Equivalance of Cournot and Bertrand equilibria in differentiated duopoly under relative profit maximization with linear demand

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Abstract

In this note we investigate the relation between a Cournot equilibrium and a Bertrand equilibrium in a duopoly with differentiated goods in which each firm maximizes its relative profit that is the difference between its profit and the profit of the rival firm. We will show that when firms maximize relative profits, a Cournot equilibrium and a Bertrand equilibrium coincide, and the equilibrium outputs under relative maximization is larger than both of the equilibrium outputs at the Cournot equilibrium and the Bertrand equilibrium under absolute profit maximization. We assume that demand functions for the goods of the firms are linear, the marginal costs of the firms are constant and the fixed costs are zero.

1. Introduction

In this note we investigate the relation between a Cournot equilibrium and a Bertrand equilibrium in a duopoly with differentiated substitutable goods in which each firm maximizes its relative profit that is the difference between its profit and the profit of the rival firm. We show that when firms maximize relative profits, a Cournot equilibrium and a Bertrand equilibrium coincide, and the equilibrium outputs under relative profit maximization are larger than the equilibrium outputs at the Cournot equilibrium and those at the Bertrand equilibrium under absolute profit maximization. We assume that demand functions for the goods of the firms are linear, the marginal costs of the firms are common and constant and the fixed costs are zero.

In recent years, maximizing relative profit instead of absolute profit has aroused the interest of economists. From an evolutionary perspective, Schaffer(1989) demonstrates with a Darwinian model of economic natural selection that if firms have market power, profit-maximizers are not necessarily the best survivors. According to Schaffer(1989), a unilateral deviation from Cournot equilibrium decreases the profit of the deviator, but decreases the other firm's profit even more. On the condition of being better than other competitors, firms that deviate from Cournot equilibrium achieve higher payoffs than the payoffs they receive under Cournot equilibrium. In Vega-Redondo(1997), it is argued that, under a general equilibrium framework, if firms maximize relative profit, a Walrasian equilibrium can be induced.

On the other hand, Lundgren(1996) shows that by making managerial compensation depend on relative profits rather than absolute profits, the incentives for oligopoly collusion can be eliminated. Kockesen et. al.(2000) have shown that under some conditions a firm which strives to maximize relative profit will outperform a firm which maximizes absolute profit. Bolton and Ockenfels(2000) and Fehr and Schmidt(1999) conducted an analysis considering an individual utility function that brings about a feeling of compassion toward an individual with a relatively lower material payoff and simultaneously brings about envy of other individuals with a higher material payoff.

2. The model

There are two firms, A and B. They produce differentiated substitutable goods. Notations are as follows.

Output of Firm A	x_A
Output of Firm B	x_B
Price of the good of Firm A	p_A
Price of the good of Firm B	p_B

The marginal costs of the firms are common, and equal $c > 0$. There is no fixed cost.

The inverse demand functions of the goods produced by the firms are

$$p_A = a - x_A - bx_B,$$

and

$$p_B = a - x_B - bx_A,$$

where $a > c$ and $0 < b < 1$. x_A represents the demand for the good of Firm A, and x_B represents the demand for the good of Firm B. The prices of the goods are determined so that demand of consumers for each firm's good and supply of each firm are equilibrated.

The ordinary demand functions for the goods of the firms are obtained from those inverse demand functions as follows,

$$x_A = \frac{1}{1-b^2}[(1-b)a - p_A + bp_B],$$

and

$$x_B = \frac{1}{1-b^2}[(1-b)a - p_B + bp_A].$$

3. Absolute profit maximization

3.1 Cournot equilibrium

The profits of Firm A and B are written as

$$\pi_A = (a - x_A - bx_B)x_A - cx_A,$$

and

$$\pi_B = (a - x_B - bx_A)x_B - cx_B.$$

Each firm determines its output given the output of the rival firm so as to maximize its (absolute) profit. The conditions for profit maximization of the firms are

$$a - 2x_A - bx_B - c = 0,$$

and

$$a - 2x_B - bx_A - c = 0.$$

From these conditions the equilibrium outputs of the firms are obtained as follows,

$$x_A^C = x_B^C = \frac{a-c}{2+b}.$$

The equilibrium prices of the goods of the firms are as follows,

$$p_A^C = p_B^C = \frac{a+(1+b)c}{2+b}$$

3.2 Bertrand equilibrium

The profits of Firm A and B are written as

$$\pi_A = \frac{1}{1-b^2}[(1-b)a - p_A + bp_B](p_A - c),$$

and

$$\pi_B = \frac{1}{1-b^2}[(1-b)a - p_B + bp_A](p_B - c).$$

Each firm determines the price of its good given the price of the rival firm's good so as to maximize its (absolute) profit. The conditions for profit maximization of the firms are

$$(1-b)a - 2p_A + bp_B + c = 0,$$

and

$$(1-b)a - 2p_B + bp_A + c = 0.$$

From these conditions the equilibrium prices of the goods of the firms are obtained as follows,

$$p_A^B = p_B^B = \frac{(1-b)a+c}{2-b}.$$

The equilibrium outputs of the firms are as follows,

$$x_A^B = x_B^B = \frac{a-c}{(1+b)(2-b)}$$

4. Relative profit maximization

4.1 Cournot equilibrium

The relative profit of Firm A (or B) is the difference between its profit and the profit of Firm B (or A). We denote the relative profit of Firm A by Π_A and that of Firm B by Π_B. They are written as follows,

$$\begin{aligned}\Pi_A =&\pi_A - \pi_B = (a - x_A - bx_B)x_A - (a - x_B - bx_A)x_B - c(x_A - x_B)\\ =&(a - c)(x_A - x_B) - x_A^2 + x_B^2,\end{aligned}$$

and

$$\begin{aligned}\Pi_B =&\pi_B - \pi_A = (a - x_B - bx_A)x_B - (a - x_A - bx_B)x_A - c(x_B - x_A)\\ =&(a - c)(x_B - x_A) - x_B^2 + x_A^2,\end{aligned}$$

Each firm determines its output given the output of the rival firm so as to maximize its relative profit. Thus, Firm A determines x_A so as to maximize Π_A. The condition for relative profit maximization of Firm A is

$$a - c - 2x_A = 0.$$

And the condition for relative profit maximization of Firm B is

$$a - c - 2x_B = 0.$$

Thus, the equilibrium outputs of the firms are

$$\tilde{x}_A^C = \tilde{x}_B^C = \frac{a - c}{2}.$$

The equilibrium prices of the goods of the firms are derived as follows,

$$\tilde{p}_A^C = \tilde{p}_B^C = \frac{(1 - b)a + (1 + b)c}{2}.$$

4.2 Bertrand equilibrium

Similarly to the previous subsection the relative profits of Firm A and B are denoted by Π_A and Π_B. They are written as follows,

$$\begin{aligned}\Pi_A =&\pi_A - \pi_B\\ =&\frac{1}{1 - b^2}[(1 - b)a(p_A - p_B) - p_A^2 + p_B^2 + (1 + b)c(p_A - p_B)],\end{aligned}$$

and

$$\begin{aligned}\Pi_B =&\pi_B - \pi_A\\ =&\frac{1}{1-b^2}[(1-b)a(p_B-p_A)-p_B^2+p_A^2+(1+b)c(p_B-p_A)],\end{aligned}$$

Each firm determines the price of its good given the price of the rival firm's good so as to maximize its relative profit. Thus, Firm A determines p_A so as to maximize Π_A. The condition for relative profit maximization of Firm A is

$$(1-b)a-2p_A+(1+b)c=0.$$

And the condition for relative profit maximization of Firm B is

$$(1-b)a-2p_B+(1+b)c=0.$$

Thus, the equilibrium prices of the goods of the firms are

$$\tilde{p}_A^B=\tilde{p}_B^B=\frac{(1-b)a+(1+b)c}{2}.$$

The equilibrium outputs of the firms are obtained as follows,

$$\tilde{x}_A^B=\tilde{x}_B^B=\frac{a-c}{2}$$

Since $0<b<1$ we can show

$$\tilde{x}_A^C=\tilde{x}_A^B>x_A^C>x_A^B.$$

5. Conclusion and future research plans

From the results of the previous sections we get the following conclusion.

> In a duopoly with differentiated substitutable goods the equilibrium outputs at the Cournot equilibrium under relative profit maximization, $\tilde{x}_A^C$ and $\tilde{x}_B^C$, and the equilibrium outputs at the Bertrand equilibrium under relative profit maximization, $\tilde{x}_A^B$ and $\tilde{x}_B^B$, are equal.
>
> Therefore, the equilibrium prices at the Cournot equilibrium under relative profit maximization, $\tilde{p}_A^C$ and $\tilde{p}_B^C$, and the equilibrium

prices at the Bertrand equilibrium under relative profit maximization, $\tilde{p}_A^B$ and $\tilde{p}_B^B$, are also equal.

The equilibrium outputs under relative profit maximization are larger than the equilibrium outputs at the Cournot equilibrium and those at Bertrand equilibrium under absolute profit maximization

We plan to research the various themes about relative profit maximization under imperfect competition. Especially;

1. The relation between Cournot and Bertrand equilibria in a duopoly or oligopoly under relative profit maximization with general demand functions.

2. Stackelberg equilibrium in a duopoly under relative profit maximization with Cournot behavior or Bertrand behavior.

3. The effects of trade policies such as tariffs and export subsidies in a duopoly or oligopoly under relative profit maximization with segmented markets and integrated markets.

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