Relative profit maximization in asymmetric oligopoly: Cournot and Bertrand equilibria

Satoh, Atsuhiro and Tanaka, Yasuhito

11 May 2014

Relative profit maximization in asymmetric oligopoly: Cournot and Bertrand equilibria

Atsuhiro Satoh*
Faculty of Economics, Doshisha University,
Kamigyo-ku, Kyoto, 602-8580, Japan.
and
Yasuhito Tanaka†
Faculty of Economics, Doshisha University,
Kamigyo-ku, Kyoto, 602-8580, Japan.

Abstract

We analyze Bertrand and Cournot equilibria in an asymmetric oligopoly with more than two firms in which the firms produce differentiated substitutable goods and seek to maximize their relative profits instead of their absolute profits. Assuming linear demand functions and constant marginal costs we show the following results. If the marginal cost of a firm is lower (higher) than the average marginal cost over the industry, its output at the Bertrand equilibrium is larger (smaller) than that at the Cournot equilibrium, and the price of its good at the Bertrand equilibrium is lower (higher) than that at the Cournot equilibrium.

Keywords: relative profit maximization, asymmetric oligopoly, Cournot and Bertrand equilibria

JEL Classification code: D43, L13.

*atsato@mail.doshisha.ac.jp
†yasuhito@mail.doshisha.ac.jp

1. Introduction

We analyze Bertrand and Cournot equilibria in an asymmetric oligopoly with more than two firms in which the firms produce differentiated substitutable goods and seek to maximize their relative profits instead of their absolute profits.

For analyses about relative profit maximization please see Gibbons and Murphy (1990), Lu (2011), Matsumura, Matsushima and Cato (2013) and Schaffer (1989). Some other related results are mentioned in Section 6.

We think that seeking for relative profit or utility is based on the nature of human. Even if a person earns a big money, if his brother/sister or close friend earns a bigger money than him, he is not sufficiently happy and may be disappointed. On the other hand, even if he is very poor, if his neighbor is more poor, he may be consoled by that fact. About the behavior of firms, we think that firms in an industry not only seek its own performance but also want to outperform the rival firms. TV audience-rating race and market share competition by breweries, automobile manufacturers, convenience store chains and mobile-phone carriers, especially in Japan, are examples of such behavior of firms.

In the next section we present the model, in Section 3 and 4 we investigate the outputs and prices at Bertrand and Cournot equilibria, and in Section 5 we compare Bertrand and Cournot equilibria. In Section 6 we mention some related results in other works.

2. The model

There are n firms. n is an integer larger than 1. They produce differentiated substitutable goods. The output and the price of the good of Firm i are denoted by x_i and p_i. The inverse demand functions of the goods are

$$p_i = a - x_i - b\sum_{j=1,j\neq i}^{n} x_j,\ i = 1, 2, \dots, n, \tag{1}$$

We assume $a > 0$ and $0 < b < 1$. From (1) we obtain the following ordinary demand functions (See Appendix 1).

$$x_i = \frac{1}{(1-b)[1+(n-1)b]}\left[(1-b)a - [1+(n-2)b]p_i + b\sum_{j=1,j\neq i}^{n} p_j\right], \tag{2}$$
$$i = 1, 2, \dots, n.$$

The inverse and ordinary demand functions are symmetric for the firms.

3. Cournot equilibrium under relative profit maximization

In this section we assume that each firm determines its output given the outputs of other firms so as to maximize its relative profit. Let denote the absolute profit of Firm i by π_i. Then,

$$\pi_i = \left(a - x_i - b\sum_{j=1, j\neq i}^{n} x_j\right)x_i - c_i x_i,\ i = 1, 2, \dots, n.$$

The relative profit of Firm i is defined as the difference between its absolute profit and the average of the absolute profits of other firms. Denote it by Π_i. Then,

$$\Pi_i = \left(a - x_i - b\sum_{j=1, j\neq i}^{n} x_j\right)x_i - c_i x_i - \frac{1}{n-1}\sum_{j=1, j\neq i}^{n}\left[\left(a - x_j - b\sum_{k=1, k\neq j}^{n} x_k\right)x_j - c_j x_j\right],$$
$$i = 1, 2, \dots, n.$$

Differentiating Π_i with respect to x_i for each i, the conditions of relative profit maximization for the firms are obtained as follows.

$$a - 2x_i - c_i - \frac{(n-2)b}{n-1}\sum_{j=1, j\neq i}^{n} x_j = 0,\ i = 1, 2, \dots, n.$$

From this, we have

$$x_i = \frac{n-1}{2(n-1)-(n-2)b}(a - c_i) - \frac{(n-2)b}{2(n-1)-(n-2)b}\sum_{j=1}^{n} x_j,$$

and

$$na - 2\sum_{j=1}^{n} x_i - \sum_{j=1}^{n} c_i - (n-2)b\sum_{j=1}^{n} x_i = 0.$$

The latter equation means

$$\sum_{j=1}^{n} x_i = \frac{1}{2+(n-2)b}\left(na - \sum_{j=1}^{n} c_i\right).$$

Then, we get the equilibrium output of Firm i as follows.

$$x_i^C = \frac{n-1}{2(n-1)-(n-2)b}(a - c_i)$$
$$- \frac{(n-2)b}{[2(n-1)-(n-2)b][2+(n-2)b]}\left(na - \sum_{j=1}^{n} c_i\right),\ i = 1, 2, \dots, n.$$

C indicates *Cournot*. The equilibrium price of the good of Firm i is

$$p_i^C = a - x_i^C - b\sum_{j=1, j\neq i}^{n} x_j^C = \frac{n-1+b}{2(n-1)-(n-2)b}(a-c_i)$$
$$-\frac{nb}{[2(n-1)-(n-2)b][2+(n-2)b]}\left(na-\sum_{j=1}^{n} c_j\right)+c_i,\ i=1,2,\dots,n.$$

4. Bertrand equilibrium under relative profit maximization

In this section each firm determines the price of its good given the prices of the goods of other firms so as to maximize its relative profit. The absolute profit of Firm i is written as

$$\pi_i = \frac{1}{(1-b)[1+(n-1)b]}\left[(1-b)a-[1+(n-2)b]p_i+b\sum_{j=1, j\neq i}^{n} p_j\right](p_i-c_i).$$

The relative profit of Firm i is

$$\Pi_i = \frac{1}{(1-b)[1+(n-1)b]}\left[(1-b)a-[1+(n-2)b]p_i+b\sum_{j=1, j\neq i}^{n} p_j\right](p_i-c_i)$$
$$-\frac{1}{(1-b)(n-1)[1+(n-1)b]}\sum_{j=1, j\neq i}^{n}\left\{(1-b)a-[1+(n-2)b]p_j\right.$$
$$\left.+b\sum_{k=1, k\neq j}^{n} p_k\right\}(p_j-c_j).$$

Differentiating Π_i with respect to p_i, the conditions of relative profit maximization for the firms are obtained as follows.

$$(1-b)a-2[1+(n-2)b]p_i+b\sum_{j=1, j\neq i}^{n} p_j+[1+(n-2)b]c_i \tag{3}$$
$$-\frac{b}{n-1}\sum_{j=1, j\neq i}^{n}(p_j-c_j)=0,\ i=1,2,\dots,n.$$

Then, we get the equilibrium price of the good of Firm i as follows (See Appendix 2).

$$p_i^B = \frac{(n-1)[1+(n-1)b]}{2(n-1)+(n-2)(2n-1)b}(a-c_i)$$
$$-\frac{nb[1+(n-2)b]}{[2(n-1)+(n-2)(2n-1)b][2+(n-2)b]}\left(na-\sum_{j=1}^{n} c_j\right)+c_i,\ i=1,2,\dots,n.$$

B indicates *Bertrand*. The equilibrium output of Firm i is

$$x_i^B = \frac{1}{(1-b)[1+(n-1)b]}\left\{[1+(n-1)b](a-c_i) - [1+(n-1)b](p_i - c_i) - b\left(na - \sum_{j=1}^{n} c_j\right) + b\sum_{j=1}^{n}(p_j - c_j)\right\}$$
$$= \frac{[n-1+(n^2-3n+1)b]}{(1-b)[2(n-1)+(n-2)(2n-1)b]}(a-c_i) - \frac{(n-2)[1+(n-1)b]b}{(1-b)[2(n-1)+(n-2)(2n-1)b][2+(n-2)b]}\left(na - \sum_{j=1}^{n} c_j\right),$$
$$i = 1, 2, \dots, n.$$

5. Comparison of Cournot and Bertrand equilibria

Let us compare the outputs and prices at the Bertrand equilibrium and those at the Cournot equilibrium. Comparing the output of Firm i at the Bertrand equilibrium and that at the Cournot equilibrium,

$$x_i^B - x_i^C = \frac{n(n-2)b^2(\sum_{j=1}^{n} c_j - nc_i)}{(1-b)[2(n-1)+(n-2)(2n-1)b][2(n-1)-(n-2)b]}. \quad (4)$$

Comparing the price of the good of Firm i at the Bertrand equilibrium and that at the Cournot equilibrium,

$$p_i^B - p_i^C = \frac{n(n-2)b^2(nc_i - \sum_{j=1}^{n} c_j)}{[2(n-1)+(n-2)(2n-1)b][2(n-1)-(n-2)b]}. \quad (5)$$

5.1 A special case 1: duopoly

Since in a duopoly $n = 2$, from (4) and (5) we get

$$x_i^B - x_i^C = 0,$$

and

$$p_i^B - p_i^C = 0.$$

Therefore, the following proposition holds.

Proposition 1 *In a duopoly Bertrand and Cournot equilibria coincide whether* $c_1 = c_2$ *or* $c_1 \neq c_2$.

5.2 A special case 2: symmetric oligopoly

In a symmetric oligopoly all c_i are equal, and so

$$\sum_{j=1}^{n} c_j = nc_i \text{ for all } i.$$

Then,

$$x_i^B - x_i^C = 0, \text{ and } p_i^B - p_i^C = 0.$$

Thus, the following proposition holds.

Proposition 2 *In a symmetric oligopoly Bertrand and Cournot equilibria coincide regardless of the number of firms.*

5.3 A general case: asymmetric oligopoly

Assume that $n \geq 3$, and the marginal costs of the firms may be different each other. From (4) we find that $x_i^B = x_i^C$ if and only if $c_i = \frac{\sum_{j=1}^{n} c_j}{n}$. Also from (5) $p_i^B = p_i^C$ if and only if $c_i = \frac{\sum_{j=1}^{n} c_j}{n}$. If $c_i < \frac{\sum_{j=1}^{n} c_j}{n}$ we have $x_i^B > x_i^C$ and $p_i^B < p_i^C$. And if $c_i > \frac{\sum_{j=1}^{n} c_j}{n}$ we have $x_i^B < x_i^C$ and $p_i^B > p_i^C$. Therefore, we obtain the following results.

Proposition 3 *In an asymmetric oligopoly, if the marginal cost of a firm is lower than the average marginal cost over the industry, its output at the Bertrand equilibrium is larger than that at the Cournot equilibrium, and the price of its good at the Bertrand equilibrium is lower than that at the Cournot equilibrium.*

On the other hand, if the marginal cost of a firm is higher than the average marginal cost over the industry, its output at the Bertrand equilibrium is smaller than that at the Cournot equilibrium, and the price of its good at the Bertrand equilibrium is higher than that at the Cournot equilibrium.

6. Relations with other results

Absolute profit maximization If firms in an oligopoly seek to maximize their absolute profits, the Bertrand and Cournot equilibria do not coincide whether the goods of firms are differentiated or homogeneous.

Relative profit maximization with a homogeneous good By Vega-Redondo (1997), in a framework of evolutionary game theoretic model, it was shown that in an oligopoly in which firms produce a homogeneous good and seek to maximize their relative profits, the Cournot equilibrium coincide with the outcome of perfect competition.

With differentiated goods, however, the Cournot equilibrium under relative profit maximization is not equivalent to perfect competition.

Delegation problem Miller and Pazgal (2001) has shown the equivalence of price strategy and quantity strategy in a delegation game when owners of firms control managers of firms seek to maximize an appropriate combination of absolute and relative profits.

But in their analyses owners of firms themselves still seek to maximize absolute profits of their firms. On the other hand, in this paper we do not consider a delegation problem, and we assume that owners of firms seek to maximize their relative profits.

Duopoly In Tanaka (2013) and Satoh and Tanaka (2014), assuming linear demand functions and constant marginal costs, it was shown that in a duopoly, in which firms produce differentiated goods, and maximize their relative profits, Bertrand and Cournot equilibria are equivalent in the sense that the output and the price of each firm's good at the Bertrand equilibrium are equal to those at the Cournot equilibrium whether they have the same cost functions or different cost functions.

Since $n = 2$ in duopoly, the conditions of relative profit maximization at the Cournot equilibrium are reduced to

$$a - 2x_i - c_i = 0,\ i = 1, 2.$$

Then

$$x_i = \frac{a - c_i}{2}.$$

Thus, the equilibrium output does not depend on the cost of the rival firm. On the other and, the conditions of relative profit maximization for the firms at the

Bertrand equilibrium are reduced to

$$(1-b)a - 2p_i + bp_j + c_i - b(p_j - c_j) = (1-b)a - 2p_i + c_i + bc_j = 0,\ i = 1,2, j \neq i.$$

By the inverse demand functions this is rewritten as

$$a - ba - 2a + 2x_i + 2bx_j + c_i + bc_j = -a + 2x_i + c_i + b(-a + 2x_j + c_j) = 0,\ i = 1,2, j \neq i.$$

Since $0 < b < 1$ this equation is equivalent to the condition of relative profit maximization at the Cournot equilibrium for Firm i and j.

The result of this paper is an extension and generalization of the result in a duopoly to an asymmetric oligopoly.

Relation between relative profit maximization and zero-sum game A game of relative profit maximization by firms in oligopoly (or duopoly) is an n-person (or two-person) zero-sum game. Let u_1 and u_2 be the payoffs of two players in a two-person zero-sum game, the relation $u_2 = -u_1$ is satisfied. But in an oligopoly such a relation does not hold. Let $u_i,\ i = 1, 2, \dots, n$ be the payoff of Player i in an n-person zero-sum game. Then, $\sum_{i=1}^{n} u_i = 0$, and $u_i = -\sum_{j=1, j\neq i} u_j$ are satisfied. u_i is not the opposite of another player's payoff. This fact seems to be the reason that coincidence of Cournot and Bertrand equilibria does not hold in oligopoly.

Appendix 1: Calculations of the ordinary demand functions

For $j \neq i$, we have

$$p_j = a - x_j - bx_i - b \sum_{k=1, k\neq i,j}^{n} x_k.$$

Thus,

$$\sum_{j=1, j\neq i}^{n} p_j = (n-1)a - (n-1)bx_i - [1 + (n-2)b] \sum_{j=1, j\neq i}^{n} x_j.$$

From this

$$\sum_{j=1, j\neq i}^{n} x_j = \frac{1}{1 + (n-2)b} \left[(n-1)a - (n-1)bx_i - \sum_{j=1, j\neq i}^{n} p_j \right].$$

Substituting this into (1),

$$x_i = a - p_i - \frac{b}{1+(n-2)b}\left[(n-1)a - (n-1)bx_i - \sum_{j=1, j\neq i}^{n} p_j\right].$$

Then, we obtain the following ordinary demand functions.

$$x_i = \frac{1}{(1-b)[1+(n-1)b]}\left[(1-b)a - [1+(n-2)b]p_i + b\sum_{j=1, j\neq i}^{n} p_j\right], \quad i = 1, 2, \dots, n.$$

Appendix 2: Calculations of the Bertrand equilibrium prices

(3) is rewritten as

$$[1+(n-2)b](a-c_i) - 2[1+(n-2)b](p_i - c_i) + b\sum_{j=1, j\neq i}^{n}(p_j - c_j)$$
$$- b\left[(n-1)a - \sum_{j=1, j\neq i}^{n} c_j\right] - \frac{b}{n-1}\sum_{j=1, j\neq i}^{n}(p_j - c_j) = 0,\ i = 1, 2, \dots, n.$$

From this we obtain

$$p_i - c_i = \frac{n-1}{2(n-1)+(n-2)(2n-1)b}\left\{[1+(n-1)b](a-c_i) - b\left(na - \sum_{j=1}^{n} c_j\right)\right\}$$
$$+ \frac{(n-2)b}{2(n-1)+(n-2)(2n-1)b}\sum_{j=1}^{n}(p_j - c_j),$$

and

$$[1+(n-2)b]\left(na - \sum_{j=1}^{n} c_i\right) - 2[1+(n-2)b]\sum_{j=1}^{n}(p_i - c_i) + (n-1)b\sum_{j=1}^{n}(p_i - c_i)$$
$$- (n-1)b\left(na - \sum_{j=1}^{n} c_i\right) - b\sum_{j=1}^{n}(p_i - c_i) = 0.$$

The latter equation means

$$\sum_{j=1}^{n}(p_i - c_i) = \frac{1-b}{2+(n-2)b}\left(na - \sum_{j=1}^{n} c_i\right).$$

Then, we get the equilibrium price of the good of Firm i as follows.

$$p_i^B = \frac{(n-1)[1+(n-1)b]}{2(n-1)+(n-2)(2n-1)b}(a-c_i) - \frac{nb[1+(n-2)b]}{[2(n-1)+(n-2)(2n-1)b][2+(n-2)b]}\left(na - \sum_{j=1}^{n} c_j\right) + c_i,\ i = 1, 2, \dots, n.$$

Acknowledgment

The authors would like to thank the referee for his/her valuable comments which helped to improve the manuscript.

References

Gibbons, R and K. J. Murphy (1990), ``Relative performance evaluation for chief executive officers'', *Industrial and Labor Relations Review*, **43**, 30S-51S.

Lu, Y. (2011), ``The relative-profit-maximization objective of private firms and endogenous timing in a mixed oligopoly'', *The Singapore Economic Review*, **56**, 203-213.

Matsumura, T., N. Matsushima and S. Cato (2013), ``Competitiveness and R&D competition revisited'' *Economic Modeling*, **31**, 541-547.

Miller, N. H. and A. I. Pazgal (2001), ``The equivalence of price and quantity competition with delegation'', *Rand Journal of Economics*, **32**, 284-301.

Schaffer, M.E. (1989), ``Are profit maximizers the best survivors?'' *Journal of Economic Behavior and Organization* **12**, 29-35.

Satoh, A. and Y. Tanaka (2014), ``Relative profit maximization and equivalence of Cournot and Bertrand equilibria in asymmetric duopoly'', *Economics Bulletin*, **34**, 819-827, 2014.

Tanaka, Y. (2013), ``Equivalence of Cournot and Bertrand equilibria in differentiated duopoly under relative profit maximization with linear demand'', *Economics Bulletin*, **33**, 1479-1486, 2013.

Vega-Redondo, F. (1997), ``The evolution of Walrasian behavior'' *Econometrica* **65**, 375-384.