A demand-driven search model with self-fulfilling expectations: The new ‘Farmerian’ framework under scrutiny

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A Demand-Driven Search Model with Self-Fulfilling Expectations: The New ‘Farmerian’ Framework Under Scrutiny*

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Abstract

In this paper, we implement Bayesian econometric techniques to analyze a theoretical framework built along the lines of Farmer’s micro-foundation of the General Theory. Specifically, we test the ability of a demand-driven search model with self-fulfilling expectations to match the behaviour of the US economy over the last thirty years. The main findings of our empirical investigation are the following. First, all over the period, our model fits data very well. Second, demand shocks are the most relevant in explaining the variability of concerned variables. In addition, our estimates reveal that a large negative demand shock caused the Great Recession via a sudden drop of confidence. Overall, those results are consistent with the main features of the New ‘Farmerian’ Economics as well as to latest demand-side explanations of the finance-induced recession.

JEL Classification: E24, E32, E52, J64.

Keywords: New Farmerian Economics; Competitive search; Dynamic models; Bayesian estimation.

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1 Introduction

In recent works, Farmer (2008a-b, 2010a-b, 2012a-b) provides a new micro-foundation of the General Theory grounded on modern search and business cycle theories. In Farmer’s (2008a-b, 2010a-b, 2012a-b) view, the labour market is not Walrasian and does not clear. Instead, self-fulfilling expectations select an equilibrium (un)employment rate each period. The output of this ambitious research agenda is a competitive-search framework with multiple steady-states in which production and employment are demand driven, labour instead of output is used to post vacancies, the money wage is used as numeraire and prices are flexible. In the remainder of the paper, this set-up will be dubbed as New ‘Farmerian’ – or ‘Old Keynesian’ – in order to distinguish the mentioned proposal from the traditional New Keynesian paradigm grounded on nominal and/or real rigidities.\(^1\)

Despite the strong theoretical emphasis of seminal papers, empirical contributions rooted in the New Farmerian Economics are still in their infancy. On the one hand, Farmer (2012a) compares a three-equation monetary version of his framework with a companion New Keynesian model and he finds that the former outperforms the latter in fitting US data. On the other hand, calibrating a New Farmerian model, Guerrazzi (2011) shows that such a theoretical proposal can provide a rationale for the Shimer (2004, 2005) puzzle, i.e., the relative stability of labour productivity in spite of the large volatility of labour market tightness indicators that standard search models are unable to replicate unless augmented with nominal and/or real rigidities (e.g. Shimer 2004, 2005, Hall 2005a-b, Hagedorn and Manovskii 2008 and Gertler et al. 2008).

In this paper we try to move one step forward by estimating a version of the New Farmerian framework developed in Guerrazzi (2011) and applying diagnostic tools typically exploited in the literature of medium-scale DSGE models (e.g. Smets and Wouters 2003). Specifically, providing estimates of a demand-driven competitive-search framework obtained with Bayesian techniques, we test the ability of this theoretical setting to match the dynamic behaviour of the US economy over the last thirty years by focusing on key labour market variables such as real wages, unemployment and job vacancies.\(^2\)

More deeply, in this work we try to provide a quantitative assessment to a number of theoretical attributes of the New Farmerian Economics, namely, bringing the model to data, we investigate on the main determinants of business cycles. First, given that equilibrium unemployment is assumed to be a recurrent feature of real-world economies, unemployment rates are explicitly exploited in the estimation process (e.g. Gali et al. 2011). Second, since in Farmer’s (2008a-b, 2010a-b, 2012a-b) proposal a fraction of employed workers is assumed to be allocated in recruiting activities without considering unfilled job openings, corporate recruiters

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1An extensive review of the Farmerian model is given by Guerrazzi (2012a).
2The monetary side of the economy is left to further research.
are exploited as proxies of observed job vacancies (e.g. Guerrazzi 2011). In addition, retrieving a definition of potential output consistent with the postulates of the theoretical framework under examination, we provide an estimation of such a critical measure of production loss together with a shock decomposition aimed at discerning its stochastic determinants.

The main results of our empirical investigation are the following. First, our model provides a good fit of US labour market data all over the full sample. Moreover, demand shocks turn out to be extremely relevant in accounting for the variance of most observed variables and they are largely the main determinants of the Great Recession of 2008-2009. Overall, our findings suggest that US data are consistent with the main theoretical attributes of the New Farmerian Economics as well as to latest demand-side explanations of the finance-induced recession (e.g. Furlanetto and Groshenny 2013, Perri and Quadrini 2011 and Plotnikov 2013).

The paper is arranged as follows. Section 2 provides the theoretical framework. Section 3 provides an overview of data and fixed parameters as well as a description of the estimation methodology. Section 4 presents and comments the estimation results. Finally, section 5 concludes.

2 The model

The theoretical framework exploited for empirical estimations draws on Guerrazzi (2011) who builds a competitive two-sided search model where time is discrete. This proposal mirrors some features of the finance-constrained economy put forward by Woodford (1986) and implements the choice of wage measurement units pioneered by Keynes (1936) recently revisited by Farmer (2008a-b, 2010a-b, 2012a-b). Specifically, Guerrazzi (2011) assumes that economic agents are divided in two broad income categories, i.e., wage and profit earners, which also are assumed to differ in their propensities to consume and their tasks.

On the one hand, wage earners, i.e., the owners of a fixed amount of labour services, are assumed to dislike saving and consume the whole income earned by offering their labour endowments in a non-Walrasian labour market characterized by search frictions. As a consequence, in our stylized model economy aggregate consumption coincides in each period with the overall wage bill and the hazard rate for employment, coherently with search thick market externalities, is positively correlated with aggregate employment.

On the other hand, profit earners, i.e., the owners of productive capital and/or overhead workers, are assumed to save and invest the total income obtained by employing a given number of wage earners and arranging a stochastic production process aimed at fostering capital accumulation. Therefore, in our setting the aggregate investment expenditure is always identically equal to the difference between the value of output and the wage bill placing the economy in a neutral region between dynamic efficiency and inefficiency (e.g. Abel et al. 1989). Moreover,
consistently with a search framework in which labour instead of output is used to post vacancies, profit earners are assumed to employ wage earners alternatively in recruiting or production activities without any wage discrimination.\(^3\) Furthermore, search congestion externalities implies that the higher (lower) the employment level, the higher (lower) the number of wage earners that the owners of the capital stock have to allocate in their recruiting departments.

In this model economy, under the conventional assumptions that the capital stock is predetermined and wage earners receive their marginal contribution to produced output, the market clearing condition of the market for goods conveys an equilibrium level of employment, i.e., the number of employed wage earners, for each possible amount of the investment expenditure put forward by profit earners. Specifically, in adherence to standard demand-driven models such as the textbook Keynesian cross, the higher (lower) the investment expenditure, the higher (lower) equilibrium employment. In order to solve such an indeterminacy, similarly to Farmer (2008a), we follow Guerrazzi (2011) who formalizes the ‘animal spirits’ of profit earners by assuming that their investment expenditure measured in wage units follows an autonomous stochastic process such as those usually exploited for total factor productivity (TFP) in conventional real business cycle (RBC) models.

The theoretical framework employed for our estimations can be summarized in four distinct blocks. The first three parts provide the conditions for a symmetric demand constrained equilibrium (DCE), i.e., feasibility and market clearing in the market for goods, consistency with the optimal choices of wage and profit earners and search market equilibrium (e.g. Farmer 2008a-b, 2010a-b, 2012a-b). Moreover, block four provides the laws of motion of the model economy. In what follows, we provide a formal description for each of them.

### 2.1 Feasibility and market-clearing in the market for goods

A distinctive feature of DCE is that all the purchased goods, i.e., consumption plus investment, are produced by profit earners while there is no certainty that all wage earners are actually employed.

Consider a given time period, say quarter \( t \). Profit earners output \( (Y_t) \) is described by a constant-returns-to-scale Cobb-Douglas production function. Hence,

\[
Y_t = A_t K_t^\alpha X_t^{1-\alpha} \quad 0 < \alpha < 1
\]

where \( A_t \) is a common-knowledge stochastic productivity disturbance, \( K_t \) is the predetermined stock of capital, \( X_t \) is the fraction of wage earners employed in production activities while \( \alpha (1 - \alpha) \) is the output elasticity with respect to capital (labour).

\(^3\)In other words, the wage paid to wage earners is the same no matter the activity to which they are actually allocated.
Wage earners can be alternatively allocated to production or recruiting activities. As a consequence, total employment \((L_t)\) breaks down as

\[ L_t = X_t + V_t \]  \(\text{(2)}\)

where \(V_t\) is the fraction of wage earners employed as corporate recruiters.

Labour service endowment of wage earners is normalized to one. As a consequence, the unemployment rate is given by

\[ U_t = 1 - L_t \]  \(\text{(3)}\)

Moreover, the national account identity implies that the value of aggregate demand has to be identically equal to the value of produced output. Therefore,

\[ p_t (C_t + I_t) \equiv p_t Y_t \]  \(\text{(4)}\)

where \(p_t\) is the general price level, \(C_t\) is aggregate consumption in real terms while \(I_t\) is the real investment expenditure.

### 2.2 Consistency with optimal choices of wage and profit earners

As stated above, wage earners dislike saving and consume the whole wage income. Therefore, aggregate consumption equals the aggregate wage bill all the times. Hence,

\[ C_t = \left( \frac{w_t}{p_t} \right) L_t \]  \(\text{(5)}\)

where \(w_t\) is the nominal wage.

Plugging (5) into (4) allows to derive the value of aggregate demand \((AD_t)\) measured in wage units. Specifically,

\[ \frac{AD_t}{w_t} = \tilde{I}_t + L_t \]  \(\text{(6)}\)

where \(\tilde{I}_t \equiv I_t (w_t/p_t)^{-1}\) is the investment expenditure in wage units.

Profit earners employ a given fraction of wage earners by aiming at maximizing their net-of-wage payments. Obviously, this means that marginal product of labour will equal the real wage all the times. Thereafter, taking into account the production function in (1), it will always hold that

\[ (1 - \alpha) \frac{Y_t}{L_t} = Q_t \]  \(\text{(7)}\)

where \(Q_t \equiv w_t/p_t\).

The expression in (7) is a conventional first-order condition for labour allocation. As a consequence, with a predetermined capital stock, it implies that that the real wage is univocally determined by the level of employment and the actual realization of the technology shock.
Moreover, (7) allows to derive the value of aggregate supply ($AS_t$) measured in wage units. Specifically,

$$\frac{AS_t}{w_t} = \frac{1}{1 - \alpha} L_t$$ (8)

Obviously, equalizing (6) and (8) gives back the equilibrium output in wage units and the equilibrium level of employment as a function of the value of $\tilde{I}_t$. Consistently with the choice of units adopted for aggregate demand and aggregate supply, this composite index conveys the genuine autonomous component of aggregate demand, i.e., the component of aggregate expenditure that does not depend on employment.

In the remainder of this paper, $\tilde{I}_t$ will be used as a proxy of the animal spirits of profit earners. In other words, we will assume that the mentioned variable is the latent signal that mirrors the state of long run expectations that drives the short-run determination of the equilibrium (un)employment rate. This choice is consistent with the empirical evidence provided by Farmer (2010a) who shows that in the US all over the period 1950-2005 investment per member of the labour force measured in wage units moved quite closely and in the opposite direction of observed unemployment.

### 2.3 Search market equilibrium

The search framework set forth by Guerrazzi (2011) implies that in each period wage earners experience a certain hazard rate for employment while profit earners have a corresponding hazard rate of filling their demand-driven vacant positions by allocating wage earners to recruiting activities.

On the one hand, the current employment hazard rate for wage earners ($H_t$) equals aggregate employment. Hence,

$$H_t = L_t$$ (9)

On the other hand, the recruiting hazard rate of a single wage earner employed as a recruiter is given by

$$R_t = \frac{L_t}{V_t}$$ (10)

Furthermore, once (un)employment is determined by the level of aggregate demand prevailing in the market for goods, the fraction of wage earners allocated to recruiting activities is

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4Its counterpart for capital is missing because profit earners are assumed to invest the whole net-of-wage payments in productive capital. However, the real interest rate consistent with a zero-profit condition on profit earners’ side would be equal to $Q_t\tilde{I}_tK^{-1}_t$.

5Respectively, $\alpha^{-1}\tilde{I}_t$ and $(1 - \alpha)\alpha^{-1}\tilde{I}_t$.

6Conversely, the current hazard rate for unemployment is one minus $L_t$. 
determined by a non-stochastic version of the Beveridge curve such as

\[ V_t = (1 - U_t)^{\frac{1}{1-\gamma}} \quad 0 < \gamma < 1 \]  

(11)

where \( \gamma \) denotes the matching elasticity.

The Beveridge curve in (11) summarizes the operation of search and production externalities in the whole economy and stresses that in Farmer’s (2008a-b, 2010a-b, 2012a-b) framework labour instead of output is used to post vacancies.\(^7\)

Since recruiters are assumed to be essential to hire wage earners, (11) - together with (1)–(3) - also implies that in each period the relation between realized output and employment is given by a reverse-u; indeed, when \( L_t \) is equal to zero, nothing can be produced because no wage earner is employed. However, \( Y_t \) is equal to zero also when \( L_t \) is equal to one because in this case our version of the Beveridge curve would lead profit earners to allocate all the wage earners in the recruiting department preventing the production of goods. The level of (un)employment that maximizes output period by period is the first-best allocation of our model economy and its actual realization depends on the value taken by the autonomous component of aggregate expenditure defined in (6). In section 4, this allocation will be used to retrieve the output gap implied by the model economy.

The expression in (11) can be combined with the results in (1)–(3) and (7) in order to derive the equilibrium wage function. Specifically,

\[ Q_t = \frac{(1 - \alpha) A_t K_t^\alpha \left(1 - L_t\right)^{\gamma\gamma}}{L_t^\alpha} \]  

(12)

Guerrazzi (2011) shows that (12) has an inflection point for the demand-driven allocation that implements the first-best level of (un)employment. Such an inflection is the endogenous source of real stickiness that in a reasonably calibrated model allows the Shimer (2004, 2005) puzzle to be resolved.

### 2.4 Laws of motion

Equations (1) – (11) determine a real wage rate \( Q_t \), a production plan \( \{Y_t, V_t, X_t, L_t\} \), a consumption allocation \( C_t \) and a pair of hazard rates \( \{H_t, R_t\} \) as a function of the parameter set \( \{\alpha, \gamma\} \) and the set of state variables \( \{\tilde{I}_t, A_t, K_t\} \). As a consequence, the definition of the exogenous laws of motion of those three state variables supplemented with their initial values allows to close the model without any indeterminacy.\(^8\)

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\(^7\)Barron et al. (2007) present evidence on labour efforts spent by firms in hiring activities.

\(^8\)Obviously, the levels of state variables also provide the value of aggregate demand and supply in wage units and the unemployment rate.
First, in the model economy under examination, investment is not derived from rational optimization. Following Farmer (2008a), Guerrazzi (2011) formalizes profit earners’ animal spirits by assuming that their investment expenditure measured in wage units is driven by an autonomous stochastic AR(1) process perturbed by white-noise disturbances that convey confidence shocks. In the remainder of our work, we maintain the same autoregressive structure. However, we assume that such a recursive structure also holds for confidence shocks. The underlying idea is that a period of bull (or bear) markets might induce agents to believe that such a situation will be characterized by a certain degree of persistence.\(^9\) Hence,

\[ \tilde{I}_t = \kappa + \rho \tilde{I}_{t-1} + \varepsilon_t \]  

(13)

where \(\kappa\) is a positive drift, \(\rho\) measures the persistence of the exogenous investment sequence while \(\ln \varepsilon_t = \rho \ln \varepsilon_{t-1} + u_t^\varepsilon\) with \(u_t^\varepsilon \sim N(0, \sigma^2_{\varepsilon})\) is the serially correlated confidence shock.

Since \(\tilde{I}_t\) conveys the autonomous component of aggregate demand, the expression in (13) pins down the DCE of the model economy and formalizes in a very simple way a central message of the General Theory, i.e., the idea that investment expenditure may exogenously evolve with no regard to fundamental signals conveyed by endowments, technologies and preferences.\(^10\) Moreover, given the non-linear shape of equilibrium output discussed above, splitting the determination of the investment expenditure from its real amount allows the model to catch divergent patterns among expectations and realized outcomes. Specifically, at low level of the economic activity, profit earners’ euphoria may lead to welfare-improving allocations because higher levels of employment are characterized by higher output. By contrast, when the level of the economic activity is near to (or above) its first best allocation, the (irrational) exuberance of profit earners may lead to welfare-reducing equilibria because higher rates of employment are characterized by a larger share of recruiters and lower output.

Second, as in conventional RBC models, the log of TFP is assumed to follow a stochastic process. Thus,

\[ \ln A_t = \rho_a \ln A_{t-1} + \ln \varepsilon_t^a \]  

(14)

where \(\rho_a\) measures the persistence of productivity disturbances while \(\ln \varepsilon_t^a = \rho_a \ln \varepsilon_{t-1}^a + u_t^a\) with \(u_t^a \sim N(0, \sigma^2_{\varepsilon})\) is the serially correlated innovation shocks.

The expression in (14) provides the stochastic trend that drives output, real investment, capital and real wages. Moreover, in adherence to the news-shock hypothesis, innovations shocks are assumed to follow the same autoregressive structure followed by confidence shocks in (13).

\(^9\) A straightforward empirical test for this hypothesis is the strong autocorrelation displayed by confidence indexes. For instance, the consumer confidence index provided by the Conference Board has a 94% first-order correlation coefficient (authors calculations).

\(^10\) Along these lines, Kurz (2008) provides a piece of micro-foundation for (13) by deriving a similar first-order autoregressive process as a limit posterior of a Bayesian learning inference in non-stationary environments.
The intuition for this assumption is that business cycle fluctuations may be driven by changes in expectations, but these changes in expectations may also shape changes in productivity (e.g. Beaudry and Portier 2006).\textsuperscript{11}

Furthermore, productive capital evolves according to the usual dynamic accumulation law. Hence,

\begin{equation}
K_t = Q_{t-1} - I_{t-1} + (1 - \delta_K) K_{t-1} \quad 0 < \delta_K < 1
\end{equation}

where $\delta_K$ is the depreciation rate of capital.

\begin{figure}[h]
\centering
\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{figure1a}
\caption{}
\end{subfigure} \hspace{0.05\textwidth}
\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{figure1b}
\caption{}
\end{subfigure}
\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{figure1c}
\caption{}
\end{subfigure} \hspace{0.05\textwidth}
\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{figure1d}
\caption{}
\end{subfigure}
\caption{Demand-constrained equilibrium}
\end{figure}

The specification of real investment conveyed by the difference equation in (15) reveals that our model economy displays a demand-driven version of the golden rule on capital accumulation. Specifically, whenever $\tilde{I}_t$ fails to deliver the social optimal level of (un)employment, the economy is under-consuming and under-investing at the same time. As a consequence, the assumption that the animal spirits of profit earners drive the actual realization of $\tilde{I}_t$ allows us to take into consideration the fact in many circumstances the short-run equilibrium dynamics of the

\textsuperscript{11}In order to offer an empirical test for this hypothesis, in section 3 the estimation of the correlation between $u_t^f$ and $u_t^r$ will be explicitly provided.
economy cannot be easily reconciled with its optimal long-run trends, a recurrent feature of Keynesian models of growth (e.g. Grabowski and Shields 2000).

An illustration of a DCE is given in the four panels of figure 1. Specifically, in panel (a) there is the equilibrium in the market for goods and services represented by the intersection of the value of aggregate demand with the value of aggregate supply both conveyed in wage units. In panel (b) there is the negative one-to-one relationship between employment and unemployment. In panel (c) there is a 45-degree line. Moreover, in panel (d) there is the Beveridge curve and the (mirrored) equilibrium wage function.

The set of diagrams in figure 1 can be used to describe the theoretical effects of demand (or confidence) and supply shocks. Specifically, everything else being equal, a positive (negative) demand shock increases (decreases) employment and the number of recruiters and reduces (increases) unemployment and the level of the real wage. Moreover, ceteris paribus, a positive (negative) supply shock leads only to an increase (decrease) of the real wage by leaving all the other variables unaltered.

3 Data, calibration and estimation methodology

In this section, we describe the data-set exploited to run our estimations. Thereafter, we provide the calibration for the model parameters that we decided to take as given. In addition, we review the implemented estimation methodology.

3.1 Data

Aiming at estimating the model developed in section 2, we rely on quarterly time series data for the following US macroeconomic real aggregates: GDP, consumption, investment, wages, unemployment and job vacancies, i.e., our empirical counterpart of corporate recruiters (e.g. Guerrazzi 2011).

The whole set of observations is retrieved from the Bureau of Economic Analysis with the exception of the series for vacancies - which comes from the Conference Board archives - and real wages - retrieved instead from the Bureau of Labor Statistics. All series are taken in per-capita terms and de-trended with their respective linear trend.

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12 The series for GDP is constructed as the sum of consumption and investment data.

13 Our measure of real wages is the series of quarterly average hourly earnings of production workers divided by the GDP deflator. We also tried the estimation with the help-wanted index built by Barnichon (2010) and results are robust to such an alternative.

14 We checked the robustness of our results to the detrending strategy. Specifically, we performed the estimation detrending data with the HP filter. Results are very robust to that alternative. Moreover, we also tried the estimation taking the observables in growth rates. Results seem less robust, but still acceptable. In addition,
The estimation period is 1980q1 to 2008q4. This time span collects interesting stylized facts such as the productivity slow-down of the 80s and a number of NBER recessions including the initial stages of the Great Recession mentioned in the introduction.

3.2 Calibration

Taking into account their well established quantitative assessment, some parameters are calibrated before the estimation. Specifically, the output elasticity with respect to labour ($\alpha$) is set like in Guerrazzi (2011) while the capital depreciation rate ($\delta_K$) is calibrated as in Kydland and Prescott (1982).\(^\text{15}\) Moreover, the value for the consumption-output ratio follows from the choice of $\alpha$; indeed, in our proposal such a parameter plays the role of the main determinant of the marginal and average propensity to consume. Those values are summarized in table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output elasticity to capital ($\alpha$)</td>
<td>0.300</td>
</tr>
<tr>
<td>Capital depreciation rate ($\delta_K$)</td>
<td>0.025</td>
</tr>
</tbody>
</table>

Table 1: Calibration

3.3 Methodology

The model economy is brought to data by assuming that per capita de-trended observed values are equal to their theoretical counterpart augmented by a measurement error (e.g. Boivin and Giannoni 2006). In other words, we suppose that

$$\hat{x}_t^{\text{OBS}} = \hat{x}_t + \hat{\varepsilon}_t$$

$$\hat{x}_t = \{\hat{y}_t^{\text{OBS}}, \hat{c}_t^{\text{OBS}}, \hat{t}_t^{\text{OBS}}, \hat{q}_t^{\text{OBS}}, \hat{u}_t^{\text{OBS}}, \hat{v}_t^{\text{OBS}}\}$$

(16)

where $\hat{x}_t^{\text{OBS}}$ is a generic observed per capita de-trended data, $\hat{x}_t$ is its log-linearized theoretical counterpart while $\hat{\varepsilon}_t$ is the corresponding measurement error.

In addition, measurement errors are assumed to follow zero-mean autoregressive processes. As a consequence,

$$\hat{\varepsilon}_t^x = \rho_x \hat{\varepsilon}_{t-1}^x + u_t^x$$

(17)

where $u_t^x \sim N(0, \sigma_x^2)$.

\(^\text{15}\)In order to address the robustness of our choice for $\alpha$, we estimate the model assigning to that parameter a normal distribution as a prior with mean 0.3 and standard deviation 0.05. The posterior estimated mean is 0.2807, and the probability interval (0.2037 – 0.3559). Given the small difference between the estimated and the calibrated value, all results are basically identical under the two alternatives.
The estimation strategy conveyed by (16) and (17) allows to overcome the stochastic singularity problem that would have emerged in the attempt of estimating a model with two structural shocks, i.e., demand and supply shocks, using six observables. The whole set of steady-state and log-linearized equations can be found in Appendix.

The Bayesian approach requires specifying a prior for each estimated parameter. In our work, the choice of priors has been done according to the following criteria. First, we choose an inverse gamma distribution for the standard deviation of shocks because it has a positive support. We allowed for an infinite variance to allow for the computing algorithm to explore a big portion of the support, although lower weights are given to higher values. Since they are bounded between zero and one, persistence parameters of demand and supply shocks are assumed to have a beta distribution. Moreover, this allows higher prior weights to be put on values close to unity, i.e., implying more persistence as we actually expect.

Thereafter, the matching elasticity $\gamma$ is assumed to be distributed as a beta with a mean equal to one-half. The most appropriate prior for that parameter would have been a uniform distribution because we do not have strong a priori beliefs about it. Nevertheless, this type of prior puts the same weights on all the possible values in the support. By definition, such a Beveridge curve parameter belongs to the simplex interval $(0,1)$. Furthermore, if values too close to those bounds are selected, then the steady-state of the model may be strongly affected.\footnote{For instance, according to equation (11), when $\gamma$ is close to one, corporate recruiters (or job vacancies) tend to explode.} Hence, in order to reduce the number of draws too close to the bounds (or - at least - to give them less importance), we impose a beta distribution.

The aim of our estimation procedure is to obtain posterior distributions of the free model parameters and make inference out of them. Since the posterior distributions are unknown, we used a Markov-Chain Monte Carlo simulation method, namely the so-called random walk Metropolis–Hasting algorithm, which uses an acceptance/rejection rule to converge towards the posterior distribution.

Before simulating, the maximization of the posterior kernel has been done in order to find the posterior modes and the variance-covariance matrix (evaluated at the modes) to be used in the Metropolis-Hasting algorithm initialization. The whole procedure is implemented in DYNARE (e.g. Adjemian et al. 2011 and Canova 2007).

4 Estimation results

In this section, we present and review our estimation results. First, we focus on the fit of the theoretical framework as well as on its estimated parameters. A variance decomposition analysis is also provided. Thereafter, we use the impulse response analysis to evaluate the
dynamics implied by demand and supply shocks. Furthermore, we offer an estimation of the output gap together with its shock decomposition.

4.1 Fitting and posterior estimation

The overall fitting of the model economy is evaluated through two different ways. The former is a comparison between the Kalman-filtered one-sided estimates of the observed variables with their estimated counterpart while the latter is a moments comparison based on volatility, autocorrelation and cross-correlations. We focus on labour market variables because the absence of empirically relevant real rigidities (e.g. habit formation in consumption, investment adjustment costs, etc.) harms the ability of the model to properly fit the other macroeconomic aggregates.\footnote{Those important extensions are left to further developments.} In fact, the theoretical intent of our model is to explain observed labour market dynamics while our empirical goal is to highlight how it properly captures such a dynamics.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Actual (solid line) and (dashed line) fitted values}
\end{figure}
On the one hand, figure 2 shows fitted values (straight line) together with actual data (dotted line) as percent deviation from their respective steady-state references. Straightforward observation reveals that they look pretty similar, indicating that for the last thirty years our theoretical model provides a good fit of US actual macroeconomic patterns. Interestingly, the model is able to match (inter alia) the productivity slow-down of the 80s as well as the early (un)employment effects of the Great Recession.

On the other hand, switching to moments comparison, table 2 shows observed and estimated standard deviations. The figures suggest that the model matches quite closely the pattern of observed data, although it vaguely tends to overstate wage stickiness. Moreover, our framework replicates the Shimer (2004, 2005) puzzle; indeed, all over the period, the labour market tightness indicator is much more volatile than labour productivity - as measured by real wages - by a factor of about fifteen.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
</tr>
<tr>
<td>Wages</td>
<td>2.95</td>
</tr>
<tr>
<td>Unemployment</td>
<td>1.02</td>
</tr>
<tr>
<td>Recruiters</td>
<td>33.63</td>
</tr>
<tr>
<td>Tightness</td>
<td>47.13</td>
</tr>
</tbody>
</table>

Table 2: Observed and estimated volatilities

Furthermore, figure 3 shows a comparison between observed and estimated values based on auto- and cross-correlations evaluated up to five lags. The 4x4 panel reveals that the 5th and the 95th percentiles accommodate most of the observed values with the exception of the auto- and cross-correlations of unemployment with respect to recruiters and labour market tightness. Both values are somehow underestimated, but, consistently with our theoretical guess, they are negative in a significant manner.

18The ratio between vacancies and unemployment is not used as an observable. Anyhow, the model does remarkably well in generating a series for that ratio which closely replicate the observed one.

19Model correlations are computed on simulated data generated from 250 draws from the posterior parameter distribution and 100 simulated samples for each draw of 117 observations. We estimate a VAR with three lags using those simulated data to compute our statistics. Moreover, in order to have consistency, we also estimate a companion VAR with actual values to compute data correlations.
In table 3 we summarize the priors chosen for the parameters and their moments, and the posterior means with the 5th and the 95th quantiles. Some results are remarkable. For instance, the posterior mean of the matching elasticity is 89.80%. There are two reference values for that parameter. First, Farmer (2008a) uses a value of one-half in his seminal simulations. This partially justifies our prior mode. Moreover, in the attempt to find a rationale for the Shimer (2004, 2005) puzzle, Guerrazzi (2011) calibrates it at 98.60%. Given those computational references, our point estimate is quite reasonable. Furthermore, the correlation between confidence \(u_t^c\) and innovation \(u_t^a\) shocks is positive in a statistically significant manner. This result provides an empirical corroboration of news shock hypothesis sketched in section 2 by suggesting that confidence shocks spill over - at least in part but in the same direction - into productivity disturbances (e.g. Beaudry and Portier 2006).

\(^{20}\)Further details are given in Appendix.
### Standard deviations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Prior Mean</th>
<th>Std. dev.</th>
<th>Posterior mean</th>
<th>Probability interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Innovation supply shock ($\sigma_u^a$)</td>
<td>IG</td>
<td>0.087 Inf</td>
<td>0.164</td>
<td>0.124</td>
<td>0.203</td>
</tr>
<tr>
<td>Innovation demand shock ($\sigma_\epsilon$)</td>
<td>IG</td>
<td>0.055 Inf</td>
<td>0.047</td>
<td>0.039</td>
<td>0.054</td>
</tr>
<tr>
<td>Output measur. error ($\sigma_y$)</td>
<td>IG</td>
<td>0.022 Inf</td>
<td>0.424</td>
<td>0.352</td>
<td>0.498</td>
</tr>
<tr>
<td>Wage measur. error ($\sigma_q$)</td>
<td>IG</td>
<td>0.022 Inf</td>
<td>0.180</td>
<td>0.133</td>
<td>0.226</td>
</tr>
<tr>
<td>Consumption measur. error ($\sigma_c$)</td>
<td>IG</td>
<td>0.030 Inf</td>
<td>0.449</td>
<td>0.386</td>
<td>0.511</td>
</tr>
<tr>
<td>Investment measur. error ($\sigma_{inv}$)</td>
<td>IG</td>
<td>0.101 Inf</td>
<td>3.644</td>
<td>3.240</td>
<td>4.011</td>
</tr>
<tr>
<td>Unempl. measur. error ($\sigma_u$)</td>
<td>IG</td>
<td>0.010 Inf</td>
<td>0.188</td>
<td>0.161</td>
<td>0.215</td>
</tr>
<tr>
<td>Vacancies measur. error ($\sigma_v$)</td>
<td>IG</td>
<td>0.184 Inf</td>
<td>3.178</td>
<td>2.405</td>
<td>3.956</td>
</tr>
</tbody>
</table>

### Persistence

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Prior Mean</th>
<th>Std. dev.</th>
<th>Posterior mean</th>
<th>Probability interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply shock ($\rho_a$)</td>
<td>B</td>
<td>0.800</td>
<td>0.050</td>
<td>0.905</td>
<td>0.863</td>
</tr>
<tr>
<td>Exogenous investment ($\rho_i$)</td>
<td>B</td>
<td>0.750</td>
<td>0.050</td>
<td>0.843</td>
<td>0.792</td>
</tr>
<tr>
<td>Innov. supply shock ($\rho_u^a$)</td>
<td>B</td>
<td>0.750</td>
<td>0.050</td>
<td>0.839</td>
<td>0.780</td>
</tr>
<tr>
<td>Demand shock ($\rho_\epsilon$)</td>
<td>B</td>
<td>0.500</td>
<td>0.100</td>
<td>0.832</td>
<td>0.767</td>
</tr>
<tr>
<td>Output measur. error ($\rho_y$)</td>
<td>B</td>
<td>0.750</td>
<td>0.050</td>
<td>0.811</td>
<td>0.731</td>
</tr>
<tr>
<td>Wage measur. error ($\rho_q$)</td>
<td>B</td>
<td>0.750</td>
<td>0.050</td>
<td>0.812</td>
<td>0.712</td>
</tr>
<tr>
<td>Consump. measur. error ($\rho_c$)</td>
<td>B</td>
<td>0.750</td>
<td>0.050</td>
<td>0.857</td>
<td>0.806</td>
</tr>
<tr>
<td>Invest. measur. error ($\rho_{inv}$)</td>
<td>B</td>
<td>0.750</td>
<td>0.050</td>
<td>0.839</td>
<td>0.793</td>
</tr>
<tr>
<td>Unempl. measur. error ($\rho_u$)</td>
<td>B</td>
<td>0.750</td>
<td>0.050</td>
<td>0.911</td>
<td>0.879</td>
</tr>
<tr>
<td>Vacancies measur. error ($\rho_v$)</td>
<td>B</td>
<td>0.750</td>
<td>0.050</td>
<td>0.889</td>
<td>0.826</td>
</tr>
<tr>
<td>Matching elasticity ($\gamma$)</td>
<td>B</td>
<td>0.500</td>
<td>0.200</td>
<td>0.898</td>
<td>0.882</td>
</tr>
<tr>
<td>Investments drift ($\kappa$)</td>
<td>U</td>
<td>0 - 3</td>
<td>–</td>
<td>0.019</td>
<td>0.012</td>
</tr>
<tr>
<td>Correlation ($u_t^\epsilon, u_t^a$)</td>
<td>U</td>
<td>0 - 1</td>
<td>–</td>
<td>0.489</td>
<td>0.308</td>
</tr>
</tbody>
</table>

**IG** = inverse gamma, **B** = beta, **U** = uniform. Upper and lower bound are indicated for the Uniform distribution.

**Table 3: Estimated parameters**

Another relevant tool for our analysis is variance decomposition. Its quantitative importance is twofold. First, exploiting this empirical diagnostic we can evaluate the relevance of measurement errors. Moreover, we can test how demand shocks shape the path of observed variables. The results of variance decomposition are illustrated in table 4.

---

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Table 4: Asymptotic variance decomposition

<table>
<thead>
<tr>
<th>Variable</th>
<th>$u^q$</th>
<th>$u^c$</th>
<th>$u^i$</th>
<th>$u^y$</th>
<th>$u^u$</th>
<th>$u^v$</th>
<th>$u^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{q}^{OBS}$</td>
<td>0.00</td>
<td>1.34</td>
<td>0.00</td>
<td>0.00</td>
<td>17.51</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$\hat{c}^{OBS}$</td>
<td>0.00</td>
<td>0.00</td>
<td>3.40</td>
<td>0.00</td>
<td>77.14</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$\hat{y}^{OBS}$</td>
<td>2.31</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>78.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$\hat{v}^{OBS}$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>94.33</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$\hat{i}^{OBS}$</td>
<td>0.00</td>
<td>0.00</td>
<td>66.78</td>
<td>0.00</td>
<td>26.53</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$\hat{o}^{OBS}$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>90.25</td>
<td>0.00</td>
<td>9.75</td>
</tr>
<tr>
<td>$\hat{q}$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>17.75</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$\hat{c}$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>79.85</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$\hat{y}$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>79.85</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$\hat{i}$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>79.85</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

The figures of the asymptotic variance decomposition suggest that measurement errors explain only a small part of the variance of the respective variables. This testifies that although the model is fairly simple it is still able to fit data in a consistent manner. By contrast, demand shocks are by far the most relevant shocks in explaining the variability of endogenous variables, with the exception of real wages that - according to our theoretical framework - are also directly affected by productivity shocks.

4.2 Impulse response analysis

Now we present the implied impulse response functions to dig into the dynamics of our model when either demand or supply shocks hit the economy.

On the one hand, the last three panels of figure 4 suggest that positive (negative) demand shocks decrease (increase) unemployment and they increase (decrease) recruiters and the labour market tightness indicator. Those movements are consistent with the thick (thin) market externality that is assumed to characterize a demand-driven search economy such as the one sketched in figure 1. It is worth noting that the response of real wages is not monotonic. At the beginning, consistently with the first postulate of the classical economy, a positive (negative) demand shock decrease (increase) real wages (e.g. Keynes 1936). Thereafter, boosting (depressing) capital accumulation, positive (negative) demand shocks increase (decrease) real wages via higher (lower) productivity. On the whole, such a non-monotonic path is responsible for the high degree of real wage stickiness already stressed above.
Figure 4: Demand shocks

The diagrams describing demand shocks also allow us to highlight another important result. Specifically, response functions to demand shocks reveal that our model economy is able to generate hump–shaped patterns for all the examined variables. In the DSGE literature, hump–shaped patterns are usually implied by the introduction of specific types of dynamic microfoundations and/or price-wage rigidities. For instance, consumption responses can be hump–shaped combining habit formation and forward looking behaviour of households; indeed, the resulting dynamic Euler equation provides the desired response of consumption over time. Moreover, hump–shaped investment responses can be obtained from ad-hoc investment Euler equations, which, in turn, are the upshot of the interaction between investment adjustment costs and forward looking behavioural paths followed by investors. In addition, price-wage rigidities only account for the persistence and impulse response shape of inflation and labour market variables; indeed, forward looking and indexation assumptions lead to one equation for inflation and one for wages which resemble very much Euler equations (e.g. Smets and Wouters 2003).

In our set-up things are quite different. There are no Euler equations; indeed, the responsible for the hump-shaped pattern of the diagrams in figure 4 is the shock structure conveyed by
(13) and (14). Specifically, as shown by Perron (1993), the hump-shaped behaviour of macroeconomic time series can be the upshot of a linear combination between stationary AR(1) processes. Therefore, the hypothesis that exogenous shocks are serially correlated allows us to derive hump-shaped impulse response functions similar to those generated by DSGE models with more well-structured dynamic micro-foundations.

On the other hand, the effects of technology shocks are described in the four panels of figure 5. Specifically, the diagrams reveal that positive (negative) technology shocks increase (decrease) real wages by leaving unaltered all the other variables. This unconventional result is due to the choice of measure units, indeed, in the model economy developed in section 2, the equilibrium in the goods market determines the ratio between output and real wages, i.e., the value of output in wage units, as well as (un)employment (e.g. Keynes 1936). In this context, a positive (negative) supply shock increases (reduce) output and real wages by the same amount by leaving unaltered their ratio and the (un)employment rate.

\[ \text{Figure 5: Supply shocks} \]

\[ ^{21}\text{Obviously, in the model economy under examination such a smooth dynamic behaviour is strengthened by the endogenous source of real wage stickiness delivered by the inflection point in the equilibrium wage function.} \]
4.3 Output gap and shock decomposition

In conclusion, we present the path of output gap together with its shock decomposition. However, before entering the details of figures, it is necessary to provide a definition of potential output consistent with the postulates of the New Farmerian Economics and compare it with more traditional definitions exploited in the DSGE literature.

In general terms, the output gap is defined as the deviation of actual output from its potential value. In the context of our model, as already mentioned in section 2, potential output is defined as the level of output that would prevail if an omniscient social planner could internalize search externalities by maximizing output period by period. Such a level of production corresponds to the first-best allocation but it also entails a positive equilibrium unemployment which depends on the value of the matching elasticity only (e.g. Guerrazzi 2011).

This definition is somehow different from the one recurring in DSGE literature. For instance, in Smets and Wouters (2003) output gap is defined as the level of output that would prevail under flexible prices and wages in the absence of cost push shocks, i.e., price and wage mark-ups disturbances. Since the framework developed in this paper is quite different from the theoretical settings usually exploited in the DSGE literature, we abstract from a direct comparison. By contrast, we provide the path of the output gap generated by our New Farmerian model together with its shock decomposition.

Figure 6: Actual (dashed line) and potential (solid line) output
First, figure 6 shows the paths of actual and potential output in percent deviation from steady state (shaded areas denote NBER recessions). The diagram shows that the model economy is able to replicate a number of NBER recessions. Specifically, the model replicates the two recessions occurred in the US at beginning of the 80s, the one just after 2000 and the GDP contraction triggered by the Great Recession of 2008-2009.

The output gap is computed as the difference between actual and potential output. Its path as well as its shock decomposition are illustrated in figure 7. The graph allows us to highlight an important aspect of the end-of-sample financial crisis; indeed, the severe increase of the output gap is determined by contractionary demand shocks.\footnote{Matching this pattern with the observed declining patterns of asset prices during 2008, this finding suggests that the finance-induced recession can be portrayed as the upshot of strong negative demand shocks triggered by a self-fulfilling drop of confidence in the stock market value that lead to a sudden aggregate demand contraction (e.g. Farmer 2010b, 2012b and Guerrazzi 2012b).}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{output_gap_shock_decomposition.png}
\caption{Output gap shock decomposition}
\end{figure}

The rationale underlying this view of the recent finance-induced recession can be given as

\footnote{The same is true for actual and potential output decompositions where supply shocks play an active role.}

\footnote{In a companion Euro Area model, Gelain and Guerrazzi (2010) find that in Europe the financial crisis exacerbated productivity problems. This is in line with Sala et al. (2012) who find that for Sweden and UK the technology shock is the most important determinant of the recent financial crisis. By contrast, in the US they find that the risk premium shock is the most relevant contractionary factor.}
follows. According to the model developed in section 2, the transmission mechanism from the financial sector to the real sector of the economy may work through a self-fulfilling effect of asset prices on the state of long run expectations that is assumed to affect aggregate demand via the investment expenditure measured in wage units. In other words, a financial crash (boom) is assumed to depress (boost) the expected financial prospects of firms by leading to a sudden reduction (increase) of investment expenditure that leads to a reduction (increase) in output and employment.\(^{24}\) Those causal links provide a straightforward explanation of the output gap path followed by the model economy at the end of the sample.

Demand-side explanations of the latest finance-induced recession have also been put forwards in other brand-new contributions. For instance, similar evidence for the US has been recently found by Furlanetto and Groshenny (2013). Specifically, exploiting a richer DSGE model featuring the traditional labour market framework à la Mortensen and Pissarides (1994), those authors find that the drop in output growth during the financial crisis was driven by increasingly negative risk premium shocks that according to their expected negative effects on the propensity to invest can be classified as demand disturbances. By contrast, technology shocks appear to be much less relevant and they are found to marginally contribute positively to output growth. Similarly, in our setting supply shocks provide a very small contribution in the pattern of the output gap decomposition (not shown in the paper).

In the same direction, Perri and Quadrini (2011) also stress the self-fulfilling nature of the recent financial crisis, although related to other factors rather than confidence in the stock market value. Specifically, they propose a theoretical framework in which endogenous credit booms and crashes can result from self-validating expectations. This is important to account for sharp contractions in real activity and asset prices like those observed during the recent financial turmoil.

More recently, Plotnikow (2013) develops a New Farmerian model with investment in which private consumption depends on wealth through the permanent income hypothesis. From a theoretical point of view, this assumption allows to resolve the indeterminacy implied by search externalities in the labour market and show that non-fundamental demand disturbances may have permanent effects on both output in wage units and unemployment.

5 Concluding remarks

In this paper we apply typical Bayesian diagnostic tools of DSGE models to the New Farmerian framework developed by Guerrazzi (2011). Specifically, estimating a simple demand-driven competitive-search model, we test the ability of this further micro-foundation of the General

\(^{24}\)Recall that in our simplified model firms invest all their net-of-wage payments. As a consequence, the lower (higher) firms’ profits, the lower (higher) investments.
Theory to match the dynamic behaviour of the US economy over the last thirty years by focusing on the patterns of key labour market variables such as real wages, unemployment and corporate recruiters measured as job vacancies.

The main results achieved in this empirical investigation are the following. First, the New Farmerian model developed by Guerrazzi (2011) provides a good fit of US data by replicating (inter alia) the productivity slowdown of the 80s as well as the early (un)employment effects of the Great Recession. Second, under the hypothesis that confidence and technology innovation shocks are serially correlated (e.g. Beaudry and Portier 2006), the suggested framework reacts to demand shocks in the same way as more micro-founded DSGE models by delivering hump-shaped impulse response functions. Furthermore, the shock decomposition of the output gap reveals that the finance-induced recession was driven by a strong negative demand shock probably triggered by a self-fulfilling drop of confidence in the stock market value (e.g. Farmer 2010b, 2012b and Guerrazzi 2012b).

On the whole, those findings are consistent with the main theoretical attributes of the New Farmerian Economics as well as to latest demand-side explanations of the finance-induced recession (e.g. Furlanetto and Groshenny 2012, Perri and Quadrini 2012 and Plotnikov 2013).

A Appendix: Steady-state equations

Here we derive the steady-state values around which the model is log-linearized. We start from the steady-state equation for the investment expenditure in wage units. Specifically, (13) implies that

\[ \tilde{I}^* = \frac{\kappa}{1 - \rho_I} \]  

where \( \tilde{I}^* \) is the steady-state investment expenditure.

Taking the results in (6), (8) and (A1) into account, it follows that steady-state employment is given by

\[ L^* = \frac{1 - \alpha}{\alpha} \tilde{I}^* \]  

Given (3), steady-state unemployment is equal to

\[ U^* = 1 - L^* \]  

Taking the result in (11) and (A3) into account, it follows that the steady-state rate of corporate recruiters is given by

\[ V^* = (1 - U^*)^{\frac{1}{1 - \gamma}} \]  

Furthermore, the steady-state rate of wage earners allocated to production activities is equal to

\[ X^* = L^* - V^* \]  

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All the remaining steady-state values follow straightforwardly.

**B Appendix: Log-linearized Equations**

Now we provide all the log-linearized equations that describe the model economy developed in section 2. Lower case-hatted-variables denote log-deviations from the corresponding steady-state reference.

Production function

\[ \hat{y}_t = \hat{a}_t + \alpha \hat{k}_t + (1 - \alpha) \hat{x}_t \]  
\[ (B1) \]

Total employment

\[ \hat{t}_t = \frac{X^*}{L^*} \hat{x}_t + \frac{V^*}{L^*} \hat{v}_t \]  
\[ (B2) \]

Unemployment rate

\[ \hat{u}_t = -\frac{L^*}{U^*} \hat{l}_t \]  
\[ (B3) \]

National account identity

\[ \frac{C^*}{Y^*} \hat{c}_t + \frac{I^*}{Y^*} \hat{i}_t = \hat{y}_t \]  
\[ (B4) \]

Real investment

\[ \hat{i}_t = \hat{\hat{i}}_t + \hat{q}_t \]  
\[ (B5) \]

Wage earners budget constraint

\[ \hat{c}_t = \hat{q}_t + \hat{l}_t \]  
\[ (B6) \]

First-order condition for labour allocation

\[ \hat{y}_t - \hat{l}_t = \hat{q}_t \]  
\[ (B7) \]

Hazard rate for employment

\[ \hat{h}_t = \hat{l}_t \]  
\[ (B8) \]

Recruiting effectiveness

\[ \hat{r}_t = \hat{l}_t - \hat{\hat{v}}_t \]  
\[ (B9) \]
\[ \hat{\nu}_t = -\frac{U^*}{(1 - \gamma) (V^*)^{(1 - \gamma)}} \hat{u}_t \]  
\[ (B10) \]

Capital accumulation

\[ \hat{k}_t = (1 - \delta_K) \hat{k}_{t-1} - \delta_K \hat{\nu}_{t-1} \]  
\[ (B11) \]

Investment expenditure in wage units

\[ \hat{\iota}_t = \rho \hat{\iota}_{t-1} + \hat{\varepsilon}_t \]  
\[ (B12) \]

Demand and supply shocks

\[ \hat{\varepsilon}_t = \rho \hat{\varepsilon}_{t-1} + \hat{w}_t \]
\[ \hat{a}_t = \rho \hat{a}_{t-1} + \hat{\varepsilon}_t \]  
\[ (B13) \]

Measurement errors

\[ \hat{\varepsilon}_t^y = \rho_y \hat{\varepsilon}_{t-1}^y + u_t^y \]
\[ \hat{\varepsilon}_t^c = \rho_c \hat{\varepsilon}_{t-1}^c + u_t^c \]
\[ \hat{\varepsilon}_t^{\text{inv}} = \rho_{\text{inv}} \hat{\varepsilon}_{t-1}^{\text{inv}} + u_t^{\text{inv}} \]
\[ \hat{\varepsilon}_t^q = \rho_q \hat{\varepsilon}_{t-1}^q + u_t^q \]
\[ \hat{\varepsilon}_t^u = \rho_u \hat{\varepsilon}_{t-1}^u + u_t^u \]
\[ \hat{\varepsilon}_t^v = \rho_v \hat{\varepsilon}_{t-1}^v + u_t^v \]  
\[ (B14) \]

where \( u_t^y \sim N (0, \sigma_y^2) \), \( u_t^c \sim N (0, \sigma_c^2) \), \( u_t^{\text{inv}} \sim N (0, \sigma_{\text{inv}}^2) \), \( u_t^q \sim N (0, \sigma_q^2) \), \( u_t^u \sim N (0, \sigma_u^2) \) and \( u_t^v \sim N (0, \sigma_v^2) \).

To bring the model to the data, we provide the relations between the observed and the corresponding theoretical variables. Specifically,

\[ \hat{\gamma}_t^{\text{OBS}} = \hat{\gamma}_t + \hat{\varepsilon}_t^y \]
\[ \hat{\iota}_t^{\text{OBS}} = \hat{\iota}_t + \hat{\varepsilon}_t^c \]
\[ \hat{\iota}_t^{\text{OBS}} = \hat{\iota}_t + \hat{\varepsilon}_t^{\text{inv}} \]
\[ \hat{\iota}_t^{\text{OBS}} = \hat{\iota}_t + \hat{\varepsilon}_t^q \]
\[ \hat{\iota}_t^{\text{OBS}} = \hat{\iota}_t + \hat{\varepsilon}_t^u \]
\[ \hat{\iota}_t^{\text{OBS}} = \hat{\iota}_t + \hat{\varepsilon}_t^v \]  
\[ (B15) \]
C Appendix: Prior and posterior distributions

Figure C.1 illustrates prior and posterior distributions. In order to test the convergence of the model we run two chains of 750,000 draws each and we discarded the first 50% as burn–in. The acceptance rate has been tuned to be around 30% while the convergence of the chains has been evaluated with the checks proposed by Brooks and Gelman (1998).
Figure C.1: Prior (grey) and posterior (black) distributions

References


