

The advantage of multiple sourcing to protect against supplier failures arising from *undependable products* due to latent defects is examined using a model with non-linear external failure costs. Prior research has focused only on supplier failures arising from *unreliable supply*, such as late/insufficient/no delivery. I derive a closed-form characterization of the optimal production quota allocation for the *LUX* (*L*atent defect-*U*ndependable product-e*X*ternal failure) setting. The allocation determines the optimal supply base, with intuitive properties that hold under a mild requirement. The requirement includes the special case of equal procurement costs charged by suppliers but also allows unequal costs without any particular order. The key result of the paper is a necessary and sufficient condition determining whether single or multiple sourcing is optimal. Another condition is obtained to determine the exact size of the optimal supply base, provided the mild requirement holds. With minor modifications, the results also hold when a buyer-initiated procurement contract can be used to elicit private information on the suppliers' unit variable production costs. (Keywords: Supplier selection, latent defects, quality cost, total cost of purchasing; *JEL* L22, L24, M11, M21, M40)

INTRODUCTION

Latent defects are flaws or weaknesses in product items that could not be discovered by reasonable inspection prior to the sale. They can result from design flaws, systematic manufacturing faults, or the like (Thirumalai and Sinha 2011). *Undependable* (or even unsafe) *products* due to latent defects can result in huge *external failure costs* caused by warranty repairs, product recalls, defect liability claims, reputation damage, loss of sale, customer confidence restoration efforts, etc (Nagar and Rajan 2001). For example, in 2000 Bridgestone/Firestone recalled 6.5 million tires that seemed to have an unusually high risk of tread failures (Bridgestone/Firestone 2000). Tires with this defect were linked to crashes killing 271 people and causing over 800 injuries (NHTSA 2001). Many of the tires were installed on Ford Motor's vehicles because Firestone is a long-term major supplier of Ford. To restore customer confidence, Ford later announced the replacement of about 13 million Firestone tires installed on Ford vehicles by non-Firestone brands (Bradsher 2001). This move ended the nearly century-long buyer-supplier relationship between Ford and Firestone (Hakim 2004).

Although quality issues arising from latent defects are important, models analyzing the protection advantage of multiple sourcing have concentrated on supplier failures due to unreliably supply, such as late/insufficient/no delivery caused by machine breakdowns, labor strikes, natural disasters, and financial defaults (e.g., Federgruen and Yang 2008, 2009, Babich, Burnetas, and Ritchken 2007, Burke, Carrillo, and Vakharia 2007, Dada, Petruzzi, and Schwarz 2007, and others reviewed in Snyder et al 2010). In contrast, I use a latent-defect model with non-linear external failure costs to capture the sort of supplier failure risks relevant to product dependability issues. To my knowledge, the model is the first of its kind in the context of a *LUX* (*L*atent defect-*U*ndependable product-e*X*ternal failure) setting.

The key result of this paper is a condition determining whether single or multiple sourcing is optimal. By analyzing the advantage of multiple sourcing to protect against the failure risk arising in a LUX setting, this paper contributes to the literature on multiple sourcing (e.g., Benjaafar et al. 2007 and Inderst 2008), as well as on supplier selection (e.g., Ittner et al. 1999) and quality management (e.g., Hwang et al. 2006, Balachandran and Radhakrishnan 2005, Baiman et al. 2000, Ittner et al. 2001). My model highlights the sourcing decision as one about selecting the right *mix* of suppliers to balance between risk diversification and quality cost reduction. In contrast, prior studies on multiple sourcing concern mainly the *number* of suppliers that determines the level of competition, with or without externalities among units or suppliers (e.g., Baiman and Netessine 2004 and Klotz and Chatterjee 1995; see also Elmaghraby 2000).

This paper adds a new perspective to the debate on single versus multiple sourcing. Deming (1986) argues that a buying company should take quality costs into consideration and choose single sourcing by selecting a high-quality supplier that minimizes *total cost* of sourcing. The results of this paper suggest that in minimizing total cost of sourcing, one might need to use multiple sourcing and include suppliers of lower quality.

A QUALITY-COST MODEL OF SUPPLY BASE COMPOSITION

Consider a setting with a single buyer and $n \geq 2$ available suppliers, indexed by $i \in N = \{1, 2, \ldots, n\}$. The buyer designs a finished product, owns the brand, and operates as a make-to-order manufacturer. Each unit of the product needs a component part to make (e.g., the accelerator pedal of a vehicle). The manufacturing of the part is outsourced to the suppliers. Other than this, the rest of the product is manufactured by the buyer. For expositional simplicity, I normalize the buyer's fixed and marginal costs of production to zero. To focus on the component part in concern without being complicated by other details like the buyer's assembly and other parts, I assume that the production quality of the buyer is virtually perfect and the designs of the rest of the product are foolproof. In other words, if a product failure occurs, it must be due to the failure of the component part in concern.

To eliminate any inherent bias toward/against multiple sourcing arising from scale diseconomies/economies, I assume the suppliers have constant marginal costs of production and no fixed costs. Let $c_i > 0$ denote the price quotation provided by supplier i for each unit of the part ordered from it. My analysis focuses on the production quota allocation $\mathbf{Q} = (Q_i)_{i \in N}$ to outsource the part production given the price quotations provided to the buyer, where $Q_i \geq 0$ is the quota for supplier i. Nesting the model into an extended setting to analyze the strategic pricing of the suppliers is left for future research.

For simplicity, I assume that if a latent defect exists in a particular unit of the part, the defect will surely reveal itself through a field failure. The amount of defective parts in Q_i, denoted by D_i, depends on supplier i's production quality (affected by a parameter δ_i), as well as the buyer's design quality (affected by a parameter μ). Specifically, defects occur in a manner following stochastically proportional yield loss, i.e., $D_i = R_i \delta_i Q_i$, where $0 < \delta_i \leq 1$. The random variables R_i's are independently and identically distributed with mean $E(R_i) = \mu$, where $0 < \mu \leq \bar{\mu} << 1$, and variance $\text{var}(R_i) = \sigma^2 > 0$. Intuitively, $R_i\delta_i$ may be referred to as the *random yield loss*, although for convenience I sometimes use this to refer to R_i alone. I refer to $1-\delta_i$ as the *quality-based scoring index* of supplier i, or simply its *quality* level. For analytical convenience, assume δ_i's are distinct. Without loss of generality, I suppose $\delta_1 < \delta_2 < \ldots < \delta_n$ so that supplier 1 has the highest quality level, and other suppliers are ordered accordingly.

Self-reported information collected from the suppliers may be used to determine the parameters δ_i's. Such potentially biased information can be crossed-checked with independent observations from other sources, e.g., direct visits at suppliers' production facilities. Therefore, the δ_i's should be interpreted as the ultimate figures used after adjusting for strategic concerns not explicitly modeled here, rather than self-reported figures taken at face value.

Let $Q > 0$ denote the total quantity of the finished product ordered by and sold to the end customers of the buyer. By definition, a part, or a finished product, with a latent defect cannot be discovered by inspection prior to the sale. Moreover, inventory holding issues are suppressed to focus on other economic forces driving the sourcing decision. Thus, Q is also the total quantity of the part ordered by the buyer and delivered by the suppliers. Feasibility requires $\sum_{i=1}^{n} Q_i = Q$.

Denote by $D = \sum_{i=1}^{n} D_i$ the total amount of defective products sold. Each customer with a defective product will experience a field failure and take the product back for warranty repair. Knowing that the product has a defective part, the buyer finds it in its best interest to apply a costly remedy to address the problem. For simplicity, I assume the fix is effective: after the warranty repair, the product will not fail again.

Suppose that all warranty related costs are proportional to the amount of products returned for repair, i.e., ωD, where $\omega > 0$. Moreover, the defective products will lead to customer dissatisfaction and eventually some additional external failure costs borne by the buyer. For tractability, a quadratic *other external failure cost* function is assumed: $C_E(D) = c_E D^2$, where $c_E > 0$. Similar to the Taguchi (1990) quality loss function, the quadratic cost captures the intuition that as the number of field failures grows, mass media coverage is increasingly more likely, and the resulting reputation damage is increasingly more detrimental. Because the increasing marginal cost of C_E is with respect to D (i.e., the sum of the D_i's), not its individual constituents, the quadratic functional form does not by itself favor multiple sourcing.

Summarized below is the event sequence of the model (with table 1 summarizing the notations used):

1. The suppliers provide price quotations on the per-unit procurement costs c_i's they will charge to the buyer.
2. The buyer outsources the production of the component part to the suppliers, with a production quota Q_i allocated to supplier i.
3. Suppliers manufacture the parts according to the allocated quotas. The ordered amounts, Q_i's, are delivered to the buyer. Out of the amount Q_i, $D_i = R_i\delta_iQ_i$ are defective parts. (The values of D_i's are unobservable to the buyer until later.)
4. The buyer uses the parts to manufacture the finished products and sells them to end customers.
5. Customers who purchased the $D = \sum_{i=1}^{n} D_i$ units of defective products eventually experience field failures and return the products for warranty repair. The values of D_i's become known to the buyer. The buyer incurs warranty related expenses of ωD and suffers reputation damage equivalent to a dollar cost of $C_E(D) = c_E D^2$.

OPTIMAL QUOTA ALLOCATION: SINGLE VERSUS MULTIPLE SOURCING

The buyer's problem is to choose a production quota allocation to minimize the expected total cost of sourcing, i.e., the sum of the procurement cost $\sum_{i=1}^{n} c_iQ_i$ and the expected total external failure cost $E[\omega D + c_E D^2] = \omega\mu[\sum_{i=1}^{n} \delta_iQ_i] + c_E\mu^2[\sum_{i=1}^{n} \delta_iQ_i]^2 + c_E\sigma^2[\sum_{i=1}^{n} \delta_i^2Q_i^2]$ (with the derivation provided in an appendix available upon request). To simplify the expression of this sum, I define the following notations: $\eta \equiv (\mu/\sigma)^2$ and $s_i \equiv (c_i+\omega\mu\delta_i)/c_E\sigma^2$. The parameter η is the *squared standardized mean* of the "random yield loss" R_i, which means η^{-1} is the *squared coefficient of variation*. The parameter s_i is a ratio representing the relative unimportance of the quadratic other external failure cost, characterized by c_E, in constituting the buyer's expected total cost. With these notations, the expected total cost of sourcing can be expressed as

$$C(\mathbf{Q};\, \mathbf{s}) \equiv c_{\mathrm{E}}\sigma^2[\, \textstyle\sum_{i=1}^{n} s_i Q_i + \eta(\, \sum_{i=1}^{n} \delta_i Q_i)^2 + \ \sum_{i=1}^{n} \delta_i^2 Q_i^2], \tag{1}$$

where $\mathbf{s} = (s_i)_{i\in N}$. Clearly, $C(\mathbf{Q};\, \mathbf{s})$ is strictly convex in $\mathbf{Q}$ and linear in $\mathbf{s}$.

The optimization program below formally summarizes the buyer's sourcing problem:

[SB] $$\min_{Q_i} \ C(\mathbf{Q};\, \mathbf{s})$$

subject to the quota constraint QC: $$\textstyle\sum_{i=1}^{n} Q_i = Q$$

with non-negative Q_i's. The choice of the production quotas Q_i's indirectly determines the *supply base* $B = \{\, i \in N \mid Q_i > 0\, \}$, i.e., the set of suppliers selected by the buyer. Let b denote the *size* of the supply base, i.e., the number of selected suppliers. Whether *multiple sourcing* ($b \geq 2$) has advantages over *single sourcing* ($b = 1$), or the other way around, depends on the optimal choice of Q_i's and the resulting size of B. The following result tells us when the protection advantage of multiple sourcing may fail to exist. (Proofs of the results are provided in an appendix available upon request.)

PROPOSITION 1 (CONDITIONS FOR NON-EXISTENCE OF PROTECTION ADVANTAGE OF MULTIPLE SOURCING): *Suppose the ascending ranking of the suppliers based on s_i also has supplier 1 ranked highest. Or, alternatively, suppose that for any distinct j and k with $(\delta_k - \delta_j)(s_k - s_j) \leq 0$,*

$$(s_j - s_k)/(\delta_k - \delta_j) < 2\eta\delta_1 Q. \tag{2}$$

Then multiple sourcing has no advantage over single sourcing if one of the following holds:

(a) *The variance of the "random yield loss" is negligible, i.e., $\sigma^2 \to 0$;*

(b) *The marginal other external failure cost is negligible, i.e., $c_{\mathrm{E}} \to 0$.*

The result of this proposition needs one of two preconditions: either that both the s_i-based and δ_i-based rankings have supplier 1 ranked highest, or that whenever they differ in ranking suppliers j and k, the cardinal difference $s_j - s_k$, relative to $\delta_k - \delta_j$, is not too large. When one of these preconditions holds, the reason for multiple sourcing to be advantageous comes solely from the protection against supplier failure risk due to latent defects. Multiple sourcing becomes unattractive if there is little risk to protect against, or the benefit (i.e., external failure cost saved) from the protection is tiny.

Below I will characterize the unique optimal production quota allocation for the buyer's sourcing problem, identify some intuitive features of the allocation, and derive conditions for determining the exact size of the optimal supply base and whether single or multiple sourcing is optimal. These results are stated in terms of the *quality-adjusted cost-based scoring index* defined as follows:

$$S_i\,(W^*) \equiv \left[\frac{c_i + (\omega + 2c_{\mathrm{E}}\mu\; W^*)\mu\delta_i}{c_{\mathrm{E}}\sigma^2}\right]. \tag{3}$$

It will be clear shortly that the μW^* in the index is simply the expected number of external failures given the optimally allocated production quotas.

If suppliers' qualities are nearly identical (i.e., δ_i's are almost the same), the ranking by $S_i(W^*)$ is not much different from that by c_i. Alternatively, if production costs are equal, $S_i(W^*)$ and δ_i give the same ranking. Even when the costs are unequal, the ranking by $S_i(W^*)$ and by δ_i can still be the same, provided c_i's are "not too unequal." Specifically, this means

$$\max_{i\in\{2,\ldots,n\}}\left(\frac{c_{i-1}-c_i}{\delta_i-\delta_{i-1}}\right) \le (\omega + 2c_{\mathrm{E}}\mu\delta_1 Q)\mu. \tag{4}$$

In words, the condition requires that the cost saving from using a lower-quality supplier is not too attractive given the loss in quality.

The first major result below characterizes the optimal quota allocation without imposing the condition of "not too unequal" costs. Subsequently, it is added to put more structure on the optimal quota allocation.

PROPOSITION 2 (OPTIMAL QUOTA ALLOCATION): *A quantity vector* $\mathbf{Q}^* = (Q_i^*)_{i\in N}$ *is the unique optimal quota allocation for the buyer's sourcing problem if and only if for some* $\theta^* > 0$, *the following marginal conditions are satisfied:*

$$Q_i^* \ge \left[\frac{\theta^* - S_i(W^*)}{2\delta_i^2}\right] \qquad \text{for all } i \in N \tag{5}$$

with the equality holding for all i's in the supply base $B^* \equiv \{\, i\in N \mid Q_i^* > 0 \,\}$, *where*

$$S_i(W^*) \equiv \left[\frac{c_i + (\omega + 2c_{\mathrm{E}}\mu\, W^*)\mu\delta_i}{c_{\mathrm{E}}\sigma^2}\right] \tag{6}$$

is a quality-adjusted cost-based scoring index and θ^* *and* W^* *are given by the following formulas*:

$$\theta^* = \frac{(\eta^{-1}+b^*)[2Q+\sum_{B^*}(s_l/\delta_l^2)] - [\sum_{B^*}(1/\delta_l)][\sum_{B^*}(s_l/\delta_l)]}{(\eta^{-1}+b^*)[\sum_{B^*}(1/\delta_l^2)] - [\sum_{B^*}(1/\delta_l)]^2} \tag{7}$$

$$W^* = \frac{[\sum_{B^*}(1/\delta_l)][2Q+\sum_{B^*}(s_l/\delta_l^2)] - [\sum_{B^*}(1/\delta_l^2)][\sum_{B^*}(s_l/\delta_l)]}{2\eta[\,(\eta^{-1}+b^*)[\sum_{B^*}(1/\delta_l^2)] - [\sum_{B^*}(1/\delta_l)]^2\,]}\ , \tag{8}$$

with $b^* \equiv |B^*|$ *denoting the size of the supply base and* $W^* = \sum_{B^*}\delta_l Q_l^* = \sum_{i=1}^{n}\delta_i Q_i^*$.

This proposition provides a closed-form characterization of the unique optimal quota allocation $\mathbf{Q}^*$. Once the supply base B^* is determined, the optimal quota for a selected supplier i can be computed with the simple formula $Q_i^* = \left[\frac{\theta^* - S_i(W^*)}{2\delta_i^{\,2}}\right]$ whose key constituents, θ^* *and* W^*, are given by another two formulas specified in the proposition. Although the procedure is straightforward, determining the optimal mix of suppliers to constitute the supply base can be prohibitively complex. This is especially so when the number of available suppliers is large. In a different but related setting, Federgruen and Yang (2008) show that a similar combinatorial optimization problem of supplier selection is NP-complete.

However, suppose that the ranking of the suppliers by the *quality-adjusted cost-based scoring index* $S_i(W^*) = [c_i + (\omega+2c_{\mathrm{E}}\mu W^*)\mu\delta_i]/c_{\mathrm{E}}\sigma^2$ is the same as that by δ_i. This would be the case if the differences among the costs c_i's are sufficiently small and the marginal other external failure cost c_{E}, or the production target Q and hence $W^* = \sum_{i=1}^{n}\delta_i Q_i^*$, is sufficiently large. Under such circumstances, the weight attached to the second component of $S_i(W^*)$ will be large enough to let δ_i dominate this scoring index. Consequently, the unique optimal quota allocation $\mathbf{Q}^*$ will have some simple, intuitive properties.

The first property is a positive association between the quality of a selected supplier and the quota assigned to it. That is to say, the higher the quality of a supplier (i.e., with a lower δ_i), the (weakly) larger the quota assigned to it. As a result, if a supplier is selected into the supply base, any suppliers of higher quality will also be selected. These intuitive properties are defined below.

DEFINITION 1: The supply base B^* of the optimal quota allocation $\mathbf{Q}^*$ is *weakly quality-driven* if the selection of a supplier into the supply base implies also the selection of any higher-quality suppliers, i.e., $j \in B^* \Rightarrow j-1 \in B^*$ for all $j \in B^*\backslash\{1\}$, or equivalently $B^* = \{1, 2, \ldots, b^*\}$.

DEFINITION 2: The supply base B^* of the optimal quota allocation $\mathbf{Q}^*$ is *strongly quality-driven* if the quota allocated to a supplier is at least as high as those allocated to any lower-quality suppliers, i.e., $Q_j^* \leq Q_{j-1}^*$ for all $j \in B^*\backslash\{1\}$.

Obviously, a strongly quality-driven B^* is also (weakly) quality-driven but not necessarily the other way around.

PROPOSITION 3 (QUALITY-DRIVEN SUPPLY BASE): *Suppose* $\max_{i\in\{2,\ldots,n\}} [(c_{i-1} - c_i)/(\delta_i - \delta_{i-1})] \leq (\omega + 2c_{\mathrm{E}}\mu\delta_1 Q)\mu$. *Then the supply base B^* of the optimal quota allocation $\mathbf{Q}^*$ is strongly quality-driven. Consequently, it is also (weakly) quality-driven.*

Despite no fixed costs for selecting more suppliers, expanding the supply base can be costly because it means using suppliers of lower quality than the incumbent ones. The increase in this cost as lower-quality suppliers are included into the supply base eventually may limit its size. The next proposition provides a characterization of b^*, the size of the optimal supply base.

PROPOSITION 4 (CONDITION FOR DETERMINING THE SIZE OF THE OPTIMAL SUPPLY BASE): *Suppose* $\max_{i\in\{2,\ldots,n\}} [(c_{i-1} - c_i)/(\delta_i - \delta_{i-1})] \leq (\omega + 2c_{\mathrm{E}}\mu\delta_1 Q)\mu$. *Then the size of the (smallest) optimal supply base is j (i.e., $b^* = j$), where $j \in N\backslash\{n\}$, if and only if there exist positive θ_j and W_j defined by formulas (7) and (8) with B^* substituted by $B_j \equiv \{1, \ldots, j\}$ such that*

$$[\theta_j - S_j(W_j)]/2\delta_j^2 > 0 \geq [\theta_j - S_{j+1}(W_j)]/2\delta_{j+1}^2, \tag{9}$$

where $S_i(W) \equiv [c_i + (\omega+2c_{\mathrm{E}}\mu W)\mu\delta_i]/c_{\mathrm{E}}\sigma^2$. If the condition above is not satisfied by any $j< n$, $b^*=n$.

Determining b^* can be rather complex owing to the combinatorial nature of the supplier selection problem. However, if the precondition of the proposition holds, i.e., the differences between consecutive c_i's are "not too large," then the problem can be reduced to simply comparing the n quality-driven supply bases, i.e., $B_j \equiv \{1, \ldots, j\}$ for $j \in N$. This comparison only requires solving for each j a linear equation system with two unknowns, i.e., θ_j *and* W_j, and then search for the j with the lowest positive value of $[\theta_j - S_j(W_j)]/2\delta_j^2$. Such a problem takes much less time to solve than the original problem.

Suppose one only needs to determine whether single sourcing or multiple sourcing is optimal, without actually identifying the size of the optimal supply base for the latter case. Then the problem can be simplified even without a precondition. Why? Proposition 2 already provides the necessary and sufficient condition for checking whether any given supply base B is optimal. For the case of single sourcing, there are only n such candidate supply bases to check. If none of them is optimal, the optimal arrangement must be multiple sourcing. To check whether a singleton supply base is optimal, it does not require the remaining unselected suppliers to be lined up in certain orders. One only needs to ensure that the buyer cannot be better off by shifting some quota away from a candidate single-sourcing supplier. This result is the next proposition.

PROPOSITION 5 (NECESSARY AND SUFFICIENT CONDITION FOR SINGLE SOURCING TO BE OPTIMAL): *Let* $\theta^h \equiv s_h + 2(1+\eta)Q\delta_h^2$ *and* $W^h \equiv \delta_h Q$ *for all* $h \in N$. *Then single sourcing is optimal if and only if there exists* $h \in N$ *such that*

$$\max_{i\in N\setminus\{h\}} [(\theta^h - S_i(W^h))/2\delta_i^2] \leq 0, \tag{10}$$

where $S_i(W) \equiv [c_i + (\omega+2c_E\mu W)\mu\delta_i]/c_E\sigma^2 = s_i + 2\eta W\delta_i$ and $s_i \equiv (c_i+\omega\mu\delta_i)/c_E\sigma^2$. *The h satisfying the condition above is the only selected supplier of the single-sourcing supply base.*

In the condition of this proposition, there is no counterpart of the $[\theta_j - S_j(W_j)]/2\delta_j^2 > 0$ required in Proposition 4. The reason is that such a requirement is automatically satisfied for the case of single sourcing. Using the definitions of θ^h and W^h, it is easy to verify that $[\theta^h - S_h(W^h)]/2\delta_h^2 = Q > 0$, regardless of the $h \in N$ in concern. The requirement of $0 \geq [\theta_j - S_{j+1}(W_j)]/2\delta_{j+1}^2$ in Proposition 4 as well as its precondition $\max_{i\in\{2,\ldots,n\}} [(c_{i-1} - c_i)/(\delta_i - \delta_{i-1})] \leq \omega\mu + 2\delta_1 Q c_E \mu^2$ are substituted by the condition that $\max_{i\in N\setminus\{h\}} [(\theta^h - S_i(W^h))/2\delta_i^2] \leq 0$ for some h. While this looks more complicated, it does not impose any ordering on the c_i's or restrictions on their differences. In terms of computational complexity, checking the condition requires calculating $n-1$ values of

$[(\theta^h - S_i(W^h))/2\delta_i^2]$ for each $h \in N$. This amounts to a total of $n(n-1)$ calculations, a more complicated task but still manageable within a reasonable time.

I end my analysis with the following comparative statics result on the optimal quota allocation characterized in Proposition 2. This is a general result without requiring "not too unequal" costs. It confirms the intuition of reducing the quota allocated to a supplier if it charges a higher cost.

PROPOSITION 6 (NON-INCREASING RESPONSE OF A SUPPLIER'S QUOTA TO THE COST IT CHARGES): *For any given* $\mathbf{c}_{-i} = (c_j^*)_{j \in N \setminus i}$ *charged by other suppliers, the optimal production quota* Q_i^* *allocated to supplier i is non-increasing in the cost* c_i *charged by the supplier*.

In proving this proposition, it would be nice if some function involved is differentiable. However, this is not obvious. To get around the problem, I apply the technique of supermodularity. This makes the proof simple even without differentiability.

CONCLUDING REMARKS

Sourcing decisions form an important part of supply chain management (Mabert and Venkataramanan 1999). Earlier studies on sourcing decisions concern outsourcing for strategic reasons versus in-house production, this decision's impacts on firm performance, and its linkage with other factors. For example, Narasimhan and Jayaram (1998) empirically investigate, among other things, the causal link between the use of outsourcing for strategic reasons and the degree of manufacturing goal achievement (in terms of cost, flexibility, dependability, and quality). Narasimhan and Das (1999) Examine whether outsourcing strategically can achieve manufacturing flexibilities and result in manufacturing cost reduction. Using an agency model, Chalos and Sung (1998) study the conditions under which outsourcing is strictly preferred to in-house production and also the tradeoff between the incremental coordination costs of outsourcing and the improved incentive structure. More recently, Murthy, Soni, and Ghosh (2004) establish a framework for supplier selection and allocation decisions and provide computational results to show the effectiveness of a heuristic procedure for applying the framework under a variety of scenarios. Although some of the studies above have considered quality issues, none of them focuses on supplier failures arising from undependable products due to latent defects or provides analytical results on the single versus multiple sourcing decision.

Various reasons for favoring single or multiple sourcing have been studied in the literature. They include supplier capacity constraints, saving in outgoing order/incoming inspection/transportation costs, saving in inventory holding costs by shortening the delivery lead time, quantity discounts due to production scale economies, saving in purchasing costs by supplier competition (with or without information asymmetry), and encouraging investment by suppliers to reduce production costs or to improve product quality (see reviews in Berger and Zeng 2006 and Mishra and Tadikamalla 2006).

To highlight the benefit from risk diversification as a reason for multiple sourcing, the model of this paper assumes away other reasons such as capacity constraints. Moreover, certainty in lead time and delivery is assumed to avoid overlapping with prior studies' emphases. To focus on the choice of single versus multiple sourcing, the model also abstracts away from other aspects of supply chain management already extensively studied in the literature, such as coordination for information sharing (e.g., Chung, Talluri, and Narasimhan 2010, Kouvelis, Chambers, and Wang 2006, and Cachon 2003).

I take the perspective of viewing the choice of single versus multiple sourcing as a supply base composition problem. The question asked is about what combination of suppliers can diversify the risk of latent defects most efficiently, in terms of the incremental quality cost to pay as a sacrifice. Instead of formulating a very general model that requires combinatorial mathematical techniques to solve, I structure the model in a tractable stylized fashion, yet rich enough to capture the fundamental economic tradeoff. The tractability of the model provides the potential of using it as a building block to integrate with another model (e.g., Chao et al. 2009 or Arya and Mittendorf 2007) to study interesting questions. An example is the joint use of multiple sourcing and product recall cost sharing to reduce external failure risks. Another is the interplay between internal transfers and external procurement in controlling quality costs in a LUX setting.

Recently, Federgruen and Yang (2008) have examined a general setting of the supplier selection problem that also emphasizes the optimal mix of suppliers. Their focus is to develop an accurate approximation method to overcome the computational complexity of the problem. Dada et al. (2007) and Federgruen and Yang (2009) have considered similarly general settings, with a focus on solving the supplier selection problem with random yields and uncertain demand.

By contrast, I consider a simple setting with no random yields nor uncertain demand but only latent defects. Complementing prior studies, this paper emphasizes the linkage between supplier selection and external failure costs. My model allows a closed-form characterization of the optimal quota allocation through which the main economic driving forces can be clearly seen.

Besides the main analysis presented above (where the buyer is supposed to take the per-unit procurement costs charged by the suppliers as given and allocate production quotas accordingly), I have considered buyer-initiated procurement contracts in an additional analysis (available upon request). There I continue to assume the quality parameters are known to the buyer. However, owing to the changing environments specific to the individual suppliers, they have private information on their unit variable costs of production.

In the additional analysis, the buyer can use full-commitment, take-it-or-leave-it contracts to elicit the private information from the suppliers. The basic setup remains the same, except that the roles of the procurement costs taken as given in the main analysis are substituted by the contractual payments specified in the optimal procurement contracts. With this substitution, all the findings here continue to hold under asymmetric information on the unit variable production costs, provided the *virtual* unit variable production cost (as usually defined in auction design analysis) is non-decreasing in the unit variable production cost privately known to a supplier.

There are several interesting directions for extending the analysis of this paper. A possible extension is to incorporate the buyer's quality improvement effort to raise the design quality level μ and the suppliers' efforts to reduce the defect rate parameter δ_i's. Another is to nest the model into an extended setting to analyze the strategic pricing and competition among the suppliers. A third direction is to identify circumstances where even without the condition of “not too unequal” costs, the optimal supply base is still quality-driven. Given the limited space here, these interesting extensions are left for future research.

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APPENDIX A: DERIVATION AND PROOFS

Derivation of the Expected Total External Failure Cost: To derive $E[\omega D + c_{\mathrm{E}}D^2]$, first note that $E(D^2) = \mathrm{var}(D) + E(D)^2$. Hence,

$$
\begin{aligned}
&E[\omega D + c_{\mathrm{E}}D^2] \\
&\quad = \omega E[D] + c_{\mathrm{E}}E[D^2] \\
&\quad = \omega E[D] + c_{\mathrm{E}}E[D]^2 + c_{\mathrm{E}}\mathrm{var}[D] \\
&\quad = \omega E[\textstyle\sum_{i=1}^n D_i] + c_{\mathrm{E}}(E[\sum_{i=1}^n D_i])^2 + c_{\mathrm{E}}\mathrm{var}[\sum_{i=1}^n D_i] \\
&\quad = \omega \textstyle\sum_{i=1}^n E[D_i] + c_{\mathrm{E}}(\sum_{i=1}^n E[D_i])^2 + c_{\mathrm{E}}\sum_{i=1}^n \mathrm{var}[D_i] \\
&\quad = \omega \textstyle\sum_{i=1}^n \mu\delta_i Q_i + c_{\mathrm{E}}[\sum_{i=1}^n \mu\delta_i Q_i]^2 + c_{\mathrm{E}}\sum_{i=1}^n \sigma^2(\delta_i Q_i)^2 \\
&\quad = \omega\mu[\textstyle\sum_{i=1}^n \delta_i Q_i] + c_{\mathrm{E}}\mu^2[\sum_{i=1}^n \delta_i Q_i]^2 + c_{\mathrm{E}}\sigma^2[\sum_{i=1}^n \delta_i^2 Q_i^2].
\end{aligned}
$$

Q.E.D.

Proof of Proposition 1 (Conditions for Non-existence of Protection Advantage of Multiple Sourcing): When one of the two conditions holds, i.e., either $\sigma^2 \to 0$ or $c_{\mathrm{E}} \to 0$, the buyer's expected total cost becomes $c_{\mathrm{E}}\mu^2[\sum_{i=1}^n \Delta_i Q_i + (\sum_{i=1}^n \delta_i Q_i)^2]$ or simply $\sum_{i=1}^n (c_i+\omega\mu\delta_i)Q_i$, where $\Delta_i \equiv (c_i+\omega\mu\delta_i)/c_{\mathrm{E}}\mu^2$. Suppose the ascending ranking of the suppliers based on Δ_i's also has supplier 1 ranked highest. Then obviously setting $Q_1 = Q$ minimizes $\sum_{i=1}^n \delta_i Q_i$ as well as $\sum_{i=1}^n \Delta_i Q_i$ individually. Consequently, the expected total cost must also be minimized when $Q_1 = Q$. Thus, multiple sourcing cannot be better than single sourcing.

Alternatively, suppose that for any distinct j and k with $(\delta_k - \delta_j)(s_k - s_j) \le 0$, $(s_j - s_k)/(\delta_k - \delta_j) < 2\eta\delta_1 Q$. Then if multiple sourcing is better than single sourcing, the supply base must not contain any j and n with the property above. Otherwise, assuming without loss of generality that $\delta_j < \delta_k$, I can rearrange the allocation by shifting some amount of Q_k to Q_j and thereby reducing the sum $\sum_{i=1}^n \Delta_i Q_i + (\sum_{i=1}^n \delta_i Q_i)^2$ in the expected total cost.

To see this, simply differentiate the sum with respect to Q_i to get the derivative $\Delta_i + 2\delta_i(\sum_{i=1}^n \delta_i Q_i)$. Note that $s_i \equiv (c_i+\omega\mu\delta_i)/c_{\mathrm{E}}\sigma^2 = \eta\Delta_i$. So for $\delta_j < \delta_k$, $(s_j-s_k)/(\delta_k-\delta_j) < 2\eta\delta_1 Q$ implies $(\Delta_j-\Delta_k) < 2(\delta_k-\delta_j)\delta_1 Q < 2(\delta_k-\delta_j)(\sum_{i=1}^n \delta_i Q_i)$. Hence,

$$\Delta_j + 2\delta_j(\textstyle\sum_{i=1}^n \delta_i Q_i) < \Delta_k + 2\delta_k(\sum_{i=1}^n \delta_i Q_i),$$

implying that shifting some amount of Q_k to Q_j will reduce the expected total cost further. This leads to the conclusion that any multiple-sourcing supply base must include only suppliers with δ_i's and s_i's showing exactly the same ranking.

However, with such a ranking of the selected suppliers, the expected total cost can be minimized with Q assigned solely to the supplier ranked highest in the supply base, i.e., the one with the lowest baseline defect rate among the suppliers selected. This contradicts the initial supposition that multiple sourcing can be better than single sourcing if $(s_j - s_k)/(\delta_k - \delta_j) < 2\delta_1 Q$ for any distinct j and k with $(\delta_k - \delta_j)(s_k - s_j) \le 0$. *Q.E.D.*

Proof of Proposition 2 (Optimal Quota Allocation): The existence of an optimal quota allocation is guaranteed because any feasible allocation must be from the closed and bounded domain $[0,Q]^n$ and the objective function and constraints of the optimization problem are concave and linear, respectively. The following is the Lagrangian of program SB (with constraint QC decomposed into two inequality constraints):

$$\boldsymbol{L} = -c_{\mathrm{E}}\sigma^2[\sum_{i=1}^n s_iQ_i + \eta(\sum_{i=1}^n \delta_iQ_i)^2 + \sum_{i=1}^n \delta_i^2Q_i^2] + \overline{\theta}(\sum_{i=1}^n Q_i - Q) + \underline{\theta}(Q - \sum_{i=1}^n Q_i)$$

with $\eta \equiv (\mu/\sigma)^2$ and $s_i \equiv (c_i+\omega\mu\delta_i)/c_{\mathrm{E}}\sigma^2$. Since $\boldsymbol{L}$ is strictly concave in $\mathbf{Q} = (Q_i)_{i\in N}$, a $\mathbf{Q}^*$ is the unique optimal quota allocation for the program if and only if the first-order conditions of the program are satisfied (see Takayama 1985, Chapter 1, Section D).[1]

Differentiating the Lagrangian with respect to Q_i yields the first-order partial derivative below:

$$\boldsymbol{L}_i = \overline{\theta} - \underline{\theta} - c_{\mathrm{E}}\sigma^2[s_i + 2\eta\delta_i(\sum_{h=1}^n \delta_hQ_h) + 2\delta_i^2Q_i].$$

The first-order conditions require that if $\mathbf{Q}^*$ has some $Q_i^* > 0$, then $\mathbf{Q}^*$ has to satisfy the equation $\boldsymbol{L}_i = 0$ for some $\overline{\theta} \geq 0$ and $\underline{\theta} \geq 0$. These $\overline{\theta}$ and $\underline{\theta}$ must be the same for all such i's with $Q_i^* > 0$. In addition, if $\mathbf{Q}^*$ has some $Q_j^* = 0$, then $\mathbf{Q}^*$ has to satisfy the inequality $\boldsymbol{L}_j \leq 0$ for any such j's for the same $\overline{\theta}$ and $\underline{\theta}$. Moreover, $\mathbf{Q}^*$ must satisfy constraint QC.

As some Q_i^* has to be positive, so must the difference $\overline{\theta} - \underline{\theta}$. Define $\theta^* = (\overline{\theta} - \underline{\theta})/c_{\mathrm{E}}\sigma^2$. The first-order conditions are equivalent to the following ones:

$$\sum_{i=1}^n Q_i^* = Q$$

and some $\theta^* > 0$ exists such that

$$[\mathrm{MC}_i]: \qquad \theta^* \leq s_i + 2\eta\delta_i(\sum_{h=1}^n \delta_hQ_h^*) + 2\delta_i^2Q_i^* \qquad \forall\, i \in N$$

with the equality holding for all $i \in B^* \equiv \{\, i\in N \mid Q_i^* > 0 \,\}$. Another expression of the marginal condition MC_i is as follows:

$$Q_i^* \geq \left[\frac{\theta^* - S_i(W^*)}{2\delta_i^{\,2}}\right]$$

with $S_i(W^*) = [c_i + (\omega+2c_{\mathrm{E}}\mu W^*)\mu\delta_i]/c_{\mathrm{E}}\sigma^2$ and $W^* \equiv \sum_{i=1}^n \delta_iQ_i^* = \sum_{B^*} \delta_lQ_l^*$.

To determine the values of θ^* and W^*, I divide MC_i by $2\delta_i$ and then sum over the equality marginal conditions, i.e., MC_i's $\forall i\in B^*$. This yields the following equation of θ^* and W^*:

$$\theta^*\,[(1/2)\sum_{B^*}(1/\delta_l)] = [(1/2)\sum_{B^*}(s_l/\delta_l)] + W^*\,[1 + \eta b^*],$$

where $b^* \equiv |B^*|$ is the size of the supply base. Similarly, divide MC_i by $2\delta_i^2$ and sum over the equality marginal conditions. Then incorporate the quota constraint, QC. This gives a second equation of θ^* and W^*:

$$\theta^*\,[(1/2)\sum_{B^*}(1/\delta_l^2)] = [Q + (1/2)\sum_{B^*}(s_l/\delta_l^2)] + W^*\,[\eta\sum_{B^*}(1/\delta_l)].$$

The solution of the two equations is as follows:

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[1] Takayama, A. 1985. *Mathematical Economics, Second Edition*. Cambridge: Cambridge Univ. Press.

$$\theta^* = \frac{(\eta^{-1}+b^*)[2Q+\sum_{B^*}(s_l/\delta_l^2)] - [\sum_{B^*}(1/\delta_l)][\sum_{B^*}(s_l/\delta_l)]}{(\eta^{-1}+b^*)[\sum_{B^*}(1/\delta_l^2)] - [\sum_{B^*}(1/\delta_l)]^2}$$

$$W^* = \frac{[\sum_{B^*}(1/\delta_l)][2Q+\sum_{B^*}(s_l/\delta_l^2)] - [\sum_{B^*}(1/\delta_l^2)][\sum_{B^*}(s_l/\delta_l)]}{2\eta[(\eta^{-1}+b^*)[\sum_{B^*}(1/\delta_l^2)] - [\sum_{B^*}(1/\delta_l)]^2]} .$$

In summary, the first-order conditions imply the conditions specified in this proposition. The reverse also holds with $\overline{\theta}$ set to $c_{\mathrm{E}}\sigma^2\theta^*$ and $\underline{\theta}$ set to zero. *Q.E.D.*

PROOF OF PROPOSITION 3 (QUALITY-DRIVEN SUPPLY BASE): For any multiple-sourcing supply base B^*, let supplier j be a selected supplier other than the highest-quality supplier in the supply base. Suppose

$$\max_{i\in\{2,\ldots,n\}} [(c_{i-1} - c_i)/(\delta_i - \delta_{i-1})] \le (\omega + 2c_{\mathrm{E}}\mu\delta_1 Q)\mu.$$

Because $W^* = \sum_{B^*} \delta_l Q_l^* \ge \delta_1 Q$,

$$(c_{j-1} - c_j)/(\delta_j - \delta_{j-1}) \le \omega\mu + 2\delta_1 Q c_{\mathrm{E}}\mu^2 \le \omega\mu + 2W^* c_{\mathrm{E}}\mu^2.$$

Hence, $c_j + (\omega\mu + 2W^*c_{\mathrm{E}}\mu^2)\delta_j \ge c_{j-1} + (\omega\mu + 2W^*c_{\mathrm{E}}\mu^2)\delta_{j-1}$, or equivalently,

$$S_j(W^*) \ge S_{j-1}(W^*),$$

where $S_i(W^*) = [c_i + (\omega+2c_{\mathrm{E}}\mu W^*)\mu\delta_i]/c_{\mathrm{E}}\sigma^2$. By Proposition 2,

$$Q_j^* = [\theta^* - S_j(W^*)]/2\delta_j^2 > 0 \text{ and}$$
$$Q_{j-1}^* \ge [\theta^* - S_{j-1}(W^*)]/2\delta_{j-1}^2.$$

Thus, $Q_{j-1}^* \ge [\theta^* - S_{j-1}(W^*)]/2\delta_j^2 \ge [\theta^* - S_j(W^*)]/2\delta_j^2 = Q_j^* > 0$. Since $b^* \equiv |B^*|$, it has to be that $B^* = \{1, 2, \ldots, b^*\}$. *Q.E.D.*

PROOF OF PROPOSITION 4 (CONDITION FOR DETERMINING THE SIZE OF THE OPTIMAL SUPPLY BASE): Suppose $\max_{i\in\{2,\ldots,n\}} [(c_{i-1} - c_i)/(\delta_i - \delta_{i-1})] \le (\omega + 2c_{\mathrm{E}}\mu\delta_1 Q)\mu$. If the size of the optimal supply base is $j \in N\backslash\{n\}$, Proposition 3 implies $B^* = \{1, \ldots, j\}$. Consequently, Proposition 2 implies the existence of positive θ^* and W^*, as defined by formulas (7) and (8), such that

$$Q_j^* = [\theta^* - S_j(W^*)]/2\delta_j^2 > 0 \text{ and}$$
$$0 = Q_{j+1}^* \ge [\theta^* - S_{j+1}(W^*)]/2\delta_{j+1}^2.$$

Define $\theta_j = \theta^*$ and $W_j = W^*$. The condition of this proposition is thus satisfied.

For the "if" part, suppose there exist positive θ_j and W_j defined by formulas (7) and (8) with B^* substituted by $B_j \equiv \{1, \ldots, j\}$ such that $[\theta_j - S_j(W_j)]/2\delta_j^2 > 0 \ge [\theta_j - S_{j+1}(W_j)]/2\delta_{j+1}^2$. Define $\theta^* = \theta_j$ and $W^* = W_j$. Then θ^* and W^* by construction satisfy formulas (7) and (8) for $B^* = B_j$. Moreover, define a quota allocation $\mathbf{Q}^*$ with $Q_i^* = [\theta^* - S_i(W^*)]/2\delta_i^2$ for all $i \le j$ and $Q_i^* = 0$ for all $i > j$. Because $\max_{i\in\{2,\ldots,n\}} [(c_{i-1} - c_i)/(\delta_i - \delta_{i-1})] \le \omega\mu + 2\delta_1 Q c_{\mathrm{E}}\mu^2$, a procedure similar to the proof of Proposition 3 will show that $S_{i+1}(W^*) \ge S_i(W^*)$ for all $i \in N\backslash\{n\}$. Consequently, $[\theta^* - S_j(W^*)]/2\delta_j^2 > 0$ implies $Q_i^* = [\theta^* - S_i(W^*)]/2\delta_i^2 > 0$ for all $i \le j$. Similarly, $0 \ge [\theta^* -$

$S_{j+1}(W^*)]/2\delta_{j+1}^2$ implies $Q_i^* = 0 \geq [\theta^* - S_i(W^*)]/2\delta_i^2$ for all $i \geq j+1$. Thus, the marginal conditions in Proposition 2 are satisfied by the $\mathbf{Q}^*$ defined above. This means it is the unique optimal allocation for the buyer's sourcing problem, and B_j is the optimal supply base. Hence, $b^* = j$.

Q.E.D.

PROOF OF PROPOSITION 5 (CONDITION FOR SINGLE SOURCING TO BE OPTIMAL): The "only if" part follows directly from Proposition 2. For the "if" part, it is straightforward to verify that for each $h \in N$, the θ^h and W^h defined in this proposition satisfy the formulas (7) and (8) of Proposition 2 for the single-sourcing supply base $B^h \equiv \{h\}$. Define for each h a quota allocation $\mathbf{Q}^h$ with $Q^h{}_h \equiv [\theta^h - S_h(W^h)]/2\delta_h^2$ and $Q^h{}_i \equiv 0$ for all $i \in N\backslash\{h\}$. Because $\theta^h \equiv s_h + 2(1+\eta)Q\delta_h^2$ and $W^h \equiv \delta_h Q$, clearly $Q^h{}_h = Q > 0$.

To see if one or none of the B^h's is the optimal supply base, it suffices to check whether one or none of them meets the remaining marginal conditions of Proposition 2, i.e., for any given h,

$$Q^h{}_i \geq [(\theta^h - S_i(W^h))/2\delta_i^2 \text{ for all } i \in N\backslash\{h\}.$$

Some h will meet this last requirement if the condition of the proposition is fulfilled. When this is the case, the $\{h\}$ is the optimal supply base and single sourcing is optimal. *Q.E.D.*

PROOF OF PROPOSITION 6 (NON-INCREASING RESPONSE OF A SUPPLIER'S QUOTA TO THE COST IT CHARGES): Because of the quota constraint, the feasible choice set defined on $\mathbf{Q}$ is not a sublattice of $\Re^n$. This prevents applying some supermodularity result directly. To get around the problem, I consider an equivalent formulation of the buyer's sourcing problem as follows.

First, the buyer takes Q_i as given and choose $\mathbf{Q}_{-i} = (Q_j^*)_{j\in N\backslash i}$ to minimize the part of the expected total cost that depends on $\mathbf{Q}_{-i}$, i.e.,

$$C_{-i}(\mathbf{Q}_{-i}; Q_i) = \textstyle\sum_{j\in N\backslash i} c_j Q_j + \omega\mu[\sum_{i=1}^n \delta_i Q_i] + c_E\mu^2[\sum_{i=1}^n \delta_i Q_i]^2 + c_E\sigma^2[\sum_{i=1}^n \delta_i^2 Q_i^2],$$

subject to the constraint that $\sum_{j\in N\backslash i} Q_j \geq Q - Q_i$. This constraint is equivalent to the quota constraint because $C_{-i}(\mathbf{Q}_{-i}; Q_i)$ being increasing in $\mathbf{Q}_{-i}$ ensures that the inequality constraint will bind at optimum. This first-stage minimization problem is analogous to the original problem. Following the same proof as for Proposition 2, I can show that a unique minimizer $\mathbf{Q}_{-i}^*(Q_i)$ exists. By the maximum theorem (Sundaram 1996, p. 235), the minimized value $C_{-i}^*(Q_i) = C_{-i}(\mathbf{Q}_{-i}^*(Q_i); Q_i)$ is continuous in Q_i.[2] The non-negativity of $Q_j^*(Q_i)$'s and Q_i ensures that $C_{-i}^*(Q_i)$ is non-negative and hence bounded from below. Moving on to the second stage, the buyer chooses $Q_i \in [0, Q]$ to minimize $c_i Q_i + C_{-i}^*(Q_i)$. A minimizer Q_i^* clearly exists.

The two-stage formulation above is equivalent to the original problem, which has a unique solution characterized in Proposition 2. The quota allocation $(Q_i^*, \mathbf{Q}_{-i}^*(Q_i^*))$ obtained

[2] Sundaram, R. K. 1996. *A First Course in Optimization Theory*. Cambridge University Press.

from the two-stage formulation must be the same unique solution. In order to apply below a theorem directly, it is convenient to define the following: $\theta \equiv -c_i$, $x \equiv Q_i$, and $f(x, \theta) \equiv \theta x - C_{-i}^*(x)$. Note that for any (x, θ) and (x', θ') with $x \geq x'$ and $\theta \geq \theta'$,

$$
\begin{aligned}
&f(x, \theta) - f(x', \theta) \\
&\qquad = [\theta x - C_{-i}^*(x)] - [\theta x' - C_{-i}^*(x')] \\
&\qquad = (\theta x - \theta x') - (\theta' x - \theta' x') + [\theta' x - C_{-i}^*(x)] - [\theta' x' - C_{-i}^*(x')] \\
&\qquad = (\theta - \theta')(x - x') + f(x, \theta') - f(x', \theta') \\
&\qquad \geq f(x, \theta') - f(x', \theta'),
\end{aligned}
$$

with the inequality holding strictly whenever $x > x'$ and $\theta > \theta'$. So f satisfies strictly increasing differences in (x, θ). By Theorem 10.6 of Sundaram (1996, p. 258), the x^* maximizing f over the feasible set $[0, Q] \subset \Re$ is non-decreasing in θ. In other words, Q_i^* is non-increasing in c_i given $\mathbf{c}_{-i} = (c_j^*)_{j \in N \backslash i}$. *Q.E.D.*

Table 1: Notations

N	=	$\{1, 2, \ldots, n\}$ is the index set of the n suppliers ($n \geq 2$).
Q_i	=	the production quota allocated to supplier i ($Q_i \geq 0$).
Q	=	$\sum_{i=1}^{n} Q_i$ is the total procurement quantity ($Q > 0$).
$\mathbf{Q}$	=	$(Q_i)_{i\in N}$ is the production quota allocation.
c_i	=	per-unit procurement cost charged by supplier i.
D_i	=	$R_i\delta_i Q_i$ is the amount of defective parts manufactured by supplier i.
R_i	$\geq$	0, or more precisely $R_i\delta_i$, is referred to as the *random yield loss* of supplier i. The random variables R_i's are independently and identically distributed with mean $E(R_i) = \mu$, where $0 < \mu \leq \bar{\mu} << 1$, and variance $\mathrm{var}(R_i) = \sigma^2 > 0$.
η	=	$(\mu/\sigma)^2$ is the *squared standardized mean* of the "random yield loss" R_i, or equivalently, η^{-1} is referred to as the *squared coefficient of variation*.
δ_i	>	0 is a parameter affecting the random yield loss of supplier i ($\delta_i \leq 1$). The value $1-\delta_i$ is referred to as the *quality-based scoring index* of the supplier, or simply its *quality* level. It is assumed that $\delta_1 < \delta_2 < \ldots < \delta_n$.
D	=	$\sum_{i=1}^{n} D_i$ is the total amount of defective products sold to end customers by the buyer. It becomes observable after the customers have experienced field failures of the products and take them back for warranty repair.
ω	>	0 is the constant marginal cost of warranty repair.
$C_{\mathrm{E}}(D)$	=	$c_{\mathrm{E}}D^2$ is the *other external failure cost* (e.g., reputation damage) borne by the buyer, in addition to the warranty-related cost, as a result of the defective products sold to customers (where $c_{\mathrm{E}} > 0$).
s_i	=	$(c_i+\omega\mu\delta_i)/c_{\mathrm{E}}\sigma^2$ is a ratio representing the relative unimportance of the quadratic other external failure cost, characterized by c_{E}, in constituting the buyer's expected total cost.
$S_i(W^*)$	=	$[c_i + (\omega+2c_{\mathrm{E}}\mu W^*)\mu\delta_i]/c_{\mathrm{E}}\sigma^2 = = s_i + 2\eta W^*\delta_i$ is referred to as the *quality-adjusted cost-based scoring index* for supplier i, evaluated at $W^* = \sum_{i=1}^{n} \delta_i Q_i^*$ based on the optimal quota allocation $\mathbf{Q}^* = (Q_i^*)_{i\in N}$.
B	=	$\{\, i\in N \mid Q_i > 0 \,\}$ is the set of selected suppliers constituting the supply base.
b	=	$\lvert B\rvert$ is the size of the supply base B.
$C(\mathbf{Q}; \mathbf{s})$	=	$c_{\mathrm{E}}\sigma^2[\sum_{i=1}^{n} s_i Q_i + \eta(\sum_{i=1}^{n} \delta_i Q_i)^2 + \sum_{i=1}^{n} \delta_i^2 Q_i^2]$, where $\mathbf{s} = (s_i)_{i\in N}$, is the buyer's expected total cost of sourcing.