On the Redistributional Effects of Long-Run Inflation in a Cash-in-Advance Economy

Kakar, Venoo

San Franciso State University

3 May 2014

On the Redistributional Effects of Long-Run Inflation in a Cash-in-Advance Economy*

Venoo Kakar†
San Francisco State University

April 9, 2014

Abstract

This paper analyses the redistributional effects of long-run inflation on income, wealth and consumption in the United States in a model economy with heterogeneous agents where money is introduced via a cash-in-advance constraint. In the case with transfers, we find that consumption inequality reduces as inflation increases since the low income households hold a relatively higher cash-wealth ratio. The bottom 60% of the population gains and the top 40% loses. In the case without transfers, we find that all income groups lose with the losses being more pronounced in the low income households.

Keywords: Consumption, Inequality, Inflation, Heterogeneity
JEL classification: E4, E5, E21, E31, E52

*I would like to thank Richard M. H. Suen for his invaluable advice and help throughout the progress of this paper. I also thank Jang-Ting Guo, Saqib Jafarey and Neil R. Ericsson and conference participants at the 20th Symposium of the Society of Nonlinear Dynamics and Economterics (SNDE), 2012 and Georgetown Center for Economic Research Conference, 2013 for their insightful suggestions. Any remnant errors are my own.

†Department of Economics, San Francisco State University, CA 94132. E-mail: vkakar@sfsu.edu

1 Introduction

With over three rounds of quantitative easing by the Federal Reserve to combat the Great Recession of 2007-09, policymakers have raised serious concerns about the impact of the rise in future inflation. Much of the existing literature has discussed how rising inflation impacts the economy as a whole. These studies measure the welfare costs of inflation and find that in general, a rise in long-run inflation reduces social welfare. In particular, households would give up some consumption to achieve zero inflation from a moderate level of inflation. This has been quantified as being less than 1% of consumption which is a fairly small cost. However, this literature has largely ignored the distributional impact of inflation on different income groups. There are only very few studies that have addressed this issue. For instance, Easterly and Fischer (2001) use polling data for a large number of households in thirty-eight countries and find evidence that inflation is a relatively bigger concern for low-income households than high-income households. They report that the change in low-income households' share of national income, the percent decline in poverty and other measures of improvements in their well-being are negatively correlated with inflation[1]. Amongst the recent quantitative and sectoral studies, Doepke and Schneider (2006) suggest that in the United States, a moderate episode of inflation causes significant redistribution of wealth amongst rich, middle-class and poor households[2]. They find that in a 5 percent inflation experiment, a coalition of rich and old households loses, in present-value terms, between 5.7 and 15.2 percent of GDP. They also find that about two-thirds of this loss accrues to households in the top 10 percent of the wealth distribution. On the winners side, about 75 percent of the total gains in the household sector benefit middle-class households under the age of 45, which receive a gift worth up to 45 percent of mean cohort net worth.

The focus of this paper is to analyze the redistributional effects of long-run inflation among different income brackets in the United States. Our paper builds on previous studies. Similar to Stockman (1981), we introduce money into the model via a cash-in-advance constraint that applies to consumption and investment. However, we extend his representative agent framework to allow for consumer heterogeneity so that we can assess the distributional impact of inflation. In our model, consumer heterogeneity in labor productivity and subjective discount factors is introduced amongst ten income groups. The labor productivity and subjective discount factors are chosen to match the income and wealth distribution in the United States in 2007. As an another interesting feature of the U.S. economy, we introduce progressive tax structure[3] into our model, following Li and Sarte (2004) and Lansing and Guo (1998). Moreover, this modeling assumption ensures that not all households eventually face the highest marginal tax rate simply as a result of economic growth[4]. Long-run

[1] They examine inflation's effects on the poor in two ways. First, by using a global survey which asked whether individuals think inflation is an important national problem. Second, by assessing the effects of inflation on direct measures of inequality and poverty in various cross-country and cross-time samples.

[2] They emphasize the role of money as a unit of account for assets and liabilities: inflation affects all nominal asset positions, not just cash positions. As a result, they find that even moderate inflation leads to substantial wealth redistribution

[3] In particular, a statutory tax schedule is said to be progressive whenever the marginal rate exceeds the average rate at all levels of income.

[4] As in Sarte (1997), a progressive tax schedule helps avoid the kind of degenerate equilibrium as analyzed in Becker (1980).

inflation, which results from lump-sum monetary injections by the central bank, alters the distribution of disposable income, of wealth and of consumption.

There are only a few studies that analyze the effects of inflation on redistribution of income and wealth in a heterogenous-agent economy. Broadly, these can be categorized into three areas: studies with cash-in advance models Imrohoroglu (1992),Erosa and Ventura (2002)); studies using matching models of money (Molico (2006), Boel and Camera (2009), Chiu and Molico (2010)); and models where money plays a precautionary role (Akyol (2004) and Wen (2010)).

The studies typically differ in the way they introduce money. However, most of these studies do introduce asset(s) so that agents can protect themselves against inflation and money is valued because agents can self-insure against some idiosyncratic shock. In Cash-in-advance (CIA) models, however, agents are not able to switch from holding money to holding assets. Thus, welfare costs could be higher in these models. The welfare losses predicted by these studies differ significantly. For instance, Wen (2010) introduces money as having a precautionary role and reports that to avoid a 10% increase in inflation, agents reduce consumption by 8%. In another study by Erosa and Ventura (2002) in a CIA economy with cash and credit goods, 10% inflation is worth 1.6% consumption[5]. Boel and Camera (2009) obtain similar results although they introduce money in a matching model. This suggests that the financial structure of the economy is important in analyzing these effects. For instance, Akyol (2004) reports that 10% inflation maximizes social welfare, whereas another study by Chiu and Molico (2010) reports that 10% inflation is worth 0.6% of average consumption in the U.S.. Imrohoroglu (1992) considers a pure exchange cash-in-advance economy with idiosyncratic endowment risk. He finds that inflation lowers welfare, but the area below the money demand curve, underestimates the welfare cost by several times. Camera and Chien (2011) report that when shocks are sufficiently persistent, moderate inflations can generate welfare gains whereas large inflations are always costly. They offer several insights about the impact of long-run inflation on key macroeconomic variables and suggest that disparities in earlier results can be reconciled with disparities in either the assumed financial structure or in the persistence of shocks. They report that when inflation is generated through lump-sum money creation, higher inflation lowers inequality in disposable income, but it permanently reduces overall income and, hence, depresses aggregate consumption. Therefore, inflation can improve average welfare only if it is capable to sufficiently reduce consumption inequality, which is zero in the efficient allocation.

We find that consumption inequality reduces as inflation increases. In general, we find that as inflation rises, the bottom 60% of the population gains and the top 40% loses. This phenomenon is more pronounced in the bottom 20% of the population. Even though the top 40% of the distribution loses, their consumption patterns do not change as much because of their large wealth holdings.

The rest of the paper is organized as follows: Section 2 introduces the model, Section 3 discusses the competitive equilibrium, Section 4 describes our calibration exercise, Section 5 presents our findings and Section 6 concludes. The derivations and tables can be found in

[5]They find that inflation may lead to a substantial concentration in the distribution of wealth and, assuming increasing returns to scale from credit transactions, inflation acts as a regressive consumption tax because low-income agents use mostly cash for trade.

Appendix A.

2 The Model

2.1 Preferences

We consider a model economy which is populated by a continuum of infinitely lived households. The size of the population is normalized to one. The population is divided into s groups where the size of each group i in total population is denoted by $\mu_i \in (0,1)$, for $i \in \{1,2,...,s\}$ and $\sum_{i=1}^{s}\mu_i = 1$. The groups differ from each other in terms of labor productivity and rate of time preference. Agents within each group are identical. The labor productivity e_i and discount factor $\beta_i \in (0,1)$ of a type i individual is deterministic and known for each of the s groups. The preferences of a typical agent in group i is given by:

$$\sum_{t=0}^{\infty} \beta_i^t u(c_{i,t}), \tag{1}$$

where $c_{i,t}$ is the consumption of an individual in group i at time t. The (period) utility function u (c) is identical for all types of consumers and is given by

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma} \; if \; \sigma \neq 1; \sigma \geq 0$$

2.2 The Agent's Problem

Agents receive total taxable income $y_{i,t}$ which is composed of labor income from work and interest income from savings. The agent can hold two types of assets: real assets denoted by $a_{i,t}$ that give a rate of return r_t and nominal money holding at the beginning of period t denoted by $M_{i,t}$.The real asset depreciates at a rate $\delta \in [0,1]$. There is a progressive tax τ_t which is a function in total taxable income $y_{i,t}$. The properties of the tax schedule are discussed in detail in Section 2.4. The fraction of investment which is subject to the cash-in-advance constraint is controlled by $\psi \in [0,1]$, as in Dotsey and Sarte (2000). The values $\psi = 0$ and $\psi = 1$ correspond to the Clower (1967) and Stockman (1981) versions, respectively. When $\psi = 0$, the CIA constraint only applies on consumption purchases and when $\psi = 1$, the CIA constraint applies to both consumption and investment. Given a sequence of wages (w_t), rental rate of capital (r_t) and price level (P_t) at time t, the agents' problem is to maximize their discounted lifetime utility, subject to sequences of budget constraints and CIA constraints. Formally, a type i household solves:

$$\max_{\{c_{i,t},a_{i,t+1},M_{i,t+1}\}} \sum_{t=0}^{\infty} \beta_i^t u(c_{i,t}) \; where \; \beta_i \in (0,1) \tag{2}$$

s.t.

$$c_{i,t} + \psi[a_{i,t+1} - a_{i,t}] \leq \frac{M_{i,t}}{P_t}, \tag{3}$$

and

$$c_{i,t} + a_{i,t+1} - a_{i,t} + \frac{M_{i,t+1}}{P_t} = y_{i,t} - \tau_t(y_{i,t}) + \frac{M_{i,t}}{P_t} + \zeta_t \ , \tag{4}$$

where $w_t e_i + r_t a_{i,t} = y_{i,t}$. The timing of production and trade follows that of the cash-in-advance economy described in Stockman (1981). Each agent allocates his/her income between money and asset holdings. (3) represents the liquidity or the CIA constraint which states that the individual must be able to finance his purchases of current consumption and gross investment out of money balances carried over from the previous period plus transfers received at the beginning of the period.

Let real money holdings on period t and real transfers in period t be denoted by $m_{i,t}$ *and* ζ_t, respectively. Let the gross inflation factor be defined as $\pi_{t+1} = P_{t+1}/P_t$. Also, let $\lambda_{i,t}$ *and* $\theta_{i,t}$ denote the Lagrangian multipliers on the CIA and budget constraints, respectively.

Then, the first order conditions of the agents' problem with respect to $c_{i,t}$, $m_{i,t+1}$ and $a_{i,t+1}$ are given by the following, respectively:

$$u'(c_{i,t}) = \lambda_{i,t} + \theta_{i,t} \ , \tag{5}$$

$$\beta_i(\lambda_{i,t+1} + \theta_{i,t+1}) = \pi_{t+1}\theta_{i,t} \ , \tag{6}$$

$$\beta_i[\psi\lambda_{i,t+1} + \theta_{i,t+1}\{1 + r_{t+1}(1 - \tau'_{t+1}(y_{i,t+1}))\}] = \psi\lambda_{i,t} + \theta_{i,t} \ , \tag{7}$$

(5) equates the marginal utility of current consumption to the marginal cost of current consumption which is the marginal indirect utility of having an additional real dollar. (6) equates the marginal value of having an additional nominal dollar at the beginning of the next period, deflated by the gross inflation factor, to the marginal cost of having that additional dollar. (7) equates the marginal benefit of an additional unit of capital which consists of the discounted value of goods it produces next period to the marginal cost of an additional unit of capital. Then, the agents' first order conditions are combined to yield the following Euler equation:

$$\psi u^{'}(c_{i,t}) = [\beta_i\psi - (1-\psi)\frac{\beta_i}{\pi_{t+1}}]u'(c_{i,t+1}) + \beta_i^2\{\frac{1}{\pi_{t+1}}(r_{t+1} - \tau'_{t+1}(y_{i,t+1})r_{t+1} + 1) - \frac{\psi}{\pi_{t+2}}\}u'(c_{i,t+2}). \tag{8}$$

This equation governs the law of motion for consumption. The Euler equation can be interpreted as follows: the marginal cost of foregoing one unit of consumption at time t, is equal to the discounted marginal benefit the agent receives from consuming in period t+1 and t+2. The discounted marginal benefit is also deflated by next period's inflation.

2.3 Production

Output is produced according to the standard neoclassical production function:

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}, \alpha \in (0, 1), \tag{9}$$

where Y_t is aggregate output at time t, K_t is aggregate capital at time t, L_t is aggregate labor and A_t is the level of labor augmenting technology at time t. The labor augmenting technology grows at a constant exogenous rate $\gamma \geq 1$,which implies $A_t \equiv \gamma^t$ for all t. The share of capital in total income is given by α. The gross return on physical capital is given by R_t.

Since we assume constant returns to scale in our production, the representative firm maximizes profits as follows:

$$\max_{K_t, L_t} \{F(K_t, A_t L_t) - w_t L_t - R_t K_t\}. \tag{10}$$

The solution of the firms problem is then characterized by the following first order conditions:

$$w_t = A_t F_L(K_t, A_t L_t) = (1-\alpha)\frac{Y_t}{L_t}, \tag{11}$$

$$R_t = F_K(K_t, A_t L_t) = \alpha \frac{Y_t}{K_t}. \tag{12}$$

Depreciation in capital can be viewed as a reduction of rate of return obtained from holding physical capital and thus we have,

$$r_t = R_t - \delta. \tag{13}$$

2.4 Government Policies

We consider two kinds of policies here: Monetary and Fiscal policy.

2.4.1 Monetary Policy

Let the nominal money supply at period t be given by $\overline{M}_t^s$, real money supply by $m_{i,t}$, price level of output at time t by P_t.

$$\overline{M}_{t+1}^s = g\overline{M}_t^s = \overline{M}_t^s + P_t \zeta_t = P_{t+1} \sum_{i=1}^{s} \mu_i m_{i,t+1} \tag{14}$$

Assuming that the central bank issues transfers at the rate of g every period, we have $\overline{M}_{t+1}^s = g\overline{M}_t^s$. Since, money is introduced via lump sum transfers ζ_t at the end of each period, we have, $\overline{M}_{t+1}^s = \overline{M}_t^s + P_t \zeta_t$. The last term equates the nominal value of all real money holdings in period $t+1$, $P_{t+1} \sum_{i=1}^{s} \mu_i m_{i,t+1}$ to the nominal value of the money supply in $t+1$, $\overline{M}_{t+1}^s$.

2.4.2 Fiscal Policy

The government imposes a progressive tax τ_t which is a function in total taxable income $y_{i,t}$ on agent's every period to finance it's expenditure on goods and services in period t denoted by G_t. The government balances its budget in each period and chooses a tax schedule summarized by the average tax rate (ATR)

$$\text{Tax schedule} = ATR = \eta(\frac{y_{it}}{\gamma^t})^{\phi} \; with \; 0 \leq \eta < 1, \; \phi > 0 \tag{15}$$

just like in Lansing and Guo (1998) and Li and Sarte (2004). Here, the parameters η and ϕ determine the level and the slope of the tax schedule, respectively. With a progressive tax system, households with higher taxable income are subject to higher tax rates, so that

$$\text{Total tax paid} = \tau_t(y_{it}) = y_{it}\eta(\frac{y_{it}}{\gamma^t})^{\phi}. \tag{16}$$

Once we know our tax schedule, we can discuss the progressivity of the tax structure by calculating the ratio of the marginal and average tax rates.

$$MTR = \tau_t'(y_{it}) = (1+\phi)\eta(\frac{y_{it}}{\gamma^t})^{\phi} = (1+\phi)ATR.$$

Since the parameter ϕ captures the degree of progressivity of the tax structure, progressive, proportional and regressive tax structures would correspond to $\phi > 0$, $\phi = 0, \phi < 0$, respectively. A tax schedule is said to be progressive whenever the marginal tax rate exceeds the average tax rate at all levels of taxable income.

3 Competitive Equilibrium

Let $\mathbf{c}_t = (c_{1,t}, \; c_{2,t}...c_{s,t})$, $\mathbf{a}_t = (a_{1,t}, \; a_{2,t}...a_{s,t}) \; and \; \mathbf{m}_t = (m_{1,t}, \; m_{2,t}...m_{s,t})$ denote a distribution of consumption, capital and money across the s groups at time t, respectively. The competitive equilibrium consists of a sequence of distributions of consumption and capital, $\{\mathbf{c}_t, \mathbf{a}_t, \mathbf{m}_t\}_{t=0}^{\infty}$, sequences of aggregate inputs, $\{K_t, L_t\}_{t=0}^{\infty}$ and sequences of prices, $\{w_t, r_t, P_t\}_{t=0}^{\infty}$, so that:

1. Given prices $\{w_t, r_t, \pi_{t+1}\}_{t=0}^{\infty}$, the sequences $\{c_{it}, a_{it}, m_{it}\}_{t=0}^{\infty}$ solve each type-i agent's problem.

2. Given prices $\{w_t, R_t\}_{t=0}^{\infty}$ and the aggregate inputs K_t and L_t solve the representative firm's problem.

3. The government's budget is balanced every period.

4. All markets clear every period so that,

Equilibrium in Labor market:

$$L_t = \sum_{i=1}^{s} \mu_i e_i = \overline{e} \tag{17}$$

where $\overline{e}$ represents the aggregate level of labor productivity.
Equilibrium in Capital market:

$$K_t = \sum_{i=1}^{s} \mu_i a_{i,t} \tag{18}$$

By Walras' Law, the goods market clears.

Equilibrium in money market:

$$\overline{M}^s_{t+1} = g\overline{M}^s_t = \overline{M}^s_t + P_t\zeta_t = P_{t+1}\sum_{i=1}^{s}\mu_i m_{i,t+1} \tag{19}$$

Also, the government budget constraint is satisfied:

$$G_t = \sum_{i=1}^{s}\mu_i \tau_t(y_{i,t}) \tag{20}$$

3.1 Balanced Growth Path

This paper focuses on a balanced growth path. A balanced growth path is a competitive equilibrium along which (i) all variables are growing at the constant growth rate $\gamma \geqslant 1$, and (ii) the real rate of return, r, is constant over time.

Thus,

$$\frac{m_{i,t+1}}{m_{i,t}} = \frac{g}{\pi} = \gamma \tag{21}$$

For CIA constraint to be binding along a balanced growth path, we must have $\lambda_{i,t} > 0$ which implies the following:

$$u'(c_{i,t}) - \frac{\beta}{\pi_{t+1}}u'(c_{i,t+1}) > 0$$

The above inequality suggests that for the CIA constraint to be binding, the marginal benefit that the agent receives by increasing consumption at time t by one unit must exceed the marginal cost the agent incurs due to a decrease cash holdings at time t that results in the loss of utility at time t+1 discounted by the rate of time preference and inflation. We can further write the inequality as follows:

$$(\frac{c_{i,t+1}}{c_{i,t}})^\sigma > \frac{\beta_i}{\pi} \ \forall i \tag{22}$$

$$or\ \gamma^\sigma > \frac{\beta_i}{\pi} \ \forall i \tag{23}$$

For our exercise, we assume that this inequality is binding.

4 Calibration

We now assess our model to see the redistributional effects of inflation in the United States. There are three parts to our quantitative exercise that are explained in the following subsections. We construct a benchmark balanced-growth equilibrium to match some of key features of the U.S. economy. In our analysis, the model period is assumed to be one year.

All benchmark parameter values are summarized in Table 1. The benchmark balanced growth equilibrium is constructed to match the following features of the U.S. economy: the capital-output ratio, capital's share of income, average annual growth rate of per-capita GDP, average annual inflation rate, the progressive tax structure and the income and wealth distributions in the United States. More specifically, the labor productivities and the subjective discount factors are calibrated to match the U.S. income and wealth distributions in 2007, using data from the Survey of Consumer Finance as reported in Díaz-Giménez et al. (2011).

Parameter	Description	Value
σ	Inverse of IES	1
γ	Common growth factor	1.018
α	Share of capital income in total output	0.36
δ	Depreciation rate	0.08
$\pi - 1$	Average annual Inflation Rate (1950-07)	0.038
r^*	Equilibrium interest rate	0.04
ψ	Fraction of investment subject to CIA	0.01
β_{10}	Subjective discount factor	0.9948
$1 + \phi$	Ratio of marginal to average tax rate (1960-05)	1.738
η	Scalar in income tax schedule	0.0197

Table 1: Benchmark parameters

In our benchmark case, the parameter σ that measures the inverse of the intertemporal elasticity of substitution (IES) in the utility function is set to one. The average annual growth rate of per capita variables ($\gamma - 1$) is 1.8%, which is the annual growth rate of real per-capita GDP in the United States over the period 1950-2007. The share of capital income in total output (α) is 1/3. The depreciation rate is calibrated so that the capital-output ratio is 3.0. The average annual inflation rate ($\pi - 1$) is 3.8% which is calculated using the Consumer Price Index (CPI) over the period 1950-2007 . Dotsey and Sarte (2000) mention that for the US and most OECD economies, the fraction of investment subject to the Cash-in-advance constraint (ψ) is probably close to zero because of high financial sophistication. In our benchmark case, we assume a very small value of (ψ)=0.1. Empirical findings show that a higher ψ is empirically plausible supported by a significant increase in consumer credit in the last two decades (Ludvigson (1999)). Also,the average cash-to-assets ratio for U.S. industrial firms more than doubled from 1980 to 2006 that could help firms to pay off their debt entirely in cash (Bates et al. (2009)). Later, we do a sensitivity analysis by considering higher values of ψ corresponding to lower degrees of financial sophistication.

Based on the data reported in Table 5 of Díaz-Giménez et al. (2011), we first divide the U.S. income distribution into ten groups (namely, 1-5%, 5-10%, 10-20%, 20-40%, 40-60%, 60-80%, 80-90%, 90-95%, 95-99%) implying s=10. The bottom 1% of the distribution is discarded because the average income for this group turns out to be negative. The subjective discount factors are chosen so as to match the share of total income held by each group[6]. In other words, the subjective discount factors are chosen so as to match the Lorenz curve for income in the 2007 Survey of Consumer Finances (SCF) sample in Díaz-Giménez et al.

[6]This involves solving a set of nonlinear equations for the subjective discount factors. The technical details can be found in the appendix.

(2011). After the subjective discount factors are determined, we then choose the labor productivities so as to match the share of total wealth owned by the ten income groups. The data are again taken from Table 5 of Díaz-Giménez et al. (2011). This procedure is further discussed in the Appendix A.

The ratio of marginal to average tax rates $(1 + \phi)$ are computed between the period 1950-2005[7] The average of the degree of progressivity $(1 + \phi = 1.738)$ for the specified time period is used in our exercise. The scalar in income tax schedules η are calibrated to match the average tax rate (ATR) of 13.8% for the same time period.

5 Findings

This section reports findings regarding the redistributional effects of inflation on consumption, disposable income and cash holdings among the ten income groups as well as overall inequality. In order to do this, we consider a number of counterfactual experiments in which the long-run inflation rate is changed.

5.1 Benchmark results

Table 2 summarizes the Gini coefficients for income, wealth, cash-holdings and consumption as suggested by our benchmark model[8] vs. the data. The data is taken from Díaz-Giménez et al. (2011) and Wen (2010). We find that our model is able to match the Gini coefficient of income close to the one found in the data. As for the wealth distribution, the model predicts a more equal distribution as compared to that observed in the data. Since, we have ignored the analysis for the bottom 1% of the population in our study due to their average income being negative[9], our prediction of a more equal distribution for both income and wealth may be a direct result of having incomes as strictly positive. Table 3 reports the share of wealth held as money for the ten income brackets. We find that as income increases, households tend to hold less money as a fraction of their total wealth. The bottom 5% holds 14% of their total wealth as money as compared to the top 1%, who hold only 6%[10]. This finding is also supported by the Flow of Funds data for the household sector that contain a detailed breakdown for the assets and liability positions for the households.

Following Stockman (1981), we can recall that since money has a negative rate of return during inflation, agents have a higher opportunity cost of holding money. This reduces the willingness to hold money. In other words, the incentive to hold money beyond the mere transaction need for consumption and investment declines. Due to the cash- in- advance constraint on consumption and investment, in periods of inflation, the households would decrease both consumption and lower investment. The real purchases of both consumption and investment goods fall with decreased money holdings at higher rates of inflation since money is more costly to hold. This results in the net return from investment to be lower in utility terms. Inflation acts as tax on investment.

Our main results are analyzed in the following subsections. We perform five counterfac-

[7]Feenberg and Coutts (1993) and Taxsim are the sources used for computing the degree of progressivity.

[8]These results correspond to the long-run inflation rate of 3.8% for the United States.

[9]In the actual data, the average earnings of the bottom 1% of the distribution are negative

[10]Note, however that if we consider aggregate money holdings for each income bracket, they are obviously much lower for the bottom 5% and much higher for the top 1%, see Table 7.

tual experiments in which we change the long-run inflation rate from 3.8% to 1.8%, 2.8%, 4.8%, 5.8% and 8.8%. We then report the redistributional effects for disposable income, consumption and money and analyze our sensitivity experiments.

5.2 Redistribution of Disposable Income, consumption and money

We find that disposable income[11] inequality, consumption inequality and inequality in cash-holdings decreases as inflation rises. Tables 5, 6 and 7 provide evidence. The rich hold more cash than the poor and so inflation is more likely to hurt the rich than the poor in terms of cash holdings. This is evident in Table 7. The bottom 5% hold 0.15 worth of real money balances and the top 1% hold 13.43. Since, inflation is a direct result the lump-sum money creation that is distributed evenly among households, the bottom 60% gains and top 40% loses in terms of cash holdings as inflation rises. This results in making the distribution of cash holdings more even, as reflected in the lower Gini coefficient with higher inflation.

Consumption inequality reduces as inflation increases however, we find that overall consumption falls. Since inflation rate of 3.8% is our benchmark for inflation, we look at both deflationary and inflationary episodes. Since, consumption is financed by money holdings only in a cash-in-advance framework, we find that once again the bottom 60% gain and top 40% lose with inflation. With reference to Tables **??** and 8, we can talk discuss who loses and wins from these episodes of an increase in long-run inflation. Even though the top 40% of the distribution loses, their consumption patterns do not change as much because of their large wealth holdings. We find a similar result for the net disposable income. The Lorenz curves for disposable income, consumption and money holdings are depicted in Figures 1, 2, 3, and 4. In order to make the curves visible, we consider hyperinflationary episodes. We find that with rising inflation, inequality in disposable income, consumption and money holdings fall. The lorenz curves move towards the line of perfect equality. Our results are similar to Doepke and Schneider (2006) who find that the losers in the economy are the old and rich households at the top of the distribution. The winners are the young middle class that have substantial fixed-rate mortgage positions and the poor who have a sizeable amount of debt[12].

5.3 Results without transfers

We find that as inflation rises, all income groups suffer a loss in terms of consumption, as suggested by Table 7. This is because the purchasing power of their disposable income falls. We find that inequality in consumption increases but is not significant. Similar results hold for inequality in disposable income and money. We do not find a significant change in the Gini coefficients of either of these variables.

5.4 Robustness Checks

The third part of our quantitative exercise is to assess whether our results are sensitive to changes in three factors: the inverse of intertermporal elasticity of substitution, σ; the fraction of investment subject to the cash-in-advance constraint, ψ; and hyperinflationary

[11] Disposable income is measured as income plus transfers less taxes.

[12] They also report similar results for having surprise inflation. In order to compare our results with Doepke and Schneider (2006), we only focus on the experiments when inflation is fully anticipated

episodes. The results are depicted in Tables 9, 10 and 11. We consider $\sigma = 1$ as our benchmark case and for our sensitivity analysis, we change σ to 0.5 and 1.5. Let's consider an increase in the value of σ that would lower the intertemporal elasticity of substitution. Ceteris paribus, each consumer would want to have less savings. The reduction in savings would be larger for the rich relative to the poor. Also, as aggregate savings decrease the real rate of interest would adjust in order to keep the capital to output ratio constant. This would encourage the rich to increase asset holdings. We find that overall, with an increase in inflation, these two effects cancel out. However, we do observe the distribution becoming more equal with inflation in which case the first effect outweighs the latter.

We consider $\psi = 0.01$ as our benchmark case and for our sensitivity analysis, we change ψ to 0.1, 0.2 and 0.3. These values do not affect our results. Consumption inequality still falls with inflation even if the degree of financial sophistication in the economy is lower. This should be the case as our model does not assume any access to sophisticated credit or financial markets. We consider inflation rates that are 10%, 20%, 30%, 40%, and 50% above the benchmark inflation rate of 3.8% Our results are robust to changes in the inverse of intertemporal elasticity of substitution (σ), the fraction of investment subject to the CIA constraint (ψ) and the hyperinflationary episodes. We also check the case when transfers are zero and find that the Gini coefficient for disposable income change by less than 0.1% when inflation rises by 5%. Thus, in our model, the redistributional effects are driven by the lump-sum transfers each period that result in inflation.

6 Conclusion

This paper contributes to the scant literature on the redistributional effects of inflation in the U.S. economy. The model presented in this paper is an extension of the standard cash-in-advance model by Stockman (1981). Our model allows for heterogeneity in the rate of time preference and labor productivities. We use this heterogeneity to match the income and wealth distributions in the U.S. among different income groups. In our model, the cash-in-advance constraint on consumption and investment is introduced. We find that a rise in inflation benefits the bottom 60% of the distribution and hurts the top 40% in terms of consumption, welfare, cash-holdings and disposable income. We also acknowledge that contrasting our model with other studies, the results presented in this paper depend on the financial structure of the economy.

A Appendix

A.1 Results

Gini Coefficients		
	Model	Data
Income	0.56	0.575
Wealth	0.67	0.816
Cash-Holdings	0.467	-
Consumption	0.465	0.28

Table 2: Benchmark Results; Data Sources: Díaz-Giménez et al. (2011) and Wen (2010)

	Model
Bottom 1-5%	0.1481
5-10%	0.3229
10-20%	0.2369
20-40%	0.2639
40-60%	0.2735
60-80%	0.2347
80-90%	0.2212
90-95%	0.1452
95-99%	0.0922
Top 99-100%	0.0632

Table 3: Share of wealth held as money

Income groups	$\pi = 1.018$	$\pi = 1.028$	$\pi = \mathbf{1.038}$	$\pi = 1.048$	$\pi = 1.058$	$\pi = 1.088$
Bottom 1-5%	0.136	0.146	0.155	0.165	0.174	0.201
5-10%	0.199	0.208	0.217	0.226	0.234	0.259
10-20%	0.279	0.287	0.296	0.304	0.312	0.334
20-40%	0.463	0.469	0.475	0.482	0.488	0.506
40-60%	0.743	0.747	0.751	0.754	0.758	0.768
60-80%	1.161	1.160	1.160	1.159	1.159	1.158
80-90%	1.712	1.706	1.700	1.694	1.688	1.673
90-95%	2.351	2.339	2.326	2.314	2.303	2.270
95-99%	4.105	4.075	4.045	4.016	3.988	3.907
Top 99-100%	13.396	13.272	13.151	13.032	12.916	12.508
Aggregate	24.484	24.413	24.280	24.150	24.023	23.661
Gini_C	0.474	0.469	0.465	0.460	0.455	0.443

Table 4: Consumption Inequality

Gain/loss in consumption					
Income groups	$\pi = 1.018$	$\pi = 1.028$	$\pi = 1.048$	$\pi = 1.058$	$\pi = 1.088$
Bottom 1-5%	-12.38	-6.09	6.03	11.93	29.07
5-10%	-8.33	-4.14	4.05	8.01	19.56
10-20%	-5.57	-2.76	2.7	5.36	13.1
20-40%	-2.69	-1.32	1.32	2.6	6.38
40-60%	-0.97	-0.47	0.47	0.94	2.34
60-80%	0.08	0.04	-0.04	-0.07	-0.13
80-90%	0.69	0.34	-0.33	-0.67	-1.57
90-95%	1.06	0.52	-0.51	-1.01	-2.42
95-99%	1.47	0.73	-0.71	-1.41	-3.4
Top 99-100%	1.86	0.92	-0.9	-1.78	-4.88

Table 5: Percentage change in consumption from Benchmark

Income groups	$\pi = 1.018$	$\pi = 1.028$	$\pi = \mathbf{1.038}$ *Benchmark*	$\pi = 1.048$	$\pi = 1.058$	$\pi = 1.088$
Bottom 1-5%	0.155	0.168	0.179	0.191	0.202	0.236
5-10%	0.213	0.225	0.237	0.248	0.26	0.294
10-20%	0.304	0.317	0.328	0.340	0.351	0.385
20-40%	0.498	0.513	0.524	0.535	0.547	0.581
40-60%	0.798	0.814	0.825	0.837	0.848	0.882
60-80%	1.257	1.275	1.286	1.298	1.309	1.343
80-90%	1.859	1.880	1.892	1.903	1.915	1.949
90-95%	2.639	2.664	2.676	2.687	2.698	2.732
95-99%	4.842	4.877	4.890	4.901	4.912	4.946
Top 99-100%	16.704	16.749	16.787	16.798	16.808	16.841
Gini_DY	0.491	0.486	0.482	0.478	0.474	0.461

Table 6: Disposable Income Inequality

Income groups	$\pi = 1.018$	$\pi = 1.028$	$\pi = \mathbf{1.038}$ *Benchmark*	$\pi = 1.048$	$\pi = 1.058$	$\pi = 1.088$
Bottom 1-5%	0.137	0.147	0.157	0.166	0.175	0.202
5-10%	0.198	0.209	0.218	0.226	0.235	0.260
10-20%	0.279	0.289	0.297	0.305	0.313	0.336
20-40%	0.462	0.471	0.478	0.484	0.490	0.508
40-60%	0.742	0.750	0.754	0.757	0.761	0.771
60-80%	1.161	1.665	1.166	1.165	1.165	1.164
80-90%	1.712	1.7152	1.709	1.703	1.698	1.682
90-95%	2.361	2.36	2.348	2.336	2.324	2.290
95-99%	4.147	4.134	4.106	4.077	4.049	3.967
Top 99-100%	13.653	13.536	13.439	13.320	13.203	12.867
Gini_m	0.476	0.471	0.467	0.462	0.458	0.445

Table 7: Inequality in Cash Holdings

Income groups	$\pi = 1.038$	$\pi = 1.088$
Bottom 1-5%	0.7848	0.8885
5-10%	0.9383	1.0206
10-20%	1.0845	1.1503
20-40%	1.3717	1.4124
40-60%	1.7197	1.7377
60-80%	2.1270	2.1240
80-90%	2.5686	2.5467
90-95%	2.9665	2.9294
95-99%	3.8295	3.7630
Top 99-100%	6.7055	6.5562
Total welfare	24.0961	24.1288

Table 8: Welfare analysis

A.2 Sensitivity Analysis

Gain/loss in consumption (% change from benchmark)					
	Net Inflation Rate ($\pi - 1$) (Above benchmark)				
Income Groups	10%	20%	30%	40%	50%
Bottom 1-5%	56.22	102.82	142.16	176.76	207.06
5-10%	37.98	69.24	95.71	119.01	139.45
10-20%	25.56	46.47	64.16	79.83	93.58
20-40%	12.71	22.86	31.44	39.17	45.98
40-60%	4.95	8.6	11.66	14.62	17.24
60-80%	0.18	-0.12	-0.41	-0.38	-0.32
80-90%	-2.57	-5.19	-7.45	-9.12	-10.56
90-95%	-4.21	-8.19	-11.59	-14.27	-16.58
95-99%	-6.12	-11.66	-16.38	-20.24	-23.56
Top 99-100%	-7.91	-14.89	-20.9	-25.87	-30.14
Gini_C	0.4229	0.3878	0.3579	0.3323	0.31

Table 9: Sensitivity Analysis: Hyperinflation

$\sigma = \mathbf{1.5}$	$\pi = 1.018$	$\pi = 1.028$	$\pi = \mathbf{1.038}$ *Benchmark*	$\pi = 1.048$	$\pi = 1.058$	$\pi = 1.088$
Gini_C	0.4751	0.4704	0.4658	0.4612	0.4567	0.4437
Gini_DY	0.4913	0.4868	0.4825	0.4782	0.474	0.4617
Gini_Y	0.5595	0.5595	0.5595	0.5595	0.5595	0.5595
Gini_W	0.6701	0.6701	0.6701	0.6701	0.6701	0.6701
$\sigma = \mathbf{0.5}$	$\pi = 1.018$	$\pi = 1.028$	$\pi = \mathbf{1.038}$ *Benchmark*	$\pi = 1.048$	$\pi = 1.058$	$\pi = 1.088$
Gini_C	0.4734	0.4687	0.464	0.4595	0.455	0.4421
Gini_DY	0.49	0.4871	0.4827	0.4785	0.4743	0.4621
Gini_Y	0.5595	0.5595	0.5595	0.5595	0.5595	0.5595
Gini_W	0.6701	0.6701	0.6701	0.6701	0.6701	0.6701

Table 10: Sensitivity Analysis: changing the inverse of the intertemporal elasticity of substitution, σ

Income groups	$\psi = 0.1$	$\psi = 0.2$	$\psi = 0.3$
Bottom 1-5%	0.136	0.156	0.157
5-10%	0.199	0.217	0.218
10-20%	0.279	0.296	0.297
20-40%	0.463	0.476	0.477
40-60%	0.743	0.751	0.753
60-80%	1.161	1.16	1.162
80-90%	1.712	1.699	1.703
90-95%	2.351	2.325	2.330
95-99%	4.105	4.042	4.048
Top 99-100%	13.396	13.141	13.136
Gini_C	0.474	0.464	0.464

Table 11: Sensitivity Analysis with limited or ineffectual credit markets

A.3 Figures

Disposable Income

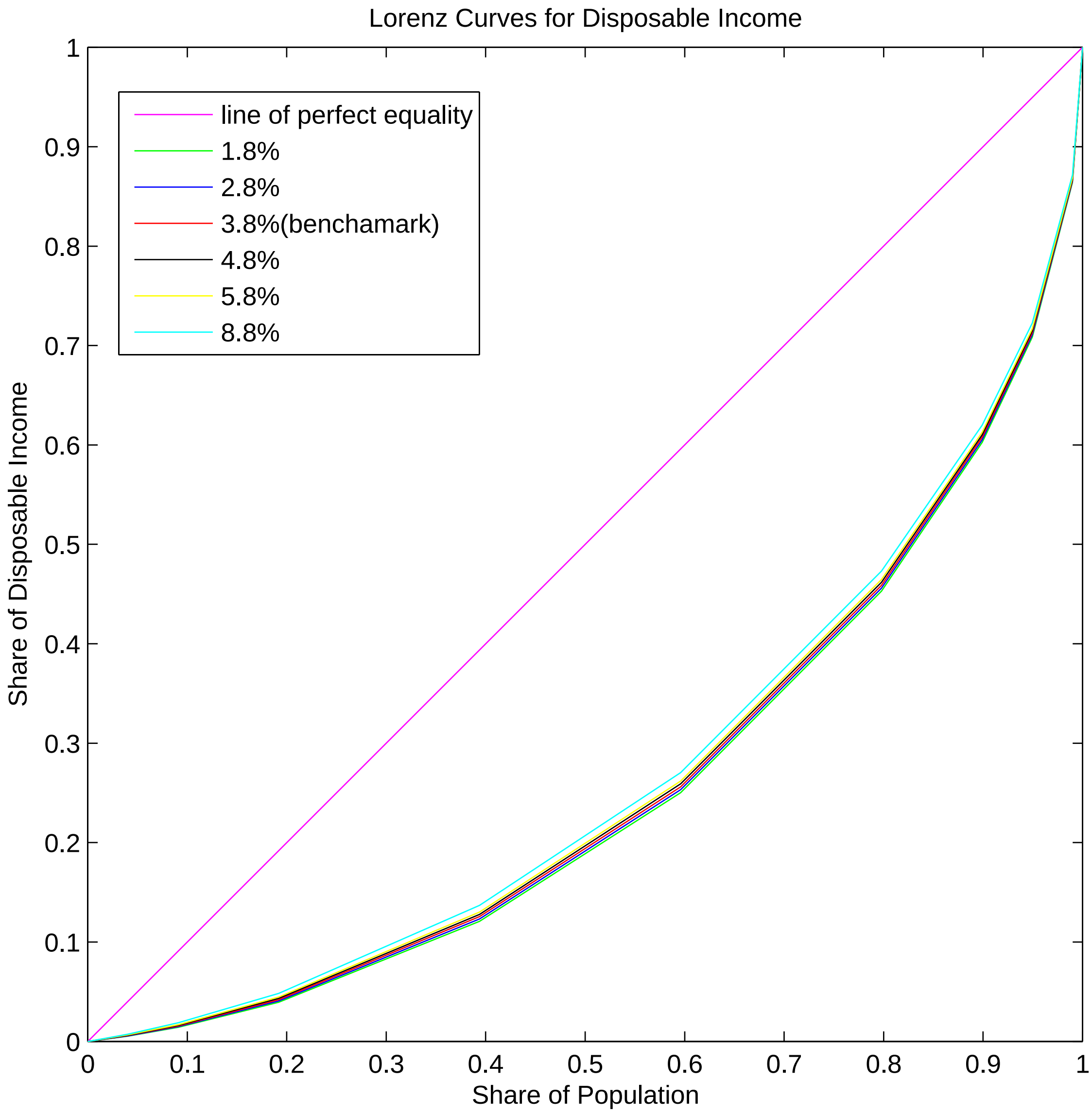

Figure 1: Lorenz Curves for Disposable Income

Disposable Income with Hyperinflation

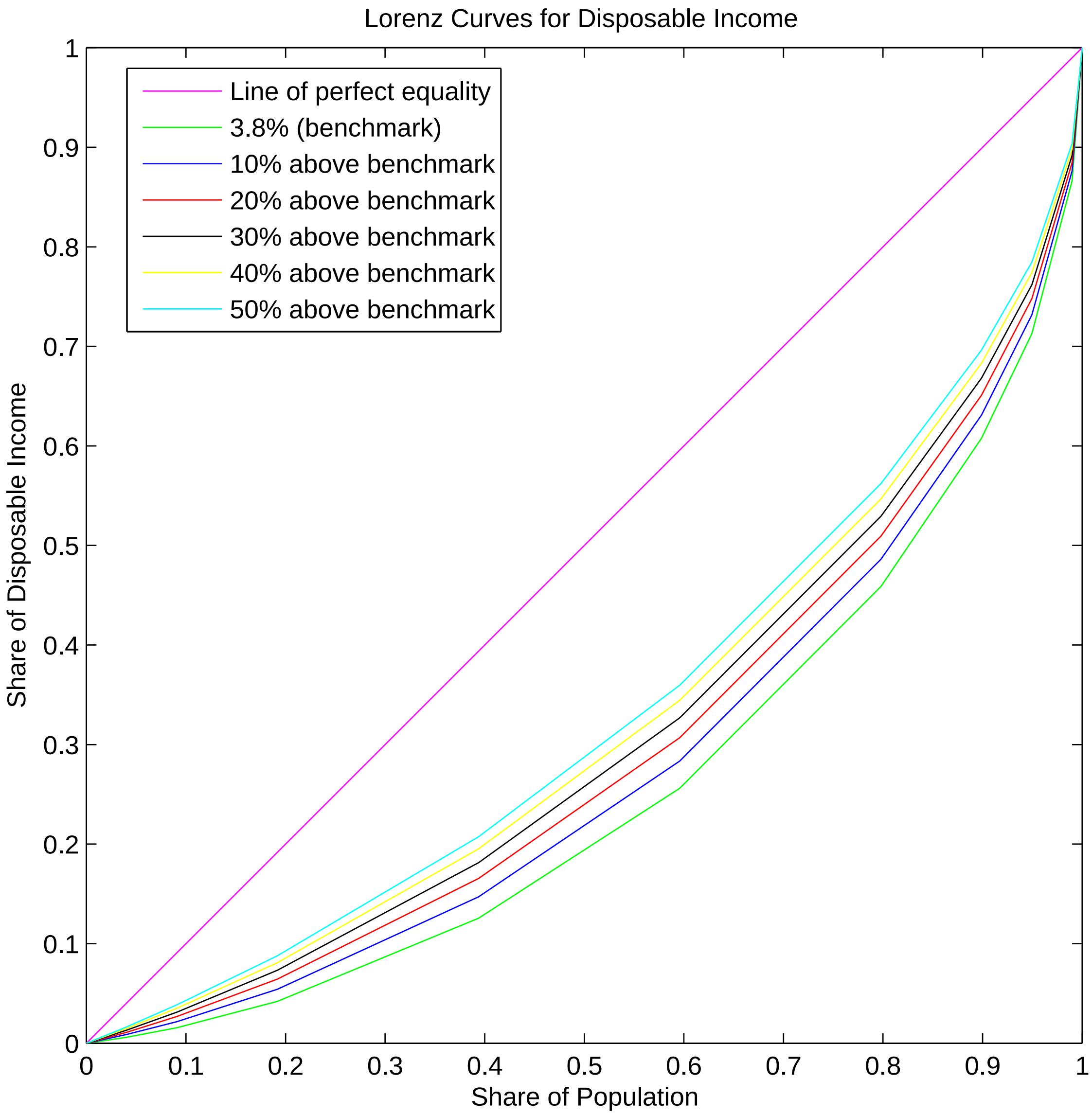

Figure 2: Lorenz Curves for Disposable Income

Consumption

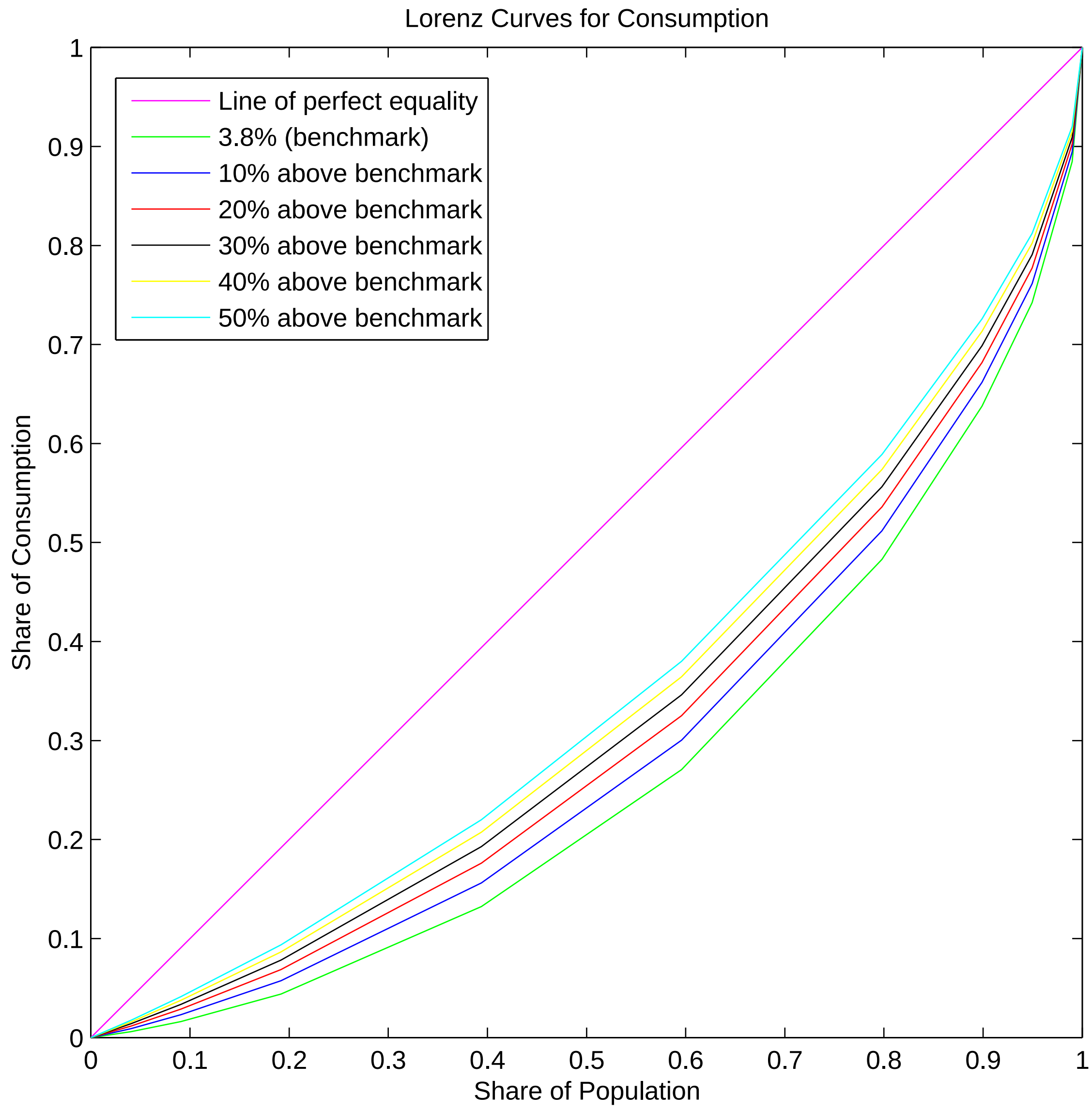

Figure 3: Lorenz Curves for consumption

Money

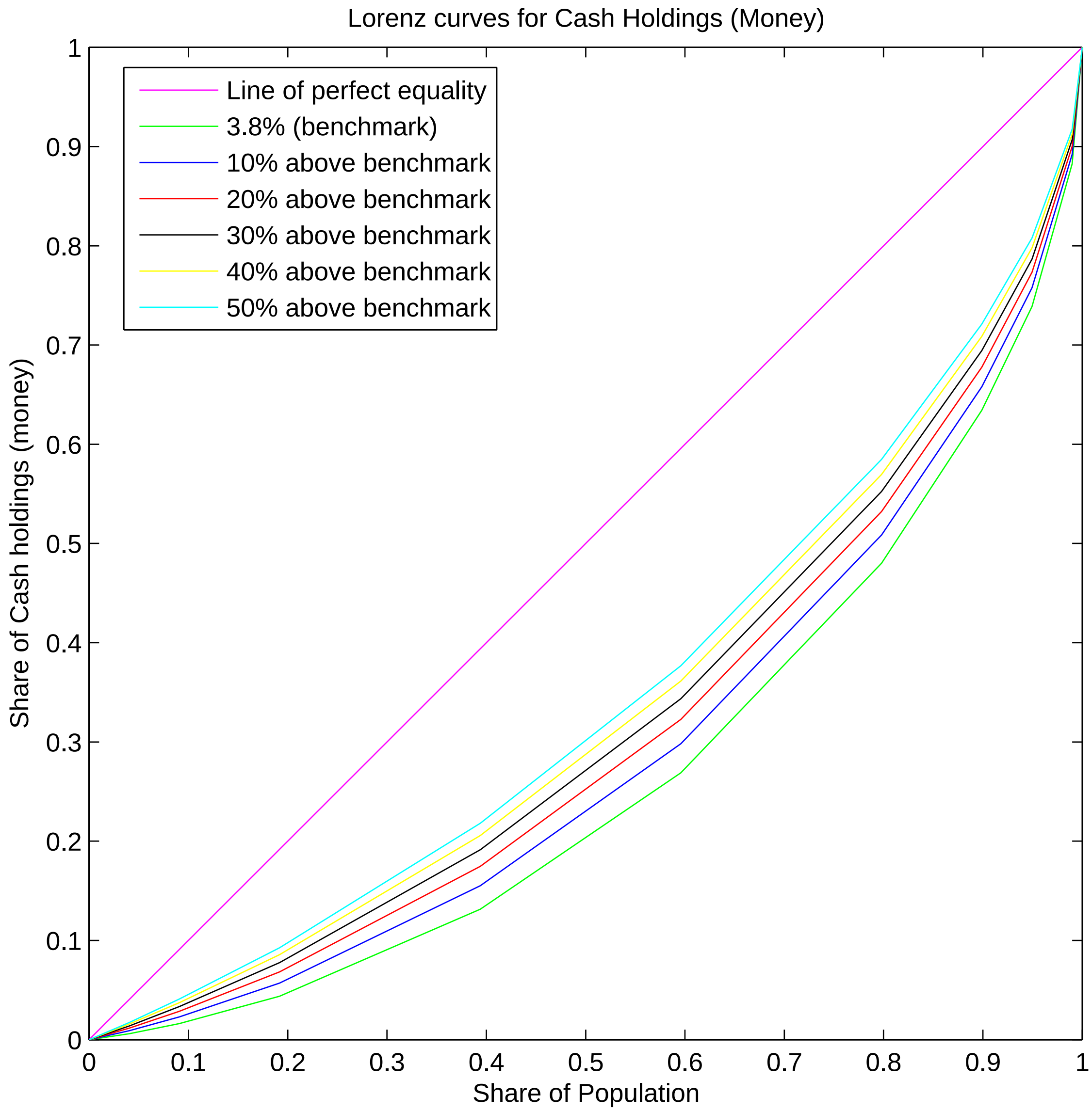

Figure 4: Lorenz Curves for money holdings

A.4 Kuhn-Tucker Conditions:

The agents problem can be re-written in real terms as:

$$\max_{(c_{i,t},a_{i,t+1},m_{i,t+1})} \sum_{t=0}^{\infty} \beta_i^t u(c_{i,t}) where \beta_i \in (0,1) \tag{24}$$

s.t.

$$c_{i,t} + \psi[a_{i,t+1} - a_{i,t}] \leq m_{i,t} where \psi, \delta \in [0,1] \tag{25}$$

and

$$c_{i,t} + a_{i,t+1} - a_{i,t} + \pi_{t+1} m_{i,t+1} = y_{i,t} - \tau_t(y_{i,t}) + m_{i,t} + \zeta_t \tag{26}$$

For the derivation of the first order conditions consider the Lagrangian:

$$\begin{aligned} L = \sum_{t=0}^{\infty} \beta_i^t [u(c_{i,t}) + \lambda_{i,t}\{m_{i,t} - c_{i,t} - \psi(a_{i,t+1} - a_{i,t})\} + \\ \theta_{i,t}\{y_{i,t} - \tau_t(y_{i,t}) + m_{i,t} + \zeta_t - c_{i,t} - a_{i,t+1} + a_{i,t} - \pi_{t+1} m_{i,t+1}\}] \end{aligned}$$

Assuming $c_{i,t} > 0; \lambda_{i,t} \geq 0; \theta_{i,t} > 0; m_{i,t+1} \geq 0$

$c_{i,t}$:

$$u'(c_{i,t}) = \lambda_{i,t} + \theta_{i,t}$$

$m_{i,t+1}$:

$$\theta_{i,t}\pi_{t+1} - \beta_i(\lambda_{i,t+1} + \theta_{i,t+1}) = 0 \tag{27}$$

$a_{i,t+1}$:

$$\psi\lambda_{i,t} + \theta_{i,t} - \beta_i[\psi\lambda_{i,t+1} + \theta_{i,t+1}\{r_{t+1}(1 - \tau'(y_{i,t})) + 1\}] \leq 0$$

$$and\ a_{i,t+1}[\psi\lambda_{i,t} + \theta_{i,t} - \beta_i[\psi\lambda_{i,t+1} + \theta_{i,t+1}\{r_{t+1}(1 - \tau'(y_{i,t})) + 1\}] = 0 \tag{28}$$

$\lambda_{i,t}$:

$$m_{i,t} - c_{i,t} - \psi(a_{i,t+1} - a_{i,t}) \geq 0\, and\ \lambda_{i,t}[m_{i,t} - c_{i,t} - \psi(a_{i,t+1} - a_{i,t})] = 0 \tag{29}$$

$\theta_{i,t}$:

$$y_{i,t} - \tau_t(y_{i,t}) + m_{i,t} - c_{i,t} - a_{i,t+1} + a_{i,t} - \pi_{t+1} m_{i,t+1} + +\zeta_t = 0 \tag{30}$$

If money is held, we have:

$$\theta_{i,t} = u'(c_{i,t}) - \lambda_{i,t} = \frac{\beta_i}{\pi_{t+1}} u'(c_{i,t+1}) \tag{31}$$

Combining the above equations we get the following, Euler equation:

$$\psi u^{'}(c_{i,t}) = [\beta_i\psi - (1-\psi)\frac{\beta_i}{\pi_{t+1}}]u'(c_{i,t+1}) + \beta_i^2\{\frac{1}{\pi_{t+1}}(r_{t+1} - \tau_t'(y_{i,t+1})r_{t+1} + 1) - \frac{\psi}{\pi_{t+2}}\}u'(c_{i,t+2})$$

For the cash-in-advance constraint to be binding, we must have $\lambda_{i,t} > 0$; which implies

$$u'(c_{i,t}) - \frac{\beta}{\pi_{t+1}} u'(c_{i,t+1}) > 0 \tag{32}$$

The above condition can be interpreted as the following: For the CIA to be binding we must have, the marginal benefit (achieved from increasing consumption by one unit today) exceed the marginal cost to the agent (due to the discounted value of the decrease in the money holdings by π_{t+1} units today).

A.5 Aggregation:

Aggregating the individual budget constraints over the entire economy we get,

$$\sum_{i=1}^{s}\mu_i[c_{i,t} + a_{i,t+1} - a_{i,t} + \pi_{t+1}m_{i,t+1}] \leq \sum_{i=1}^{s}\mu_i[y_{i,t} - \tau_t(y_{i,t}) + m_{i,t}] + \zeta_t \tag{33}$$

$$\sum_{i=1}^{s}\mu_i[c_{i,t}+a_{i,t+1}-a_{i,t}+\pi_{t+1}m_{i,t+1}] \leq \sum_{i=1}^{s}\mu_i(w_te_i+r_ta_{i,t})-\sum_{i=1}^{s}\mu_i\tau_t(y_{i,t})+\sum_{i=1}^{s}\mu_im_{i,t}+\zeta_t \tag{34}$$

Using market clearing conditions and re-writing we get back the goods market clearing condition:

$$C_t + K_{t+1} - (1-\delta)K_t + G_t = Y_t \tag{35}$$

A.6 Transformed variables

Consider the following transformed variables:

$$\widehat{c_i} = \frac{c_{i,t}}{\gamma^t}; \widehat{a_i} = \frac{a_{i,t}}{\gamma^t}; \widehat{m_i} = \frac{m_{i,t}}{\gamma^t}; \widehat{w} = \frac{w_t}{\gamma^t}; \widehat{y_i} = \frac{y_{i,t}}{\gamma^t}; R = R_t; r = r_t; \pi = \pi_t\ ; \frac{\zeta_t}{\gamma^t} = \zeta \forall t \tag{36}$$

Also, rewriting the tax function from the text we have,

$$Tax\ schedule = ATR = \eta(\frac{y_{it}}{\kappa\gamma^t})^\phi = \eta(\frac{\widehat{y_i}}{\kappa})^\phi with\ 0 \leq \eta < 1,\ \phi > 0, \kappa > 0 \tag{37}$$

$$\text{Total tax paid} = \tau_t(y_{it}) = y_{it}\eta(\frac{y_{it}}{\kappa\gamma^t})^\phi \tag{38}$$

gives

$$MTR = \tau_t'(y_{it}) = (1+\phi)\eta(\frac{\widehat{y_i}}{\kappa})^\phi = (1+\phi)ATR \tag{39}$$

Rewriting the first order conditions in terms of the transformed variables, we get 3 equations in 3 unknowns $(\widehat{m}_i, \widehat{c}_i, \widehat{a}_i)$:

$$\widehat{m}_i - \widehat{c}_i - \psi(\gamma-1)\widehat{a}_i = 0$$

$$\widehat{y_i} - \widetilde{\tau}(\widehat{y_i}) + (1-\pi\gamma)\widehat{m}_i - \widehat{c}_i + (1-\gamma)\widehat{a}_i + \zeta = 0$$

$$\psi = \beta_i[\psi - \frac{(1-\psi)}{\pi_{t+1}}]\gamma^{-\sigma} + \beta_i^2\frac{1}{\pi}[r - r(1+\phi)\eta(\frac{\widehat{y_i}}{\kappa})^\phi + 1 - \psi]\gamma^{-2\sigma}$$

where $\widehat{y_i} = \widehat{w}e_i + r\widehat{a}_i$

Now substituting this back into the Euler equation we get :

$$\widehat{y_i} = \kappa[\frac{(r+1-\psi)\beta_i^2 + \gamma^2(\pi\psi - 1 + \psi)\beta_i - \psi\pi\gamma^{2\sigma}}{\beta_i^2 r\eta(1+\phi)}]^{\frac{1}{\phi}} = h(r, \beta_i) \tag{40}$$

A.7 Calibration procedure

Using the functional forms and parameters discussed in the text, we fix β_{10}, to compute $\widehat{y}_{10}$ along a balanced growth path according to (40).

Since,

$$\widehat{y_s} = h(r, \beta_s)$$

Using data on $\{\widehat{y_i}\}_{i=1}^{10}$ we can compute: the following ratio:

$$\frac{\widehat{y_i}}{\widehat{y_s}} = \frac{h(r, \beta_i)}{h(r, \beta_s)} \tag{41}$$

This can be used to compute $\{\widehat{\beta_i}\}_{i=1}^{9}$. Then we can find the respective average incomes $\{\widehat{y_i}\}_{i=1}^{9}$ according to (40)

Also,

$$\widehat{w} = (1-\alpha)(\frac{r^* + \delta}{\alpha})^{\dfrac{\alpha}{\alpha - 1}} \tag{42}$$

$$\widehat{y_i} = \widehat{w}(r)e_i + r\widehat{a}_i$$

$$\widehat{a}_i = \frac{\widehat{y_i}}{r^*} - \frac{\widehat{w}}{r^*}e_i = \frac{h(r, \beta_i)}{r^*} - \frac{\widehat{w}}{r^*}e_i$$

$$\sum_{i=1}^{s} \mu_i \widehat{a}_i = \sum_{i=1}^{s} \mu_i [\frac{h(r, \beta_i)}{r^*} - \frac{\widehat{w}}{r^*}e_i] = \widehat{K}^s(r) \tag{43}$$

The above equation tells us the supply of assets in the economy. We also normalize $\sum_{i=1}^{s} \mu_i e_i = 1$.

$$r^* = \alpha(\frac{K_t}{A_t L_t})^{\alpha - 1} - \delta = aR - \delta \tag{44}$$

$$\frac{K_t}{A_t L_t} = (\frac{\alpha}{r^* + \delta})^{\frac{1}{1-\alpha}} = \widehat{K}^d(r) \tag{45}$$

The above equation tells us the demand for assets in the economy. In order to find the equilibrium interest rate, we find r* such that the supply of assets equals demand of assets in the economy, that is:

$$\widehat{K}^s(r) = \widehat{K}^d(r) \tag{46}$$

If $\widehat{K}^s(r) > \widehat{K}^d(r)$, we will decrease the rate of interest and if $\widehat{K}^s(r) < \widehat{K}^d(r)$, then we will increase the rate of interest, till supply equals demand.

Now eliminating $\widehat{c}_i$ from the following equations :

$$\widehat{m}_i - \widehat{c}_i - \psi(\gamma - 1)\widehat{a}_i = 0 \tag{47}$$

$$\widehat{y}_i - \widetilde{\tau}(\widehat{y}_i) + (1 - \pi\gamma)\widehat{m}_i - \widehat{c}_i + (1 - \gamma)\widehat{a}_i + \zeta = 0 \tag{48}$$

we get,

$$\widehat{m}_i - \psi(\gamma - 1)\widehat{a}_i = \widehat{y}_i - \widetilde{\tau}(\widehat{y}_i) + (1 - \pi\gamma)\widehat{m}_i + (1 - \gamma)\widehat{a}_i + \zeta \tag{49}$$

Aggregating the above equation and re-arranging we get,

$$\pi\gamma\sum_{i=1}^{s}\mu_i\widehat{m}_i + (\gamma - 1)(1 - \psi)\sum_{i=1}^{s}\mu_i\widehat{a}_i = \sum_{i=1}^{s}\mu_i(\widehat{y}_i - \widetilde{\tau}(\widehat{y}_i)) + \zeta \tag{50}$$

Substituting the following expression for transfers in the above equation:

$$\zeta = (g - 1)\sum_{i=1}^{s}\mu_i\widehat{m}_i \tag{51}$$

we get,

$$[\frac{\pi\gamma}{g - 1} - 1]\zeta = \sum_{i=1}^{s}\mu_i(\widehat{y}_i - \widetilde{\tau}(\widehat{y}_i)) - (\gamma - 1)(1 - \psi)\sum_{i=1}^{s}\mu_i\widehat{a}_i \tag{52}$$

Since $\pi\gamma = g$,

$$\widehat{\zeta} = (g - 1)[\sum_{i=1}^{s}\mu_i\widehat{y}_i\{1 - \eta(\frac{\widehat{y}_i}{\kappa})^{\phi}\} - (\gamma - 1)(1 - \psi)\sum_{i=1}^{s}\mu_i\widehat{a}_i \tag{53}$$

Now, we define wealth of individual i as the sum of individual assets and money holdings, that is

$$\widehat{W}_i = \widehat{a}_i + \widehat{m}_i \tag{54}$$

$$\widehat{W}_i(r^*, \beta_i, e_i) = \frac{1}{g}[\widehat{y}_i\{1 - \eta(\frac{\widehat{y}_i}{\kappa})^{\phi}\} + \widehat{\zeta} + \underbrace{\{g - (\gamma - 1)(1 - \psi)\}}_{\chi}\widehat{a}_i] \tag{55}$$

$$\widehat{W}_i(r^*, \beta_i, e_i) = \frac{1}{g}[\underbrace{\widehat{y}_i\{1 - \eta(\frac{\widehat{y}_i}{\kappa})^{\phi}\} + \widehat{\zeta} + \frac{\chi}{r^*}\widehat{y}_i}_{\Omega_i} - \chi\frac{\widehat{w}}{r^*}e_i] \tag{56}$$

Now let us define:

$$\rho_i = \frac{\widehat{W}_i(r^*, \beta_i, e_i)}{\widehat{W}_1(r^*, \beta_1, e_1)} = \frac{\Omega_i - \chi\frac{\widehat{w}}{r^*}e_i}{\Omega_1 - \chi\frac{\widehat{w}}{r^*}e_1} \tag{57}$$

The above ratio can be computed from the data on wealth of the ten income groups from in the 2007 Survey of Consumer Finances (SCF) sample Díaz-Giménez et al. (2011). We can find e_1 by aggregating the above equation. Similarly, we can find labor productivity using the same equation for $\{e_i\}_{i=2}^{i=s}$

References

Akyol, A. (2004). Optimal monetary policy in an economy with incomplete markets and idiosyncratic risk. *Journal of Monetary Economics 51*(6), 1245–1269.

Bates, T., K. Kahle, and R. Stulz (2009). Why do us firms hold so much more cash than they used to? *The Journal of Finance 64*(5), 1985–2021.

Becker, R. A. (1980). On the long-run steady state in a simple dynamic model of equilibrium with heterogeneous households. *The Quarterly Journal of Economics 95*(2), 375–382.

Boel, P. and G. Camera (2009). Financial sophistication and the distribution of the welfare cost of inflation. *Journal of Monetary Economics 56*(7), 968–978.

Camera, G. and Y. Chien (2011). Understanding the Distributional Impact of Inflation.

Chiu, J. and M. Molico (2010). Liquidity, redistribution, and the welfare cost of inflation. *Journal of Monetary Economics 57*(4), 428–438.

Clower, R. (1967). A reconsideration of the microfoundations of monetary theory. *Economic Inquiry 6*(1), 1–8.

Díaz-Giménez, J., A. Glover, and J. Ríos-Rull (2011). Facts on the distributions of earnings, income, and wealth in the united states: 2007 update. *Federal Reserve Bank of Minneapolis Quarterly Review 34*(1), 2–31.

Doepke, M. and M. Schneider (2006). Inflation and the redistribution of nominal wealth. *Journal of Political Economy 114*(6).

Dotsey, M. and P. Sarte (2000). Inflation uncertainty and growth in a cash-in-advance economy* 1. *Journal of Monetary Economics 45*(3), 631–655.

Easterly, W. and S. Fischer (2001). Inflation and the Poor. *Journal of Money, Credit and Banking 33*(2), 160–178.

Erosa, A. and G. Ventura (2002). On inflation as a regressive consumption tax. *Journal of Monetary Economics 49*(4), 761–795.

Feenberg, D. and E. Coutts (1993). An introduction to the taxsim model. *Journal of Policy Analysis and Management 12*(1), 189–194.

Imrohoroglu, A. (1992). The welfare cost of inflation under imperfect insurance. *Journal of Economic Dynamics and Control 16*(1), 79–91.

Lansing, K. and J. Guo (1998). Indeterminacy and stabilization policy. *Journal of Economic Theory 82*(2), 481–490.

Li, W. and P. Sarte (2004). Progressive taxation and long-run growth. *The American Economic Review 94*(5), 1705–1716.

Ludvigson, S. (1999). Consumption and credit: a model of time-varying liquidity constraints. *Review of Economics and Statistics 81*(3), 434–447.

Molico, M. (2006). The distribution of money and prices in search equilibrium. *International Economic Review 47*(3), 701–722.

Sarte, P.-D. G. (1997). Progressive taxation and income inequality in dynamic competitive equilibrium. *Journal of Public Economics 66*(1), 145–171.

Stockman, A. (1981). Anticipated inflation and the capital stock in a cash in-advance economy* 1. *Journal of Monetary Economics 8*(3), 387–393.

Wen, Y. (2010). Liquidity Demand and Welfare in a Heterogeneous Agent Economy. *Working Paper Series*.