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Abstract

This paper explores a one-sector AK model with a cash-in-advance (CIA) constraint that itself depends on relative income, which implies status (we call this constraint the "CIA-status constraint"). The CIA-status constraint means that agents with higher income are more creditworthy and can make purchases with fewer money holdings. Under the Clower-Lucas-type CIA-status constraint, we show that the endogenous growth rate and money growth are positively correlated. On the other hand, under the Stockman-type CIA-status constraint, we confirm that the relationship between the endogenous growth rate and money growth changes from negative to positive when the elasticity of the CIA constraint with respect to status exceeds one.

Keywords: Cash-in-advance constraint; Status; Endogenous growth; Money growth

JEL classification: E41; E52; O42

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1 Introduction

The roles of status in economic growth models have actively been explored. Existing studies capture the roles of status in terms of sociology, in which status is defined as one of the components which generate utility (this type of preference is referred to as status preference). Zou (1994) is one of the pioneering studies concerning this concept of status. Zou (1994) focuses on the spirit of capitalism, based on Weber (1958), and assumes that the utility function depends on status, defined as the level of capital stock holdings, as well as consumption. Under this setting, Zou (1994) shows that endogenous growth can arise even if the interest rate is smaller than the time preference rate.[1]

As for an extension of this stream, the relationship between status and money has been discussed recently. Specifically, recent studies introduce status preference into a model with a cash-in-advance (CIA henceforth) constraint in order to consider the channel through which status preference encourages the portfolio substitution — the shift from money to capital when the inflation arises. They examine how status has an impact on the effect of higher inflation (money growth) on the level of capital stock or on an economic growth rate, which has been one of the important issues in macroeconomics. Gong and Zou (2001) and Chang and Tsai (2003) deal with this issue in a neoclassical framework, while Chang et al. (2000) and Chen and Guo (2009, 2011) tackle this issue in an AK framework.[2] In particular, concerning the AK model, Chang et al. (2000) employ the Clower-Lucas-type CIA constraint and show that an endogenous growth rate and money growth are positively correlated. Chen and Guo (2009) clarify the negative relationship between an endogenous growth rate and money growth under the Stockman-type CIA constraint, while Chen and Guo (2011) confirm both positive and negative relationships between them under the generalized Stockman type.[3] [4]

On the other hand, the roles of status can be also captured in terms of a social system, in which status can work as credit. From this perspective, Kaminoyama and

[1]This preference had already been constructed mathematically by Kurz (1968).

[2]Chang et al. (2000) also consider a neoclassical framework.

[3]The Clower-Lucas type means that only consumption is constrained by cash, while the Stockman type implies that both consumption and investment are constrained by cash. Under the generalized Stockman-type CIA constraint, consumption and a fraction of investment are constrained by cash.

[4]Chang et al. (2000) and Chen and Guo (2009) employ the utility function involving additive status, while Chen and Guo (2011) consider the utility function involving multiplicative status.

Kawagishi (2013) construct the "CIA-status constraint," which reflects the observations of the existing studies (e.g., Avery et al. (1987), Wolff (1983), Kessler and Wolff (1991), Kennickell and Starr-McCluer (1996)).[5] Specifically, Kaminoyama and Kawagishi (2013) assume that an agent faces a CIA constraint which binds more loosely with a rise in the agent's own status, defined as relative income. Thus, the CIA-status constraint implies that agents with higher income are more creditworthy and can make purchases with fewer money holdings. Under this framework, unlike the existing studies which consider the portfolio substitution through status preference (i.e., in terms of sociology), their study analyzes the portfolio substitution through the CIA-status constraint (i.e., in terms of a social system).

This study also focuses on the roles of status in terms of a social system, and conducts the further analysis with the CIA-status constraint under an endogenous growth model.[6] Additionally, we employ the two types of CIA-status constraints — the Clower-Lucas-type and the Stockman-type CIA-status constraints. We introduce each CIA-status constraint into a one-sector AK model and examine how status which affects a CIA constraint has an influence on the relationship between higher inflation (money growth) and an economic growth rate.[7]

Under the Clower-Lucas-type CIA-status constraint, we show that there exists a unique balanced-growth-path (BGP henceforth) equilibrium in which the equilibrium path is determinate, and that the endogenous growth rate and money growth are positively correlated. Under the Stockman-type CIA-status constraint, on the other hand, we also confirm that there exists a unique BGP equilibrium in which the equilibrium path is determinate. However, the relationship between the endogenous growth rate and money growth changes from negative to positive when the elasticity of the CIA constraint with respect to status exceeds one.[8] This result is the main

[5]The observations are summarized as follows: (i) "high income individuals use cash and cash plus checks for a smaller fraction of their total transactions than low income individuals (Avery et al. (1987));" (ii) "the fraction of household wealth held in liquid assets decreases with income and wealth (Wolff (1983), Kessler and Wolff (1991), Kennickell and Starr-McCluer (1996))."

[6]Kaminoyama and Kawagishi (2013) introduce the CIA-status constraint into a neoclassical growth model, and examine the effects of money growth on capital accumulation.

[7]We analyze the existence and the uniqueness of a BGP equilibrium and its stability as well.

[8]Suen and Yip (2005) analyze a one-sector AK model with the Stockman-type CIA constraint. Chen and Guo (2008) extend Suen and Yip (2005) by positing that consumption and a fraction of investment are constrained by cash. These studies assume that the instantaneous utility function is the constant-intertemporal-elasticity-of-substitution (CIES) type, and show that faster money growth depresses an endogenous growth rate when the intertemporal elasticity of substitution in

finding in the present study.

The brief intuition for the above results is as follows. A higher status makes the CIA-status constraint less restricted. In the present study, this is referred to as the status effect, which implies that the agent can recover the loss of the net rate of return on capital induced by the inflation tax effect (note that the inflation tax is caused by a rise in the money growth rate). For instance, if the status effect is larger than the inflation tax effect, then the net rate of return on capital ultimately rises. In this case, the agent invests more in order to obtain a higher status, so that capital accumulation is accelerated and the growth rate rises.

This paper is organized as follows. Section 2 explains the CIA-status constraint in detail, and provides the basic framework. Section 3 considers the model with the Clower-Lucas-type CIA-status constraint and analyzes the effects of money growth on the endogenous growth rate as well as uniqueness and stability of a BGP equilibrium. In Section 4, this paper constructs the model with the Stockman-type CIA-status constraint and conducts the same analysis as in Section 3. The conclusion of this paper is presented in Section 5.

2 The model

We consider a continuous-time, infinite-horizon, and one-sector growth model with inelastic labor supply. The size of the population is constant and is normalized to unity. We employ a simple AK production technology:

$$y(t) = Ak(t), \tag{1}$$

where A is a positive constant which reflects the level of the technology, y is per capita output, and k is per capita capital.

2.1 CIA-status constraint

In this subsection, we explain about the CIA-status constraint. This constraint is formulated based on the assumption that the CIA constraint itself depends on relative income, which implies status. Note that since the CIA-status constraint employed in this study is basically along the lines of Kaminoyama and Kawagishi (2013), the formulation of the CIA-status constraint is almost the same process as in that existing study.

consumption is small.

Let us now introduce the ratio of goods which *require* cash to total goods which *may require* cash, and denote this ratio by Ω. Taking the observations of the existing studies into consideration, we assume that Ω depends on the agent's own relative income, and that Ω lies in the following range along the lines of the standard CIA model:

$$0 < \Omega\left(\frac{y}{\bar{y}}\right) < 1, \tag{2}$$

where y and $\bar{y}$ are private income and average income in the economy respectively, and $y/\bar{y}$ stands for the agent's own relative income. Additionally, we assume that

$$\Omega'\left(\frac{y}{\bar{y}}\right) \leq 0, \quad \Omega''\left(\frac{y}{\bar{y}}\right) \geq 0. \tag{3}$$

(3) implies that agents with higher relative income are more creditworthy and can purchase more cash goods with fewer money holdings.

Since we have assumed that $\Omega(\cdot)$ is the ratio of puchased goods which require cash to total purchased goods which may require cash, we find that

$$\frac{G_c}{G} = \Omega\left(\frac{y}{\bar{y}}\right), \tag{4}$$

where G_c and G are puchased goods which require cash in transaction and total purchased goods which may require cash, respectively. By this definition of G_c, the following constraint holds:

$$m \geq G_c, \tag{5}$$

where m is real money balances defined as the nominal money balances divided by the price level.

In Section 3, we consider the Clower-Lucas type, in which G_c and G are denoted as follows:

$$G_c = c_c, \quad G = c, \tag{6}$$

where c_c is consumption of cash goods and c is total consumption. Thus, it follows from (4), (5) and (6) that

$$m \geq \Omega\left(\frac{y}{\bar{y}}\right) c. \tag{7}$$

On the other hand, in Section 4, we focus on the Stockman type, in which G_c and G are represented by

$$G_c = c_c + i_c, \quad G = c + i, \tag{8}$$

where i_c is investment in cash goods and i is total investment (c_c and c are the same definitions as in the Clower-Lucas type). Hence, it follows from (4), (5) and (8) that

$$m \geq \Omega\left(\frac{y}{\bar{y}}\right)(c + i). \tag{9}$$

Under the AK production technology, (7) and (9) are respectively expressed as follows:

$$m \geq \Omega\left(\frac{k}{\bar{k}}\right)c, \tag{10}$$

$$m \geq \Omega\left(\frac{k}{\bar{k}}\right)(c + i), \tag{11}$$

where $\bar{k}$ is the average level of capital in the economy. In this paper, therefore, (10) and (11) are referred to as the Clower-Lucas-type CIA-status constraint and the Stockman-type CIA-status constraint, respectively.

2.2 Other settings

The representative agent maximizes its lifetime utility

$$\int_0^\infty \frac{c(t)^{1-\sigma} - 1}{1 - \sigma} e^{-\theta t} dt, \quad \sigma \geq 1, \tag{12}$$

where σ is the inverse of the intertemporal elasticity of substitution and θ is the rate of time preference. The overwhelming preponderance of empirical evidence suggests that $1/\sigma$ is relatively small, so that we assume that $\sigma \geq 1$.

The budget constraint of the representative agent is

$$\dot{m}(t) = Ak(t) - c(t) - i(t) - \pi(t)m(t) + \tau(t), \tag{13}$$

where i is investment and π is the rate of inflation. In addition, τ is the seigniorage that the agent receives from the monetary authority as a lump-sum transfer:

$$\tau(t) = \phi m(t), \tag{14}$$

where ϕ is the constant, time-invariant money growth rate. By using ϕ, the nominal money supply, M, is expressed as

$$M(t) = M(0)e^{\phi t}, \quad \text{given } M(0) > 0. \tag{15}$$

The law of motion of the capital stock is given by

$$\dot{k}(t) = i(t). \tag{16}$$

For simplicity, the depreciation rate of capital is assumed to be zero.

Finally, in what follows, we assume that

$$A - \theta > 0. \tag{17}$$

This is the standard assumption in the studies on an AK model. We assume that the CIA-status constraint is binding in equilibrium, as is common in the CIA literature.[9]

3 Clower-Lucas type

3.1 Optimal conditions and dynamic system

In the case of the Clower-Lucas type, the representative agent's maximization problem is as follows:

$$\begin{aligned}
&\max \int_0^\infty \frac{c(t)^{1-\sigma} - 1}{1-\sigma} e^{-\theta t} dt, \\
&\text{s.t. } \dot{m}(t) = Ak(t) - c(t) - i(t) - \pi(t)m(t) + \tau(t) \ , \quad \text{given } k(0) > 0, \\
&\qquad \dot{k}(t) = i(t), \\
&\qquad m(t) = \Omega\left(\frac{k(t)}{\bar{k}(t)}\right) c(t).
\end{aligned}$$

In this problem, the representative agent is assumed to take the sequences, $\{\bar{k}(t)\}_{t=0}^{\infty}$, as given. In what follows, we drop time index from the endogenous variables. To derive the necessary conditions for an optimum, we set up the current-value Hamiltonian function:

$$H = \frac{c^{1-\sigma} - 1}{1-\sigma} + \lambda\big[Ak - c - i - \pi m + \tau\big] + \mu\big[\, i \,\big] + \eta\left[m - \Omega\left(\frac{k}{\bar{k}}\right) c\right],$$

[9] In our model, the inequality, $\phi > A$, is the sufficient condition under which the CIA-status constraint is binding in equilibrium. In addition, note that $\phi > A$ ensures $\pi > 0$.

where λ and μ are the shadow prices associated with m and k, respectively, and η is the Lagrange multiplier associated with the CIA-status constraint. The first-order conditions are

$$c^{-\sigma} - \lambda - \eta\,\Omega\left(\frac{k}{\bar{k}}\right) = 0, \tag{18a}$$

$$-\lambda + \mu = 0, \tag{18b}$$

$$\lambda A + \dot{\mu} - \mu\theta - \eta\,\Omega'\left(\frac{k}{\bar{k}}\right)\frac{c}{\bar{k}} = 0, \tag{18c}$$

$$-\lambda\pi + \dot{\lambda} - \lambda\theta + \eta = 0. \tag{18d}$$

Furthermore, the transversality conditions are

$$\lim_{t\to\infty} e^{-\theta t}\mu k = 0, \tag{19a}$$

$$\lim_{t\to\infty} e^{-\theta t}\lambda m = 0. \tag{19b}$$

(18a) equates the marginal benefit to the marginal cost of consumption. (18b) and (18c) govern the evolution of physical capital over time, where the standard Euler equation of the representative agent is modified to reflect status which affects a CIA constraint. (18d) implies that the marginal values of real money holdings are equal to their marginal costs.

Since it is assumed that the total size of population is constant and normalized to unity, the following conditions hold in equilibrium:

$$k = \bar{k}. \tag{20}$$

Additionally, in equilibrium, the goods market clears, and money demand is equal to money supply:

$$\dot{k} = Ak - c, \tag{21}$$

$$\dot{m} = (\phi - \pi)m. \tag{22}$$

We assume that the Clower-Lucas-type CIA-status constraint is binding in equilibrium, as is common in the CIA literature. Thus, from (10) and (20), we obtain

$$m = \Omega\,(1)\,c. \tag{23}$$

We here introduce χ and ψ, which are defined as

$$\chi \equiv \frac{c}{k}, \quad \psi \equiv \frac{\eta}{\lambda}.$$

From these definitions, (18a)-(19b), and (20)-(23), we obtain the following dynamic system after some manipulation:[10]

$$\frac{\dot{\chi}}{\chi} = \{1 - \Omega'(1)\psi\}\chi - \psi + \phi, \tag{24}$$

$$\frac{\dot{\psi}}{\psi} = \{\Omega(1)^{-1}\psi^{-1} + 1\}\left\{(\sigma - 1)\left(\chi - \frac{\dot{\chi}}{\chi} - A\right) + (\psi - \phi - \theta)\right\}. \tag{25}$$

3.2 BGP equilibrium and stability

We first examine the existence and the uniqueness of a BGP equilibrium. Note that (17) ensures that the BGP equilibrium value of χ is in $(0, A)$, so that the endogenous growth rate is positive.

The $\dot{\chi} = 0$ locus and the $\dot{\psi} = 0$ locus are respectively given by[11]

$$\chi = \frac{\psi - \phi}{1 - \Omega'(1)\psi}, \tag{26a}$$

$$\chi = \left(\frac{1}{1-\sigma}\right)\psi - \frac{\phi + \theta}{1 - \sigma} + A + \frac{\dot{\chi}}{\chi}. \tag{26b}$$

Moreover, the $\dot{\chi} = \dot{\psi} = 0$ locus is

$$\chi = \left(\frac{1}{1-\sigma}\right)\psi - \frac{\phi + \theta}{1 - \sigma} + A. \tag{27}$$

The intersection between (26a) and (27) gives a BGP equilibrium.

The $\dot{\chi} = 0$ locus (26a) is monotonically increasing in ψ, while the $\dot{\chi} = \dot{\psi} = 0$ locus (27) is monotonically decreasing in ψ since $\sigma \geq 1$.[12] Thus, there exists a unique BGP equilibrium under (17). Furthermore, from Appendix A.1 and Fig. 1, we see that the BGP equilibrium is a source. Hence, the equilibrium path is determinate because χ and ψ are jumpable variables.

Proposition 1. *In the AK model with the Clower-Lucas-type CIA-status constraint, there exists a unique BGP equilibrium in which the equilibrium path is determinate.*

(Insert Fig. 1 here.)

[10] The inflation rate (π) is endogenously determined: $\pi = -A + \psi\{1 + \Omega'(1)\chi\}$.

[11] Regarding the shapes of the $\dot{\chi} = 0$ locus and the $\dot{\psi} = 0$ locus and the dynamics of χ and ψ, see Appendix A.1.

[12] When $\sigma = 1$, the $\dot{\chi} = \dot{\psi} = 0$ locus is given by $\psi = \phi + \theta$.

3.3 Effects of money growth

Let us denote the BGP equilibrium value of χ by χ^*. From (13), (14), (20), and (22), the endogenous growth rate g is

$$g = A - \chi^*. \tag{28}$$

As mentioned in Section 3.2, (17) ensures that χ^* is in $(0, A)$, so that $g > 0$. In what follows, we first derive χ^*. Then, we examine the effect of money growth on the endogenous growth rate.

Eliminating ψ from (26a) and (27), we have

$$X\chi^2 + Y\chi + Z = 0, \tag{29}$$

where

$$\begin{aligned} X &\equiv \Omega'(1)(1-\sigma) > 0, \\ Y &\equiv -\sigma + \Omega'(1)\big\{\phi + \theta - (1-\sigma)A\big\} < 0, \\ Z &\equiv \theta - (1-\sigma)A > 0. \end{aligned}$$

Here, we define D as the discriminant of the equation (29). Under $\sigma \geq 1$, it follows that

$$D \equiv Y^2 - 4XZ > 0. \tag{30}$$

Solving (29), we obtain[13]

$$\chi^* = -\frac{Y + \sqrt{D}}{2\Omega'(1)(1-\sigma)} > 0. \tag{31}$$

Thus, the relationship between the endogenous growth rate and money growth is

$$\frac{dg}{d\phi} = -D^{-\frac{1}{2}}\Omega'(1)\chi^* > 0. \tag{32}$$

Proposition 2. *In the AK model with the Clower-Lucas-type CIA-status constraint, the endogenous growth rate and money growth are positively correlated.*

[13] The equation (29) has two positive solutions. However, the larger solution is not valid because this solution makes the value of ψ^* minus.

The intuition for this result is as follows. Suppose that the economy is in a BGP equilibrium initially, and that the money growth rate rises. This leads to a rise in the inflation rate, so that the demand for current consumption declines through the CIA constraint. Furthermore, the agent knows that capital accumulation causes a higher status, which makes the CIA constraint less restricted. This provides incentives to invest in capital, because accumulating capital enables the agent to increase future consumption through the CIA constraint. Thus, since the agent shifts his/her demand from current consumption to capital, the endogenous growth rate rises.

4 Stockman type

4.1 Optimal conditions and dynamic system

In the case of the Stockman type, the representative agent's maximization problem is expressed as follows:

$$\begin{aligned}
&\max \int_0^\infty \frac{c(t)^{1-\sigma}-1}{1-\sigma} e^{-\theta t} dt, \\
&\text{s.t. } \dot{m}(t) = Ak(t) - c(t) - i(t) - \pi(t)m(t) + \tau(t) \ , \quad \text{given } k(0) > 0, \\
&\qquad \dot{k}(t) = i(t), \\
&\qquad m(t) = \Omega\left(\frac{k(t)}{\bar{k}(t)}\right)\big(c(t) + i(t)\big).
\end{aligned}$$

As mentioned in the Clower-Lucas type, the representative agent is assumed to take the sequences, $\{\bar{k}(t)\}_{t=0}^{\infty}$, as given. To derive the necessary conditions for an optimum, we set up the current-value Hamiltonian function (in what follows, we drop time index from the endogenous variables):

$$H = \frac{c^{1-\sigma}-1}{1-\sigma} + \lambda\big[Ak - c - i - \pi m + \tau\big] + \mu\big[\, i \,\big] + \zeta\left[m - \Omega\left(\frac{k}{\bar{k}}\right)(c+i)\right],$$

where λ and μ are the shadow prices associated with m and k, respectively, and ζ is the Lagrange multiplier associated with the CIA-status constraint. The first-order conditions are given as follows:

$$c^{-\sigma} - \lambda - \zeta\,\Omega\left(\frac{k}{\bar{k}}\right) = 0, \tag{33a}$$

$$-\lambda + \mu - \zeta\,\Omega\left(\frac{k}{\bar{k}}\right) = 0, \tag{33b}$$

$$\lambda A + \dot{\mu} - \mu\theta - \zeta\,\Omega'\left(\frac{k}{\bar{k}}\right)\frac{1}{\bar{k}}(c+i) = 0, \tag{33c}$$

$$-\lambda\pi + \dot{\lambda} - \lambda\theta + \zeta = 0. \tag{33d}$$

Moreover, the transversality conditions are

$$\lim_{t\to\infty} e^{-\theta t}\mu k = 0, \tag{34a}$$

$$\lim_{t\to\infty} e^{-\theta t}\lambda m = 0. \tag{34b}$$

(33a) equates the marginal benefit to the marginal cost of consumption. (33b) and (33c) govern the evolution of physical capital over time, where the standard Euler equation of the representative agent is modified to reflect status which affects a CIA constraint. (33d) implies that the marginal values of real money holdings are equal to their marginal costs.

As in the preceding section, (20), (21), and (22) hold in equilibrium. We assume that the CIA constraint is binding in equilibrium, as is common in CIA literature. Thus, it follows from (11) and (20) that

$$m = \Omega\,(1)\,(c+i). \tag{35}$$

In addition, from (33a)-(33c), the Euler equation is given by

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}\left\{A(1-\xi)\left(\frac{\mu}{\lambda}\right)^{-1} + A\xi - \theta\right\} \equiv g, \tag{36}$$

where

$$\xi \equiv -\frac{\Omega'(1)}{\Omega(1)} > 0.$$

Note that ξ expresses the elasticity of the CIA constraint with respect to status.

Along the BGP, c, k, and m grow at a common constant rate g, and the costate variables grow at a common rate. To derive the dynamic system and the BGP equilibrium, we introduce χ and ω, which are defined as

$$\chi \equiv \frac{c}{k}, \quad \omega \equiv \frac{\mu}{\lambda}.$$

Deriving the dynamic system, we have[14]

$$\frac{\dot{\chi}}{\chi} = \frac{A(1-\xi)}{\sigma}\omega^{-1} + \chi - \frac{1}{\sigma}\left\{A(\sigma-\xi)+\theta\right\}, \tag{37a}$$

$$\frac{\dot{\omega}}{\omega} = \frac{1}{\Omega(1)}\omega - A(1-\xi)\omega^{-1} - \chi + \left\{(1-\xi)A - \phi - \frac{1}{\Omega(1)}\right\}. \tag{37b}$$

4.2 Domain of ω

In the model with the Stockman-type CIA-status constraint, we need to consider the domain of ω in terms of the restriction on ζ and the endogenous growth rate.

From (33b), ζ is given by

$$\zeta = \frac{\mu-\lambda}{\Omega(1)}. \tag{38}$$

Since $\zeta > 0$ in a BGP equilibrium, it follows from (38) that $\mu > \lambda$.[15] Thus, the following condition needs to hold:

$$\omega > 1. \tag{39}$$

Furthermore, in order to ensure that the endogenous growth rate is positive (see (36)), we assume that

$$\omega < \frac{A\xi - A}{A\xi - \theta} \quad \text{if} \quad \xi < \frac{\theta}{A}, \tag{40a}$$

$$\omega > \frac{A\xi - A}{A\xi - \theta} \quad \text{if} \quad \xi \geq \frac{\theta}{A}. \tag{40b}$$

As a consequence, it follows from (39), (40a) and (40b) that the domain of ω is given as follows:

$$1 < \omega < \frac{A\xi - A}{A\xi - \theta} \quad \text{if} \quad \xi < \frac{\theta}{A}, \tag{41a}$$

$$1 < \omega \quad \text{if} \quad \xi \geq \frac{\theta}{A}. \tag{41b}$$

Note that (17) enables us to consider the case where $\xi = 0$ (i.e., the framework of Suen and Yip (2005)).

[14] Using (16), (20), (21), (22), and χ, we obtain the following inflation rate: $\pi = \phi - A + \chi$.

[15] If the Lagrange multiplier associated with (11), ζ, is zero, then the CIA-status constraint is not binding.

4.3 BGP equilibrium and stability

In a BGP equilibrium, $\dot{\chi} = 0$ and $\dot{\omega} = 0$ are satisfied. Hence, from (37a) and (37b), the following expressions hold:

$$\chi = -\frac{A(1-\xi)}{\sigma}\omega^{-1} + \frac{1}{\sigma}\left\{A(\sigma - \xi) + \theta\right\}, \tag{42a}$$

$$\chi = \frac{1}{\Omega(1)}\omega - A(1-\xi)\omega^{-1} + \left\{(1-\xi)A - \phi - \frac{1}{\Omega(1)}\right\}. \tag{42b}$$

(42a) is the $\dot{\chi} = 0$ locus, and (42b) is the $\dot{\omega} = 0$ locus. Here, regarding the $\dot{\chi} = 0$ locus (42a), the upper (resp. lower) bound exists when $\xi < 1$ (resp. when $\xi > 1$). In order to ensure that a BGP equilibrium exists, we assume that the upper or lower bound of the $\dot{\chi} = 0$ locus is positive. This assumption is equivalent to the following inequality:

$$\xi < \sigma + \frac{\theta}{A}. \tag{43}$$

Note that the right-hand side of (43) is greater than one since $\sigma \geq 1$.

We now focus on the BGP equilibrium value of ω. Using (42a) and (42b) and eliminating χ, we have

$$\Gamma(\omega) \equiv \frac{1}{\Omega(1)}\omega^2 + \left[A\xi\left(\frac{1}{\sigma} - 1\right) - \left\{\phi + \frac{1}{\Omega(1)} + \frac{\theta}{\sigma}\right\}\right]\omega + A(1-\xi)\left(\frac{1}{\sigma} - 1\right) = 0. \tag{44}$$

Since we assume that $\sigma \geq 1$, it follows that

$$\Gamma(1) = \frac{A-\theta}{\sigma} - (A+\phi) \leq A - \theta - (A+\phi) = -(\phi+\theta) < 0. \tag{45}$$

Thus, we find that the equation (44) ($\Gamma(\omega) = 0$) has two different real roots such that one solution is greater than one and another solution is less than one. Since a BGP equilibrium value of ω needs to satisfy at least (39), we see that the larger solution of the equation (44) may be valid as the BGP equilibrium value of ω. In what follows, let us denote this larger solution by ω^*:[16]

$$\omega^* = \frac{\Omega(1)}{2}\left[-B + \left\{B^2 - \frac{4A}{\Omega(1)}(1-\xi)\left(\frac{1}{\sigma} - 1\right)\right\}^{\frac{1}{2}}\right], \tag{46}$$

[16] In order for a BGP equilibrium to satisfy the transversality conditions, the rate of money supply, ϕ, must have the following upper bound:

$$\phi < \frac{\omega^* - 1}{\Omega(1)}.$$

Note that this condition is always satisfied under $\sigma \geq 1$ (see Appendix A.2).

where

$$B \equiv A\xi \left(\frac{1}{\sigma} - 1 \right) - \left\{ \phi + \frac{1}{\Omega(1)} + \frac{\theta}{\sigma} \right\} < 0.$$

As explained in Section 4.2, the domain of ω is different depending on the value of ξ. Because of this, we examine the existence and the uniqueness of a BGP equilibrium in both the case where $\xi < \frac{\theta}{A}$ and the case where $\xi \geq \frac{\theta}{A}$.

When $\xi < \frac{\theta}{A}$

In order for ω^* to be the BGP equilibrium value of ω, the following inequality must be satisfied:

$$\omega^* < \frac{A\xi - A}{A\xi - \theta}. \tag{47}$$

Here, (47) always holds under the following inequality:

$$\frac{\theta}{A} < \frac{\frac{1}{\Omega(1)}}{\phi + \frac{1}{\Omega(1)} + \theta}. \tag{48}$$

Thus, if (48) holds, then ω^* becomes the BGP equilibrium value of ω. Moreover, substituting ω^* into (42a) or (42b) yields the BGP equilibrium value of χ, denoted by χ^*. Therefore, there exists the unique BGP equilibrium (ω^*, χ^*) under (48).[17]

When $\xi \geq \frac{\theta}{A}$

In this case, under (17) and (43), ω^* becomes the BGP equilibrium value of ω. Furthermore, substituting ω^* into (42a) or (42b) yields χ^*. Thus, there exists the unique BGP equilibrium (ω^*, χ^*) under (17) and (43).

Let us move on to the stability of the unique BGP equilibrium (ω^*, χ^*). The phase diagram is illustrated in Fig. 2 (when $\xi < 1$) and Fig. 3 (when $\xi > 1$).[18] Therefore, in the unique BGP equilibrium (ω^*, χ^*), the equilibrium path is determinate because ω and χ are jumpable variables.

Proposition 3. *In the one-sector AK model with the Stockman-type CIA-status constraint, there exists a unique BGP equilibrium, in which the equilibrium path is determinate.*

[17]Note that (17) automatically holds under (48), and that (43) always holds when $\xi < \frac{\theta}{A}$.

[18]See Appendix A.3 for the derivation of the phase diagram, and note that Fig. 2 and Fig. 3 are examples.

(Insert Fig. 2 and Fig. 3 here.)

4.4 Effects of money growth

4.4.1 The relationship between g and ϕ

We analyze the effects of money growth on the endogenous growth rate. Differentiating (36) with respect to ϕ, we obtain

$$\frac{dg}{d\phi} = -\frac{A(1-\xi)}{\sigma\omega^*}\frac{1}{\sqrt{D_\omega(\xi)}}. \tag{49}$$

Here, $D_\omega(\xi)$ is the discriminant of the equation $\Gamma(\omega) = 0$:

$$D_\omega(\xi) \equiv \Lambda_1\xi^2 - \Lambda_2\xi + \Lambda_3, \tag{50}$$

where

$$\Lambda_1 \equiv A^2\left(\frac{1}{\sigma} - 1\right)^2, \quad \Lambda_2 \equiv 2A\left(\frac{1}{\sigma} - 1\right)\left\{\phi - \frac{1}{\Omega(1)} + \frac{\theta}{\sigma}\right\},$$
$$\Lambda_3 \equiv \left\{\phi + \frac{1}{\Omega(1)} + \frac{\theta}{\sigma}\right\}^2 - \frac{4A}{\Omega(1)}\left(\frac{1}{\sigma} - 1\right).$$

Since (45) holds, we see that $D_\omega(\xi)$ is always positive. Taking (49) into consideration, we are able to summarize the relationship between the endogenous growth rate and money growth as follows:

$$\frac{dg}{d\phi} = -\frac{A(1-\xi)}{\sigma\omega^*}\frac{1}{\sqrt{D_\omega(\xi)}} < 0 \quad \text{if} \quad \xi < 1; \tag{51}$$

$$\frac{dg}{d\phi} = 0 \quad \text{if} \quad \xi = 1; \tag{52}$$

$$\frac{dg}{d\phi} = -\frac{A(1-\xi)}{\sigma\omega^*}\frac{1}{\sqrt{D_\omega(\xi)}} > 0 \quad \text{if} \quad \xi > 1. \tag{53}$$

We confirm that (51) is the same result as in Suen and Yip (2005). In addition to this result, however, (52) and (53) are also obtained in our model. From (51) through (53), the relationship between the endogenous growth rate and money growth changes from negative to positive when the elasticity of the CIA constraint with respect to status, ξ, exceeds one.

Proposition 4. *In the one-sector AK model with the Stockman-type CIA-status constraint, the endogenous growth rate and money growth are negatively correlated when $\xi < 1$, while they are positively correlated when $\xi > 1$. The superneutrality of money holds when $\xi = 1$.*

4.4.2 Intuition

We can rewrite $dg/d\phi$ as follows:

$$\begin{aligned}\frac{dg}{d\phi} &= \frac{dg}{d\chi^*}\frac{d\chi^*}{d\omega^*}\frac{d\omega^*}{d\phi} \\ &= -\frac{d\chi^*}{d\omega^*}\frac{d\omega^*}{d\phi}. \end{aligned} \tag{54}$$

Suppose that the economy is in a BGP equilibrium at the initial date, and that the money growth rate, ϕ, rises.

We first consider the term $d\omega^*/d\phi$ in (54). Concerning this term, from (46), we find that the following expression holds:

$$\frac{d\omega^*}{d\phi} = \frac{\omega^*}{\sqrt{D_\omega(\xi)}} > 0. \tag{55}$$

This implies that the representative agent switches a part of real money balances into capital holdings when a higher inflation occurs, so that the shadow price of capital becomes relatively higher than that of real money balances.[19]

We next focus on the term $d\chi^*/d\omega^*$ in (54). From (42a), this term is represented by

$$\frac{d\chi^*}{d\omega^*} = \frac{1}{\sigma\omega^*}\left(\frac{A}{\omega^*} - \xi\frac{A}{\omega^*}\right). \tag{56}$$

The first term in parentheses on the right-hand side of (56), A/ω^*, captures the inflation tax effect caused by a rise in the money growth rate, ϕ. This is because the rise in ϕ raises ω^*, so that the net rate of return on capital, A/ω^*, falls. On the other hand, the second term in parentheses on the right-hand side of (56), $\xi(A/\omega^*)$, represents the status effect in our model. The reason for this is given as follows.

[19]The term $d\omega^*/d\phi$ is represented by

$$\frac{d\omega^*}{d\phi} = \frac{\Omega(1)}{\underbrace{1 + A(1-\xi)\frac{\Omega(1)}{(\omega^*)^2}}_{PSE}\underbrace{-A(1-\xi)\frac{\Omega(1)}{\sigma(\omega^*)^2}}_{ISE}}.$$

The meanings of PSE and ISE are according to Chen and Guo (2008). The PSE (the portfolio substitution effect) means that a higher inflation causes the representative agent to switch a part of real money balances into capital holdings, while the ISE (the intertemporal substitution effect) implies that a rise in the money growth rate induces the representative agent to consume less and invest more in exchange for higher future consumption. In the present study, we assume that $\sigma \geq 1$, so that the PSE dominates the ISE.

The agent can make the CIA constraint less restricted by enhancing his/her status, so that the agent can increase consumption. This process implies that the agent can recover the loss of the net rate of return on capital induced by the inflation tax effect.

Taking into account the above explanation, we consider $dg/d\phi$. Note that the rise in ϕ raises ω^* under $\sigma \geq 1$, and that the sign of $d\chi^*/d\omega^*$ depends on the value of ξ.

When $\xi < 1$
The inflation tax effect is larger than the status effect. Thus, since the net rate of return on capital ultimately falls, the agent decreases investment and increases current consumption. This leads to a rise in $\chi^*(= c^*/k^*)$ (this process is consistent with $d\chi^*/d\omega^* > 0$). Therefore, capital accumulation is depressed, so that the growth rate falls.

When $\xi = 1$
The inflation tax effect is equal to the status effect, so that the agent does not change his/her behavior. Thus, the growth rate does not change as well.

When $\xi > 1$
The status effect is larger than the inflation tax effect. Thus, since the net rate of return on capital ultimately rises through the status effect, the agent increases investment and decreases current consumption in order to enhance his/her status. This leads to a fall in $\chi^*(= c^*/k^*)$ (this process is consistent with $d\chi^*/d\omega^* < 0$). Therefore, capital accumulation is accelerated and the growth rate rises.

5 Conclusion

In this study, we have captured the roles of status in terms of a social system and have conducted the analysis with the CIA-status constraint, which implies that agents with higher income are more creditworthy and can make purchases with fewer money holdings. Specifically, we have introduced the CIA-status constraint into a one-sector AK model, and have examined how status, which affects the CIA constraint, has an impact on the relationship between the endogenous growth rate and money growth.

Under the Clower-Lucas-type CIA-status constraint, we have shown that the endogenous growth rate and money growth are positively correlated. This is the same result as in Chang et al. (2000). On the other hand, under the Stockman-type CIA-status constraint, we have confirmed that both the positive and negative effects of money growth on the endogenous growth rate arise depending only on the degree of the elasticity of the CIA constraint with respect to status. More specifically, the relationship between the endogenous growth rate and money growth changes from negative to positive when the elasticity of the CIA constraint with respect to status exceeds one.

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Appendix

A.1. $\dot{\chi} = 0$ *locus and* $\dot{\psi} = 0$ *locus in the Clower-Lucas-type model*

As for the shape of $\dot{\chi} = 0$, we find that

$$\frac{d\chi}{d\psi} > 0, \quad \frac{d^2\chi}{d\psi^2} < 0.$$

The dynamics of χ become

$$\begin{aligned} &\dot{\chi} > 0 \;\; \text{above} \;\; \dot{\chi} = 0 \text{ locus;} \\ &\dot{\chi} < 0 \;\; \text{below} \;\; \dot{\chi} = 0 \text{ locus.} \end{aligned}$$

On the other hand, concerning the shape of $\dot{\psi} = 0$, we see that

$$\frac{d\chi}{d\psi} > 0, \quad \frac{d^2\chi}{d\psi^2} < 0 \;\; (\because \sigma \geq 1).$$

The dynamics of ψ are as follows:

$$\begin{aligned} &\dot{\psi} > 0 \;\; \text{below} \;\; \dot{\psi} = 0 \text{ locus;} \\ &\dot{\psi} < 0 \;\; \text{above} \;\; \dot{\psi} = 0 \text{ locus} \;\; (\because \sigma \geq 1). \end{aligned}$$

A.2. Transversality conditions in the Stockman-type model

In order to ensure that the transversality conditions are satisfied in a BGP equilibrium, the following condition needs to hold:

$$\phi < \frac{\omega^* - 1}{\Omega(1)}.$$

This condition is equivalent to

$$A\left(\frac{1}{\sigma} - 1\right)\xi + \phi + \frac{1}{\Omega(1)} - \frac{\theta}{\sigma} < \sqrt{D_\omega(\xi)}. \tag{57}$$

Here, the following inequality always holds:

$$A\left(\frac{1}{\sigma} - 1\right)\xi + \phi + \frac{1}{\Omega(1)} - \frac{\theta}{\sigma} < A\left(\frac{1}{\sigma} - 1\right)\xi + \phi + \frac{1}{\Omega(1)} + \frac{\theta}{\sigma} \equiv \Upsilon. \tag{58}$$

From (50), under $\sigma \geq 1$, we find that

$$\Upsilon^2 < \Upsilon^2 - 4A\left(\frac{1}{\sigma} - 1\right)\left(\phi + \frac{\theta}{\sigma}\right)\xi - \frac{4A}{\Omega(1)}\left(\frac{1}{\sigma} - 1\right) = D_\omega(\xi). \tag{59}$$

When $\Upsilon > 0$, from (59), we obtain

$$\Upsilon < \sqrt{D_\omega(\xi)}. \tag{60}$$

When $\Upsilon \leq 0$, on the other hand, (60) automatically holds. Therefore, from (58) and (60), we find that (57) always holds, that is, the transversality conditions are always satisfied under $\sigma \geq 1$.

A.3. $\dot{\chi} = 0$ *locus and the* $\dot{\omega} = 0$ *locus in the Stockman-type model*

The shape of the $\dot{\chi} = 0$ locus is as follows:

$$\frac{d\chi}{d\omega} > 0\,, \quad \frac{d^2\chi}{d\omega^2} < 0 \;\text{ if }\; \xi < 1;$$
$$\frac{d\chi}{d\omega} < 0\,, \quad \frac{d^2\chi}{d\omega^2} > 0 \;\text{ if }\; \xi > 1.$$

As for the dynamics of χ, we see that

$$\dot{\chi} > 0 \;\text{ above }\; \dot{\chi} = 0 \text{ locus};$$
$$\dot{\chi} < 0 \;\text{ below }\; \dot{\chi} = 0 \text{ locus}.$$

On the other hand, the shape of the $\dot{\omega} = 0$ locus is given by

$$\frac{d\chi}{d\omega} > 0 \ , \quad \frac{d^2\chi}{d\omega^2} < 0 \ \text{ if } \ \xi < 1;$$
$$\frac{d\chi}{d\omega} \lesseqgtr 0 \ , \quad \frac{d^2\chi}{d\omega^2} > 0 \ \text{ if } \ \xi > 1.$$

Concerning the dynamics of ω, we confirm that

$$\dot{\omega} > 0 \ \text{ below } \ \dot{\omega} = 0 \text{ locus;}$$
$$\dot{\omega} < 0 \ \text{ above } \ \dot{\omega} = 0 \text{ locus.}$$

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Figures

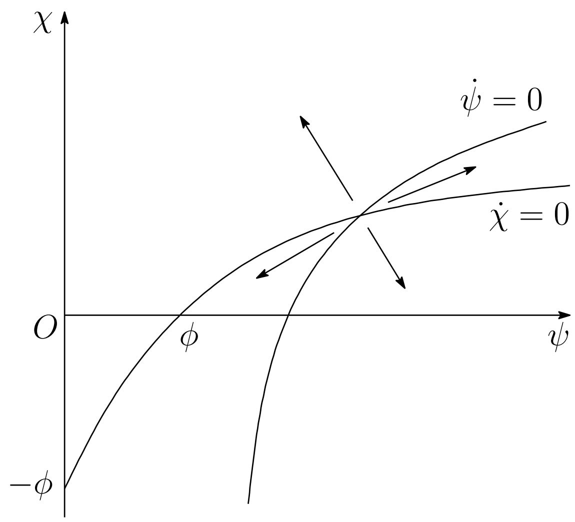

Fig. 1. Phase diagram under the Clower-Lucas-type CIA-status constraint

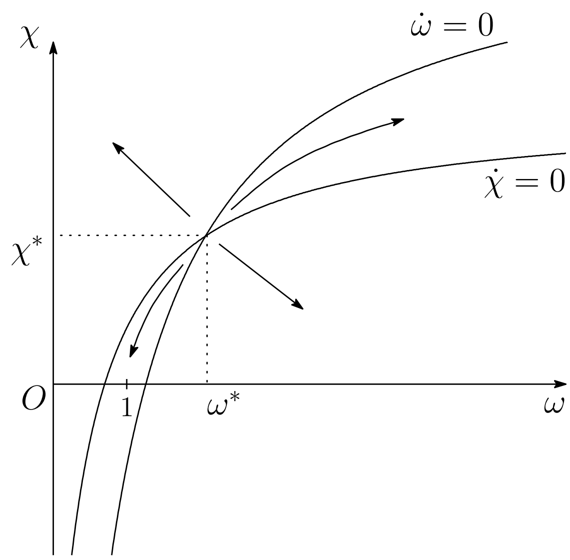

Fig. 2. Phase diagram under the Stockman-type CIA-status constraint when $\xi < 1$

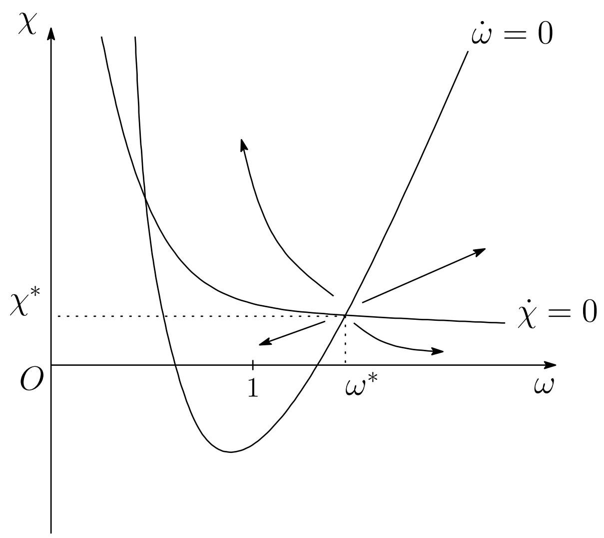

Fig. 3. Phase diagram under the Stockman-type CIA-status constraint when $\xi > 1$