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Abstract

Frequency mismatch has been a problem in econometrics for quite some time. Many monthly economic and financial indicators are normally aggregated to match quarterly macroeconomic series such as GDP when analysed in a statistical model. However, temporal aggregation, although widely accepted, is prone to information loss. To address this issue, mixed frequency modelling was employed by using state space models with time-varying parameters. Quarter-on-quarter growth rate of GDP estimates were first treated as a monthly series with missing observation. Using Kalman filter algorithm, state space models were estimated with eleven monthly economic indicators as exogenous variables. A one-step-ahead predicted value for GDP growth rates was generated and as more indicators were included in the equation, the predicted values came closer to the actual data. Further evaluation revealed that among the group competing models, using Consumer Price Index (CPI), growth rates of PSEi, exchange rate, real money supply, WPI and merchandise exports are the more important determinants of GDP growth and generated the most desirable forecasts (lower forecast errors).

Keywords: Multi-frequency models, state space model, Kalman filter, GDP forecast

I. Introduction

Econometric equations usually operate in a uniform frequency even though most macroeconomic variables are observed or collected in different regularity. For instance, gross domestic product (GDP) of a country is normally reported quarterly while leading economic indicators, such as, value of net exports is collected at monthly basis. Financial data relevant to the economy, the local composite stock price index, for instance, is even observed at a higher rate. As a result, econometricians are faced with a dilemma of constructing a model with variables sampled at different frequencies. One has to aggregate data with higher frequency to match variables with lower frequency to proceed with model building.

Temporal aggregation has been the predominant technique among practitioners up until now to address the dilemma of having multi-frequency series. Albeit practical and widely acceptable, aggregation of higher frequency data to match that of lower frequencies may result in information loss. The dynamics between two series certainly cannot be assessed if both of them are not measured at the same time period.

For example, on one hand, the PSEi – a usual barometer of capital investment in the country and one of the leading economic indicators (LEI) – can be observed intraday. On the other hand, GDP, which measures the overall economic activity is only reported every quarter. Any dynamics between the two variables is normally assessed by aligning the variables to the same frequency and in this case, quarterly. The daily or monthly fluctuations in the PSEi, therefore, are not taken into account since it had to be aggregated or trimmed down to represent the value of the index at each quarter's end.

Same goes with GDP and exports. Since the economy is affected by exports of goods to other countries, it is important to see how a drop or rise in export would affect the overall economic growth. However, GDP estimates are not available in a monthly format unlike the value of exports. Dynamics within each quarter between the two series then cannot be analyzed if models are operated in a quarterly format.

The mismatch of frequencies in most macroeconomic data potentially leads to information loss when econometric models resort to temporal aggregation motivated this study. Thus, this paper primarily aims to provide an alternative in the form of a multi-frequency model.

Multi-frequency models, such as Mixed Data Sampling (MIDAS) regression models of Ghysels, et. al (2004) and Varied Data Samling (VARDAS) model of Qian (2010), has shown to have desirable results in empirical studies when compared to the usual models with time aggregated data. Retaining the original frequency of the data in an econometric model, intra-month or intra-quarter dynamics can be analyzed.

Using multi-frequency approach can also be used to forecast end-quarter or end-day values using monthly or intraday data, respectively – or *nowcast* as Gotz and Hecq (2013) coined the procedure.

This study employed a multi-frequency procedure in forecasting instead of resorting to temporal aggregation. Specifically, quarterly GDP estimates are treated as a monthly data with missing observations and relevant macroeconomic indicators, such as total merchandise exports, imports, terms of trade index, consumer price index, wholesale price index, Meralco sales, the peso-dollar exchange rate, money supply (M1), number of new businesses from SEC, stock price index and tourist arrivals were used as explanatory variables – all of which are available at a monthly basis. The ability of the state space model in handling missing observation was utilized to predict GDP using monthly series.

II. Review of Related Literature

Even before the problem of mismatched frequencies of data, econometricians have already been challenged with the peculiarities of macroeconomic data. Practitioners have hundreds of series at their disposal, although most of them are not desirably long enough (e.g. 20 to 40 years of quarterly data). Dynamic factor models (DFM) which can produce models for datasets that has more number of series than the number of time observations. Stock and Watson (2010) discussed in detail the DFM and enumerated several related studies.

The DFM, which was based on the theory that there are latent dynamic factors that affect the comovement of a collection time-series variables, was first used by Geweke in 1977 to apply a factor model designed for cross-sectional data in a time-series analysis. Sargent and Sims in the same year showed that two latent factors were able to explain a large portion of variability of different macroeconomic variables in the United States. The technique, however, has now been used in different applications such as in two-stage regression as instruments and in forecasting (Stock and Watson, 2010).

Giannone, Reichlin and Small (2008), for instance, developed a procedure that updates current-quarter GDP forecast every time a monthly data within that quarter are released using DFM. With a large collection of monthly data with varying release dates factors were computed using principal component and then Kalaman Smoothing. The marginal impact of each data released was able to be analyzed since the model was updated every time a monthly observation was released. Their results showed that current-quarter GDP forecast's precision increases as new monthly data comes in. Moreover, empirical evaluation of their proposed model showed fare performance compared to benchmarks they used.

Arouba, Deibold and Scotti (2008) also used dynamic factor model to measure economic activity at high frequency. They used a DFM to extract latent factors from a variety of stock and flow data sampled at different frequencies as a measure of the macroeconomic state. They further suggested using higher frequency data in empirical macroeconomic studies instead of the usual monthly or quarterly data. One of the empirical examples shown in their study revealed that incorporating weekly initial jobless claims to GDP and unemployment model was better compared to GDP and unemployment model only.

Similarly, Camacho and Perez-Quiros (2008) proposed an approach in forecasting euro area quarterly GDP in real-time using dynamic factor model. They also looked at the impact of each release of new data to their forecasts. Their work primarily dealt with problems such as asynchronous macroeconomic data release and using euro area aggregated data with short time spans.

Aside from DFM, another popular approach in dealing with multi-frequency model Mixed Data Sampling (MIDAS) regression models. Ghysels, Santa-Clara and Valkanov (2004) introduced MIDAS to deal with models with varied frequencies of dependent and explanatory variables. These models use distributed lags of regressors which are sampled at a higher frequency compared to dependent variables which is sampled at a lower frequency. One of their example models involved stock market volatility. The quadratic variation over a long future horizon which was sampled at low frequency was modelled using intra-day market information.

Another example in their 2004 paper involved GDP and other macroeconomic variable sampled at higher frequency such as inflation. They suggested that instead of aggregating monthly inflation data to match the quarterly GDP estimates, one can implement a MIDAS regression to combine the two series. One of their key findings was they were able to show a more efficient estimation using MIDAS regression compared to the usual regression with time-aggregated data.

Armesto, Engemann and Owyang (2010) surveyed different procedures aside from MIDAS modeling to circumvent the dilemma of mixed frequency data. Their problem was most of the macroeconomic variables are sampled monthly or quarterly whereas financial data which were found to be related to the macroeconomy are sampled at higher frequencies. Their paper showed that in some cases, aggregating the higher frequency data (e.g. averaging) did not have any disadvantage, although in some cases, the MIDAS technique introduced by Ghysels, Santa-Clara and Valkanov showed to be more beneficial especially in intra-period analysis.

Faced with the similar problem, Clements and Galvão (2008) used monthly and weekly data to generate short-term forecasts of US output growth. Since GDP is sampled at quarterly basis, an AR process was a reasonable candidate model. Thus, they extended the distributed lag MIDAS specification of Ghysels, et. al., and introduced an AR component, resulting to a MIDAS-AR specification which was shown to have better short term forecast compared to a benchmark AR model or and AR distributed lag model.

Similarly, Tay (2006) compared an AR(1) GDP growth model following a MIDAS framework and a usual quarterly AR(1) model of GDP growth with the most recent stock price index for each quarter as an additional explanatory variable. The result of his paper showed that particularly in recent years, stock returns were useful in predicting GDP growth. His results also showed that his MIDAS model was superior to his benchmark model. Furthermore, his study suggests that mixing frequencies can lead to better forecasts.

MIDAS was also used in Asimakopoulos, Paredes and Warmedinger's (2013) study of forecasting fiscal time series of different euro area countries. Using mixed frequency fiscal variables, MIDAS was employed to analyze annual or year-end fiscal variables. Their empirical work was able to show that as quarterly information within the year was able to improve the year-end forecast.

Aside from MIDAS, another approach in multi-frequency modelling is Mixed-Frequency VAR (MF-VAR) models. Basically, MF-VARs are VAR models containing component variables with different frequency. Götz and Hecq (2013) showed that a low frequency (aggregated) data (e.g. A quarter), as a function of its lagged value and distributed lagged values of the independent variable with higher frequency (disaggregated data, e.g. monthly data within the same quarter). Disaggregated data, meanwhile, is a function of the aggregated data and its lagged values. In the same study, they introduced *nowcasting* causality for mixed-frequency VAR models.

Nowcasting was predicting the value of a certain variable observed at lower frequency using variables observed at higher frequency and available in the current period. Nowcasting causality, meanwhile, is analogous to Granger causality, but is restricted to a certain time period, say, months within each quarter. Both nowcasting and Granger causality was then tested among selected US economic data.

As an example, they showed that the weekly growth rate of the stock of money (M2) in the US does not Granger causes the monthly growth rate of industrial production index but nowcasting causality was detected between the two series. They also showed that weekly growth of M2 does not Granger causes nor nowcasting causes monthly variation in the civilian unemployment rate.

A variation of MF-VAR but in a Bayesian context was used by Qian (2010) called Varied Data Sampling (VARDAS). A key feature of the procedure was that it only requires users to provide the data and the aggregation structure of each series while the estimation of VAR is similar to the ordinary VAR model. As an example, a previous study involving demand and supply component of GNP and unemployment (both quarterly series) was replicated using monthly unemployment data. The results showed, using impulse-response function, that the dynamics between unemployment and GNP components was more evident.

Still in the context of mixed-frequency VAR, Mittnik and Zadrozny (2004) used Kalman filtering method to forecast monthly German real GDP. They argued that when quarterly GDP is regressed to monthly indicators, it may not address reverse causality. Instead, they proposed a quarterly and monthly VAR(2) models of quarterly GDP, monthly industrial production, and monthly current business condition. Their empirical work showed that the monthly VAR model produced better short-term (1 to 3 months) GDP forecast while their quarterly model produced a better long-term forecast (1 up to 24 months ahead).

A paper by Kuzin, Marcellino and Schumacher (2009) compared the performance of the two popular approaches, MIDAS and MF-VAR, in terms of forecasting and now-casting of GDP growth of euro area using 20 monthly indicators as explanatory variables. Their results showed that two competing models tend to complement each other. MF-VAR was found to perform better for longer horizons while the other approach performs better for shorter horizons.

Aside from using the MIDAS approach, some papers treated low frequency data as high-frequency data with missing observations. Those missing observations are then forecasted to proceed with model building at a high sampling frequency. Fernández (1981) suggested interpolation by estimating missing data points using relevant series. This can be applied to stock data such as demand deposits which are usually available at year end to produce quarterly or monthly series.

Aside from Macroeconomics, mixed-frequency models were also used in signal processing. The Kalman Filter algorithm was also used by Fulton, Bitmead and Williamson (2001) to reproduce missing elements in an array processing. Using a state space model, a signal model was used to reconstruct missing data streams. Kalman smoothing was implemented and showed fare performance compared to an existing process to reconstruct missing data streams.

From those literatures noted above, this study was driven in treating deseasonalized (quarter-on-quarter) GDP growth rates as a monthly series with missing observations. A one-step-ahead predicted GDP growth rates was then generated from state space models with monthly leading economic indicators as independent variables. Forecasting capabilities of competing models were then evaluated using different criteria.

III. Proposed Model and Methodology

The study treated the quarter-on-quarter growth rate of deaseasonalized GDP estimates as a monthly series to match the higher frequency of different macroeconomic variables used in the study. The quarterly observations were placed on months corresponding to end of quarters (e.g. March, June, September and December) and the rest of the months were treated as missing observations. A monthly GDP growth rate series is generated by estimating the state space model with actual GDP growth rate as the left hand side of the signal equation and leading economic indicators (e.g. Stock price index (PSEi), Peso-Dollar exchange rate, consumer price index, money supply - M1, wholesale price index, total merchandise exports, total merchandise imports, terms of trade index for merchandise goods, Meralco sales, registered stock corporations and partnership, and tourist/visitor arrivals) as exogenous variables with time-varying parameters.

With the exception of exports, these are the same set of variables used as leading economic indicators of GDP by the Philippine Statistics Authority (PSA), formerly National Statistical Coordination Board (NSCB). Since they are available on a monthly series and are used in official economic planning, the study adopted the same set of indicators and opted to add merchandise exports as well. The hotel occupancy rate was also in the list of variables considered, however, a monthly series with length suitable for the study was not available.

The indicators were entered in the model one at a time depending on the usual order of their release or frequency resulting to eleven competing state space models. To evaluate the accuracy of the monthly GDP growth rates, 3-month averages corresponding to each quarter of the year were compared to the actual data. Root mean squared error and mean percentage errors were also computed to compare the state space models.

3.1 Data definition and source

The study used data sampled at different frequencies, specifically, quarterly and monthly economic time series variables. Quarterly GDP data was the low frequency series while all other indicators were collected on a higher frequency, i.e. monthly basis. When seasonality was present, the data were seasonally adjusted using Census X12[1] program which was a built-in package in Eviews7.

[1] Census X12 is a seasonal adjustment program developed by U.S. Census Bureau and is the improved version of X-11 Variant of Census II seasonal adjustment program originally written by Shiskin, Young and Musgrave in 1967.

Real GDP level from 2000 to 2013 was downloaded from the NSCB website and was seasonally adjusted. Quarter-on-quarter growth rates were then computed by taking the first difference of logarithms of the seasonally adjusted series. The resulting series was then converted to a monthly series by placing the quarterly observations on months corresponding to quarter ends. This now served as the dependent variable or the left hand side of the signal series in the state space models.

The explanatory variables used, meanwhile, were growth rates of eleven different economic variables mentioned earlier. Their growth rates were computed by taking the first difference of logarithm of their levels as well, i.e. month-on-month growth rate. The indicators and their variable names are listed in Table 1. All time series were from January 2000 to December 2013 except for the Meralco sales which was only up to May 2013 and number of new business incorporations which was up to December 2012 only. The definition and sources of each indicators are briefly discussed in the succeeding paragraphs.

Table 1. List of economic indicators used

Indicator
Stock price index *(psei)*
Peso-Dollar exchange rate *(fx)*
Consumer price index *(cpi)*
Real Money supply - M1 *(rm1)*
Wholesale price index *(wpi)*
Total merchandise exports *(exports)*
Total merchandise imports *(imports)*
Terms of trade index for Merchandise Goods *(trade_indx)*
Tourist/visitor arrivals *(visitors)*
Meralco sales *(meralco)*
Registered stock corporations and partnership *(new_buss)*

() - variable name

Stock price index or PSEi is the main composite index of the local stock market. It primarily serves as a measure of fluctuations in average price of equities being traded in Philippine Stock Exchange (PSE) and is available by end of each business day on the PSE website (http://www.pse.com.ph). For this study, the month-end value of PSEi was used for each month from January 2000 to December 2012. The historical data were downloaded from Bangko

The program is based on the premise that economic time series can be decomposed to seasonal component, trend-cycle component, trading-day component and irregular component.

Sentral ng Pilipinas (BSP) website (http://www.bsp.gov.ph/), Yahoo!Finance website (http://finance.yahoo.com/) and Bloomberg website (http://www.bloomberg.com/).

The Peso-Dollar exchange rate (fx) is the official guiding rate of exchange of one US dollar to the local currency. It is the weighted average of all foreign exchange trades done through the Philippine Dealing System. It is reported daily and is available for download from the BSP website. The monthly exchange rate used in the study was the monthly average exchange rate which was also downloaded from the central bank's website.

Consumer price index (CPI) is a composite index that serves as an indicator of average monthly changes in retail prices of a basket of commodities purchased by households and is based on 2006 prices. The monthly series was downloaded from the PSA-National Statistics Office website (http://census.gov.ph/).

Real money supply is the ratio of money supply (M1) over CPI multiplied by 100. Money supply (M1), also called narrow money, is currency in circulation or outside depository corporations and transferable deposits. It is available as a monthly series and was downloaded from the central bank's website.

Similar to CPI, wholesale price index is also a composite index of prices, wholesale prices in particular, of certain commodities. It has a base year of 1998 and has a monthly series available for download from NSO website.

Total merchandise imports and exports are the free on board (FOB) value of goods coming in and out (respectively) of the country through a seaport or airport and are properly cleared by the Bureau of Customs. Observations in the time series are in thousands of US dollars and correspond to cumulative value for the month only. Trade data were downloaded from the NSO website as well.

A monthly series was not readily available for the terms of trade index, thus, it was computed by following the LEI technical notes posted on the NSCB website[2]. The formula below was applied to the monthly series to come up with a monthly series for terms of trade index with 2000 as the base year.

$$Terms\ of\ Trade\ index = \frac{Merchandise\ export\ price\ index}{Merchandise\ import\ price\ index} \times 100$$

[2] http://nscb.gov.ph/lei/2014/1st_Qtr/TechnicalNotes.asp

$$Merchandise\ export\ price\ index = \frac{FOB\ value\ of\ export\ current\ price}{\frac{\sum_{\forall\ months\ of\ 2000} export\ value}{12}} \times 100$$

$$Merchandise\ import\ price\ index = \frac{FOB\ value\ of\ import\ current\ price}{\frac{\sum_{\forall\ months\ of\ 2000} import\ value}{12}} \times 100$$

Data on tourist arrival pertains to the number of visitors in the country. A visitor, as defined by NSCB Resolution No. 11 Series of 2003, is anyone travelling to a place outside his/her usual environment and staying there for less than a year. The data is compiled by the Department of Tourism and can be downloaded from their website (http://www.tourism.gov.ph/).

Monthly Meralco sales (in million kilowatt per hour) and registered stock corporations and partnership from the Securities and Exchange Commission were proxy variables for electric energy consumption and number of new businesses, respectively.

3.2 *State space model and Kalman filter algorithm*

The study's main feature is its use of state space models to fit a model consisting of variables with different frequencies. A state space model is usually employed to deal with dynamic relationship of time series data with unobserved components. A wide range of literature have used such model to estimate underlying components such as rational expectations, trend and cycle and missing observations to name a few. Many time series models, such as simple linear regression and ARIMA models, can also be represented using state space models.

State space modelling has two primary benefits first of which is it can integrate unobserved components called state variables with observable series in a single system. The second advantage of the technique is it uses a recursive algorithm called Kalman filter to recursively update the state variables.

To illustrate, consider a univariate time series y_t represented as

$$y_t = \mu_t + e_t\ , \qquad e_t \sim N(0, \sigma_e^2)$$
$$\mu_{t+1} = \mu_t + \eta_t\ , \qquad \eta_t \sim N(0, \sigma_\eta^2)$$

where $\{e_t\}$ and $\{\eta_t\}$ are independent Guassian white noise series and $t=1,2,\ldots,T$. For the moment, let the initial value μ_1 be equal to zero. In this example, y_t is the observed series with an underlying or unobserved component μ_t. In state space modelling, the first equation is called the signal equation while the second, which follows a drift-less random walk and is not directly observable, is called the state equation. The signal equation incorporates the state variable with the observed series accounting for measurement error e_t while the state equation represents the time evolution of the state variable with innovation η_t. The purpose of the analysis is to recover or estimate the unobserved state μ_t from the observable data $\{y_t | t=1, \ldots, T\}$. To do this, there are three common approaches or inferences that can be employed. These are, filtering, prediction and smoothing.

The three approaches differ on how they recover the state variable using the information available. To illustrate, let $F_t = \{y_1, \ldots, y_t\}$ be the information available at time t (inclusive). Filtering uses the information F_t, i.e. removing measurement errors from the data. Prediction, meanwhile, uses a one-step ahead forecast of μ_t or y_t and smoothing estimates μ_t using F_T (where $T>t$, i.e. using all information).

Furthermore, these approaches can be done using the Kalman filter algorithm. Its main purpose is to recursively update the state variables when new information becomes available. Consider the basic filter approach where estimation of the state variable uses information up to time t. The algorithm consists of two major parts, namely, predicting and updating. In the first part, a one-step ahead prediction of y_t is estimated utilizing information from $t=1$ up to $t-1$, i.e. $y_{t|t-1}$. When y_t is realized at time t, the prediction error or innovation can be computed as $\eta_{t|t-1} = y_t - y_{t|t-1}$. This innovation, η_t, now contains information about the state variable μ_t which was not captured in $\mu_{t|t-1}$ and is incorporated in estimating μ_t with $\mu_{t|t} = \mu_{t|t-1} + K_t\eta_{t|t-1}$, where K_t is called the Kalman gain or the weight assigned to the innovation.

3.3 Generating monthly GDP estimates

One of the methodologies adopted in this paper came from Fulton, Bitmead and Williamson (2001) study on signal processing which was discussed earlier. This paper departs from theirs mainly by using economic data, specifically, quarter-on-quarter growth rates of deseasonalized GDP entered at a monthly frequency with observations placed at each month corresponding to quarter ends (e.g. March, June, September and December) and the rest of the sample were treated as missing observation. To illustrate the series, let $t=1, 2, \ldots, 165$ corresponding to 165 months from January 2000 to December 2013. The second quarter of 2000 GDP growth rate was denoted as $GDPGR_6 = 0.13$ at June 2000 – corresponding to the last month of Q2 2000. Third quarter 2000 GDP growth rate is denoted as $GDPGR_9 = 2.11$ and entered at September 2000 and so on. The observations in between quarter ends were treated as missing observations.

The study then took advantage of the state space models' ability to handle missing observations. Thus, state space models were fitted to the monthly GDP data and the Kalman Filter algorithm was used to generate monthly GDP growth rate series. Following the discussion of state space models in the previous section, let y_t or the signal equation be the monthly GDP data with missing observations and μ_t is the state equation or unobserved component. We then supposed that $\{ y_t \}_{t=l+1}^{l+h}$ were missing, where $h \geq 1$ and $1 \leq l \leq T$. For t ϵ $\{l+1,\ldots, l+h\}$, μ_t is expressed as a linear combination of μ_{l+1} and $\{\eta_j\}_{j=l+1}^{t-1}$. Thus, for t ϵ $\{l+1,\ldots, l+h\}$,

$$E(\mu_t|F_{t-1}) = E(\mu_t|F_t) = \mu_{l+1|l}$$

$$Var(\mu_t|F_{t-1}) = Var(\mu_t|F_t) = \Sigma_{l+1|l} + (t-l-1)\sigma_\eta^2.$$

Consequently, $\mu_{t|t-1} = \mu_{t-1|t-2,}$ for t = $l+1,\ldots, l+h$. In other words, the Kalman filter algorithm can still be used even with missing observations by equating the Kalman gain and prediction error (η_t, used in updating state estimates) to zero.

Similarly, the study made use of this procedure, but instead of having a signal equation stated above, a set of exogenous variables with time-varying parameters were fitted. The time-varying parameters were considered as state variables and the corresponding state equations followed a drift-less random walk process. Thus, on periods with no actual GDP growth rate is available, a one-step-ahead forecast is generated factoring in the indicators in the signal equation.

Each indicator is entered one after the other, depending on their time of release, starting with the growth rate of PSEi in the first model. The change in Peso-Dollar exchange rate is then added to that to form the second state space model and so on until all indicators were included resulting to a group of eleven different models.

The signal equation was composed of the GDP growth rate on the left hand side and growth rate of the economic indicators with time-varying parameters on the right hand side. Generally, the signal and state equations are described below and the corresponding EViews specification codes for the eleven state space models are summarized in detail in the Appendix section.

$$dlog(GDP_t) = C(1) + \begin{pmatrix} SV_{1,t} \\ \vdots \\ SV_{11,t} \end{pmatrix} (PSEi_t \quad \ldots \quad new\_buss_t) + e_t$$

$$\begin{pmatrix} SV_{1,t} \\ \vdots \\ SV_{11,t} \end{pmatrix} = \begin{pmatrix} SV_{1,t-1} + \eta_{1,t-1} \\ \vdots \\ SV_{11,t-1} + \eta_{11,t-1} \end{pmatrix}$$

To have another set of competing models, the same set of indicators were used and entered with the same manner but the error term in each signal equation is treated as a state variable and follows has an AR(1) state equation.

The initial one-step-ahead predicted value for the states and variance matrix was set to zero and 1 million, respectively. After a state space models were specified, signal series was generated using one-step-ahead forecast to represent the monthly estimates of GDP growth rates.

3.4 Evaluating the monthly estimates of GDP growth rate

To assess the performance of the proposed method, the monthly growth rates were aggregated by quarter using simple arithmetic mean and were compared to the actual data. Root mean squared error and mean absolute error were also calculated to further check the models' performance.

IV. Results and Discussion

4.1. Preliminary analysis

The study used macroeconomic variables sampled at different frequencies to illustrate a multi-frequency model. Specifically, a state space model consisting of growth rates of GDP and of leading economic indicators was generated. The GDP series was treated as a monthly series with missing observations, then a one-step-ahead predicted series was generated using a Kalman filter algorithm.

Gross domestic product from the National Income Accounts (NIA) report of the PSA, is released quarterly, specifically, a month after each quarter end. The quarterly GDP levels at constant 2000 prices were deseasonalized using Census X12 in Eviews7 then the quarter-on-quarter growth rate was calculated by taking the difference of logarithms of two succeeding quarters (i.e. $log(GDP_t) - log(GDP_{t-1})$). After which, it was treated as a monthly series and the observations were placed on months corresponding to quarter ends. This new series is then used as the left-hand side of the signal equation.

Figure 1 and 2 below illustrates the original and deseasonalized quarterly series of GDP levels and quarter-on-quarter growth rates, respectively. Figure 3, meanwhile, shows the growth rate series in monthly frequency.

Figure 1. Quarterly GDP levels at constant 2000 prices, original and seasonally adjusted series, 2000 -2013

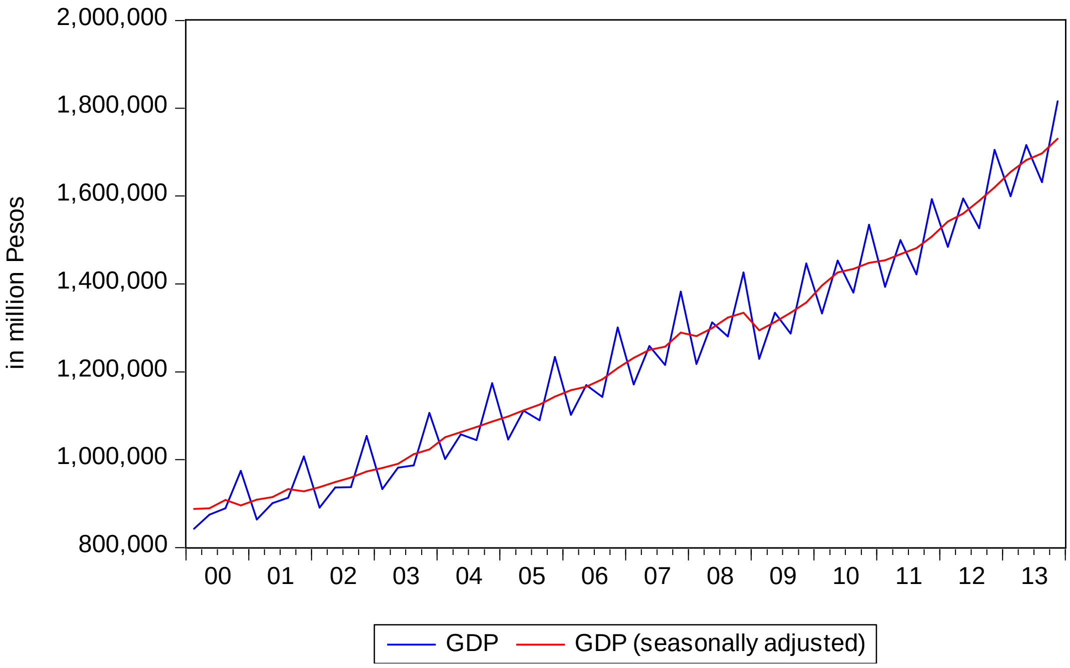


Figure 2. Quarter-on-quarter GDP (deseasonalized) growth rate, 2000-2013

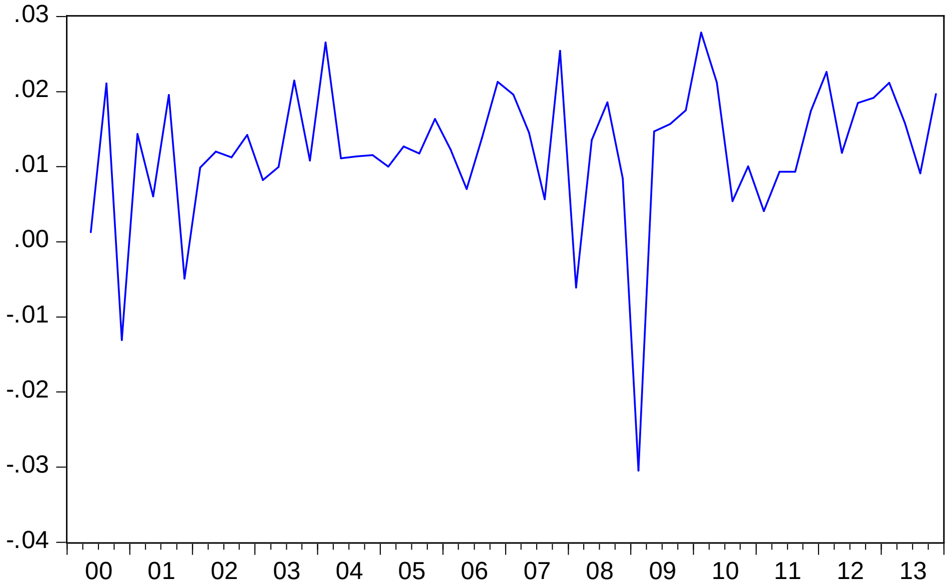
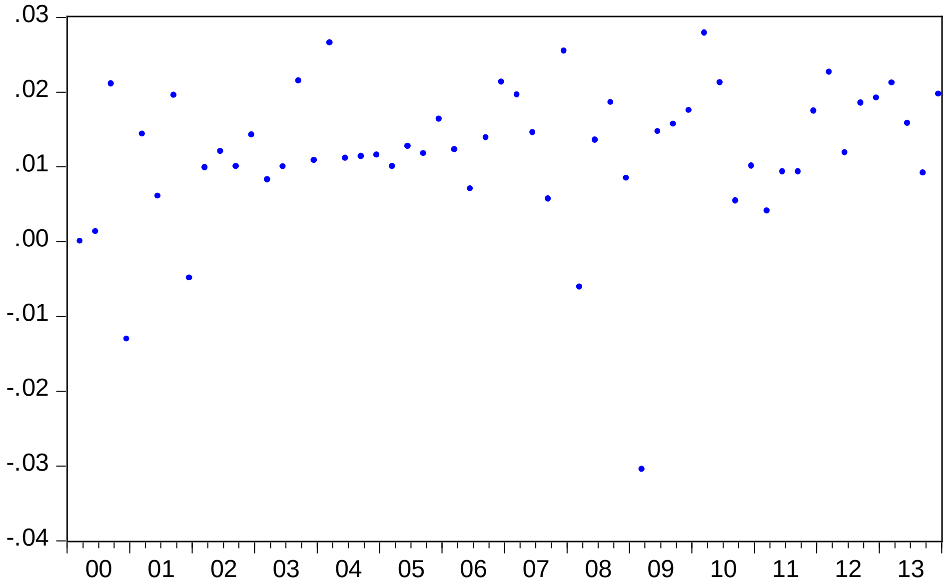


Figure 3. Monthly series of GDP (deseasonalized) growth rate, 2000-2013



Since Figure 3 is the monthly equivalent of the original quarterly growth rate series, the graph only shows the quarterly observations placed at months corresponding to quarter ends. The rest of the series is treated as missing observations. Both graphs, however, illustrate the same movement or pattern of growth. A noticeable drop in growth can be seen in both graphs during 2008 which clearly reflects the global financial crisis during the period.

Meanwhile, the leading economic indicators together with merchandise exports were also deseasonalized when seasonality was present and its growth rates were used as exogenous variables with time-varying coefficient in the signal equations.

After using Census X12 to seasonally adjust the series (except for PSEi where no seasonality was detected), presence of a unit root on their growth rates – computed as first difference of the logarithm of each variables – were tested. Results of Augmented Dickey-Fuller unit root test (see Appendix) revealed that growth rates of the indicators were all stationary.

Figure 4 and 5 illustrates the levels and growth rates of the economic indicators used in the study. Similar to the graphs of GDP, most of the graphs of monthly economic indicators reflect the 2008 financial crisis. It is apparent, especially in the PSEi, foreign exchange, exports, imports, and registered stock corporations and partnerships.

4.2 Estimation of State Space models

State space models were estimated using EViews 7 statistical package. Models were fitted by entering the indicators one by one depending on its date of release or availability (e.g. PSEi is available by end of day, thus, it was the first indicator used followed by exchange rate, and so on). Another set of competing models were estimated by replacing the error term in signal equations, which was previously set to a normalized error, with a state variable that follows an AR(1) process. Table 2 summarizes the signal equation of the eleven state space models estimated. A detailed specification of each state space model can be found in the Appendix.

Table 2. State space models and corresponding signal equation

Model	Signal equation
SS01	@signal dl_gdp_sa = c(1) + sv1*dlog(psei) + [var=1]
SS02	@signal dl_gdp_sa = c(1) + sv1*dlog(psei) + sv2*dlog(fx_sa) + [var=1]
SS03	@signal dl_gdp_sa = c(1) + sv1*dlog(psei) + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa) + [var=1]
SS04	@signal dl_gdp_sa = c(1) + sv1*dlog(psei) + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa) + sv4*dlog(rm1_sa) + [var=1]
SS05	@signal dl_gdp_sa = c(1) + sv1*dlog(psei) + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa) + sv4*dlog(rm1_sa) + sv5*dlog(wpi_sa) + [var=1]
SS06	@signal dl_gdp_sa = c(1) + sv1*dlog(psei) + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa) + sv4*dlog(rm1_sa) + sv5*dlog(wpi_sa) + sv6*dlog(exports_sa) + [var=1]
SS07	@signal dl_gdp_sa = c(1) + sv1*dlog(psei) + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa) + sv4*dlog(rm1_sa) + sv5*dlog(wpi_sa) + sv6*dlog(exports_sa) + sv7*dlog(imports_sa) + [var=1]
SS08	@signal dl_gdp_sa = c(1) + sv1*dlog(psei) + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa) + sv4*dlog(rm1_sa) + sv5*dlog(wpi_sa) + sv6*dlog(exports_sa) + sv7*dlog(imports_sa) + sv8*dlog(trade_indx_sa) + [var=1]
SS09	@signal dl_gdp_sa = c(1) + sv1*dlog(psei) + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa) + sv4*dlog(rm1_sa) + sv5*dlog(wpi_sa) + sv6*dlog(exports_sa) + sv7*dlog(imports_sa) + sv8*dlog(trade_indx_sa) + sv9*dlog(visitor_sa) + [var=1]
SS10	@signal dl_gdp_sa = c(1) + sv1*dlog(psei) + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa) + sv4*dlog(rm1_sa) + sv5*dlog(wpi_sa) + sv6*dlog(exports_sa) + sv7*dlog(imports_sa) + sv8*dlog(trade_indx_sa) + sv9*dlog(visitor_sa) + sv10*dlog(meralco_sa) + [var=1]
SS11	@signal dl_gdp_sa = c(1) + sv1*dlog(psei) + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa) + sv4*dlog(rm1_sa) + sv5*dlog(wpi_sa) + sv6*dlog(exports_sa) + sv7*dlog(imports_sa) + sv8*dlog(trade_indx_sa) + sv9*dlog(visitor_sa) + sv10*dlog(meralco_sa) + sv11*dlog(new_buss_sa) + [var=1]

Figure 4. Monthly economic indicators used as exogenous variables, levels and seasonally adjusted (SA)

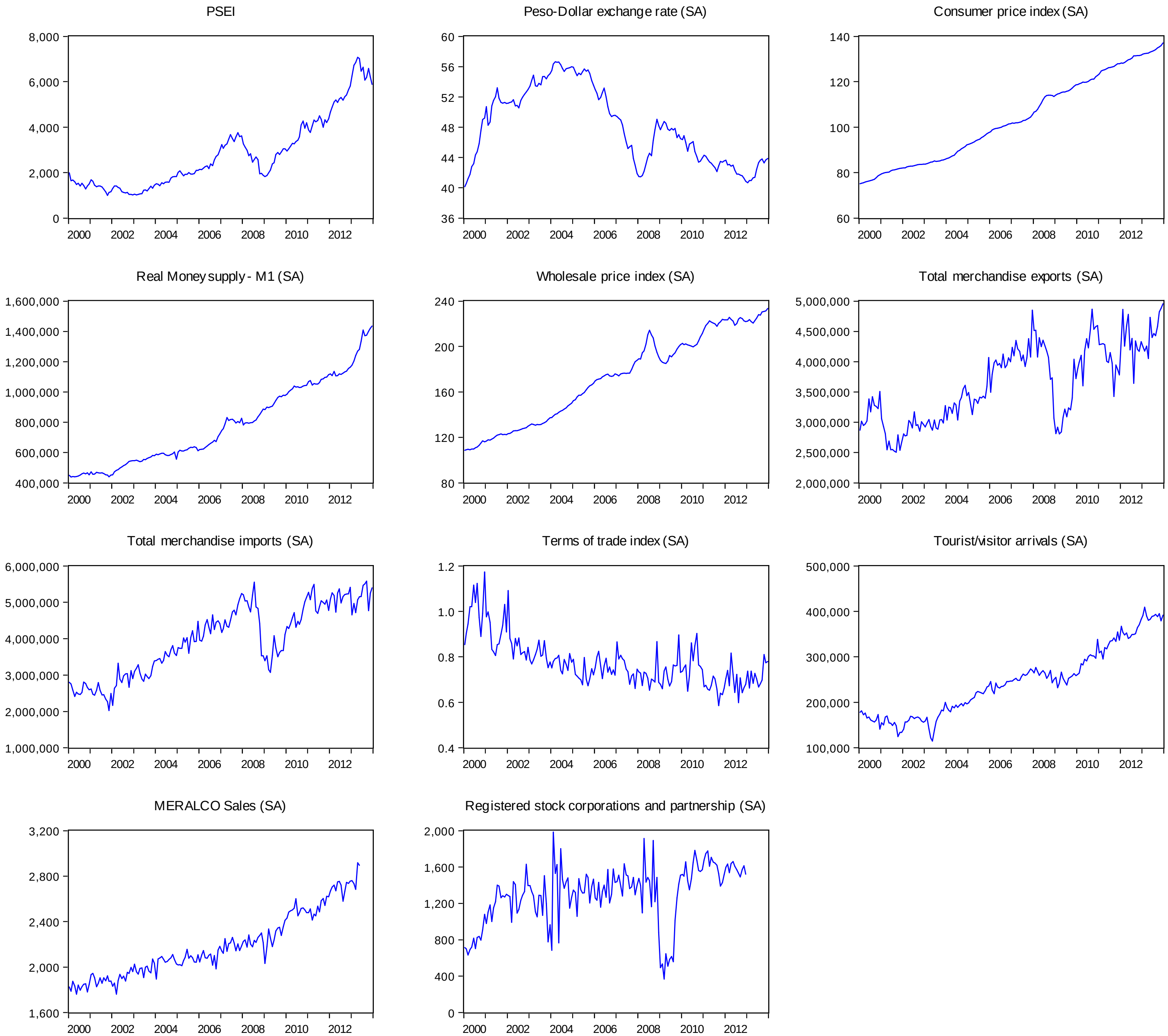
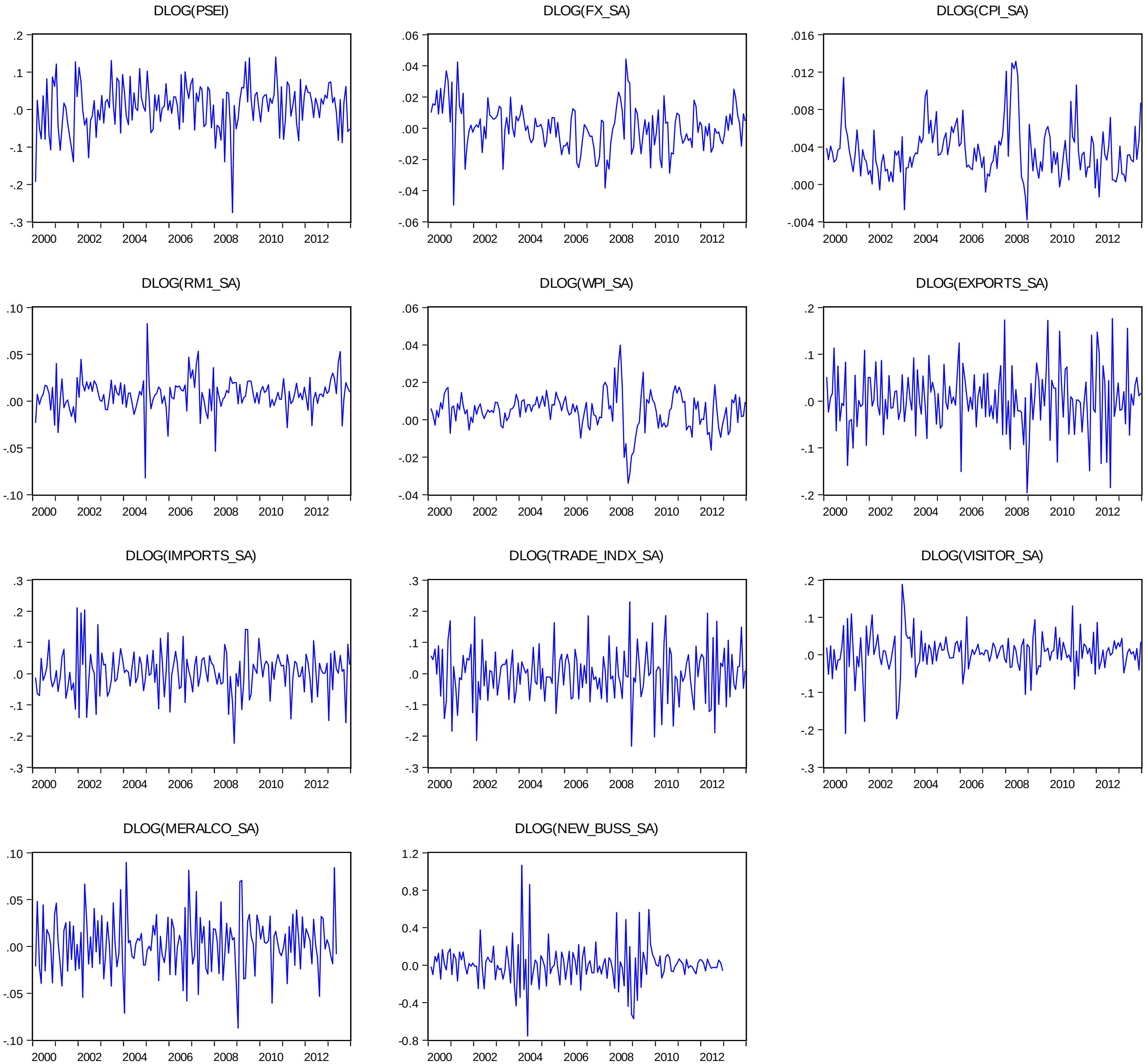


Figure 5. Monthly economic indicators used as exogenous variables, growth rates



A second set of competing models were estimated wherein the error term in each signal series (*[var=1]*) is replaced by a state variable which follows an AR(1) process.

After running the specification codes in Eviews 7, the one-step-ahead predicted signals for each state space model were generated. Table 3 summarizes a sample EViews7's estimation output of the state space model for the model with six indicators in the signal equation and normalized error. Figure 6 illustrates the corresponding one-step-ahead predicted signals from the same state space model shown in Table 3. Likewise, Table 4 and Figure 7 correspond to generated state space model with all eleven indicators in the signal equation with normalized error. The predicted signals of the other state space models are illustrated in Figure 8.

Table 3. Eviews 7 state space model estimation, SS06

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.0181	33.1971	0.0005	0.9996
	Final State	**Root MSE**	**z-Statistic**	**Prob.**
SV1	-0.0827	6.2976	-0.0131	0.9895
SV2	-0.2258	14.3358	-0.0158	0.9874
SV3	-1.3077	45.2049	-0.0289	0.9769
SV4	0.0800	10.0241	0.0080	0.9936
SV5	0.0855	19.1895	0.0045	0.9964
SV6	0.0266	5.0834	0.0052	0.9958
Log likelihood	-84.4252	Akaike info criterion		3.0509
Parameters	1.0000	Schwarz criterion		3.0871
Diffuse priors	0.0000	Hannan-Quinn criter.		3.0649

Figure 6. Monthly actual GDP growth and 1-step-ahead predicted signals from SS06

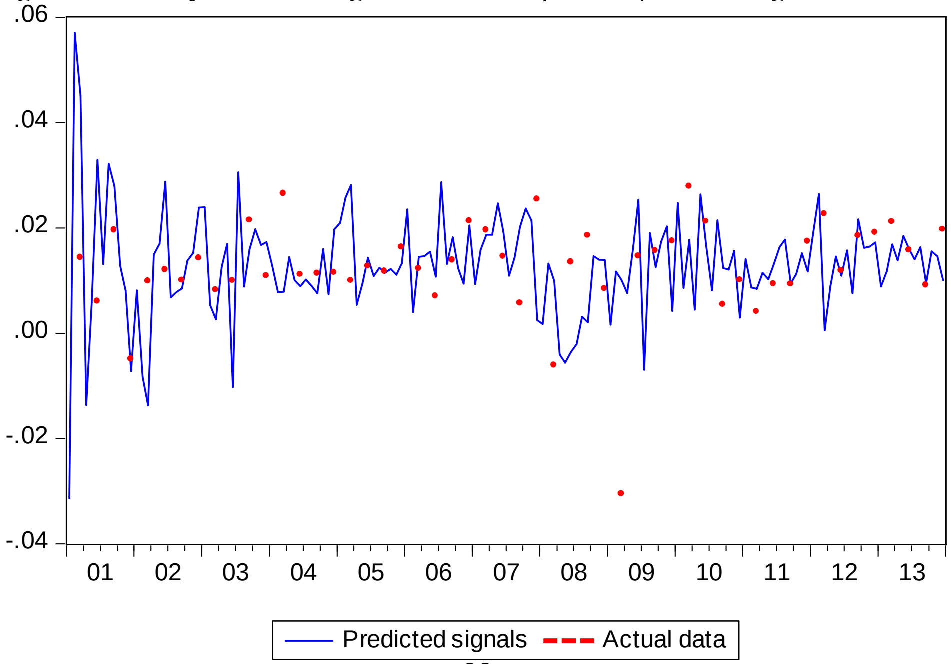


Table 4. Eviews 7 state space model estimation output, SS11

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.0181	42.1628	0.0004	0.9997
	Final State	Root MSE	z-Statistic	Prob.
SV1	-0.0419	7.3599	-0.0057	0.9955
SV2	-0.3625	19.3121	-0.0188	0.9850
SV3	-1.3848	56.8430	-0.0244	0.9806
SV4	0.0913	12.4696	0.0073	0.9942
SV5	0.1880	25.0936	0.0075	0.9940
SV6	0.0918	8.9788	0.0102	0.9918
SV7	-0.1123	10.7053	-0.0105	0.9916
SV8	-0.0883	9.5361	-0.0093	0.9926
SV9	0.0152	7.6970	0.0020	0.9984
SV10	-0.0560	10.5959	-0.0053	0.9958
SV11	0.0123	6.6167	0.0019	0.9985
Log likelihood	-117.0225	Akaike info criterion		4.5393
Parameters	1.0000	Schwarz criterion		4.5769
Diffuse priors	0.0000	Hannan-Quinn criter.		4.5537

Figure 7. Monthly actual GDP growth and 1-step-ahead predicted signals from SS11

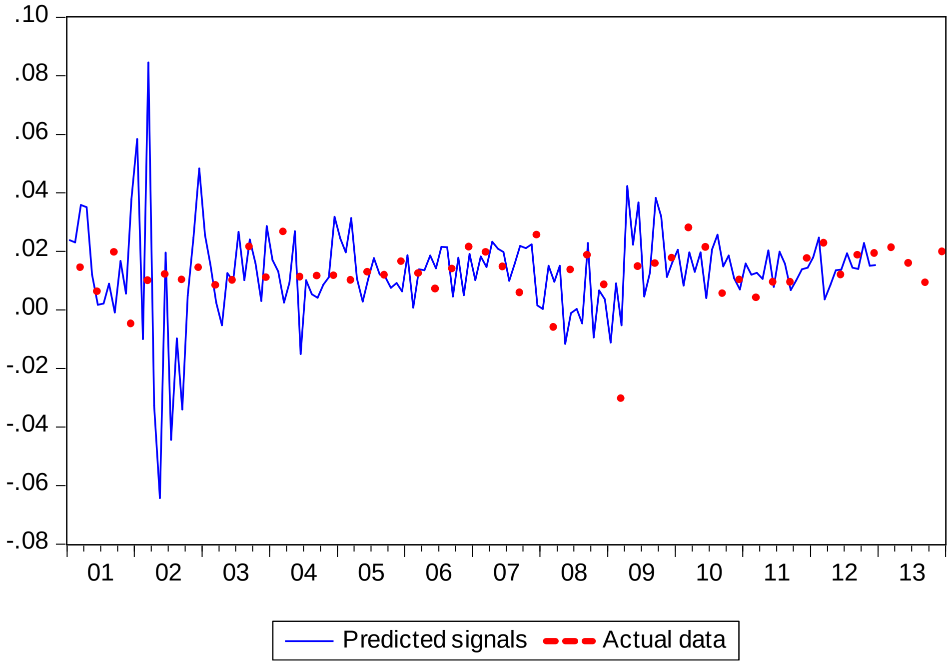
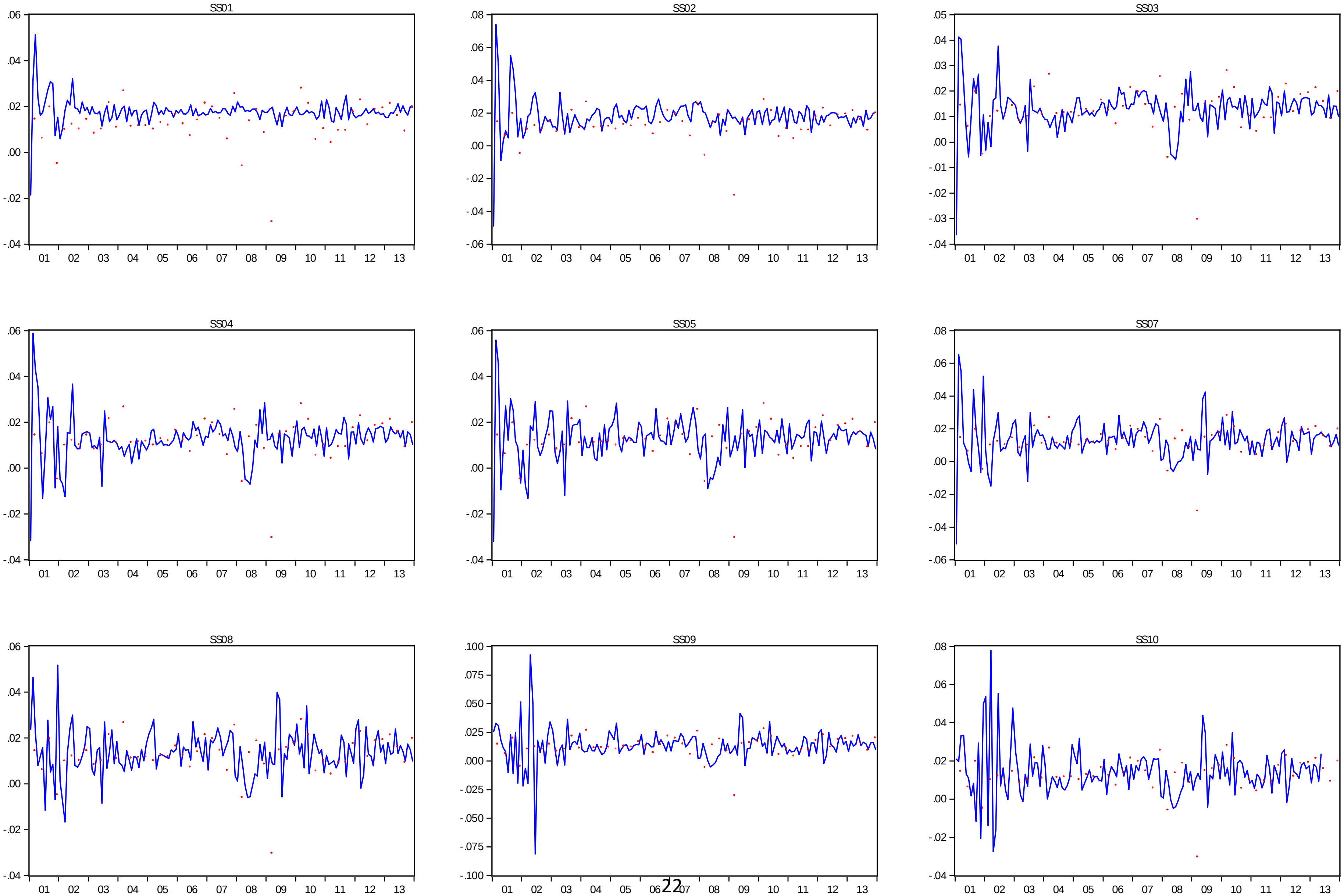


Figure 8. Monthly actual GDP growth (red) and 1-step-ahead predicted signals (blue) of state space models



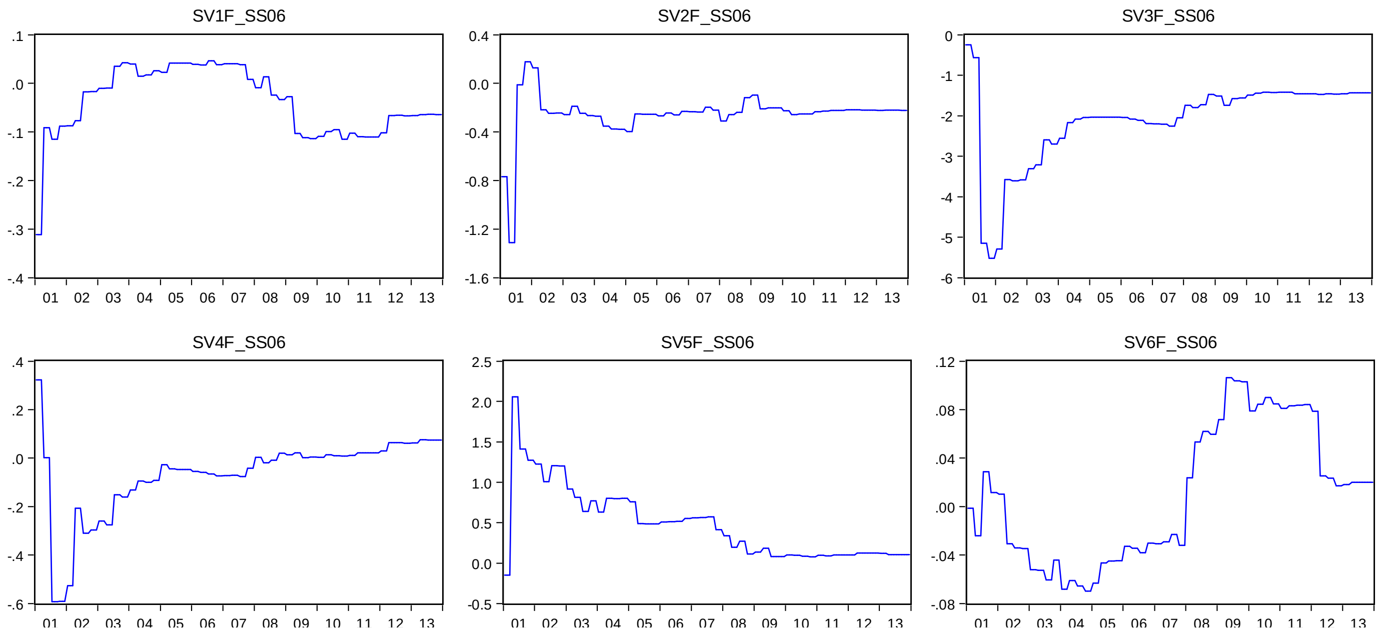
The estimated state space model which included registered stock corporations and partnerships was only up to 2012 since the series was only up to that point. Same goes with SS10 which included Meralco sales that extended up to May 2013 only. Although the state space model has the ability to handle missing data, variables in the signal equation should have at least the same time horizon.

It can be noted that the generated one-step-ahead predicted signals from the first two state space models (*SS01* & *SS02*) were not relatively far from the plotted actual data but does not exhibit much fluctuations unlike on other models. The first model's (*SS01*) signal equation is composed of the growth rate of PSEi as exogenous variable while the second model (*SS02*) has PSEi and exchange rates as explanatory variables. The models with these two indicators, apparently, were not sufficient to predict the GDP growth based on the graphs alone.

As expected, by adding more variables in the signal equation the predicted signal series came closer to the actual data. Moreover, the large dip in GDP growth during 2008 was only reflected starting from *SS03*. It can be noted that most indicators used exhibited dramatic changes during 2008 reflecting the economic turmoil in that period.

To illustrate how each indicator contributes to the fluctuation of the monthly GDP growth, i.e. the 1-step-ahead predicted signals, the predicted state variables were also plotted and are shown in Figure 9. Most parameters exhibited a notable change in regime during 2001 and 2008 when there were global economic turmoil.

Figure 9. Predicted state variables from SS06



**SV1F – psei, SV2F – fx, SV3F – cpi, SV4F – rm1, SV5F – wpi, SV6F - exports*

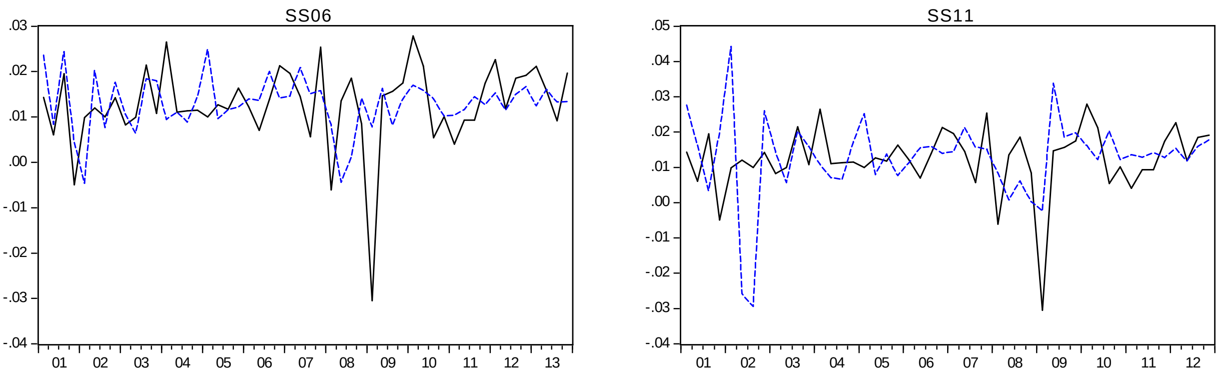
4.3. Evaluating the forecasting capabilities

To further evaluate the forecasting ability of the state space models, the predicted monthly signal series of GDP growth rates were aggregated by simple averaging to form a quarterly series and then compared to the actual data.

Figure 10a and 10b illustrates the aggregated one-step-ahead predicted signal series of each state space model. Similar to the non-aggregated data, the generated signal series from the first two models were relatively far from the actual data compared to the rest of the models. When CPI was added, though, it came closer to the actual data as shown in the third graph in Figure 10b (*SS03*) and as more indicators were added, the aggregated predicted values came closer to the actual data.

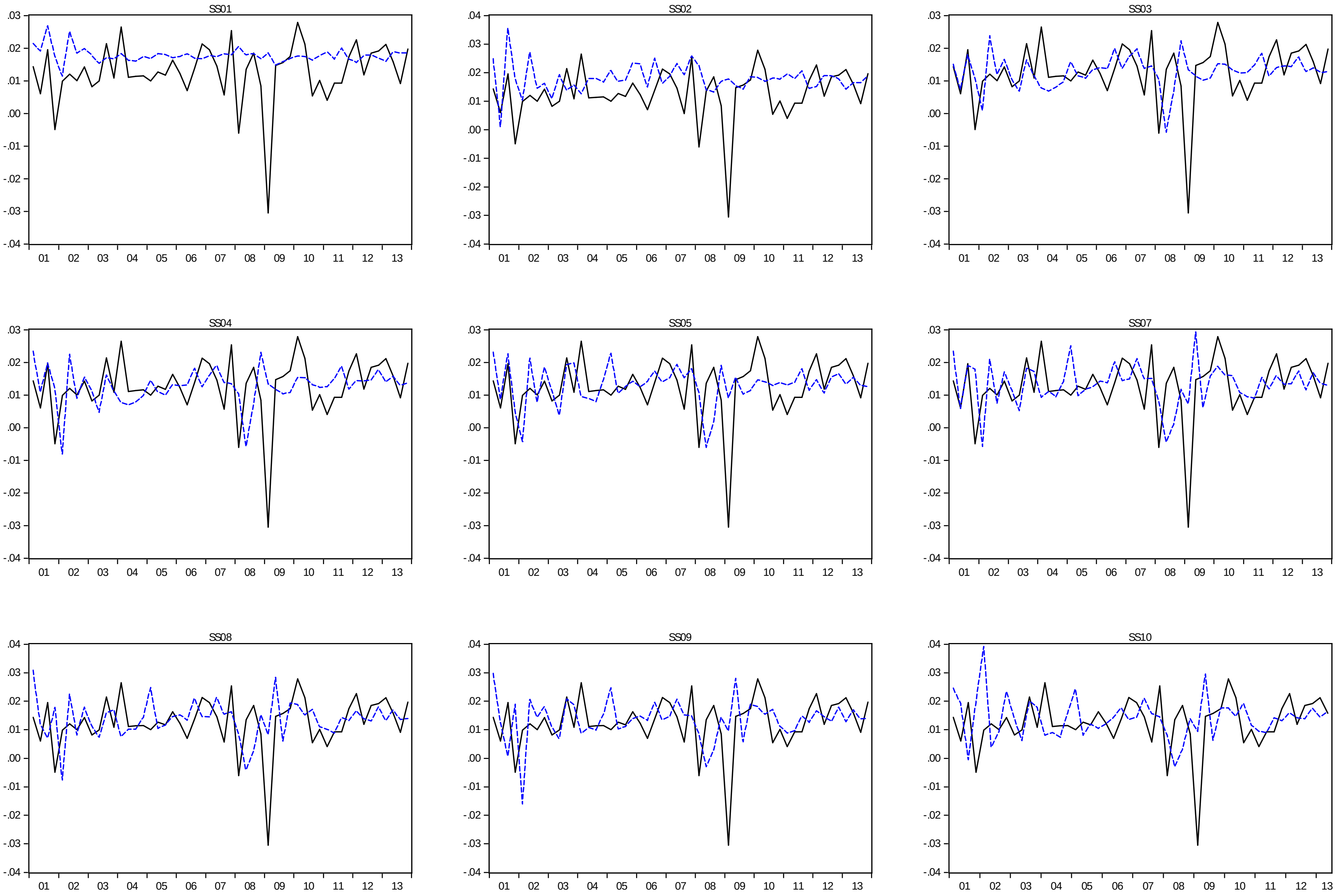
The aggregated signal series clearly follows the actual data although the huge dip in 2008 was not captured in any of the generated models. Moreover, there are more noticeable fluctuations in SS11 compared to SS11. Difference between the actual and aggregated predicted signals is summarized in Table 8 in the Appendix. Other aggregated signals from the rest of the models are shown in Figure 10b.

Figure 10a. Quarterly actual GDP growth (solid line) and aggregated 1-step-ahead predicted signals (blue) of state space models



To have another set of competing models, the state space models were re-estimated but included a signal equation with an error term represented as a state variable following an AR(1) process, i.e. *[var=1]* in each signal equation was replaced with another state variable. After re-estimation of the state space models, 1-step ahead predicted signals were also generated and then aggregated into a quarterly series. Model specifications and aggregated series resulting from these re-estimated models are posted in the Appendix.

Figure 10b. Quarterly actual GDP growth (solid line) and aggregated1-step-ahead predicted signals (blue) of state space models



To compare the predicting capabilities of the competing models, root mean squared error (RMSE) and mean absolute error (MAE) were computed for each model to evaluate which among them had the most desirable result. Formula for RMSE and MAE are summarized below.

$$RMSE = \sqrt{\sum_{t=1}^{n} (\hat{y_t} - y_t)^2/n}$$

$$MAE = \sum_{t=1}^{n} |\hat{y_t} - y_t|/n$$

Root mean squared error and mean absolute error were calculated for each aggregated series and the results are summarized in Table 5.

Table 5. RMSE and MAE of estimated models

with normalized error in signal equation					with AR(1) error in signal equation				
Model	RMSE	Rank	MAE	Rank	Model	RMSE	Rank	MAE	Rank
SS01	0.0108	7	0.0074	6	SS01	0.0161	10	0.0146	10
SS02	0.0114	9	0.0080	9	SS02	0.0167	11	0.0150	11
SS03	0.0098	3	0.0069	2	SS03	0.0129	8	0.0104	8
SS04	0.0102	5	0.0072	5	SS04	0.0111	1	0.0084	3
SS05	0.0097	2	0.0072	3	SS05	0.0111	2	0.0083	1
SS06	0.0092	1	0.0067	1	SS06	0.0117	6	0.0088	6
SS07	0.0099	4	0.0072	4	SS07	0.0112	3	0.0083	2
SS08	0.0104	6	0.0077	7	SS08	0.0113	4	0.0085	4
SS09	0.0109	8	0.0080	8	SS09	0.0115	5	0.0086	5
SS10	0.0115	10	0.0085	10	SS10	0.0119	7	0.0090	7
SS11	0.0137	11	0.0102	11	SS11	0.0135	9	0.0105	9

Among the first set of models, *SS06* had the lowest RMSE and MAE. *SS06* signal equation is composed of growth rates of PSEi, exchange rate, CPI, real money supply, WPI and exports. *SS05* was the next model with lowest RMSE and also had the 3rd least MAE. Including all indicators in the signal equation, i.e. SS11, resulted with the least desirable RMSE and MAE.

When an error term following an AR(1) process was introduced in the signal equation, *SS04* and *SS05* had the least RMSE and MAE, respectively. *SS06*, meanwhile, ranked 6th in both criteria.

To summarize vividly, RMSE and MAE of the estimated models are illustrated in Figure 11 and Figure 12, respectively. In both graphs, RMSE and MAE of the two sets of models were plotted against each other. And based on them, SS06 from the first set of state space model had the least RMSE and MAE. It can also be seen that treating the error term as a state variable did not improve the predicting capabilities of the models, although in SS10 and SS11, RMSE and MAE of the two sets of models almost coincided.

Figure 11. RMSE of estimated state space models

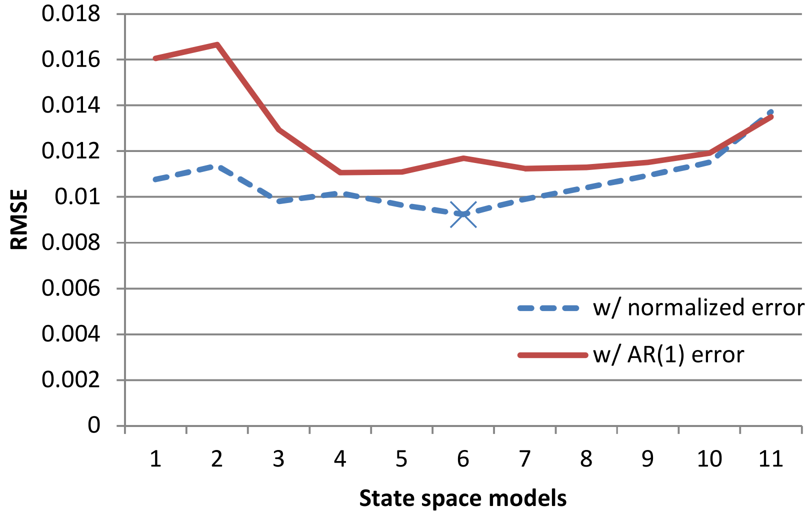
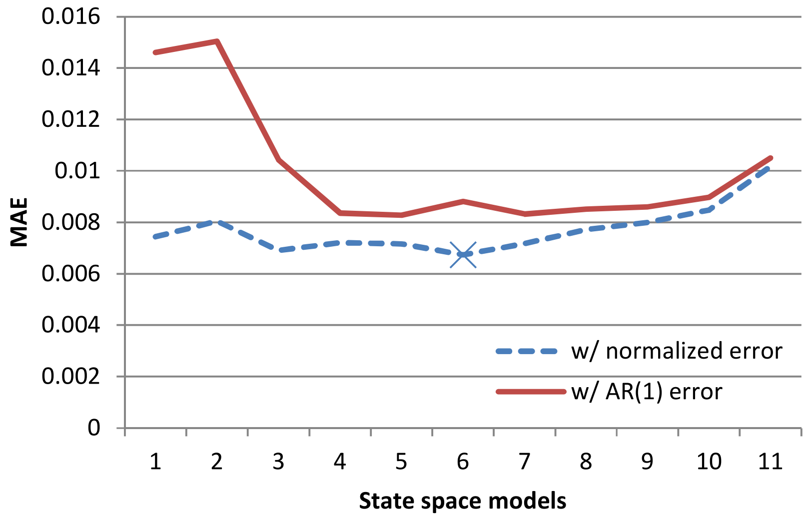


Figure 12. MAE of estimated state space models



V. Conclusion and Recommendation

Mismatch in frequency of most macroeconomic variables has been a problem of econometricians for quite some time. Surveyed economic indicators such as price indices and foreign trade data, for example, are usually reported on a monthly basis while macroeconomic variables, such as GDP, are usually reported quarterly. And to accommodate variables with different frequencies in an econometric model, researchers usually resort to temporal aggregation to match the lowest frequency in the dataset. However, based on different literature, temporal aggregation may result in information loss. To address this issue, mixed frequency models can be employed instead of resorting to temporal aggregation.

Since this field of study is relatively new, the collection of literature is fairly limited, but it has been applied in different areas of research, such as macroeconomic forecasting, financial modelling and engineering, to name a few. This study aims to contribute to the literature of mixed-frequency modelling by using state space models with time-varying parameters in generating higher frequency macroeconomic variable.

The study used quarter-on-quarter growth rate of deaseasonalized GDP estimates and treated it as a monthly series with missing observation. Using a state space model, specifically a time-varying parameter model with random walk coefficients, the quarterly GDP growth rates converted to a monthly series was fitted with eleven monthly economic indicators. A one-step-ahead predicted value for GDP growth rates was generated and as more indicators were included in the equation, the predicted values came closer to the actual data. Further evaluation revealed that among the group of models, using the PSEi, exchange rate, CPI, real money supply, WPI and exports generated the most desirable forecasts based on RMSE and MAE.

The state space models used in this study followed time-varying parameters. In future studies, a different specification of the state variable can be explored. Further research can also entail different combinations of exogenous variables in the signal equation instead of entering the indicators one by one. Lastly, other relevant variables can also be included in the model.

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Appendix

Test for unit root of eleven economic indicators

Table 6a. Summary of results of test for presence of unit root

Variable	Augmented Dickey-Fuller test	P-value	Remark
LOG(PSEI)	-3.2259	0.0830	I(1)
LOG(FX_SA)	-3.6835	0.0262	I(0)+Trend
LOG(CPI_SA)	-2.1043	0.5392	I(1)
LOG(RM1_SA)	-2.7859	0.2046	I(1)
LOG(WPI_SA)	-2.3651	0.3964	I(1)
LOG(EXPORTS_SA)	-3.7409	0.0224	I(0)+Trend
LOG(IMPORTS_SA)	-2.9799	0.1411	I(1)
LOG(TRADE_INDX_SA)	-4.5117	0.0020	I(0)+Trend
LOG(VISITOR_SA)	-4.3014	0.0040	I(0)+Trend
LOG(MERALCO_SA)	-2.3575	0.4003	I(1)
LOG(NEW_BUSS_SA)	-3.4681	0.0465	I(0)

using 5% level of significance

Table 6b. Summary of results of test for presence of unit root of growth rates

Variable	Augmented Dickey-Fuller test statistic	P-value
DLOG(PSEI)	-12.3358	0.0000
DLOG(FX_SA)	-9.3592	0.0000
DLOG(CPI_SA)	-7.0059	0.0000
DLOG(RM1_SA)	-15.1681	0.0000
DLOG(WPI_SA)	-7.1152	0.0000
DLOG(EXPORTS_SA)	-18.7891	0.0000
DLOG(IMPORTS_SA)	-16.1547	0.0000
DLOG(TRADE_INDX_SA)	-19.4828	0.0000
DLOG(VISITOR_SA)	-14.6424	0.0000
DLOG(MERALCO_SA)	-11.9915	0.0000
DLOG(NEW_BUSS_SA)	-19.5476	0.0000

using 5% level of significance

State space model specifications (with normalized error)

```
Sspace: SS01
@signal dl_gdp_sa =  c(1) + sv1*dlog(psei)  + [var=1]
@state sv1 = sv1(-1) + [var=1]
@mprior vec01
@vprior sym01
```

Sspace: SS02
@signal dl_gdp_sa = c(1) + sv1*dlog(psei) + sv2*dlog(fx_sa) + [var=1]
@state sv1 = sv1(-1) + [var=1]
@state sv2 = sv2(-1) + [var=1]
@mprior vec02
@vprior sym02

Sspace: SS03
@signal dl_gdp_sa = c(1) + sv1*dlog(psei) + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa) + [var=1]
@state sv1 = sv1(-1) + [var=1]
@state sv2 = sv2(-1) + [var=1]
@state sv3 = sv3(-1) + [var=1]
@mprior vec03
@vprior sym03

Sspace: SS04
@signal dl_gdp_sa = c(1) + sv1*dlog(psei) + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa)+ sv4*dlog(rm1_sa) + [var=1]
@state sv1 = sv1(-1) + [var=1]
@state sv2 = sv2(-1) + [var=1]
@state sv3 = sv3(-1) + [var=1]
@state sv4 = sv4(-1) + [var=1]
@mprior vec04
@vprior sym04

Sspace: SS05
@signal dl_gdp_sa = c(1) + sv1*dlog(psei) + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa)+ sv4*dlog(rm1_sa) + sv5*dlog(wpi_sa) + [var=1]
@state sv1 = sv1(-1) + [var=1]
@state sv2 = sv2(-1) + [var=1]
@state sv3 = sv3(-1) + [var=1]
@state sv4 = sv4(-1) + [var=1]
@state sv5 = sv5(-1) + [var=1]
@mprior vec05
@vprior sym05

Sspace: SS06
@signal dl_gdp_sa =   c(1) + sv1*dlog(psei) + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa)+ sv4*dlog(rm1_sa) + sv5*dlog(wpi_sa) + sv6*dlog(exports_sa) + [var=1]
@state sv1 = sv1(-1) + [var=1]
@state sv2 = sv2(-1) + [var=1]
@state sv3 = sv3(-1) + [var=1]
@state sv4 = sv4(-1) + [var=1]

```
@state sv5 = sv5(-1) + [var=1]
@state sv6 = sv6(-1) + [var=1]
@mprior vec06
@vprior sym06
```

Sspace: SS07

```
@signal dl_gdp_sa = c(1) + sv1*dlog(psei)  + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa)+
sv4*dlog(rm1_sa)  + sv5*dlog(wpi_sa) + sv6*dlog(exports_sa) +
sv7*dlog(imports_sa) + [var=1]
@state sv1 = sv1(-1) + [var=1]
@state sv2 = sv2(-1) + [var=1]
@state sv3 = sv3(-1) + [var=1]
@state sv4 = sv4(-1) + [var=1]
@state sv5 = sv5(-1) + [var=1]
@state sv6 = sv6(-1) + [var=1]
@state sv7 = sv7(-1) + [var=1]
@mprior vec07
@vprior sym07
```

Sspace: SS08

```
@signal dl_gdp_sa =  c(1) + sv1*dlog(psei)  + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa)+
sv4*dlog(rm1_sa)  + sv5*dlog(wpi_sa) + sv6*dlog(exports_sa) +
sv7*dlog(imports_sa) + sv8*dlog(trade_indx_sa) + [var=1]
@state sv1 = sv1(-1) + [var=1]
@state sv2 = sv2(-1) + [var=1]
@state sv3 = sv3(-1) + [var=1]
@state sv4 = sv4(-1) + [var=1]
@state sv5 = sv5(-1) + [var=1]
@state sv6 = sv6(-1) + [var=1]
@state sv7 = sv7(-1) + [var=1]
@state sv8 = sv8(-1) + [var=1]
@mprior vec08
@vprior sym08
```

Sspace: SS09

```
@signal dl_gdp_sa =  c(1) + sv1*dlog(psei)  + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa)+
sv4*dlog(rm1_sa)  + sv5*dlog(wpi_sa) + sv6*dlog(exports_sa) +
sv7*dlog(imports_sa) + sv8*dlog(trade_indx_sa) + sv9*dlog(visitor_sa) + [var=1]
@state sv1 = sv1(-1) + [var=1]
@state sv2 = sv2(-1) + [var=1]
@state sv3 = sv3(-1) + [var=1]
@state sv4 = sv4(-1) + [var=1]
@state sv5 = sv5(-1) + [var=1]
```

```
@state sv6 = sv6(-1) + [var=1]
@state sv7 = sv7(-1) + [var=1]
@state sv8 = sv8(-1) + [var=1]
@state sv9 = sv9(-1) + [var=1]
@mprior vec09
@vprior sym09
```

Sspace: SS10

```
@signal dl_gdp_sa =  c(1) + sv1*dlog(psei)  + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa)+
sv4*dlog(rm1_sa)  + sv5*dlog(wpi_sa) + sv6*dlog(exports_sa) +
sv7*dlog(imports_sa) + sv8*dlog(trade_indx_sa) + sv9*dlog(visitor_sa) +
sv10*dlog(meralco_sa) + [var=1]
@state sv1 = sv1(-1) + [var=1]
@state sv2 = sv2(-1) + [var=1]
@state sv3 = sv3(-1) + [var=1]
@state sv4 = sv4(-1) + [var=1]
@state sv5 = sv5(-1) + [var=1]
@state sv6 = sv6(-1) + [var=1]
@state sv7 = sv7(-1) + [var=1]
@state sv8 = sv8(-1) + [var=1]
@state sv9 = sv9(-1) + [var=1]
@state sv10 = sv10(-1) + [var=1]
@mprior vec10
@vprior sym10
```

Sspace: SS11

```
@signal dl_gdp_sa =  c(1) + sv1*dlog(psei)  + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa)+
sv4*dlog(rm1_sa)  + sv5*dlog(wpi_sa) + sv6*dlog(exports_sa) +
sv7*dlog(imports_sa) + sv8*dlog(trade_indx_sa) + sv9*dlog(visitor_sa) +
sv10*dlog(meralco_sa) + sv11*dlog(new_buss_sa) + [var=1]
@state sv1 = sv1(-1) + [var=1]
@state sv2 = sv2(-1) + [var=1]
@state sv3 = sv3(-1) + [var=1]
@state sv4 = sv4(-1) + [var=1]
@state sv5 = sv5(-1) + [var=1]
@state sv6 = sv6(-1) + [var=1]
@state sv7 = sv7(-1) + [var=1]
@state sv8 = sv8(-1) + [var=1]
@state sv9 = sv9(-1) + [var=1]
@state sv10 = sv10(-1) + [var=1]
@state sv11 = sv11(-1) + [var=1]
@mprior vec11
@vprior sym11
```

State space model specifications (error represented as state variable with AR(1) process)

```
Sspace: SS01
@signal dl_gdp_sa =  c(1) + sv1*dlog(psei)  + sv2
@state sv1 = sv1(-1) + [var=1]
@state sv2 = c(2)*sv2(-1) + [var=1]
@mprior vec02
@vprior sym02
```

```
Sspace: SS02
@signal dl_gdp_sa =  c(1) + sv1*dlog(psei)  + sv2*dlog(fx_sa) + sv3
@state sv1 = sv1(-1) + [var=1]
@state sv2 = sv2(-1) + [var=1]
@state sv3 = c(2)*sv3(-1) + [var=1]
@mprior vec03
@vprior sym03
```

```
Sspace: SS03
@signal dl_gdp_sa = c(1) + sv1*dlog(psei)  + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa) + sv4
@state sv1 = sv1(-1) + [var=1]
@state sv2 = sv2(-1) + [var=1]
@state sv3 = sv3(-1) + [var=1]
@state sv4 = c(2)*sv4(-1) + [var=1]
@mprior vec04
@vprior sym04
```

```
Sspace: SS04
@signal dl_gdp_sa =  c(1) + sv1*dlog(psei)  + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa)+
sv4*dlog(rm1_sa) + sv5
@state sv1 = sv1(-1) + [var=1]
@state sv2 = sv2(-1) + [var=1]
@state sv3 = sv3(-1) + [var=1]
@state sv4 = sv4(-1) + [var=1]
@state sv5 = c(2)*sv5(-1) + [var=1]
@mprior vec05
@vprior sym05
```

```
Sspace: SS05
@signal dl_gdp_sa =  c(1) + sv1*dlog(psei)  + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa)+
sv4*dlog(rm1_sa)  + sv5*dlog(wpi_sa) + sv6
```

```
@state sv1 = sv1(-1) + [var=1]
@state sv2 = sv2(-1) + [var=1]
@state sv3 = sv3(-1) + [var=1]
@state sv4 = sv4(-1) + [var=1]
@state sv5 = sv5(-1) + [var=1]
@state sv6 = c(2)*sv6(-1) + [var=1]
@mprior vec06
@vprior sym06
```

Sspace: SS06

```
@signal dl_gdp_sa =  c(1) + sv1*dlog(psei)  + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa)+
sv4*dlog(rm1_sa)  + sv5*dlog(wpi_sa) + sv6*dlog(exports_sa) + sv7
@state sv1 = sv1(-1) + [var=1]
@state sv2 = sv2(-1) + [var=1]
@state sv3 = sv3(-1) + [var=1]
@state sv4 = sv4(-1) + [var=1]
@state sv5 = sv5(-1) + [var=1]
@state sv6 = sv6(-1) + [var=1]
@state sv7 = c(2)*sv7(-1) + [var=1]
@mprior vec07
@vprior sym07
```

Sspace: SS07

```
@signal dl_gdp_sa = c(1) + sv1*dlog(psei)  + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa)+
sv4*dlog(rm1_sa)  + sv5*dlog(wpi_sa) + sv6*dlog(exports_sa) + sv7*dlog(imports_sa) + sv8
@state sv1 = sv1(-1) + [var=1]
@state sv2 = sv2(-1) + [var=1]
@state sv3 = sv3(-1) + [var=1]
@state sv4 = sv4(-1) + [var=1]
@state sv5 = sv5(-1) + [var=1]
@state sv6 = sv6(-1) + [var=1]
@state sv7 = sv7(-1) + [var=1]
@state sv8 = c(2)*sv8(-1) + [var=1]
@mprior vec08
@vprior sym08
```

Sspace: SS08

```
@signal dl_gdp_sa =  c(1) + sv1*dlog(psei)  + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa)+
sv4*dlog(rm1_sa)  + sv5*dlog(wpi_sa) + sv6*dlog(exports_sa) + sv7*dlog(imports_sa) +
sv8*dlog(trade_indx_sa) + sv9
@state sv1 = sv1(-1) + [var=1]
@state sv2 = sv2(-1) + [var=1]
@state sv3 = sv3(-1) + [var=1]
```

```
@state sv4 = sv4(-1) + [var=1]
@state sv5 = sv5(-1) + [var=1]
@state sv6 = sv6(-1) + [var=1]
@state sv7 = sv7(-1) + [var=1]
@state sv8 = sv8(-1) + [var=1]
@state sv9 = c(2)*sv9(-1) + [var=1]
@mprior vec09
@vprior sym09
```

Sspace: SS09

```
@signal dl_gdp_sa =  c(1) + sv1*dlog(psei)  + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa)+
sv4*dlog(rm1_sa)  + sv5*dlog(wpi_sa) + sv6*dlog(exports_sa) + sv7*dlog(imports_sa) +
sv8*dlog(trade_indx_sa) + sv9*dlog(visitor_sa) + sv10
@state sv1 = sv1(-1) + [var=1]
@state sv2 = sv2(-1) + [var=1]
@state sv3 = sv3(-1) + [var=1]
@state sv4 = sv4(-1) + [var=1]
@state sv5 = sv5(-1) + [var=1]
@state sv6 = sv6(-1) + [var=1]
@state sv7 = sv7(-1) + [var=1]
@state sv8 = sv8(-1) + [var=1]
@state sv9 = sv9(-1) + [var=1]
@state sv10 = c(2)*sv10(-1) + [var=1]
@mprior vec10
@vprior sym10
```

Sspace: SS10

```
@signal dl_gdp_sa =  c(1) + sv1*dlog(psei)  + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa)+
sv4*dlog(rm1_sa)  + sv5*dlog(wpi_sa) + sv6*dlog(exports_sa) + sv7*dlog(imports_sa) +
sv8*dlog(trade_indx_sa) + sv9*dlog(visitor_sa) + sv10*dlog(meralco_sa) + sv11
@state sv1 = sv1(-1) + [var=1]
@state sv2 = sv2(-1) + [var=1]
@state sv3 = sv3(-1) + [var=1]
@state sv4 = sv4(-1) + [var=1]
@state sv5 = sv5(-1) + [var=1]
@state sv6 = sv6(-1) + [var=1]
@state sv7 = sv7(-1) + [var=1]
@state sv8 = sv8(-1) + [var=1]
@state sv9 = sv9(-1) + [var=1]
@state sv10 = sv10(-1) + [var=1]
@state sv11 = c(2)*sv11(-1) + [var=1]
@mprior vec11
@vprior sym11
```

Sspace: SS11

```
@signal dl_gdp_sa =  c(1) + sv1*dlog(psei)  + sv2*dlog(fx_sa) + sv3*dlog(cpi_sa)+
sv4*dlog(rm1_sa)  + sv5*dlog(wpi_sa) + sv6*dlog(exports_sa) + sv7*dlog(imports_sa) +
sv8*dlog(trade_indx_sa) + sv9*dlog(visitor_sa) + sv10*dlog(meralco_sa) + sv11*dlog(new_b
@state sv1 = sv1(-1) + [var=1]
@state sv2 = sv2(-1) + [var=1]
@state sv3 = sv3(-1) + [var=1]
@state sv4 = sv4(-1) + [var=1]
@state sv5 = sv5(-1) + [var=1]
@state sv6 = sv6(-1) + [var=1]
@state sv7 = sv7(-1) + [var=1]
@state sv8 = sv8(-1) + [var=1]
@state sv9 = sv9(-1) + [var=1]
@state sv10 = sv10(-1) + [var=1]
@state sv11 = sv11(-1) + [var=1]
@state sv12 = c(2)*sv12(-1) + [var=1]
@mprior vec12
@vprior sym12
```

Figure 13. Monthly predicted signals (blue) from state space models with error term as a state variable and actual data (red)

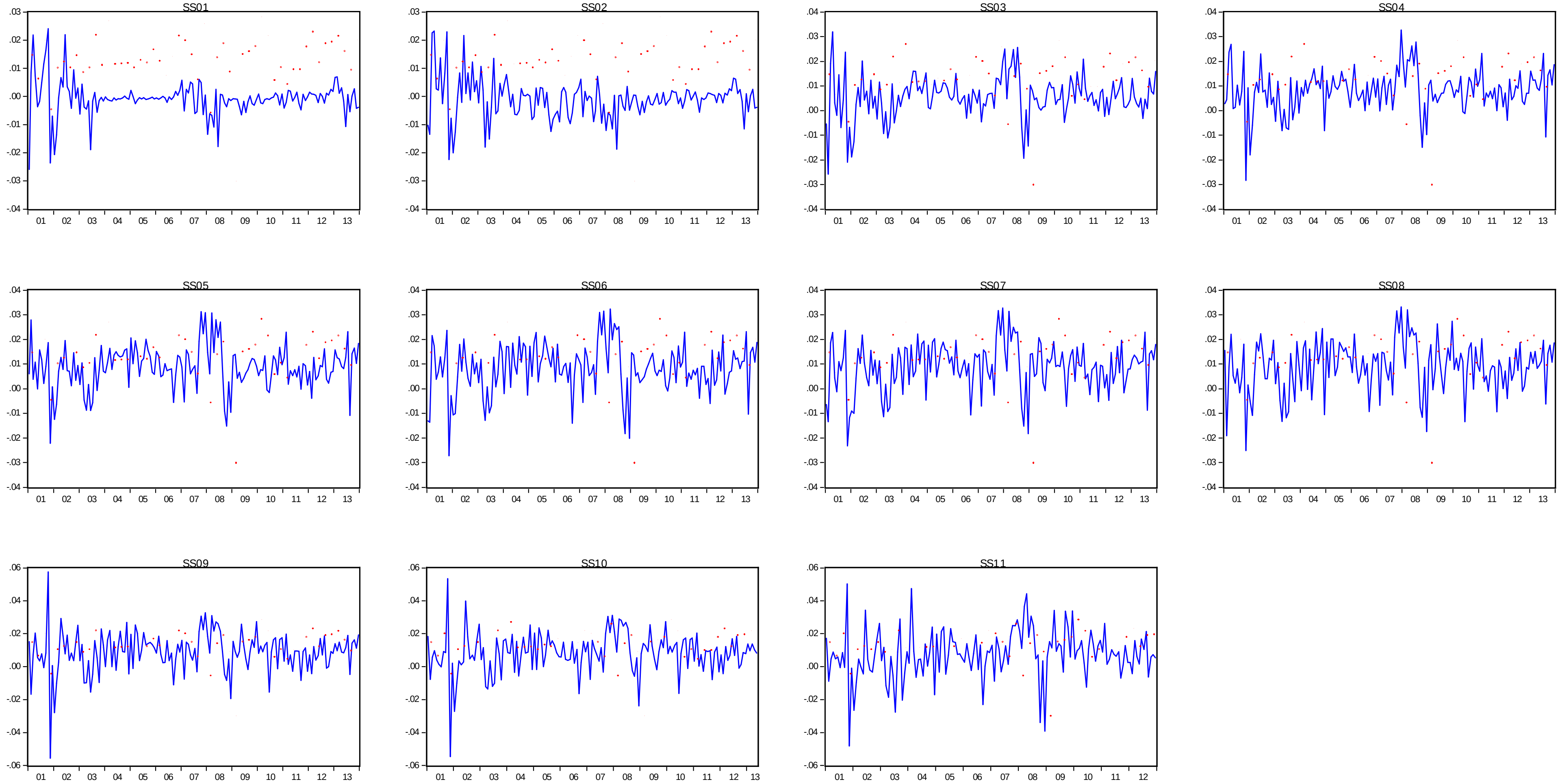

Figure 14. Aggregated predicted signals (blue) from state space models with error term as a state variable and actual quarterly data (black)

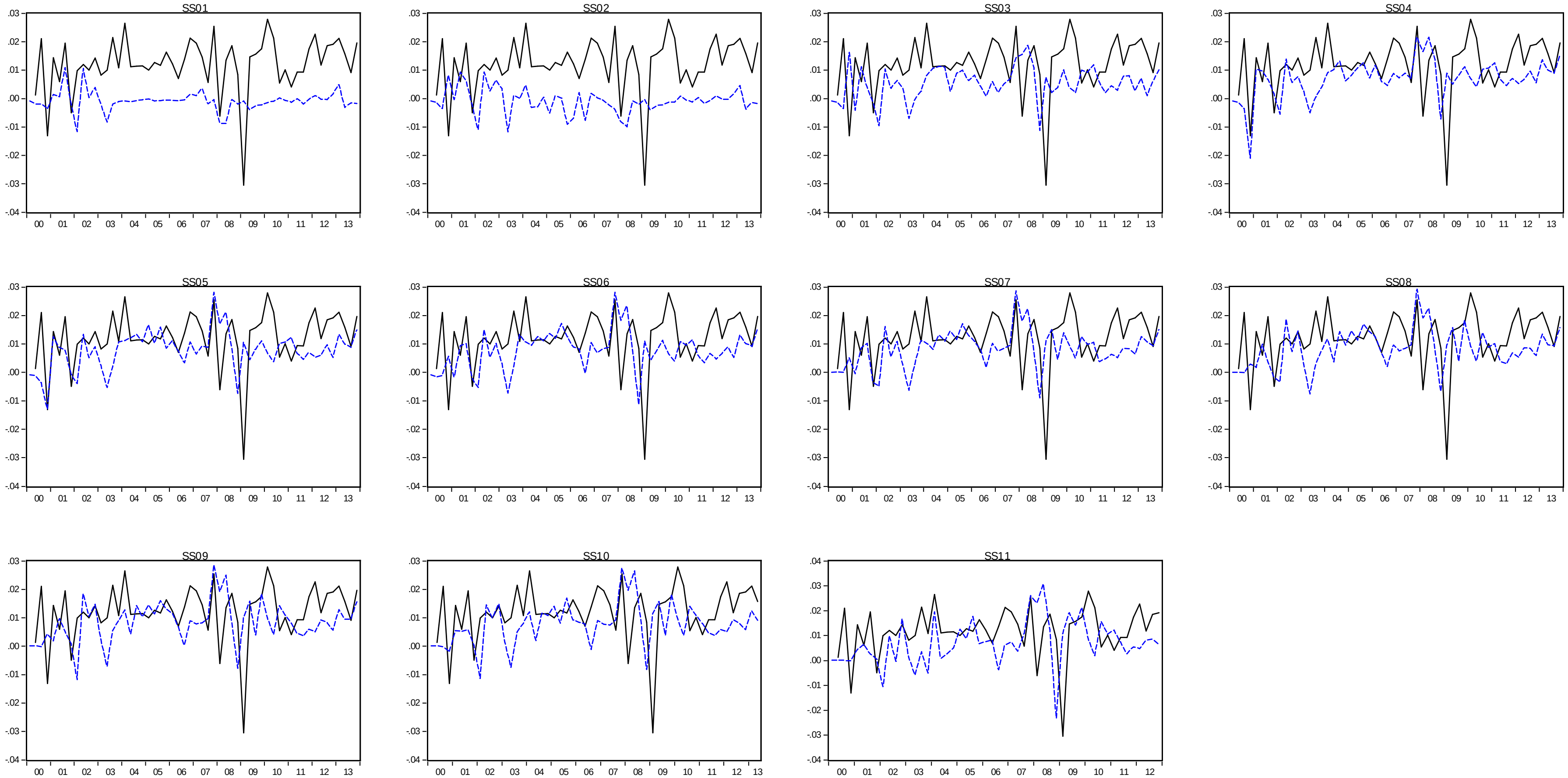

Table 7. Summary table of actual GDP growth rate data and predicted (aggregated) data, SS06 & SS11

Year/ Quarter	Actual	Predicted		Year/ Quarter	Actual	Predicted	
		SS06	SS11			SS06	SS11
2000Q1	0.0000	0.0181	0.0181	2007Q1	0.0196	0.0146	0.0144
2000Q2	0.0013	0.0374	0.0234	2007Q2	0.0145	0.0209	0.0213
2000Q3	0.0211	0.0207	0.0257	2007Q3	0.0057	0.0151	0.0158
2000Q4	-0.0131	0.0014	0.0093	2007Q4	0.0254	0.0158	0.0151
2001Q1	0.0143	0.0236	0.0276	2008Q1	-0.0061	0.0083	0.0084
2001Q2	0.0061	0.0084	0.0163	2008Q2	0.0135	-0.0044	0.0008
2001Q3	0.0196	0.0244	0.0034	2008Q3	0.0186	0.0010	0.0062
2001Q4	-0.0049	0.0046	0.0200	2008Q4	0.0084	0.0142	0.0003
2002Q1	0.0099	-0.0046	0.0443	2009Q1	-0.0305	0.0078	-0.0024
2002Q2	0.0120	0.0203	-0.0258	2009Q2	0.0147	0.0163	0.0338
2002Q3	0.0100	0.0077	-0.0294	2009Q3	0.0157	0.0082	0.0186
2002Q4	0.0142	0.0176	0.0260	2009Q4	0.0175	0.0140	0.0198
2003Q1	0.0082	0.0106	0.0145	2010Q1	0.0279	0.0170	0.0162
2003Q2	0.0100	0.0064	0.0057	2010Q2	0.0212	0.0158	0.0122
2003Q3	0.0214	0.0184	0.0203	2010Q3	0.0054	0.0140	0.0204
2003Q4	0.0108	0.0180	0.0159	2010Q4	0.0101	0.0102	0.0122
2004Q1	0.0265	0.0095	0.0109	2011Q1	0.0041	0.0104	0.0136
2004Q2	0.0111	0.0111	0.0071	2011Q2	0.0093	0.0116	0.0129
2004Q3	0.0114	0.0089	0.0066	2011Q3	0.0093	0.0145	0.0141
2004Q4	0.0115	0.0144	0.0172	2011Q4	0.0174	0.0127	0.0128
2005Q1	0.0100	0.0249	0.0252	2012Q1	0.0226	0.0153	0.0154
2005Q2	0.0127	0.0097	0.0080	2012Q2	0.0118	0.0115	0.0119
2005Q3	0.0117	0.0116	0.0138	2012Q3	0.0185	0.0150	0.0159
2005Q4	0.0163	0.0122	0.0077	2012Q4	0.0191	0.0167	0.0178
2006Q1	0.0123	0.0140	0.0112	2013Q1	0.0212	0.0125	
2006Q2	0.0070	0.0136	0.0155	2013Q2	0.0158	0.0161	
2006Q3	0.0139	0.0200	0.0159	2013Q3	0.0091	0.0133	
2006Q4	0.0213	0.0141	0.0140	2013Q4	0.0197	0.0134	

Table 8. Summary table of difference of actual and predicted (aggregated) data from SS06 & SS11

Year/ Quarter	Difference from actual SS06	SS11	Year/ Quarter	Difference from actual SS06	SS11
2000Q1	0.0181	0.0181	2007Q1	-0.0049	-0.0052
2000Q2	0.0361	0.0221	2007Q2	0.0063	0.0068
2000Q3	-0.0004	0.0047	2007Q3	0.0095	0.0101
2000Q4	0.0144	0.0224	2007Q4	-0.0096	-0.0104
2001Q1	0.0093	0.0132	2008Q1	0.0144	0.0144
2001Q2	0.0023	0.0102	2008Q2	-0.0179	-0.0128
2001Q3	0.0049	-0.0161	2008Q3	-0.0175	-0.0123
2001Q4	0.0095	0.0250	2008Q4	0.0057	-0.0081
2002Q1	-0.0145	0.0344	2009Q1	0.0383	0.0281
2002Q2	0.0082	-0.0378	2009Q2	0.0016	0.0191
2002Q3	-0.0023	-0.0394	2009Q3	-0.0074	0.0030
2002Q4	0.0034	0.0117	2009Q4	-0.0035	0.0024
2003Q1	0.0024	0.0063	2010Q1	-0.0108	-0.0116
2003Q2	-0.0035	-0.0042	2010Q2	-0.0054	-0.0090
2003Q3	-0.0030	-0.0011	2010Q3	0.0086	0.0150
2003Q4	0.0071	0.0050	2010Q4	0.0001	0.0021
2004Q1	-0.0170	-0.0157	2011Q1	0.0064	0.0095
2004Q2	0.0000	-0.0040	2011Q2	0.0023	0.0036
2004Q3	-0.0024	-0.0048	2011Q3	0.0052	0.0048
2004Q4	0.0029	0.0057	2011Q4	-0.0047	-0.0046
2005Q1	0.0149	0.0152	2012Q1	-0.0073	-0.0072
2005Q2	-0.0030	-0.0047	2012Q2	-0.0004	0.0001
2005Q3	-0.0002	0.0020	2012Q3	-0.0036	-0.0026
2005Q4	-0.0041	-0.0087	2012Q4	-0.0025	-0.0014
2006Q1	0.0018	-0.0011	2013Q1	-0.0086	
2006Q2	0.0066	0.0084	2013Q2	0.0003	
2006Q3	0.0062	0.0020	2013Q3	0.0042	
2006Q4	-0.0072	-0.0073	2013Q4	-0.0063	