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Systemic Risk of Commercial Banks: A Markov-Switching Quantile Autoregression Approach

Xiaochun Liu*

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Abstract

This paper extends the Conditional Value-at-Risk approach of Adrian and Brunnermeier (2011) by allowing systemic risk structures subject to economic regime shifts, which are governed by a discrete, latent Markov process. This proposed Markov-Switching Conditional Value-at-Risk is more suitable to Supervisory Stress Scenario required by Federal Reserve Bank in conducting Comprehensive Capital Analysis and Reveiw, since it is capable of identifying the risk states in which the estimated risk levels are characterized. Applying MSCoVaR to stress-testing the U.S. largest commercial banks, this paper finds that the CoVaR approach underestimates systemic risk contributions of individual banks by around 131 basis points of asset loss on average. In addition, this paper constructs Banking Systemic Risk Index by value-weighted individual risk contributions for specifically monitoring the systemic risk of the banking system as a whole.

Keywords: Markov-Switching Conditional Value-at-Risk, Conditional Expected Shortfall, Bayesian Quantile Inference, Stress-testing, Value-at-Risk, Commercial Banks, Banking Systemic Risk Index

JEL: C22, C58, C51, C11, G23

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1 Introduction

Recently, Adrian and Brunnermeier (2011) propose to measure systemic risk via the conditional value-at-risk (CoVaR) of the financial system, conditional on institutions being in a state of distress. In their work, an institution's contribution to systemic risk is defined as the difference between CoVaR conditional on the institution being in distress and CoVaR in the median ("normal") state of the institution. Hence, it characterizes the marginal contribution of a particular institution (in a non-causal sense) to the overall systemic risk.

The CoVaR approach is particularly appealing in that it outlines a method to construct a countercyclical, forward-looking systemic risk measure by predicting future systemic risk using current institutional characteristics. This is a time-varying systemic risk measure which does not rely on contemporaneous price movements and thus can be used to anticipate systemic risk. This method relates systemic risk measure to macroeconomic variables and the balance sheet deleveraging and characteristics of individual institutions. This is essentially a main regulatory concern of central banks.

A number of recent studies have extended and estimated the CoVaR measure of systemic risk for a variety of financial systems.[1] Adams et al. (2011) estimate a system of quantile regressions for four sets of major financial institutions (commercial banks, investment banks, hedge funds and insurance companies). Wong and Fong (2010) estimate CoVaR for the CDS of Asia-Pacific banks. Brunnermeier et al. (2012) use the CoVaR approach to examine the contribution of non-interest income to sytemic bank risk. They find that banks with a higher non-interest income to interest income ratio have a higher contribution to systemic risk and their contributions appear to be countercyclical to sytemic risk build-up. Lopez-Espinosa et al. (2012) use the CoVaR approach to identify the main factors behind systemic risk in a set of large international banks. They find that short-term wholesale funding is a key determinant in triggering systemic risk episodes.

[1] See e.g., Brunnermeier et al. (2012), Lopez-Espinosa et al. (2012), Rodriguez-Moreno and Pena (2012), Arias et al. (2010), Girardi and Ergun (2012), Roengiptya and Rungcharoenkitkul (2011), and Van Oordt and Zhou (2010), etc. Bisias et al. (2012) and Brunnermeier and Oehmke (2012) provide comprehensive reviews on systemic risk analytics.

However, Bisias et al. (2012) raise the important econometric issue of nonstationarity which is particularly relevant to systemic risk measurement. Virtually the existing methods of systemic risk estimation and inference rely on the assumption of stationarity. In other words, the joint distribution of the relevant variables is stable over time. Nonetheless, the literature has recognized the stylized fact of structural breaks in macroeconomic and financial time series, so that the distribution structures of a time series might, driven by economic states, evolve over time. Hence, the very nature of systemic risk implies a certain degree of nonstationarity that may not always be consistent with the econometric framework in which risk measures are typically estimated.

Brunnermeier and Oehmke (2012) also concern that the CoVaR approach is vulnerable to regime changes based on historical data. The estimated CoVaR value is undistinguished from the distributions associated with i.e., a good economic state or an economic downturn. In this regard, without informing its associated risk states, the CoVaR measure is at best an averaging across different economic regimes and hence less advisable to or even misleading market participants and regulators in managing risks with ambiguous targets. Evidently, Adams et al. (2011) have shown the sensitivity of systemic risk to tranquil, normal and volatile economic states, while Lopez-Espinosa et al. (2012) have found that asymmetries based on the sign of bank returns play an important role in capturing sensitivity of system-wide risk to individual bank returns. These concerns highlight the need for new systemic risk methods that are able to address nonstationarity in a more sophisticated way.

This paper specifically considers the systemic risk measure subject to regime shifts. I extend the CoVaR measure of systemic risk to a nonlinear dynamic structure, namely Markov-switching CoVaR (MSCoVaR), in which an instituion's contribution to systemic risk is measured by allowing the joint distribution evolving over time. Switching regimes is determined by the outcome of a latent, discrete Markov process, so that the conditional value-at-risk can be obtained with the filtered probabilities of risk states.

This paper characterizes two risk states: a normal risk level implied by good economic periods and a high risk level associated with economic recessions, crises or extreme events.

MSCoVaR is thus obtained for each risk state in stress-testing. Particularly, this paper obtains MSCoVaR by estimating Markov-switching quantile autoregressive models (MSQAR) recently developed by Liu (2014). MSQAR is the location-scale quantile autoregression in which the location and scale parameters are permitted to evolve over time.

The MSCoVaR measure of systemic risk appears to have the advantage of naturally fitting to the Supervisory Stress Scenario required by Federal Reserve Bank in Comprehensive Capital Analysis and Reveiw (CCAR).[2] In CCAR, a supervisory stress scenario is a hypothetical scenario to be used to assess the strength and resillience of BHC capital in a severely adverse economic environment. It represents an outcome in which the U.S. economy experiences a significant recession and economic activity in other major economies also contracts significantly, i.e., a deep recession in the United States, significant declines in asset prices and increases in risk premia, and a slowdown in global economic activity, etc. Therefore, the MSCoVaR result from a high risk episode is well-defined for the stress-testing in Fed's supervisory stress scenario since it estimates a separate set of parameters for high risk episodes.

In addition, the MSCoVaR measure of sytemic risk provides various ways to test different stress scenarios. For instance, if an instituion is systematically important, its hypothetically distressed scenario should also cause a distress in financial system. The systemic risk of a systemically important institution can thus be measured by the high risk episodes of both financial system and the insitution. By contrast, as a non-systemically important instution, its hyptothetical stress scenario, unless leading to a herding effect, does not cause a distress in financial system. Hence, its systemic risk can be measured by using the high risk episode of the insitution and the normal risk period of financial system.

Importantly, the assumption in Liu (2014) that quantile error terms follow a three-parameter asymmetric Laplace distribution (ADL) for filtering transition probabilities of regimes can also be used to simulate the Markov-switching conditional expected shortfall

[2] See Comprehensive Capital Analysis and Review 2012 : Methodology and Results for Stress Scenario Projections. Board of Governors of the Federal Reserve System: March13 , 201 2; and Comprehensive Capital Analysis and Review 2013: Assessment Framework and Results. Board of Governors of the Federal Reserve System: March 2013

(MSCoES) from the MSQAR results. This provides a natural solution to the theoretical issue that CoVaR is not a coherent risk measure due to its nonsubadditive nature.[3] Note that MSCoES takes distributional aspects within the tail into account. To this end, a banking systemic risk index by value-weighted individual contributions is constructed for monitoring systemic risk specific to the banking system as a whole.

This paper estimates MSCoVaR and MSCoES as risk contributions of the largest U.S. commercial banks. The empirical results show strong evidence that financial institutions and the banking system as a whole experience regime shifts in their lower tails. The new systemic risk measure shows that the CoVaR approach of Adrian and Brunnermeier (2011) underestimates systemic risk contributions of individual banks by around 131 basis points of asset loss on average. The empirical results also show that the banking system is more sensitive to marginal changes of an individual bank during high risk episodes than during normal risk periods. In addition, Banking Systemic Risk Index presents the high relavence of tracing financial distress situations over the sample period.

The rest of this paper is structured as follows. Section 2 defines the Markov-switching systemic risks measured by MSCoVaR and MSCoES. Markov-Switching Quantile Autoregression of Liu (2014) for estimating MSCoVaR and MSCoES are described in *Appenix A*. Section 3 applies MSCoVaR and MSCoES methods to stress-testing the U.S. largest commercial banks. In this section, the banking systemic risk index is also constructed. Section 4 concludes this paper.

2 Systemic Risk Measure

This section briefs the CoVaR measure of systemic risk and then extends it to define the Markov-Switching CoVaR to identify risk states for a potential nonstationary time series. It is followed by a discussion of simulating the Markov-switching conditional expected shortfall as a coherent risk measure.

[3]See Adrian and Brunnermeier (2011) and Artzner et al. (1999).

2.1 CoVaR

Recall that the value-at-risk of institution n given the probability of τ is

$$Pr\left(X_t^n \leq VaR_t^n\right) = \tau \tag{2.1}$$

where X_t^n denotes the asset return value of institution n at time t. The VaR of the financial system return (X_t^w) conditional on the event $\left\{\mathbb{C}(X_t^n) : X_t^n = VaR_{t,\tau}^n\right\}$, i.e., institution n's asset-return attains its VaR value, is denoted by $CoVaR_{t,\tau}^{w|n}$, such that

$$Pr\left(X_t^w \leq CoVaR_t^w|\mathbb{C}\left(X_t^n\right)\right) = \tau$$

Institution n's contribution to the system risk is thus defined as

$$\triangle CoVaR_{t,\tau}^{w|n} = CoVaR_{t,\tau}^{w|n} - CoVaR_{t,\tau}^{w|n,50\%} \tag{2.2}$$

where $CoVaR_{t,\tau}^{w|n,50\%}$ denotes the VaR of the financial system when the institution n's returns are at their median ("normal") state as $Pr\left(X_t^w \leq CoVaR_t^w|X_t^n = VaR_{t,50\%}^n\right) = \tau$. For simplicity, this paper suppresses the superscript w. Hence, $\triangle CoVaR_{t,\tau}^n$ denotes the difference between the VaR of the financial system conditional on the distress of a particular financial institution n and the VaR of the financial system conditional on the median state of the institution n. Thus, $\triangle CoVaR_{t,\tau}^n$ quantifies how much an institution n adds to overall systemic risk. It captures the amount of additional risk that an institution inflicts upon financial system when the institution attains its VaR value.

Adrian and Brunnermeier (2011) apply quantile autoregressive models (QAR) of Koenker and Xiao (2006) to estimate CoVaR in two steps as follows

$$X_t^n = \alpha_\tau^n + \rho_\tau^n X_{t-1}^n + \gamma_\tau^{'n} Z_{t-1} + \varepsilon_{t,\tau}^n \tag{2.3}$$

$$X_t^w = \alpha_\tau^{w|n} + \rho_\tau^{w|n} X_{t-1}^w + \beta_\tau^{w|n} X_t^n + \gamma_\tau^{\prime w|n} Z_{t-1} + \varepsilon_{t,\tau}^{w|n} \tag{2.4}$$

where ε_t is quantile error terms and Z_t is the predictive variables. From (2.2), the risk contribution of an institution n to financial system is then given by

$$\triangle CoVaR_{t,\tau}^n = \beta_\tau^{w|n} \left(VaR_{t,\tau}^n - VaR_{t,50\%}^n \right) \tag{2.5}$$

where $VaR_{t,\tau}^n = \alpha_\tau^n + \rho_\tau^n X_{t-1}^n + \gamma_\tau^{\prime n} Z_{t-1}$ is estimated from (2.3) and $\beta_\tau^{w|n}$ is estimated from (2.4). In this framework, the existence of risk spillovers is captured through the parameter $\beta_\tau^{w|n}$: for non-zero values of this parameter, the left tail of the system distribution can be predicted by observing the predetermined distribution of an institution's returns.

2.2 Markov-Switching CoVaR

To address the vulnerability of CoVaR to regime shifts and the requirement of stress-testing of an institution in a hypothetically stressed scenario, i.e., a deep economic recession or asset price downturn, this section defines the Markov-switching CoVaR measure of systemic risk to identify distinct risk states as CoVaR subject to regime changes.

Let $\{s_t\}$ be an ergodic homogeneous Markov chain on a finite set $K = \{1, ..., k\}$ with a transition matrix P defined by the following transition probabilities

$$\{p_{ij} = Pr\left(s_t = j | s_{t-1} = i\right)\}$$

for $i, j \in K$ and assume s_t follow a first-order Markov chain. Transition probabilities satisfy $\sum_{j \in S} p_{ij} = 1$. In this paper, I define two distinct risk regimes, $K = \{1, 2\}$. Regime 1 ($s_t = 1$) represents a normal risk level which is implied by a good economic state and regime 2 ($s_t = 2$) represents a high risk episode most likely associated with an economic recession or financial crisis. The risk structures are determined by data distributions of each regime over time. Note that economic states, s_t, are unobservable so that switching in s_t is inferred by transition probabilities which are estimated from data.

Suppose that X_t can be observed directly but can only make an inference about the value of s_t based on the observations as of date t. From (2.1), Markov-switching VaR (MSVaR) of an instituion n can be defined as

$$Pr\left(X_t^n \leq VaR_t^n | s_t^n = j\right) = \tau$$

and denoted by $MSVaR_{s_t,\tau}^n$ which represents the value-at-risk level of an institution n in its risk regime j. Accordingly, the VaR of the financial systemic returns conditional on the event $\left\{\mathbb{C}(X_t^n) : X_t^n = MSVaR_{s_t,\tau}^n\right\}$, denoted by $MSCoVaR_{s_t,\tau}^n$, is given by

$$Pr\left(X_t^w \leq CoVaR_t^w | \mathbb{C}\left(X_t^n | s_t^n = i\right), s_t^w = j\right) = \tau$$

Note that the risk states of an institution and the financial system are not necessarily coincided, i.e., $i \neq j$. For instance, a non-systemically important institution being distressed does not cause the same high risk episode to the whole financial system. However, a distressed financial system may indeed cause a high risk episode for a non-systemically important institution.

Apply the definition in (2.2) to obtain an institution n's contribution to systemic risk as

$$\triangle MSCoVaR_{s_t,\tau}^n = MSCoVaR_{s_t,\tau}^n - MSCoVaR_{s_t,\tau}^{n,50\%}$$

In this paper, MSCoVaR is estimated by Markov-Switching quantile autoregressive models (MSQAR), specified as

$$X_t^n = \alpha_{s_t,\tau}^n + \rho_{s_t,\tau}^n X_{t-1}^n + \gamma_{s_t,\tau}^{'n} Z_{t-1} + \varepsilon_{t,\tau}^n \tag{2.6}$$

$$X_t^w = \alpha_{s_t,\tau}^{w|n} + \rho_{s_t,\tau}^{w|n} X_{t-1}^w + \beta_{s_t,\tau}^{w|n} X_t^n + \gamma_{s_t,\tau}^{'w|n} Z_{t-1} + \varepsilon_{t,\tau}^{w|n} \tag{2.7}$$

such that $MSVaR^n_{s_t,\tau} = \alpha^n_{s_t,\tau} + \rho^n_{s_t,\tau} X^n_{t-1} + \gamma'^n_{s_t,\tau} Z_{t-1}$ is estimated from (2.6) and then

$$MSCoVaR^n_{s_t,\tau} = \beta^{w|n}_{s_t,\tau} MSVaR^n_{s_t,\tau}$$

can also be computed based on the estimation results of (2.7). See *Appendix A* for details of the MSQAR model estimation for (2.6) and (2.7).

In this MSCoVaR measure, $\beta^{w|n}_{s_t,\tau}$ depends on risk states. The response of financial system to a negative shock to an institution's balance sheet during a high risk episode ($\beta^{s|i}_{s_t=2,\tau}$), hence, allows to be different from a normal risk period ($\beta^{s|i}_{s_t=1,\tau}$). The set of coefficients estimated from high risk episodes describes the distributional structures of data in economic recessions, crises or extreme events. Therefore, it is suitable to be applied to stress-testing financial instituions in supervisory stress scenario required by Federal Reserve Bank. Note that if no risk regime-switching presents, MSCoVaR is equivalent to CoVaR. In this sense, the CoVaR approach is a special case of the MSCoVaR measure when there is no structural breaks. In this paper, I assume the presentence of distinct economic regimes based on the findings in literature. However, an appropriate approach of testing the number of regimes should be considered in future research.

The new framework of the MSCoVaR approach indeed provides flexibility to test different stress scenarios. For instance,

Scenario(1) An extreme scenario is that the financial system depends on the regimes of systemically important banks. This scenario describes the recent financial crisis as: the financial system is distressed once a systemically important bank is distressed, while the financial system is away from distress only if none of systemically important banks are distressed. Hence, systemic risk contribution might be measured by

$$\triangle MSCoVaR^n_{t,\tau} = \beta^{w|n}_{s_t=2,\tau} \left(MSVaR^n_{s_t=2,\tau} - MSVaR^{n,50\%}_{s_t=1,\tau} \right) \tag{2.8}$$

The first product in the right side of (2.8) is the value-at-risk of financial system conditional on hypothetically assuming both the financial system and the institution

n in their high risk episodes. The second product in (2.8) is the value-at-risk of financial system conditional on normal states of that institution.

Scenario(2) In comparison, assuming current financial system in regime 1, a distressed institution n contributes systemic risk to financial system given by

$$\triangle MSCoVaR^{n}_{t,\tau} = \beta^{w|n}_{s_t=1,\tau}\left(MSVaR^{n}_{s_t=2,\tau} - MSVaR^{n,50\%}_{s_t=1,\tau}\right) \tag{2.9}$$

This scenario implies that the institution n is assumed to be not systemically important. Its high risk state does not cause a distressed financial system. However, it might still accumulate and contribute systemic risk to financial system, especially when herding effects occurring.

Scenario(3) Even during a normal time if a systemically important financial institution reaches its VaR level, it also likely shocks financial system into its high risk episode. Hence, the systemic risk contribution of a distressed institution i can also be measured by

$$\triangle MSCoVaR^{n}_{t,\tau} = \beta^{w|n}_{s_t=2,\tau}\left(MSVaR^{n}_{s_t=1,\tau} - MSVaR^{n,50\%}_{s_t=1,\tau}\right) \tag{2.10}$$

For instance, an institution reaching its risk level during a normal period might be caused by short-term maturity mismatch, while an institution reaching its high risk episode might be caused by the large number of defaults on loans like the recent subprime crisis. Despite that risk during a normal time is less severe than during a high risk period, the highly interconnected banking system, herding effects, and market panic might contagiously amplify these negative impacts on financial system and hence lead to crises by i.e., fire-sales and domino effects, etc.

2.3 Markov-Switching CoES

VaR is not a coherent risk measure due to its nonsubadditivity and does not take distributional aspects within the tail into account. This theoretical issue to some extent makes the CoVaR and MSCoVaR measures of systemic risk invalid. However, the asymmetric Laplace distribution assumption in the MSQAR framework of Liu (2014) provides a convenient solution by obtaining expected shortfall through Monte Carlo simulation based on model estimation results. Expected shortfall computed as conditional tail expectation is a coherent risk measure and considers risks beyond the point of a VaR value. See Artzner et al. (1999).

Using the simulation method in *Appendix A* and the estimation results from (2.6), Markov-switching expected shortfall ($MSES$) for an institution n can be obtained and denoted by $MSES^n_{s_t,\tau}$. Then, conditional on the event $\left\{\mathbb{C}(X^n_t) : X^n_t = MSES^n_{s_t,\tau}\right\}$, an institution n's contribution to systemic risk is given by

$$\triangle MSCoES^n_{s_t,\tau} = MSCoES^n_{s_t,\tau} - MSCoES^{n,50\%}_{s_t,\tau}$$

with $MSCoES^n_{s_t,\tau} = \beta^{w|n}_{s_t,\tau} MSES^n_{s_t,\tau}$. Expected shortfall can also be applied to the three scenarios of measuring systemic risk discussed previously:

$$(1) \quad \triangle MSCoES^n_{t,\tau} = \beta^{w|n}_{s_t=2,\tau}\left(MSES^n_{s_t=2,\tau} - MSES^{n,50\%}_{s_t=1,\tau}\right) \tag{2.11}$$

$$(2) \quad \triangle MSCoES^n_{t,\tau} = \beta^{w|n}_{s_t=1,\tau}\left(MSES^n_{s_t=2,\tau} - MSES^{n,50\%}_{s_t=1,\tau}\right) \tag{2.12}$$

$$(3) \quad \triangle MSCoES^n_{t,\tau} = \beta^{w|n}_{s_t=2,\tau}\left(MSES^n_{s_t=1,\tau} - MSES^{n,50\%}_{s_t=1,\tau}\right) \tag{2.13}$$

3 Stress-testing Commercial Banks

In this section, the MSCoVaR and MSCoES measures of systemic risk are estimated for stress-testing the largest U.S. commercial banks, using the CoVaR measure of systemic risk as the benckmark model. In addition, given the subadditivity property, the Markov-switching expected shortfall is used to construct a banking systemic risk index (BSRI) via value-weighted individual systemic risk contributions for monitoring dynamic systemic risk of the financial system.

3.1 Data

Daily market equity data were taken from The Center for Research in Security Prices (CRSP). The universe of bank holding companies (BHCs) are the stocks corresponding to CRSP SIC codes 6000-6199 and 6712. Daily market data is used to form weekly returns on market-valued total assets of individual banks. Following Adrian and Brunnermeier (2011), a bank market-valued total asset is transformed from book-valued total assets into market-valued total assets by applying market-to-book equity ratios. Then, the financial system return is computed as a value-weighted average on the returns of the universe of banks.[4]

This paper considers the largest U.S. commercial banks since they are the targets of current regulatory efforts and would likely be considered too-big-to-fail by central banks. Table 1 provides a bank list considered for stress-testing in this paper. The ultimate criterion to configure the sample of potentially systemically important banks is the availability of comparable data over a long enough period of time. This sifting criterion rules out some large banks, i.e., HSBC, etc. The resulting sample is formed by a total of the 27 largest BHCs sampled from June 1993 to June 2012 with 1000 weekly observations. Note that this paper estimates the systemic risk contributions of the 27 commercial banks to the financial system, while the financial system is constructed by the universe of financial institutions with the SIC code of 6000-6199 and 6712. Hence, the financial system defined in this paper is equivalently referred to as the banking system hereafter.

[Table 1 about here]

The identification of risk regimes is enhanced by using a set of macro-financial predictive variables that are acknowledged to capture the expected return in financial markets. I choose a small set of predictive variables to avoid over-fitting the data. The predictive variables (Z_t) used in this paper include: (1) the change in the credit spread ($\triangle cs$) between the 10-year Moody's seasoned Baa corporate bond and the 10-year U.S. Treasury bond;

[4]See details in Adrian and Brunnermeier (2011) and Lopez-Espinosa et al. (2012).

(2) The change in the U.S. Treasury bill secondary market 3-month rate ($\triangle 3mtb$); (3) the change in the slope of the yield curve ($\triangle ys$), measured by the yield spread between the U.S. Treasury benchmark 10-year bonds and the U.S. 3-month T-bill rate; (4) liquidity spread (ls), defined as the difference between the 3-month U.S. repo rate and the 3-month T-bill rate; (5) the S&P500 Composite Index return (sp); (6) the volatility Index (vix) of the Chicago Board Options Exchange (CBOE). All these variables are sampled weekly and obtained from CBOE, the Federal Reserve Board's H.15 Release and the Datastream database, respectively.

3.2 Empirical Results

Table 2 reports the results of the MSQAR model estimation with $\tau = 5\%$. Panel A presents the results estimated from (2.6) for individual banks (X_t^n) conditional on predictive variables (Z_{t-1}), and Panel B estimated from (2.7) for the banking system (X_t^w) conditional on a individual bank n (X_t^n) and predictive variables (Z_{t-1}). This table displays the medians of the coefficient estimates, the numerical standard errors in square brackets, and the posterior credible intervals (PCI) in parentheses, across banks.[5]

[Table 2 about here]

In Table 2, the quantile intercepts ($\alpha_{s_t,\tau}$) of both individual banks and the banking system appear to have the non-overlapped PCIs between regimes ($s_t = 1$ and $s_t = 2$). This indicates an effective identification of risk regimes by the label switching restriction. The regime identification is further enhaned by predictive variables: S&P 500 returns, the changes in T-bill rates, market volatility for individual banks; and contemporaneous returns of individual banks, the lagged banking system returns, S&P 500 returns, the change in yield curve, and market volatility for the banking system. These predictive variables have non-overlapped PCIs between regimes.

[5]The detail estimation results of each bank are not reported here to save space, but available upon request. Numerical standard errors are obtained using batch mean method, e.g., Ripley (1987). The posterior credible intervals are computed using the highest posterior probability regions with the 95% credible level.

In addition, the transition probabilities, which have the non-overlapped PCIs between regimes, present a much higher level of the regime persistence during regime 1 than during regime 2. The transition probability of regime 2 at 5% VaR is around 50%, which is much lower than that at median levels around 92%.[6] The explanation to this result is that, compared to a deviation from the median or a normal risk period, whenever an individual bank attains its 5% VaR (tail risk) in a high risk episode, the bank more likely takes measures to resolve the risky situation immediately, i.e., adjusting capital structure to reduce debt levels, implementing more conservative loan policies, etc. Those measures affect the persistence of a high risk episode. Similarly, when the banking system is stressed in a high risk episode, regulators also likely intervene markets by monetary and/or fiscal policies. The scale parameters (ς_{s_t}) imply much higher standard deviations (around 20.85 and 5.988 for individual banks and the banking system, respectively) during regime 2 than those during regime 1 (around 4.447 and 1.532 for individual banks and the banking system, respectively).[7] This result is highly consistent with the findings in literature that financial returns are more volatile during economic recessions and crises than economic good times.

Panel A of Table 2 shows that the predictive variables, including S&P 500, changes in T-bill rates, changes in yield curves and market volatility, which have their PCIs excluded zero values, show the predictability for the VaRs of individual banks. By contrast in Panel B of Table 2, the predictors, including contemporaneous returns of individual banks, the lagged banking system returns, S&P 500 returns, the change in yield curve, and market volatility, which have their PCIs excluded zero values, present the predictability for the VaRs of the banking system. For instance, among these predictors, a widening of yield spreads and spikes in market volatility are generally associated with a larger one-period ahead VaR value, and hence could be used to anticipate higher levels of downside risk. As a result, the conditioning variables considered in the analysis have shown the predictability for financial systemic risk.

[6] The estimation results for $\tau = 50\%$ are not reported in this paper to preserve space, but available upon request.

[7] The implied variance is computed based on the formula provided in *Appendix A*.

Interestingly, S&P 500 returns appear to countercyclically contribute to the systemic risk of the banking system: the negative coefficient of S&P 500 return in regime 1 implies that a stock market boom accumulates tail risks in the banking system, while its positive coefficient in regime 2 provides that the increase in stock market prices recovers tail risks of the banking system. Additionally, the contemporaneous returns of individual banks appear to have a strong positive relationship with system risk. This contemporaneous effect exacerbates the downside risk level of the banking system due to the drop of a bank return. The small numerical standard errors in Table 2 indicate reasonabe model estimation accuracy.

Table 3 reports the VaR and MSVaR values of individual banks (X_t^n) estimated from (2.3) and (2.6) conditional on predictive variables (Z_{t-1}), respectively. MSES values are simulated based on the model estimation results using the approaches in *Appendix A*. Table 3 shows that given 5% probability, the worst possible outcome is MS estimated from $VaR_{t,\tau}$ (around 1,017 basis points) and STT (around 5,695 basis points) estimated from $MSVaR_{s_t=2,\tau}$. On average, $MSVaR_{s_t=2,5\%}$ values are about 800 basis points more riskier than $VaR_{t,5\%}$ results and about 1,200 basis points more riskier than $MSVaR_{s_t=1,5\%}$ results. From the coherent risk measure, $MSES_{s_t=2,5\%}$ and $MSES_{s_t=1,5\%}$ results have about 110 and 120 basis points on average more riskier than $MSVaR_{s_t=2,5\%}$ and $MSVaR_{s_t=1,5\%}$, respectively.

[Table 3 about here]

Note that these estimated values are used in (2.5), (2.8)-(2.10), and (2.11)-(2.13) to compute $\triangle CoVaR$, $\triangle MSCoVaR$, and $\triangle MSCoES$ for measuring systemic risk contributions of individual banks. Due to the clear difference between VaR and $MSVaR$ values in Table 3, this evidence shows that existing VaR methods, which provide the results averaging across different economic regimes, do not well reflect extreme risk scenarios for stress-testing purposes. In contrary, the risk levels obtained from high risk episodes (regime 2) are more suitable for measuring hypothetically distressed contributions under supervisory stress scenarios.

The disparity between regimes can also be observed in Figure 1, which plots the $MSES^n_{s_t,\tau}$ values for the six largest U.S. commercial banks. The solid dark lines are the $MSES^n_{s_t,\tau}$ estimates from regime 1 and the dashed light lines from regime 2. Generally speaking, high risk episodes show higher dynamics and larger volatilities than normal risk periods. The difference between regimes exists over time, and the gap is dramatically enlarged during recessions and financial crises. For instance, the risk level during the recent financial crisis of 2008-2009 is well reflected in regime 2 by showing a deep drop into far left tails.

[Figure 1 about here]

Table 4 reports the systemic risk sensitivities of the banking system as a whole conditional on individual banks. The banks in this table are ranked based on risk sensitivity coefficients ($\beta^{w|n}_{s_t=2,\tau}$). The risk sensitivity coefficients are the important elements for computing systemic risk contributions in (2.5), (2.8)-(2.10), and (2.11)-(2.13). For comparison, this table also includes the estimation results of the QAR model as the benchmark for 1-regime using (2.4).

[Table 4 about here]

The systemic risk sensitivity coefficients in Table 4 show that many individual banks tend to impact the banking system heavier during high risk episodes than during normal risk periods, whereas for some other banks the opposite is true. For instance, the marginal impact of BK on the banking system is 0.414 during high risk episodes much larger than 0.169 during normal risk periods. Different sensitivities across regimes show asymmetric effects of individual banks on the banking system. Generally, it is observed that the systemic risk sensitivity coefficients of $\beta^{w|n}_{s_t=2,\tau}$ are also largerly different from the sensitivity results of 1-regime estimations ($\beta^{w|n}_{t,\tau}$). The higher value of a sensitivity coefficient represents the larger response of the banking system to individual banks' shocks. The negative coefficients of BBT and CMA banks imply that during high risk episodes these banks do not worsen the systemic risk of the banking system, despite that their negative coefficients are small in magnitudes.

Table 5 reports the systemic risk contributions of individual banks to the banking system as a whole. $\triangle MSCoVaR_1$, $\triangle MSCoVaR_2$, $\triangle MSCoVaR_3$, $\triangle MSCoES_1$, $\triangle MSCoES_2$, and $\triangle MSCoES_3$, are computed in each scenario of (2.8)-(2.10) and (2.11)-(2.13), respectively. For comparison, the systemic risk contributions without switching regimes are also computed from (2.5) as benchmarks. The ingredients for computing systemic risk contributions are the systemic risk coefficients ($\beta^{w|n}_{s_t,\tau}$) and individual banks' $MSVaR^n_{s_t,\tau}$ and $MSES^n_{s_t,\tau}$ values. The banks in each scenario are ordered by their values of the systemic risk contributions.

[Table 5 about here]

On average across banks, the systemic risk contribution from scenario (1) is around 131 basis points higher than that measured by the CoVaR approach. In addition, scenario (2) generates the systemic risk contribution to the banking system about 72 basis points on average higher than that measured by the CoVaR approach. These results clearly show empirical evidence of the underestimated systemic risk contributions by the CoVaR approach.

The orders of individual banks' systemic risk contributions are very different between $\triangle MSCoVaR_1$ and $\triangle CoVaR$ measures as well. For instance, the systemic risk contribution of STT is the highest in the $\triangle MSCoVaR_1$ measure, while the highest systemic risk contribution in the $\triangle CoVaR$ measure is the AXP bank. The difference between their contributions is as large as about 827 basis points. A strong negative relationship between systemic risk contributions and bank sizes has also been found through a OLS regression (not reported here). This result indicates that the bigger the bank asset sizes are, the larger the banks impact on the banking system. This result provides quantitative evidence for the recent debate of "too big to fail" of banks.

Apparently, the $\triangle MSCoVaR_1$ measure of systemic risk provides the most extreme stressed outcomes among the 3 scenarios considered. Even in the case that a bank is not systemically important but distressed during high risk episodes (scenario (2)), the average sytemic risk contribution is around 169 basis points which cannot be neglected. The orders of systemic risk contributions also vary across the 3 scenarios.

In addition, Table 5 reports the simulated results of $MSCoES_{s_t,\tau}$. As seen, the sytemic risks are similar between $\triangle MSCoES_1$ and $\triangle MSCoVaR_1$, and between $\triangle MSCoES_2$ and $\triangle MSCoVaR_2$, while the results from scenaior (3) are very different. However, this paper suggests to adopt the systemic risk measurement results of $\triangle MSCoES_{s_t,\tau}$ since $\triangle MSCoVaR$ is not a coherent risk measure.

Figure 2 plots the dynamics of systemic risk contributions measured by $\triangle MSCoVaR_1$ and $\triangle CoVaR$ approaches along with the correlation.[8] The results show that the $\triangle MSCoVaR_1$ measure of systemic risk contributions are more dynamic than the $\triangle CoVaR$ measure. Some banks, i.e., JPMorgan Chase, Citi Financial Group and Morgan Stanley, etc., appear to have high correlations (about 83%-95%) between $\triangle MSCoVaR_1$ and $\triangle CoVaR$, while other banks, i.e., Bank of America, Well Fargo, etc., have correlations below 50%. Furthermore, $\triangle MSCoVaR_1$ and $\triangle CoVaR$ are negatively correlated for the bank of USB. These results show that systemic risk contributions measured by $\triangle MSCoVaR_1$ and $\triangle CoVaR$ are not only different in magnitudes, but also in the dynamics over sample periods.

[Figure 2 about here]

Table 6 reports the correlation matrix for banks' systemic risk contributions measured by $\triangle MSCoVaR_1$. The correlation matrix shows that banks are highly interconnected. For instance, Bank of America is positively correlated with other banks ranging from 75%-95%. Bank of America has the highest correlation of 96% with JPMorgan Chase bank. Among all the banks sampled, BBT, CMA and SCHW are the only banks negatively correlated with other banks. Table 6 shows that the potential contagious channels of a crisis are hidden behind the high interconnections between banks.

[8] Instead of $\triangle MSCoES_1$ and $\triangle CoES$, this paper makes the comparison between $\triangle MSCoVaR_1$ and $\triangle CoVaR$, because Adrian and Brunnermier (2011) approach cannot be used to compute expected shortfall.

3.3 Banking Systemic Risk Index

Figure 3 plots the quarterly systemic risk index of the banking sector (BSRI). The solid line is the quarterly Financial Stress Index constructed by Federal Reserve Bank of St. Louis (STLFSI). The dashed line is quarterly BSRI constructed by the value-weighted $\triangle MSCoES_1$ on individual banks as

$$BSRI_t = -\sum_{n=1}^{N} w_t^n \triangle MSCoES_{t,\tau}^n$$

where weekly $\triangle MSCoVaR_{t,\tau}^n$ is aggregated to quarterly frequency and w_t^n is the bank n's weight based on its market capitalization at time t. The shaded areas are NBER-dated business cycle phases. Figure 3 shows that the constructed systemic risk index for the banking sector is capable of reproducing the recent economic recession. The quarterly BSRI reaches the highest risk during the recent financial crisis of 2007-2009. The BSRI also shows a milder risk increase than STLFSI for the economic recession during the IT Bubble Bust period since it is not a recession highly related to the banking sector. Figure 3 presents a positive 61.5% comovement between the BSRI and the STLFSI. Furthermore, a simple linear regression shows that the BSRI is able to significantly explain the dynamics of the Financial Stress Index by 37.83% (R^2). Hence, the constructed BSRI index is supplementary to monitoring financial market risks by very specific to the risk nature of the banking sector.

[Figure 3 about here]

4 Conclusion

This paper has defined a Markov-switching conditional Value-at-Risk (MSCoVaR) approach to measure systemic risk of commercial banks. Applying the Markov-Switching Quantile Autoregression framework of Liu (2014), systemic risks are estimated subject to regime shifts within tails. The new method presents the advantage and flexibility in supervisory stress scenarios required by Federal Reserve Bank. I estimated systemic risk contributions of the U.S. largest commercial banks and found around 131 basis points of the underestimated asset loss by the existing CoVaR measure of systemic risk. The banking system is more sensitive to marginal changes of an individual bank during high risk episodes than during normal risk periods. In addition, systemic risk contributions of individual banks are highly interconnected. Furthermore, Banking Systemic Risk Index, constructed in this paper by value-weighted individual systemic risk contributions, presents not only a high relavence to trace financial distress situtions, but also very specific to the risk nature of the banking industry.

References

[1] Adams, Z., R. Fuss and R. Gropp (2011) Spillover effects among financial institutions: a state-dependent sensitivity Value-at-Risk (SDSVaR) Approach. Working Paper

[2] Adrian, T. and M.K. Brunnermeier (2011) CoVaR. NBER Working Paper No. 17454

[3] Arias, M., J.C. Mendoza and D. Perez-Reyna (2010) Applying CoVaR to measure systemic market risk: the Colombian case. Working paper

[4] Artzner, P., F. Delbaen, J. Eber and D. Heath (1999) Coherent Measures of Risk. Mathematical Finance 9(3): 203-228

[5] Bisias, D., M. Flood, A. W. Lo and S. Valavanis (2012) A survey of systemic risk analytics. The Annual Review of Financial Economics 4: 255-96

[6] Brunnermeier, M. and M. Oehmke (2012) Bubbles, Financial Crises and Systemic Risk. NBER Working Paper No. 18398

[7] Brunnermeier, M.K., G. Dong and D. Palia (2012) Banks' Non-Interest Income and Systemic risk. Working Paper

[8] Girardi, G. and A.T. Ergun (2012) Systemic risk measurement: Multivariate GARCH estimation of CoVaR. Working paper

[9] Hamilton, J.D. (1994) Time Series Analysis. Princeton University Press

[10] Koenker, R. and Z. Xiao (2006) Quantile Autoregression. Journal of the American Statistical Association 101(475): 980-990

[11] Liu, X. (2014) Markov-Switching Quantile Autoregression. Working Paper

[12] Lopez-Espinosa, G., A. Moreno, A. Rubia, and L. Valderrama (2012) Short-term wholesale funding and systemic risk: A global CoVaR approach. Journal of Banking & Finance 36: 3150-3162

[13] Ripley, B. (1987) Stochastic Simulation. John Wiley, New York

[14] Rodriguez-Moreno, M. and J. I. Pena (2012) Systemic risk measures: The simpler the better? Journal of Banking & Finance, forthcoming

[15] Roengiptya, R. and P. Rungcharoenkitkul (2011) Measuring Systemic Risk and Financial Linkages in the Thai Banking System. Discussion paper 02/2010. Bank of Thailand.

[16] Van Oordt, M. and C. Zhou (2010) Systematic Risk under Adverse Market Conditions. Working paper. De Nederlandsche Bank.

[17] Wong, A. and T. Fong (2010) Analysing Interconnectivity among Economies. Hong Kong Monetary Authority Working Paper 03/2010

[18] Yu, K. and J. Zhang (2005) A Three-Parameter Asymmetric Laplace Distribution and Its Extension. Communications in Statistics- Theory and Methods 34: 1867-1879

Table 1

The Sample List of the U.S. Largest Commercial Banks as of 06/30/2012 Ranked in Total Assets

Institution Name	Ticker	Total Assets in thousand dollars as of 06/30/2012
JPMORGAN CHASE & CO.	JPM	$2,290,146,000
BANK OF AMERICA CORPORATION	BAC	$2,162,083,396
CITIGROUP INC.	C	$1,916,451,000
WELLS FARGO & COMPANY	WFC	$1,336,204,000
MORGAN STANLEY	MS	$748,517,000
U.S. BANCORP	USB	$353,136,000
BANK OF NEW YORK MELLON CORPORATION, THE	BK	$330,490,000
PNC FINANCIAL SERVICES GROUP, INC., THE	PNC	$299,712,018
STATE STREET CORPORATION	STT	$200,368,976
BB&T CORPORATION	BBT	$178,560,000
SUNTRUST BANKS, INC.	STI	$178,307,292
AMERICAN EXPRESS COMPANY	AXP	$146,890,000
REGIONS FINANCIAL CORPORATION	RF	$122,344,664
FIFTH THIRD BANCORP	FITB	$117,542,579
CHARLES SCHWAB CORPORATION	SCHW	$111,816,000
NORTHERN TRUST CORPORATION	NTRS	$94,455,895
KEYCORP	KEY	$86,741,424
M&T BANK CORPORATION	MTB	$80,807,578
BBVA USA BANCSHARES, INC.	BBVA	$66,013,042
COMERICA INCORPORATED	CMA	$62,756,597
HUNTINGTON BANCSHARES INCORPORATED	HBAN	$56,622,959
ZIONS BANCORPORATION	ZION	$53,418,819
POPULAR, INC.	BPOP	$36,612,000
PEOPLE'S UNITED FINANCIAL, INC.	PBCT	$28,134,752
SYNOVUS FINANCIAL CORP.	SNV	$26,294,110
BOK FINANCIAL CORPORATION	BOKF	$25,561,731
FIRST HORIZON NATIONAL CORPORATION	FHN	$25,493,925

Note: The composition of the banks is based on consolidated assets, lagged by one quarter.

Table 2
MSQAR model estimation results

Panel A	p_{ii}^{n}	$\alpha_{s_t,\tau}$	X_{t-1}^{n}	sp	$\triangle 3mtb$	$\triangle ys$	$\triangle cs$	ls	vix	$\varsigma_{s_t}^{n}$
$s_{t,\tau}=1$	0.862	-2.418	-0.105	0.441	-1.740	-0.168	0.008	-0.893	-0.040	0.222
	[0.028]	[0.143]	[0.044]	[0.057]	[0.301]	[0.086]	[0.056]	[0.184]	[0.029]	[0.026]
	(0.823,0.900)	(-2.927,-1.944)	(-0.156,-0.055)	(0.350,0.531)	(-2.523,-0.029)	(-0.355,0.016)	(-0.071,0.096)	(-1.657,-0.076)	(-0.064,-0.018)	(0.203,0.242)
$s_{t,\tau}=2$	0.499	-10.70	-0.170	0.991	-5.404	-1.340	0.179	-1.723	-0.229	1.041
	[0.050]	[0.336]	[0.086]	[0.144]	[0.384]	[0.238]	[0.143]	[0.347]	[0.067]	[0.072]
	(0.370,0.627)	(-13.04,-8.427)	(-0.368,-0.035)	(0.517,1.582)	(-8.245,-2.461)	(-2.620,-0.146)	(-0.347,0.708)	(-4.119,0.630)	(-3.58,-0.113)	(0.870,1.222)

Panel B	$p_{ii}^{w\|n}$	$\alpha_{s_t,\tau}^{w\|n}$	X_{t}^{n}	X_{t-1}^{w}	sp	$\triangle 3mtb$	$\triangle ys$	$\triangle cs$	ls	vix	$\varsigma_{s_t}^{w\|n}$
$s_{t,\tau}=1$	0.852	-0.695	0.125	0.085	-0.109	0.238	-0.115	-0.006	-0.191	-0.026	0.077
	[0.028]	[0.080]	[0.023]	[0.043]	[0.037]	[0.172]	[0.051]	[0.033]	[0.099]	[0.017]	[0.015]
	(0.813,0.889)	(-0.850,-0.537)	(0.110,0.140)	(0.037,0.131)	(-0.145,-0.031)	(-0.324,0.827)	(-0.179,-0.050)	(-0.036,0.023)	(-0.424,0.031)	(-0.034,-0.019)	(0.070,0.083)
$s_{t,\tau}=2$	0.469	-2.824	0.164	-0.073	0.201	1.153	-0.669	0.026	0.033	-0.177	0.299
	[0.048]	[0.187]	[0.043]	[0.072]	[0.081]	[0.236]	[0.142]	[0.067]	[0.195]	[0.049]	[0.040]
	(0.350,0.589)	(-3.515,-2.129)	(0.136,0.212)	(-0.204,-0.023)	(0.042,0.361)	(0.048,2.173)	(-1.102,-0.249)	(-0.085,0.139)	(-0.710,0.762)	(-0.238,-0.123)	(0.254,0.347)

Note: Panel A reports the results estimated from (2.6)for individual banks (X_t^n) conditional on predictive variables. Panel B reports the results estimated from (2.7) for financial system (X_t^w) conditional on individual banks (X_t^n) and predictive variables. This table displays the medians of the coefficient estimates, the medians of the numerical standard errors in square brackets for evaluating the estimation accuracy, and the medians of the posterior credible intervals in parenthesis. Numerical standard errors are obtained using batch mean method, e.g., Refly (1987). The posterior credible intervals are computed using the highest posterior probability regions with the 95% confidence level. The detail estimation results for each bank are not reported here to save space, but available upon request. The results this table is for $\tau = 5\%$.

Table 3
VaR, MSVaR and MSES estimates of individual banks

	$VaR_{t,5\%}$	$MSVaR$		$MSES$	
		$MSVaR_{s_t=1,5\%}$	$MSVaR_{s_t=2,5\%}$	$MSES_{s_t=1,5\%}$	$MSES_{s_t=2,5\%}$
JPM	-6.286	-2.642	-9.204	-2.835	-9.760
BAC	-7.324	-2.919	-12.21	-3.141	-13.14
C	-9.446	-4.711	-35.62	-5.038	-37.56
WFC	-5.813	-2.096	-9.047	-2.269	-9.699
MS	-10.17	-5.384	-18.45	-5.750	-20.12
USB	-6.670	-4.141	-22.91	-4.419	-25.01
BK	-6.069	-2.482	-9.572	-2.687	-10.10
PNC	-6.172	-3.105	-10.07	-3.325	-10.79
STT	-7.273	-5.403	-56.95	-5.761	-59.75
BBT	-6.207	-2.497	-10.16	-2.686	-11.00
STI	-6.804	-2.530	-10.89	-2.712	-11.72
AXP	-5.664	-2.892	-9.574	-3.105	-10.10
RF	-9.327	-3.521	-16.27	-3.757	-17.40
FITB	-7.176	-3.360	-15.21	-3.598	-16.19
SCHW	-9.652	-5.767	-21.68	-6.158	-23.06
NTRS	-5.382	-1.600	-7.555	-1.759	-7.979
KEY	-6.730	-3.047	-12.03	-3.256	-12.83
MTB	-5.212	-2.711	-11.91	-2.914	-14.17
BBVA	-8.094	-3.991	-15.38	-4.277	-17.28
CMA	-7.454	-2.990	-11.64	-3.220	-12.44
HBAN	-7.482	-2.784	-11.81	-2.990	-12.82
ZION	-7.620	-2.709	-12.49	-2.914	-13.45
BPOP	-8.345	-2.543	-12.57	-2.729	-13.42
PBCT	-5.087	-2.865	-9.344	-3.084	-9.881
SNV	-8.161	-2.821	-12.937	-3.039	-13.80
BOKF	-5.360	-2.532	-9.309	-2.731	-9.876
FHN	-7.372	-2.421	-12.18	-2.623	-13.32

The entries are VaR and MSVaR values of individual banks (X_t^n) estimated from (2.3) and (2.6) conditional on predictive variables (Z_{t-1}), respectively. $MSES$ values are simulated based on the model estimation results using the approaches in Appendix A. The values are ordered by banks' total asset values.

Table 4
Systemic risk sensitivities

Banks	$\beta^{w\|n}_{5\%}$	$\beta^{w\|n}_{s_t=1,5\%}$	$\beta^{w\|n}_{s_t=2,5\%}$
BK	0.227	0.169	0.414
NTRS	0.208	0.178	0.374
AXP	0.272	0.213	0.359
BOKF	0.111	0.087	0.279
WFC	0.222	0.207	0.256
USB	0.185	0.116	0.238
PNC	0.209	0.196	0.231
PBCT	0.178	0.080	0.231
JPM	0.195	0.221	0.228
KEY	0.193	0.146	0.224
BBVA	0.096	0.039	0.214
STI	0.158	0.222	0.194
SNV	0.107	0.163	0.184
MTB	0.170	0.119	0.179
STT	0.152	0.071	0.173
BAC	0.143	0.197	0.147
RF	0.136	0.106	0.119
MS	0.090	0.047	0.118
ZION	0.096	0.111	0.113
BPOP	0.016	0.009	0.097
HBAN	0.068	0.162	0.058
FITB	0.112	0.123	0.055
FHN	0.115	0.059	0.049
C	0.040	0.085	0.023
SCHW	0.072	0.019	0.011
BBT	0.136	0.122	-0.048
CMA	0.129	0.110	-0.075

β_τ and $\beta_{s_t,\tau}$ are estimated from QAR and MSQAR models on (2.4) and (2.7), respectively. The banks in this table are ranked based on the risk sensitivity coefficients ($\beta^{w|n}_{s_t=2,5\%}$).

Table 5
Systemic Risk Contribution of each bank to financial system

1-regime		2-regime											
Banks	$\triangle CoVaR$	Banks	$\triangle MSCoVaR_1$	Banks	$\triangle MSCoVaR_2$	Banks	$\triangle MSCoVaR_3$	Banks	$\triangle MSCoES_1$	Banks	$\triangle MSCoES_2$	Banks	$\triangle MSCoES_3$
AXP	-1.617	STT	-9.886	STT	-4.064	AXP	-1.134	STT	-9.857	STT	-4.052	USB	-0.636
BK	-1.413	USB	-5.542	C	-3.037	BK	-1.096	USB	-5.528	C	-2.960	STT	-0.537
KEY	-1.339	BK	-4.029	USB	-2.713	USB	-1.084	BBVA	-3.241	USB	-2.705	BBVA	-0.461
WFC	-1.336	AXP	-3.533	BAC	-2.492	STT	-0.987	BK	-3.174	STI	-2.288	AXP	-0.435
PNC	-1.330	BBVA	-3.384	STI	-2.491	BBVA	-0.951	AXP	-2.947	BAC	-2.215	KEY	-0.327
RF	-1.284	NTRS	-3.116	SNV	-2.192	PNC	-0.941	KEY	-2.470	SNV	-1.974	MS	-0.316
USB	-1.272	KEY	-2.752	PNC	-2.162	NTRS	-0.887	BOKF	-2.308	JPM	-1.765	BOKF	-0.313
JPM	-1.248	BOKF	-2.691	JPM	-2.110	PBCT	-0.829	MTB	-2.252	HBAN	-1.748	PNC	-0.264
NTRS	-1.166	PNC	-2.550	AXP	-2.094	BOKF	-0.800	NTRS	-2.237	AXP	-1.747	SNV	-0.254
STT	-1.118	SNV	-2.476	HBAN	-1.933	KEY	-0.741	SNV	-2.230	FITB	-1.727	STI	-0.252
STI	-1.092	WFC	-2.373	WFC	-1.921	JPM	-0.684	WFC	-2.035	PNC	-1.686	RF	-0.246
BAC	-1.071	PBCT	-2.323	FITB	-1.898	MS	-0.664	MS	-1.996	RF	-1.664	JPM	-0.245
CMA	-0.985	MS	-2.202	KEY	-1.794	SNV	-0.619	STI	-1.996	WFC	-1.647	MTB	-0.243
PBCT	-0.948	JPM	-2.178	RF	-1.763	WFC	-0.595	PNC	-1.989	KEY	-1.611	BAC	-0.180
SNV	-0.914	MTB	-2.174	BK	-1.642	STI	-0.554	RF	-1.865	MTB	-1.497	ZION	-0.138
MS	-0.905	STI	-2.173	NTRS	-1.480	MTB	-0.532	JPM	-1.822	ZION	-1.309	WFC	-0.135
MTB	-0.904	RF	-1.976	MTB	-1.445	BAC	-0.491	PBCT	-1.670	BK	-1.293	BPOP	-0.118
BBT	-0.872	BAC	-1.857	ZION	-1.425	RF	-0.462	BAC	-1.651	BBT	-1.163	BK	-0.107
FHN	-0.857	ZION	-1.443	CMA	-1.327	ZION	-0.342	ZION	-1.325	CMA	-1.161	PBCT	-0.103
FITB	-0.819	BPOP	-1.253	BBT	-1.306	BPOP	-0.284	BPOP	-1.151	NTRS	-1.062	FITB	-0.080
BBVA	-0.805	FITB	-0.853	MS	-0.873	FITB	-0.198	C	-0.805	MS	-0.792	HBAN	-0.056
ZION	-0.747	C	-0.826	BOKF	-0.836	HBAN	-0.169	FITB	-0.777	BOKF	-0.717	C	-0.055
SCHW	-0.681	HBAN	-0.691	PBCT	-0.808	FHN	-0.137	HBAN	-0.625	FHN	-0.666	FHN	-0.031
BOKF	-0.621	FHN	-0.616	FHN	-0.739	C	-0.113	FHN	-0.555	BBVA	-0.593	SCHW	-0.026
HBAN	-0.507	SCHW	-0.235	BBVA	-0.619	SCHW	-0.062	SCHW	-0.210	PBCT	-0.581	BBT	0.058
C	-0.367	BBT	0.515	SCHW	-0.407	BBT	0.145	BBT	0.459	SCHW	-0.364	NTRS	0.092
BPOP	-0.134	CMA	0.906	BPOP	-0.120	CMA	0.256	CMA	0.792	BPOP	-0.110	CMA	0.100

Note: the systemic risk contributions are measured for 2-regimes using $MSCoVaR$ approach and for 1-regime using $CoVaR$ method. The banks are ranked based on each systemic risk measure.

Table 6
Correlation matrix of banks' systemic risk contributions measured by $\triangle MSCoVaR_1$

	BAC	BBT	BBVA	BK	BOKF	BPOP	C	CMA	FHN	FITB	HBAN	JPM	KEY	MS	MTB	NTRS	PBCT	PNC	RF	SCHW	SNV	STI	STT	USB	WFC	ZION
AXP	0.55	-0.25	0.42	0.06	0.66	0.19	0.31	-0.16	0.35	0.23	0.29	0.49	0.39	0.17	0.07	0.33	0.27	0.30	0.12	0.13	0.21	0.29	0.12	0.17	0.23	0.47
BAC	1.00	-0.66	0.57	0.74	0.74	0.72	0.83	-0.76	0.76	0.89	0.90	0.96	0.83	0.85	0.43	0.89	0.55	0.88	0.63	-0.34	0.67	0.90	-0.09	0.28	0.87	0.72
BBT		1.00	-0.84	-0.60	-0.63	-0.78	-0.37	0.87	-0.65	-0.68	-0.83	-0.57	-0.92	-0.66	-0.83	-0.78	-0.28	-0.87	-0.88	0.50	-0.81	-0.73	-0.46	-0.59	-0.68	-0.65
BBVA			1.00	0.27	0.75	0.46	0.35	0.74	0.58	0.55	0.65	0.43	0.79	0.41	0.76	0.60	0.25	0.70	0.64	-0.36	0.60	0.57	0.49	0.56	0.45	0.52
BK				1.00	0.46	0.84	0.70	-0.70	0.58	0.87	0.87	0.81	0.75	0.88	0.49	0.87	0.57	0.82	0.80	-0.49	0.78	0.91	-0.08	0.31	0.88	0.66
BOKF					1.00	0.55	0.58	-0.57	0.49	0.65	0.68	0.68	0.78	0.43	0.50	0.67	0.54	0.72	0.59	-0.29	0.68	0.72	0.34	0.56	0.59	0.78
BPOP						1.00	0.47	-0.77	0.54	0.73	0.88	0.74	0.89	0.76	0.55	0.81	0.39	0.90	0.90	-0.50	0.93	0.85	0.31	0.58	0.89	0.84
C							1.00	-0.61	0.62	0.89	0.75	0.85	0.57	0.77	0.29	0.75	0.62	0.70	0.42	0.35	0.46	0.83	-0.38	0.07	0.75	0.46
CMA								1.00	-0.62	-0.85	-0.9	-0.66	-0.89	-0.76	-0.75	-0.79	-0.45	-0.93	-0.85	0.68	-0.78	-0.85	-0.27	-0.56	-0.84	-0.65
FHN									1.00	0.71	0.78	0.77	0.69	0.85	0.49	0.88	0.17	0.69	0.56	-0.03	0.47	0.69	-0.23	0.02	0.58	0.31
FITB										1.00	0.94	0.88	0.81	0.89	0.58	0.91	0.64	0.92	0.73	-0.54	0.72	0.97	-0.12	0.31	0.91	0.63
HBAN											1.00	0.89	0.94	0.91	0.64	0.96	0.48	0.98	0.84	-0.49	0.85	0.97	0.07	0.42	0.93	0.73
JPM												1.00	0.77	0.88	0.33	0.91	0.52	0.83	0.59	-0.25	0.66	0.91	-0.21	0.17	0.87	0.69
KEY													1.00	0.75	0.73	0.88	0.44	0.97	0.91	-0.51	0.91	0.89	0.38	0.63	0.85	0.84
MS														1.00	0.50	0.94	0.40	0.84	0.69	-0.33	0.63	0.88	-0.28	0.10	0.85	0.48
MTB															1.00	0.62	0.32	0.70	0.79	-0.54	0.62	0.58	0.43	0.53	0.46	0.43
NTRS																1.00	0.47	0.92	0.80	-0.33	0.76	0.93	-0.06	0.29	0.84	0.63
PBCT																	1.00	0.53	0.43	-0.50	0.40	0.61	-0.01	0.31	0.53	0.53
PNC																		1.00	0.89	-0.58	0.89	0.96	0.22	0.55	0.93	0.80
RF																			1.00	-0.59	0.89	0.80	0.44	0.67	0.76	0.74
SCHW																				1.00	-0.55	-0.53	-0.35	-0.55	-0.57	-0.49
SNV																					1.00	0.84	0.47	0.71	0.84	0.86
STI																						1.00	0.02	0.42	0.95	0.76
STT																							1.00	0.81	0.05	0.49
USB																								1.00	0.43	0.71
WFC																									1.00	0.79

Figure 1
Plots of $MSES^n_{s_t,\tau}$ for a subset of the sample banks. The solid dark lines are the estimated $MSES^n_{s_t,\tau}$ values for $s_t = 1$, and the dash light lines for $s_t = 2$, respectively.

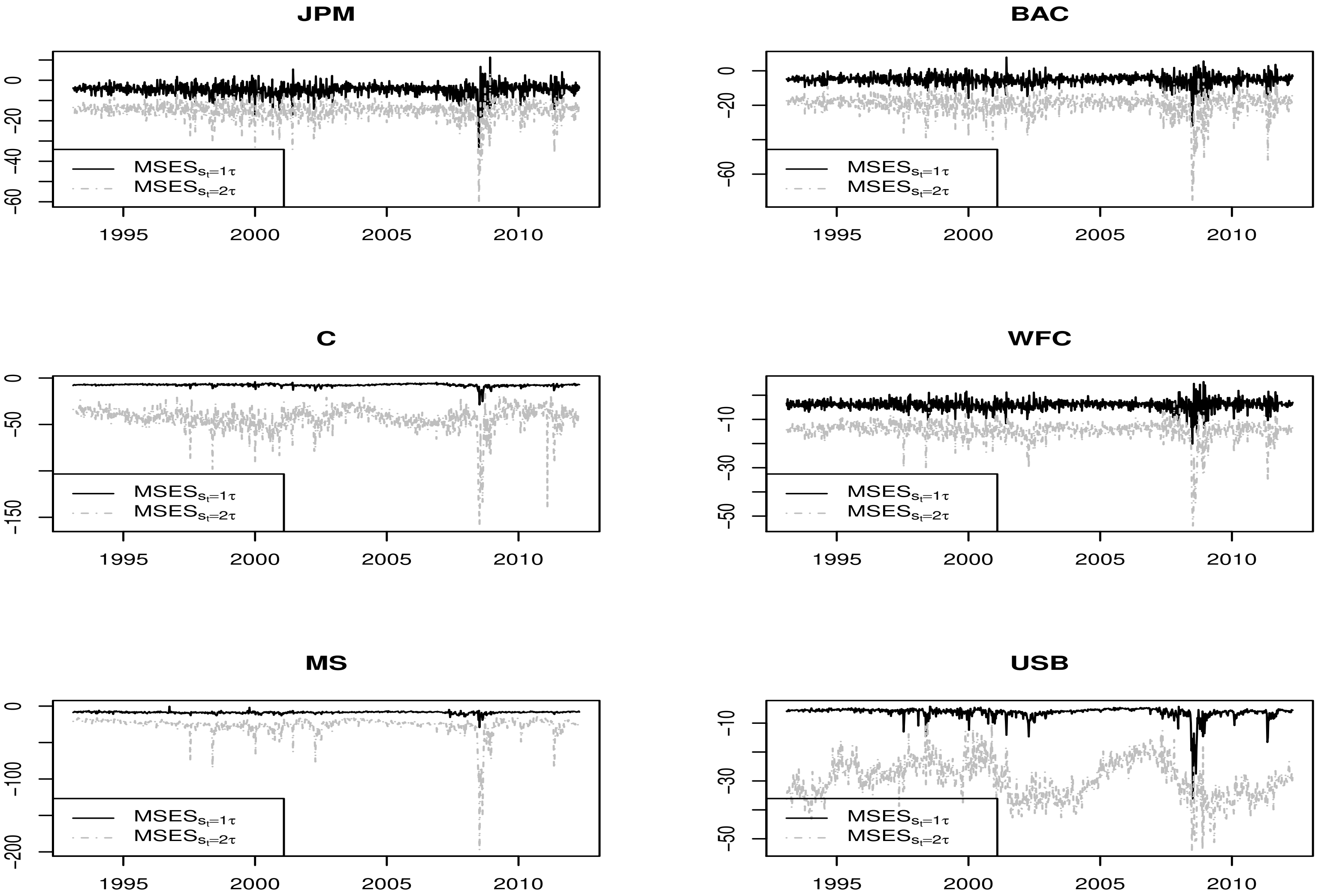


Figure 2

Plots of systemic risk contributions for a subset of the sample banks: $\triangle MSCoVaR_1$ vs $\triangle CoVaR$

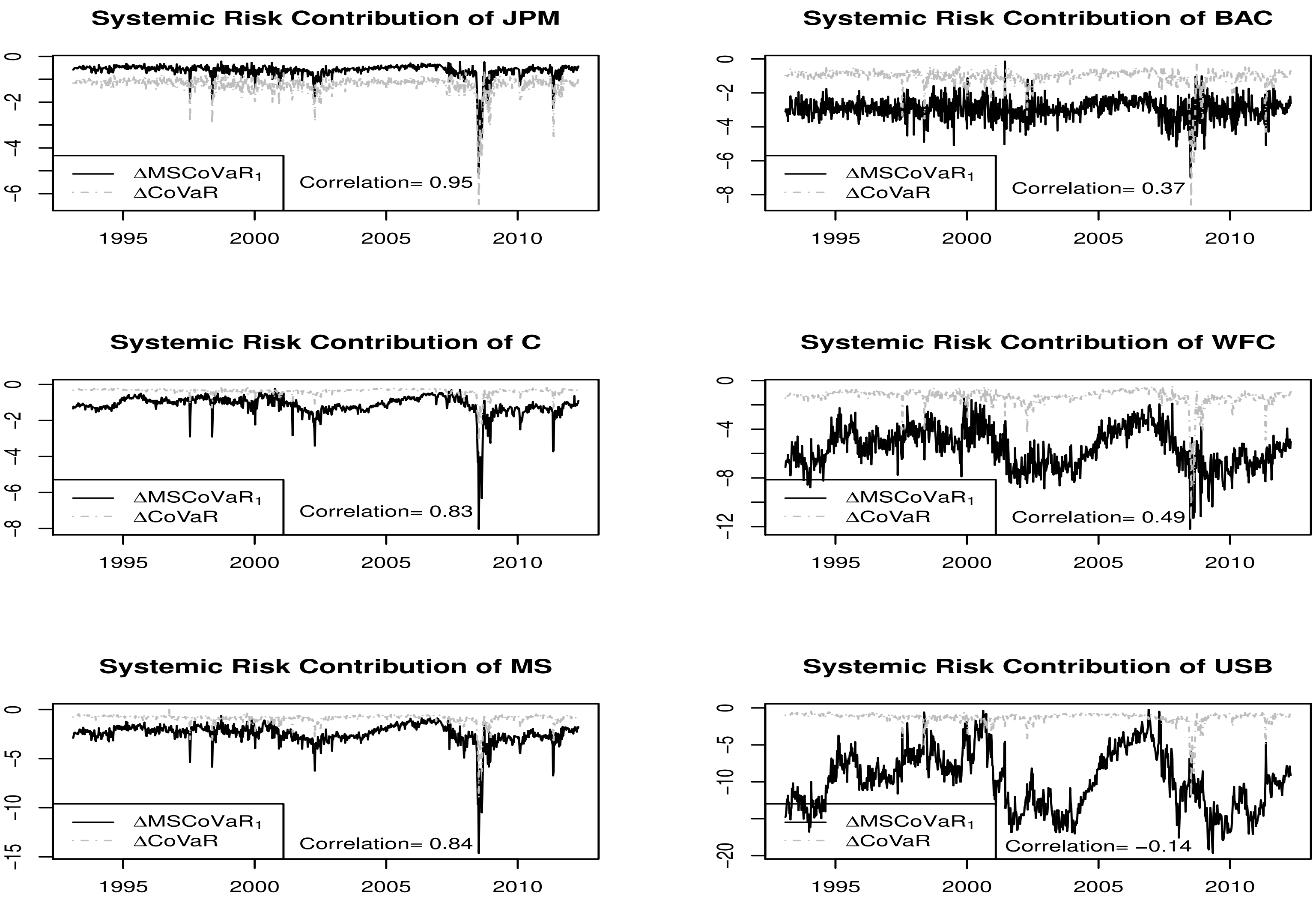
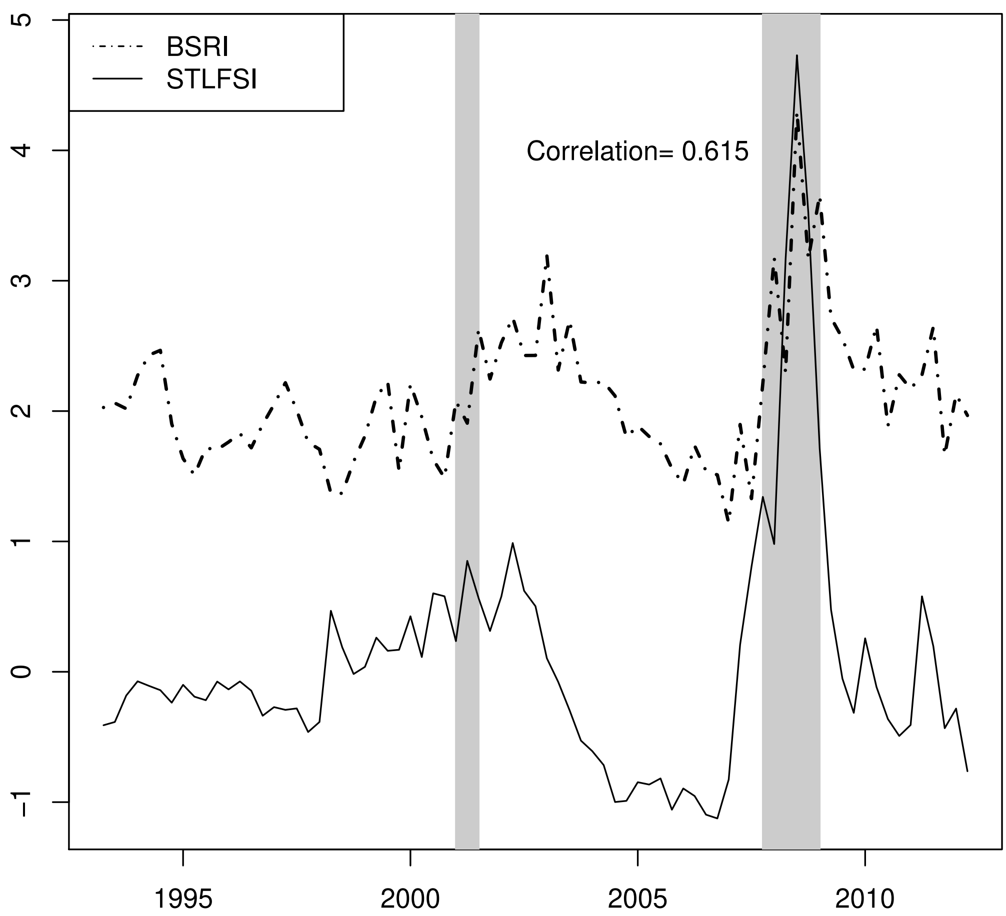


Figure 3
Banking Systemic Risk Index (BSRI). The solid line is the financial stress index constructed by Federal Reserve Bank of St. Louis and the dashed line is BSRI constructed by the value-weighted $\triangle MSCoES_1$ on individual banks.



A The Markov-Switching Quantile Autoregressive Model Estimation

This appendix briefly describes the estimation method of Markov-Switching Quantile Autoregressive models as proposed in Liu (2014), with an extension of adding exogenous variables. For more details on the model, see the author's original work.

Rewrite (2.6) and (2.7) in a general MSQAR model form as follows

$$X_t = \theta_{s_t,0}(\tau) + \sum_{l=1}^{L} \theta_{s_t,l}(\tau) X_{t-l} + \sum_{r=1}^{R} \delta'_{s_t,r}(\tau) Z_{t-r} + \varepsilon_t(\tau)$$

with the τth quantile of X_t given by

$$Q_{X_t}\left(\tau | \boldsymbol{X}_{t-1}, \boldsymbol{Z}_{t-1}, \boldsymbol{\theta}_{s_t}\right) = \theta_{s_t,0}(\tau) + \sum_{l=1}^{L} \theta_{s_t,l}(\tau) X_{t-l} + \sum_{r=1}^{R} \delta'_{s_t,r}(\tau) Z_{t-r}$$

where $\boldsymbol{\theta}_{s_t}(\tau) = \{\theta_{s_t,0}(\tau), ..., \theta_{s_t,L}(\tau), \delta'_{s_t,1}(\tau), ..., \delta'_{s_t,R}(\tau)\}$, $\boldsymbol{X}_{t-1} = \{X_{t-1}, ..., X_{t-l}\}$ and $\boldsymbol{Z}_{t-1} = \{Z_{t-1}, ..., Z_{t-R}\}$. Assume quantile error terms, $\varepsilon_t(\tau)$, follow a three parameter asymmetric Laplace distribution of Yu and Zhang (2005), $ALD(0, \varsigma, \tau)$, with the density function given by

$$\begin{aligned} f(\varepsilon; 0, \varsigma, \tau, \boldsymbol{\theta}_{s_t}) &= \frac{\tau(1-\tau)}{\varsigma_{s_t}} \\ & exp\left\{ -\frac{\left(X_t - Q_{y_t}\left(\tau|\boldsymbol{X}_{t-1}, \boldsymbol{Z}_{t-1}, \boldsymbol{\theta}_{s_t}\right)\right)\left(\tau - I\left(X_t \leq Q_{y_t}\left(\tau|\boldsymbol{X}_{t-1}, \boldsymbol{Z}_{t-1}, \boldsymbol{\theta}_{s_t}\right)\right)\right)}{\varsigma_{s_t}} \right\} \end{aligned} \quad (A.1)$$

where $I(\cdot)$ is an indicator function. τ determines the skewness of the distribution, $\varsigma > 0$ is a scale parameter. $ALD(0, \varsigma, \tau)$ with the location parameter being zero provides that the τth quantile of the distribution is zero as $Pr\left(\varepsilon_t \leq 0\right) = \tau$, which satisfies the quantile regression condition $\int_{-\infty}^{0} f_\varepsilon(q)dq = \tau$. The asymmetric-Laplace distribution with the density function of (A.1) has the mean and variance, $E(\varepsilon_t) = \varsigma(1-2\tau)/[(1-\tau)\tau]$ and $Var(\varepsilon_t) = \varsigma^2(1 - 2\tau + 2\tau^2)/[(1-\tau)^2\tau^2]$, respectively. See Yu and Zhang (2005) for details.

Suppose that X_t can be observed directly but can only make an inference about the value of s_t based on the observations as of date t. The inference for unobservable states is based on the filtering probability as

$$\begin{aligned} \xi_{j,t|t} &= Pr\left(s_t = j | \mathbf{X}_t, \boldsymbol{Z}_t; \boldsymbol{\Theta}\right) \\ &= \sum_{i \in K} Pr\left(s_t = j, s_{t-1} = i | \mathbf{X}_t, \boldsymbol{Z}_t; \boldsymbol{\Theta}\right) \end{aligned}$$

where $\sum_{j\in K} \xi_{j,t|t} = 1$ and $\boldsymbol{\Theta} = (P, \boldsymbol{\theta}_{s_t}(\tau))$ is a vector of the parameters with $s_t \in K$. The formulation of filtering probabilities is obtained by Bayes theorem as

$$\xi_{j,t|t} = \frac{\sum_{i\in K} p_{ij} \xi_{i,t-1|t-1} \eta_{j,t}}{f\left(X_t | \mathbf{X}_{t-1}, \boldsymbol{Z}_{t-1}, \tau; \boldsymbol{\Theta}\right)} \quad (A.2)$$

where $\eta_{j,t}$ is the conditional density in (A.1) given $s_t = j$, and

$$f(X_t|\mathbf{X}_t, \boldsymbol{Z}_t, \tau; \boldsymbol{\Theta}) = \sum_{j\in K}\sum_{i\in K} p_{ij}\xi_{i,t-1|t-1}\eta_{j,t}$$

Thus, the relationship between the filtering and prediction probabilities is given by

$$\xi_{j,t+1|t} = Pr\left(s_{t+1} = j|\mathbf{X}_t, \boldsymbol{Z}_t; \boldsymbol{\Theta}\right) = \sum_{i\in K} p_{ij}\xi_{i,t|t} \tag{A.3}$$

The inference, similar to Hamilton's filter (Hamilton, 1994), is performed iteratively for $t = 1, ..., T$ with the initial values, $\xi_{j,0|0}$ for $j \in K$. The sample likelihood for the τth conditional quantile of X_t is then given by

$$L(\boldsymbol{\Theta}) = \prod_{t=1}^{T} f(X_t|\mathbf{X}_t, \boldsymbol{Z}_t, \tau; \boldsymbol{\Theta}) \tag{A.4}$$

In this paper, regimes are labeled by the restrictions on quantile intercepts, for example, $\theta_{1,0}(\tau) > ... > \theta_{k,0}(\tau)$. The MSQAR model is estimated by Bayesian method. See Liu (2014) for details of the Bayesian model estimation. Note that the cummulative distribution function of $ALD(Q, \varsigma, \tau)$ is also provided in Yu and Zhang (2005) as

$$F\left(x; Q, \varsigma, \tau\right) = \begin{cases} \tau exp\left(\frac{1-\tau}{\varsigma}\left(x - Q_\tau\right)\right), & if\ x \leq Q_\tau \\ 1 - (1-\tau)exp\left(-\frac{\tau}{\varsigma}\left(x - Q_\tau\right)\right), & if\ x > Q_\tau \end{cases}$$

with the quantile function

$$F^{-1}\left(u; Q, \varsigma, \tau\right) = \begin{cases} Q_\tau + \frac{\varsigma}{1-\tau} log\left(\frac{u}{\tau}\right), & if\ 0 \leq u \leq \tau \\ Q_\tau - \frac{\varsigma}{\tau} log\left(\frac{1-u}{1-\tau}\right), & if\ \tau < u \leq 1 \end{cases}$$

The expected shortfall is defined as the tail conditional expectation by

$$\begin{aligned} ES_\tau &= E\left(X|X \leq Q_\tau\right) \\ &= E\left(F^{-1}\left(u; Q, \varsigma, \tau\right)|0 \leq u \leq \tau\right) \end{aligned} \tag{A.5}$$

Based on the model estimation results of $\hat{Q}_\tau$, the expected shortfall can be numerically obtained by Monte Carlo simulation as follows

1. Randomly draw u_i for $i = 1, ..., N$ from a uniform distribution $U = \{u_i : 0 \leq u \leq \tau\}$. In this paper, $N = 5000$.

2. Compute $ES_{i,\tau} = F^{-1}\left(u_i; \hat{Q}, \hat{\varsigma}, \tau\right)$ for $i = 1, ..., N$

3. Compute $ES_\tau = \frac{1}{N}\sum_{i=1}^{N} ES_{i,\tau}$

Note that in a dynamic model setting of MSQAR, $Q_\tau = Q_{X_t}(\tau|\boldsymbol{X}_{t-1}, \boldsymbol{Z}_{t-1}, \boldsymbol{\theta}_{s_t})$ so that the steps (1)-(3) are repeated for $t = 1, ..., T$ to obtain $MSES_{s_t,\tau}$.

Generally, the scale parameter ς is a nuisance parameter when linking the nonlinear least square (NLS) quantile autoregression of Koenker and Xiao (2006) to an asymmetric laplace distribution (see i.e., Gerlach et al. (2011)), since it does not affect quantile locations. However, in order to filtering transition probabilities, MSQAR model estimation of Liu (2014) assumes quantile error terms following an asymmetric laplace distribution. Therefore, the scale parameter ς is used to estimate the distribution shape, which is no longer nuisance and can be identified. An anolog to this sitution is the relationship between ordinary least square (OLS) estimation and a linear model with the normal distribution assumption. If assuming data following a normal distribution, its variance parameter must be estimated and identified. However, using ordinary least square (OLS), the variance parameter is nuisance and cannot be identified. In many existing studies, variance parameters of normal distributions have been modeled subject to regime shifts to describe the varying dispersions driven by different economic states. Hence, this paper allows the scale parameter subject to regime shifts as well.

More importantly, since expected shortfall takes distributional aspects within the tail into account, the shapes of distributions become highly relavant to estimate accurate expected shortfall. To simulate expected falls, the shape parameter is essential to characterize the tail distribution shapes.