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27. May 2014

Online at <http://mpra.ub.uni-muenchen.de/56213/>

MPRA Paper No. 56213, posted 9. June 2014 05:11 UTC

# A Politico-economic Approach on Public Debt in an Endogenous Growth Economy

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May 27, 2014

## Abstract

We consider an overlapping generations closed economy in which a government finances the cost of public good provision by labor income taxation and/or public debt issuance. The size of these public policies is determined in a repeated probabilistic voting game. We investigate the characteristics of a Markov perfect politico-economic equilibrium in which the size of public policies depends on both the stock of public debt and the level of physical capital, and show that individuals' stronger preferences for public good provision tighten fiscal discipline and promote economic growth.

*JEL classification:* D72; H41; H63; O43

*Keywords:* public debt; probabilistic voting; Markov perfect equilibrium; economic growth

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# 1 Introduction

In nearly every country, the government finances the cost of various types of public goods provision by issuing public debt. Public debt issuance affects patterns of physical capital accumulation, and many studies analyze its effects on economic growth. In democratic countries, on the other hand, the amount of public debt issuance is determined through voting processes, which stimulates a large body of literature investigating the determinants of public debt issuance in politico-economic frameworks. Building on these two strands of literature, this paper investigates interactions between politically implemented public debt policies and patterns of economic development.

Public debt issuance is a type of redistribution policy from younger to older generations because it imposes its repayment costs on younger generations. If parental generations are not altruistic toward their children, they would issue public debt as much as possible and put off fiscal burdens to offspring generations. In some countries, however, the amount of public debt issuance is maintained to be low: the public debt/GDP ratios in Luxembourg, Denmark, and Finland were kept below 60% in 2009.<sup>1</sup> Some studies such as Song et al. (2011) and Röhrs (2010) attempt to explain the mechanism by which public debt issuance is kept down. They construct overlapping generations models in which the size of public policies is determined through

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<sup>1</sup>The Maastricht Treaty convergence criteria requires EU countries to suppress public debt to 60% of GDP. In 2009, the public debt/GDP ratio in Luxembourg was below 20%, and the ratios in Denmark and Finland were around 45% (OECD Outlook 2009).

voting processes, and investigate the characteristics of Markov perfect equilibria. In the equilibria, future public good provision responds negatively to the amount of public debt issuance, which generates an incentive for individuals to suppress the issuance of public debt: individuals take into account that an increase in public debt issuance would decrease the level of future public good provision.

Song et al. (2011) and Röhrs (2010) analyze the intergenerational political conflicts on public debt issuance and obtain many interesting results. These studies, however, do not explicitly analyze how the extent of economic development affects the amount of politically implemented public debt issuance. Since the capacity of public debt issuance crucially depends on the scale of the economy, the extent of economic development, such as the level of physical capital, should have crucial effects on the amount of politically implemented public debt issuance. Furthermore, they do not analyze the effects of politically implemented public debt issuance on patterns of economic development, especially on physical capital accumulation. Standard textbooks such as Blanchard and Fischer (1989) mention that public debt issuance retards physical capital accumulation in a neo-classical growth economy, and Saint-Paul (1992) shows that an increase in public debt issuance lowers the rate of economic growth in an endogenous growth framework.<sup>2</sup> In contrast to the previous literature, this paper considers an overlapping

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<sup>2</sup>Elmendorf and Mankiw (1999) survey the effects of public debt issuance in the short and long run.

generations closed economy with physical capital accumulation and investigates the characteristics of a Markov perfect politico-economic equilibrium in which the size of public policies including public debt issuance depends not only on the stock of public debt but also on the level of physical capital. By doing so, we can explicitly analyze interactions between politically implemented public policies and economic development.

In this paper, individuals live for two periods (young and old) and derive utility from the consumption of private and public goods in both periods. When young, they supply labor inelastically and allocate their disposable income between consumption and savings. When old, they retire and consume the proceeds of their savings. The economy produces a final good by using physical capital and labor as inputs, and the technology is represented as a Romer (1986) type production function. The government finances the cost of public good provision by levying labor income taxation and/or issuing public debt. The size of public good provision, the labor income tax rate, and public debt issuance is determined in a repeated probabilistic voting game. When voting, individuals take into account that an increase in the current public good provision and/or a decrease in the current labor income tax rate not only accelerate public debt issuance but also retard physical capital accumulation, and would change the size of public good provision in the next period.

In this setup, we first show that there exists a Markov perfect equilibrium in which the size of public policies is represented by simple functional forms:

the size of public good provision and the amount of public debt issuance are negatively linear with respect to the stock of public debt and positively linear with respect to the level of physical capital, and the labor income tax rate is positively linear with respect to the ratio of public debt to physical capital. In the equilibrium, a rise in the size of current public good provision and/or a decline in the current labor income tax rate decrease the size of public good provision in the next period since these accelerate public debt issuance and retard physical capital accumulation. Thus, there exists an incentive to suppress public debt issuance. We next analyze the effects of some exogenous parameters on the size of equilibrium public policies. In particular, we show that individuals' stronger preferences for public good provision tighten fiscal discipline: these raise the equilibrium labor income tax rate and suppress public debt issuance.

We lastly investigate the patterns of public debt and physical capital accumulation in the Markov perfect equilibrium. It is shown that the public debt/physical capital ratio converges to a constant value within one period, and thereafter, both the stock of public debt and physical capital grow at the same rate (i.e., balanced growth path). Furthermore, we show that individuals' stronger preferences for public good provision tighten fiscal discipline and raise the economic growth rate in the balanced growth path. This result is consistent with the data from some democratic countries. Using the Corruption Perception Index (CPI) as the proxy of the extent of individuals' preferences for public good provision, we find a negative relationship between

CPI and the public debt/GDP ratio. Furthermore, some empirical studies such as Reinhart and Rogoff (2010) and Kumar and Woo (2010) provide evidence indicating that a high public debt/GDP ratio is likely to lower the economic growth rate. The two observations imply that individuals' stronger preferences for public good provision lower the public debt/GDP ratio and promote economic growth.

This paper belongs to a large body of literature investigating the determinants of public debt issuance in politico-economic frameworks. Persson and Svensson (1989), Alesina and Tabellini (1990), and Tabellini and Alesina (1990) analyze intragenerational political conflicts on public debt issuance. Battaglini and Coate (2008) and Yared (2010) are also related to this strand of literature. In the literature on optimal fiscal policy, Ortigueira and Pereira (2007) investigate the characteristics of optimal income taxation and public debt policy in a closed economy with physical capital. This paper is also related to many studies analyzing intergenerational political conflicts on social security (e.g., Forni 2005; Gonzalez-Eiras and Niepelt 2008) and redistribution policy (e.g., Hassler et al. 2003; Hassler et al. 2007). These studies, however, assume balanced budget constraints of the governments and do not consider the determinants of public debt issuance.

The rest of this paper is organized as follows. We characterize a competitive equilibrium in Section 2.1 and investigate the characteristics of a Markov perfect politico-economic equilibrium in Section 2.2. In Section 2.3, we analyze the dynamics of the stock of public debt and physical capital. We

conclude in Section 3.

## 2 Model

### 2.1 Competitive Equilibrium

We consider an overlapping generations closed economy in which individuals are homogeneous within each generation and live for two periods (young and old). There is no population growth, and the size of each generation is normalized to one. Individuals derive utility from the consumption of private and public goods in both periods, and their preferences are represented as

$$\log c_t + \beta \log d_{t+1} + \gamma (\log g_t + \beta \log g_{t+1}), \quad \beta \in (0, 1), \quad \gamma > 0, \quad (1)$$

where  $c_t$  and  $d_{t+1}$  are private consumption when young and old, and  $g_t$  and  $g_{t+1}$  are public good consumption when young and old, respectively. The parameter  $\gamma$  represents the degree of individuals' preferences for public good provision. When young, they supply one unit of labor inelastically and allocate their disposable income between consumption and savings:

$$c_t + s_t = (1 - \tau_t)w_t, \quad (2)$$



where  $s_t$ ,  $w_t$ , and  $\tau_t$  denote savings, wage, and the labor income tax rate, respectively. When old, they retire and consume the return on their savings:

$$d_{t+1} = R_{t+1}s_t, \quad (3)$$

where  $R_{t+1}$  is the interest rate. As economic agents, individuals choose consumption and savings in order to maximize their utility subject to (2) and (3), taking  $w_t$ ,  $R_{t+1}$ ,  $\tau_t$ ,  $g_t$ , and  $g_{t+1}$  as given. From the utility-maximization problem, we obtain

$$c_t^* = \frac{1}{1+\beta}(1-\tau_t)w_t, \quad d_{t+1}^* = \beta R_{t+1}c_t^*, \quad (4)$$

$$s_t^* = \frac{\beta}{1+\beta}(1-\tau_t)w_t. \quad (5)$$

A final good is produced by using physical capital and labor as inputs, and the technology is represented by a Romer (1986) type production function:

$$y_t = Ak_t^\alpha l_t^{1-\alpha} \bar{k}_t^{1-\alpha}, \quad (6)$$

where  $k_t$  and  $l_t$  are inputs of physical capital and labor, respectively, and  $\bar{k}_t$  is the aggregate physical capital. Physical capital fully depreciates within one period. Each firm chooses  $k_t$  and  $l_t$  in order to maximize profit, taking

$\bar{k}_t$ ,  $R_t$ , and  $w_t$  as given. All markets are competitive, which leads to

$$R_t = \alpha A, \quad (7)$$

$$w_t = (1 - \alpha)Ak_t, \quad (8)$$

$$y_t = Ak_t. \quad (9)$$

In the competitive equilibrium, the wage and the output are proportional to the level of physical capital.

The government finances the cost of public good provision by levying labor income tax and/or issuing public debt. The budget constraint of the government is given by

$$b_{t+1} = R_t b_t + g_t - \tau_t w_t, \quad (10)$$

where  $b_t$  is the stock of public debt. We assume that the government cannot repudiate public debt and does not hold positive assets (i.e.,  $b_{t+1} \geq 0$ ).

The capital market clearing condition is represented as

$$k_{t+1} = s_t - b_{t+1}. \quad (11)$$

From (7), (10), and (11), we obtain the transition equations of the state

variables:

$$\begin{aligned} b_{t+1} &= \alpha Ab_t + g_t - (1 - \alpha)A\tau_t k_t \\ &\equiv Z^B(g_t, \tau_t, b_t, k_t), \end{aligned} \tag{12}$$

$$\begin{aligned} k_{t+1} &= \frac{\beta}{1 + \beta}(1 - \alpha)Ak_t + \frac{1}{1 + \beta}(1 - \alpha)A\tau_t k_t - \alpha Ab_t - g_t \\ &\equiv Z^K(g_t, \tau_t, b_t, k_t). \end{aligned} \tag{13}$$

An increase in current public good provision accelerates public debt issuance and retards physical capital accumulation (i.e.,  $\partial Z^B/\partial g > 0$  and  $\partial Z^K/\partial g < 0$ ). In contrast, an increase in current labor income tax rate suppresses public debt issuance and promotes physical capital accumulation (i.e.,  $\partial Z^B/\partial \tau < 0$  and  $\partial Z^K/\partial \tau > 0$ ).

## 2.2 Politico-economic Equilibrium

### 2.2.1 Markov perfect equilibrium

We next investigate the characteristics of a politico-economic equilibrium, employing a probabilistic voting model á la Lindbeck and Weibull (1987). Under probabilistic voting, the size of public policies is determined to maximize the weighted sum of voters' welfare. The welfare of young individuals

and that of old individuals are represented as, respectively,

$$V^y(g_t, \tau_t, b_t, k_t, g_{t+1}) = C^y + (1 + \beta) \log k_t + (1 + \beta) \log(1 - \tau_t) + \gamma \{\log g_t + \beta \log g_{t+1}\}, \quad (14)$$

$$V^o(g_t, b_t, k_t) = C^o + \log(k_t + b_t) + \gamma \log g_t, \quad (15)$$

where  $C^y$  and  $C^o$  are constant variables. From (14) and (15), the weighted sum of the welfare is given by

$$\begin{aligned} W(g_t, \tau_t, b_t, k_t, g_{t+1}) &\equiv \omega V^y(g_t, \tau_t, b_t, k_t, g_{t+1}) + (1 - \omega) V^o(g_t, b_t, k_t) \\ &= C + \omega(1 + \beta) \log k_t + (1 - \omega) \log(k_t + b_t) \\ &\quad + \omega(1 + \beta) \log(1 - \tau_t) + \gamma[\log g_t + \omega\beta \log g_{t+1}], \end{aligned} \quad (16)$$

where  $C \equiv \omega C^y + (1 - \omega) C^o$ , and  $\omega \in [0, 1]$  is the weight attached to young individuals.

In order to investigate the interactions between politically implemented public policies and the patterns of economic growth, we focus on a Markov perfect equilibrium, in which the size of public policies depends only on the payoff-relevant state variables. In this paper, the payoff-relevant state variables are the stock of public debt,  $b_t$ , and physical capital,  $k_t$ . Thus the size of public good provision,  $g_t$ , the labor income tax rate,  $\tau_t$ , and the size of public debt issuance,  $b_{t+1}$ , are represented as functions of these two state

variables:

$$g_t = G(b_t, k_t), \quad \tau_t = T(b_t, k_t), \quad b_{t+1} = B(b_t, k_t). \quad (17)$$

In the Markov perfect equilibrium, individuals take into account that current public policies,  $g_t$  and  $\tau_t$ , affect the stock of public debt,  $b_{t+1}$ , and physical capital,  $k_{t+1}$ , in the next period, and would affect the size of public good provision,  $g_{t+1}$ , in the next period.

Since public debt issuance retards physical capital accumulation, an extremely large amount of public debt relative to physical capital will discreate the economy. In particular, when  $b_t/k_t \geq (1 - \alpha)/\alpha$ , the level of physical capital in the next period,  $k_{t+1}$ , becomes non-positive even if the government supplies no public good and sets the labor income tax rate as high as possible (i.e.,  $g_t = 0$  and  $\tau_t = 1$ ). We thus restrict the domain of state variables as follows:

$$S \equiv \left\{ (b, k) : 0 \leq b, 0 < k, \frac{b}{k} < \frac{1 - \alpha}{\alpha} \right\}.$$

The Markov perfect equilibrium is characterized by the following functional equation:

$$(G(b, k), T(b, k)) = \arg \max_{g \geq 0, \tau \in [0, 1]} W(g, \tau, b, k, g'),$$

subject to

$$b' = Z^B(g, \tau, b, k), \quad k' = Z^K(g, \tau, b, k), \quad g' = G(b', k'), \quad (b', k') \in S,$$

and

$$B(b, k) = \alpha Ab + G(b, k) - (1 - \alpha)AT(b, k)k.$$

In order to make our model tractable and obtain some interesting results, we focus on a situation in which the size of public good provision is represented as a linear function of the stock of public debt and physical capital. We guess that the function  $G$  is given by

$$g' = G(b', k') = \delta_1 k' - \delta_2 b', \quad (18)$$

where  $\delta_1$  and  $\delta_2$  are positive coefficients. The first-order condition with respect to  $g$  is given by

$$\underbrace{\frac{\gamma}{g}}_{MB_g} = \underbrace{\frac{\omega\beta\gamma}{g'}(\delta_1 + \delta_2)}_{MC_g}. \quad (19)$$

The left-hand side of (19) is the marginal benefit of increasing  $g$ , which results from an increase in current public good provision. The right-hand side is the marginal cost of increasing  $g$ : an increase in  $g$  accelerates public debt issuance,  $b'$ , and lowers the level of physical capital in the next period,  $k'$ , which implies a decrease in the size of public good provision in the next period,  $g'$ . The first-order condition with respect to  $\tau$  is given by

$$\underbrace{\frac{\omega(1 + \beta)}{1 - \tau}}_{MC_\tau} = \underbrace{\frac{\omega\beta\gamma}{g'} \left( \frac{\delta_1}{1 + \beta} + \delta_2 \right)}_{MB_\tau} (1 - \alpha)Ak. \quad (20)$$

The left-hand side of (20) is the marginal cost of increasing  $\tau$ , which results from a reduction in the disposable income of young individuals. The right-hand side is the marginal benefit of increasing  $\tau$ : an increase in  $\tau$  suppresses public debt issuance,  $b'$ , and raises the level of physical capital in the next period,  $k'$ , which implies an increase in public good provision in the next period,  $g'$ . Solving simultaneous functional equations (19) and (20) with respect to  $g$ , we obtain

$$g = \frac{\gamma}{\gamma(1 + \omega\beta) + \omega(1 + \beta)} [(1 - \alpha)Ak - \alpha Ab] \equiv G(b, k) > 0 \quad \forall (b, k) \in S. \quad (21)$$

Comparing the coefficients of (18) with those of (21), we obtain

$$\delta_1 = \frac{\gamma(1 - \alpha)A}{\gamma(1 + \omega\beta) + \omega(1 + \beta)}, \quad \delta_2 = \frac{\gamma\alpha A}{\gamma(1 + \omega\beta) + \omega(1 + \beta)}.$$

Substituting  $g$  of (21) into (20), we obtain

$$\tau = 1 - \frac{\omega(1 + \beta)^2}{(1 + \alpha\beta)[\gamma(1 + \omega\beta) + \omega(1 + \beta)]} \left( 1 - \frac{\alpha}{1 - \alpha} \frac{b}{k} \right) \equiv T(b, k). \quad (22)$$

Furthermore, substituting  $g$  of (21) and  $\tau$  of (22) into the transition equation of  $b$ , we obtain

$$\begin{aligned} b' &= Rb + G(b, k) - wT(b, k)k \\ &= \frac{\omega\beta[(1 - \alpha)(1 + \beta) - \gamma(1 + \alpha\beta)]}{(1 + \alpha\beta)[\gamma(1 + \omega\beta) + \omega(1 + \beta)]} [(1 - \alpha)Ak - \alpha Ab] \equiv B(b, k). \end{aligned} \quad (23)$$

The results mentioned above are summarized as the following proposition.

**Proposition 1.** *If the parameters satisfy*

$$\frac{\omega\beta(1-\alpha)(1+\beta)}{(1+\alpha\beta)(1+\omega\beta)} \leq \gamma \leq \frac{(1-\alpha)(1+\beta)}{1+\alpha\beta}, \quad (\text{A.1})$$

*then there exists a Markov perfect equilibrium in which the size of public good provision, the labor income tax rate, and the amount of public debt issuance are, respectively, represented as (21), (22), and (23).*

(A.1) ensures that the labor income tax rate given by (22) and the amount of public debt issuance given by (23) are nonnegative for any  $(b, k) \in S$ . By substituting  $g$  of (21) and  $\tau$  of (22) into the transition equation of  $k$ , we can show that the level of physical capital in the next period becomes positive in the Markov perfect equilibrium:

$$k' = \frac{\omega\beta[\gamma(1+\alpha\beta) + \alpha(1+\beta)]}{(1+\alpha\beta)[\gamma(1+\omega\beta) + \omega(1+\beta)]} [(1-\alpha)Ak - \alpha Ab] > 0 \quad \forall (b, k) \in S. \quad (24)$$

Furthermore, from (23) and (24), we obtain

$$\frac{b'}{k'} = \frac{(1-\alpha)(1+\beta) - \gamma(1+\alpha\beta)}{\alpha(1+\beta) + \gamma(1+\alpha\beta)} < \frac{1-\alpha}{\alpha}. \quad (25)$$

Thus  $(b', k') \in S$  as long as the parameters satisfy (A.1).



### 2.2.2 Properties of policy functions

The Markov perfect equilibrium policy functions given by (21), (22), and (23) have the following properties. First, whereas the tax function  $T$  is increasing in the stock of public debt,  $b$ , the public good provision function  $G$  and the public debt issuance function  $B$  are decreasing in  $b$ . A large amount of public debt raises the labor income tax rate, reduces the size of public good provision, and suppresses public debt issuance.<sup>3</sup> Second, whereas the tax function  $T$  is decreasing in the level of physical capital,  $k$ , the public good provision function  $G$  and the public debt issuance function  $B$  are increasing in  $k$ . A high level of physical capital expands the capacity of public debt issuance and loosens fiscal discipline.

We here consider the intuition about the properties of the public good provision function  $G$ . Suppose that the size of public policies except for current public good provision is given by the Markov perfect policy rule; i.e.,  $\tau = T(b, k)$  and  $g' = G(b', k')$ . Then, the marginal benefit and cost of increasing  $g$  are represented as, respectively,

$$MB_g = \frac{\gamma}{g}, \quad (26)$$

$$MC_g = \frac{\omega\beta\gamma A}{\gamma(1 + \omega\beta) + \omega(1 + \beta)} \frac{1}{g'} = \frac{\omega\beta\gamma}{(1 - \alpha)k' - \alpha b'}. \quad (27)$$

Note that a high level of physical capital in the next period,  $k'$ , and/or a

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<sup>3</sup>This result is similar to that of Song et al. (2011).

small amount of public debt issuance,  $b'$ , increase the size of public good provision in the next period,  $g'$ , and thus lower the marginal cost. In what follows, we investigate the features of the marginal benefit and cost.

First, from (22), the tax revenue  $\hat{T}$  is represented by

$$\begin{aligned}\hat{T} &\equiv wT(b, k)k \\ &= \frac{\omega(1 + \beta)^2}{(1 + \alpha\beta)[\gamma(1 + \omega\beta) + \omega(1 + \beta)]} \alpha A b \\ &\quad + \frac{\gamma(1 + \alpha\beta)(1 + \omega\beta) - \omega\beta(1 - \alpha)(1 + \beta)}{(1 + \alpha\beta)[\gamma(1 + \omega\beta) + \omega(1 + \beta)]} (1 - \alpha) A k.\end{aligned}\tag{28}$$

The tax revenue is increasing in  $b$  since a large amount of public debt raises the equilibrium tax rate,  $T$ . The tax revenue is also increasing in  $k$ . Whereas an increase in the level of physical capital lowers the equilibrium tax rate,  $T$ , it expands the tax base,  $wk$ . Under (A.1), the latter effect dominates the former one, and a high level of physical capital increases the tax revenue. Substituting  $\hat{T}$  of (28) into the transition equations of  $b$  and  $k$ , we obtain

$$b' = Rb + g - \hat{T}(b, k),\tag{29}$$

$$k' = \frac{\beta}{1 + \beta} wk + \frac{1}{1 + \beta} \hat{T}(b, k) - Rb - g.\tag{30}$$

Differentiating  $b'$  and  $k'$  with respect to  $b$ , we obtain

$$\frac{\partial b'}{\partial b} = R - \frac{\partial \hat{T}}{\partial b} = \frac{\gamma(1 + \alpha\beta)(1 + \omega\beta) - \omega\beta(1 - \alpha)(1 + \beta)}{(1 + \alpha\beta)[\gamma(1 + \omega\beta) + \omega(1 + \beta)]} \alpha A > 0,$$

$$\frac{\partial k'}{\partial b} = \frac{1}{1+\beta} \frac{\partial \hat{T}}{\partial b} - R = -\frac{\gamma(1+\alpha\beta)(1+\omega\beta) + \omega\alpha\beta(1+\beta)}{(1+\alpha\beta)[\gamma(1+\omega\beta) + \omega(1+\beta)]} \alpha A < 0.$$

Whereas a large amount of public debt increases the tax revenue,  $\hat{T}$ , it also increases the repayment cost of public debt,  $Rb$ . In our setup, the latter effect dominates the former one, and thus,  $b'$  and  $k'$  are increasing and decreasing in  $b$ , respectively. In contrast,  $b'$  and  $k'$  are decreasing and increasing in  $k$ , respectively, since a high level of physical capital increases the tax revenue,  $\hat{T}$ .

We proceed to analyze the features of the marginal cost and benefit. First, whereas the marginal benefit,  $MB_g$ , is independent of  $b$ , the marginal cost,  $MC_g$ , is increasing in  $b$ : a large amount of public debt accelerates public debt issuance,  $b'$ , lowers the level of physical capital in the next period,  $k'$ , and raises the marginal cost. Thus, an increase in  $b$  lowers the level of public good provision equalizing the cost with the benefit, which implies that the function  $G$  is decreasing in  $b$  (see Figure 1.a). Second, whereas the marginal benefit is independent of  $k$ , the marginal cost is decreasing in  $k$ : a high level of physical capital suppresses public debt issuance,  $b'$ , raises the level of physical capital in the next period,  $k'$ , and hence lowers the marginal cost. Thus, an increase in  $k$  raises the level of public good provision equalizing the cost with the benefit, which implies that the function  $G$  is increasing in  $k$  (see Figure 1.b).

We next consider the intuition about the properties of the equilibrium tax function  $T$ . Suppose that the size of public policies except for the current

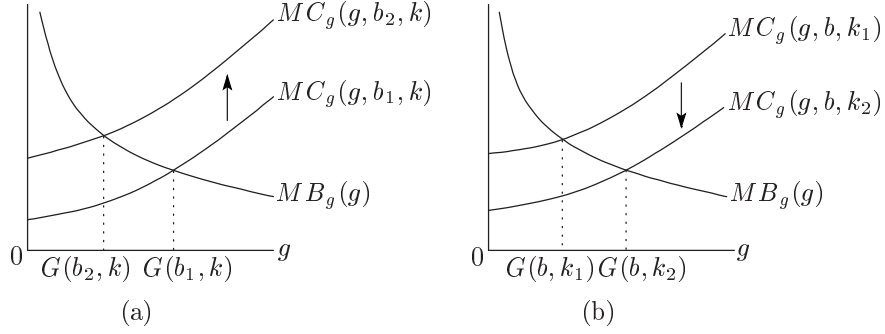


Figure 1: Properties of the Function  $G$

labor income tax rate is given by the Markov perfect policy rule; i.e.,  $g = G(b, k)$  and  $g' = G(b', k')$ . The marginal cost and benefit of increasing  $\tau$  are represented as, respectively,

$$MC_\tau = \frac{\omega(1 + \beta)}{1 - \tau}, \quad (31)$$

$$MB_\tau = \frac{\omega\beta\gamma(1 + \alpha\beta)}{(1 - \alpha)(1 + \beta)} \frac{1}{\hat{k}' - \frac{\alpha}{1 - \alpha}\hat{b}'}, \quad (32)$$

where  $\hat{b}' \equiv b'/(wk)$  and  $\hat{k}' \equiv k'/(wk)$  are the ratio of public debt issuance to wage and the ratio of physical capital in the next period to wage, respectively.

We then denote the ratio of public good provision to wage by  $\hat{G}$ :

$$\hat{G} \equiv \frac{G(b, k)}{wk} = \frac{\gamma}{\gamma(1 + \omega\beta) + \omega(1 + \beta)} \left(1 - \frac{\alpha}{1 - \alpha} \frac{b}{k}\right). \quad (33)$$

The public good/wage ratio,  $\hat{G}$ , is increasing in  $k$ : a high level of physical capital increases the size of public good provision more elastically than the

wage. In contrast,  $\hat{G}$  is decreasing in  $b$  since a large amount of public debt reduces the size of public good provision,  $G$ . Substituting  $\hat{G}$  of (33) into the transition equations of  $b$  and  $k$ , we obtain

$$\hat{b}' = \frac{Rb}{wk} + \hat{G} - \tau, \quad (34)$$

$$\hat{k}' = \frac{\beta}{1+\beta} + \frac{1}{1+\beta}\tau - \frac{Rb}{wk} - \hat{G}. \quad (35)$$

Differentiating  $\hat{b}'$  and  $\hat{k}'$  with respect to  $b$ , we obtain

$$\frac{\partial \hat{b}'}{\partial b} = \frac{\partial}{\partial b} \left( \frac{Rb}{wk} \right) + \frac{\partial \hat{G}}{\partial b} = \frac{\omega(1+\beta+\beta\gamma)}{\gamma(1+\omega\beta) + \omega(1+\beta)} \frac{\alpha}{1-\alpha} \frac{1}{k} > 0,$$

$$\frac{\partial \hat{k}'}{\partial b} = -\frac{\partial}{\partial b} \left( \frac{Rb}{wk} \right) - \frac{\partial \hat{G}}{\partial b} = -\frac{\omega(1+\beta+\beta\gamma)}{\gamma(1+\omega\beta) + \omega(1+\beta)} \frac{\alpha}{1-\alpha} \frac{1}{k} < 0.$$

While a large amount of public debt lowers the public good/wage ratio,  $\hat{G}$ , it raises the repayment cost/wage ratio,  $(Rb)/(wk)$ . In our setup, the latter effect dominates the former one, and thus,  $\hat{b}'$  and  $\hat{k}'$  are increasing and decreasing in  $b$ , respectively. Differentiating  $\hat{b}'$  and  $\hat{k}'$  with respect to  $k$ , we obtain

$$\frac{\partial \hat{b}'}{\partial k} = \frac{\partial}{\partial k} \left( \frac{Rb}{wk} \right) + \frac{\partial \hat{G}}{\partial k} = -\frac{\omega(1+\beta+\beta\gamma)}{\gamma(1+\omega\beta) + \omega(1+\beta)} \frac{\alpha}{1-\alpha} \frac{b}{k^2} < 0,$$

$$\frac{\partial \hat{k}'}{\partial k} = -\frac{\partial}{\partial k} \left( \frac{Rb}{wk} \right) - \frac{\partial \hat{G}}{\partial k} = \frac{\omega(1+\beta+\beta\gamma)}{\gamma(1+\omega\beta) + \omega(1+\beta)} \frac{\alpha}{1-\alpha} \frac{b}{k^2} > 0.$$

Whereas a high level of physical capital raises the public good/wage ratio,

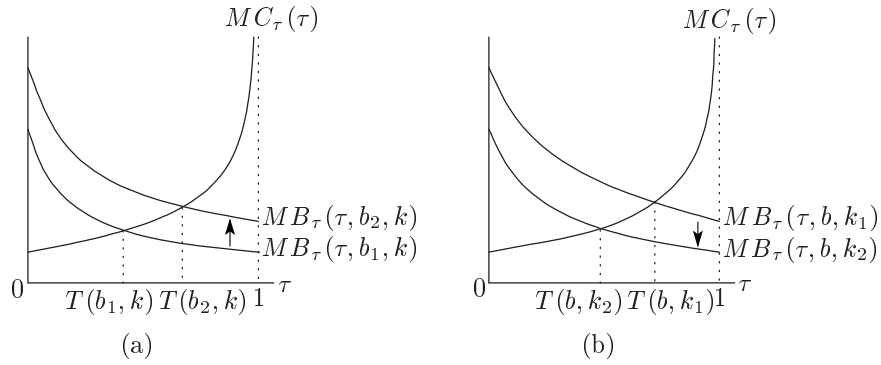


Figure 2: Properties of the Function  $T$

$\hat{G}$ , it lowers the repayment cost/wage ratio,  $(Rb)/(wk)$ . Since the latter effect dominates the former one,  $\hat{b}'$  and  $\hat{k}'$  are decreasing and increasing in  $k$ , respectively.

We proceed to investigate the features of the marginal cost and benefit. First, while the marginal cost is independent of  $b$ , the marginal benefit is increasing in  $b$ : a large amount of public debt raises the public debt issuance/wage ratio,  $\hat{b}'$ , lowers the ratio of the physical capital in the next period to wage,  $\hat{k}'$ , and thus raises the marginal benefit. Thus, an increase in  $b$  raises the labor income tax rate equalizing the cost with the benefit, which implies that the function  $T$  is increasing in  $b$  (see Figure 2.a). Second, while the marginal cost is independent of  $k$ , the marginal benefit is decreasing in  $k$ : a high level of physical capital lowers  $\hat{b}'$ , raises  $\hat{k}'$ , and thus lowers the marginal benefit. Thus, an increase in  $k$  lowers the labor income tax rate equalizing the cost with the benefit, which means that the function  $T$  is decreasing in  $k$  (see Figure 2.b).

### 2.2.3 Comparative statics

The features of the equilibrium policy functions given by (21), (22), and (23) also depend on some exogenous parameters. For instance, individuals' stronger preferences for public good provision,  $\gamma$ , raise not only the level of public good provision,  $G$ , but also the labor income tax rate,  $T$ . Furthermore, in our setup, a rise in  $\gamma$  increases the tax revenue more elastically than the level of public good provision, and thus suppresses public debt issuance,  $B$ .<sup>4</sup> In contrast, an increase in the weight attached to young individuals,  $\omega$ , lowers not only the level of public good provision,  $G$ , but also the labor income tax rate,  $T$ . Since the latter effect dominates the former one, an increase in  $\omega$  accelerates public debt issuance,  $B$ .<sup>5</sup>

In order to analyze the effects of the exogenous parameters on the feature of the public good provision function,  $G$ , suppose that the size of public policies except for current public good provision is given by the Markov perfect policy rule. The tax revenue  $\hat{T}$  given by (28) is increasing in  $\gamma$  and decreasing in  $\omega$ , since an increase in  $\gamma$  and/or a decrease in  $\omega$  raises the equilibrium tax rate,  $T$ . Whereas an increase in  $\gamma$  suppresses public debt issuance,  $b'$ , and raises the level of physical capital in the next period,  $k'$ , an increase in  $\omega$ , increases  $b'$  and reduces  $k'$ . We next analyze the effects of the exogenous parameters on the marginal benefit and cost of increasing in

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<sup>4</sup>By differentiating  $B$  of (23) with respect to  $\gamma$ , it is shown that the function  $B$  is decreasing in  $\gamma$ .

<sup>5</sup>By differentiating  $B$  of (23) with respect to  $\omega$ , it is shown that the function  $B$  is increasing in  $\omega$  under (A.2).

*g.* First, an increase in  $\gamma$  raises both the marginal benefit and cost directly, but lowers only the marginal cost indirectly: it reduces public debt issuance,  $b'$ , and raises the level of physical capital in the next period,  $k'$ , and thus indirectly lowers the marginal cost. Thus, an increase in  $\gamma$  raises the benefit more elastically than the cost and raises the level of public good provision equalizing the cost with the benefit, which implies that the function  $G$  is increasing in  $\gamma$ . Second, while the marginal benefit is independent of  $\omega$ , the marginal cost is increasing  $\omega$ : an increase in  $\omega$  raises the marginal cost directly, accelerates public debt issuance,  $b'$ , and lowers the level of physical capital in the next period,  $k'$ . Thus, an increase in  $\omega$  lowers the level of public good provision equalizing the cost with the benefit, which implies that the function  $G$  is decreasing in  $\omega$ .

As for the effects of the exogenous parameters on the feature of the tax function  $T$ , suppose that the size of public policies except for current labor income tax rate is given by the Markov perfect policy rule. The ratio of public good provision to wage,  $\hat{G}$ , given by (33), is increasing in  $\gamma$  and decreasing in  $\omega$  since an increase in  $\gamma$  and/or a decrease in  $\omega$  raises the level of public good provision,  $G$ . Whereas an increase in  $\gamma$  raises  $\hat{b}'$  and lowers  $\hat{k}'$ , an increase in  $\omega$  lowers  $\hat{b}'$  and raises  $\hat{k}'$ . We lastly analyze the effects of the exogenous parameters on the marginal benefit and cost of increasing  $\tau$ . The marginal cost is independent of  $\gamma$ , but the marginal benefit is increasing in  $\gamma$ : an increase in  $\gamma$  raises the marginal benefit directly, lowers  $\hat{b}'$  and raises  $\hat{k}'$ . Thus, an increase in  $\gamma$  raises the labor income tax rate equalizing the



cost with the benefit, which implies that the function  $T$  is increasing in  $\gamma$ . Furthermore, an increase in  $\omega$  raises both the marginal benefit and cost directly, but it lowers only the marginal benefit indirectly: it lowers  $\hat{b}'$ , raises  $\hat{k}'$ , and thus lowers the marginal benefit. Since an increase in  $\omega$  raises the marginal cost more elastically than the marginal benefit, it lowers the labor income tax rate equalizing the cost with the benefit, which implies that the function  $T$  is decreasing in  $\omega$ .

### 2.3 Dynamics and Economic Growth

In this section, we investigate the dynamic pattern of public debt and physical capital accumulation in the Markov perfect equilibrium described in proposition 1. Note first that (25) states that the public debt/physical capital ratio in the next period becomes constant for any  $(b_t, k_t) \in S$ :

$$\frac{b_{t+1}}{k_{t+1}} = \frac{(1 - \alpha)(1 + \beta) - \gamma(1 + \alpha\beta)}{\alpha(1 + \beta) + \gamma(1 + \alpha\beta)} \equiv z. \quad (36)$$

The public debt/physical capital ratio converges to  $z$  within one period, and thereafter, the stock of public debt and physical capital grow at the same rate; i.e., a balanced growth path (BGP). The ratio  $z$  is decreasing in the degree of individuals' preferences for public good provision,  $\gamma$ , because an increase in  $\gamma$  raises the level of physical capital in the next period,  $k'$ , more elastically than the amount of public debt issuance,  $b'$ .

We lastly analyze the effects of the exogenous parameters on the pattern

of economic growth. The growth rate of physical capital in the BGP is given by

$$\begin{aligned}
\left. \frac{k_{t+1}}{k_t} \right|_{b_t/k_t=z} &= (1 - \alpha)A \left( \frac{\beta}{1 + \beta} + \frac{1}{1 + \beta} \tau^* - \frac{\alpha}{1 - \alpha} z - \hat{g}^* \right) \\
&= (1 - \alpha)A \frac{\omega\beta[\gamma(1 + \alpha\beta) + \alpha(1 + \beta)]}{(1 + \alpha\beta)[\gamma(1 + \omega\beta) + \omega(1 + \beta)]} \left( 1 - \frac{\alpha}{1 - \alpha} z \right) \\
&= \frac{\omega\beta\gamma A}{\gamma(1 + \omega\beta) + \omega(1 + \beta)},
\end{aligned} \tag{37}$$

where  $\tau^*$  and  $\hat{g}^*$  are the labor income tax rate and the public good/wage ratio in the BGP:

$$\tau^* = 1 - \frac{\omega(1 + \beta)^2}{(1 + \alpha\beta)[\gamma(1 + \omega\beta) + \omega(1 + \beta)]} \left( 1 - \frac{\alpha}{1 - \alpha} z \right), \tag{38}$$

$$\hat{g}^* = \frac{\gamma}{\gamma(1 + \omega\beta) + \omega(1 + \beta)} \left( 1 - \frac{\alpha}{1 - \alpha} z \right). \tag{39}$$

From a simple calculation, this growth rate is shown to be increasing in the strength of individuals' preferences for public good provision,  $\gamma$ . An increase in  $\gamma$  lowers the public debt/physical capital ratio,  $z$ , and tends to raise the growth rate. In addition, taking  $z$  as given, an increase in  $\gamma$  raises the labor income tax rate,  $\tau^*$ , and raises the public good/wage ratio,  $\hat{g}^*$ . In our setup, the sum of the first and second effects dominates the third one, and thus, the growth rate is increasing in  $\gamma$ . It is also shown that the growth rate of physical capital is increasing in the weight attached to young individuals,  $\omega$ .

An increase in  $\omega$  lowers not only the labor income tax rate,  $\tau^*$ , but also the public good/wage ratio,  $\hat{g}^*$ . Since the latter effect dominates the former one, an increase in  $\omega$  raises the growth rate. We summarize the results obtained in this section as Proposition 2.

**Proposition 2.** *In the BGP, the public debt/physical capital ratio is decreasing in  $\gamma$ , and the economic growth rate is increasing in  $\gamma$  and  $\omega$ .*

As mentioned above, individuals' stronger preferences for public good provision tighten fiscal discipline and promote economic growth. This result is consistent with the data from some democratic countries. Like Song et al. (2011), we focus on the Corruption Perception Index (CPI) as the proxy of the extent of individuals' preference for public good provision. In less-corrupt countries, wherein CPI is high, individuals are likely to trust the government and to have strong preferences for public good provision. In corrupt countries, wherein CPI is low, the government cannot win the confidence of individuals, and the strength of individuals' preferences for public good provision are likely to be low.

Using the CPI data for 2011 and the OECD outlook data for 1995–2006, we find a positive correlation between the total tax revenue/GDP ratio and CPI and a negative correlation between the public debt/GDP ratio and CPI.<sup>6</sup> While the average of the tax revenue/GDP ratio and that of the gross general government debt/GDP ratio in the high-CPI group are 18.37 % and 50.87

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<sup>6</sup>Song et al. (2011) also argue that the central government debt/GDP ratio is negatively correlated with the strength of individuals' preferences for public good provision.

%, respectively, those in the low-CPI group are 9.675 % and 73.34 %, respectively.<sup>7</sup> Furthermore, some empirical studies such as Reinhart and Rogoff (2010) and Kumar and Woo (2010) argue that the economic growth rate is negatively correlated with the public debt/GDP ratio.<sup>8</sup> Taking account of the negative relationship between the public debt/GDP ratio and CPI, individuals' stronger preferences for public good provision are likely to promote economic growth.

### 3 Conclusion

We construct a simple overlapping generations economy with physical capital accumulation in which the size of public policies including public debt issuance is determined in a repeated probabilistic voting game. We investigate interactions between politically implemented public policies and patterns of economic development by focusing on a Markov perfect equilibrium and show that individuals' stronger preferences for public good provision tighten fiscal discipline and promote economic growth.

We conclude by discussing possible directions for future research. First, in

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<sup>7</sup>We include countries with CPI higher than 8.0 in the high-CPI group (New Zealand, Denmark, Finland, Sweden, Norway, Netherlands, Australia, Switzerland, Canada, and Luxembourg) and countries with CPI lower than 7.0 in the low-CPI group (Spain, Portugal, Poland, Hungary, Czech Republic, Italy, and Greece).

<sup>8</sup>Reinhart and Rogoff (2010) present a new time-series dataset of public debt and show that the economic growth rate in countries with a public debt/GDP ratio higher than 90 % tend to be lower than those in the other countries. Using several estimation methods, Kumar and Woo (2010) show that an increase in the initial public debt/GDP ratio lowers the economic growth rate.

addition to increasing individuals' welfare directly, public goods can raise the productivity of production as mentioned in Barro (1990). Considering the productivity-enhancing effect of public goods might shed light on different characteristics of the politico-economic equilibria from ours. Furthermore, this paper considers a simple tax system comprising only labor income taxation. A natural extension is to investigate the characteristics of Markov perfect equilibria under more general tax systems that include, for instance, capital income and consumption taxation.

## Acknowledgements

We are grateful to Akihisa Shibata, Kazuo Mino, Tetsuo Ono, and Koichi Kawamoto for their helpful comments and suggestions. We also thank the participants of ETPW at Tokyo Metropolitan University, the CAES Seminar at Fukuoka University, 5th macroeconomics conference for young economists, and the 2011 JEA Spring Meeting at Kumamoto Gakuen University.

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