Irrelevance of conjectural variation in duopoly under relative profit maximization and consistent conjectures

Satoh, Atsuhiro and Tanaka, Yasuhito

Faculty of Economics, Doshisha University, Japan

11 May 2014

Irrelevance of conjectural variation in duopoly under relative profit maximization and consistent conjectures

Atsuhiro Satoh*
Faculty of Economics, Doshisha University,
Kamigyo-ku, Kyoto, 602-8580, Japan.
and
Yasuhito Tanaka†
Faculty of Economics, Doshisha University,
Kamigyo-ku, Kyoto, 602-8580, Japan.

Abstract

We study the equilibrium with quantity setting behavior and price setting behavior of firms in duopoly under relative profit maximization with constant conjectural variations, and show mainly the following results. 1) Conjectural variations of firms are irrelevant to the equilibrium of a duopoly. 2) Quantity setting behavior and price setting behavior are equivalent with any conjectural variation of each firm. 3) Any pair of conjectural variations of firms which satisfies some relation is consistent. In particular, if firms have the same cost functions or the cost functions are linear, and both firms determine the outputs or both firms determine the prices, any conjectural variations which are common to both firms are consistent. Therefore, there are multiple consistent conjectures.

Keywords: duopoly, relative profit maximization, conjectural variation, consistent conjecture.

JEL Classification code: D43, L13, L21.

1 Introduction

We study the equilibrium with quantity setting behavior and price setting behavior of firms in duopoly under relative profit maximization with constant conjectural variations. Conjectural variation in oligopoly and its consistency have been studied in many papers such as Bresnahan (1981),Bresnahan (1983), Robson (1983), Perry (1982),

*atsato@mail.doshisha.ac.jp
†yasuhito@mail.doshisha.ac.jp

Boyer and Moreaux (1983), Tanaka (1985) and Tanaka (1988). Bresnahan (1981) defines a consistent conjectural equilibrium (CCE) and has shown the existence of CCE in a case where demand and cost functions are linear and conjectural variations are polynomial. Bresnahan (1983) has shown the existence of CCE when demand functions are quadratic using a formulation by Robson (1983). In Perry (1982) it was shown that in a duopoly producing a homogeneous good under constant marginal costs the competitive conjecture is the only consistent conjecture. In Tanaka (1985) it was shown that in a free entry oligopoly only the competitive conjecture is consistent. Klemperer and Meyer (1988) presented an interpretation of Bresnahan's formulation of consistent conjectures as a dominant strategy equilibrium and showed multiplicity of consistent conjectures. Kamien and Schwartz (1983), among others, showed that if the demand is linear, constant consistent conjectural variations in quantity correspond to constant consistent conjectural variations in price in the sense that the same equilibrium price and quantity will be attained if either conjectural variation is constant.

We consider a duopoly in which firms produce differentiated substitutable goods under linear demand functions and general cost functions so as to maximize their relative profits, and we consider constant conjectural variations about outputs and prices. We consider three frameworks.

1. (Quantity setting framework) Both firms determine their outputs (quantities).
2. (Price setting framework) Both firms determine the prices of their goods.
3. (Mixed framework) One firm determines its output, and the other firms determines the price of its good.

We show the following results.

1. Conjectural variations of firms are irrelevant to the equilibrium of a duopoly.
2. The equilibrium outputs and prices in all of three frameworks are equal, that is, quantity setting behavior and price setting behavior are equivalent with any conjectural variation of each firm.
3. Any pair of conjectural variations of firms which satisfies some relation is consistent, and in particular, if firms have the same cost functions or the cost functions are linear, and both firms determine their outputs or both firms determine their prices, any conjectural variations which are common to both firms are consistent. Therefore, there are multiple consistent conjectures.

In recent years, maximizing relative profit instead of absolute profit has aroused the interest of economists. Please see Gibbons and Murphy (1990), Lu (2011), Matsumura, Matsushima and Cato (2013), Miller and Pazgal (2001), Vega-Redondo (1997) and Schaffer (1989).

In Vega-Redondo (1997), it is argued that, in a homogeneous good case, if firms maximize the relative profits, a competitive equilibrium can be induced. But in the case of differentiated goods, the result under relative profit maximization is different from the competitive result. Miller and Pazgal (2001) has shown the equivalence of price strategy and quantity strategy in a delegation game when owners of firms control managers of firms seek to maximize an appropriate combination of absolute and relative profits. But in their analyses owners of firms themselves still seek to maximize absolute profits of their firms. On the other hand, we do not consider a delegation problem, and assume that owners of firms seek to maximize their relative profits.

We think that seeking for relative profit or utility is based on the nature of human. Even if a person earns a big money, if his brother/sister or close friend earns a bigger money than him, he is not sufficiently happy and may be disappointed. On the other hand, even if he is very poor, if his neighbor is more poor, he may be consoled by that fact. About the behavior of firms, we think that firms in an industry not only seek its own performance but also want to outperform the rival firms. TV audience-rating race and market share competition by breweries, automobile manufacturers, convenience store chains and mobile-phone carriers, especially in Japan, are examples of such behavior of firms.

In Section 3 we consider a case where both firms determine their outputs (quantities). In Section 4 we consider a case where both firms determine the prices of their goods. And in Section 5 we consider a case where one firm determines its output and the other firm determines the price of its good.

2 The model

There are two firms, A and B. They produce differentiated substitutable goods. Denote the output and price of the good of Firm A by x_A and p_A, and the output and price of the good of Firm B by x_B and p_B. The cost functions of Firm A and and B are, respectively, denoted by $c(x_A)$ and $c(x_B)$. They may be different. The constant conjectural variations of Firm A and B in the quantity setting framework are denoted, respectively, by δ_A and δ_B. In the price setting framework they are denoted, respectively, by η_A and η_B. Assume $-1 < \delta_A < 1$, $-1 < \delta_B < 1$, $-1 < \eta_A < 1$ and $-1 < \eta_B < 1$. In the mixed framework the conjectural variations are denoted by ζ_A and ζ_B. We do not assume $-1 < \zeta_A < 1$ nor $-1 < \zeta_B < 1$.

The inverse demand functions of the goods produced by firms are

$$p_A = a - x_A - bx_B,$$

and

$$p_B = a - x_B - bx_A,$$

where $a > c$ and $0 < b < 1$. x_A represents demand for the good of Firm A, and x_B represents demand for the good of Firm B. The prices of the goods are determined so that demand of consumers for each firm's good and supply of each firm are equilibrated.

The ordinary demand functions for the goods of firms are obtained from these inverse demand functions as follows,

$$x_A = \frac{1}{1-b^2}[(1-b)a - p_A + bp_B],$$

and

$$x_B = \frac{1}{1-b^2}[(1-b)a - p_B + bp_A].$$

Next consider a case where Firm A determines its output and Firm B determines the price of its good. Then, the ordinary demand function for Firm A is

$$x_A = a - p_A - bx_B, \tag{1}$$

and the inverse demand function for Firm B is

$$p_B = (1-b)a - (1-b^2)x_B + bp_A. \tag{2}$$

3 Quantity setting framework

Assume that the strategic variable of each firm is the output of its good. The relative profit of Firm A (or B) is the difference between its profit and the profit of Firm B (or A). We denote the relative profit of Firm A by Π_A and that of Firm B by Π_B. They are written as follows,

$$\begin{aligned}\Pi_A =&\pi_A - \pi_B = (a - x_A - bx_B)x_A - (a - x_B - bx_A)x_B - c(x_A) + c(x_B)\\ =&a(x_A - x_B) - x_A^2 + x_B^2 - c(x_A) + c(x_B),\end{aligned}$$

and

$$\begin{aligned}\Pi_B =&\pi_B - \pi_A = (a - x_B - bx_A)x_B - (a - x_A - bx_B)x_A - c(x_B) + c(x_A)\\ =&a(x_B - x_A) - x_B^2 + x_A^2 - c(x_B) + c(x_A).\end{aligned}$$

Firm A determines its output so as to maximize its relative profit assuming that the reaction of the output of Firm B to the output of Firm A is

$$\frac{\partial x_B}{\partial x_A} = \delta_A,$$

and Firm B determines its output so as to maximize its relative profit assuming that the reaction of the output of Firm A to the output of Firm B is

$$\frac{\partial x_A}{\partial x_B} = \delta_B.$$

The conditions for relative profit maximization of Firm A and B are, respectively,

$$a - c'(x_A) - 2x_A - (a - c'(x_B))\delta_A + 2\delta_A x_B = 0, \tag{3}$$

and

$$a - c'(x_B) - 2x_B - (a - c'(x_A))\delta_B + 2\delta_B x_A = 0. \tag{4}$$

From (3) and (4) we have

$$(1 - \delta_A\delta_B)(a - c'(x_A)) = 2(1 - \delta_A\delta_B)x_A,$$

and

$$(1 - \delta_A\delta_B)(a - c'(x_B)) = 2(1 - \delta_A\delta_B)x_B.$$

Then, the equilibrium outputs are

$$x_A = \frac{a - c'(x_A)}{2}.$$

and

$$x_B = \frac{a - c'(x_B)}{2}.$$

They do not depend on the values of δ_A or δ_B. Thus, the conjectural variations of firms are irrelevant to the equilibrium outputs under relative profit maximization in the quantity setting framework.

The equilibrium prices are obtained as follows.

$$p_A = \frac{(1-b)a + c'(x_A) + bc'(x_B)}{2}.$$

and

$$p_B = \frac{(1-b)a + c'(x_B) + bc'(x_A)}{2}.$$

Again from (3) and (4) the real reaction of the output of Firm A to the output of Firm B and the real reaction of the output of Firm B to the output of Firm A are obtained as follows,

$$\frac{\partial x_A}{\partial x_B} = \frac{2 + c''(x_B)}{2 + c''(x_A)}\delta_A.$$

and

$$\frac{\partial x_B}{\partial x_A} = \frac{2 + c''(x_A)}{2 + c''(x_B)}\delta_B,$$

The conditions for consistency of conjectural variations are

$$\frac{2 + c''(x_B)}{2 + c''(x_A)}\delta_A = \delta_B,$$

and

$$\frac{2 + c''(x_A)}{2 + c''(x_B)}\delta_B = \delta_A.$$

They are the same equations. Therefore, any pair of conjectural variations, δ_A and δ_B, which satisfy the condition,

$$\frac{\delta_A}{\delta_B} = \frac{2 + c''(x_A)}{2 + c''(x_B)}, \tag{5}$$

are consistent. If the firms have the same cost functions, the equilibrium outputs of firms are equal, and (5) is reduced to

$$\delta_A = \delta_B. \tag{6}$$

Also if the cost functions are linear, the second order derivatives of the cost functions are zero, and we get (6). Thus, if the firms have the same cost functions, or the cost functions are linear. any common conjectural variations are consistent.

4 Price setting framework

In this section we assume that the strategic variable of each firm is the price of its good. Similarly to the previous section, the relative profits of Firm A and B are denoted by Π_A and Π_B. They are written as follows,

$$\begin{aligned}\Pi_A =&\pi_A - \pi_B\\ =&\frac{1}{1-b^2}[(1-b)a(p_A - p_B) - p_A^2 + p_B^2] - c(x_A) + c(x_B),\end{aligned}$$

and

$$\begin{aligned}\Pi_B =&\pi_B - \pi_A\\ =&\frac{1}{1-b^2}[(1-b)a(p_B - p_A) - p_B^2 + p_A^2] - c(x_B) + c(x_A),\end{aligned}$$

with

$$x_A = \frac{1}{1-b^2}[(1-b)a - p_A + bp_B],$$

and

$$x_B = \frac{1}{1-b^2}[(1-b)a - p_B + bp_A].$$

Firm A determines the price of its good so as to maximize its relative profit assuming that the reaction of the price of the good of Firm B to the price of the good of Firm A is

$$\frac{\partial p_B}{\partial p_A} = \eta_A,$$

and Firm B determines the price of its good so as to maximize its relative profit assuming that the reaction of the price of the good of Firm A to the price of the good of Firm B is

$$\frac{\partial p_A}{\partial p_B} = \eta_B.$$

The conditions for profit maximization of Firm A and B are, respectively,

$$\begin{aligned}&(1-b)a - 2p_A + c'(x_A) + bc'(x_B)\\ &- [(1-b)a - 2p_B + c'(x_B) + bc'(x_A)]\eta_A = 0,\end{aligned} \tag{7}$$

and

$$\begin{aligned}&(1-b)a - 2p_B + c'(x_B) + bc'(x_A)\\ &- [(1-b)a - 2p_A + c'(x_A) + bc'(x_B)]\eta_B = 0.\end{aligned} \tag{8}$$

From (7) and (8) we have

$$(1-\eta_A\eta_B)[(1-b)a + c'(x_A) + bc'(x_B)] - 2(1-\eta_A\eta_B)p_A = 0,$$

and

$$(1-\eta_A\eta_B)[(1-b)a + c'(x_B) + bc'(x_A)] - 2(1-\eta_A\eta_B)p_A = 0.$$

Then, the equilibrium prices are

$$p_A = \frac{(1-b)a + c'(x_A) + bc'(x_B)}{2},$$

and

$$p_B = \frac{(1-b)a + c'(x_B) + bc'(x_A)}{2}.$$

They do not depend on the values of η_A or η_B. Thus, the conjectural variations of firms are irrelevant to the equilibrium prices under relative profit maximization in the price setting framework. The equilibrium outputs are obtained as follows.

$$x_A = \frac{a - c'(x_A)}{2}.$$

and

$$x_B = \frac{a - c'(x_B)}{2}.$$

Therefore, the equilibrium prices and outputs under relative profit maximization in the price setting framework are equal to those in the quantity setting framework.

Again from (7) and (8) the real reaction of the price of the good of Firm A to the price of the good of Firm B and the real reaction of the price of the good of Firm B to the price of the good of Firm A are obtained as follows,

$$\frac{\partial p_A}{\partial p_B} = \frac{[2 + c_B''(x_B) - b^2 c_A''(x_A)]\eta_A - b(c_B''(x_B) - c_A''(x_A))}{2 + c_A''(x_A) - b^2 c_B''(x_B) - b\eta_A(c_A''(x_A) - c_B''(x_B))},$$

and

$$\frac{\partial p_B}{\partial p_A} = \frac{[2 + c_A''(x_A) - b^2 c_B''(x_B)]\eta_B - b(c_A''(x_A) - c_B''(x_B))}{2 + c_B''(x_B) - b^2 c_A''(x_A) - b\eta_B(c_B''(x_B) - c_A''(x_A))}.$$

In these calculations we used the ordinary demand functions. The conditions for consistency of conjectural variations are

$$\frac{[2 + c_B''(x_B) - b^2 c_A''(x_A)]\eta_A - b(c_B''(x_B) - c_A''(x_A))}{2 + c_A''(x_A) - b^2 c_B''(x_B) - b\eta_A(c_A''(x_A) - c_B''(x_B))} = \eta_B,$$

and

$$\frac{[2 + c_A''(x_A) - b^2 c_B''(x_B)]\eta_B - b(c_A''(x_A) - c_B''(x_B))}{2 + c_B''(x_B) - b^2 c_A''(x_A) - b\eta_B(c_B''(x_B) - c_A''(x_A))} = \eta_A.$$

They are the same equations, and are reduced to

$$\begin{aligned}&[2 + c_B''(x_B) - b^2 c_A''(x_A)]\eta_A + bc_A''(x_A) + b\eta_A\eta_B c_A''(x_A) \\ &= [2 + c_A''(x_A) - b^2 c_B''(x_B)]\eta_B + bc_B''(x_B) + b\eta_A\eta_B c_B''(x_B).\end{aligned} \tag{9}$$

Therefore, any pair of conjectural variations, η_A and η_B, which satisfy (9) are consistent. If the firms have the same cost functions, then the equilibrium outputs and the equilibrium prices of the goods of firms are, respectively, equal, and (9) is reduced to

$$\eta_A = \eta_B. \tag{10}$$

Also if the cost functions are linear, the second order derivatives of the cost functions are zero, and we get (10). Thus, if the firms have the same cost functions, or the cost functions are linear, any common conjectural variations are consistent.

5 Mixed framework

In this section we assume that the strategic variable of Firm A is the price of its good and the strategic variable of Firm B is its output. Similarly to the previous sections, the relative profits of Firm A and B are denoted by Π_A and Π_B. Using (1) and (2), they are written as follows,

$$\begin{aligned}\Pi_A =& \pi_A - \pi_B = (a - p_A - bx_B)p_A \\ &- [(1-b)a - (1-b^2)x_B + bp_A]x_B - c_A(x_A) + c_B(x_B),\end{aligned}$$

and

$$\begin{aligned}\Pi_B =& \pi_B - \pi_A = [(1-b)a - (1-b^2)x_B + bp_A]x_B \\ &- (a - p_A - bx_B)p_A - c_B(x_B) + c_A(x_A),\end{aligned}$$

with

$$x_A = a - p_A - bx_B.$$

Firm A determines the price of its good so as to maximize its relative profit assuming that the reaction of the output of the good of Firm B to the price of the good of Firm A is

$$\frac{\partial x_B}{\partial p_A} = \zeta_A,$$

and Firm B determines its output so as to maximize its relative profit assuming that the reaction of the price of the good of Firm A to the output of Firm B is

$$\frac{\partial p_A}{\partial x_B} = \zeta_B.$$

The conditions for relative profit maximization of Firm A and B are, respectively,

$$\begin{aligned}&a - 2p_A - 2bx_B + c_A'(x_A) + [-2bp_A - (1-b)a + 2(1-b^2)x_B + c_B'(x_B) \\ &+ bc_A'(x_A)]\zeta_A = 0,\end{aligned} \tag{11}$$

and

$$\begin{aligned}&-2(1-b^2)x_B + (1-b)a + 2bp_A - c_B'(x_B) - bc_A'(x_A) \\ &+ [-a + 2bx_B + 2p_A - c_A'(x_A)]\zeta_B = 0.\end{aligned} \tag{12}$$

From (11) and (12) we have

$$(1 + b\zeta_A)[2p_A - c_A'(x_A)] = 2[(1-b^2)\zeta_A - b]x_B + c_B'(x_B)\zeta_A + a - (1-b)a\zeta_A,$$

and

$$(b+\zeta_B)[2p_A - c'_A(x_A)] = 2[(1-b^2) - b\zeta_B]x_B + a\zeta_B - (1-b)a + c'_B(x_B).$$

Then, the equilibrium price of the good of Firm A is

$$p_A = \frac{(1-b)a + c'_A(x_A) + bc'_B(x_B)}{2}.$$

and the equilibrium output of Firm B is

$$x_B = \frac{a - c'_B(x_B)}{2}.$$

They do not depend on the values of ζ_A or ζ_B. Thus, the conjectural variations of firms are irrelevant to the equilibrium prices under relative profit maximization in the mixed framework. The equilibrium output of Firm A and the equilibrium price of the good of Firm B are obtained as follows.

$$x_A = \frac{a - c'_A(x_A)}{2}.$$

and

$$p_B = \frac{(1-b)a + c'_B(x_B) + bc'_A(x_A)}{2}.$$

Therefore, we find that the equilibrium prices and outputs under relative profit maximization in the mixed framework are equal to those in the quantity setting framework and the price setting framework.

Again from (11) and (12) the real reaction of the price of the good of Firm A to the output of the good of Firm B and the real reaction of the output of Firm B to the price of the good of Firm A are obtained as follows,

$$\frac{\partial p_A}{\partial x_B} = \frac{2[(1-b^2)\zeta_A - b] + \zeta_A c''_B(x_B) - b(1+b\zeta_A)c''_A(x_A)}{(1+b\zeta_A)(2+c''_A(x_A))},$$

and

$$\frac{\partial x_B}{\partial p_A} = \frac{(b+\zeta_B)(2+c''_A(x_A))}{2[(1-b^2) - b\zeta_B] + c''_B(x_B) - b(b+\zeta_B)c''_A(x_A)}.$$

Therefore, the conditions for consistency of conjectural variations are

$$\frac{2[(1-b^2)\zeta_A - b] + \zeta_A c''_B(x_B) - b(1+b\zeta_A)c''_A(x_A)}{(1+b\zeta_A)(2+c''_A(x_A))} = \zeta_B,$$

and

$$\frac{(b+\zeta_B)(2+c''_A(x_A))}{2[(1-b^2) - b\zeta_B] + c''_B(x_B) - b(b+\zeta_B)c''_A(x_A)} = \zeta_A.$$

We find that they are the same conditions and are reduced to

$$2[(1-b^2)\zeta_A - b] + \zeta_A c''_B(x_B) - b(1+b\zeta_A)c''_A(x_A) - (1+b\zeta_A)(2+c''_A(x_A))\zeta_B = . \quad (13)$$

Thus, any pair of conjectural variations which satisfy (13) are consistent under relative profit maximization in the mixed framework.

6 Concluding Remarks

In this paper, assuming constant conjectural variations, we have shown the irrelevance of conjectural variations to the equilibrium outputs and prices in duopoly under relative profit maximization, and that there are multiple consistent conjectures. We hope to generalize these results to the case of non-constant conjectural variations.

In Tanaka (2013) it was shown that under relative profit maximization Cournot and Bertrand equilibria in duopoly are equivalent. The results of this paper is an extension of that result to a duopoly with arbitrary conjectural variations. The equivalence of Cournot and Bertrand equilibria under relative profit maximization, however, may not hold in oligopoly with more than two firms.

A game of relative profit maximization in a duopoly is a two-person zero-sum game. In a two-person zero-sum game the payoff of one player is an opposite of the payoff of the other player. It seems to be a reason why conjectural variations are irrelevant to the equilibrium of duopoly under relative profit maximization. A game of relative profit maximization in an oligopoly is a multi-person zero-sum game. If the number of players is larger than two, the payoff of one player is not an opposite of the payoff of another player. Thus, probably in an oligopoly the irrelevance of conjectural variation does not hold unless the oligopoly is symmetric (all firms have the same cost functions).

Acknowledgment

The authors would like to thank the referees of this journal for their valuable comments which helped to improve the manuscript.

References

Boyer, M. and M. Moreaux (1983), "Consistent versus non-consistent conjectures in duopoly theory: some examples", *Journal of Industrial Economics* **32**, 97-110.

Bresnahan, T. (1981), "Duopoly models with consistent conjectures", *American Economic Review* **71**, 934-945.

Bresnahan, T. (1981) ,"Existence of Consistent Conjectures: Reply", *American Economic Review* **73**, 457-458.

Gibbons, R and K. J. Murphy (1990), "Relative performance evaluation for chief executive officers", *Industrial and Labor Relations Review*, **43**, 30S-51S.

Lu, Y. (2011), "The relative-profit-maximization objective of private firms and endogenous timing in a mixed oligopoly", *The Singapore Economic Review*, **56**, 203-213.

Kamien, M. Y. and N, Schwartz (1983), "Conjectural variations", *Canadian Economic Journal*, **16**, 191-211.

Klemperer, P. D. and M. A. Meyer (1988), "Consistent conjectures equilibria: A reformulation showing non-uniqueness", *Economics Letters*, **27**, 111-115.

Matsumura, T., N. Matsushima and S. Cato (2013), "Competitiveness and R&D competition revisited" *Economic Modeling*, **31**, 541-547.

Miller, N. H. and A. I. Pazgal (2001), "The equivalence of price and quantity competition with delegation", *Rand Journal of Economics*, **32**, 284-301.

Perry, M. (1982), "Oligopoly and consistent conjectural variations", *Bell Journal of Economics* **13**, 197-205.

Robson, A. (1983), "Existence of Consistent Conjectures: Comment", *American Economic Review* **73**, 454-456.

Schaffer, M.E. (1989), "Are profit maximizers the best survivors?", *Journal of Economic Behavior and Organization* **12**, 29-45.

Tanaka, Y. (1985), "Consistent conjectures in free entry oligopoly", *Economics Letters*, **17**, 15-18.

Tanaka, Y. (1988), "On multiplicity of consistent conjectures in free entry oligopoly", *Economics Letters*, **28**, 109-115.

Tanaka, Y. (2013), "Equivalence of Cournot and Bertrand equilibria in differentiated duopoly under relative profit maximization with linear demand", *Economics Bulletin*, **33**, 1479-1486.

Vega-Redondo, F. (1997), "The evolution of Walrasian behavior", *Econometrica* **65**, 375-384.