Analysis of Short-Term Asset Concentration in Islamic Banking

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Analysis of Short-Term Asset Concentration in Islamic Banking

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Abstract

In general the process of implementation of Islamic banking in the Islamic Republic of Iran and Pakistan appears to be proceeding with relative success. However, number of problems have surfaced during the transition period, among which is a tendency for short-term assets to dominate commercial bank portfolios. The negative effects on capital formation is one result of this portfolio behavior. The cause of this behavior is a set of regulations constraining profit-sharing activities of commercial banks. It is shown here that such regulations rather than reducing the risks of bankruptcies in the banking system may well increase them.

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Contents

Summary

An economic objective of Islamic banking is to have the rate of return from real activities become the allocative mechanism in the financial sector. To achieve this objective, Islamic law proposes a configuration of various modes of transactions as a replacement for interest-based activities. Among these are methods referred to as "strongly" Islamic because they are based on risk and profit-sharing arrangements. Where profit-sharing modes cannot be employed, other methods are proposed which, in appearance, resemble interest-based modes of transaction, hence, they are referred to as "weakly" Islamic. Among the latter are modes employed in short-term trade transactions.

The process of implementation of Islamic banking in the Islamic Republic of Iran and Pakistan is proceeding with relative success. In the transition period, however, a number of problems have emerged among which is a tendency for short-term (weakly Islamic) assets to dominate commercial bank portfolios. The negative effects on capital formation is one result of this behavior. Among the causes of this portfolio behavior is a set of regulations constraining profit-sharing activities of commercial banks. It is shown, in this paper, that such regulations do not necessarily reduce risks of bankruptcies but may well increase such risks.

I. Introduction

The Islamic Prohibition against charging of interest can be interpreted as a situation in which there are no risk-free assets in the portfolio of investors. 1/ Islamic law proposes a configuration of various modes of transactions to replace an interest-based financial system. Among these are principal modes which can be referred to as "strongly" Islamic because they are based on risk and profit sharing relationships between the owner of financial resources and the entrepreneur. Two of these modes are called "musharakah" and "mudarabah." 2/ In transactions where risk and profit sharing methods cannot be employed, such as purchases of consumer durables, other modes of financing are proposed that in appearance resemble financial transactions in an interest-based system, hence, they can be referred to as "weakly" Islamic.

In an ideal Islamic system the rate of return from the activities in the real sector becomes the allocative mechanism in the financial sector. Recent theoretical studies have shown that such a system can be implemented without any a priori demonstrable negative effects on the saving-investment process, 3/ or on financial intermediation and monetary policy. 4/ The validity of these assertions has, to a large degree, been borne out by the recent experiences of the Islamic Republic of Iran and Pakistan which have adopted an Islamic financial system. 5/

The fact that optimal portfolios can be constructed in the absence of risk-free assets has been frequently demonstrated in the portfolio theory literature at least since the late 1950s. 6/ In its simplest form the problem can be stated as follows: for N risky assets, choose the proportion f_i of the portfolio invested in the ith asset such that the portfolio risk is minimized. The search is for an efficient portfolio defined as the locus of all feasible portfolios that have the smallest variance for a prescribed expected return. The solution to the problem assumes that the investor is risk averse; meaning that (a) if two portfolios have the same standard deviation and different expected returns, the one with the larger expected return is preferred, (b) if two portfolios have the same expected returns but different standard deviation, the one with the smaller standard deviation is preferred, and (c) if two portfolios have different standard deviation and expected returns, the one with larger expected return and lower standard deviation is preferred. Graphically,

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1/ Haque and Mirakhor (1986b).
2/ See Iqbal and Mirakhor (1987) for definition of the various modes.
3/ Haque and Mirakhor (1986; a and b).
4/ Khan (1986), and Khan and Mirakhor (1987).
5/ Iqbal and Mirakhor (1987).
6/ See, for example, Martin (1955), Markowitz (1959), and Mossin (1966).

this implies that so long as expected return is desired and variance is not, every indifference curve in the expected return-standard deviation space will be upward sloping. Once the efficient frontier is obtained, the optimal portfolio can be determined and is represented by the point of tangency between the efficiency frontier and an indifference curve as shown in Figure 1. The point C would represent an optimal portfolio, with risk equal to σ^* and return equal to π^*.

The above problem has been solved by a number of authors 1/ and successfully extended to derive the efficient frontier and comparative static properties of optimal portfolio behavior of depository financial intermediaries in the absence of risk-free assets. 2/ Clearly then at a theoretical level an Islamic financial system does not create difficulties for formation of optimal and well-diversified portfolios, either for individual investors or the banks. However, the experiences of Iran and Pakistan, thus far, indicate that the implementation of Islamic banking may involve certain problems. A majority of these problems relate to a lack of legal and institutional framework that would allow closer bank-client partnership on the basis of risk and profit sharing thus accommodating the objectives of the Islamization process. One such problem is an observed tendency on the part of banks to concentrate their asset portfolios in short-term and relatively low-risk assets acquired through weakly Islamic modes of finance such as mark-up and installment sales.

Many of the factors which have been instrumental in encouraging short-term asset concentration are symptomatic of the state of transition from an interest-based system to Islamic banking. 3/ One factor, which is policy induced, is a set of regulations imposed on the portfolio behavior of banks in the two countries intended to limit the exposure of banks' portfolios to relatively high-risk, high-return assets which the banks can obtain through musharakah and mudarabah financing. 4/

The authorities, while agreeing that Islamic banking system must operate on the basis of risk-return sharing arrangements in which the relationship between a bank and its client is one of partnership rather than creditor-debtor, argue that the structure which must be maintained is one which "does not lead to the collapse of the banking system." 5/

1/ See for example Martin (1955), Sharpe (1970), Fama (1971), Fama and Miller (1972), Merton (1972), and Black (1972). (a standard solution is given in the Appendix)

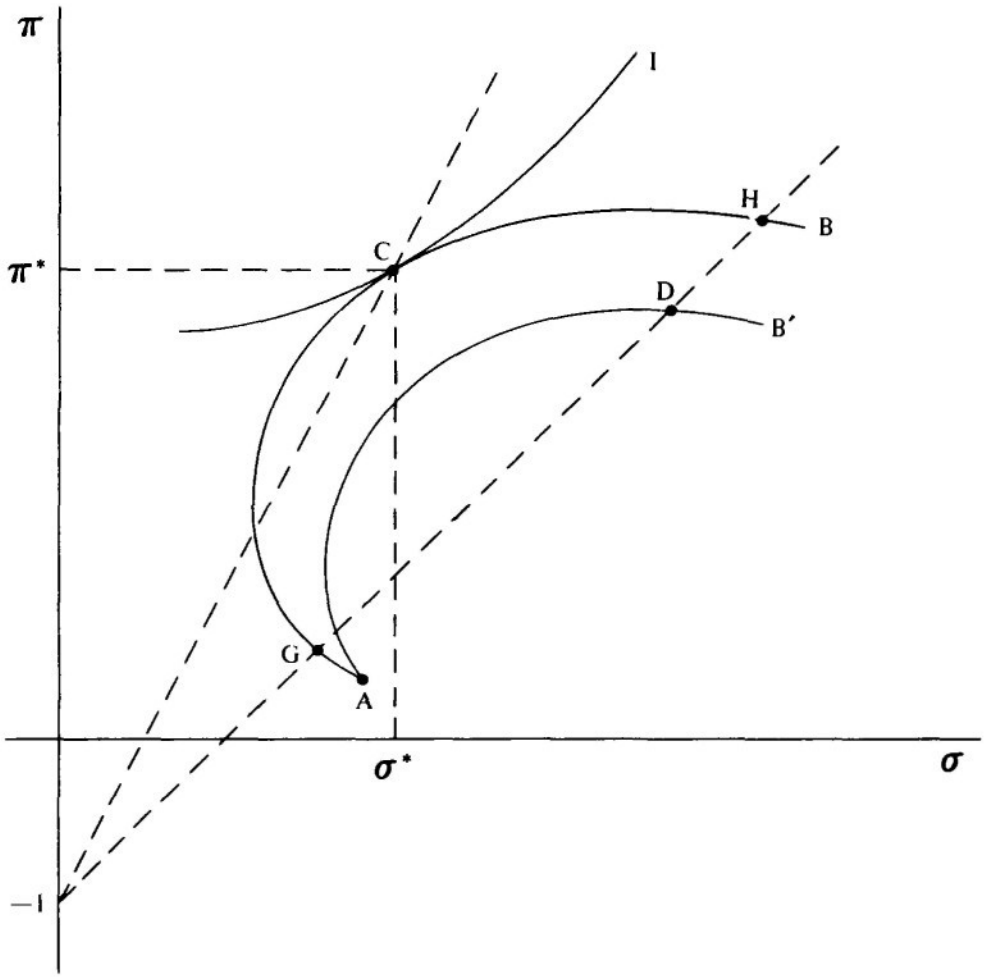
2/ See for example Hart and Jaffee (1974).

3/ See Iqbal and Mirakhor (1987), p. 24.

4/ See the Appendix in Iqbal and Mirakhor (1987) for some of these regulations.

5/ For a strong and clear statement of Pakistan authorities' concern with the risk of bank failures as a result of exposure to risk and profit sharing activities, see Kazi (1984).

FIGURE 1



The concern with the safety of the banking system is essentially based on a type of moral hazard argument that, in the absence of operating Islamic values in the economy, 1/ engaging in high-risk, high-return activities by the banks may lead to bank failures. 2/ This legitimate concern of the authorities stems, in part, from a perception that the removal of interest rate increases risk in the financial system in general and in the banking system in particular. One way to reduce the probability of bank failure, according to the authorities, is to reduce this probability and enhance bank safety by limiting mudarabah and musharakah financing. The regulators' encouragement, plus the relative ease of low-risk methods of financing, has led to an overwhelming dominance of short-term assets, acquired through trade financing, in the asset portfolio of the banking system in the two countries. 3/

Persistence of short-term asset concentration is worrisome on at least two grounds. First, because institutional structures are not developed to facilitate the growth of investment-type lending (in particular the private capital markets in the two countries do not have the required depth and breadth to accommodate the instruments necessary for long-term investment) the banks are the only source of funds for long-gestating projects. The second reason for concern is that, even without existing regulation to encourage minimal exposure to longer-term risk-return sharing financing, a tendency naturally exists among the bankers to opt for short-term trade financing. The result of bias against musharakah and mudarabah threatens investment-type financing and thus capital accumulation. 4/

The purpose of this paper is to demonstrate that the implementation of portfolio regulation via restrictions placed on high-risk, high-return asset acquisition through musharakah and mudarabah financing may produce results not intended by authorities, i.e., there is a distinct possibility that the risk of bank failure may in fact increase.

II. Bank Portfolio and Probability of Failure

It is because financial markets are imperfect that the financial intermediaries have a role to play. 5/ Intermediaries, through exploitation of these imperfections, alter the relationship between

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1/ Qureshi (1984), p. 89, and Kazi (1984), p. 10.
2/ Kazi (1984), p. 13.
3/ Short-term assets constitute about 85 percent of financial transactions, in form of mark-up, in Pakistan. In Iran, this figure was about 60 percent in the first year of the operation of the banking system (1984-85).
4/ Ahmed, et. al. (1983) and Ahmed (1984).
5/ Stigler (1967).

lenders and borrowers. Thus, intermediaries provide higher returns to lenders and lower costs to borrowers than would be possible with direct finance. In the process of intermediation a bank becomes unsafe if it cannot liquidate enough assets to supply deposit withdrawals. If a bank suffers severe losses so that the value of its asset portfolio declines below the level of its debts to depositors, i.e., it has a negative capital position, then under massive withdrawals it may fail. In this context bank earnings are important because a profitable operation is the bank's first line defense against occasional shrinkage in asset value and because profitable banks have higher capacity to attract deposits and can better raise new capital when desired in order to take advantage of profitable opportunities. On the other hand, the greater the risk of a bank's assets, the higher the level of capital required to avoid potential failures due to asset losses. Hence, the probability of failure is related to the ability of a bank's capital to absorb losses.

Failure, then, can be defined as when a bank's losses exceed its total capital. Given a bank's capital position and its asset portfolio characteristics, defined by its expected return and its variance, an upper boundary can be estimated for the probability of failure. 1/ One method of estimation is to use Chebyshev Inequality according to which if y is a random variable with mean m and variance σ^2, then

$$P = PR(|y - m| \geq d) \leq \frac{\sigma^2}{d^2}$$

where d is any positive number and $PR(|y - m| \geq d)$ is the probability that y will differ from m by at least $\pm d$. 2/

If the regulators designate some disaster level r for expected net profits $\bar{\Pi}$, i.e., $d = \bar{\Pi} - r$, then the probability of failure will be given as:

$$(1) \qquad P = PR[\Pi \leq r] = \frac{\sigma^2}{(\bar{\Pi} - r)^2}$$

1/ This is a familiar definition of failure; see Roy (1952), Telser (1955) Kahane (1977), Blair and Heggestad (1978), Koehn and Santomero (1980), and Allen (1983).

2/ See Roy (1952) for proof.

so that if $r = -1$, then

$$(2) \qquad P = PR[\Pi \leq -1] = \frac{\sigma^2}{(\bar{\Pi}+1)},$$

which can be graphically represented as the square of the reciprocal of the slope of a ray in expected return-standard deviation space with an intercept of -1 as in Figure 1. From (2) it can be seen that an increase in the expected return decreases failure risk while an increase in variance increases the probability of failure. The bank's portfolio thus has an upper limit on its probability of failure which is constant along the ray that intersects the efficient frontier at point C; where the optimal allocation is indicated by the tangency point and where the portfolio is characterized by proportion $f_i (i=1,2,\ldots,n)$ of each asset and Π^* and σ^*. The steeper the ray to the portfolio selected, the lower is the probability of its failure for any given specification of disaster level.

III. Bank Portfolio Regulation and Failure Risk

When the regulation takes the form of restricting the role of risk and profit sharing assets in the portfolio of banks, it has the effect of shifting the frontier downward. The new frontier would coincide with the old at the minimum-risk asset as in Figure 1 where the frontier in the absence of regulation is AB and the regulated frontier is AB'. The result is that, in the presence of regulation, not only are profits lower but the probability of failure is higher. This is because at point D, a point consistent with an optimal portfolio on the new frontier, the new ray intersecting AB' at D is less steep than the old ray, i.e., there is a higher probability of failure. Moreover, since the new optimal portfolio selected at D is on a lower indifference curve the bank is worse off. Additionally, the variance is larger as well. Consequently the effect of portfolio regulation is unambiguously perverse. 1/

Moreover, by focusing their attention solely on risks of individual assets, the regulators ignore the pooling effects of diversification. To illustrate we assume that, in order to safeguard against the risk of bank failure the regulators set a minimum capital to asset ratio, k. 2/ The

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1/ This graphic analysis was first utilized by Blair and Heggestad (1978) to show the perverse effects of regulations to limit the role of high-risk, high-return assets, such as common stock, in the portfolio of banks.

2/ See Fama (1971), and Shapiro (1982).

decision problem facing the bank involves selecting the optimal level of its assets and its distribution between the available assets. It is assumed that the bank is operating in a perfectly competitive market, which implies that risk and return per unit of capital do not vary as a function of bank capital thus avoiding concerns about scale effects.

For simplicity of analysis, it is assumed that there are only two assets available, those acquired via mark-up (denoted by h) and those obtained via mudarabah (denoted by m) financing--representing weak and strong Islamic modes--respectively. The fractions of bank capital committed to the two assets are f_h and f_m. The overall and individual net return per unit of bank capital are Π, Π_h, and Π_m, so that:

$$\Pi = f_h\Pi_h + f_m\Pi_m \tag{3}$$

The sum of f_h and f_m together represent the bank's degree of leverage, i.e., assets per unit of capital. The capital to asset ratio k constrains the bank's degree of leverage such that: 2/

$$f_h + f_m = \frac{1}{k} \tag{4}$$

and (4) is assumed to be a binding constraint.

The variance of overall return Π is a function of variances of returns from each of the two assets as well as the covariance between them. That is:

$$\sigma^2 = f_h^2\,\sigma_h^2 + f_m^2\sigma_m^2 + 2f_hf_m\sigma_{hm} \tag{5}$$

where σ^2, σ_h^2, σ_m^2 are the variances of overall returns and variances of the two assets respectively 2/; and σ_{hm} is the covariance between the mark-up and mudarabah assets. 3/

1/ Ibid.

2/ Even though mark-up resembles an interest-based asset, it is still not riskless; see Khan (1983).

3/ Markowitz (1959), Tobin (1966), Babcock (1972), Shapiro (1982) and Allen (1983).

Recall from (2) that the probability of bankrupcy is:

$$P = PR(\Pi \leq -1) = \frac{\sigma^2}{(\overline{\Pi} + 1)^2}$$

Where $\overline{\Pi}$, the expected overall return, is equal to $f_h\overline{\Pi}_h + f_m\overline{\Pi}_m$. Substitution and expansion of (2) yields an expression for the probability of failure for this problem as:

$$(6) \qquad P = PR(\Pi \leq -1) = \frac{f_h^2\ \sigma_h^2 + f_m^2\ \sigma_m{}^2 + 2f_h f_m\ \sigma hm}{[f_h\overline{\Pi}_{\ h} + f_m\overline{\Pi}_{\ m} + 1]^2}$$

This expression shows the relationship between expected returns to mark-up and mudarabah activities, the variance and covariance of returns, the bank's capital position, and the probability of bank failure. The expression further illustrates the narrowness of focusing primarily on the risk of mudarabah assets. It can be seen that the effect on bank's risk of failure when the proportion f_m of these assets is increased is a function of the current level of f_m as well as the variance-covariance structure of asset returns and their means. Depending on the bank's objective function, then, there is an optimal level of mudarabah assets for a bank.

If the bank chooses to minimize the variance of its overall returns, then the values of f_h and f_m which minimize (5) can be determined. Accordingly, the first order conditions are:

$$(7) \qquad \frac{\delta\sigma^2}{\delta f_h} = 2f_h\sigma_h^2 + 2f_m\sigma_{hm} = 0$$

$$(8) \qquad \frac{\delta\sigma^2}{\delta f_m} = 2f_m\sigma_m^2 + 2f_m\sigma_{hm} = 0,$$

From (7) and (8) we obtain:

$$(9) \qquad f_h^* = \frac{\sigma_m - \sigma_{mh}}{k(\ \sigma_h^2 + \sigma_m^2 - 2\sigma_{hm}\ )}$$

$$(10) \qquad f_m^* = \frac{\sigma_h - \sigma_{hm}}{k(\sigma_h^2 + \sigma_m^2 - 2\sigma_{hm}\ )}$$

It can be shown that (9) and (10) satisfy the condition that $f_h^* + f_m^* = \frac{1}{k}$.

A sufficient condition for $f_{h_2}^*$ to be an interior point, thus yielding a global minimum, is that $\sigma_h^2, \sigma_m^2 > \sigma_{hm}$. However, since the probability of bank failure is a function of expected return as well as its variance, minimizing σ^2 could increase the risk of failure. 1/ The alternative is to choose f_h and f_m such that the right-hand side of (2) is minimized. This occurs at the point:

$$(11) \qquad f_h^{**} = \frac{\sigma_m^2 (\bar{\Pi} + k) - \sigma_{hm} (\bar{\Pi} + k)}{k \left[\sigma_h^2(\bar{\Pi}_m + k) + \sigma_m^2 (\bar{\Pi}_h + k) - 2\sigma_{hm} (\bar{\Pi}_m + \bar{\Pi}_h + 2k)\right.}$$

from (11) it can be seen that:

$$f_h^{**} = f_h^* \text{ if } \bar{\Pi}_m = \bar{\Pi}_h$$
$$f_h^{**} > f_h^* \text{ if } \bar{\Pi}_h > \bar{\Pi}_m$$
$$f_h^{**} < f_h^* \text{ if } \bar{\Pi}_h < \bar{\Pi}_m$$

and that f_h^{**} is an increasing function of σ_m^2 and $\bar{\Pi}_h$ and a decreasing function of σ_h^2 and $\bar{\Pi}_m$. f_h^{**} is an increasing function of σ_{hm} if, and only if,

$$(\bar{\Pi}_h + k)\sigma_m^2 > (\bar{\Pi}_m + k)\sigma_h^2$$

The major implication of the above analysis is that attempts to restrict the banks from engaging in risk-return sharing financing, such as mudarabah, in order to reduce the variance of bank returns, could increase the probability of failure if the expected return from mudarabah assets exceed the expected return from mark-up, i.e., $\bar{\Pi}_m > \bar{\Pi}_h$; assuming that the variances are the same.

1/ See Shapiro (1982) p. 733.

For regulations on portfolio composition to be effective in reducing failure risks, their effect on overall return and its variance as well as covariance of expected returns from various assets must be considered. A constraint on mudarabah assets will lower the probability of failure if the actual proportion of these assets in the bank's portfolio is greater than its optimal value given by (11). If the expected returns from weakly Islamic modes is different from musharakah and mudarabah then the proportion of short-term and minimum risk assets in the portfolio will affect its overall expected return and minimization of risk of bank failures will lead to lower returns.

Consequently, imposing restrictions on risk-return activities of banks simply because of their risk alone may result in nonoptimal decisions. To the extent that the portfolio decisions of individual banks are biased in favor of low-risk assets, financial intermediation in the aggregate and, concomitantly, the allocation of real resources in the economy is affected. The evidence available thus far indicates that the portfolio behavior of banks in Iran and Pakistan may entail undesirable consequences for capital formation. It is noteworthy that in early stages in the process of deliberations regarding the Islamization of banking some economists foresaw the possibility that conditions may arise which may lead to short-term asset concentration, and pointed out its harmful effects. 1/ For example, Karsten ((1982), p. 132) argued that the intermediation process in an Islamic banking system "could be affected if Islamic banks were somehow constrained in carrying out the task of transforming short-term liabilities into long-term advances." Clearly, and at least in the case of banks in an Islamic system, in countries where capital markets are not sufficiently deep and wide to accommodate long-term investment financing instruments on the basis of profit sharing, the portfolio behavior which is not optimal for the banks may also not be optimal for the economy as a whole. 2/ Moreover, even the perception of riskiness of mudarabah and musharakah financing by the regulators, in isolation from their expected returns and covariances with other assets, may represent an overestimate.

The risks which a bank takes in entering into musharakah and mudarabah activities with a given firm can be decomposed into risks that relate to market conditions;including socio-political conditions and general economic fluctuations, and risks which are firm-specific. The first type of risks will exist regardless of whether the bank engages in low-risk financing or mudarabah and musharakah activities. The firm-specific risks can be further

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1/ See, for example, Ahmed, et. al., (1983, p.12), Ahmed (1984, p.10), Siddiqi (1983).

2/ See Pringle (1972).

decomposed into investments and fraud risks. The former is clearly a function of viability and profitability of the proposed project as well as the abilities of the entrepreneur. The banks should be able to develop the expertise necessary for project appraisal as well as monitoring the competence of the entrepreneur. One way of doing so is to allow the development of specialized banks whose main function would be investment-type financing on the basis of profit sharing. These banks could then issue their own liabilities to the commercial banks. It is not unrealistic, moreover, to expect the banks, as partners, to be represented in the managerial decision-making processes of the firm in order to monitor the projects in which they have taken equity positions.

The risk of fraud, which is especially worrisome to the regulators, 1/ seems to have two sources. One is the possibility of underreporting of profits earned by the firm via maintenance of two sets of books which is in turn motivated by tax-avoidance. Although there are no precise measures of the extent of presence of this practice, this risk can be minimized by a system of audit stipulated in musharakah and mudarabah contracts. 2/ The other source of risk of fraud is the perception that since in risk-return sharing arrangements the banks will have to carry the burden of potential financial losses there is an element of moral hazard involved in these transactions. 3/ Due to the short history of Islamic banking practices and the minimal size of profit-sharing assets, there is no practical way of estimating the magnitude of this risk. However, there are at least three possible ways in which this residual risk can be minimized. First is by implementing the Islamic law of contracts which requires that stipulations of agreements entered into must be faithfully observed, and which proposes well-defined retributive judicial measures to safeguard the terms of the contract. Second is the possibility of third-party insurance schemes with cost participation by the central bank and commercial banks. Third is maintenance of loss-compensating reserves by the banks. 4/ It must also be noted that hardly any bank can be expected to finance a risk-return sharing project without sufficient information regarding the managerial ability, competence and character of the entrepreneur.

By relaxing regulations on portfolio composition the banks will be allowed additional opportunity to diversify their asset portfolios, thus affecting both the overall expected net returns as well as their variance. It has long been recognized that, depending on the degree of correlation

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1/ See Iqbal and Mirakhor (1987).

2/ The contracts can stipulate bank participation in managerial decision-making processes of the firm in order to give banks the necessary ability to monitor firm's behavior.

3/ Haque and Mirakhor (1986,b)

4/ Siddiqi (1982 and 1983)

between returns, portfolio risk can be reduced by diversification. 1/ It is almost a tautology that a more diversified bank represents a more stable source of funds and a more stable institution for depositors.

IV. Conclusions

There seems to be little doubt among Muslim economists and bankers that, given a favorable policy and institutional framework, the banks in an Islamic system should and will undertake financing of risk-return projects. Thus far, however, the policy stance of regulators has been to encourage short-term and low-risk financing which has resulted in concentration of bank portfolios in trade-type financing rather than long-term investment projects. The primary cause of this policy stance has been the perception, possibly overestimated, that the removal of the interest rate increases the potential of moral hazard, thus making profit-sharing investment projects risky.

This paper has shown that the overall risk of banks' portfolios and their expected returns are what must be considered rather than the risk of individual assets. If the regulators' concern is the failure of banks, one way of insuring their safety without harmful effects on resource allocation is for regulators to determine an acceptable probability of failure and then allow the banks the freedom of determining the optimal composition for their portfolios. Each bank can then determine the appropriate trade-off between risk and return as well as asset diversification in order to assess how the various modes of asset acquisition are related in their prospects for timely payment of returns and principal. If the banks are constrained to concentrate their portfolios heavily in favor of one type of asset, e.g., mark-up, then that portfolio might turn out to be very risky. In the short life of Islamic banking, a configuration of various types of regulations have been divised which impose guidelines--including restrictions on amounts and rate of return--on profit-sharing asset acquisition behavior of banks for each mode of financing which is deemed relatively risky by the regulators. Moreover, other policy measures such as credit rationing and moral suasion are used as controls on banks' participation in long-term profit-sharing and investment-type financing. Given the natural propensity of the banks for low-risk and easily administered modes of financing--which is partially due to lack of expertise in portfolio management and project appraisal--these regulations may hold further negative prospects for capital formation.

1/ For proof, see Lintner (1965), Samuelson (1967), Evans and Archer (1968) and Allen(1983). Muslim economists have recognized the need for portfolio diversification in Islamic banking; see, for example, Siddiqi (1982) and Qureshi (1985).

This paper has attempted to argue that the authorities' legitimate concerns should be directed at the safety of bank's overall portfolio of assets rather than the risk of a particular mode of financing considered in isolation. Thus, to restrain the banking system from employing profit-sharing modes, such as musharakah and mudarabah--methods most appropriate for investment-type financing--simply because of perceptions of high risks without sufficient analysis of their effects on the risk and return characteristics of the banks' overall portfolio may neither allow Islamic banking to take root nor permit the banking system to play an effective role in the development process. Coupled with poorly developed equity markets, the dynamic effects of regulations discouraging exposure of bank portfolios to risky investment projects on economic growth should be of serious concern because of the negative effects on capital formation, innovation, and the process of adjustment in evolving industrial structure in the economy.

APPENDIX

Definition of symbols:

Π = overall expected return from portfolio.

σ^2 = variance of the overall expected return (measures the riskiness of the portfolio).

Π_i = expected return from the ith asset, where i = 1, 2,..., N.

σ_{ij} = covariance of returns between ith and jth asset.

$\sigma_i^2 = \sigma_{ii}$ = variance of the ith asset, assumed > 0 since all assets are assumed risky.

$C = [c_{ij}]$ = the variance-covariance matrix of returns. In line with the requirements of Islamic banking, it is assumed here that no asset can be represented as a linear combination of other assets. This assumption makes C a nonsingular, symmetric and positive-defenite matrix.

$C^{-1} = [d_{ij}]$ = the inverse of the variance-covariance matrix which is also nonsingular, symmetric and positive-definite matrix.

λ_1, λ_2 = Lagrange multipliers.

f_i = the proportion of portfolio invested in the ith asset.

The problem is to find an efficient portfolio composed of N risk assets subject to conditions that:

$$\sigma^2 = \sum_i^N \sum_j^N f_i \, f_j \, \sigma_{ij} \quad , \tag{12}$$

$$\Pi = \sum_i^N f_i \, \Pi_i \quad , \text{ and} \tag{13}$$

$$\sum_i^N f_i = 1 \tag{14}$$

APPENDIX

Forming the Lagrangian, we have:

$$(15) \qquad \text{Min } L = \sum_i^N \sum_j^N f_i f_j \sigma_{ij} + \lambda_1 [\Pi - \sum_i^N f_i \Pi_i] + \lambda_2 [1 - \sum_i^N f_i]$$

First order conditions yield:

$$(16) \qquad \sum_i^N f_j \sigma_{ij} - \lambda_1 \Pi_i - \lambda_2 = 0 \qquad , i = 1, 2, \ldots, N$$

$$(17) \qquad \Pi - \sum_i^N f_i \Pi_i = 0 \qquad , i = 1, 2, \ldots, N$$

$$(18) \qquad 1 - \sum_i^N f_i = 0 \qquad , i = 1, 2, \ldots, N$$

Because of the assumption regarding C, the f_is which minimize σ_2 are unique. The system (16) - (18) is linear in the f_is. For the proportion of the efficient portfolio invested in the kth asset from (16) we have:

$$(19) \qquad f_\kappa = \lambda_1 \sum_i^N d_{\kappa j} \Pi_j + \lambda_2 \sum_i^N d_{\kappa j}$$

multiplying (19) by Π_κ and summing over $\kappa = 1, 2, \ldots, N$, yields:

$$(20) \qquad \sum_i^N f_\kappa = \lambda_1 \sum_i^N \sum_i^N d_{\kappa j} \Pi_j + \lambda_2 \sum_i^N \sum_i^N d_{\kappa j}$$

$$(21) \qquad \sum_i^N f_\kappa \Pi_\kappa = \lambda_1 \sum_i^N \sum_j^N d_{\kappa j} \Pi_j \Pi_\kappa + \lambda_2 \sum_i^N \sum_i^N d_{\kappa j} \Pi_\kappa$$

from (13), (14), (20) and (21) we obtain a linear system in λ_1 and λ_2 as:

$$(22) \qquad \Pi = \lambda_1 \sum_{i}^{N} \sum_{i}^{N} d_{\kappa j}\, \Pi_j\, \Pi_\kappa + \lambda_2 \sum_{i}^{N} \sum_{i}^{N} d_{\kappa j}\, \Pi_j$$

$$(23) \qquad \lambda_1 \sum_{i}^{N} \sum_{i}^{N} d_{\kappa j}\, \Pi_j\, \Pi_\kappa + \lambda_2 \sum_{i}^{N} \sum_{i}^{N} d_{\kappa j} = 1$$

From the assumption on C and its inverse, we have $d_{kj} = d_{jk}$ for all j and κ and also that:

$$\sum_{i}^{N} \sum_{i}^{N} d_{\kappa j}\, \Pi_j = \sum_{i}^{N} \sum_{i}^{N} d_{\kappa j}\, \Pi_\kappa$$

Also $\sum_{i}^{N} \sum_{i}^{N} d_{\kappa j}\, \Pi_j$ and $\sum_{i}^{N} \sum_{i}^{N} d_{\kappa j}$ are quadratic forms of the inverse matrix, therefore, they are strictly positive. Allowing

$$M = \sum_{i}^{N} \sum_{i}^{N} d_{\kappa j}\, \Pi_j \; ; \qquad N = \sum_{i}^{N} \sum_{i}^{N} d_{\kappa j}\, \Pi_j\, \Pi_\kappa$$

and $P = \sum_{i}^{N} \sum_{i}^{N} d_{\kappa j}$ we can rewrite (22) and (23) as:

$$(22') \qquad \Pi = \lambda_1 N + \lambda_2 M$$

$$(23') \qquad \lambda_1 N + \lambda_2 P = 1$$

solving (22') and (23') for λ_1 and λ_2 we have:

$$(24) \qquad \lambda_1 = \frac{P\Pi - M}{Q}$$

$$(25) \qquad \lambda_2 = \frac{N - M\Pi}{Q}$$

where $Q = NP - M^2$ and is positive because of the assumption that the inverse matrix is positive-definite. Substitution of (24) and (25) in (19) gives optimal f_κ^*, the proportion of the efficient portfolio invested in the kth asset as:

$$f_k^* = \frac{\Pi \sum_i^N d_{\kappa j} (P\Pi_j - M) + \sum^N d_{\kappa j}(N - \Pi_j)}{Q} \tag{26}$$

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