Credit Derivatives in Managing Off Balance Sheet Risks by Banks

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July 2001

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MSc in Finance 2001

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This project is submitted as part of the requirements for the award of the MSc in Finance.

July 2001

ABSTRACT

Credit risk has been a worrying type of risk for financial managers. Fortunately, a recent market development –credit derivatives- has made the credit risk more manageable. The loan portfolio management has become more practicable than it used to be in the past. However, credit derivatives are still not well examined. There are uncertainties about and difficulties in the pricing and portfolio management of credit derivatives due to the non-normality in probability distribution of credit risk.

Various models have been developed for credit derivatives pricing. After having drawn the general picture for the credit derivatives, we have studied some recent pricing models in a Das (1999) framework, in this study. Also appended is a an attempt to a step forward for simulating the risk-free rates and spreads, to test how powerful simulation can be in modelling the credit risk and pricing of it. Moreover, with highly developed computer technology, it is possible to make sensitivity analysis under several scenarios, to form imaginary loan portfolios, find their risk exposures, and perform a successful risk management practice.

Acknowledgements

I am grateful to my supervisor for his help, and to all those who have supported me throughout the preparation of this work.

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I. INTRODUCTION

Banks and other non-bank financial institutions are today's most complicated economic units. Their overall portfolios are more so... The management of these portfolios is one of the greatest challenges facing the financial manager. With the introduction of credit derivatives and mechanisms that allow institutions to unbundle the credit risk (CR) portion of traditional debt instruments from market risk (in an effort to improve pricing efficiency), this challenge could have been better and more easily handled in the last decade.

The goal of a portfolio manager - regardless of whether the portfolio is composed of equities or credit assets- should be to create an "efficient" portfolio. With equities, the manager can usually buy and sell assets until he attains the optimal level of diversification. However, the manager of a loan portfolio typically faces several constraints and conflicting objectives while managing this portfolio. For example, some of the loans in the portfolio may be liquid, and those that are may not be truly saleable because of restrictions in the documentation or the effect of a sale on the relationship with the borrower(s).

Credit derivatives (CDs) provide loan portfolio managers with a number of ways of constructing and shaping a portfolio and managing the conflicting objectives they have to face on both micro and macro levels. Firstly, CDs can be used to reduce the portfolio's exposure to specific borrowers (obligors) or to diversify the portfolio by synthetically accepting CR from different industries or geographic regions that were previously underweighted in the portfolio, on the micro level. On the macro level, CDs can be used to create "synthetic" securitized positions that alter the risk and return characteristics of a large number of exposures at once.

The development of CDs is a logical extension of two of the most significant developments of the recent past: securitization and derivatives. The concept of derivative is to create a contract that derives from an original contract or asset. For example, stock market derivatives are contracts that are settled based on movements in prices of stocks, without transferring the underlying stock. Similarly, a credit derivative is a contract that involves a contract between parties in relation to the returns from a credit asset, without transferring it.

This work has two main parts. In Part II, we first analyse the major aspects of credit derivatives briefly. We then examine the pricing particulars of credit derivatives in more detail, focusing on simple spread models, Jarrow-Lando-Turnbull (JLT) model, Das-Tufano extension of JLT, and Duffie-Singleton analysis, in Part III.[1]

[1] Part III is largely based on a framework set up within two articles by Sanjiv Ranjan Das referred to in the bibliography

II. CREDIT DERIVATIVES IN GENERAL

II.1. Definition of Credit Risk

Every financial asset is a bundle of risks - interest rate risk, liquidity risk, price risk, exchange rate risk etc., - of which credit risk (CR) is but one. CDs enable banks to strip the CR from an underlying asset (such as a loan or a bond) and trade or manage it separately.

Credit risk itself can be further decomposed into its constituent elements – market risk, specific risk, settlement risk, default risk, sectorial risk and country risk – each of which in turn can be priced, hedged and traded as discrete items.

Credit risk is the possibility that a borrower will fail to service or repay a debt on time. The degree of risk is reflected in the borrower's credit rating, which defines the premium over the riskless borrowing rate it pays for funds and ultimately the market price of its debt. CR has two main components: market risk and firm-specific risk. CDs allow users to isolate, price and trade firm-specific CR by unbundling a debt instrument or a basket of instruments into its component parts and transferring each risk to those best suited or most interested in managing it. Various traditional mechanisms have been used to reduce the CR including the refusal to make a loan, insurance products, factoring, financial guarantees, and letters of credit. Even so, these mechanisms are less effective and have proved to be very expensive during periods of economic downturn when risks that normally offset each other may simultaneously default and may cause financial institutions to suffer substantial loan losses.

II.2. Definition of Credit Derivatives

Derivatives such as swaps, options and forwards have long been used to transfer interest rate, exchange rate and other risks. CDs use similar, but specially adapted, instruments to trade and hedge CR. Techniques such as cash settlement - and payoffs linked to the market valuation of defaulted assets - facilitate the separation of defined CRs from the underlying instruments in which they are normally embedded.

A credit asset is the extension of credit in some form: normally a loan, instalment credit or a financial leasing contract. Every credit asset is a bundle of risks and returns: every credit asset is acquired to make certain returns on the asset, and the probability of not making the expected return is the risk inherent in a credit asset. There are several reasons due to which a credit asset may not yield the expected return to the holder. These include delinquency, default, losses, foreclosure, prepayment, sharp interest rate movements and erratic exchange rate movements etc.

CDs are privately negotiated bilateral contracts that allow users to manage their exposures to CR. They separate the ownership and management of CR from other qualitative and quantitative aspects of ownership of financial assets. A credit derivative, being usually a composite product composed of plain vanilla counterparts, are over the counter (OTC) instruments that are designed to manage the

CR in the same way as to manage currency and interest rate exposures, as well as to exploit profit opportunities arising on the market. With the use of CDs, given the default risk (DR), the CR can be transferred to another party in return for a fee - which can be considered as an insurance fee against the adverse credit market conditions, and the premium paid for the eligibility to exploit the profit opportunity. The determinants of this fee are the credit rating of probable swap counterparty, maturity, probability of default, and the expected post default value of the underlying. A bank that is concerned that one of its customers may not be able to repay a loan can protect itself against losses by transferring the CR to another party while keeping the loan on its books. This mechanism can be used for any debt instrument or a basket of instruments for which an objective default price can be determined. In this process, buyers and sellers of the CDs can achieve various objectives, including reduction of risk concentrations in their portfolios, and access to a portfolio without actually making the loans. CDs offer a flexible way of managing CR and provide opportunities to enhance yields by purchasing credit synthetically. CDs cannot eliminate all CR, because inherent in the transfer of a loan exposure to Company A, is the introduction of a new exposure to Company B because of the use of a derivative with the latter. Generally, AAA-rated Special Purpose Corporations or Vehicles (SPCs or SPVs) are created to enter into such transactions to reduce the new exposure. Examples of CDs include Credit Linked Notes (CLNs), Total Return Swaps (TRSs), Credit Default Puts, Credit Spread Options and others. Figure 1 demonstrates the relationship between the general derivatives classification and triggering events. Any type of arrangement can be carried out based on these products and credit events.

Figure 1. Triggering Events and Credit Derivatives

Triggering Event	General Classification			
	Options	Swaps	Forwards	Securitization
Default	✔	✔	NA	✔
Spread Changes	✔	NA	✔	?
Rating Changes	✔	✔	✔	✔

CDs can also be classified according to the underlying reference entity (the entity whose CR is being traded - who is rarely a party to the transaction) as follows:

1. Single credit: where the payout is linked to the defaulted value of a defined debt obligation of a single reference credit

2. Basket: several reference credits, with a single payment linked either to the occurrence of a first default by any one of them or a multiple default by two or more

3. Index: where the payout is linked to one or more indices (e.g. an emerging market or high yield bond index).

Reference entity may be banks, corporates or sovereign states.

Though CDs are described as bilateral transactions, they can be completely marketable contracts; the CR inherent in a portfolio can be securitised and sold in the capital markets just like any other capital market security. So, anyone who buys such a security is inherently buying a fragment of the risk inherent in the portfolio, and the buyers of such securities are buying a fraction of the risks and returns of a portfolio held by the originating bank. Thus, the concept of derivatives and securitization have joined together to make risk a tradable commodity.

CDs bring several advantages to those wishing to actively manage their CR:

1. The "reference entity" whose CR is being transferred, does not need to be party to, or even aware of, the credit derivative transaction. This enables banks and corporate treasurers to manage their CR without affecting customer relationships.

2. The terms of the derivative transaction (tenor, seniority etc.) can be customized to suit the buyer and seller of risk, rather than the borrower.

3. CDs allow short selling of credit - selling CR that you do not own. While it is impossible to short sell a bank loan, the effect of a short position can be achieved synthetically by using derivatives to buy credit protection. The buyer pays a small premium in return for the opportunity of receiving a large gain in the event of credit deterioration.

II.3. Users of Credit Derivatives

The appeal of CDs is obvious. Not only do they offer a new range of credit enhancement techniques, but they also revolutionize the trading and management of CR. Just as the stripping of interest rate risk through derivatives in the 1980s opened up a wealth of now familiar investment, arbitrage and hedging techniques, CDs offer a host of opportunities with respect to CR. These include the shorting of credit exposure, the creation of synthetic assets and the manipulation of term credit exposures through mismatched maturities between debt and credit derivative holdings.

Once largely confined to banks, due to their need to meet their capital adequacy requirements, the market participants have expanded to include insurance companies, hedge funds, mutual funds, pension funds, corporate treasuries and other investors looking for yield enhancement or CR transference. The market has evolved from the financial institutions' needs to manage their illiquid credit concentrations and their use of default puts to hedge their credit exposure. Existing derivative techniques have been used for emerging market debt and have further been applied to corporate bonds and syndicated bank loans. Total Return Swaps, for example, were developed to sell customized exposures to investors looking for an increase in the yields on their portfolios. These structures enable investors to have exposure to portfolios that previously were not available to them and provide them with new diversification opportunities.

Several factors have contributed to the development of the credit derivative market. Investors have shown keen interest in these products for yield enhancement because of narrowing credit margins on conventional corporate and emerging market sovereign issues. As investors have come to understand these products more fully, trading volumes have increased. Now dealers are more frequently warehousing trades in the same way they warehouse and manage interest rate risk.

Most market participants indicate a consensus expectation of continued growth and increased liquidity for the credit derivatives markets in the future. CDs will make CR pricing more efficient, and help segregate CR from market risk in bond and loan pricing. Institutions best suited to handle the CR component of these debt instruments will be able to buy only that portion of the risk and warehouse it.

II.4. Uses of Credit Derivatives

There are four main reasons of using CDs in bank financial management. A financial manager may use CDs to manage his credit risk. He may also use them to optimize his balance sheet management. Tailoring investments and taking advantage of the relative value are the other two options for the managers.

II.4.1. Management of Credit Risk

II.4.1.1 Managing Illiquid Credit Exposure

Markets are assuming an increased burden of credit exposure that is not particularly liquid. There is illiquid credit exposure that is not readily apparent because it is not associated with a standard, readily transferable credit instrument. Fixed-price supply contracts, trade receivables, or insurance contracts are a few examples. Or illiquidity of a CR may arise because relationship, regulatory, or tax considerations make it hard for the owner to liquidate the asset.

Managing Client Relationships: Portfolio managers can find themselves locked into huge credit exposures arising from client transactions. Lending is a primary resource in maintaining client relationships, yet when the credit lines run out, banks are themselves in need of liquidity. Selling loans to free up capacity may be just as harmful to relations as refusing funding outright.

Banks can employ Credit Swaps to reduce credit exposure without physically removing assets from their balance sheet.

Reducing Portfolio Concentrations: Concentration risk can arise from increased exposure to one reference entity, or it can arise from one or more exposures to a group of highly correlated credits. Credits or loans concentrated in one particular industry or a particular location would be an example of the latter kind of concentration or correlation. It is economically rational to pay a premium to reduce exposure to over-concentrated credits. Credit Swaps reduce targeted exposures.

Credit Downgrades: A down-grade or expected down-grade by rating agencies will be reflected in the secondary market as prices of liquid instruments fall. Portfolios of liquid assets may be forced to make mandatory liquidations. Even the holders of an illiquid loan portfolio may be required to recognize a loss. If loan holders cannot sell the underlying assets, the economic capital that needs to be set aside against these riskier assets will be greater. Pre-emptive measures can be taken by structuring a credit derivative to provide down-grade protection, reducing the risk of forced sales at distressed prices and enabling a portfolio manager to own assets of marginal credit quality at lower risk.

II.4.1.1 Hedging Against Future Borrowing Costs

The use of CDs to hedge dynamic exposure is a complex application, but their use to hedge against future borrowing costs is relatively more straightforward. The desire to hedge the future costs of borrowing may reflect a wish to guard against or benefit from a narrowing or widening of the credit spread between debt instruments. Alternatively, borrowers may wish to lock in future borrowing costs without enlarging the volume of their balance sheet now. An acceptable hedge in these situations is a Credit Spread Forward in which a Protection Buyer would receive the difference between a floating spread of the reference security and some benchmark yield and the strike, if positive, and would pay the difference, if negative.

II.4.2. Optimization of Balance Sheet Use

II.4.2.1 Funding

Banks with high funding costs often buy risky assets to generate income from credit spread. Consequently, they are open to an alternative that would take on credit exposure in off-balance-sheet positions that do not need to be funded. On the other hand, a bank with low funding costs may want to capitalize on this advantage by buying balance sheet assets. A Credit Swap may help both of them meet their objective. For the bank with the high funding level, there is no upfront principal outlay in assuming a Credit Swap position. This approach is an important source of portfolio diversification for banks and institutional investors who would otherwise continue to accumulate concentrations of lower quality assets. And for a low cost investor, the premium for buying protection on balance sheet assets may be less than the bank's spread over funding.

II.4.2.2 Economic Capital

Consider a downgrade of a credit already in a loan portfolio. The expected loss on this position rises to reflect an increased probability of default. Higher expected losses directly affect loan loss reserves, which are a counter asset (liability side) charge on the balance sheet. There is an indirect cost as well - further costs are incurred by the need to set aside more economic capital in recognition of the greater volatility in default probability for lower rated credits. In other words, there is less confidence in loan loss projections since lower quality credits exhibit more disparate default behaviour. CDs can provide a solution by reducing exposure and freeing economic capital, which may be used in more capital efficient investments.

II.4.2.3 Total Return Swaps

Occasionally a party has a reason to exchange the total economic performance of an asset for another cash flow without balance sheet impact. Alternatively, the desire may be to remove effectively all economic exposure to an underlying asset for a given term, and perhaps to effect the transfer with confidentiality and without the need for a cash sale. Using a Total Return Swap, the TR Receiver can develop exposure to the underlying asset without the initial outlay required to purchase it. The maturity of the swap does not have to match the maturity of the underlying asset. The TR Receiver in a swap with maturity less than that of the underlying asset may benefit from the positive carry associated with being able to roll forward short-term financing of a longer term investment. The TR Payer may benefit from being able to purchase protection for a limited period without having to liquidate the asset permanently.

II.4.3. Tailoring Investments

Using CDs to tailor investments can involve complex applications making new asset classes accessible to investors for whom administrative complexity or restrictions imposed by borrowers have traditionally presented barriers. Or they can be used to isolate recovery rate expectations and interact to mutual benefit with a counterparty with a different expectation. A more straightforward example involves tailoring an investment's term. It is often difficult for investors to tailor the term of investments to meet their own needs, or to extract value for developing focused views on the term structure of CR. Consider an exposure to a corporation that does not issue debt in less than eight-year increments. The predominance of investors limited to terms inside five years, and the absence of shorter term debt than eight years means that the term structure of credit spreads is likely to reflect a tighter spread for the first five years and a wider spread for the last three than would be expected. In other words, the term structure reflects a technical imbalance between supply of and demand for short term versus long term investment.

Credit derivatives offer negotiable maturity profiles. Using a Credit Swap, for example, it is possible to break an eight-year position into a five-year position and a three-year position. A bank investor limited to five-year terms is able to take the first five years of risk, while another investor is able to take the last three. Both investors are satisfied since the forward investor is able to focus his exposure in the area in which he is able to extract most value for his purpose, while the other investor is able to generate an exposure which is not otherwise available in the cash markets.

II.4.4. Exploiting Relative Value

From an investor's perspective, CDs may be valuable simply by providing credit exposure in a form that would not otherwise be available. However, where alternative investments offer essentially similar risks, an investor needs to ascertain relative value to justify using CDs instead of more traditional or more liquid assets. The outcome is the opportunity to exploit any relative value arising from risk and return differences. Methodologies of varying complexity have been developed to help decide whether a derivative is an attractive investment in a given specific situation.

Credit derivatives provide a means of earning an income without requiring large initial cash outlays. This is a great advantage for investors (mostly banks and financial institutions) that face a relatively high cost of capital. However, selling CDs purely for generating income is not recommended for companies. These are highly leveraged contracts, and in the worst case can lead to heavy losses for the seller. The right time where it may be appropriate for a company to sell a CD is when transferring credit exposure from one reference entity to another, i.e. selling CDs on one reference entity to subsidize the purchase of credit protection on another reference entity.

In a Credit Swap, the Protection Buyer must deal with another type of risk, that of the default of the Protection Seller. The Implications for the buyer range from having to find alternative protection, to facing the situation where both the reference entity and the Protection Seller default. This affects pricing of credit derivatives.

Protection bought from higher rated counterparties will involve a higher premium. Correlations between the reference entity and the Protection Seller which make simultaneous default more likely suggest lower premia - for example, protections bought from one bank against another bank in the same country.

II.5. Types of Credit Derivatives

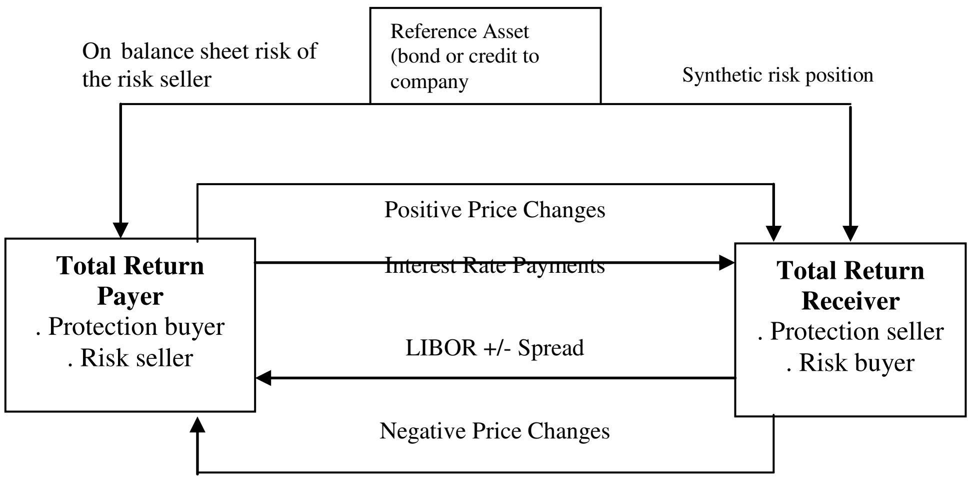
The easiest and the most traditional form of a credit derivative is a guarantee. Financial guarantees have existed for centuries. However, today's concept of CDs goes much farther than a simple bank guarantee. The CDs being currently used in the market can be broadly classified into the following: Total Return Swaps, Credit Default Swaps, Credit Spread Options, Credit Linked Notes, and Credit Forwards. Practitioners provide detailed descriptions of a transaction-specific payoff profile so it is of more value to understand under what circumstances one will receive a payment, or be required to make one, than it is to know a list of product names; that is the payoff profiles, and the pricing of CDs. However, due to their extremely sophisticated structures, and a lack of a clear-cut industry standard, the pricing of CDs is left out of the scope of this work.

II. 5.1.Total Return Swaps (TRSs)

A Total Return Swap is a derivative instrument that allows an investor to receive the total economic return of an asset (income plus or minus any change in capital value) without actually buying the asset. Figure 2 is a diagram of TRS cash flows. One party pays the total economic return on a notional amount of principal to another party in return for periodic fixed or floating rate payment (plus some spread).

The underlying reference credit (e.g. LIBOR) can be any financial asset, basket of assets or an index. There can be many variations on the basic TRS structure. One can, for instance, use a basket of assets instead of a single credit. Maximum and minimum levels for the floating rate leg of the structure can be set via embedded caps on a reference credit.

Figure 2. Total Return Swap Cash Flows



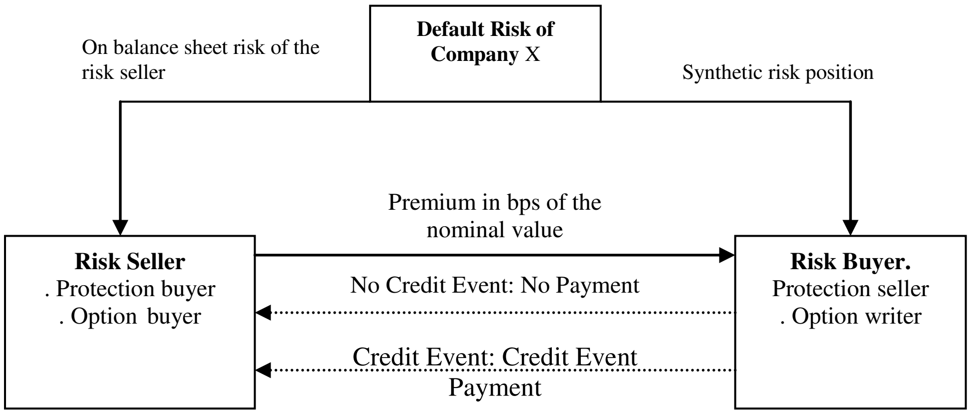
Banks use this product as a way of transferring the risk exposure of an asset to another interested party. Investors seeking exposure to a bank portfolio use TRSs to enhance their yield. The swap enables banks to keep the entire asset on their books, but maintain only the desired amount of credit exposure. This is important and valuable for the banks because they can look to move as much off their balance sheet as possible. In many cases, banks may want to keep the loans on their books to avoid jeopardizing their relationship with a customer or breaching client confidentiality.

Investors can leverage and diversify their portfolios to achieve higher yields by taking on this credit exposure. A TRS enables the investor to make loans synthetically without the administrative burden of documenting the loan agreement and periodically resetting the interest rate. TRSs can provide an economic way of using leverage to maximize return on capital. The exposure on an interest rate is not as large as the notional principal amount since only the respective interest payments are made. Only the total return of the portfolio is exchanged with the fixed or floating payments.

II. 5.2. Credit Default Swaps (CDSs)

A Credit Default Swap is another mechanism for distributing the default risk of securities and loans, enabling lenders and investors to improve risk management and better achieve their financial goals. In this case, one party makes periodic basis points payments (bps) and another party makes payments for the principal if the "credit default" event occurs. The pricing of such a derivative depends upon the credit quality of the reference credit, supply and demand for the reference credit, and prevailing credit spreads. The objective might be any of the following: to sell a specific risk, e.g. country risk in a project finance transaction, to free up credit lines for a specific customer, to obtain additional yield by assuming the CR, to improve portfolio diversification, to gain exposure to credits without buying the assets or to assume an off balance sheet synthetic position. Figure 3 is a CDS’s cash flow.

Figure 3. Credit Default Swap Cash Flows



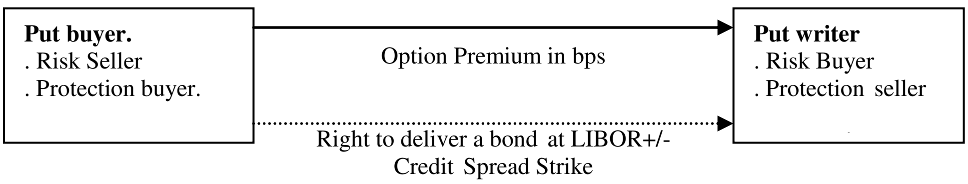
Banks that want to reduce or eliminate their exposure to a particular loan or basket of loans can buy a CDS without the borrower's knowledge or consent which may be required when the loans are sold outright. Manufacturing companies that depend upon a limited number of customers for revenue can buy a CDS on their customers' payment obligations. Investors who need to protect themselves against default but cannot or do not want to sell the risky security for accounting, tax or regulatory reasons, can buy a CDS. Investors can obtain additional yield without buying an asset, holding it on their balance sheet and funding it. Building on the basic swap structure, investors can swap the default risk of one credit with that of another credit. This can help companies diversify their portfolios while avoiding the transaction costs associated with buying and selling many individual securities or loans.

Credit events in such transactions are pre-defined in the agreement, which could include a payment default, bankruptcy or debt rescheduling. The credit event must be material and objectively measurable. The reference credit can be almost any loan or security, a basket of loans or securities, regardless of the currency, and the tenure of the swap can match or be shorter than the tenure of the reference credit.

II. 5.3. Credit Spread Options (CSOs)

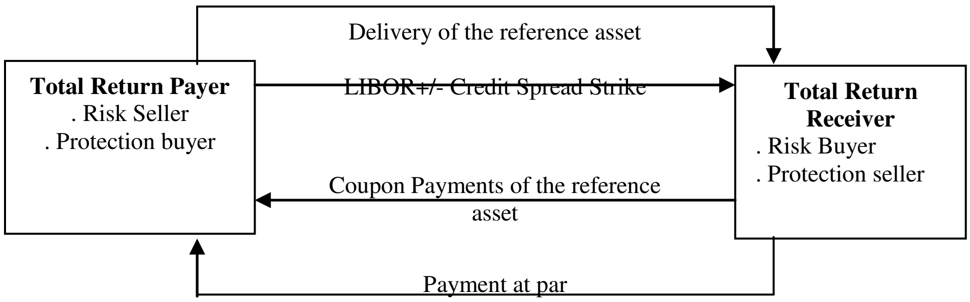
Buying or selling an option on a borrower's credit spread provides an opportunity to gain exposure on the borrower's future CR. A bank can lock in the current spread or earn premium for the risk of adverse movement of credit spreads. They also present a method of buying securities on a forward basis at favourable prices. Credit Spread Options are normally associated with bonds, which are priced and traded at a spread over a benchmark instrument of comparable maturity. The yield spread represents the risk premium the market demands for holding the issuer's bond(s) relative to holding risk free assets, such as U.S. Treasury Bonds. Options can refer to the borrower's spread over U.S. Treasuries, LIBOR or any other relevant benchmark. Figure 4 shows the cash flows from a CS Put

Figure 4. Credit Spread Put



Credit Spread Option structures allow investors to buy the bonds at attractive terms. If the option expires worthless, the total cost of the bond is reduced by the amount of the premium. Otherwise, the investor pays for the bond at the contracted strike price. There could be different strategic variations of this, such as using options on credit spreads to take position on the relative performance of two different bonds, and locking in the current spread by buying calls and selling puts on the spread with the possibility of earning a premium in the transaction. The cash flows in case of exercise are depicted in Figure 5. Again, this derivative structure allows investors to take a position in the underlying assets synthetically rather than buying assets in the cash market.

Figure 5. CSO Cash Flows in Exercise



Credit spread options also give the users protection in the event of a large, unfavourable credit shift, which falls short of default. Spreads should move to reflect any downgrading in the credit rating. End users who purchased spread options will be able to cash in even though the referenced credit has not defaulted.

II. 5.4. Credit Forwards (CFs)

A credit forward is a forward agreement used to hedge against an increase in default risk on a loan (decline in credit quality of a borrower) after the loan rate is determined and the loan has been issued. The CF agreement specifies a credit spread (a risk premium above the risk free rate, to compensate for default risk) on benchmark bond issued by the borrower (S_F). S_T is the actual credit spread on the bond when the CF matures. MD is the modified duration on the benchmark bond rated R, and A is the principal amount of the forward agreement.

The CF buyer bears the risk of an increase in default risk on the benchmark bond of the borrowing firm, and the CF seller hedges itself against an increase in the borrower's DR. If the borrower's DR increases, when the CF matures the market will require a higher CS on the borrower's benchmark bond (S_T) than was originally agreed to in the contract S_F ($S_F < S_T$). The CF buyer pays the CF seller (S_F - S_T)*MD*A. This amount could be used to offset the loss in market value of the loan due to the rise in the borrower's DR.

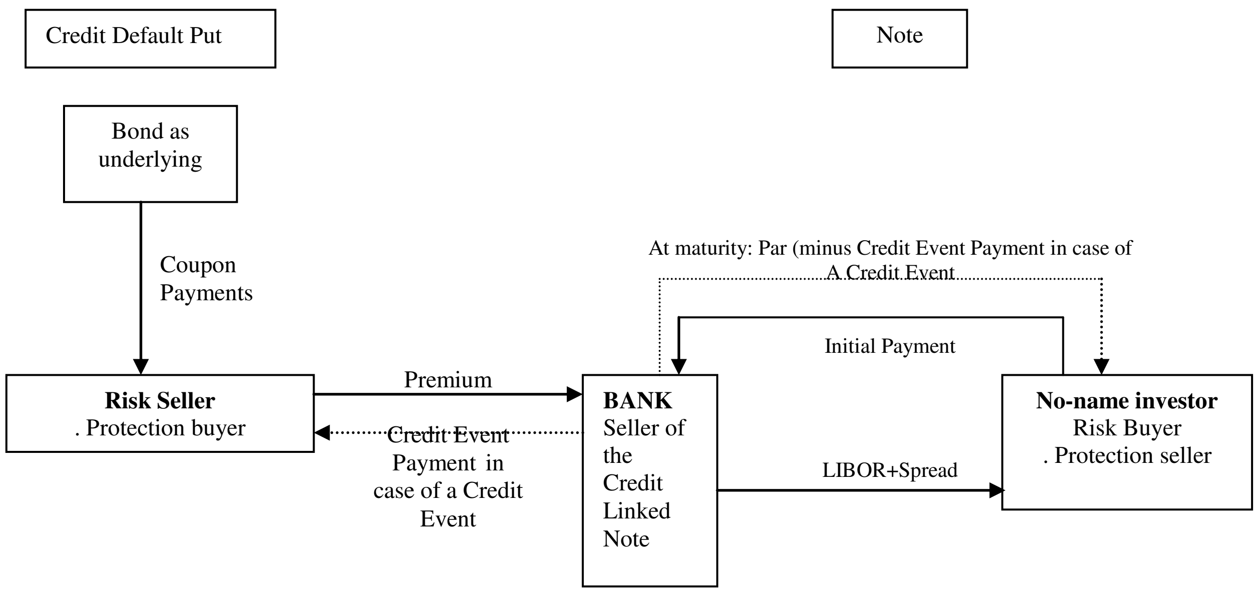
II. 5.5. Credit-Linked Notes (CLNs)

Under this structure, the coupon or price of the note is linked to the performance of a reference asset. It offers borrowers a hedge against CR and investors a higher yield for buying a credit exposure synthetically, rather than buying it in the publicly traded debt.

CLNs are created through a Special Purpose Company (SPC) or trust, which is collateralized with AAA-rated securities. Investors buy the securities from the trust that pays a fixed or floating coupon during the life of the note. At maturity, the investors receive face value unless the referenced credit defaults or declares bankruptcy, in which case they receive an amount equal to the recovery rate. Here the investor is, in fact, selling the credit protection in exchange for higher yield on the note.

The trust on the one hand enters into a default swap with a deal arranger. In the case of default, the trust pays the dealer par minus the recovery rate in exchange for an annual fee. This annual fee is passed on to the investors in the form of a higher yield on the notes. In this structure, the investors can obtain higher yield for taking the same risk as the holder of the underlying reference credit. The investor does, however, take the additional risk, though limited, of its exposure to the AAA-rated trust. The CLN allows a bank to lay off its credit exposure to a range of credits to other parties. The structure of a credit default note is outlined in Figure 6.

Figure 6. The Structure of a Credit Default Note



II.6. Credit Derivatives in Practice

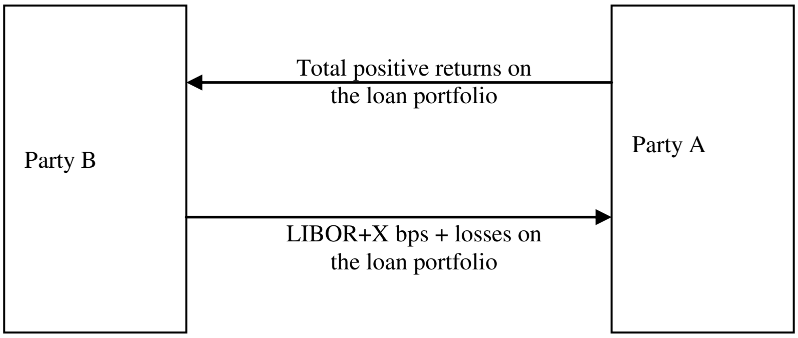
II.6.1. Loan Portfolio Management

CR is one of the most significant classes of risk for financial institutions. Until recently, there has not been a developed liquid market for trading CR. Financial institutions have concentrated on their net market exposures sometimes at the expense of increasing the CR to certain companies. CDs allow financial institutions to change their exposure to a range of credit-related risks. As outlined above, there are different structures that allow the transference of CR from one party to another. The choice of the product depends upon the goals a financial institution is looking to achieve. In some cases, the bank can buy protection in the form of default puts to transfer the CR to an insurance company or other institutional investors. In any case, the bank may swap one credit for another of equal rating, just to reduce its exposure to one party. By using CDs, a loan portfolio manager can achieve any of the following objectives:

1. Control CR of any debt instrument or basket of instruments by selling or transferring the credit exposure of the portfolio,
2. Reduce a particular risk concentration in the portfolio,
3. Create synthetic assets tailored to meet their needs,
4. Provide a diverse menu of global exposures to achieve portfolio diversification, or

5. Gain exposure to another bank's loan portfolio without participating in the syndicate. Figure 7. shows a loan portfolio’s cash flows

Figure 7. TRS Loan Portfolio Cash Flows



II.6.2. Mergers and Acquisitions Transactions

CDs can be used for leveraged mergers and acquisitions (M&A) transactions. Lenders who finance such transactions can use credit protection to manage exposure to the acquirer of a target company. The funding exposure can be in terms of bridge financing or a permanent syndicated loan used to finance the transaction.

Another arena where CDs can be used in an M&A transaction is the merger of a stronger credit with a weaker one that will potentially downgrade the credit of the combined firm. A lender that is exposed to stronger credit can buy credit protection or buy a put option on the credit spread of Company A to protect itself from any downgrading of the referenced credit.

Another application of CDs in an M&A transaction is to free credit constraints. For example, Bank C may not be able to provide bridge or permanent financing to the acquirer Company A since it has reached the maximum credit limit with A. To free this lending constraint, it can transfer the risk of the existing credit lines by entering into a default swap with other credit dealers. By doing so it will expand the bank's capacity to assume additional lending and provide the needed M&A financing to Company A.

To hedge the risk of a credit derivative in a large M&A transaction, one can diversify the CR by entering into syndication or repackaging the CR and selling it on the credit markets. The pricing of these products is generally done using the benchmarks in the cash markets. If such cash market benchmarks are not available for any particular market, then default probability and recovery rate models are used to price CDs.

II.7.Pricing and Hedging Considerations

II.7.1. Pricing Issues

CDs lie at the junction of traditional insurance or guarantee products and financial derivatives. Each of these products has their own valuation methodology, neither of which is entirely satisfactory for the valuation of CDs. The insurance industry typically uses historical data to value insurance policies relying on actuarial science and the probability of payment-triggering events.

Credit rating agencies have tables of probability of downgrading or default by maturity, which some practitioners use. These tables, however, are based largely on inflexible assumptions: they assume that the future will be like the past (No Markov Process), they do not take into account market information available in the form of credit spreads and they assume that exposure to different entities is unrelated (No Correlation). On the other hand, derivative dealers use market-based information to price their products. The derivatives practitioners use this information based on the assumptions of risk neutral valuation and arbitrage-free complete markets. Credit markets are not liquid enough to be perfect, nor is there a complete set of financial instruments available for precise valuation. There is also the question of which stochastic process to assume for different credit events. A more comprehensive approach to pricing is taken in Part III.

The pricing models can be grouped under four main categories:

1. Ratings-Based Default Probability Models: These pricing models rely on credit ratings and published data on default losses, to approximate the probability of default of a given issuer. This data is then supplemented by the dealer's assumption about what the likely recovery rate will be in the event of a default, i.e. how far below par will the debt be trading when the company announces its default. Some models for determining the recovery rates in default use fixed percentages based on industry or credit ratings, while others rely on random, stochastic processes for default. These pricing models are good in that they are not overly data intensive and rely more on aggregate statistics. With regards to a new issuer, this model is good in that it does not require issuer-specific data sets. It does however limit one’s ability to introduce specifics about a particular issuer.

2. Credit-Spread Based Default Probability Models: These models track an issuer's credit spread over time and for different maturities to establish a term-structure for their CR. Once this term structure is established, it is then used to estimate the probability of default of the issuer for a specific term. One of the advantages of this approach is that it allows for the use of issuer-specific data. Some weaknesses of this approach include the fact that a complete term structure of credit spreads for most issuers is not available, i.e. a company might have only three tranches of public debt of maturities of 1, 5 and 15 years. To use this data one must interpolate between these small number of points. Another assumption included in this model is that the entire spread over risk free assets is due to credit and does not consider market risk.

3. Pricing Based on Guaranteed Product Markets: Pricing based on guaranteed product markets is perhaps the simplest approach but is very limited in that it requires comparison to a credit default instrument already priced in the market. For example, if two counterparties have an agreement whereby one party is paying the other a margin of 100 basis points to guarantee the debt of a third party, then any similar default products on the third party should be priced similarly.

4. Replication/Cost-of-Funds Models: This model uses the hedging costs of a credit derivative as the basis for its pricing. Basically, the dealer decides – using probability models, default ratings etc.– under what portfolio of assets he requires to hedge the payments. If the dealer uses a Total Return Swap, for example, he decides what kind of margin he requires on the TRS. The combination of the cost of constructing a hedge, along with the dealer's required return, establishes a price for the credit derivative. This is perhaps the most straightforward approach for cases when a hedge can be constructed and hedging or replication-pricing methods are common in all derivative areas. However, problems arise when a good hedge is not available or the costs associated with putting it together are too expensive.

A more theoretical categorization of the pricing models can be made as below:

1. Firm Value Based Models: Models that use Merton's approach aimed at valuing the CDs indirectly by taking the firm value in to account are considered in this category. These include works by Merton (1974), Black and Cox (1976), Bhattacharya and Mason (1981), Shimko, Tejima and VanDeventer (1993), Longstaff and Schwartz (1995), Das (1995).

2. Reduced Form Models: They are broken down into three headings according to their focus of interest. Default Models, concentrate on the default rate of firm where default is depicted through a gradual change in ratings driven by a Markov transition matrix (e.g. Jarrow and Turnbull (1995), Longstaff and Scwartz (1995) etc.). Spread Models use the credit spreads of the firm in valuing the CDs (e.g. Longstaff (1995)). Credit Rating Models use the same transition matrix in the Default Models in their pricing methodology (e.g. Jarrow, Lando and Turnbull (1997), Das and Tufano (1996), etc.).

II.7.2. Hedging Issues

The majority of CDs outlined in this study have themselves been the hedge to an existing exposure of the dealer, i.e. the TRS issued on a bank's existing loan portfolio serves as a reduction of the bank's risk and does not require hedging. What about stand-alone trades where the dealer has no exposure to a particular area but is asked by a client to provide default protection against someone's receivables?

The main method available to CDs dealers in these cases involve constructing replicating portfolios, using either the company's publicly traded bonds, equity or by taking positions in a comparable company in the same industry. The techniques employed are comparable to the delta hedging of an options portfolio and require the dealer to make assumptions about the volatility (i.e. the probability of default) of the guaranteed company. The dealer builds an offsetting portfolio whose positive return will mimic the losses incurred on the default protection in the event of default. Typical hedges involve shorting the companies bonds or equity. The particular characteristics of an industry might allow a dealer to use a less-conventional hedge. For example, if the dealer suspects that the default of Company A on its bonds is a function of whether they win an future contract on which they are bidding against Company B and Company C, the dealer might establish a small long position in Company B's stock as an additional hedge. The hedging of CDs, or any financial derivatives for that matter, is not an exact science.

III. PRICING CREDIT DERIVATIVES

In this part we will summarize some of the latest and the leading pricing models of credit derivatives.[2] They are presented in a simplified manner in order to be more understandable and easier to handle, even by the layman. These models involve the inclusion of the interest rate risk, default risk, and recovery risk; they are practically more implementable using observed data and easy to use for arbitrage free risk neutral pricing.

Risk on a defaultable bond may be divided into three different components:

1. Interest rate risk: Any bond, defaultable or not, has the interest rate risk. Several models deal with interest rate risk (e.g. Heath-Jarrow-Morton (1992)).
2. Default risk: It refers to the possibility of the default of a bond, irrespective of the magnitude of loss from default. Rating agencies such as S&P and Moodys have usually been concerned with default risk; ratings usually refer to the likelihood of default. Hence, as a wealth of information is available on credit ratings, they can be a good proxy for default risk.
3. Recovery Risk: Being different from default risk, recovery risk refers to the residual market value for the assets of a firm when it defaults.

The combination of default and recovery risks determines the credit spread on a bond. Because credit derivatives may be traded using either risks, it is essential to separate them both. Therefore, from a modelling viewpoint, they cannot be treated as one composite entity. Besides, if they are not treated separately, the sources of

[2]This part is based on the structure taken up by three works, Das (1998), Das and Sundaram (1999), and Francis, et al (1999) pp 101-138 (Chapter 5 by Das).

empirical information in this process would be inconsistent. While ratings and other industry-level information are quite effective in providing market participants with a good idea of default likelihood, a more precise evaluation of the recovery risk of an individual firm is necessary to explain why spreads are different from those of other firms in the same rating category and industry.

In this part, we will investigate a span of different models that deal with the pricing of credit-sensitive securities. Starting with simple models of credit spread, we will proceed to more complicated ones in which all three sources of risk are incorporated. The latter includes the Jarrow-Lando-Turnbull (1997) model, the Das-Tufano (1996) extension of the Jarrow-Lando-Turnbull, and the Duffie-Singleton (1995) framework. In the last section of this part we look at some conceptual issues on relative pricing and its relationship to pricing to provide a rate of return on capital. We also briefly discuss the application of these methods to credit portfolios.

In order to effectively run a portfolio of credit derivatives, we need a model that will accommodate a wide variety of products, and encompasses all the risk categories possible. Such a model would enable the investor to avail himself of a desirable credit structure, and offer the dealer a working solution from which new products can easily be offered.

Credit risk is not a well traded risk; it is also complex, and can result in several modelling difficulties. Some of these problems can be enumerated as follows:

- CDs are still non-standardized;
- Historical information on credit risks is poor due to changes in economic conditions. Hence past data is unreliable.

- Prices are slow in adjusting to changes in ratings; so too are ratings following real credit quality changes.
- Replication of CDs is difficult due to the illiquidity of the underlyings.
- There are not many indexes for credit sensitive products.
- The majority of deals take place in the emerging markets where data is noisy and the statistical properties of asset prices are not well examined.

III.1. Technicalities in Pricing Credit Derivatives

A practical model should be able to use observable data, and employ a lattice scheme to price securities. This scheme should also be consistent in risk neutrality (absence of arbitrage). In this section, we look at the whole range of models that can be calibrated by using a bootstrapping method, proceeding from shorter to longer maturities. All three risks discussed above are incorporated into these models. The approach to modelling starts from the simple to the more complex.

III.1.1 Spread Models

These models consider the spread as a combination of default and recovery risks. Spread can be modelled directly. Being ideal for pricing credit spread options, this approach involves the definition of the model for the spread by choosing an appropriate stochastic process. The assumptions about the underlying stochastic processes behind the risks that are being dealt with are vital in designing the pricing models. Though principally the tools used in modern finance are continuous-time, the models in this part are based on discrete mathematics. Therefore, the calculations in this part will be simple arithmetic computations.

The square-root diffusion provides one possible example of a process that can be used in modelling the credit spreads. The following stochastic may be a possible candidate for credit spread modelling

$$ds = k(\theta - s)\,dt + \sigma\sqrt{s}\,dz \qquad (1)$$

where s is the spread, k is the rate of mean reversion, θ is the long run mean of the spread, and σ is the volatility coefficient. The Wiener increment is dz. The model assumes in its simplest form that interest rates are constant: when the spread option has a short maturity, this is a reasonable assumption. In addition, the volatility of the risk-free rate of interest is low in comparison to that of the spread making such an assumption justifiable. To represent this stochastic process in a more simple fashion, we discretize it

$$s(t) = s(0) + k[\theta - s(0)]t \pm \sigma\sqrt{s(0) \times t} \qquad (2)$$

indicating that next period's spread s(t) given this period's spread s(0) will be given by the equation above. This equation results in an "up" value or a "down" value determined by the sign of the shock term σ√s (0) x t. The spread changes by an amount k[θ - s(0)]t which is positive when the spread is below its average level θ and negative when the spread is above its average. This term represents the "mean reversion" in the spread. Therefore, the process above is a reasonably good modelling device for the spread.

In order to price options on spreads, we need the range of outcomes of the spread at the option maturity T, and the probabilities assigned to these outcomes. This is contingent upon the choice of the spread model. For instance, in the discrete version depicted earlier, in each period there are two outcomes, and the probabilities are equal (i.e., 0.5).

Given the Black-Scholes option pricing model in order to price derivatives, it is essential to discount the expected cash flows of the option, assuming that the cash flows at maturity are determined by using arbitrage free processes. These processes are pseudo-processes that make the existing prices of securities equal to the discounted, expected future values of the security. The process above in (Equation 2) is the statistical process. This equation must be refined to make it risk-neutral. We shall add an adjustment term (γ) to its drift, and work out for this term such that the expected values of securities under the modified process turn out equivalent to the current price to incorporate the risk into it (i.e. to make it risk neutral).

III.1.1.1. One-Factor Spread Model

Let us assume a two-period model with each period equal to one year. The current observed term structure of interest rates from the government bond market (the risk-free zero coupon rate curve) is made up of two rates: 5 per cent (for one year) and 6 per cent (over two years). Therefore,

$$\mathrm{r} = \begin{bmatrix} 0.05 \\ 0.06 \end{bmatrix}$$

Similarly, assume that the credit spreads for two periods are given by

$$\mathrm{s} = \begin{bmatrix} 0.008 \\ 0.009 \end{bmatrix}$$

Therefore, the risky zero coupon curve is simply the sum of these two curves, i.e.,

$$r + s = \begin{bmatrix} 0.058 \\ 0.069 \end{bmatrix}$$

Therefore, the price of a risky zero coupon bond of maturity two years (face value $1,000) is

$$B(0) = 875.0736 = 1{,}000 / (1.069)^2$$

The riskless forward rate between year 1 and year 2

$$f_{12} = (1.06)^2/1.05 - 1 = 0.0701$$

Assuming that

$$k = 0.15,\ \theta = 0.0083, \text{ and } \sigma = 0.0092$$

A simple discrete binomial representation of the model is

$$s(t) = s(0) + k[\theta - s(0)] \pm \sigma \sqrt{s(0)}$$

since the unit of time is 1 year (i.e., t = 1). Hence, the one period credit spread after a year would take one of two values (with equal probability) based on the equation above, where s(0) = 0.008. To make the pricing risk neutral (arbitrage-free), the stochastic process must be adjusted by modifying its drift term to include an adjustment term γ.

$$s(t) = s(0) + k[\theta - s(0)] + \gamma \pm \sigma \sqrt{s(0)}$$

Given the parameter values, by using the difference equation, the spread at the end of period 1 will be equal to either of the following two values:

$$s = \begin{bmatrix} \gamma + 0.00887 \\ \gamma + 0.00722 \end{bmatrix}$$

Thus, at the end of one period, the price of a two period zero coupon risky bond will be equal to one of the following two values:

$$B(1) = \begin{bmatrix} \dfrac{1{,}000}{1+f_{12}+\gamma+0.00887} \\ \dfrac{1{,}000}{1+f_{12}+\gamma+0.00722} \end{bmatrix}$$

As a result, by discounting these two values back at the risky rate with equal probabilities, the price of the bond at time 0 will be found as

$$B(0) = (1/2)[1{,}000/(1.07897+\gamma)+1{,}000/(1.07732+\gamma)]*1/(1.058) = 875.0736$$

We are left with a single equation in the unknown parameter γ, which we solve to obtain the risk adjustment term, computed as

$$\gamma=0.00198$$

By using this, the spread after one period is found as one of the following two values:

$$s = \begin{bmatrix} 0.0108 \\ 0.0092 \end{bmatrix}$$

Spread options can be effortlessly priced on this lattice. To illustrate, a one period call option on the spread with an exercise price of K = 0.01 would pay off when the spread was equal to 0.0108 and would expire worthless otherwise. The price of this spread option would be equal to

Spread (K = 0.01) option = 1/(1.05) x *N* x (0.0108 - 0.01) x 0.5

where *N* is the face value of the contract, (0.0108 - 0.01) is the final intrinsic value per dollar, and 1/(1.05) is the discount factor to present value the option. The probability of this option to end up in the money is 0.5.

The model above can be used to price risky and riskless bonds. These models can be extended to accommodate more periods, and it is possible to incorporate an additional factor to them.

III.1.1.2.Two-Factor Model

By extension of the one-factor model the interest rate can be made stochastic and correlated with the credit spread. Assume that we have a square root process for the interest rate similar to the stochastic process (Equation1) for the spread. This the interest rate process can be simplified using the same discretization procedure earlier

$$r(t) = r(0) + \alpha\left[\beta - r(0)\right] \pm \eta\sqrt{r(0)}$$

We also assume that changes in the spread and risk-free rate are correlated with parameter ρ. The rate of mean reversion is α, the long run mean of the risk-free rate is β, and the volatility coefficient is η. Assume the following parameters for this process:

$$\alpha = 0.2,\ \beta = 0.11,\ \eta = 0.02, \text{ and } \rho = 0.25$$

In order to establish the lattice for pricing credit sensitive debt, the lattice for riskless debt is first built, and then used as a base to build the lattice for the risky debt.

Because the risk-free rate of interest is stochastic, so as to do risk-neutral pricing, a risk-adjustment to the drift of the process must be made using an additional parameter (γ) as below

$$r(t) = r(0) + \alpha[\beta - r(0)] + \delta \pm \eta\sqrt{r(0)}$$

Given the parameter values, the one period interest rate will take one of the following two values after one period:

$$r = \begin{bmatrix} \delta + 0.0665 \\ \delta + 0.0575 \end{bmatrix}$$

Therefore, the price of a two-period riskless bond after one period will be

$$B(1) = \begin{bmatrix} \dfrac{1{,}000}{1+\delta \ + 0.00887} \\ \dfrac{1{,}000}{1+\delta \ + 0.00722} \end{bmatrix}$$

And as before, the price of the riskless bond at time 0 will be

$$B(0) = (1/2)[1{,}000/(1.0665+\delta)+1{,}000/(1.0575+\delta)]*1/(1.05)$$

$$= 1{,}000/(1.06)^2 = 889.996$$

Solving this equation we get δ = 0.00811 which is roughly the value of the term premium for the second period in the model. Plugging it back into the model we end up with the possible values of the risk-free rate over time.

$$r = \begin{bmatrix} 0.0746 \\ 0.0656 \end{bmatrix}$$

After the derivation of the process for the risk-free interests, the complete tree can be established by extending four branches at each node, as we are dealing with the combination of two binomial processes for two factors. This can be accomplished as below. Assuming the interest rates are in the up position in the first two states, and down in the last two, the state-space for the risk-free rate of interest will be

$$r(1) = \begin{bmatrix} 0.0746 \\ 0.0746 \\ 0.0656 \\ 0.0656 \end{bmatrix}$$

In a similar fashion, the state space for the credit spread will be:

$$s(1) = \begin{bmatrix} \gamma + 0.00887 \\ \gamma + 0.00722 \\ \gamma + 0.00887 \\ \gamma + 0.00722 \end{bmatrix}$$

The state space in spreads is expressed as a function of γ, the parameter for the risk adjustment.

In order to ensure that the correlation between r and s is achieved, it is also imperative to determine the probabilities. The two risk-neutral processes for r and s could be written in discrete-time as

$$r(t) = r(t-\Delta) + \alpha\left[\beta - r(t-\Delta)\right]\Delta + \delta + \eta\sqrt{r(t-\Delta)} \times w\sqrt{\Delta}$$

$$s(t) = s(t-\Delta) + k\left[\theta - s(t-\Delta)\right]\Delta + \gamma + \sigma\sqrt{s(t-\Delta)} \times z\sqrt{\Delta}$$

$$\text{corr}(z,w) = \rho$$

where Δ is the discrete time interval on the lattice and (w, z) are shocks to (r, s). Under this structure, the state-space for the random shocks (w, z) to the two processes may be discretized as:

$$w' = \left[+1 \;\; +1 \;\; -1 \;\; -1\right] \qquad\qquad z' = \left[+1 \;\; -1 \;\; +1 \;\; -1\right]$$

The following probability structure achieves the desired correlation between **w** and **z**:

$$q = \begin{bmatrix} \frac{1+\rho}{4} \\ \frac{1-\rho}{4} \\ \frac{1-\rho}{4} \\ \frac{1+\rho}{4} \end{bmatrix}$$

Note that **q** is a risk-neutral probability vector; this scheme makes sure that probabilities lie in [0,1], and also that the correlation is in the range [-1, 1], Assuming that the correlation is $\rho = 0.25$, the probabilities are found as

$$q' = [0.3125\ 0.1875\ 0.1785\ 0.3125]$$

We have solved for all desired values except the one for risk adjustment for credit spreads (γ). Now, we calculate the prices of the two period bond at the end of the first period. Given that there are four states, there will be four different prices.

$$B(1) = \begin{bmatrix} \frac{1{,}000}{1+0.0746+\gamma+0.00887} \\ \frac{1{,}000}{1+0.0746+\gamma+0.00722} \\ \frac{1{,}000}{1+0.0656+\gamma+0.00887} \\ \frac{1{,}000}{1+0.0656+\gamma+0.00722} \end{bmatrix}$$

We can solve for the risk adjustment y using the no-arbitrage equation below:

$1{,}000/(1.069)^2 = (1/1.058)*q'B(1)$

$$= \frac{1}{1.058}\begin{bmatrix}0.3125 & 0.1875 & 0.1785 & 0.3125\end{bmatrix}\begin{bmatrix}\dfrac{1{,}000}{1+0.0746+\gamma+0.00887}\\ \dfrac{1{,}000}{1+0.0746+\gamma+0.00722}\\ \dfrac{1{,}000}{1+0.0656+\gamma+0.00887}\\ \dfrac{1{,}000}{1+0.0656+\gamma+0.00722}\end{bmatrix}$$

where q and B are vector forms of the probabilities and bond prices respectively. The solution from this model is $\gamma = 0.00198$. The state space for the spread is

$$s(1) = \begin{bmatrix}0.0109\\ 0.0092\\ 0.0109\\ 0.0092\end{bmatrix}$$

This lattice can be extended in the same way to more periods, though it may not recombine, and, therefore, the calculations may be quite difficult.

The extension of the one-factor model to two factors gives rise to various advantages. The new model accounts for correlation between interest rates and spreads which is often the driving factor behind some of the derivative structures we encounter in practice. It is useful when credit swaps are being priced where the swap involves a payment of LIBOR versus a fixed rate plus spread. These swaps are sensitive to the stochastic processes of both the risk-free rate and credit spread. With a one-factor model, pricing of credit swaps would not be very easy if not possible.

III.2. Modelling of Spread in Detail

In one and two factor spread models aggregate spread is considered without decomposing it into its default and recovery components. However, the spread should be broken down into its components to price more complex forms of CDs. To illustrate, the one factor model allows the pricing of spread options but not credit default swaps, and total return swaps. Upon the extension of the model to two factors, some forms of credit swaps, but not default swaps can be priced, because the model does not depict the event of default. In order to make the components of the spread separately tradable default itself has to be modelled.[3]

Decomposition of the spread involves modelling choices of the processes for default (λ), and recovery rates in the event of default (ϕ).

This default (hazard) rate may be a constant or a function of time to maturity of the bond, the level of the current interest rate, or some other factors in the economy. The recovery rate is denoted ϕ, and represents the percentage of the face value of the bond recovered in the event of default ($\phi \, \varepsilon \, [0,1)$).[4] The relationship between these parameters and spreads is such that given the one period risk-free rate r, the risk-neutral value of a credit risky bond with a time to maturity of one period will be equal to the discounted value of expected cash flows in the future. The pricing equation is[5]

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[3] Often, buyers of credit derivatives are seeking default protection, and do not mind tolerable levels of overall spread risk.
[4] For a more detailed and in-depth discussion of hazard-rate models the reader is referred to the work of Madan and Unal (1994, 1998).
[5] We have taken the expected value, risk aversion is ignored (i.e. the investor is risk-neutral) as the expected value is taken

$$B= (\lambda\phi+(1-\lambda))/(1+r)$$

It is possible to price the risky bond off the spread curve directly, meaning that

$$B=1/(1+r+s)$$

Equating the two equations for *B,* a relationship between the spread and its determinants can be developed. Solving for s, we obtain:

$$s= (\lambda(1-\phi)(1+r))/(1-\lambda(1-\phi)), \qquad \partial s/\partial\lambda>0, \qquad \partial s/\partial\phi<0.$$

What we observe from this relationship is that the spread is a function of the term $\lambda(1-\phi)$, not the individual parameters λ, and ϕ. This renders the separate determination of these rates non-viable.[6]

In this part, three recent models that have tried to model the default risk in detail are examined. The JLT model[7] models in great detail the event of default and does not focus extensively on recovery rate risk. The DT model[8] concentrates on the modelling of the recovery process in addition together with the risk neutrality conditions. The DS model[9] uses the connection between spreads and its components in modelling the term structure of swap yields. In the next section, we will examine the JLT model.

[6] One way to separate the two rates is to use debt of different seniority from the same firm, since presumably the debt would have the same hazard rate but different recovery rates (see Duffie and Singleton [1995] for a discussion of this issue).

[7] Jarrow, Lando, and Turnbull (1997)

[8] Das and Tufano (1996)

[9] Duffie and Singleton (1995,1996)

III.3. The Jarrow-Lando-Turnbull (JLT) Model

The JLT model focuses on modelling default and credit migration rather than modelling recovery rates. Therefore, in this model changes in spreads will be a function of changes in credit rating and the event of default.

The information on rating changes (rating transition matrix) is almost readily available. This model focuses on the changes in bond rating through the use of these rating changes. The transition matrix is a square matrix that portrays the probability in one period of migrating from any given credit rating to another (including the default class). Consider the following matrix

$$d = \begin{bmatrix} 0.87 & 0.08 & 0.05 \\ 0.07 & 0.85 & 0.08 \\ 0.00 & 0.00 & 1.00 \end{bmatrix}$$

It has three states, I, J, and D, standing for "investment grade (I)," "junk grade (J),' and "default (D)." When we read across the rows, the probabilities of going from one state to another in one period can be obtained. For example, the first row provides the probabilities of the rating level changing from I to any of the three possible states. It may remain in state I with a probability of 0.87, downgrade to level J with a probability of 0.08, and go into default with a probability of 0.05. The last row is a special row in the sense that it manifests the assumption that once the default state is reached, the system will remain in default for sure (i.e., state D is an absorbing state).[10]

[10] The transition matrix here is a simplified version of the true matrix, which usually comprises many different rating levels, ranging from AAA to default (D).

So as to establish the model, we also need information on risk-free interest rates, and the spreads for each of the rating levels, I and J. As before, we shall assume a two period model, and the risk-free zero coupon rates are

$$r = \begin{bmatrix} 0.05 \\ 0.06 \end{bmatrix}$$

The spreads for each rating level are as follows:

$$S_I = \begin{bmatrix} 0.008 \\ 0.009 \end{bmatrix}, \quad S_J = \begin{bmatrix} 0.010 \\ 0.015 \end{bmatrix}$$

A necessary assumption of the JLT model is that rating migration and interest rates are not correlated. For simplicity, we shall also assume that the risk-free rates are non-stochastic. The prices of risky debt of maturity of one and two periods are calculated as below

$B_I(1) = 1/1.058$, $B_I(2) = 1/(1.069)^2$, $B_J(1) = 1/1.06$, and $B_J(2) = 1/(1.075)^2$,

The JLT model assumes a recovery rate (ϕ) for default. It also makes the assumption that the recoverable amount is received at the maturity of the bond, not at the time of default.[11] For our example, we assume that

$$\phi=0.35$$

At maturity, the bond will be in one of the three states (I, J, or D). If it is in the first two states, the payoff on a zero coupon bond will be 1. In the event of default, the payoff is ϕ. The payoff or cash flows for all possible three states can be written as

$$\mathbf{C}' = \begin{bmatrix} 1 & 1 & \phi \end{bmatrix}$$

[11] This assumption is a simplification which makes the numerics of the model fairly manageable.

In order to get the risk-neutral values of the bonds, which must equal the prices above, we need to get expected cash flows at maturity and discount them back to the present using risk-neutral default probabilities. However, in transition matrix d we have the statistical probabilities that must be converted into risk-neutral probabilities to adjust them for the risk. Hence, in the JLT model, the off-diagonal probabilities in d will be multiplied by an adjustment, denoted Π. Assume a one-period bond is currently in state I. At maturity, the three states have probabilities given by vector

$$d' = [.0.87 \ \ 0.08 \ \ 0.05]$$

The statistical probability vector *d* is transformed into the risk-neutral vector q, with an adjustment Π_I as follows

$$q_I = \begin{bmatrix} 1 - 0.13\Pi_I \\ 0.08\Pi_I \\ 0.05\Pi_I \end{bmatrix}$$

so that the last two elements of d, have been multiplied by Π_I, and the first element is simply the plug required to make the probabilities add up to 1. To solve for the risk-adjustment, we find the value of Π_I, which makes the expected value of discounted cash flows equal to the traded price of the bond:

$$B_I = \frac{1}{1+r} C' q_I$$

$$\frac{1}{1.058} = \frac{1}{1.05} [1 \ \ 1 \ \ 0.35] \begin{bmatrix} 1 - 0.13\Pi_I \\ 0.08\Pi_I \\ 0.05\Pi_I \end{bmatrix}$$

The solution is

$$\Pi_I(1) = 0.232678$$

Similarly, we may carry out the same calculations for one-period junk debt, so as to get the required risk adjustment (Π_I).

$$B_J = \frac{1}{1+r} C' q_J$$

$$\frac{1}{1.06} = \frac{1}{1.05} [1 \;\; 1 \;\; 0.35] \begin{bmatrix} 0.07\Pi_J \\ 1 - 0.15\Pi_J \\ 0.08\Pi_J \end{bmatrix}$$

which ends up in

$$\Pi_J\,(1) = 0.18142$$

In general, if we define the cumulative probability of default over n periods from any state i = {I, J} as $q_{di}(n)$, then the risk adjustment for the nth period for a bond initially in state i can be written as follows

$$\Pi_i(n) = \left[1 - \left(\frac{1+r(n)}{1+r(n)+s(n)}\right)^n\right] \frac{1}{(1-\phi)q_{di}(n)}$$

We now obtain the risk-neutral transition matrix for one period from these computations denoted as *Q(n)*.

$$Q(1) = \begin{bmatrix} 0.9698 & 0.0186 & 0.0116 \\ 0.0127 & 0.9728 & 0.0145 \\ 0.0000 & 0.0000 & 1.0000 \end{bmatrix}$$

Similarly, by the same logic we can obtain the two-period risk-neutral transition matrix. Assuming that the periods are identical in terms of default risk, by squaring the transition matrix we can get the two period cumulative default probability transition matrix. In like fashion, the n period matrix would be simply the one period matrix taken to the power n.

$$d^2 = \begin{bmatrix} 0.87 & 0.08 & 0.05 \\ 0.07 & 0.85 & 0.08 \\ 0.00 & 0.00 & 1.00 \end{bmatrix} \begin{bmatrix} 0.87 & 0.08 & 0.05 \\ 0.07 & 0.85 & 0.08 \\ 0.00 & 0.00 & 1.00 \end{bmatrix} = \begin{bmatrix} 0.7625 & 0.1376 & 0.0999 \\ 0.1204 & 0.7821 & 0.1515 \\ 0.0000 & 0.0000 & 1.0000 \end{bmatrix}$$

Therefore,

$$q_I(2) = \begin{bmatrix} 1 - 0.2375\prod_I(2) \\ 0.1376\prod_I(2) \\ 0.1173\prod_I(2) \end{bmatrix}$$

and

$$q_J(2) = \begin{bmatrix} 0.1204\prod_I(2) \\ 1 - 0.2179\prod_I(2) \\ 0.1529\prod_I(2) \end{bmatrix}$$

Using our equation, we can solve for the risk adjustments

$$\prod_I(2) = \left[1 - \left(\frac{1+0.06}{1+0.06+0.009}\right)^2\right] \frac{1}{(1-0.35)\times 0.0999} = 0.258216$$

$$\prod_J(2) = \left[1 - \left(\frac{1+0.06}{1+0.06+0.015}\right)^2\right] \frac{1}{(1-0.35)\times 0.1515} = 0.281414$$

that are substituted back to adjust the statistical matrix d^2 to give the two period risk-neutral transition matrix Q(2)

$$Q(2) = \begin{bmatrix} 0.9387 & 0.0355 & 0.0258 \\ 0.0339 & 0.9235 & 0.0426 \\ 0.0000 & 0.0000 & 1.0000 \end{bmatrix}$$

Given the implementation of the JLT model in this simple form, the risk-neutral rating transition matrices computed from observed term structures make the pricing of various credit risk derivatives practicable. For instance, in order to price a default swap, the cumulative probability of default and the loss on default should be multiplied and discounted at the interest rate for two periods.

The JLT model brings about several advantages. These include the following:

- It accounts for default risk.
- It models default as a process of migration from higher credit quality to default. By modelling a range of credit ratings, it also enables one to value CDs indexed to ratings without difficulty (e.g. credit-sensitive notes)
- As the event of default is explicitly modelled, it allows the pricing of default swaps, whilst with pure spread models this was not a possibility.

However, we must also point out that the simple version of the model has some weaknesses:

- The assumption of no correlation between risk-free rates and default is not realistic
- As the transition matrix is obtained from past observations, it may not give the correct picture of the future credit event. Still, it may be adjusted to incorporate the trader's view or forecasts without altering the structure of the model.
- The assumption that upon default cash flows are received at maturity- though necessary – is not realistic, but the model may be changed without great harm to its structure.

- All securities within the same rating class have the same spread is implied by the model. Even so, it can explain the average spreads for a rating class in preference to spreads for individual bonds.

This structure may be made more sophisticated to reflect most features of credit risky instruments, and to price CDs. By making the recovery rate stochastic, the DT model extended the JLT model, and removed the latter's problematic shortcomings.[12]

III.4. The Das-Tufano (DT) Model

Being an extension of JLT, this model uses credit ratings to depict the probability of default. In the JLT model, on default, the recovery rate is constant, and it is characterized by a variability in spreads that depend on changes in credit ratings. The DT model, however, makes the recovery rate in the event of default stochastic, providing at the same time a two-factor decomposition of credit spreads. Therefore:

- More variability in the spreads on risky debt is allowed.
- Spreads have become a function of factors not only the ratings.
- Recovery rates and credit spreads are now correlated with interest rates.
- Spread variability may be made specific to the firm, by choosing different recovery rate processes for firms belonging to the same rating class.

[12] Another approach is to make the probability of default correlated with the risk-free interest rate. Those models, which make the hazard (default) rate uncertain, are those of Jarrow and Turnbull (1995) and Duffie-Singleton (1995).

- By making recovery rates stochastic, the pricing of a wide span of spread-based contracts is made feasible. This enables one to reset the price of debt and the valuation of risky debt for counterparties with different credit risks (e.g., in the pricing of risky coupon swaps).

The connection of ratings and credit spreads makes the implementation of the model feasible, enables the valuation of standard bonds, and credit contingent instruments.

In general, models for pricing risky debt can be expressed simply using the following equation

$$B(r, t, .) = P(r, t) - L(.)Q(.)P(r, t)$$

where r is the risk-free interest rate, t is maturity, B(.) is the price of zero coupon risky debt, P(.) is the price of riskless debt of the same maturity, Q(.) is the pseudo probability of default and L(.) = 1 - ϕ(.), the loss rate. As before, ϕ(.) is the recovery rate in the event of default; here it is assumed to be stochastic.[13]

Notice that the spread is a function of the composite L(.)Q(.) (i.e., recovery rate and probability of default), and that none of them can be separately identified.[14] It is possible to make credit spreads correlated with the risk-free rate if any of the two components L(.) or Q(.) can be made correlated with r. Recovery rate ¢(.) is

[13] This conceptual specification was first examined and modeled in detail by Longstaff and Schwartz (1995).

[14] This corresponds to a similar point made in the Duffle-Singleton (1995) framework.

assumed as stochastic and correlated with r, so that we can get the necessary correlation.[15]

Model

The pricing model has two parts: (1) the term structure model, and (2) the default model. For the term structure any model may be used.[16]

The transition matrix provides the probabilities of migration. DT model involves a mixture of existing models and an extension which accommodates stochastic default recovery rates[17] which may be correlated with the term structure of interest rates.

First, we get the risk-neutral setup for the progress in the term structure of interest rates, and then find the risk-neutral probabilities of the default process. With these two, the stochastic framework for the arbitrage-free pricing of risky debt is established.

Implementation

We again make a two-period example. Suppose we use the same initial term structure variables as before. The risk-free rates and spreads are

$$r = \begin{bmatrix} 0.05 \\ 0.06 \end{bmatrix}, S_I = \begin{bmatrix} 0.008 \\ 0.009 \end{bmatrix}, S_J = \begin{bmatrix} 0.01 \\ 0.015 \end{bmatrix}$$

Assume a binomial lattice with equal probabilities on each branch. As before, the interest rate process is a discrete square-root model.

$$r(t) = r(0) + \alpha\,[\beta - r(0)] + \delta \pm \eta \sqrt{r(0)}$$

[15] It is usually more tractable to do it this way than to make Q(.) correlated with r (to see the alternative approach refer to Lando [1994]).
[16] Heath-Jarrow-Morton [1992], Hull-White [1990], Black-Derman-Toy [1990], etc.

We assume that $\alpha = 0.3$, $\beta= 0.10$, and $\eta = 0.1$. Then δ is the risk adjustment, which is found as 0.001976. As we found before, the two risk-neutral spot interest rates in period 1 are

$$r(1) = \begin{bmatrix} 0.0746 \\ 0.0656 \end{bmatrix}$$

With this setup of the risk-free interest rate we can price riskless debt up to a maturity of two periods.

Now we develop a stochastic process for recovery rates. Any process can be chosen for recovery rates as long as they stay within the range [0,1]. We choose recovery rates so that they are correlated with risk-free rates, thereby the correlation of credit spreads with the term structure can be achieved. At time 0, the recovery rates are assumed to be

$$\phi_I (0)=0.55, \text{ and } \phi_J (0)=0.39$$

We then choose a stochastic process for recovery rates. A process such as $\phi_i(t) = \phi_i(0) \pm \sigma_i$ may be chosen. Still, we just choose values for the evolution of the spread over time as a binomial process. As before, we have two binomial stochastic variables; the joint evolution of the lattice is a tree with a four-way branching scheme. We assume that the recovery rates are negatively correlated with risk-free interest rates with is $\rho= -0.4$. At the end period 1, four possible states arise. The values of the model variables in these four states are as follows

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[17] As in Jarrow and Turnbull (1995)

$$r(1)=\begin{bmatrix}0.0746\\0.0746\\0.0656\\0.0656\end{bmatrix},\theta_I(1)=\begin{bmatrix}0.58\\0.54\\0.58\\0.54\end{bmatrix},\theta_J(1)=\begin{bmatrix}0.42\\0.35\\0.42\\0.35\end{bmatrix},prob=\begin{bmatrix}0.15\\0.35\\0.35\\0.15\end{bmatrix}$$

The details of how we obtain the state space, is not critical to the theory of the model. The example above has negative correlation between risk-free rates and the spreads.[18] Now in order to facilitate the computations in the model, we have to find the state prices (w) off the risk-free interest rate tree. At time zero the state price is unity, i.e., w(0) = 1. The state price vector is given by

$$w(t+1)=prob\times w(t)\times\frac{1}{1+r(t)}$$

where prob is the probability at the particular branch. We obtain the state prices at time 1 as

$$w(1)=\begin{bmatrix}0.142857\\0.333333\\0.333333\\0.142857\end{bmatrix}$$

The same transition matrix as before is assumed

$$d=\begin{bmatrix}0.87\ 0.08\ 0.05\\0.07\ 0.85\ 0.08\\0.00\ 0.00\ 1.00\end{bmatrix}$$

In order to price various credit sensitive derivatives the risk-neutral transition matrix must be obtained. We start with the one period risky bond of rating class I. At the end of a period, it may default, with risk-neutral first passage probability $q_i°(t)$ in

[18] This is easily seen from the fact that when risk-free rates and interest rates move (from time 0) in the same direction, lower probabilities are assigned than when they move in opposite directions.

which case it will yield an amount $\phi_i(1)$. The first-passage probabilities are different from cumulative probabilities denoted as $[q_i(t)]$. The "first-passage" probabilities are the probability of default in period t, conditional on no prior default. Since there are four states of the world at time 1, in each state the expect cash flow is

$$C_i(1) = q_i^{\circ}(1)\phi_i(1) + (1 - q_i^{\circ}(1))1$$

$C_i(1)$ is the "expected" cash flow. The present value of this expected cash flow for a bond of rating class I is

$$w(1)'C_I(1) = [0.142857\ 0.333333\ 0.333333\ 0.142857]\begin{bmatrix} 0.58q_I^0(1)+1-q_I^0(1) \\ 0.54q_I^0(1)+1-q_I^0(1) \\ 0.58q_I^0(1)+1-q_I^0(1) \\ 0.54q_I^0(1)+1-q_I^0(1) \end{bmatrix}$$

In equilibrium, this must equal the price of the one year risky bond, which is

$$1/1.058 = w(1)'C_I(1)$$

Solution to this gives the value of the risk-neutral first passage default probability for the rating class I

$$q_I^o(1) = 0.0172$$

The solution for the risk-neutral first passage default probability for the rating class J is obtained by doing similar calculations as

$$q_J^o(1) = 0.01534$$

By using these probabilities we get the risk parameter $\Pi i(t)$ to adjust to the statistical probability of default

$$\Pi_I(1) = \frac{q_I^0(1)}{d_I(1)} = \frac{0.0172}{0.05} = 0.344$$

$$\Pi_J(1) = \frac{q_J^0(1)}{d_J(1)} = \frac{0.01534}{0.08} = 0.19173$$

Following the same procedure taken by the JLT model, we adjust the matrix d to find the entire risk-neutral rating change matrix for one period as

$$Q(1) = \begin{bmatrix} 0.9553 & 0.0275 & 0.0172 \\ 0.0135 & 0.9712 & 0.0153 \\ 0.0000 & 0.0000 & 1.0000 \end{bmatrix}$$

In a similar fashion, we find the values for period 2. At the end of period 2 there are in total 16 end nodes. Now we have to obtain the expected cash flows in each end node (incorporating default and recovery), and discount these cash flows to time 0. Given that default is possible at the end of the first period; a cash flow may arise in any of the four nodes after one period. This possible cash flow must also be considered in calculations.

Assuming that recovery rates enter the cash flows in a linear way,[19] given the simplification above we do not need to examine each node in detail to do the calculations. Now, we assume the average recovery rates over the time period 2, originating from the appropriate state at the end of period 1 as

$$\overline{\theta}_{\mathrm{I}}(1) = \begin{bmatrix} 0.60 \\ 0.55 \\ 0.60 \\ 0.55 \end{bmatrix}, \overline{\theta}_{\mathrm{J}}(1) = \begin{bmatrix} 0.45 \\ 0.34 \\ 0.45 \\ 0.34 \end{bmatrix}$$

[19] They are linear in default probabilities, or are not functions of default probabilities.

The value of the two period risky zero coupon bond is the present value of cash flows at time 1 and time 2. At time 1 a cash flow is generated only if default occurs. The present value of this cash flow for both classes of bonds is

PV[Cash flow] = prob(default) x average recovery x 1/(1 + r)

$PVC_I(1) = 0.0172*0.56*(1/1.05) = 0.00917$

$PVC_J(1) = 0.01534*0.385*(1/1.05) = 0.005625$

Considering that recovery rates enter the cash flow equation linearly, the present value of the probability weighted cash flows cash flows is

$$\mathbf{PVC}_i(2) = \sum_{k=1}^{4}\left(\frac{\mathbf{w}_k(1)}{1+\mathbf{r}_k(1)}\left[\overline{\varphi_k}\,\mathbf{q}_i^{o}(2)+1-\mathbf{q}_i^{o}(2)\right]\right)$$

where k simply indexes the four states, and ϕ is the average recovery rate from the each of the four states. By using the actual parameter values we get the cash flows for each rating class at maturity, and the following two equations give the solutions for $\{q_I^o(2), q_J^o(2)\}$.

$$PVC_I(1) + PVC_I(2) = 1/(1.069)^2$$

$$PVC_J(1) + PVC_J(2) = 1/(1.075)^2$$

Solving we obtain

$$q_I^o(2) = 0.0637$$

$$q_J^o(2) = 0.0563$$

These probabilities are "first passage" probabilities. In order to develop the cumulative ratings transitions matrix Q(2), these risk-neutral first-passage default probabilities must be converted into cumulative ones by using

$$q_i(2) = q_i(1) + [(1 - q_i(1)]\, q_i^{\circ}(2)$$

$= q_i^o(1) + [(1 - q_i^o(1)] q_i^o(2)$

The values are found to be

$q_I(2) = 0.0172 + [1 - 0.0172](0.0751) = 0.0798$

$q_J(2) = 0.01534 + [1 - 0.01534](0.0631) = 0.0707$

Therefore the risk adjustments for the second period are:

$\Pi_I(2) = 0.091/0.1173 = 0.7986$

$\Pi_J(2) = 0.0775/0.1529 = 0.4669$

By exerting these adjustments to the statistical matrix

$$d^2 = \begin{bmatrix} 0.7625 & 0.1376 & 0.0999 \\ 0.1204 & 0.7821 & 0.1515 \\ 0.0000 & 0.0000 & 1.0000 \end{bmatrix}$$

We will obtain the risk-neutral cumulative transition matrix

$$Q(2) = \begin{bmatrix} 0.8103 & 0.1099 & 0.0798 \\ 0.0562 & 0.8730 & 0.0707 \\ 0.0000 & 0.0000 & 1.0000 \end{bmatrix}$$

After all the calculations we have obtained a number of items which will render the pricing of almost any kind of credit derivative possible. We have now

- Risk-neutral ratings transition matrices for both periods [i.e., Q(1), Q(2)]. These, of course, contain the cumulative probabilities of default [$q_i(t)$]. The required risk adjustments [$\Pi_i(t)$] to the statistical transition matrix (d) are also achieved as a by-product.

- A bivariate lattice of risk-free interest rates (r) and recovery rates (ϕ) that satisfy risk-neutrality conditions, assuming a correlation between recovery rates and interest rates.
- State prices (w) which help speed up calculations on the lattice.
- First-passage probabilities of default for each rating class, $[q_i^{\circ}(t)]$, i = I,J.

Using this information, the following derivative products are priced by generating the necessary cash flows at each node on the lattice and discounting the cash flows back by multiplying them with the state prices to obtain present values:

- Plain-vanilla risky debt for any rating class
- Rating-sensitive debt
- Spread-adjusted notes
- Spread options
- Total return swaps
- Credit default swaps
- Floating rate debt
- Swaps by counterparties with different credit rating

III.5. The Duffie-Singleton (DS) Model

DS analysis shows how the pricing of risky debt may be analyzed in the same way as riskless debt, where the discount rate is made up of the risk-free rate plus an adjustment involving the hazard and recovery rates. Keeping the same notation we used before, and assuming risk neutrality, it can be argued that, in a very small interval of time Δ the value of a risky bond is equal to

$$B = e^{-r\Delta}[(1-\lambda\Delta)+(\lambda\Delta)\phi]$$

$$\cong (1-r\Delta)\ [(1-\lambda\Delta)+(\lambda\Delta)\phi]$$

Here λ is the annualized hazard rate, and r is the annualized risk-free rate. Expanding this expression and then eliminating terms in Δ^2 (in continuous time would be zero), we get

$$B \cong 1-\Delta\ [r+\lambda(1-\phi)]$$

$$\cong e^{-\Delta[\ r+\lambda(1-\phi)\ ]}$$

$$= e^{-\ R\Delta}$$

where R may be thought of as the "risky" interest rate. Using this model, the risky interest rate of may be written as

$$R = r+\lambda(1-\phi)$$

$$= r + s$$

As we saw before, the spread cannot be decomposed into default and recovery risk components, unless additional information is brought into the model[20].

The DS model is a completely general one. R is composed of three elements of risk, namely the risk-free rate, hazard rate, and recovery rate. They all may be treated as stochastic and correlated with one another. Lattice approach can be implemented in these models, where a branching process must be chosen to provide for a default event branch.

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[20] The spread is a composite of the two risks, and each elements is not separately identifiable.

Let us give a small example. Let $\lambda = 0.15$ (i.e., a 15 per cent chance of default in a year). Also, assume that the recovery rate is $\phi = 0.4$. Assume one-year risk-free interest rate today as 7.5 per cent, and after a year assume that this changes to either 9.5 per cent or 5 per cent with equal probability. The price of a two-period riskless zero-coupon bond will be

$$100 \times 0.5 \times [e^{-0.095} + e^{-0.05}] \times e^{-0.075} = 86.31$$

The risky discount rates are simply the risk-free rate plus spread, i.e.,

$$R = r + 0.15(1-0.4) = r + 0.09$$

Hence, to price risky bonds the current rate of 16.5 per cent (0.075 + 0.09) is used, and the rates after one year will be 18.5 per cent (0.095 + 0.09) and 14 per cent (0.05 + 0.09) with probability of 0.5 each. The price of the bond will be

$$100\,(0.5)\,[e^{-0.185} + e^{-0.14}]\,e^{-0.165} = 72.09$$

At the end of two years, the expected cash flow is

$$100\,e^{-0.15} + 40(1 - e^{-0.15}) = 91.64$$

where $(1 - e^{-0.15})$ is the probability of default. Discounting this back at the two possible values of the risk-free rate gives

$$91.64e^{-0.05} = 87.173$$

and

$$91.64e^{-0.095} = 83.337$$

Taking 91.6 per cent of these values and discounting back gives

$$0.5[87.173 + 83.337]0.9164e^{-0.075} = 72.48$$

roughly equal to that from the direct calculation.[21] The DS model thus enables users of regular term structure models for government bonds to directly apply them to risky debt, by replacing the risk-free rates with risky rates from the formula above.[22]

III.6. Conclusions and Other Issues for Credit Risk Models

In Part III. we have mainly focused on pricing models. Previously we carried out a survey on a range of different approaches credit derivatives modelling, and we also examined simple versions of some major extant models. One strand of the literature was given short shrift note: the Merton (1974) models the value of the firm instead of the prices of risky and riskless securities, which are functions of the value of the firm. The following are theoretically appealing papers of this class: Longstaff and Schwartz (1995), Bhattacharya and Mason (1981), Black and Cox (1976), Shimko, Tejima, and Van Deventer (1993), Nielsen, Saa-Requejo, and Santa-Clara (1993), Das (1995). Nevertheless, these papers still pose enormous implementation problems, especially when dealing with risky bonds of firms with complicated capital structures. Unfortunately examination of this entire stream of work would have caused enormous diversion from the simple models pursued in this piece of work. Without a doubt, the Merton approach is enormously important however; before making an engineering decision, on pricing approach other models should be carefully examined in addition to those in this part.

While this part aims to focus primarily on the pricing of credit derivatives, there are several other issues that bear addressing in conjunction with pricing. In this

[21] Rounding occurs since the recovery rate has not been handled in continuous time

section, we are going to focus on only two of these issues, which pertain directly to pricing models. These are (a) portfolio credit risk and (b) pricing to obtain a rate of return on capital. So far we have discussed pricing models, that envisage a framework for the no-arbitrage pricing of derivatives with credit risk. Therefore, the lattices that are developed may be used to price several differently structured products, since these models fit the existing term structure of interest rates and credit spreads. Moreover, for each security that is priced, the initial conditions (parameters) may be perturbed and numerical derivatives calculated, so that "hedge" ratios with respect to all the underlying sources of risk may be obtained, and then, these hedge ratios may be used to put in place the required hedges. This clearly shows that, the pricing technology directly supports portfolio risk management. In addition, bears no difference to the technology already used for equity, foreign exchange, commodity, and interest rate derivatives.

Simulation methods have also been developed in recent past, to carry out credit portfolio management. These methods have a common feature of the generation of a wide variety of default scenarios, and thus also enable carry out a complete and satisfactory stress testing of credit portfolios. In addition to that, other risk management methods can be combined with the simulation approach and it allows for a large number of sources of risk. For example, the melting of price and credit risk into one risk management system can be one with this approach. Simulation is a very powerful approach towards credit risk management. Moreover,

[22] These models are now known as "reduced-form" models of risky debt pricing.

it can support large portfolios adequately and it is directly extensible. Every institution dealing in credit derivatives faces the choice between a pricing technology and risk management by simulation. If the pricing technology has been developed, it would make perfect sense to use it for risk management, thus it would ensure consistency of the front end dealing process with the ex-post management of positions. Since the model accounts for no-arbitrage this would also ensure that the risk measures are also consistent with this issue. Risk measures do not in any way conform to risk-neutral (i.e., no-arbitrage pricing), with simulation off the statistical matrices. (See the Appendix A)

Using pricing technologies, like one developed in this part, is often called "relative pricing". The models here used observed values of riskless rates and spreads, developed a lattice, and then enabled pricing off the lattice, which ensured (a) that no arbitrage was admitted, and (b) that all securities were priced correctly relative to each other. This directs us to the second major issue of this section (i.e., an often adopted approach by most financial institutions): when not availing of a relative pricing model. This is to ensure a risk-adjusted rate of return on capital, when pricing a security. If two financial houses with differing balance sheets price the same derivative, quite clearly the same security would have two different prices. In fact, because there is little agreement on price it deprives expectations on to see price formation and it makes it hard to create a market. However, using relative pricing technology helps establishment of markets due to the consistency achieved across security prices.

Important point to remember is to recognize the fact that, when the inputs used by participants differ relative pricing fails, because they have differential information. On the other hand, relative pricing markets shows evidence of a convergence of information, which does not come with pricing for a rate of return. The relative pricing mechanism offers the hope a convergence of all prices, models, and information to the optimal maximizing welfare.

IV. CONCLUSION

In today's complex economic environment, bank loan portfolio managers have to deal with credit risk in their day to day operations. Credit risk is so hard to measure and manage that bank managers have to come up with sound procedures that will help them survive in this complicated environment.

In dealing with credit risk, bank financial managers may use credit derivatives. Credit derivatives are complicated financial structures originally designed to mitigate the credit risk and its components, and now used for active loan portfolio management purposes to further boost incomes of the banks, with an optimal portfolio approach.

By using various types of credit derivatives, a one can hedge away the credit risk, and manage the portfolio of the financial institution more efficiently. Still, the credit risk cannot be totally offset, nor can incomes be increased to the limit.

Currently, these instruments are perceived so complicated that no one could come up with an exact formula or model to measure the risks involved and payoffs anticipated. The pricing of credit derivatives is still a challenge, so is the design of the loan portfolio management strategies using them.

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APPENDICES

APPENDIX A

SIMULATION AND SENSITIVITY RESULTS[23]

We have carried out simulations for the risk free interest rates and credit spreads.

The assumptions:

1. The underlying process under risk free rates and spreads is a Wiener Process.
2. The parameters are assumed so that they enter the equations as given.
3. The correlation between the risk-free rates and spreads are assumed.
4. The transitions matrices are assumed to follow a Markov transition process. (They are also assumed not simulated)
5. The recovery rate is given.

The equation used for simulating risk-free interest rates is

$$r(t) = r(t-\Delta) + \alpha\left[\beta - r(t-\Delta)\right]\Delta + \delta + \eta\sqrt{r(t-\Delta)} \times w\sqrt{\Delta} \times \xi(t-\Delta)$$

where $\xi(t-\Delta)$ is the random number generated by using Excel Random Number Generator drawn from a normal distribution with mean 0 and standard deviation 1 (i.e. $\xi(t-\Delta) \sim N(0,1)$). The equation used for simulating spread is

$$s(t) = s(t-\Delta) + k\left[\theta - s(t-\Delta)\right]\Delta + \gamma + \sigma\sqrt{s(t-\Delta)} \times z\sqrt{\Delta} \times \wp(t-\Delta)$$

where $\wp(t-\Delta)$ is the random number generated by using Excel Random Number Generator drawn from a normal distribution with mean 0 and standard deviation 1 (i.e. $\wp(t-\Delta) \sim N(0,1)$).

[23] Simulations are carried out through Excel, and Solver function is used for the other computations

Table 1. Sample Simulation for Risk-free Interest Rates

	r(0)	0,05	alpha	0,15
	T	0,004	beta	0,0083
	eta	0,0092		
day	interest	Random	change	int(riskfree)
0	0,0500	-3,0230149	-0,000418337	0,050000
1	0,0497	0,1600654	-4,19427E-06	0,049582
2	0,0497	-0,8657844	-0,000137665	0,049577
3	0,0495	0,873265	8,85985E-05	0,049440
4	0,0496	0,2147249	2,91735E-06	0,049528
5	0,0496	-0,0504724	-3,15868E-05	0,049531
6	0,0496	-0,3845344	-7,50509E-05	0,049500

Table 2. Sample Simulation for Spread

	s(0)	0,01	k	0,15
	T	0,004	theta	0,0083
	sigma	0,0092		
day	spread	Random	change	spread
0	0,0100	-3,0230149	-0,000176917	0,010000
1	0,0099	0,1600654	8,31695E-06	0,009823
2	0,0099	-0,8657844	-5,08688E-05	0,009831
3	0,0098	0,873265	4,93627E-05	0,009781
4	0,0099	0,2147249	1,14693E-05	0,009830
5	0,0099	-0,0504724	-3,83821E-06	0,009841
6	0,0099	-0,3845344	-2,31145E-05	0,009838

Time increments are taken daily (i.e. T = 1/250). 500 days are used and the rates and spreads for the days 250 and 500 are the ones for period 1 (T=1) and period 2 (T=2). The assumption here is that “holidays are not counted as a part of the time” which is not realistic. Hence, the year is composed of 250 working days. These rates and spreads are found to be

$$r = \begin{bmatrix} 0.0523 \\ 0.0588 \end{bmatrix}, \qquad S_I = \begin{bmatrix} 0.0086 \\ 0.0087 \end{bmatrix}, \qquad S_J = \begin{bmatrix} 0.0098 \\ 0.0093 \end{bmatrix}$$

Spread Models
One Factor Spread Model

1. At t = 0 with assumed values

$$r = \begin{bmatrix} 0.05 \\ 0.06 \end{bmatrix}, \qquad s = \begin{bmatrix} 0.008 \\ 0.009 \end{bmatrix}, \qquad r + s = \begin{bmatrix} 0.058 \\ 0.069 \end{bmatrix}$$

Drift adjustment parameter (γ) is found to be $\gamma = 0.00198$ and the spread is found as

$$s = \begin{bmatrix} 0.0108 \\ 0.0092 \end{bmatrix}$$

We then performed sensitivity analysis for the risk-free rates and spreads; this revealed that when $\Delta r = 0.01r$ and $\Delta s = 0$ spreads remained the same. Whereas when $\Delta r = 0$ and $\Delta s = 0.01s$, and $\gamma = 0.002007$

$$s = \begin{bmatrix} 0.0109 \\ 0.0093 \end{bmatrix} \qquad (1)$$

When $\Delta r = 0.01r$ and $\Delta s = 0.01s$ we obtained the same results as (1). Therefore, the changes in the spreads are not sensitive to changes in the risk-free interest rates.

2. At t = 0 with simulated values

$$r = \begin{bmatrix} 0.0523 \\ 0.0588 \end{bmatrix}, \qquad s = \begin{bmatrix} 0.0086 \\ 0.0087 \end{bmatrix}, \qquad r + s = \begin{bmatrix} 0.0609 \\ 0.0675 \end{bmatrix} \qquad (2)$$

Drift (γ) is found to be $\gamma = -\ 0.00451$ and the spread is found as

$$s = \begin{bmatrix} 0.0049 \\ 0.0032 \end{bmatrix}$$

We then performed sensitivity analysis for the risk-free rates and spreads. When $\Delta r = 0.01r$ and $\Delta s = 0$, $\gamma = -\ 0.00385$ and spreads

$$s = \begin{bmatrix} 0.00555 \\ 0.00385 \end{bmatrix}$$

and when $\Delta r = 0.01r$ and $\Delta s = 0.01s$ $\gamma = -\ 0.00384$ and spreads

$$s = \begin{bmatrix} 0.00565 \\ 0.00393 \end{bmatrix}$$

lastly, when $\Delta r = 0$ and $\Delta s = 0.01s$, and $\gamma = -0.00449$ and spreads

$$s = \begin{bmatrix} 0.00499 \\ 0.00328 \end{bmatrix}$$

Two Factor Model

1. One Period

a. With assumed values

We found at t = 0 the drift adjustment parameter (δ) is found as $\delta = 0.00811$, and the risk free-rates are

$$r = \begin{bmatrix} 0.0746 \\ 0.0656 \end{bmatrix}$$

With $\Delta r = 0.01r$ and $\Delta s = 0$ δ is found as $\delta = 0.00842$ and the risk free-rates are

$$r = \begin{bmatrix} 0.0753 \\ 0.0663 \end{bmatrix} \qquad (3)$$

When $\Delta r = 0.01r$ and $\Delta s = 0.01s$ the rates are found the same as (3). Hence, the risk-free rates are not sensitive to changes in spreads.

a. With simulated values given in (2) at t = 0
the drift parameter is found as $\delta = 0.00152$ and the risk-free rates are

$$r = \begin{bmatrix} 0.0699 \\ 0.0608 \end{bmatrix} \qquad (4)$$

Sensitivity analysis reveals that when $\Delta r = 0.01r$ and $\Delta s = 0$, the drift parameter is found to be $\delta = 0.00176$ and the risk-free rates remain the same as in (4).

$$r = \begin{bmatrix} 0.0706 \\ 0.0614 \end{bmatrix}$$

When $\Delta r = 0.01r$ and $\Delta s = 0.01s$ the results do not change. This leads up to the fact that the risk-free rates are again not sensitive to changes in spreads.

2. Two Period Model

a. With assumed values

We found at t = 0 the drift adjustment parameter for spread (γ) is found as γ = 0.00811,
and the spreads and risk free-rates are

$$s(1) = \begin{bmatrix} \gamma + 0.00887 \\ \gamma + 0.00722 \\ \gamma + 0.00887 \\ \gamma + 0.00722 \end{bmatrix}, \qquad r(1) = \begin{bmatrix} 0.0746 \\ 0.0746 \\ 0.0656 \\ 0.0656 \end{bmatrix}$$

Sensitivity analysis results are as follows,
With $\Delta r = 0.01r$ and $\Delta s = 0$ γ is found as $\gamma = 0.00249$, and the spreads and the risk free-rates are

$$s(1) = \begin{bmatrix} 0.01136 \\ 0.00971 \\ 0.01136 \\ 0.00971 \end{bmatrix}, \qquad r(1) = \begin{bmatrix} 0.0753 \\ 0.0753 \\ 0.0663 \\ 0.0663 \end{bmatrix} \qquad (5)$$

When $\Delta r = 0.01r$ and $\Delta s = 0.01s$ the risk-free rates are found the same as (5), while the spreads.

$$s(1) = \begin{bmatrix} 0.01154 \\ 0.00989 \\ 0.01154 \\ 0.00989 \end{bmatrix}$$

With $\Delta r = 0$ and $\Delta s = 0.01s$

$$s(1) = \begin{bmatrix} 0.01103 \\ 0.00938 \\ 0.01103 \\ 0.00938 \end{bmatrix}, \qquad r(1) = \begin{bmatrix} 0.0746 \\ 0.0746 \\ 0.0656 \\ 0.0656 \end{bmatrix}$$

b. With simulated values

We found at t = 0 the drift adjustment parameter for spread (γ) is found as γ = 0.000248,
and the spreads and risk free-rates are

$$s(1) = \begin{bmatrix} \gamma + 0.0094 \\ \gamma + 0.0077 \\ \gamma + 0.0094 \\ \gamma + 0.0077 \end{bmatrix}, \qquad r(1) = \begin{bmatrix} 0.0699 \\ 0.0699 \\ 0.0608 \\ 0.0608 \end{bmatrix}$$

Sensitivity analysis results are as follows,
With $\Delta r = 0.01r$ and $\Delta s = 0$ γ is found as $\gamma = 0.00248$, and the spreads and the risk free-rates are

$$s(1) = \begin{bmatrix} 0.00966 \\ 0.00795 \\ 0.00966 \\ 0.00795 \end{bmatrix}, \qquad r(1) = \begin{bmatrix} 0.0706 \\ 0.0706 \\ 0.0614 \\ 0.0614 \end{bmatrix} \qquad (6)$$

When $\Delta r = 0.01r$ and $\Delta s = 0.01s$ the risk-free rates are found the same as (6), while the spreads.

$$s(1) = \begin{bmatrix} 0.00975 \\ 0.00803 \\ 0.00975 \\ 0.00803 \end{bmatrix}$$

With $\Delta r = 0$ and $\Delta s = 0.01s$, $\gamma = 0{,}000028$

$$s(1) = \begin{bmatrix} 0.00951 \\ 0.00779 \\ 0.00951 \\ 0.00779 \end{bmatrix}, \qquad r(1) = \begin{bmatrix} 0.0702 \\ 0.0702 \\ 0.0610 \\ 0.0610 \end{bmatrix}$$

The Jarrow Lando Turnbull Model

In JLT Model different from the spread models the outcome of the pricing process is the Risk Neutral Transition Matrices (RNTM). Assuming the same interest rates above and different spreads for the two rating classes (i.e. the latter assumption postulates that all the firms within the same rating class have the same spread on the average, which is a simplifying one). The assumed values for the risk-free rates and spreads for classes I and J are

$$r = \begin{bmatrix} 0.0523 \\ 0.0588 \end{bmatrix}, \qquad S_I = \begin{bmatrix} 0.0086 \\ 0.0087 \end{bmatrix}, \qquad S_J = \begin{bmatrix} 0.0098 \\ 0.0093 \end{bmatrix}$$

whereas, with the simulations we have carried out they turned out to be

$$r = \begin{bmatrix} 0.0523 \\ 0.0588 \end{bmatrix}, \qquad S_I = \begin{bmatrix} 0.0086 \\ 0.0087 \end{bmatrix}, \qquad S_J = \begin{bmatrix} 0.0098 \\ 0.0093 \end{bmatrix}$$

The assumed Markov Transition Matrix

$$d = \begin{bmatrix} 0.87\,0.08\,0.05 \\ 0.07\,0.85\,0.08 \\ 0.00\,0.00\,1.00 \end{bmatrix}$$

At t = 1, the calculations give us the adjustment parameters for RNTM as

$\Pi_I(1) = 0.232678$, and $\Pi_J(1) = 0.181427$ and the RNTM is obtained as

$$Q(1) = \begin{bmatrix} 0.9898\,0.0186\,0.0116 \\ 0.0127\,0.9728\,0.0145 \\ 0.0000\,0.0000\,1.0000 \end{bmatrix}$$

When we carry out the sensitivity analysis, with 1 per cent change in risk-free rates we observe that the RNTM remains unchanged. On the other hand, a 1 per cent change the RNTM changes. This reveals the fact that the RNT probabilities insensitive to changes in risk-free rates, but sensitive to changes in spreads. This same explanation can be made for the results using the simulated rates and spreads.

The sensitivity analysis using the assumed values suggests the same fact for the two period JLT model. However, for the simulated values we find that the RNT probabilities are sensitive to both types of changes.

The Das Tufano Model

As an extension to JLT Model, DT model exploits the fact that the interest rates and spreads may be correlated (and are in reality, mostly). For this, as we stated in the beginning, we assume a correlation coefficient (which itself can be changed as a part of a scenario analysis or forecast from the past-though may not reflect the reality).

Due to space, and time constraints we leave the computations out. The output of the sensitivity analysis for both assumed and simulated values for all periods, is such that the RNT probabilities are sensitive to both interest rates and spreads, individually and as a group.

The basic assumption we have made in this analysis is that only the interest rates and spreads are stochastic and can be simulated. All other parameters must be assumed at start. Even so, this is a very simplifying assumption though one that must be made for the manageability of the problem. Everything but the initial conditions (they can be obtained from the market) can be made stochastic and simulated as long as the underlying process is known or can be replicated in one way. Some candidates for simulations can be initial Markov Transition Matrices, recovery rates, etc.

In this appendix we tried to briefly show that the simulation is a powerful tool in pricing the CDs, given the current computing power we have. This analysis was simplified. It can be extended to a more sophisticated one with more accurate and satisfactory results.

APPENDIX B

PRICING FORMULAE FOR CREDIT DERIVATIVES[24]

An exchange option

An option to exchange a risky zero coupon bond for a riskless zero coupon bond at time T has a payoff
Max(qZ-Z^*,0)
for some fixed q.

Assumptions:

1. Risk free rates and hazard rate are deterministic.
2. One, but not both of the interest rate and risk of default is stochastic.

1.The price of risk free bond satisfy

$$\frac{\partial Z}{\partial t}+\frac{1}{2}w^2\frac{\partial^2 Z}{\partial r^2}+(u-\lambda z)\frac{\partial Z}{\partial r}-rZ=0$$

with
$Z(r,T_B) = 1$

u-λw and the w are the risk adjusted drift and the volatility of the spot rate respectively.
The risky bond will similarly satisfy

$$\frac{\partial Z}{\partial t}+\frac{1}{2}w^2\frac{\partial^2 Z}{\partial r^2}+(u-\lambda z)\frac{\partial Z}{\partial r}-(r+p)Z^*=0$$

with

$Z^*(r,T_B) = 1.$

The solution of this equation is simply

$Z^*(r,t)=e^{-p(T_B-t)}Z(r,t)$

This deterministic relationship between the two bonds with the assumption of constant hazard rate shadows the pricing of the exchange option. The complexity of

[24] Appendix B is prepared as a supplement by using Willmott (1998), Ch 44. Credit Derivatives

this contract is due to the *randomness in the risk of default.* Hence, the constant hazard rate assumption is not appropriate for credit derivatives.

2. Given the second assumption, a better approach is to take the interest rates given by the forward rates and hazard rate p some stochastic differential equation.

Assuming that

$dp = \gamma dt + \delta dX$

and, interest rates are constant

$Z(r,t)=e^{-p(T_B-t)}$

and

$$\frac{\partial Z^*}{\partial t} + \frac{1}{2}\delta^2 \frac{\partial^2 Z^*}{\partial p^2} + \delta \frac{\partial Z^*}{\partial r} - (r+p)Z^* = 0$$

with

$Z^*(p,T_B) = 1.$

The payoff of the exchange option has the value V(p,t)

$$Max(qe^{-r(T_B-t)} - Z^*(p,t),0)$$

$qe^{-r(T_B-t)\,v}$ being constant this looks like a put option on a zero coupon bond. Taking the factor $e^{-r(T_B-t)}$ out from Z* in the partial differential equation

$$\frac{\partial Z}{\partial t} + \frac{1}{2}\delta^2 \frac{\partial^2 Z}{\partial p^2} + \delta \frac{\partial Z}{\partial r} - 2pZ = 0$$

As δ and γ are chosen in practice explicit solutions can be found.

3. Stochastic interest and hazard rates

Finally assuming that interest rates and hazard rate are stochastic, both Z and Z* satisfy

$$\frac{\partial V}{\partial t} + \frac{1}{2}w^2 \frac{\partial^2 V}{\partial r^2} + \rho w \delta \frac{\partial^2 V}{\partial r \partial p} + \frac{1}{2}\delta^2 \frac{\partial^2 V}{\partial p^2} + (u - \lambda w)\frac{\partial V}{\partial r} + \gamma \frac{\partial V}{\partial p} - (r+p)V = 0$$

The risk free bond is independent of the default risk so that we have Z(r,t) with no p dependence. The risky bond depends on the default risk and is therefore a function of three variables, Z*(r,p,t).

Using relationship we solve the underlying bonds

$$\mathbf{Z(r,t) = Z^*(r,p,t) = 1}$$

And then solve for the exchange option V(r,p,t) satisfying with

$$\mathbf{V(r,p,t) = Max\ (\ qZ(r,T) - Z^*(r,p,t),\ 0)}$$

As this exchange option is a second order contract the price may be quite sensitive to the model.

Payoff of a Change in Rating

Payoffs take values according to

1. Ratings taking values at expiry
2. Ratings are realised at any time before expiry

In this category, we need to intoduce the rating migration into the valuation model. Assuming the process is Markov and the interest rates are constant the equation

$$\frac{d\mathrm{V}}{dt} + (\mathrm{Q} - r\mathrm{I})\mathrm{V} = 0$$

V(T) = $\mathbf{e_R}$ **R** being the rating category, and there is no payment unless the issuer is rated **R**. $\mathbf{e_R}$ is

$$[0\ldots.1..00]^T$$

which is a vector of transition matrix that relates to rating class R.