Relative profit maximization and equivalence of Cournot and Bertrand equilibria in asymmetric duopoly

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Abstract

We study the relation between a Cournot equilibrium and a Bertrand equilibrium in an *asymmetric* duopoly with differentiated goods in which each firm maximizes its relative profit that is the difference between its profit and the profit of the rival firm. Both demand and cost functions are linear but asymmetric, that is, demand functions for the goods are asymmetric and the firms have different marginal cots. We will show that a Cournot equilibrium and a Bertrand equilibrium coincide even in an asymmetric duopoly.

Keywords: asymmetric duopoly, relative profit maximization, equivalence of Cournot and Bertrand equilibria.

JEL Classification code: D43, L13.

1 Introduction

We study the relation between a Cournot equilibrium and a Bertrand equilibrium in an *asymmetric* duopoly with differentiated goods in which each firm maximizes its relative profit that is the difference between its profit and the profit of the rival firm. Both demand and cost functions are linear but asymmetric, that is, demand functions for the goods are asymmetric and the firms have different marginal costs. We will show that a Cournot equilibrium and a Bertrand equilibrium coincide even in an *asymmetric* duopoly.

In recent years, maximizing relative profit instead of absolute profit has aroused the interest of economists. From an evolutionary perspective, Schaffer (1989) demonstrates with a Darwinian model of economic natural selection that if firms have market power, profit-maximizers are not necessarily the best survivors. According to Schaffer (1989), a unilateral deviation from Cournot equilibrium decreases the profit of the deviator, but decreases the other firm's profit even more. On the condition of being better than other competitors, firms that deviate from Cournot equilibrium achieve higher payoffs than the payoffs they receive under Cournot equilibrium. In Vega-Redondo (1997), it is argued that, in a homogeneous good case, if firms maximize relative profit, a Walrasian equilibrium can be induced.

> Note that if the goods produced by the firms are homogeneous, relative profit maximization leads to the competitive output and price (equal to marginal cost). But in the case of differentiated goods, the results under relative profit maximization is different from the competitive result.

For other analyses of relative profit maximization see Lundgren (1996), Kockesen et. al. (2000), Matsumura, Matsushima and Cato (2013), Gibbons and Murphy (1990) and Lu (2011).

In another paper Tanaka (2013) we have shown that in a *symmetric* duopoly with differentiated goods and linear demand functions when firms maximize relative profits, a Cournot equilibrium and a Bertrand equilibrium coincide. The result of this paper is an extension of this result to an *asymmetric* duopoly.

2 The model and analyses

There are two firms, A and B. They produce differentiated substitutable goods. Denote the outputs of Firm A and B by, respectively, x_A and x_B, the prices of the goods of Firm A and B by, respectively, p_A and p_B, the marginal costs of Firm A and B by, respectively, c_A and c_B. Of course, $c_A > 0$ and $c_B > 0$. They may be different. There is no fixed cost.

The inverse demand functions of the goods produced by the firms are

$$p_A = a - x_A - b_A x_B,$$

and

$$p_B = a - x_B - b_B x_A,$$

where $a > c$, $0 < b_A < 1$ and $0 < b_B < 1$. b_A and b_B may be different. x_A represents the demand for the good of Firm A, and x_B represents the demand for the good of Firm B. The prices of the goods are determined so that demand of consumers for each firm's good and supply of each firm are equilibrated.

From these inverse demand functions the following ordinary demand functions are derived.

$$x_A = \frac{1}{1 - b_A b_B}[(1 - b_A)a - p_A + b_A p_B],$$

and

$$x_B = \frac{1}{1 - b_A b_B}[(1 - b_B)a - p_B + b_B p_A].$$

3.1 Cournot equilibrium

The relative profit of Firm A (or B) is the difference between its profit and the profit of Firm B (or A). We denote the relative profit of Firm A by Π_A and that of Firm B by Π_B. They are written as follows.

$$\Pi_A = \pi_A - \pi_B = (a - x_A - b_A x_B)x_A - (a - x_B - b_B x_A)x_B - c_A x_A + c_B x_B$$

and

$$\Pi_B = \pi_B - \pi_A = (a - x_B - b_B x_A)x_B - (a - x_A - b_A x_B)x_A - c_B x_B + c_A x_A$$

Each firm determines its output given the output of the rival firm so as to maximize its relative profit. The condition for relative profit maximization of Firm A is

$$a - c_A - 2x_A - (b_A - b_B)x_B = 0. \tag{1}$$

Similarly the condition for relative profit maximization of Firm B is

$$a - c_B - 2x_B - (b_B - b_A)x_A = 0. \tag{2}$$

Then, the equilibrium outputs of Firm A and B are derived as follows.

$$\tilde{x}_A^C = \frac{(2 - b_A + b_B)a - 2c_A + (b_A - b_B)c_B}{4 + (b_A - b_B)^2}, \tag{3}$$

and

$$\tilde{x}_B^C = \frac{(2 - b_B + b_A)a - 2c_B + (b_B - b_A)c_A}{4 + (b_A - b_B)^2}. \tag{4}$$

The equilibrium prices of the goods of Firm A and B are

$$\tilde{p}_A^C = \frac{(2 - b_A - b_B - b_A b_B + b_B^2)a + (2 - b_A b_B + b_A^2)c_A + (b_A + b_B)c_B}{4 + (b_A - b_B)^2}, \tag{5}$$

and

$$\tilde{p}_B^C = \frac{(2 - b_A - b_B - b_A b_B + b_A^2)a + (2 - b_A b_B + b_B^2)c_B + (b_A + b_B)c_A}{4 + (b_A - b_B)^2}. \tag{6}$$

About calculations of (3)~(6) see Appendix A.

3.2 Bertrand equilibrium

The relative profits of Firm A and B are also denoted by Π_A and Π_B. Using the ordinary demand functions, they are written as follows.

$$\begin{aligned}\Pi_A = \pi_A - \pi_B = & \frac{1}{1 - b_A b_B}[(1 - b_A)a - p_A + b_A p_B](p_A - c_A) \\ & - \frac{1}{1 - b_A b_B}[(1 - b_B)a - p_B + b_B p_A](p_B - c_B),\end{aligned}$$

and

$$\begin{aligned}\Pi_B = \pi_B - \pi_A = & \frac{1}{1 - b_A b_B}[(1 - b_B)a - p_B + b_B p_A](p_B - c_B) \\ & - \frac{1}{1 - b_A b_B}[(1 - b_A)a - p_A + b_A p_B](p_A - c_A).\end{aligned}$$

Each firm determines the price of its good given the price of the rival firm's good so as to maximize its relative profit.

The condition for relative profit maximization of Firm A is

$$(1 - b_A)a - 2p_A + b_A p_B + c_A - b_B(p_B - c_B) = 0.$$

And the condition for relative profit maximization of Firm B is

$$(1 - b_B)a - 2p_B + b_B p_A + c_B - b_A(p_A - c_A) = 0.$$

Substituting the inverse demand functions into them and arranging the terms, we obtain

$$-(a - 2x_A - b_A x_B - c_A + b_B x_B) - b_B(a - 2x_B - b_B x_A - c_B + b_A x_A) = 0,$$

and

$$-(a - 2x_B - b_B x_A - c_B + b_A x_A) - b_A(a - 2x_A - b_A x_B - c_A + b_B x_B) = 0.$$

Since $0 < b_A < 1$ and $0 < b_B < 1$, these equations imply

$$a - 2x_A - b_A x_B - c_A + b_B x_B = 0,$$

and

$$a - 2x_B - b_B x_A - c_B + b_A x_A = 0.$$

They are identical to the conditions for relative profit maximization at the Cournot equilibrium in (1) and (2). Therefore, the equilibrium outputs of Firm A and B are obtained as follows.

$$\tilde{x}_A^B = \frac{(2 - b_A + b_B)a - 2c_A + (b_A - b_B)c_B}{4 + (b_A - b_B)^2},$$

and

$$\tilde{x}_B^B = \frac{(2 - b_B + b_A)a - 2c_B + (b_B - b_A)c_A}{4 + (b_A - b_B)^2}.$$

And the equilibrium prices of the goods of Firm A and B are

$$\tilde{p}_A^B = \frac{(2 - b_A - b_B - b_A b_B + b_B^2)a + (2 - b_A b_B + b_A^2)c_A + (b_A + b_B)c_B}{4 + (b_A - b_B)^2},$$

and

$$\tilde{p}_B^B = \frac{(2 - b_A - b_B - b_A b_B + b_A^2)a + (2 - b_A b_B + b_B^2)c_B + (b_A + b_B)c_A}{4 + (b_A - b_B)^2}.$$

We have

$$\tilde{x}_A^C = \tilde{x}_A^B,\ \tilde{p}_A^C = \tilde{p}_A^B,$$

and

$$\tilde{x}_B^C = \tilde{x}_B^B,\ \tilde{p}_B^C = \tilde{p}_B^B.$$

Thus, in an asymmetric duopoly with linear demand functions the Cournot equilibrium and the Bertrand equilibrium are equivalent under relative profit maximization in the sense that the equilibrium outputs and prices at the Cournot equilibrium and those at the Bertrand equilibrium are equal.

4 Interpretation

The result of this paper implies that when firms seek to maximize their relative profits, distinction of price competition and quantity competition in duopoly does not make sense.

A game of relative profit maximization in duopoly is interpreted as a two-person zero-sum game with two sets of strategic variables. Consider an example of two-person zero-sum game as follows. There are two players, A and B. They have two sets of strategic variables, (s_A, s_B) and (t_A, t_B). The relations of them are represented by

$$s_A = f_A(t_A, t_B), \text{and } s_B = f_B(t_A, t_B).$$

f_A and f_B are assumed to be linear, so $\frac{\partial f_A}{\partial t_A}$, $\frac{\partial f_A}{\partial t_B}$, $\frac{\partial f_B}{\partial t_A}$ and $\frac{\partial f_B}{\partial t_B}$ are constant. We assume that the payoff function of Player A is the following quadratic function.

$$u_A = \alpha + \beta_A s_A^2 + \beta_B s_B^2 + \gamma s_A s_B + \delta_A s_A + \delta_B s_B.$$

α, β_A, β_B, γ, δ_A and δ_B are constants. The payoff function of Player B is $u_B = -u_A$. The condition for maximization of u_A with respect to s_A and the condition for maximization of u_B with respect to s_B are

$$2\beta_A s_A + \gamma s_B + \delta_A = 0, \tag{7}$$

and

$$2\beta_B s_B + \gamma s_A + \delta_B = 0. \tag{8}$$

We assume the existence of the maximums of u_A and u_B. Substituting f_A and f_B into u_A and u_B yields

$$u_A = \alpha+\beta_A[f_A(t_A, t_B)]^2+\beta_B[f_B(t_A, t_B)]^2+\gamma f_A(t_A, t_B)f_B(t_A, t_B)+\delta_A f_A(t_A, t_B)+\delta_B f_B(t_A, t_B).$$

Since f_A and f_B are linear, u_A and u_B are quadratic with respect to t_A and t_B. The condition for maximization of u_A with respect to t_A and the condition for maximization of u_B with respect to t_B are

$$2\beta_A f_A(t_A,t_B)\frac{\partial f_A}{\partial t_A}+2\beta_B f_B(t_A,t_B)\frac{\partial f_B}{\partial t_A}+\gamma f_A(t_A,t_B)\frac{\partial f_B}{\partial t_A}+\gamma f_B(t_A,t_B)\frac{\partial f_A}{\partial t_A}$$
$$+\delta_A\frac{\partial f_A}{\partial t_A}+\delta_B\frac{\partial f_B}{\partial t_A}=0,$$

and

$$2\beta_A f_A(t_A,t_B)\frac{\partial f_A}{\partial t_B}+2\beta_B f_B(t_A,t_B)\frac{\partial f_B}{\partial t_B}+\gamma f_A(t_A,t_B)\frac{\partial f_B}{\partial t_B}+\gamma f_B(t_A,t_B)\frac{\partial f_A}{\partial t_B}$$
$$+\delta_A\frac{\partial f_A}{\partial t_B}+\delta_B\frac{\partial f_B}{\partial t_B}=0.$$

They are rewritten as

$$\begin{aligned}&\left[2\beta_A f_A(t_A,t_B)+\gamma f_B(t_A,t_B)+\delta_A\right]\frac{\partial f_A}{\partial t_A}\\&+\left[2\beta_B f_B(t_A,t_B)+\gamma f_A(t_A,t_B)+\delta_B\right]\frac{\partial f_B}{\partial t_A}=0,\end{aligned}\tag{9}$$

and

$$\begin{aligned}&\left[2\beta_A f_A(t_A,t_B)+\gamma f_B(t_A,t_B)+\delta_A\right]\frac{\partial f_A}{\partial t_B}\\&+\left[2\beta_B f_B(t_A,t_B)+\gamma f_A(t_A,t_B)+\delta_B\right]\frac{\partial f_B}{\partial t_B}=0.\end{aligned}\tag{10}$$

Under the assumption that $\frac{\partial f_A}{\partial t_A}\frac{\partial f_B}{\partial t_B}-\frac{\partial f_A}{\partial t_B}\frac{\partial f_B}{\partial t_A}\neq 0$, (9) and (10) are equivalent to (7) and (8). Therefore, competition by (s_A,s_B) and competition by (t_A,t_B) are equivalent.

Assuming $s_A=x_A$, $s_B=x_B$, $t_A=p_A$, $t_B=p_B$, $\alpha=0$, $\gamma=b_B-b_A$, $\beta_A=-1$, $\beta_B=1$, $\delta_A=a-c_A$, $\delta_B=-a+c_B$, f_A and f_B be the ordinary demand functions, we obtain the model of this paper.

We plan to generalize discussions of this paper to a case of general demand and cost functions,

Appendix: Calculations of (3)~(6)

From the condition for relative profit maximization of Firm B in the Cournot equilibrium

$$x_B = \frac{1}{2}[a - c_B + (b_A - b_B)x_A]$$

Substituting this into the condition for relative profit maximization of Firm A yields

$$a - c_A - 2x_A - \frac{b_A - b_B}{2}[a - c_B + (b_A - b_B)x_A] = 0$$

Rearranging the terms,

$$[4 + (b_A - b_B)^2]x_A = 2a - 2c_A - (b_A - b_B)(a - c_B)$$

Then, we get (3). Calculation of (4) is similar.

From the inverse demand function for the good of Firm A

$$p_A = a - x_A - b_A x_B.$$

Substituting (3) and (4) into this,

$$\begin{aligned} p_A =& a - \frac{(2 - b_A + b_B)a - 2c_A + (b_A - b_B)c_B}{4 + (b_A - b_B)^2} \\ & - b_A \frac{(2 - b_B + b_A)a - 2c_B + (b_B - b_A)c_A}{4 + (b_A - b_B)^2} \end{aligned}$$

Then, we get (5). Calculation of (6) is similar.

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