Optimal Unemployment Insurance with Private Insurance

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Optimal Unemployment Insurance with Private Insurance *

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Abstract

I present a model of optimal contracts between firms and workers, under limited commitment and with worker savings. A central feature of the model is that firms can provide insurance against unemployment, by targeting a path of wages that encourages wealth accumulation. I provide an analytical expression for the scope of private insurance measured in the drop of consumption that the worker suffers when the match terminates. I then consider how government policy affects risk sharing through private markets. I find that unemployment benefits should be large and frontloaded. The government has the incentive to drive the allocation to the point where the firm's participation constraint binds. At this point wages are equal to the match productivity in every period and thus private risk sharing is crowded out. However, the drop in consumption in unemployment is minimized. Moreover, the implications of the theory of optimal contracts are assessed relative to the standard model of heterogeneous households, whereby wealth is utilized for self-insurance purposes. I show that under the optimal UI policy, the contract model and the heterogeneous households model are equivalent.

JEL codes: D52, E21, H31, H53, J41

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1 Introduction

Whether governments should provide insurance against the risk that individuals face in the labor market has been a long standing debate in economics. Much of the discussion over the scope of public policy centers around the notion that it can crowd out the private insurance arrangements of individuals, with a widespread belief that the scope of policy is limited when such arrangements are in place. Theoretical work seeking to describe this tradeoff is ample: For example Attanasio and Rios Rull (2000) and Krueger and Perri (2011) consider economies where private insurance is a contract under limited commitment between risk averse households, and show that public insurance can crowd out private risk sharing with adverse effects on welfare. Moreover, Hansen and Imhrohoroglu (1992), Wang and Williamson (2002) among others, study the properties of optimal unemployment insurance within the heterogeneous households model of precautionary savings. They illustrate that unemployment benefits discourage workers from accumulating wealth, thus reducing the scope of self insurance in the model.

This paper studies the impact of public insurance in the form of unemployment benefits, in an economy where private insurance is a contract under limited commitment offered by firms to their workforce. The contract considered is broadly similar to the self enforcing wage models of Thomas and Worrall (1988) and Rudanko (2009, 2011) whereby limited commitment means that the firm and the worker have to be better off in the match than to separate. As a consequence private risk sharing is partial because the allocation has to satisfy two participation constraints.

In contrast to Thomas and Worrall (1988) and Rudanko (2009, 2011), I assume that workers have access a storage technology and therefore can accumulate assets over time. Allowing for workers to save is novel in the literature, and therefore I study thoroughly the properties of the optimal allocation. I show that the optimal contract offers a higher wage, to some workers, at the start of the job, in order to encourage wealth accumulation. By this initial investment in wealth, the worker is (partially) insured against the risk of unemployment, and in particular she can ward off the risk of an early dismissal. I obtain an analytical expression for the scope of private risk sharing, measured in terms of the percentage drop in consumption in unemployment. Moreover, I show that this quantity depends on the firm's and the worker's relative discount rates, and on whether the participation constraints bind.

I then turn to investigate the properties of the optimal UI scheme. I obtain the following results: First, it is optimal to target a UI scheme that maximizes the region over which firms earn zero profits and their participation constraints bind, in the steady state and over the stationary distribution. This becomes particularly evident when I consider the optimal timing of unemployment insurance: I show that it is preferable to pay out benefits in the first period (quarter) of an unemployment spell as frontloading benefits is most effective in tightening the firm's constraint. Second, I establish that under the optimal policy the contract offered by firms to their workforce is essentially flat and wages are set equal to productivity in each period. In this sense there is a complete crowding out of wealth accumulation, through the upfront wage arrangement described previously.

To understand these results it is important to outline several properties of the model. First, as noted previously the relative discount rates of workers and firms exert an influence over the allocation. Under the standard notation let β be the discount rate of the worker, $\frac{1}{R}$ the discount factor of the firm and r be the market interest rate on savings. Further note that an equilibrium under incomplete financial markets exists if it holds that $\beta r < 1$ (see Huggett (1993)). I prove analytically that in the case where $R = \frac{1}{\beta}$, (as is customary to assume in the literature of optimal contracts) the allocation features: (1) underinsurance against unemployment, in the sense that consumption drops when the firm and the worker separate, and (2) the drop in consumption is less when the firm's participation constraint is binding. With $R = \frac{1}{\beta} > r$ the firm has access to a technology that offers a higher rate of return than r. It is then worthwhile to give lower wages initially to some workers, (in particular workers with high initial wealth) run down assets and finance a higher consumption path subsequently. I show that in the model this arrangement makes the firm's participation constraint bind, because promising to increase wages above productivity in the future, gives a negative value to the firm. As the firm's constraint rules out negative profits, the firm ends offering a flat contract where wages are equal to productivity. In contrast, when individuals are relatively poor and the participation constraint is slack, the wage profile is frontloaded, leading to asset accumulation.

Consider now a UI scheme that gives a benefit level higher than wages for the first period of unemployment, and subsequently reduces unemployment income considerably. Given this policy the newly unemployed agent is induced to save in order to ward off the risk of a prolonged unemployment spell. Since typically, a large fraction of the unemployed find a job after one quarter, these individuals arrive to their new jobs with a high wealth endowment. As argued previously, having high wealth initially, implies that the firms participation constraint is binding. For this reason in the model, UI policies that concentrate payments to the first period of unemployment, perform considerably better. Under the optimal policy, all individuals in the economy receive a wage equal to the match productivity.

This result, that wages are equal to productivity under the optimal UI scheme, makes the model equivalent to a heterogeneous households model whereby wealth is accumulated for precautionary savings purposes. As discussed previously, there is a sizable literature which considers the role optimal unemployment insurance in this class of models (for example Hansen and Imhrohoroglu (1992), Wang and Williamson (2002) among others). One of the key innovations of my paper is therefore to replace the assumption that workers are paid their marginal product each period, with the optimal contracting scheme, and to study its implications over several important dimensions. I provide, with the use of analytical examples, a thorough description of the differences for the consumption and savings behavior of individuals, between the two environments. I show that under the optimal contract and when the participation constraint of the firm is slack, consumption is constant over the life of the match and wealth accumulation takes place in one period, through the frontloaded wage property discussed previously. In contrast, under precautionary savings it takes several periods for the agent to build a buffer stock of assets, and for consumption to reach the stationary point. This implies that the optimal contract is particularly useful as an insurance device against early separation shocks. It also implies that the steady state distributions of wealth in the two models are quite different (for benefit levels different than the optimal policy). However, when the initial wealth endowment is above the stationary level, and the firms participation constraint is binding I show that the two models deliver essentially the same (decreasing) path for consumption and wealth. The equivalence carries over to the optimal UI policy.

This paper is also related to a recent literature studying the effect of public policy on private insurance assuming limited commitment frictions in contracts between risk averse individuals (eg. Krueger and Perri (2011), Di Tella and MacCulloch (2002), Attanasio and Rios Rull (2001)). Di Tella and MacCulloch (2002) use a model where unemployment insurance is provided by the government but also by extended families. They show that government benefits lead to more than one for one reductions in intrafamily insurance when the later is subject to limited commitment considerations. In the context of redistributive taxation, Krueger and Perri (2011) reach a similar conclusion. In Attanasio and Rios Rull (2000) public insurance takes the form of a reduction in aggregate uncertainty. Their results highlight that such interventions may reduce welfare, through adverse effects in the realm of private risk sharing. My paper is related to this work though the focus here is risk sharing provided by firms to their workforce. This focus is well grounded given the empirical support for the self enforcing wage model that I utilize (see for example Thomas and Worrall (2007) and the references therein).

The paper proceeds as follows: Section 2 presents the economic environment. Section 3 discusses the implications of the optimal contract. Section 4 contains the main results. Section 5 discusses several extensions of the baseline model. A final section concludes.

2 The Model

There is a continuum (measure one) of infinitely lived, risk averse agents with preferences of the following form:

$$(1) \qquad E_0 \sum_{t=0}^{\infty} \beta^t (\log(c_t) - v(s_t))$$

where c_t denotes the consumption of a general multipurpose good and $v(s_t)$ (where $v(0) = 0$, $v_s(s_t) > 0$ and $v_{ss}(s_t) > 0$) denotes the disutility of search. β is the discount factor.

Each period, a fraction $e = 1 - u$ of all individuals in the economy are employed, matched with firms in joint production, and the remaining u agents are unemployed waiting for a job offer to arrive. The arrival rate of job offers to an unemployed agent is given by $\gamma(s_t)$, where γ is a technology that maps search effort s_t, to the job finding probability. When matched with a firm the employed agent (worker) produces y units of output per period. The firm does not search actively for a worker, and when the job starts the firm is assumed to earn zero profits in expectation. Moreover, employed agents also don't search (hence $s_t = 0$ for them) but their matches terminate at an exogenous rate λ per unit of time. When this occurs they become unemployed.

Let $j = 0, 1, 2, ...$ denote the number of periods that an individual has spent in unemployment prior to the current period. An agent with an index j is at her $j + 1$ period as unemployed. Therefore $j = 0$ applies to a newly unemployed agent. The government provides insurance against unemployment in the form of benefits denoted by b_j. Benefits depend on the index j to show that the income received from the government varies with the duration of the spell. Not all unemployed individuals are eligible for the scheme: There is a maximum horizon m (duration of non-employment spell) beyond which the unemployment income is assumed to be zero (i.e. $b_j = 0 \;\; \forall j \geq m$) and for all $j < m$ the level of income is a constant b. To finance benefits, taxes are levied on employed individuals in the amount τ each period. It is assumed that the government runs a balanced budget. Therefore it must be that $e\tau = \sum_{j<m} u_j b$ where u_j denotes the total number of unemployed agents who are running their $j + 1$ period of joblessness.

Finally, financial markets are incomplete and agents can a trade non-contingent asset subject to an ad hoc (no borrowing) constraint $\overline{a} = 0$. The gross interest rate on the asset is denoted by r. Note that the equilibrium in this economy with incomplete financial markets requires that $\beta r < 1$ to be well defined (for asset positions to not diverge).

2.1 Value Functions

2.1.1 Employed Agents

As discussed previously, each match generates a per period output equal to y. Because firms earn zero profits at the start of the match, one possible firm-worker contract is to pay a wage y each period and let the worker rely purely on savings to insure against the job separation shock. This arrangement, however, is not necessarily optimal: Insofar as the firm is risk neutral, and the worker is risk averse, there ought to be a different arrangement, that (Pareto) dominates the flat wage contract.

This section characterizes recursively the optimal contract, as a Pareto optimal allocation, following the literature on self enforcing labor contracts. I let J_t be the present discounted (profit) value to the firm at time t, a_t the stock of wealth of the worker, and W_t the lifetime utility of the worker. The latter will be specified as a function of a_t and J_t. The optimal program consists of choosing a sequence of control variables, to maximize W_t subject to a sequence of constraints. Note that it is important to consider values of J_t that are different from zero, even though it is assumed that all matches start at $J_0 = 0$. As will become evident shortly, for a wage profile that is not constant (flat) over the life of the match, we anticipate that $J_0 = 0$ but $J_t \neq 0$ for some $t > 0$, i.e. that the total wage paid to the worker between periods 0 and $t-1$ is not (necessarily) equal to ty. Moreover, to ensure that the sequence of payments is such that both the firm and the worker (weakly) prefer to be matched, rather than to separate, I impose two sustainability constraints on the equilibrium allocation: at each point in time it must be that $J_t \geq 0$, and $W_t \geq U(a_t, 0)$ where $U(a_t, 0)$ is the lifetime utility of a newly unemployed individual.

My formulation here of the worker's program follows closely the work of Thomas and Worrall (1988), Ligon et al (2000, 2002) and Rudanko (2009). As is customary I drop all time subscripts in writing the recursive representation of the program as follows:

$$W(a, J) = \max_{a' \geq \bar{a}, J'} \log(c) + \beta(\lambda U(a', 0) + (1 - \lambda) W(a', J')) \tag{2}$$

Subject to the constraint set:

$$a' = ra + w - \tau - c \tag{3}$$

$$J \leq y - w + \frac{1-\lambda}{R} J' \tag{4}$$

$$J' \geq 0 \quad \text{and} \quad W(a', J') \geq U(a', 0) \tag{5}$$

Primes denote next period variables. Equation (4) is the so called promise keeping constraint which imposes that the firm's expected profit is at least J over the life of the match. Under the contract the firm earns $y - w$ this period, where w denotes the wage, and discounts the future profit value (J') at a rate $\frac{1-\lambda}{R}$. Notice that the discount rate for the firm, R, maybe different than the market interest rate r. In equation (2) the worker solves for next period wealth a', and for a continuation utility J' for the firm. [1] Finally, the two sustainability constraints described previously, ensuring that the allocation does not violate participation are imposed in equation (5).

2.1.2 Unemployed Agents

Equilibrium payoffs for unemployed individuals solve the following functional equation:

$$U(a, j) = \max_{a' \geq \bar{a}, s} \log(c) - v(s) + \beta\, \gamma(s)\, W(a', 0) + \beta(1 - \gamma(s))\, U(a', j+1)) \tag{6}$$

Subject to the constraint set:

$$a' = ra + b_j - c \tag{7}$$

As discussed above, when the job starts it must always be that $J = 0$ so that firms make zero profits in expectation.

[1] Notice that under the specification of the utility function of the worker, the promise keeping and the budget constraints hold with equality. Therefore consumption and wages could be eliminated as control variables from (2).

2.2 Competitive Equilibrium

This section describes the stationary competitive equilibrium. It consists of a set of value functions $\{U(a,j), W(a,J)\}$ for unemployed and employed workers respectively, and a set of decision rules on asset holdings $\{a'_e(a,J), a'_u(a,j)\}$, the firms continuation value $J'(a,J)$, and search the intensity $s(a,j)$. It also consists of a level of taxes τ and an invariant measure μ of agents across assets, employment status and J such that:

1) Agents optimize: $\{U(a,j), W(a,J)\}$ solve functional equations 2 and 6 above and optimal policies derive.

2) Taxes and benefits are consistent with Budget Balance: $e\tau = \sum_{j<m} u_j b_j$

3) The measure μ is consistent: In particular the law of motion of μ can be represented as:

$$\mu_{(e,\mathcal{A},\mathcal{J})} = (1-\lambda)\int_{a'(e,a,J)\in\mathcal{A},J'(a,J)\in\mathcal{J}} d\,\mu_{(e,a,J)} + \mathcal{I}_{0\in\mathcal{J}}\sum_j \int_{a'_u(a,j)\in\mathcal{A}} \gamma(s(a,j))d\,\mu_{(u,a,j)}$$

$$\mu_{(u,\mathcal{A},j)} = \mathcal{I}_{j=0}(\lambda)\int_{a'_e(a,J)\in\mathcal{A}} d\,\mu_{(e,a,J)} + \mathcal{I}_{j>0}\int_{a'_u(a,j-1)\in\mathcal{A}} (1-\gamma(s(a,j-1)))d\,\mu_{(u,a,j-1)}$$

where $\mathcal{A}$ and $\mathcal{J}$ are subsets of the relevant state space and $\mu_{(u,\mathcal{A},j)}$ and $\mu_{(e,\mathcal{A},\mathcal{J})}$ are the probability distributions conditional on employment status. [2]

3 Implications

This section studies the implications of the optimal contract. I show how the sustainability (participation) constraints and the model parameters impact risk sharing and the sequence of wages that the worker receives. Moreover, I compare the properties of the optimal allocation that solves program (2) with those of a model where the worker receives y each period. As discussed previously, the latter case corresponsds to the standard environment of heterogeneous households with wealth accumulation and unemployment risks (for example Hansen and Imhrohoroglu (1992) and Wang and Williamson (2002)).

[2] For brevity I use the same number of arguments in $\mu_{(e,.,.)}$ and $\mu_{(u,.,.)}$. The third argument however is J (promised utility) if the agent is employed and j (duration of the spell) if she is unemployed.

3.1 Intertemporal Behavior

In the Appendix I derive the first order conditions from program (2). I establish that the allocation satisfies the following equations:

$$u'(c_t^e)\frac{1-\lambda}{R} = \beta(1-\lambda)u'(c_{t+1}^e) + \phi_{t+1}^1 u'(c_{t+1}^e) - \phi_{t+1}^2 \tag{8}$$

$$u'(c_t^e) = \beta r \lambda u'(c_{t+1}^u) + \beta r(1-\lambda)u'(c_{t+1}^e) + \chi_t + \beta r \phi_{t+1}^1(u'(c_{t+1}^e) - u'(c_{t+1}^u)) \tag{9}$$

where u' denotes the worker's marginal utility (under log utility the inverse of consumption), ϕ_{t+1}^1 and ϕ_{t+1}^2 are the multipliers on the participation constraints for the worker and the firm respectively, and χ_t is the multiplier on the borrowing constraint. I use time subscripts to avoid double primes in the next periods marginal utility of consumption. c_t^e is the consumption of an employed agent in t, and c_t^u is the analogous object of an unemployed individual.

Equation (8) gives the optimal consumption path. Note that it has been customary in the literature of optimal contracts to assume that workers and firms have equal discount rates, so that $R = \frac{1}{\beta}$. If in addition we posit that $\phi_{t+1}^1 = \phi_{t+1}^2 = 0$ then the optimal contract gives a constant consumption path to the worker. If however, $R < \frac{1}{\beta}$ (for example in the case of $R = r$) then (8) implies that the consumption sequence is decreasing over the life of the match.

To understand how the participation constraints influence the allocation assume first that $\phi_{t+1}^2 > 0$: In this case the worker's marginal utility of consumption in period t exceeds the marginal utility in $t+1$ and therefore the level of consumption must drop, giving a higher share to the firm in the future. The converse holds if $\phi_{t+1}^1 > 0$. In this case it is the worker that needs to be made better off, and therefore consumption must increase tomorrow, when the worker's participation constraint binds. Notice that it can never be that both ϕ_{t+1}^1 and ϕ_{t+1}^2 are greater than zero simultaneously in a match with positive surplus.

Equation (9) is a *modified* Euler equation (see Ligon et al (2000)). Off corners, when $\phi_{t+1}^1 = 0$ and $\chi_t = 0$ it equates the cost of a unit of savings today measured in terms of the marginal utility, with the benefit from the additional unit, expressed as the discounted (expected) future marginal utility. However, equating the cost and benefit from savings does not hold if $\phi^1_{t+1} > 0$ i.e. when the participation constraint of the worker binds. Note that since assets influence the utility levels $W(a, J)$ and $U(a, 0)$, they also influence the tightness of the participation constraint. Therefore, the Euler equation needs to be augmented to include the last term. Assuming that $u'(c^e_{t+1}) - u'(c^u_{t+1}) < 0$ i.e. that consumption drops when the agent becomes unemployed, (9) suggests that when $\phi^1_{t+1} > 0$ (and $\chi_t = 0$) the worker is savings constrained in the sense that $u'(c^e_t) < \beta r \lambda u'(c^u_{t+1}) + \beta r(1-\lambda)u'(c^e_{t+1})$. In this case, the influence of the participation constraint is to reduce the amount of wealth accumulated, because more wealth would further tighten the constraint. [3]

Finally, note that the value of ϕ^2_{t+1} exerts no influence on the inter-temporal Euler equation. Actual savings are equal to desired savings when $\phi^2_{t+1} > 0$. This result is important for the following reason: It suggests that in solving for the optimal contract in (2) it is not necessary to consider the Euler equation as an additional constraint, if we focus on cases where only the firm's participation constraint may bind. As I will later illustrate, the equilibrium in the model features this property. Therefore the Euler equation will hold in the model.

3.2 Insurance Against Unemployment

I now consider the properties of the optimal allocation focusing on the implications for the agent's consumption path in unemployment. By deriving the ratio of the worker's consumption in employment, in a given period, relative to her consumption if she were to lose her job in that period, I characterize the insurance value of the optimal contract. In the Appendix I show that rearranging (8) and (9) we can get the following expression:

$$\text{(10)} \qquad \frac{(c^e_{t+1})}{(c^u_{t+1})} = 1 + \frac{(R-r)}{r(\lambda - \phi^1_{t+1})} - \omega + \frac{R}{1-\lambda}\frac{1}{r(\lambda - \phi^1_{t+1})}(\phi^1_{t+1} - \frac{\phi^2_{t+1}}{u'(c^e_{t+1})})$$

[3] Consider the following argument: Since c^e_{t+1} must increase to make the worker better off in employment, and since $u'(c^e_{t+1}) - u'(c^u_{t+1}) < 0$ and marginal utilities are the partial derivatives of the lifetime utilities with respect to wealth, should the rise in consumption be financed by savings it would tighten the constraint (see Ligon et al (2000)).

where $\omega = \frac{\chi_t}{u'(c^e_{t+1})\beta r(\lambda - \phi^1_{t+1})}$.

There are several noteworthy features: First, note that off corners, the right hand side of (10) becomes: $1 + (\frac{R}{r} - 1)\frac{1}{\lambda}$. In the case where $R = r$ this condition implies that the firm insures the worker perfectly against the event of separation, as consumption does not fall when in unemployment. However, if $R = \frac{1}{\beta} > r$, the unemployment spell leads to a drop of consumption that is proportional to the difference in the rates of return.

To understand how relative discounting affects the allocation, note that since the storage technology possessed by the worker delivers a return equal to r and because the firm is risk neutral and is assumed not to face borrowing constraints, the firm effectively has access to a storage technology that earns a superior rate of return if $R = \frac{1}{\beta}$. It is then worthwhile for the worker to accept a lower wage when the match starts (say in period $t = 0$), and enjoy higher wages in the future. Notice that under the previous results, in this case consumption stays constant over the life of the match (i.e. $c^e_t = c^e_{t+1}$). It follows that the lower wage in $t = 0$ does not result to lower consumption but rather the impact is to reduce the wealth invested for period $t + 1$. As (10) reveals assets are then held constant in all future periods, since the ratio of marginal utilities in (10) will also be constant.

Note that the above argument should not be construed to mean that the initial wage offered to the worker is below productivity, in which case assets are run down when the match starts. Generally, because wealth is the instrument via which the risk of unemployment can be mitigated, should the firm offer initially a wage greater than y and thereby encourage asset accumulation, the drop in consumption in unemployment is less. This will be the case, even if we assume $R = \frac{1}{\beta}$. The argument above highlights that the investment in wealth is smaller than if we have $R = r$.

To explain better the behavior of wages under the assumption $R = \frac{1}{\beta}$, and in the case where the participation constraints don't bind I provide the following proposition:

Proposition 1. *Assume that* $R = \frac{1}{\beta}$ *and* $\phi^1_{t+1} = \phi^2_{t+1} = 0$ *for* $t = 0, 1, 2, ...$ *Let* $t = 0$ *denote the initial period of the match. Analogously let* w_t *be the wage rate offered by the firm to the worker in* t. *It is possible to show that*:

i) *Wages are constant for periods* $t = 1, 2, ...,$ *i.e.* $w_t = \overline{w}$ *(a constant) for all* $t \geq 0$

ii) *The initial wage* w_0 *could either satisfy* $w_0 > \overline{w}$ *or* $w_0 = \overline{w}$*. In the latter case it must be that* $\overline{w} = y$*.*

As mentioned previously i) follows from the fact that $c_t^e = c_{t+1}^e$ under the optimal contract. Then since the ratio of the marginal utility of consumption in employment and unemployment is constant in $t + 1$ (as equation (10) shows), by the envelope condition in the unemployed agent's value function, it must be that assets are constant after $t = 1$. This implies that the wages offered are also constant (from the budget constraint).

Part ii) of the proposition states that the initial wage could be of a larger value than $\overline{w}$. In fact by the property of stationarity of wages from $t = 1$ onwards it follows that if $w_0 > \overline{w}$ then $w_0 > y$ and $\overline{w} < y$ (otherwise profits would not be zero). Notice that such a scheme does not violate the firm's participation constraint. This holds since firms make profits greater than zero after the initial period. Moreover, note that because of the stationarity of assets, wages and consumption for $t \geq 1$, a higher wage in period zero leads to a larger wealth stock a_1.

Could the initial wage be smaller than y thus leading to an extraction of the worker's wealth endowment? The answer is no. In such a case we can claim that the implied wage profile is one that violates the firms participation constraint. Assume the converse: Let the firm pay $w_0 < y$ in the initial period. Under zero initial profits it follows that the firm must then offer $\overline{w} > y$ (i.e. wages greater than output in every subsequent period) The firm earns a negative present value of profits from $t = 1$ onwards thus violating participation.

The suggested wage profile in proposition 1 is feasible if we can show that there are values of a_1 such that the constraint $W(a_1, J_1) > U(a_1, 0)$ is slack. It is however very difficult to provide general conditions for this to be the case, especially in light of the nonlinearities involved. Obviously feasibility depends on the initial wealth endowment of the agent and the overall environment, i.e. the shape of the payoff functions $W(a, J)$ and $U(a, 0)$. It turns out that this is the case in the simulation results I provide below. Moreover, in the next paragraph I provide analytical examples to support the argument.

I now turn to the effect of the participation constraints on the consumption ratio in equation (10). Assume that the firm needs to be made better off so that $\phi^2_{t+1} > 0$. Then as (10) reveals if $R = r$, the worker maybe overinsured against the job separation as consumption could rise if she becomes unemployed. In the more relevant case of $R = \frac{1}{\beta}$ the drop in consumption is less (the ratio $\frac{c^e_{t+1}}{c^u_{t+1}}$ is smaller). The converse holds if $\phi^1_{t+1} > 0$. In this case the worker needs to be made better off under the contract, and her consumption drops when she becomes unemployed even if we assume $R = r$. Notice that this echoes to the previous result that the worker is savings constrained in this case. The following proposition summarizes these results:

Proposition 2. *Consider the unemployment insurance properties of the optimal contract as shown in equation (10).*

i) *If $\chi_t = 0$ and $\phi^1_{t+1} = \phi^2_{t+1} = 0$ and $r = R$ then the worker is perfectly insured against unemployment (in the sense that $c^u_{t+1} = c^e_{t+1}$). On the other hand with sufficient discounting $r < R = \frac{1}{\beta}$ the agent is underinsured almost everywhere on the optimal contract (unless $\chi_t > 0$ in which case it is impossible to sign the difference in marginal utilities.)*

ii) *Under a binding participation constraint for the worker ($\phi^1_{t+1} > 0$) the drop in consumption in the event of separation is larger. Under a binding constraint for the firm, the drop in consumption is less.*

Proof: See text.

3.2.1 Two Analytical Examples

I derive here two examples, that illustrate the properties of the allocation under the assumption $r < R = \frac{1}{\beta}$. Example 1 illustrates the properties of the optimal wage schedule in closed form. Example 2 explains the decreasing consumption profile property in the case where the firm's participation constraint is binding.

Example 1: Unemployment as an absorbing state. For simplicity consider the following version of the model: Assume that an employed individual faces a constant probability of unemployment λ each period but assume that when the separation shock arrives the agent stays unemployed forever (hence $v(s) = 0$). Moreover, assume that benefits are zero at all horizons and therefore taxes are also zero. For simplicity let $r = 1$.

Under these assumptions it is possible to derive the value function $U(a, 0)$ as follows:

$$\begin{aligned} U(a,0) &= \frac{\log(a)}{1-\beta} + \frac{\log(1-\beta)}{1-\beta} + \beta\log(\beta) + 2\beta^2 \log(\beta) + 3\beta^3\log(\beta) + ... = \\ &= \frac{\log(a)}{1-\beta} + \frac{\log(1-\beta)}{1-\beta} + \beta\log(\beta)\frac{1}{(1-\beta)^2} \end{aligned} \tag{11}$$

Now consider the worker's initial employment period with a value function $W(a_0, 0)$. As described previously the worker must decide on an initial wage w_0 and a constant wage $\overline{w}$, from period one onwards. Moreover, by the firm's promise keeping constraint it must be that $w_0 = y + (y - \overline{w})\frac{1-\lambda}{R-1+\lambda} = y + (y - \overline{w})\epsilon$. Letting the worker have initial assets of a_0, and a constant consumption of c^e during employment it must then be that:

$$a_1 = a_1 + \overline{w} - c^e \rightarrow c^e = \overline{w}$$

$$a_1 = a_0 + w_0 - c^e = a_0 + y + (y - \overline{w})\epsilon - \overline{w} = a_0 + (y - \overline{w})(1 + \epsilon) \tag{12}$$

Finally, notice that since a_1 is constant the worker's value function satisfies: $W(a_0, 0) = W(a_1, J_1)$. In particular it holds that:

$$W(a_0, 0) = \frac{1}{1-\beta(1-\lambda)}(\log(\overline{w}) + \beta\lambda U(a_1, 0)) \tag{13}$$

The first order condition that defines the optimum is given by: $\frac{1}{\overline{w}} = \beta\lambda\frac{1}{a_1}\frac{1}{1-\beta}$, which yields that:

$$\overline{w} = a_0(1-\beta) + y\frac{1-\beta}{1+\beta\lambda-\beta}$$

Notice that $\overline{w} \leq y$ if and only if the following condition is met:

Condition 1. *The firm's participation constraint is slack if it holds that:*

$$a_0(1-\beta) \leq y\frac{\beta\lambda}{1+\beta\lambda-\beta} \tag{14}$$

Intuitively Condition 1 states that if the worker has low initial wealth, she is underinsured against unemployment. It is then optimal to borrow from the firm in period 0 and accumulate assets. If on the other hand a_0 is high enough, then the difference in the rates of return induce the agent to want save with the firm and enjoy higher wages and consumption in the future. As the previous discussion indicated in this case it will be that $w_0 = \overline{w} = y$ (or the firm's participation constraint will bind). I will later show that this property also holds in the simulations of the model.

The allocation defined above is optimal if we can illustrate that the worker's participation constraint is slack. Given the solution to the worker's program we can demonstrate that this is the case if it holds that:

$$W(a_1, J_1) > U(a_1, 0) \quad \text{or} \quad \frac{1}{1-\beta(1-\lambda)}(\log(\overline{w}) + \beta\lambda U(a_1, 0)) > U(a_1, 0) \tag{15}$$

Making use of the above formulas (15) becomes:

$$\frac{\log(\overline{w})}{1-\beta} > \frac{\log(a_0\beta\lambda(1+\epsilon) + y\beta\lambda(1+\epsilon)^2)}{1-\beta} + \frac{\log(1-\beta)}{1-\beta} + \beta\log(\beta)\frac{1}{(1-\beta)^2} \tag{16}$$

(16) then gives:

$$\frac{\log(1-\beta)}{1-\beta} > \frac{\log(\beta\lambda(1+\epsilon))}{1-\beta} + \frac{\log(1-\beta)}{1-\beta} + \beta\log(\beta)\frac{1}{(1-\beta)^2}$$

or

$$0 > \log(\frac{\lambda}{1-\beta+\beta\lambda}) + \log(\beta)\frac{1}{(1-\beta)} \tag{17}$$

Note that the second term on the LHS in (17) is negative as β is less than one. Moreover, the leading term is also negative if λ is less than one. Therefore the inequality in (17) holds, proving that the allocation does not violate the worker's participation constraint. This provides an example of the wage scheme in proposition 1.

Example 2: Decreasing consumption under a binding participation constraint. Consider the model of example 1, however assume that $\lambda = 0$ (no separations). In this case it is evident that the first order condition of the worker's program derived previously, does not have an interior solution. For a worker with wealth a_0 at the initial date of the match, the optimum is to set $a_1 = 0$. The implied initial wage, w_0, is below productivity to finance a higher wage and consumption profile in the future. It must then be that $\overline{w} > y$ thus violating the firm's participation constraint. Under this condition wages have to be equal to productivity in every period. The optimal consumption path can be shown to satisfy the following equations:

$$\frac{1}{c_t^e} = \beta \frac{1}{c_{t+1}^e} = \frac{1}{c_{t+1}^e} - \phi_{t+1}^2 \quad \text{if} \quad a_{t+1} > 0 \tag{18}$$

$$\frac{1}{c_t^e} = \beta \frac{1}{c_{t+1}^e} + \chi_t = \frac{1}{c_{t+1}^e} - \phi_{t+1}^2 + \chi_t \quad \text{if} \quad a_{t+1} = 0 \tag{19}$$

Notice that both (18) and (19) are consistent with equation (9) so long as $\phi_{t+1}^1 = 0$. (18) suggests that consumption is given by the standard Euler equation implying a decreasing profile insofar as assets are greater than zero, and a constant consumption profile (obviously equal to y) if assets are at the zero bound. Therefore, under a binding participation constraint and when assets are above the desired level (here zero) consumption is high and decreasing over time, until the desired wealth level is hit.

We can derive the optimal path applying the following arguments: First, note that insofar as wealth is positive, the worker's budget constraint gives:

$$a_{t+1} = a_t - c_t^e + y = ... = ty + a_0 - \sum_{j=0}^{t} c_j^e = ty + a_0 - c_0^e \sum_{j=0}^{t} \beta^j \tag{20}$$

where the last equality follows from (18). Second, assume that from $t = 0$ to some $t = T - 1$ the optimal consumption path is indeed given by $c_t^e = \beta^t c_0^e$ for some c_0^e. This implies that $c_{T-1}^e = a_{T-1} + y$ and that $a_T = 0$. Given this path and since consumption is given by y for any $t \geq T$ we can derive the worker's lifetime utility as follows:

$$W_T = \mathcal{I}(T>1)\sum_{t=0}^{T-2}\beta^t \text{log}\beta^t(c_0^e) + \beta^{T-1}\text{log}(a_{T-1}+y) + \frac{\beta^T}{1-\beta}\text{log}(y)$$

or making use of the formulas above and rearranging:

$$\begin{aligned} W_T = {} & \mathcal{I}(T>1)(\frac{1-\beta^{T-1}}{1-\beta}\text{log}(c_0^e) + \frac{\beta}{1-\beta}\text{log}(\beta)(\frac{1-\beta^{T-2}}{1-\beta} - (T-2)\beta^{T-1})) \\ & + \beta^{T-1}\text{log}(a_0 + Ty - \frac{1-\beta^{T-1}}{1-\beta}c_0^e) + \frac{\beta^T}{1-\beta}\text{log}(y) \end{aligned} \tag{21}$$

Notice that the optimal allocation is one that maximizes (21) with respect to T and c_0^e subject to $a_t \geq 0$ for $t = 1, 2, ...T-1$ and subject to $a_T = a_0 + Ty - \frac{1-\beta^t}{1-\beta}c_0 \geq 0$. The problem is then trivial since when T is constrained to be an integer, the constraints $a_t \geq 0$ are violated for any T other than the optimal one. For example, assume $a_0 < \frac{1-\beta}{\beta}y$. It is then evident that setting $T = 2$ would violate the asset bound as in this case $a_1 = y + a_0 - c_0 = a_0 + y - \frac{2y+a_0}{1+\beta} < 0$. Moreover, we can establish that:

$$W_1 = \text{log}(a_0+y) + \frac{\beta}{1-\beta}\text{log}(y) > W_2 = (1+\beta)\text{log}((a_0+2y)) + \beta\text{log}(\beta) + \frac{\beta^2}{1-\beta}\text{log}(y)$$

whenever $a_0 < \frac{1-\beta}{\beta}y$. Similarly $W_2 > W_3$ when $a_0 < \frac{1+\beta-2\beta^2}{\beta^2}y$ and so on.

One final comment is in order: Note that example 2 applies also to the case where $\lambda > 0$. The difference is that the optimal stationary asset level is then positive and equal to $y\frac{\beta\lambda}{(1+\beta\lambda-\beta)(1-\beta)}$, as the previous results indicate. [4] If the worker starts the job with assets above the that level, the firm's participation constraint binds, assets are gradually run down and consumption falls over time.

[4] When $\lambda > 0$ the drop of consumption will be slower than at rate β because it is optimal to hold assets for self insurance purposes. The Euler equation will then contain an additional term, the marginal utility of consumption in unemployment, (see the derivations in example 1).

3.2.2 Optimal Compensation

Figure 1 illustrates the wage profile of the employed agent as a function of her initial wealth, under the baseline version of the model (see section 4 for details on the parameter values). The top left panel plots the wage in the first period of the match. The bottom panel represents the analogous wage schedule in period two and every other period.

Consistent with previous theoretical results, the baseline model produces a region where individuals take an upfront payment (higher wages in the first period) which helps to build a stock of assets. In the second period, wages drop permanently so that firms make positive profits in the match. At higher (initial) wealth, the allocation is such that the firm's participation constraint binds, and wages are equal to $y = 1$ throughout. Morever, since it is assumed that $R = \frac{1}{\beta}$, the optimal consumption path stays constant and the optimal allocation features stationarity, after the initial period, meaning that wealth and the level of utility are also constant. This stationary region in the figure, corresponds to any value of assets that gives an initial wage greater than one. Conversely, if a worker starts at a very high level of wealth, the firm's participation constraint binds and consumption drops overtime. The allocation will then (gradually) converge to the lowest wealth level such that $w_0 = y$ (around 0.8 in terms of asset income).

[Figure 1 About Here]

3.2.3 Why the worker's participation constraint is slack

Under the assumption $R = \frac{1}{\beta} > r$ the model doesn't give a region where the worker's participation binds. This was also shown to be the case previously under examples 1 and 2. [5] To better understand this feature of the model, assume that instead we had $R = r$. As established previously in this case the worker's consumption drops over time, and if it were not for the explicit participation constraint, the marginal utility would tend to infinity (consumption would tend to zero). Under the limited commitment contract, however, such paths can be ruled out since at or beyond the borrowing limit, the worker becomes

[5] For example 1 this property was established. For example 2 it follows from the fact that the firm's constraint binds and the worker's constraint cannot bind since the match surplus is strictly positive.

eventually better off in unemployment (at least if government benefits are positive). [6] In this model the stationary point of the allocation, is where the worker's participation constraint binds. To put this differently the worker's constraint is particularly relevant when $R = r$ but not when $R = \frac{1}{\beta} > r$ as I have assumed.

3.3 Comparison with flat wage contracts

I have thus far illustrated that the optimal contract under the assumption $R = \frac{1}{\beta}$, is such that firms frontload wages to encourage asset accumulation, and that provided the participation constraints are slack, consumption and wealth are constant, for a long as the job lasts. Employed individuals therefore, accumulate assets up to the stationary point, which is reached in one period. Individuals that lose their job but are lucky not to suffer from a prolonged unemployment spell will eventually have higher wealth, since assets are run down in unemployment.

These properties can be contrasted to the typical shape of private consumption and wealth accumulation during employment, in models of heterogeneous agents without optimal contracts (for example Wang and Williamson (2002)). Since in these models, the labor income is higher in employment (assuming that the wage is equal to y each period), workers accumulate wealth over time and consumption grows during employment. This occurs until a buffer stock of assets is built and then consumption remains constant. Therefore, in contrast to the optimal allocation, under a flat contract it may take considerable time to build that buffer stock.

To clarify this intuition it is useful to first consider the derivations of examples 1 and 2 from the previous subsection. Note that in example 1 it was established that insofar as the the firms participation constraint is slack (i.e. at an initial wealth level $a_0 \leq \tilde{a} = y\frac{\beta\lambda}{(1+\beta\lambda-\beta)(1-\beta)}$ the optimal policy was to have: $a_1 = a_0\frac{\beta\lambda}{1+\beta\lambda-\beta} + y\frac{\beta\lambda}{(1+\beta\lambda-\beta)^2}$. When the firm's constraint was binding (i.e. when $a_0 > \tilde{a}$) it was optimal to decrease assets up to the point $\tilde{a}$. One can arguably make the case that for any initial wealth level exceeding this threshold, the optimal contract gives a solution identical to the standard

[6]The borrowing constraint precludes to have marginal utility in unemployment tending to infinity (eg. assets tend to a natural borrowing limit of minus the present value of unemployment benefits).

of heterogeneous agents model i.e. a solution to the following program:

$$\mathcal{W}(a_0) = \max_{a_1 \geq 0} \log(-a_1 + a_0 + y) + \beta(1-\lambda)\mathcal{W}(a_1) + \beta\lambda U(a_1, 0) \tag{22}$$

where $U(a_1, 0)$ is given by (11). Under the previous results it holds that: $W(a_0, 0) = \mathcal{W}(a_0)$ if $a_0 \geq \overline{a}$, but also that $W(a_0, 0) \geq \mathcal{W}(a_0)$ if $a_0 \leq \overline{a}$. [7]

In figure 2 I illustrate the wealth accumulation paths under the baseline calibration of the model. The solid line corresponds to the optimal contract, the crossed line shows asset growth for the flat contract and the dashed line is at 45 degrees. Notice that in contrast to the case of example 1, the stationary wealth levels are different between the two allocations and also the rate of asset de-cumulation is different. This property derives from the fact that in the baseline model the shape of the unemployment payoff function is not identical in the two models, since the analogous value functions for employed individuals also differ. In example 1 I had assumed that the lifetime utility of unemployment is given, by making unemployment an absorbing state. In the baseline model this property obviously does not hold.

[Figure 2 About Here]

There are several noteworthy features: First, notice that where asset growth is positive, a newly employed agent experiences considerably stronger wealth growth over the first period, under the optimal contract. For instance, if the worker starts the job with zero wealth after one period, her wealth endowment increases to roughly 0.47. If she starts with assets of 0.2, her wealth goes to 0.5. The analogous figures for asset accumulation are substantially smaller in the case of the flat contract. From zero initial wealth, next period's wealth is roughly at 0.08. Second, note that since wealth accumulation under the optimal contract takes place in the first period of the match, eventually the wealth level could be greater under a flat wage contract. This is so in the case of zero initial assets: The optimal contract delivers 0.47 whereas the stationary wealth level for the flat contract is 0.62. Obviously this does not hold when initial wealth is high.

[7] This follows from the fact that a flat wage contract is Pareto dominated by the optimal allocation. Note that under both arrangements firms make zero profits but the worker optimizes in $W(a_0, 0)$.

These properties are useful to think about the steady state distribution of assets in the two cases. Under the optimal allocation, for employed individuals the stationary distribution will be concentrated in points above 0.47 whereas under the standard model of heterogeneous agents, for some individuals wealth levels could be considerably low. Moreover, under the optimal contract individuals are obviously much better placed to deal with early separation shocks, whereas if wages are flat, it will take several periods until a buffer stock of assets is built.

To further illustrate this point, but also to illustrate the scope of (private) insurance through wealth in the two models in figure 3, I show the consumption ratio $\frac{c^u_{t+1}}{c^e_{t+1}}$ as a function of the individuals initial wealth endowment. Under the solid line I represent the optimal contract. Under the dashed line the flat contract. Moreover, the top panel shows the consumption ratios, when the worker loses her job after one period, and the bottom panel shows the expected consumption loss, that is the weighted average of the events that the match survives for x periods, weighted by the probability of duration x. As the results suggest the optimal contract provides considerably more insurance (under the baseline calibration) over all wealth levels and across both horizons. [8]

4 Numerical Analysis

4.1 Calibration

I briefly explain the choice of parameters and functional forms.

Following Wang and Williamson (2002) I assume that the search function is of the following form: $\gamma(s) = 1 - e^{(-\overline{\gamma}s)}$, where $\overline{\gamma}$ is a constant. The cost of search is given by $v(s) = s^{\delta}$. I set $\delta = 2$ (quadratic cost).

In order to pin down the separation rate λ I assume as in the search and matching literature that over a monthly horizon workers face a probability of 2.5% of losing their job. Since one period in the model corresponds to one quarter, I have to recover from this assumption the quarterly value for λ. To accomplish this I assume that the stationary

[8]Notice that if initial wealth is high, the consumption loss could increase over time if the worker runs down her stock of assets. This is typically the case in the upper part of the wealth grid.

unemployment rate is 6.2 % (average in the CPS over the years 1994-2011). This gives me a value for the monthly job finding probability of 0.3782 (denote this by γ^{monthly}). Then the number of unemployed individuals that have a duration of up to one quarter is given by $(1-u)\lambda^{\text{monthly}}(1+1-\gamma^{\text{monthly}}+(1-\gamma^{\text{monthly}})^2)$. This gives a (quarterly) value for λ equal to $\lambda^{\text{monthly}}(1+1-\gamma^{\text{monthly}}+(1-\gamma^{\text{monthly}})^2)$ since the stationary unemployment rate is the same at both horizons. The corresponding value is 0.0503. Moreover, I set $\overline{\gamma}$ so that the model produces an unemployment rate of 6.2%. This gives a value of 2.54 for $\overline{\gamma}$.

The baseline unemployment benefit scheme is such that each agent earns 50 % of gross income for the first two quarters in unemployment and zero afterwards. Formally $b_j = .5y$ For $j = 0, 1$ and $b_j = 0$ For $j \geq 2$. [9] I normalize the value of y to unity.

Finally, I calibrate the discount rates and the market interest rates as follows: I choose a value for r equal to one as Wang and Williamson (2002) do. This practically means that workers have access to a storage technology, and they earn zero return on their savings. For parameters R and β, which given r govern consumption loses in unemployment, I target values so that the model yields an average (over an annual horizon) consumption loss suffered by workers that experience an unemployment spell of 6.8% consistent with the empirical evidence in Gruber (1997). This procedure gives $\beta = 0.990675$. Then $R = \frac{1}{\beta}$ is roughly equal to 1.00941.

4.2 Optimal Unemployment Benefit Scheme

This section turns to the evaluation of the optimal unemployment benefit scheme. I assume that the government implements a change in policy, and offers a different UI schedule than the baseline. The government takes private behavior as given and therefore it does not exert direct control over the risk sharing arrangement between workers and firms. To characterize the optimal policy I focus on the steady steady outcome. Therefore the evaluation of the optimal policy is made, assuming that the economy has settled to

[9] Note that this corresponds to the current policy in most states in the US, assuming that benefits are not extended as usual in periods of "high unemployment". Since my target rate of unemployment is 6.2% the baseline benefit scheme is realistic.

the new steady state distribution.

Moreover, I consider a restricted class of UI schemes here, and in particular schemes that give out different levels of unemployment benefits over two time intervals: the government pays out $b_{\leq m}$ for any duration smaller that m, and it pays $b_{>m}$ for durations exceeding m. One example of such a policy, is the current UI benefit schedule in the US that pays a constant replacement ratio for up to two quarters, and no benefits subsequently (hence $b_{\leq 1} = b$ and $b_{>1} = 0$). I maximize welfare over $b_{\leq m}$ and $b_{>m}$ considering cases where where m is either 0 or 1 that is benefits are given for either one or two quarters. [10] I restrict the analysis along these lines for two reasons: First, because it is computationally very difficult to consider policies that optimize benefits over many different time intervals. Second, because in the model most unemployment spells end after two quarters. Therefore extending to consider a more complex benefit scheme would not affect my conclusions.

In table 1 I report the results from various UI schemes. "Zero benefits" sets the levels of both $b_{\leq m}$ and $b_{>m}$ equal to zero. "Optimal Timing" shows the outcome from maximizing welfare over $b_{\leq m}$ and $b_{>m}$. The welfare effects reported in the second column of the table, are shown relative to the baseline scheme and are stated in terms of the compensating variation (hereafter CV): Practically this measures how much more consumption (in percentage terms) individuals require in the original steady state to be as well off as under the new UI scheme. [11] For each of the policies considered columns three and four report percentage changes in the unemployment rate and the required tax revenue to balance the budget.

[10] As previously the m denotes the number of periods in unemployment prior to the current quarter. Therefore, $m = 0$ corresponds to a newly unemployed individual.

[11] To evaluate the welfare effects of different policies I assume that the social planner assigns equal weight to all agents in the economy. The welfare criterion is of the form:

$$\Theta = \int W(a, J)\, d\, \mu_{e,a,J} + \sum_j \int \; U(a, j) d\, \mu_{u,a,j}$$

I convert the welfare numbers in terms of percentage consumption using the following calculation:

$$\Theta_1 = \Theta_0 + \frac{1}{1-\beta} \log(1+\epsilon)$$

Where Θ_0 is the expected utility in the baseline regime and Θ_1 is the analogous object under a different policy regime. The fraction ϵ is therefore the standard measure of compensated variation.

Within the class of UI policies considered, the optimal scheme is one that pays for the first quarter of unemployment, benefits equal to 1.165 and subsequently (for all durations greater than a quarter) it pays 0.40. According to the results in the table under this policy there is a welfare gain of 0.75% in terms of CV. Other policies considered may also improve considerably on the baseline UI scheme, however, the welfare gains are smaller. For instance an "Optimal Timing" which sets $m = 1$ (i.e. chooses benefits optimally for two quarters and separately for longer durations) gives a gain of 0.50%. Moreover, setting optimal benefits for the first m quarters, but subsequently restricting benefits to zero (last two rows of the table) also does better than the baseline and delivers similar gains for $m = 0, 1$.

4.2.1 Understanding the result

These patterns can be explained using the results of previous sections. An important implication of the analysis, was that the scope of insurance is maximized when the firms participation constraint binds. It was in that region that assets were beyond the desired stationary point (the buffer stock level) and consumption declined over time. If the worker lost her job, the drop in consumption relative to employment was less, because savings were effective in mitigating the risk of unemployment. Moreover, these properties were common with the model of heterogeneous agents; in fact the optimal allocation was to offer a wage equal to y (flat contract) in that region.

Given these remarks we anticipate that the government would have an implicit incentive to maximize the frequency with which the firms' participation constraints bind over the stationary distribution. Figure 4 shows on the left panel the wage profile under the optimal UI scheme, and on the right, the stationary wealth distribution of employed individuals. Notice that the optimal contract is effectively a flat wage contract: the entire distribution falls in the region where a wage equal to y is offered to the worker in all periods. Therefore, under the optimal policy the firms participation constraint is tight independent of the initial wealth endowment of the worker.

Frontloading UI payments accomplishes to tighten the firm's constraint for two reasons: First, benefits that exceed y in the first period of unemployment but fall rapidly for the remaining periods of a spell, induce individuals to save. This effectively means that a considerable fraction of workers who manage to find jobs after one quarter, will start their matches with wealth above the stationary point. This effect operates through the ergodic distribution.

To further explain this point, assume that unemployment benefits were equal to zero at all horizons. In the simulations such as scheme implies that over the entire ergodic distribution the optimal contract is one that offers a frontloaded wage and a constant consumption path over time (i.e. the constraint is slack everywhere). Workers accumulate assets when the match formulates, through the wage schedule properties, but when the unemployment shock arrives they run them down. Over the stationary distribution there is no one with wealth at or above the buffer stock level. The opposite holds under the suggested UI scheme. Individuals save in unemployment and a large fraction find new jobs with high wealth.

The second reason for why the frontloaded UI scheme is most effective in tightening the firms constraint is that in the model, wealth is the vehicle via which firms can contract on the worker's value in unemployment, and control their consumption in that state. However, since the agent is effectively beyond the reach of the firm when she becomes unemployed, it is only consumption in the first period that may be influenced, or the payoff $U(a, 0)$. Frontloading benefits reduces the value of wealth in $U(a, 0)$ but not in $U(a, 1)$, $U(a, 2)$ and so on. Therefore the optimal investment in wealth is less under this policy.

The combined effect of these channels, is important to get the results in figure 4. To illustrate this point in figure 5 I show the wage profile and the stationary distribution in the case where UI payments are received only in the first quarter of unemployment and subsequently they are set to zero (second to last column in table 1). Notice that in this case in the stationary distribution there is still a considerable mass of individuals whose wealth level is below the buffer stock level. This is so because individuals reduce their savings very rapidly, if their spell lasts for longer than one quarter. The welfare gains implied are smaller than under the optimal policy, as unemployment benefits extended to longer durations are particularly useful in mitigating the risk from prolonged spells.

Finally, note that policies setting benefits optimally for the first two quarters (i.e. $m = 1$ in the table) do not induce individuals to save sufficiently in quarter one. In effect these schemes produce very few individuals with sufficient wealth to reach the point where the firm's participation constraint is binding. This also reduces the welfare gains from UI.

4.3 Benefits under flat wage contracts

The previous paragraph showed that the optimal UI scheme is frontloaded and effectively one that makes the optimal contract offered to all workers, a flat wage contract. Moreover, the analysis of section 3 demonstrated that when the firm pays a constant wage the allocation is essentially equivalent to the model of heterogeneous households, when the value function of the unemployed agent is common in the two models. This property holds in the steady state, under the optimal policy.

In now turn to the properties of the optimal UI scheme under heterogeneous households. I establish that the optimal policy is the very one that maximizes welfare in the optimal contract economy. The results are shown in table 2. Notice that under "Optimal Timing" with $m = 1$, the target benefit levels $b_{\leq m}$ and $b_{>m}$, are identical to the previous case of the optimal contract. Moreover, this policy of frontloading benefits in the first quarter of the unemployment spell delivers the highest welfare gains. [12]

Notice that obtaining the same benefit schedule in the two models is far from being an obvious result. To put this differently though the allocation is equivalent under the optimal policy (or in a region close to it) the outcomes can differ substantially away from the optimal benefit scheme. Therefore it is necessary to explain why in the heterogeneous agent model the planner wants to frontload benefits.

The crucial observation is that the government wishing to provide insurance against unemployment is generally better placed to do so if it utilizes state contingent benefits,

[12] Notice that the welfare gains across all schemes are more modest now than under the optimal contract economy. The reason is that in the benchmark economy, in the case of the heterogeneous household model, aggregate unemployment is slightly less than 6.2% and therefore taxes are also smaller. I however have chosen not to adjust the value of $\overline{\gamma}$, in order to keep the optimal policy implications comparable across the two models.

rather than non contingent assets. For example, in the case where benefits are set equal to zero workers have a stronger incentive to save. But as savings are not particularly useful in insuring consumption against early separation shocks (this case was made previously in comparison to the optimal contract model), consumption loses for a fraction of individuals may be considerably large. In contrast, unemployment benefits can provide an insulation of the worker's consumption, since unemployment income is given contingent on a separation.

Note that under the optimal UI scheme the stationary (buffer stock) wealth level is effectively zero. Workers that begin their job with an asset level exceeding zero are effectively insured sufficiently against unemployment and run down their wealth endowment over time. The model, therefore, implies that the cost of accumulating precautionary savings during employment (in terms of consumption smoothing) is greater than the cost of higher taxes during employment which are required to finance higher benefits.

Finally, in figure 6 I show the optimal consumption schedules as a function of assets and at the optimal policy. For the sake of clarity I have included one curve which now represents both the optimal contract economy and the heterogeneous household model. Note that there is considerably more insurance now than in the benchmark UI scheme in both cases. However it is still not optimal to insure the worker perfectly against unemployment in the initial period of her spell. The intuition is that partial insurance is optimal in the presence of moral hazard concerns.

5 Discussion and Extensions

In this section I discuss the assumptions of the model with particular focus on the role of relative discounting and the zero profit condition imposed on firms at the start of each match. Moreover, I present two extensions: One applies the analysis to the economic environment of the directed search equilibrium of Moen (1997) and Acemoglu and Shimer (1999), whereas the second derives the implications for the optimal contract assuming that firms and workers bargain over the allocation in each period. I consider how the main take away of the paper about the role of public policy is impacted by these considerations.

Finally, I explicitly address whether the model presented in this paper is empirically relevant. The discussion is based on the analysis presented in Oikonomou (2010, Ch. 1 and 2).

5.1 Discount Rates and General Equilibrium

As discussed in the previous sections, the relative discount factors of firms and workers bear important implications for the optimal allocation. The main results of the paper derive under the assumption that $R = \frac{1}{\beta} > r$ which as the analysis of section 3 has demonstrated, limits the scope of private insurance against unemployment. If I had assumed $R = r$ instead, then the optimal allocation would feature complete insurance over part of the state space, but also a decreasing profile of consumption for the worker. So long as the match survives for a long period, the stationary point of the allocation is reached when the worker becomes indifferent between employment and unemployment. Therefore, the participation constraint, which is crucial in equilibrium in this model, is on the worker's side, rather than on the firm's side. As equation (10) suggests when the worker's participation constraint binds, it is likely that there is underinvestment in wealth.

A completely different set of results could arise in this type of environment as the government is concerned to drive the allocation away from that point. In Oikonomou (2010, Ch 2) I found that contrary to the optimal policy in this paper, it is optimal to backload unemployment benefits. In fact a scheme that sets benefits equal to zero for the first or first two quarters of an unemployment spell, delivers the largest welfare gains. Through such a policy the government effectively accomplishes to not crowd out private risk sharing. It therefore seems that in terms of the limited commitment model and its implications the crucial feature is which of the two participation constraints is particularly relevant.

5.1.1 General Equilibrium

Assuming a production technology without capital in this paper, enabled me to fix the rate of return r. Instead, if I had assumed, that aggregate worker savings form the economy's capital stock, then varying the UI scheme would have an effect on the interest rate r and thus on the optimal allocation.

There is a particular concern related to the presence of general equilibrium effects in the model: Reductions in wealth, driven by higher unemployment benefits would lead to increases in the rate of return r. Even though the property $R = \frac{1}{\beta} > r$ would hold, the distance between R and r would be less. This could in principle lead to a wider range over which the firm's participation constraint is non binding. [13]

If this effect is overwhelming it could imply that no policy is able to deliver the prediction that the wage schedule is equal to y in every period, and therefore the equivalence of the optimal contract model, with the heterogeneous household model. Moreover, as highlighted by Young (2004), when general equilibrium effects are accounted for, typically they are powerful enough to make optimal a benefit level equal to zero at all horizons. It is certainly interesting to extend the analysis presented here to this case. However, it is also important to point out that not accounting for general equilibrium considerations, is typical in models which consider the effect of individual savings on UI (see for example Shimer and Werning (2008) and Werning (2002) among others). Therefore, though endogenizing interest rates is a valid extension of the model, the assumption made here is common in much of the related literature.

5.2 Directed Search

One of the important assumptions made in this paper, is that at the start of the match, the firm earns zero profits in expectation. This assumption was motivated by the fact that search costs were entirely borne by the agents in the model. Thus the paper describes the effects of policy focusing on the supply side of the labor market. I briefly discuss the

[13] The argument could be traced to the previous analysis, showing that the firm's constraint binds when the optimal contract calls for an initial wage below productivity (and an extraction of the wealth of the worker). Since higher values of r reduce the difference between R and r, the incentive to save with the firm is weaker.

implications of assuming that matching costs are borne by firms, suggesting to extend the directed search model of Moen (1996), Acemoglu and Shimer (1999) and Rudanko (2009, 2011) to the optimal contract with worker savings presented here. The economic environment is the following: Each firm in the economy offers a different wage level, which is assumed to be constant throughout the life of the match. The different wage levels form different segments in the labor market and agents channel their search to the segment that offers the highest payoff in terms of wages and the duration of unemployment. Higher wages attract more unemployed individuals and fewer firms and therefore are associated with longer durations.

The framework of optimal contracts presented in this paper can be embedded in the directed search equilibrium, replacing the flat wage with an optimal contract. [14] In Oikonomou (2010, Ch 1), following the work of Rudanko (2009, 2011) I describe the equilibrium where firms post a present value J_0, which is a sufficient statistic for the optimal allocation. Workers of lower wealth then choose a higher value of J_0, since typically poorer individuals prefer to get a job offer fast, rather than wait for higher wages. If in this model we assume $R = \frac{1}{\beta} > r$, there is the following implications: Because jobs start at $J_0 > 0$ (otherwise firms would not bear the vacancy costs) it is possible to see firms offering very low wages initially, consistent with an asset extraction (see section 3), and without violating their participation constraints. This may give a different role to government provided unemployment insurance but also give a richer pattern of wages at the cross-section (most notably in some cases workers will experience positive wage growth). This is therefore an extension of the model worthy of considering in future work.

5.3 Are Frontloaded Wage Profiles Reasonable?

One of the main goals of this paper was to extend limited commitment model of wages, to allow for savings. The key prediction is that when wealth can be utilized to insure the worker against the risk of unemployment, the implied wage profile may be frontloaded,

[14] Rudanko (2009, 2011) was the first to incorporate the limited commitment model without savings, in the directed search equilibrium.

meaning that the firm gives a loan to the worker.

At a first glance the frontloaded wage property seems to predict that wages drop over the life of the match. However the model could be extended to produce a rising wage profile, if we postulate that the match productivity increases over time. Therefore the correct interpretation of the wage schedule is rather that wages rise at a slower pace than productivity. If productivity is steeply rising, the worker would like to borrow from the firm for consumption smoothing purposes, in the same sense that high initial wages help to mitigate the risk of unemployment in the model. These implications seem reasonable when matched against the wages of young individuals which are relatively flat in the data, even though initial productivity is typically low in part due to training and human capital accumulation. [15]

5.3.1 Decentralizing through severance payments

A further reason for which the implications of the limited commitment model with savings are empirically relevant, is that the allocation under certain conditions, can be shown equivalent to a flat wage schedule with a severance payment upon separation. [16] This holds in the model when $R = r$ but not more generally when $R > r$. To see this let severance payments be denoted by ξ. Moreover, assume for the moment that the participation constraints for the firm and the worker can be ignored. The optimal allocation then solves the following functional equation:

$$W(a,J) = \max_{a',J',\xi} \log(-a' + ra - \tau - J + y + \frac{1-\lambda}{R}J' - \frac{\lambda\xi}{R}) \tag{23}$$

$$+ \beta(\lambda U(a' + \frac{\xi}{R}, 0) + (1-\lambda)W(a', J')) \tag{24}$$

[15] It is important to stress that the usual contractual considerations put forward by the personnel economics literature (see for example Hutchens (1989)) and which make productivity rise faster than wages are missing from the model. The focus here is on one particular aspect of the wage setting process, but the results should not be misconstrued to mean that other considerations are not important.

[16] Insurance against unemployment through severance compensation is rather common in the US data (see Pissarides (2004) and Chetty (2008)).

Subject to the constraint set:

$$a' \geq \overline{a} \qquad a' + \frac{\xi}{R} \geq \overline{a}$$

Note that in (24) the promise keeping constraint (which holds as an equality) is given by: $J = y - w + \frac{1-\lambda}{R} J' - \frac{\lambda}{R}\xi$. In the event of a separation the firm pays to the worker income equal to ξ.[17] Moreover, note that after replacing consumption with the promise keeping and budget constraints in (24) what remains in the constraint set is the boundary conditions $a' \geq \overline{a}$ and $a' + \frac{\xi}{R} \geq \overline{a}$. The latter requires that total assets in the event of separation to not violate the borrowing limit. To show that program (24) is the same as program (2) consider the following Ricardian equivalence argument: Increase wealth for the worker by $\frac{\xi}{R}$ today and let the new level of assets be $\tilde{a}' = a' + \frac{\xi}{R}$. Also increase the continuation (promised) utility J' by ξ and define $\tilde{J}' = J' + \xi$. Then clearly $W(\tilde{a}', \tilde{J}') = W(a', J')$ since the amount of resources available to finance consumption for the worker next period is unchanged. Thus a program that sets $\xi = 0$ and uses next period's wealth as a single control variable as in equation (2) is payoff equivalent to one where both investment in wealth and severance payments are allowed.

Under limited commitment matters are more complicated but we can always construct examples where the constraint set is such that allowing for severance payments makes no difference for the optimal allocation. Most notably this requires to assume that severance payments are fully enforceable, but also that the worker's outside option is such that she always gets ξ when the match is destroyed. (i.e $U(a' + \frac{\xi}{R}, 0)$). Note, however, that the result above is nevertheless pertinent for the limited commitment contract, for the following reason: Under $R = r$ and full commitment the resulting wage profile will be even more frontloaded leading to a full insulation of the worker's consumption against unemployment. Therefore wage paths which are even more volatile than the analogous paths derived in this paper, are empirically relevant, if the frontloaded wage profile is interpreted as severance compensation. A similar argument can be made for the case

[17] Note that if the firm pays ξ to the worker in the event of a separation the workers wealth is equal to $ra + \xi$ and not $a + \xi$ given the market structure assumed. Therefore utility in the state of unemployment is $U(a' + \frac{\xi}{R}, 0)$ as suggested by equation (24).

$R > r$ where severance payments only provide partial insurance against unemployment, reflecting the previous results of section 3. In this case the limited commitment coupled with the difference in the rates of return, also give less volatile wages.

5.4 Nash Bargaining

The baseline model of section 2 assumed that the firm and the worker could commit to a given set of policies without ever renegotiating the optimal allocation. Though the worker's program was represented recursively, the allocation may be equivalently described as an optimization at date 0, deriving a policy rule for every future period in which the match survives. Renegotiating this rule is not feasible; implicitly commitment is sustained by the threat of mutual reversion to autarky. This paragraph illustrates that the framework presented in this paper, and the recursive representation of the problem, can be extended to include renegotiation and bargaining between workers and firms in every period. I focus here on Nash Bargaining following the bulk of the literature of search theoretic models.

Before proceeding it is useful to consider a case in which the allocation described in sections 2 and 3 presents an opportunity to the worker to renegotiate. In particular consider where the frontloaded wage property. It was argued that after the first period, wages were constant and equal to some level $\overline{w} < y$. Note that in this case the worker would benefit from having a higher wage $\overline{w}_t > \overline{w}$ in every $t > 0$ period with $\overline{w}_t < y$. The firm would have no incentive to destroy the match since the present value of profits is still positive, and the initial investment in the worker's assets is sunk. This example also gives an illustration for why the type of allocation presented in this section will rule out frontloaded wages. The equilibrium that arises is one were wages, and payoffs, depend on wealth as the only state variable.

Let the equilibrium, under bargaining give rise payoffs $\Omega(a)$ and $\Phi(a)$ to the worker and the firm respectively. In order to uncover $\Omega(a)$ and $\Phi(a)$ I consider a representation of the program as the (dual) problem of maximizing the firm's profit function (see section 7.2 in the appendix). Moreover, I impose that the continuation policies are consistent with the equilibrium under Nash Bargaining. The program maybe written as follows:

$$\Pi(W,a) = \max y - w + \frac{1-\lambda}{R}\Phi(a') \tag{25}$$

subject to:

$$W \leq u(\frac{-a'}{r} + w - \tau + a) + \beta((1-\lambda)\Omega(a') + \lambda U(a',0)) \tag{26}$$

$$\Omega(a) \in arg\max_{W}\,(W - U(a,0))^{\eta}(\Pi(W,a))^{1-\eta} \tag{27}$$

$$\Phi(a) = \Pi(\Omega(a),a) \tag{28}$$

The parameter η determines the share of the surplus that accrues to the worker. Note that equation (26) is the analogous object to the promise keeping constraint considered in text. It requires that at least a level of lifetime utility W be delivered to the worker although in this case the continuation utility must be consistent with the equilibrium payoff $\Omega(a')$. The firms profit is defined in (28) imposing that $W = \Omega(a)$.

This type of contract does not appear new in the literature. In fact Krusell et al (2010) construct a model with search frictions in the labour market and incomplete insurance, assuming that rents are bargained for each period with a Nash protocol. However, their approach is different from mine; they approximate the Nash sharing rule with an invariant function $w(a)$ and solve the workers value function. Instead I treat allocations as part of a more general contracting problem offering the possibility of incorporating additional features to the model, such as considering separations endogenously determined by the worker's effort (see Wang and Williamson (2002) and Oikonomou (2010, Ch. 2)).

It can be shown that optimal choices of w and a' satisfy the following first order conditions:

$$\kappa_t u'(c_t^e) = 1 \tag{29}$$

$$\kappa_t\beta(\lambda U_{a_{t+1}} + (1-\lambda)\Omega_{a_{t+1}}) - \frac{1}{r} + \frac{1-\lambda}{R}(1 - \kappa_{t+1}\Omega_{a_{t+1}}) \leq 0 \tag{30}$$

with strict equality if $a_{t+1} > 0$. κ_t represents the multiplier on the promise keeping constraint. The envelope condition is given by: $\Phi_{a_t} = 1 - \kappa_t \Omega_{a_t}$. These equations have the following interpretation: An increment is wealth in equation (30) has two distinct effects on the firms profits: it lowers required wages to finance a given consumption stream, but also increases the level of promised utility that the firm must deliver to the worker (according to the derivative $\Omega_{a_{t+1}}$). The latter effect would tend to dominate the closer the wealth is to the borrowing constraint since it is precisely there that an increment in assets encounters the highest marginal utility gains. Rearranging (30) and making use of the envelope conditions we get the following Euler condition for the model with Nash Bargaining:

$$u'(c_t^e) \geq \beta r(\lambda U_{a_{t+1}} + (1-\lambda)\Omega_{a_{t+1}}) + \frac{1-\lambda}{R}\frac{r}{\kappa_t}\Phi_{a_{t+1}} \tag{31}$$

Equation (31) sets the marginal cost of saving an extra unit today, equal to the future marginal benefit, and an extra term that pertains to the shape of the profit function. Should $\Phi_{a_{t+1}}$ be less than zero, the marginal cost would be less than the marginal benefit and the agent would be savings constrained. The converse holds if $\Phi_{a_{t+1}} > 0$.

It is perhaps more relevant to consider cases where the derivative $\Phi_{a_{t+1}}$ is less than zero. This is a common property of the numerical solutions to this model (see Krusell et al (2010) and Oikonomou (2010 , Ch 1)). In such a case we can show from the Nash rule that $\Omega_{a_{t+1}} - U_{a_{t+1}} < 0$ (i.e. the marginal increment from an extra unit of wealth is higher for an unemployed that for an employed agent). Rearranging (30) we get:

$$\kappa_{t+1}\Omega_{a_{t+1}} = 1 + \frac{\kappa_t R}{r(1-\lambda)}\beta r(\lambda U_{a_{t+1}} + (1-\lambda)\Omega_{a_{t+1}} - u'(c_t^e)) < \kappa_{t+1}U_{a_{t+1}} \tag{32}$$

Equation (32) gives the underinsurance result for Nash Bargaining contracts. Whenever $\Phi_{a_{t+1}} < 0$ the term in the parenthesis is positive and consumption falls as the agent becomes unemployed. If on the other hand $\Phi_{a_{t+1}} > 0$ underinsurance is impossible to prove.

Consider now the case where $\eta = 1$. The worker gets the entire surplus as we have assumed in text. It is possible to argue that the Nash bargaining contract described above is a flat wage contract that sets wages equal to productivity each period. Assume the contrary: Let wages be frontloaded (i.e. $y < w_0$ initially) so that the worker receives a loan that finances wealth accumulation. Assume without loss of generality that the choice of assets is a_1. The firms payoff is then $\Phi(a_1)$. It must be that $\Phi(a_1) = 0$ since under $\eta = 1$ firms break even under the rebargained allocation. However note that the equilibrium payoff satisfies $\Phi(a_0) = y - w_0 + \frac{1-\lambda}{R}\Phi(a_1) < 0$. Note that this is a contradiction since the equilibrium payoff must also satisfy $\Phi(a_0) = 0$.

Result 1. *In an equilibrium under Nash Bargaining with $\eta = 1$ the only incentive compatible allocation has wages equal to productivity each period (flat wage contract).*

The above result states that it is not possible to get a higher wage initially, as was the case under the limited commitment contract. As explained previously under Nash bargaining, the worker cannot commit to a lower wage $\overline{w}$ in every subsequent period.

6 Conclusions

This paper studies the optimal provision of unemployment benefits, in an economy with private risk sharing. In particular firms offer to their workforce a contract subject to limited commitment. It is shown that when workers have access to a storage technology, savings are utilized to provide (partial) insurance against the risk of a job separation. The participation constraints implied by limited commitment influence the scope of risk sharing.

In this environment the government has the explicit goal to drive the allocation to the point where the scope of private risk sharing is maximized. I illustrate that this corresponds to the point where the firm's participation constraint binds. The implied optimal UI scheme entails large and frontloaded benefits. Moreover, under the optimal public policy the allocation becomes identical to the model of heterogeneous households, whereby assets are utilized for self insurance purposes.

On the methodological side this paper is the first to introduce savings in a limited commitment model within the labor market context. I characterize analytically the optimal behavior offering a comparison with the standard model of heterogeneous households.

Finally, a number of interesting extensions that illustrate the general applicability of the framework utilized, are provided.

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7 Appendix

7.1 Derivations in the Model of Section 3

The first order conditions from (2) are given by the following equations:

$$
\begin{gathered}
-u'(c) + \beta\lambda U_{a'}(a',0) + \beta(1-\lambda)W_{a'}(a',J') + \phi_1(W_{a'}(a',J') - U_{a'}(a',0)) + \chi = 0 \\
u'(c)\frac{(1-\lambda)}{R} + \beta(1-\lambda)W_{J'}(a',J') + \phi^1 W_{J'}(a',J') + \phi^2 = 0 \\
W_a(a',J') = ru'(c) \quad \text{and} \quad W_J(a',J') = -u'(c)
\end{gathered}
$$

where u' denotes the worker's marginal utility, W_x is the partial derivative of W with respect to argument x, ϕ^1 and ϕ^2 are the multipliers on the participation constraints for the worker and the firm respectively, and χ is the multiplier on the borrowing constraint.

To derive equations (9) and (8) let c_t^e denote the consumption of the employed agent in t and c_t^u the analogous object for the unemployed agent (in the first period of unemployment). Notice also that the partial derivative value function of the unemployed with respect to wealth is given by $U_a(a,0) = ru'(c_t^u)$ by the standard envelope condition. With the appropriate substitutions we have:

$$
\begin{gathered}
u'(c_t^e) = \beta\lambda ru'(c_{t+1}^u) + \beta(1-\lambda)ru'(c_{t+1}^e) + \phi_{t+1}^1 r(u'(c_{t+1}^e) - u'(c_{t+1}^u)) + \chi_t \\
u'(c_t^e)\frac{(1-\lambda)}{R} - \beta(1-\lambda)u'(c_{t+1}^e - \phi_{t+1}^1 u'(c_{t+1}^e) + \phi_{t+1}^2 = 0
\end{gathered}
$$

which is equations (9) and (8) in text. Moreover, rearranging we get:

$$
\begin{aligned}
&\frac{R}{(1-\lambda)}(\beta(1-\lambda)u'(c_{t+1}^e + \phi_{t+1}^1 u'(c_{t+1}^e) - \phi_{t+1}^2) - \beta(1-\lambda)ru'(c_{t+1}^e) \\
&\qquad -\phi_{t+1}^1 r(u'(c_{t+1}^e) - u'(c_{t+1}^u)) - \chi_t = u'(c_{t+1}^u)\beta\lambda r
\end{aligned}
\tag{33}
$$

From (33) it is straightforward to derive (10).

7.2 A Dual Representation of the Worker's Program

In order to solve the model I utilize the standard value function iteration approach. However, for simplicity I cast the program in its dual form, considering an optimal contract that maximizes the firms profit subject to delivering a given level of utility to the worker. This representation is standard in the literature. The program may be written as follows:

$$
\Pi(W,a) = \max y - w + \frac{1-\lambda}{R}\Pi(W',a')
$$

subject to:

$$
\begin{gathered}
W \leq u(\frac{-a'}{r} + w - \tau + a) + \beta((1-\lambda)W' + \lambda U(a',0)) \\
W' \geq U(a',0) \quad \text{and} \quad \Pi(W',a') \geq 0
\end{gathered}
$$

In order to reduce the number of control variables I make use of the following properties: First, I utilize the promise keeping constraint to express wages as:

$$w = u^{-1}(W - \beta((1-\lambda)W'\lambda U(a',0))) + \frac{a'}{r} + \tau - a)$$

which allows to write:

(34)

$$\Pi(W,a) = \max y - u^{-1}(W - \beta((1-\lambda)W'\lambda U(a',0)))\frac{-a'}{r} - \tau + a) + \frac{1-\lambda}{R}\Pi(W',a')$$

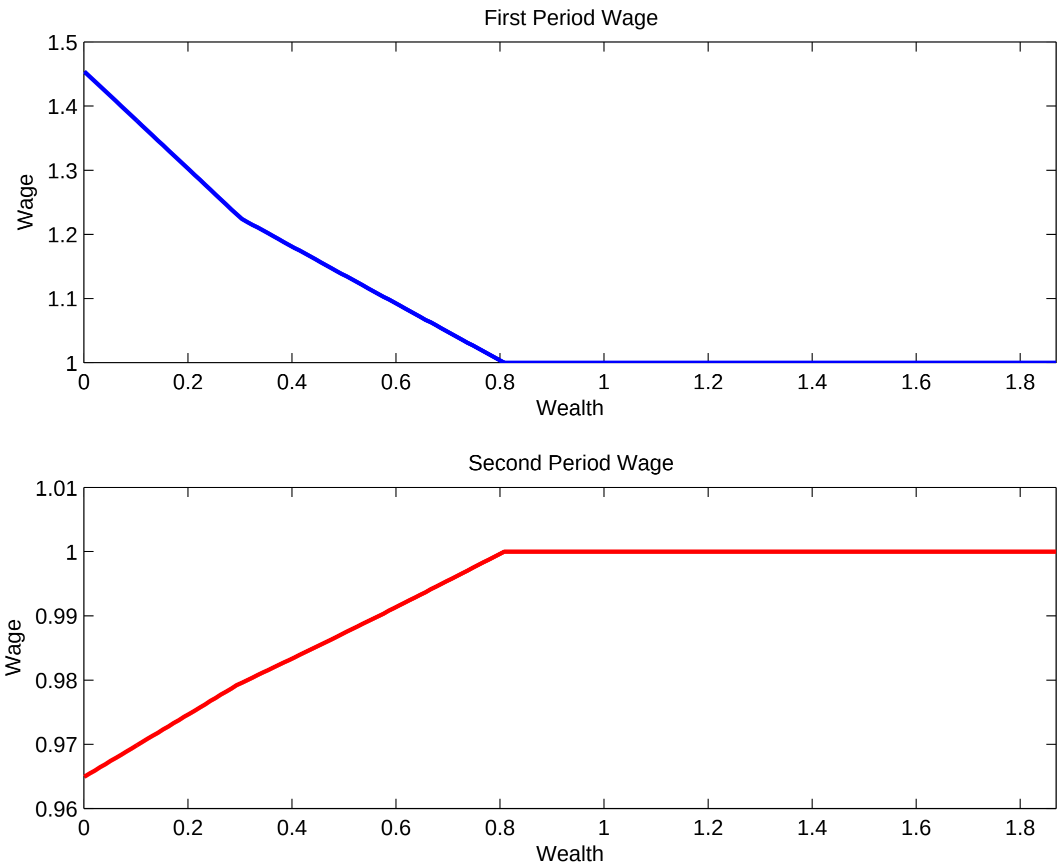
Second, from (34) it is possible to show that the firms value function is linear homogeneous in wealth. Therefore we can solve:

(35)

$$\Pi(W,0) = \max y - u^{-1}(W - \beta((1-\lambda)W'\lambda U(a',0)))\frac{-a'}{r} - \tau) + \frac{1-\lambda}{R}\Pi(W',0) + a'$$

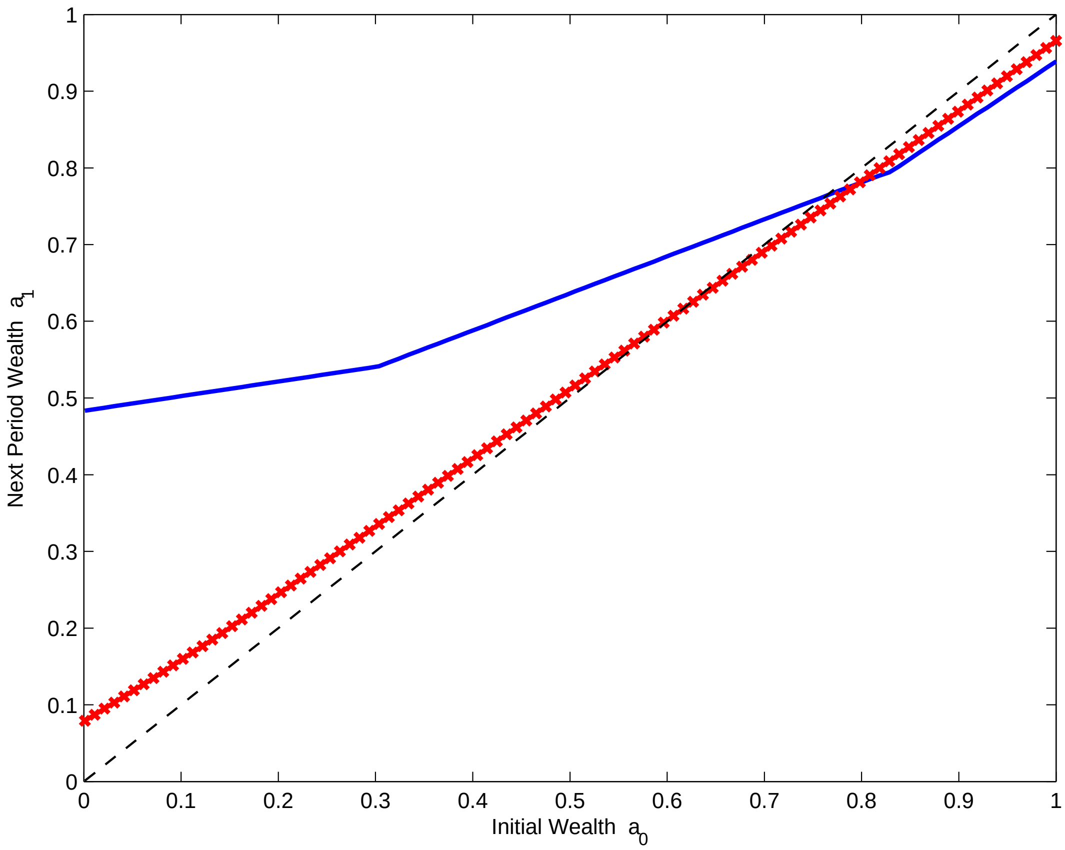
to recover the policy rules. Equivalently the participation constraints are $W' \geq U(a',0)$ and $\Pi(W',0) + a' \geq 0$.

Figure 1: **Optimal Contract- Wage Profiles**



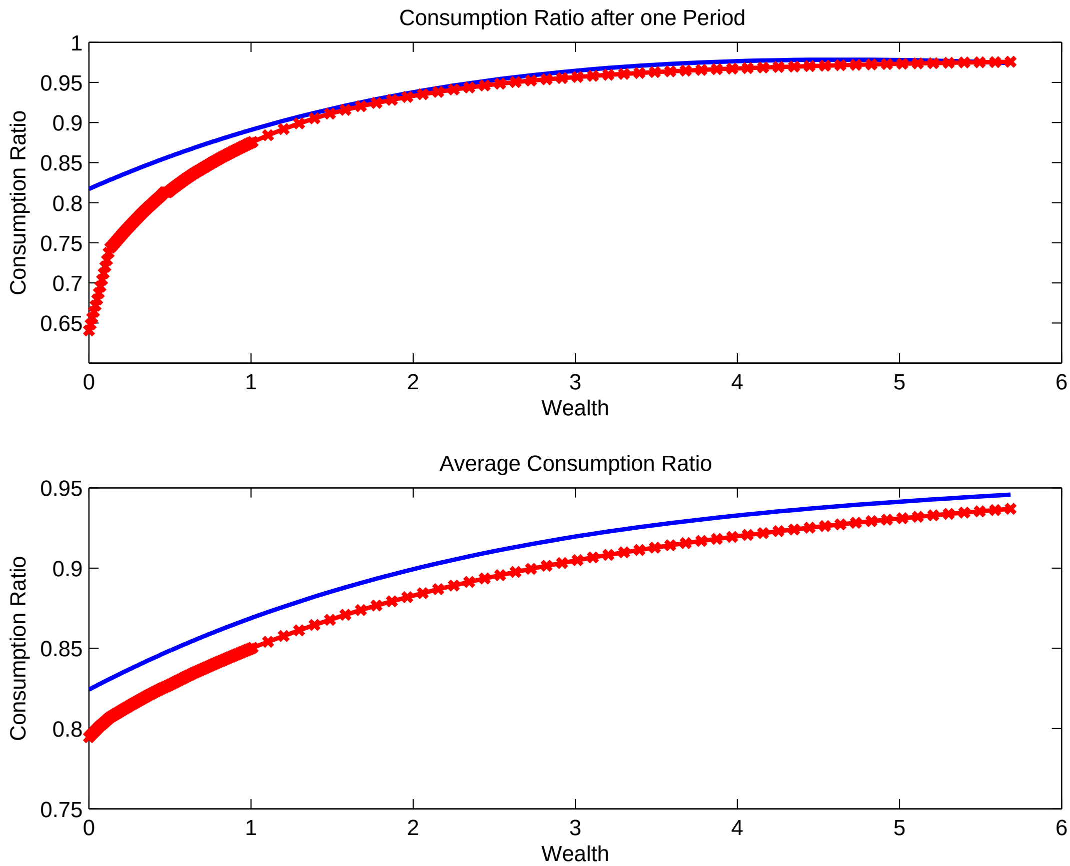
Notes: The top panel shows wages at the start of the match, as a function of the wealth endowment of the newly employed agent. The bottom panel shows the wage schedule for every period after period one.

Figure 2: **Wealth Accumulation- Optimal vs. Flat Contract**



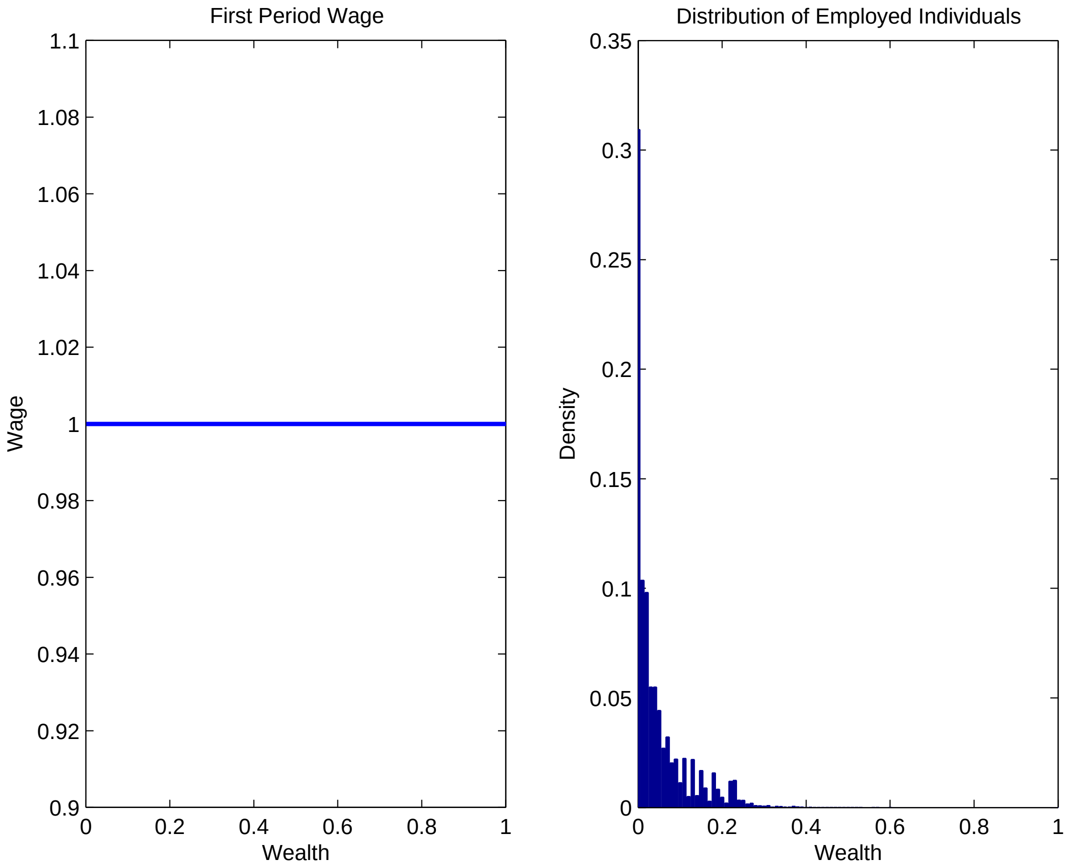
Notes: The solid line plots next period assets against initial wealth under the optimal contract. The crossed line corresponds to the analogous object for the model of precautionary savings. The dashed line is the 45 degree line which indicates the stationary wealth level.

Figure 3: **Consumption Ratio - Optimal vs. Flat Contract**



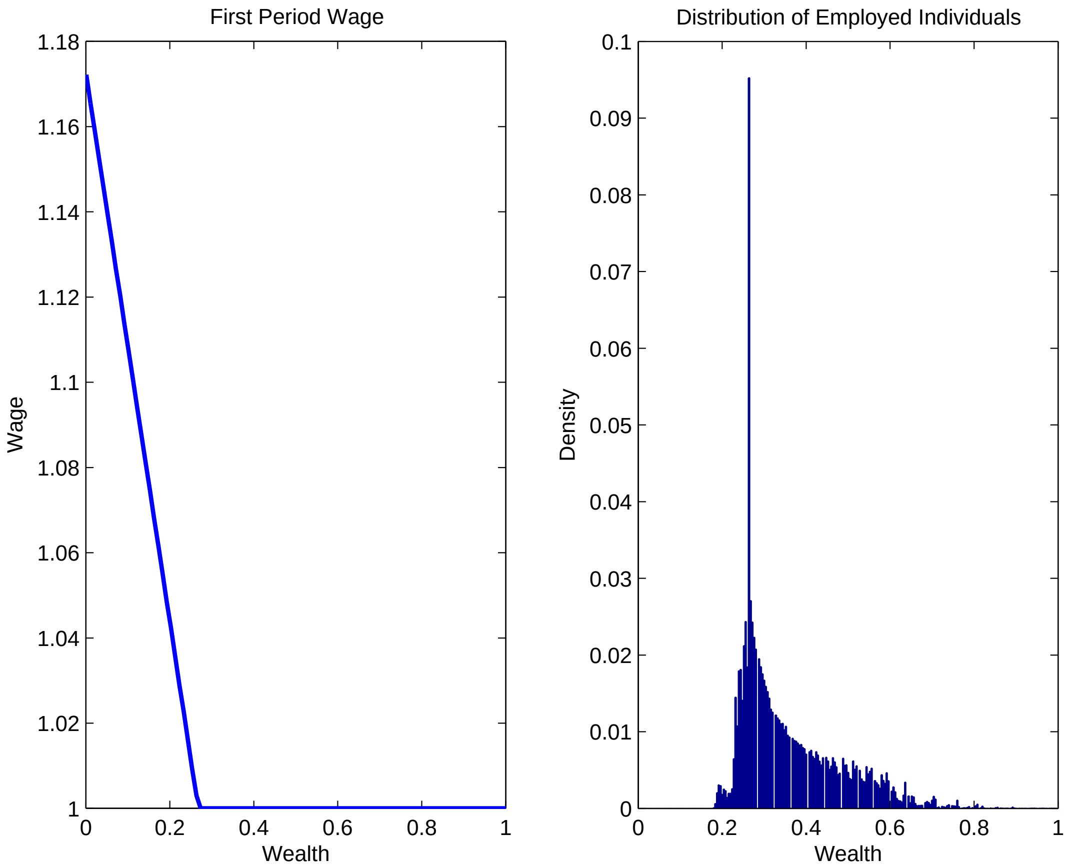
Notes: The figure plots the ratio of consumption in unemployment over consumption in employment. The solid line represents the optimal allocation. The crossed line represents the optimal contract. The top panel shows the ratio as a function of wealth after one period on the job. The bottom panel shows the average consumption loses (weighted average across periods).

Figure 4: **Optimal Policy: Wage Path and Distribution**



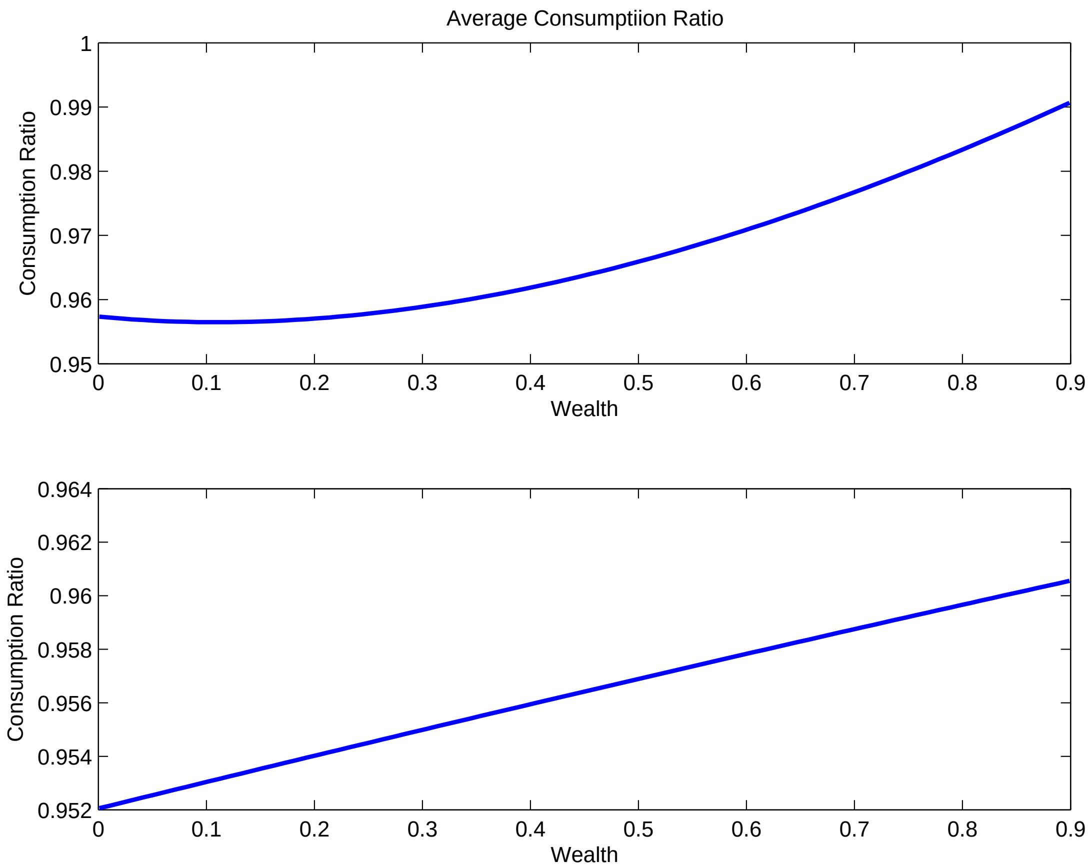
Notes: The figure plots under the optimal UI scheme the wage policy of the firm (right) and the distribution of employed individuals over the wealth space.

Figure 5: **One Quarter Policy: Wage Path and Distribution**



Notes: The figure plots shows the wage policy of the firm (right) and the distribution of employed individuals over the wealth space. The UI scheme considered is one that sets benefits optimally for one quarter and then offers zero benefits for longer durations.

Figure 6: **Consumption Ratios under the Optimal Policy**



Notes: The figure plots the ratio of consumption in unemployment over consumption in employment. The top panel shows the ratio as a function of wealth after one period on the job. The bottom panel shows the average consumption loses (weighted average across periods).

	$(b_{\leq m}, b_{>m})$.	Welfare	u	τ
Zero Benefits	$(0, 0)$	-0.47%	-4.3%	-100%
Optimal Timing				
$m = 0$	$(1.165, 0.400)$	+0.75%	+6.45%	+103%
$m = 1$	$(0.758, 0.701)$	+0.50%	+8.38%	+57%
Zero Benefits after $m + 1$ quarters				
$m = 0$	(1.168.0)	0.39%	-3.70%	+77%
$m = 1$	$(0.78, 0.0)$	0.40%	+ 3.87%	+50%

Table 1: **Effects of UI schemes: Optimal Allocation**

Notes: The table considers the effects of various UI schemes on economic outcomes. Zero benefits refers to a policy that eliminates unemployment insurance. Zero Benefits after m quarters sets the UI payment equal to zero for durations longer than m quarters. I report the case where $m = 1$ and the case where $m = 2$ Optimal timing chooses optimally $(b_{\leq m}, b_{>m})$..

	$(b_{\leq m}, b_{>m})$.	Welfare	u	τ
Zero Benefits	$(0, 0)$	-0.22%	-3.88%	-100%
Optimal Timing				
$m = 0$	$(1.165, 0.400)$	+0.40%	+ 7.60%	+111%
$m = 1$	$(0.517, 0.389)$	+0.17%	+7.92%	+13%
Zero Benefits after $m + 1$ quarters				
$m = 0$	$(1.301, 0.0)$	0.035%	-3.55%	+105%
$m = 1$	$(0.897, 0.0)$	0.064%	+ 6.95%	+ 89%

Table 2: **Effects of UI schemes: Flat Wages**

Notes: The table considers the effects of various UI schemes on economic outcomes. Zero benefits refers to a policy that eliminates unemployment insurance. Zero Benefits after m quarters sets the UI payment equal to zero for durations longer than m quarters. I report the case where $m = 1$ and the case where $m = 2$ Optimal timing chooses optimally $(b_{\leq m}, b_{>m})$.